An Analysis of Preparatory Measures for Mathematical Proof Courses

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Bio:

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Abstract:

Many mathematically minded students struggle in courses that require proofs. This paper will explore the ways in which mathematics majors prepare for their first proof course in two ways. The first is a review of the literature regarding the best practices used in preparation. These practices have been devised after researching difficulties students have with proofs as well as the reasons behind these difficulties, such as recurring misconceptions. This review will also include the courses that may be suggested or required by other major public universities. The second part is an analysis of whether or not the current prerequisites at the University of North Texas are effectively preparing students for their first proof course.

Introduction

When I decided to major in mathematics, I thought I would be working with numbers and solving practical equations. I had seen proofs and theorems in calculus, but was unable to realize their importance at the time. Many people have this same misconception about mathematics, that it is numbers and calculations. However, there is much more to it than that. It requires a different way of thinking.

When it comes to proving a statement, lemma, theorem, or other fact, you must not only be sure to include every possible case but also try to create the proof in the most direct manner, leaving out any irrelevant or unnecessary steps. Even in beginning levels there are concepts that are more abstract than people realize. When constructing a proof, this different mindset must be used in order to complete the task efficiently. It is a combination of the problem-solving techniques learned in mathematics as well as the intuition that is gained only through practice and experience. There are many different approaches to proofs including induction, definitions, contradiction, negation, and more, and it is this intuition that must be used in order to choose the most efficient technique.

At the University of North Texas, the first proof course is called Real Analysis (MATH 2510). I remember signing up for the course and hearing rumors about how hard it would be and that it was the “make you or break you” class for mathematics majors. Students have struggled with this course for a while. Since the intuition required to construct a proof cannot be taught, students must receive enough exposure in the courses they have previously taken in order to get started. Therefore, we must look at the classes a UNT student must complete and pass before enrolling in Real Analysis. Hopefully it will be clear whether or not the current prerequisites properly prepare students for this proof course.

Review of the Literature

General Problems

Mathematics is not just “the deduction of a result from axioms” (Spencer, 1970, p. 754), but it is also not “a collection of tricks for calculating and solving” (Moses, 1951, p. 186). It is a continuous thought process from which ideas are developed. Mathematicians must have the ability to construct proofs efficiently (Weber, 2001). However, throughout the literature it is clear that even mathematics majors struggle with proofs and that the many observed difficulties are present due to a variety of reasons. Overall, mathematics is not an easily understood discipline.

Throughout the literature it is noted that there is a noticeable gap between computational classes and abstract proof classes. This goes along with the idea that many times prerequisite courses do not fully prepare students to work with the reasoning used in proofs (Epp, 2003). A person cannot blindly create a proof; the statement that is to be proven must first be understood. Only then can the next steps of figuring out which approach is most efficient, be taken. This understanding also includes the ability to correctly apply the fact. If a student does not understand the application of a statement, the proof will most likely be either insufficient or completely incorrect. Students must be able to recall the facts necessary to create a proof and also be able to apply these facts efficiently (Weber, 2001). This means that if a student has not learned the facts related to the proof, they will be unable to complete the proof successfully.

The knowledge of which facts should be recalled and how they should be used in the proof becomes part of the intuition that is needed when constructing proofs. Weber (2001) compared doctoral students in mathematics to the undergraduates and found that, as a group, the doctoral students proved 95% of the statements while the undergraduates were only able to prove 30%. The undergraduates chosen for the study did not correctly apply their knowledge of mathematics in 57% of the incorrect proofs. However, they were able to construct the proof after being told to use statements that they had previously classified as true or false. They also made 35 irrelevant inferences while the doctoral students only made 9. This means that mathematical knowledge and an understanding of proof construction are not enough to successfully prove a statement (Weber, 2001). There is a certain amount of experience needed in order to gain the intuition, referred to as metacognition, to successfully complete a proof (Vobejda, 1987). Metacognition, also known as strategy selection, is crucial when proving mathematical facts considering how many inferences can be drawn and how many approaches can be attempted. Mathematicians must be able to know which theorems will be necessary for each proof. In Weber's study the doctoral students used powerful theorems because they recognized the usefulness of the theorem, considering the statement that they were proving fit a general set of conditions. While undergraduates usually began their proofs by writing out a list of inferences, graduate students appeared to first try making sense of the statement. Undergraduates, as they have less mathematical experience, attempted to prove statements using only the manipulation of various definitions (Weber, 2001). This is evidence that experience and practice are necessary to understand proofs and be able to construct them correctly.

Proof Requirements and Structural Issues

Students must be aware of what the requirements are for a mathematical proof to be valid (Weber, 2001) but many times proof construction is not taught (Gries & Schneider, 1995). A formal proof should make connections between various facts to give necessary evidence that the statement is true. If the differences between formal and informal proofs are discussed, students will gain a better understanding of both and will, therefore, do better in later courses. Some people have been given the impression that formal proofs are not worth the extra effort involved (Gries & Schneider, 1995). Misconceptions about proofs may arise when the teacher uses an informal, or even an incorrect proof, in order to use a specific case or because the correct proof is thought to be too long or too difficult (Richardson, 1951). Using an example as an informal proof causes some students to think that empirical evidence is equivalent to a proof. Logically, however, showing that a mathematical statement is true in one case does not, generally, prove it to be true for all cases. Even some textbooks have been found to show truth by an example (Epp, 2003). In situations like this, it would be better to overlook the proof entirely for the time being and simply assume a statement to be true rather than give false information (Richardson, 1951). Meanings can change depending on whether the statement is made formally or informally. Some mathematicians even write definitions using if-then statements expecting them to be interpreted as if-and-only-if statements (Epp, 2003).

Misconceptions

One of the major reasons students have difficulties with proofs is that they have misconceptions about mathematics. Throughout their learning experiences students will make assumptions about mathematics, many of which are detrimental to future situations. Some students conceive math as hard and out of their reach, boring, or irrelevant to the real world. Since proofs are sometimes only stated rather than explained, some students come to the incorrect conclusion that all proofs are abstract and that they are not new discoveries, but only verify mathematical statements that are already known to be true. Furthermore, students view the information necessary in proofs as irrelevant. Other assumptions are that if the covered material is understood, a student should be able to solve a problem in a small amount of time and that only a genius will be able to understand mathematics (Vobejda, 1987). All of these take away any motivation to continue on with the learning process as far as mathematics is concerned, and especially concerning a proof course. Students may do well in lower-level computational classes but then finish the remaining courses required for graduation by barely achieving the minimum. Many of these students are secondary mathematics education majors. These students were probably not taught general principles when they were in secondary school, so through experience they come to the conclusion that proofs are unnecessary and therefore do not need to be understood. This destructive belief causes these students to dislike higher-level and more abstract mathematics courses, which is then passed down to the next generation, the students that they teach in the future, and the cycle continues (Epp, 2003).

Another misconception is in the language of mathematics. This has to do with the myriad of statements including “ifs” as well as “if-and-only-ifs.” In everyday life the phrase, if and only if, is not used, even when it is the implication. Epp (2003) gives a few examples of this language difference and the effects it could have on the meaning of a statement and, therefore, the way in which the proof is constructed. The words “all” and “some” also create difficulties for students. In mathematics, “some” means that there exists at least one but it does not exclude all. In English, this is not the meaning. “Some” is taken to mean at least one but not all. Finally, the distinction between “and” and “or” is important in mathematics. The word “and” implies both, whereas “or” implies one, the other, or possibly both. Students have difficulty negating statements and this confusion of language could be part of the reason (Epp, 2003).

Some mistakes are made because of misunderstandings of simple arithmetic operations. If these errors are not caught and corrected quickly, students will make a habit of incorrectly applying an operation without even knowing a mistake is being made (Vobejda, 1987). When a student enters a proof course, these arithmetic misconceptions should already be corrected. Otherwise, the student could be left behind due to lack of mastery of the techniques learned in previous courses.

Classrooms today

Undergraduates studying mathematics should be able to gain some understanding of the internal structure of mathematics as well as how it relates to other disciplines. Although students should study some areas of interest in depth, they should also study to gain a more global view of mathematics as a whole. Many students do not realize the importance of proofs because they are unable to see the bigger picture, "what it's good for" (Spanier, 1970). However, without knowing details in some topics an understanding can be hard to grasp in those areas. Thus, there must be a balance between detail and general knowledge, as well as between pure and applied mathematics, in order for the learning process to be efficient. The observation of this necessary balance appears in most of the literature.

Much of the decision to focus on fundamentals or strategies lies with the teacher (Epp, 2003). The classroom has the potential to create weaknesses in students. When it comes to learning mathematics, many students are conditioned to "plug-and-chug" without actually understanding the reasons behind the techniques they are using. This way of solving problems is taught as early as elementary school, but causes major problems later on in education, especially in proof courses. There are reasons to teach using these sorts of methods. Students must have had enough previous exposure to particular concepts before a full understanding of a complex idea can be gained. However, when students are given formulas without explanations, they may begin to think of mathematics as meaningless because they are able to solve problems without thinking through the process (Richardson, 1951).

Many teachers spend most of the time in the classroom on techniques to solve problems, with applications coming in second, and the fundamentals almost non-existent in lesson plans. Mathematics majors should strive for a balance between these three aspects. Otherwise students become fairly good at crunching numbers, but are unable to explain to others how they came to their final answer (Richardson, 1951). This also means that although students will know the steps to various problem solving techniques, they will not necessarily be able to make connections between the different procedures (Vobejda, 1987). Without knowing why a certain technique works or why it is considered a good approach, students cannot possibly have a full understanding of the concept. It also adds another obstacle when choosing the correct approach to construct a proof, which is already a difficult task (Weber, 2001).

There are many reasons the fundamentals of mathematics tend to be overlooked at the undergraduate level. Some teachers believe that they should be completely avoided at this level, while others believe that teaching fundamentals should be done even though students will not yet be able to make sense of it all. Another reason is that techniques and applications are much easier taught than the fundamentals of mathematics. This also means that teachers struggle to get all of the material in the allotted time and, therefore, must choose what gets covered which usually results in not having enough time to cover the fundamentals. Finally, it is not an easy task to create tests over the fundamentals and even more difficult to distribute grades (Richardson, 1951). The increasing amount of mandatory, standardized tests puts pressure on teachers. In order to help students succeed, techniques and strategies become the focus of the class rather than the fundamentals on which they are based (Epp, 2003).

Other teachers do not allow their students to simply plug numbers into a formula or solve geometric problems simply by looking at the shapes and figures. A new concept is put into the most general, and therefore usually abstract, form in the hopes that it can then be presented in a more correct way (Spanier, 1970). The problem with this approach is that students begin to think that mathematics is mostly made up of complex abstract structures and, therefore, does not serve an applicable purpose. Another problem is that teaching fundamentals first has a tendency to leave students confused about how to actually apply the concept. When teachers use this approach, students are forced to accept information without understanding what it means or how it can be applied (Vobejda, 1987). Although mathematicians need to be detailed in order for a proof to be satisfactory, it is not always necessary or worthwhile in lower-level classes (Spanier, 1970). On the other hand, students that are taught to apply a general formula will be working with this one formula more than each specialized formula and, therefore, will gain a better understanding of it. "Familiarity creates fertile ground for actual learning to grow," meaning practice is necessary to gain an understanding of mathematics (Epp, 2003, p. 891].

Students must be engaged and interested in a topic in order to really understand. They must be given a chance to make certain discoveries on their own (Vobejda, 1987). One study resulted in indications that traditional teaching methods are failing. Intuition and understanding cannot be taught. Instead, they must be developed by the student through exploration and discovery (Epp, 2003). Although this may mean struggling, it is a necessary struggle because the discovered information will be remembered and more easily recalled than the facts that were merely copied down and memorized. Learning is also a social process, meaning that having interactive discussions in class with experts or outside of class with peers, would be helpful (Martin, Soucy McCrone, Wallace Bower, & Dindyal, 2005). This also means that studying in groups can be much more beneficial than studying alone (Vobejda, 1987). Although this will take more time outside of class, the benefits make it a worthwhile investment (Epp, 2003). When students work through problems on their own, they are given the opportunity to find their own problem solving techniques. In mathematics, there is usually more than one way to solve a problem, so allowing students to make their own discoveries will give them a chance to come up with alternative techniques and help them to see the connections between different concepts. A problem arises when students are left alone because, in most cases, undergraduates are not capable of discovering the structure of a proof on their own (Leron, 1983). Students also tend to be unable to learn the most efficient techniques through experience by itself. They must be given assignments that will push them in the right direction (Weber, 2001).

In the study done by Vobejda (1987), approximately 60% of the time students began solving a problem using a single technique without first evaluating whether or not it would be the most efficient method. This shows that once students are taught to solve problems in a certain way, they have a harder time using their abilities to decide on their own what kind of approach would result in the completion of the problem in the most direct manner. Adding to this is the tendency of teachers to automatically solve problems in class, using a specific approach without first allowing the class to think about which technique would be most efficient or explaining to the class why that approach was chosen. This explains why students are not as likely to successfully complete a problem if it is not given in the context in which it was first introduced. Many teachers assign problems that are grouped together by the approach that should be used and are, therefore, unable to see this weakness in their students.

Other Schools

In order to major in mathematics it is necessary to take courses that involve proofs. Students must learn how to create these proofs for themselves as well. However, universities present this material at different times and under a variety of course names. At the University of

North Texas, the first proof course, as stated before, is Real Analysis I (MATH 2510) with Calculus II (MATH 1720) as the prerequisite. The University of Texas calls their first proof course Discrete Mathematics (M325K) and requires Calculus II or Integral Calculus (M408D or M408L) to be completed (M325K discrete mathematics, n.d.). Texas A&M University also uses Fundamentals of Discrete Mathematics (MATH 220) as the first proof course but only requires Calculus (MATH 172) as a prerequisite (Course descriptions, n.d.). The University of Texas at Arlington requires Calculus I (MATH 1426) as a prerequisite for Introduction to Proofs (MATH 3300) (Course descriptions for mathematics, n.d.). The University of Houston uses Intermediate Analysis (MATH 3333) and requires Calculus III (MATH 2433) (Student syllabus for MATH 3333, n.d.). Texas Tech University in Lubbock calls the first proof course Introduction to Mathematical Reasoning and Proof (MATH 3310) and allows concurrent enrollment in Calculus III (MATH 2350) (Mathematics, Texas Tech University, n.d.).

As for a few major public universities outside of Texas, Oklahoma University also requires their students to take Discrete Mathematical Structures (MATH 2513) as the first proof course, but allows students to take Calculus and Analytic Geometry II (MATH 2423) concurrently (Mathematics, Oklahoma University, n.d.). Oklahoma State University requires Introduction to Modern Algebra (MATH 3613) and Introduction to Modern Analysis (MATH 4023) as their proof courses. MATH 3613 must be taken before MATH 4023 while the prerequisite to MATH 3613 is Linear Algebra (MATH 3013) and, therefore, Calculus II (MATH 2153) (Mathematics, n.d., Oklahoma State University, n.d.). The University of Louisiana calls their first proof course Fundamentals of Mathematics (MATH 360) which has a prerequisite of Survey of Calculus (MATH 250) or Calculus I (MATH 270) (Course offerings, n.d.). Finally, the University of Tulsa requires students to take Calculus II (MATH 2024) before taking the proof course called Introduction to Advanced Mathematics (MATH 3033) (MATH 3033-Introduction to advanced mathematics, n.d.).

Suggestions for Improvement

Since there are so many misconceptions due to the misunderstanding of logic and the language of mathematics, it would be extremely helpful to take time at the beginning of the first proof course to discuss logic and mathematical reasoning. Students should spend time practicing with these new terms and meanings in formal and informal settings. Having a background in logic will help students make connections and, therefore, allow them to avoid mistakes when solving problems [4].

Other suggestions are for the structure of the proof itself. Gries and Schneider argue that once students have acquired a grasp on the necessary mathematical logic, the structure of rigorous proofs should be covered and then the thought process of calculational proofs can be taught. In only four weeks students studying the calculational approach will be exposed to a variety of proofs and learn strategies for constructing proofs. This approach will also allow students to gain confidence in formal proofs [5]. Although proofs are usually written linearly, in steps from the hypothesis to the conclusion, Leron gives a method he calls the “structural method” in which the proof is organized into levels that start at the top and become more detailed at each lower level [6]. Each level is also broken up into “modules” that each present a single idea of the proof. Thus, the bottom level closely resembles a linear proof. By introducing major ideas in upper levels, students are more able to see the role they will play in the proof and will, therefore, understand why each step in the bottom level is taken [6].

Methodology

Data has been collected on the grades of 705 students that have taken MATH 2510 (Real Analysis I) between fall 1992 and spring 2008. Identification numbers were assigned in order to keep track of students' repetition of courses; however, since they were randomly generated all students have remained anonymous. The data consists of the grades each student made in MATH 2510 (Real Analysis I), the prerequisite courses MATH 1710 (Calculus I), MATH 1720 (Calculus II), MATH 2700 (Linear Algebra), and MATH 2730 (Multivariable Calculus), as well as the semester in which each course was taken. The data was analyzed in order to determine the effects the following factors have on the grades made in MATH 2510: the length of time between taking MATH 1710 and MATH 2510, the grades made in prerequisite courses, and the repetition of prerequisite courses. For the purposes of this study a W, WF, I, and F, all give the same result of failure to complete the course which is denoted by a 0. An A in the course is denoted by a 4, B is 3, C is 2, and D is 1.

First, the data were sorted in order to separate the students who took at least three of the prerequisite courses at the University of North Texas and received either an A or a B in all of them. An average MATH 2510 grade was taken from this group.

In order to test the relationship between the amount of time between MATH 1710 and the grade received in MATH 2510, a linear correlation coefficient was used. Each term was numbered beginning with spring 1988 coded as 1. All summer courses were counted together as one term, creating a total of three terms per year. Data showing a student took MATH 1710 after MATH 2510 was not included in this analysis. For students who repeated courses, the data from the most recent attempt were used.

Linear correlation coefficients were also used to test the relationship between the grade received in MATH 2510 and the grade received in each of the prerequisite courses. Once again, data from the last repetition of each course were used.

A chi-squared test was used to check the independence of each of the prerequisites to MATH 2510. In order to have expected numbers of at least five, the categories of D and F were combined.

The hypotheses were as follows:

> H_o: The grade received in MATH 2510 is independent of the grade received in the prerequisite course.
>
> H_a: The grade received in MATH 2510 is dependent on the grade received in the prerequisite course.

For each prerequisite course, a one-tailed t-test was used to determine whether the average MATH 2510 grade was greater for students who made an A in the prerequisite course than for those who made a B. This test was also used to compare the average MATH 2510 grade for students who made a B in the prerequisite course rather than a C, and a C rather than a D or F.

The hypotheses were as follows:

> H_o: There is not a statistically significant difference in the average grade made in MATH 2510.
>
> H_a: The average grade made in MATH 2510 is greater for students who made a higher grade in the prerequisite course.

For each prerequisite course, a two-tailed t-test was used to determine whether there was a statistically significant difference between average MATH 2510 grade for students who made an A the first time they took the prerequisite course and for students who made an A eventually. This two-tailed t-test was repeated for those who made a B the first time versus those who eventually made a B in the prerequisite course. Only students who repeated the course before taking MATH 2510 for the first time were included in counting the number of students who eventually made an A or a B. The hypotheses were as follows:

> H_o: There is not a statistically significant difference in the average grade made in MATH 2510 when comparing students who made an A or B the first time they took the prerequisite course compared to those who eventually made an A or a B.
>
> H_a: There is a statistically significant difference in the average grade made in MATH 2510 when comparing students who made an A or B the first time they took the prerequisite course compared to those who eventually made an A or a B.

Results

From the chi-squared tests we come to the following conclusions: It appears that the grade received in MATH 2510 is dependent upon the grade received in MATH 1710. We reject the null hypothesis at the 0.01 significance level. The average grade received in MATH 2510 is 2.1 (a grade of C) with a standard deviation of 1.5473. Of the 116 students fitting the criteria, the average grade received in MATH 2510 is 3 (a grade of B) with a standard deviation of 1.2786.Refer to Table 1.

It appears that the grade received in MATH 2510 is dependent upon the grade received in MATH 1720. We reject the null hypothesis at the 0.01 significance level. There appears to be a positive correlation between the grade received in MATH 1720 and the grade received in MATH 2510 ($r = 0.4382$). Refer to Table 2.

It appears that the grade received in MATH 2510 is dependent upon the grade received in MATH 2700. We reject the null hypothesis at the 0.01 significance level. There appears to be a positive correlation between the grade received in MATH 2700 and the grade received in MATH 2510 (r = 0.5187). Refer to Table 3.

It appears that the grade received in MATH 2510 is dependent upon the grade received in MATH 2730. We reject the null hypothesis at the 0.01 significance level. There appears to be a positive correlation between the grade received in MATH 2730 and the grade received in MATH 2510 (r = 0.4630). Refer to Table 4.

It appears that the average grade received in MATH 2510 is higher for the students who received an A in MATH 1710 than for those who received a B in MATH 1710. We reject the null hypothesis at both the 0.01 and 0.1 significance levels. There appears to be a slight positive correlation between the grade received in MATH 1710 and the grade received in MATH 2510 (r = 0.2790). Refer to Table 5.

There does not appear to be a statistically significant difference in the average grade received in MATH 2510 between the students who received a B in MATH 1710 and those who received a C in MATH 1710. We accept the null hypothesis at both the 0.01 and 0.1 significance levels. Depending on the significance level, there may or may not be significant difference in the average grade received in MATH 2510 between the students who received a C in MATH 1710 and those who received either a D or whose grade resulted in a failure for the purpose of this study. We accept the null hypothesis at the 0.01 significance level and reject the null hypothesis at the 0.1 significance level. Refer to Table 6.

It appears that the average grade received in MATH 2510 is higher for the students who received an A in MATH 1720 than those who received a B in MATH 1720. We reject the null hypothesis at both the 0.01 and 0.1 significance levels. Refer to Table 7.

It appears that the average grade received in MATH 2510 is higher for the students who received a B in MATH 1720 than those who received a C in MATH 1720. We reject the null hypothesis at both the 0.01 and 0.1 significance levels. Refer to Table 8.

Depending on the significance level, there may or may not be a significant difference in the average grade received in MATH 2510 between the students who received a C in MATH 1720 and those who received either a D or whose grade resulted in a failure for the purpose of this study. We accept the null hypothesis at the 0.01 significance level and reject the null hypothesis at the 0.1 significance level. Refer to Table 9.

It appears that the average grade received in MATH 2510 is higher for the students who received an A in MATH 2700 than those who received a B in MATH 2700. We reject the null hypothesis at both the 0.01 and 0.1 significance levels. Refer to Table 10.

Depending on the significance level, there may or may not be a significant difference in the average grade received in MATH 2510 between the students who received a B in MATH 2700 and those who received a C in MATH 2700. We accept the null hypothesis at the 0.01 significance level and reject the null hypothesis at the 0.1 significance level. Refer to Table 11.

It appears that the average grade received in MATH 2510 is higher for the students who received a C in MATH 2700 and those who received either a D or whose grade resulted in a failure for the purpose of this study. We reject the null hypothesis at both the 0.01 and 0.1 significance levels. Refer to Table 12.

It appears that the average grade received in MATH 2510 is higher for the students who received an A in MATH 2730 than those who received a B in MATH 2730. We reject the null hypothesis at both the 0.01 and 0.1 significance levels. Refer to table 13.

It appears that the average grade received in MATH 2510 is higher for the students who received a B in MATH 2730 than those who received a C in MATH 2730. We reject the null hypothesis at the 0.01 and 0.1 significance levels. Refer to Table 14.

Depending on the significance level, there may or may not be a significant difference in the average grade received in MATH 2510 between the students who received a C in MATH 2730 and those who received either a D or whose grade resulted in a failure for the purpose of this study. We accept the null hypothesis at the 0.01 significance level and reject the null hypothesis at the 0.1 significance level. Refer to Table 15.

There appears to be a statistically significant difference between the average MATH 2510 grade received by students who received an A the first time MATH 1710 was taken and those who eventually received an A after repeating the course. We reject the null hypothesis at both the 0.01 and 0.1 significance levels. Refer to Table 16.

Depending on the significance level, there may or may not be a statistically significant difference between the average MATH 2510 grade received by students who received a B the first time MATH 1710 was taken and those who eventually received a B after repeating the course. We accept the null hypothesis at the 0.01 significance level and reject the null hypothesis at the 0.1 significance level. Refer to Table 17.

Depending on the significance level, there may or may not be a statistically significant difference between the average MATH 2510 grade received by students who received an A the first time MATH 1720 was taken and those who eventually received an A after repeating the course. We accept the null hypothesis at the 0.01 significance level and reject the null hypothesis at the 0.1 significance level. Refer to Table 18.

There appears to be a statistically significant difference between the average MATH 2510 grade received by students who received a B the first time MATH 1720 was taken and those who eventually received a B after repeating the course. We reject the null hypothesis at both the 0.01 and 0.1 significance levels. Refer to Table 19.

There does not appear to be a statistically significant difference between the average MATH 2510 grade received by students who received an A the first time MATH 2700 was taken and those who eventually received an A after repeating the course. We accept the null hypothesis at both the 0.01 and 0.1 significance levels. There does not appear to be a statistically significant difference between the average MATH 2510 grade received by students who received a B the first time MATH 2700 was taken and those who eventually received a B after repeating the course. We accept the null hypothesis at both the 0.01 and 0.1 significance levels.

Since there were only four cases in which a student repeated MATH 2730 before taking MATH 2510 for the first time and received either an A or B in the repeated course, a two-tailed t-test would not give an accurate result and therefore could not be used. Whether or not there would be a statistically significant difference between the average MATH 2510 grade received by students who received an A or B the first time MATH 2730 was taken and those who eventually received an A or B after repeating the course could not be tested.

Conclusions

Although there is not a strong linear correlation, it is clear that how well students perform in the prerequisite courses will impact their performance in MATH 2510. Overall, receiving higher grades in the prerequisite courses seems to be beneficial to students considering they seem to receive higher grades in MATH 2510. The repetition of MATH 1710 and MATH 1720 seems to have more of an effect on grades received in MATH 2510 than the repetition of MATH 2700 and MATH 2730. This, however, could be due to the ability to take MATH 2700 and MATH 2730 concurrently with MATH 2510 rather than being required to complete the courses before the first attempt of MATH 2510. This is also why there was not enough data to test the effect of repetition for MATH 2730.

References

Course Descriptions. Dept. of Mathematics, Texas A&M University. Retrieved April 27, 2009, from http://www.tamu.edu/admissions/catalogs/08-09_UG_Catalog/course_descriptions/math.htm

Course Descriptions for Mathematics (MATH). Dept. of Mathematics, University of Texas at Arlington. Retrieved April 27, 2009, from http://www.uta.edu/catalog/courses/course-catalog.php?subject=MATH

Course Offerings. Dept. of Mathematics, University of Louisiana at Lafayette. Retrieved April 27, 2009, from http://bulletin.louisiana.edu/UN/84/Bulletin2009.pdf

Epp, S. S. (2003). The role of logic in teaching proof [Electronic version]. *The American Mathematical Monthly*, *110*, 886-899.

Gries, D. & Schneider., F.B.(1995). Teaching math more effectively through calculational proofs [Electronic version]. *The American Mathematical Monthly*, *102*, 691-697.

Leron, U. (1983). Structuring mathematical proofs [Electronic version]. *The American Mathematical Monthly*, *90*, 174-185.

M325K Discrete Mathematics. Dept. of Mathematics, University of Texas at Austin. Retrieved April 27, 2009, from http://www.ma.utexas.edu/dev/math/Courses/Descriptions/M325K.html

Martin, T. S., Soucy McCrone, S. M., Wallace Bower, M.L., & Dindyal, J.. (2005). The interplay of teacher and student actions in the teaching and learning of geometric proof [Electronic version]. *Educational Studies in Mathematics*, *60*, 95-124.

Math 3033-Introduction to Advanced Mathematics. Dept. of Mathematics, University of Tulsa. Retrieved April 27, 2009, from http://www.utulsa.edu/academics/colleges/College-of-Engineering-and-Natural-Sciences/Departments-and-Schools/Department-of-Mathematical-and-Computer-Sciences/Math-Programs-of-Study/Undergraduate-Majors-and-Minors/BS-in-Mathematics

Mathematics (MATH). Dept. of Mathematics, Oklahoma State University. Retrieved April 27, 2009, from http://www.okstate.edu/registrar/Course%20Listings/MATH.html

Mathematics (MATH). Dept. of Mathematics, Oklahoma Universit Retrieved April 27, 2009, from http://catalog.ou.edu/courses/courses.htm

Mathematics (MATH). Dept. of Mathematics, Texas Tech University. Retrieved April 27, 2009, from http://www.depts.ttu.edu/officialpublications/courses/MATH.php

Richardson, M. (1951). Fundamentals in the teaching of undergraduate mathematics [Electronic version]. *The American Mathematical Monthly*, *58*, 182-186.

Spanier, E. (1970). The undergraduate program in mathematics [Electronic version]. *The American Mathematical Monthly*, 77, 752-755.

Student Syllabus. Retrieved April 27, 2009, from http://www.math.uh.edu/Matweb/syllabi/3333/3333.html

Vobejda, B. (1987). A mathematician's research on math instruction [Electronic version]. *Educational Researcher*, *16*, 9-12.

Weber, K. (2001). Student difficulty in constructing proofs: The need for strategic knowledge [Electronic version]. *Educational Studies in Mathematics*, *48*, 101-119.

Observed	2510 Grade	A	B	C	D+FWI	D	FWI	Totals
1710 Grade	A	46	29	31	37	6	31	143
	B	16	21	17	40	9	31	94
	C	7	2	9	21	8	13	39
	D+FWI	0	4	3	15	3	12	22
	D	0	2	2	7	3	4	11
	FWI	0	2	1	8	0	8	11
	Totals	69	56	60	113	26	87	298

Chi-squared	31.69272
Critical value	21.66599
P-value	0.000225

Table 1. Testing independence of grades in MATH 1710 to grades in MATH 2510

Observed	2510 Grade	A	B	C	D+FWI	D	FWI	Totals
1720 Grade	A	61	28	20	22	7	15	131
	B	22	20	24	36	6	30	102
	C	5	13	16	35	8	27	69
	D+FWI	2	6	12	39	9	30	59
	D	1	3	6	15	4	11	25
	FWI	1	3	6	24	5	19	34
	Totals	90	67	72	132	30	102	361

Chi-squared	81.92528
Critical value	21.66599
P-value	6.69E-14

Table 2. Testing independence of grades in MATH 1720 to grades in MATH 2510

Observed	2510 Grade	A	B	C	D+FWI	D	FWI	Totals
2700 Grade	A	82	27	18	14	3	11	141
	B	18	36	41	32	8	24	127
	C	12	16	16	40	17	23	84
	D+FWI	4	14	16	65	7	58	99
	D	1	4	5	12	2	10	22
	FWI	3	10	11	53	5	48	77
	Totals	116	93	91	151	35	116	451

Chi-squared	168.0571
Critical value	21.66599
P-value	1.57E-31

Table 3. Testing independence of Grades in MATH 2700 to grades in MATH 2510

Observed	2510 Grade	A	B	C	D+FWI	D	FWI	Totals
2730 Grade	A	63	21	12	15	4	11	111
	B	19	27	19	24	7	17	89
	C	8	13	21	33	9	24	75
	D+FWI	7	7	21	49	11	38	84
	D	4	0	7	16	7	9	27
	FWI	3	7	14	33	4	29	57
	Totals	97	68	73	121	20	52	359

Chi-squared	107.0242
Critical value	21.66599
P-value	5.93E-19

Table 4. Testing independence of grades in MATH 2730 to grades in MATH 2510

MATH 1710 Grades	Average MATH 2510	SD	n
A	2.370629	1.509026	143
B	1.808511	1.518889	94

Degrees of Freedom	198
t	2.794329
t-critical at 0.01	2.345328
t-critical at 0.1	1.285842
P-value	0.002856

Table 5. Testing average MATH 2510 grade when MATH 1710 grade is A or B

MATH 1710 Grades	Average MATH 2510	SD	n
C	1.538462	1.466219	39
DFWI	0.954545	1.214095	22

Degrees of Freedom	50
t	1.670897
t-critical at 0.01	2.403272
t-critical at 0.1	1.298714
P-value	0.050495

Table 6. Testing average MATH 2510 grade
when MATH 1710 grade is C or D/failure/withdrew

MATH 1720 Grades	Average MATH 2510	SD	n
A	2.862595	1.363047	131
B	1.980392	1.521987	102

Degrees of Freedom	204
t	4.593024
t-critical at 0.01	2.344766
t-critical at 0.1	1.285715
P-value	3.82E-06

Table 7. Testing average MATH 2510 grade when MATH 1720 grade is A or B

MATH 1720 Grades	Average MATH 2510	SD	n
B	1.980392	1.521987	102
C	1.434783	1.366385	69

Degrees of Freedom	156
t	2.445713
t-critical at 0.01	2.350489
t-critical at 0.1	1.287002
P-value	0.007784

Table 8. Testing average MATH 2510 grade when MATH 1720 grade is B or C

MATH 1720 Grades	Average MATH 2510	SD	n
C	1.434783	1.366385	69
DFWI	1	1.203443	59

Degrees of Freedom	125
t	1.913929
t-critical at 0.01	2.35655
t-critical at 0.1	1.288361
P-value	0.028958

Table 9. Testing average MATH 2510 grade when MATH 1710 grade is C or D/failure/withdrew

MATH 2700 Grades	Average MATH 2510	SD	n
A	3.177305	1.214694	141
B	2.125984	1.290946	127

Degrees of Freedom	258
t	6.845432
t-critical at 0.01	2.340888
t-critical at 0.1	1.284841
P-value	2.77E-11

Table 10. Testing average MATH 2510 grade when MATH 2700 grade is A or B

MATH 2700 Grades	Average MATH 2510	SD	n
B	2.125984	1.290946	127
C	1.72619	1.417202	84

Degrees of Freedom	166
t	2.077513
t-critical at 0.01	2.349021
t-critical at 0.1	1.286672
P-value	0.019647

Table 11. Testing average MATH 2510 grade when MATH 2700 grade is B or C

MATH 2700 Grades	Average MATH 2510	SD	n
C	1.72619	1.417202	84
DFWI	0.979798	1.301332	99

Degrees of Freedom	170
t	3.685452
t-critical at 0.01	2.348483
t-critical at 0.1	1.286551
P-value	0.000153

Table 12. Testing average MATH 2510 grade when MATH 2700 grade is C or D/failure/withdrew

MATH 2730 Grades	Average MATH 2510	SD	n
A	3.09009	1.311138	111
B	2.269663	1.396131	89
Degrees of Freedom	183		
t	4.242999		
t-critical at 0.01	2.346897		
t-critical at 0.1	1.286195		
P-value	1.75E-05		

Table 13. Testing average MATH 2510 grade when MATH 2730 grade is A or B

MATH 2730 Grades	Average MATH 2510	SD	n
B	2.269663	1.396131	89
C	1.626667	1.373232	75

Degrees of Freedom	158
t	2.964518
t-critical at 0.01	2.35018
t-critical at 0.1	1.286933
P-value	0.001751

Table 14. Testing average MATH 2510 grade when MATH 2730 grade is B or C

MATH 2730 Grades	Average MATH 2510	SD	n
C	1.626667	1.373232	75
DFWI	1.214286	1.326935	84

Degrees of Freedom	153
t	1.920547
t-critical at 0.01	2.350967
t-critical at 0.1	1.287109
P-value	0.028324

Table 15. Testing average MATH 2510 grade
when MATH 2730 grade is C or D/failure/withdrew

MATH 1710 Grades	Average MATH 2510	SD	n
A Finally	1.473684	1.263523	19
A Originally	2.508065	1.500656	124

Degrees of Freedom	26
t	3.235808
t-critical at 0.01	2.778715
t-critical at 0.1	1.705618
P-value	0.003296

Table 16. Testing average MATH 2510 grade when repetition of MATH 1710 is necessary for an A

MATH 1710 Grades	Average MATH 2510	SD	n
B Finally	1.083333	1.311372	12
B Originally	1.914634	1.525109	82

Degrees of Freedom	15
t	2.00635
t-critical at 0.01	2.946713
t-critical at 0.1	1.75305
P-value	0.063197

Table 17. Testing average MATH 2510 grade when repetition of MATH 1710 is necessary for a B

MATH 1720 Grades	Average MATH 2510	SD	n
A Finally	2.066667	1.533747	15
A Originally	2.965517	1.311693	116

Degrees of Freedom	16
t	2.169481
t-critical at 0.01	2.920782
t-critical at 0.1	1.745884
P-value	0.045453

Table 18. Testing average MATH 2510 grade when repetition of MATH 1720 is necessary for an A

MATH 1720 Grades	Average MATH 2510	SD	n
B Finally	1	1.414214	15
B Originally	2.149425	1.482681	87

Degrees of Freedom	19
t	2.886203
t-critical at 0.01	2.860935
t-critical at 0.1	1.729133
P-value	0.009461

Table 19. Testing average MATH 2510 grade when repetition of MATH 1720 is necessary for a B