NYO-9771
Waste Disposal and Processing

STATISTICAL ANALYSIS OF THE FREQUENCY AND SEVERITY OF ACCIDENTS TO POTENTIAL HIGHWAY CARRIERS OF HIGHLY RADIOACTIVE MATERIALS

by

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PREFACE

It seems desirable to make a short statement regarding the nature and purposes of this report. First, it is an assemblage of data bearing on the probabilities of accidents to heavy highway vehicles which are potential carriers of radioactive materials, and on the severity of such accidents, because the latter is germane to the probability of a release of activity from a container in transit. Second, these data are analyzed statistically to reveal their significance.

Beyond this, no attempt is made to draw conclusions or to point out possible specific uses, the authors preferring to present the material in a very general form. The report is, in effect, a tool.

However, one specific use should be mentioned. The report has already been employed in working out a number of decision rules in which the alternative costs are compared of shipping during intervals of low-accident rates or by low-accident rate routes or of permitting such shipments under less propitious circumstances. In the application referred to the costs to be considered are those directly connected with accidents against those of increased container inventory, increased storage capacity, etc. The operations research analysis in which this application is made is soon to be reported (see item 4, page ii).

And in like manner it is believed that others interested in the highway safety movement may adapt the findings of the report to their specific needs. Those who may profit by so doing include insurance groups; motor vehicle administrators; traffic managers of general, as well as dangerous cargo haulers; heavy vehicle designers; and highway planners.

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ABSTRACT

The probability of accidents to tractor semitrailers is developed through analysis of accident frequency data in relation to season; geographical factors; road type, traffic and population density; and type of carrier business. Maximum likelihood rates are developed for the potential carriers of radioactivity. Impact characteristics of accidents are studied through the analysis of mass, speed, and energy relations and the effect of these on vehicle and cargo damages are explored.

I. GENERAL INTRODUCTION

This report is one of several resulting from a study of the transportation of fission materials at The Johns Hopkins University under a research contract sponsored by the U. S. Atomic Energy Commission and administered by the Nuclear Safety Group of the Division of Reactor Development. These reports, exclusive of the one in hand, are listed below both as a record for interested parties and to show the diversity of areas which must be explored before one can hope for success in the culminating operations analysis which attempts to make use of ideas and facts uncovered in the several areas. Publications which have already appeared or are scheduled for early release are:

1. "An Operations Research Study of the Transportation of Highly Radioactive Materials, A Progress Report," by J. T. Thompson and F. F. Leimkuhler, April 1959, NYO 7832. (published)

2. "Structural Analysis and Design Considerations for Shipping Containers of Highly Radioactive Materials," by Robert G. Sanford, May 1961, NYO 9374. (published)

3. "A Study of the Possible Consequences and Costs of Accidents in the Transportation of High Level Radioactive Materials," J. M. Morgan, Jr., John W. Knapp, and J. T. Thompson. (in preparation)

4. "An Operations Research Study of the Potential Accident Experience and Total Cost of Truck Shipments of Highly Radioactive Materials," F. F. Leimkuhler. (in preparation)

In item 1 of the above list the model of the proposed analysis operation was set forth. Reduced to its simplest terms, it may be stated thus,

$$C_t = C_v + C_p + C_e + C_h \quad \text{where} \quad C_h = p_a \, p_r \, c_c$$

C_t = the total cost of transportation

C_v = cost associated with the vehicle (fuel, labor, etc.)

C_p = cost of packaging (shielded containers)

C_e = cost of escort

C_h = cost of hazard

P_a = probability of an accident

P_r = probability of a release from containers

c_c = cost of protecting and/or rehabilitating the environment following possible release.

The terms of the model are related in a complex manner so that variation in one may have a pronounced effect upon one or perhaps all of the others. It is by studying their relations with a set of controllable variables that operations research attempts to determine that combination of variables which yields minimum expected total cost subject to an acceptable level of risk.

As one might suppose, the data for making such an analysis were almost wholly non-existent and, therefore, in the project's first release (item 1), in order to show that the model was a workable one, figures were used which were in some cases fictitious. Since that time much of the needed data have been uncovered, collected and analyzed. Although organizations and individuals too numerous to record here were helpful in many ways, the data actually used were secured mainly from the Interstate Commerce Commission and the U. S. Bureau of Public Roads. Dr. Acheson Duncan of the Department of Industrial Engineering of The Johns Hopkins University served as consultant in in statistical analysis of the data. To all of these the authors are deeply grateful.

Although it is expected, ultimately, that other modes of transportation will be studied, the Hopkins group has been mainly highway oriented. This is because of the greater experience and familiarity possessed by its personnel with the highway field.

The report in hand concerns itself mainly with the probability of accidents to tractor semitrailers, which are the type of vehicle contemplated for use in transporting high level radioactivity, and the probability of release of material from containers. The latter probability may be estimated in more than one way; obviously, containers may be analyzed from the structural viewpoint or tests may be conducted on them.[1/] Another method is by inference from the study of vehicle and cargo damage in relation to speed, mass, and energy. The latter method has been employed herein.

Summarizing, the material in this report deals with the frequency and severity of highway accidents in which heavy vehicles and their cargos are involved. The Table of Contents is sufficiently detailed to give the reader an understanding of the relationships which are explored.

It is hoped that this report may be of interest and use not only to those responsible for decisions in the transportation of fission or other dangerous materials, but to others as well, such as general transportation companies, insurance groups, and even the designers of roads and vehicles.

1/ Under a contract with the U. S. Atomic Energy Commission, Franklin Institute, Philadelphia, Pa., is making a study of the possibilities of model and prototype testing of containers.

II. ANALYSIS OF THE FREQUENCY OF TRUCK ACCIDENTS AND THEIR IMPACT CHARACTERISTICS.

1.0. Introduction

A study was made of the accident experience of large, commercial carriers engaged in interstate commerce in order to better understand the sequence of events which may lead to a serious accident in the highway transportation of radioactive materials, to discover methods of controlling both the frequency and severity of such accidents, and to estimate the potential effectiveness of such methods in reducing the accident risk. In the analysis of accident frequency, data were obtained from the Bureau of Motor Carriers of the Interstate Commerce Commission, which covered a four year period (1956-59) with a total of 111,120 accidents experienced by approximately 2500 large motor carriers of property in more than 30.5 billion vehicle miles of inter-city travel. A reportable accident is defined as one from which there results an injury or death, or property damage to an apparent extent of $100 or more. These data were classified by quarter of year, regional location in the United States, and type of carrier or service rendered. Significant annual, quarterly, regional, and carrier-type differences in accident rate were found in the data; however, the annual bias could be attributed to a change in the reporting procedures which was co-incident with a change in accident rate.

The quarterly or seasonal variation in accident rate followed a cyclic pattern throughout the four years, rising in the Winter and falling in the Spring and Summer months. The differences in accident rates among the various ICC regions in the United States could be associated with differences in the highway characteristics for these regions, principally in terms of the traffic congestion on the highways.

When the carriers were analyzed according to the type of haulage engaged in, a large proportion, accounting for almost two-thirds of the total vehicle mileage, was found to have a common accident expectancy with no significant differences in accident rate among them. The remaining carriers appeared to have a significantly higher accident rate. Included in the former and larger group were carriers of explosives and other dangerous articles, the category under which shipments of radioactive materials are currently classified. The accident experience of this group was taken to be representative of the accident frequency to be expected in the highway transportation of radioactive materials.

A further analysis of accident frequency was made for various days of the week and hours of the day, from a recent report by the ICC in which the accident data were classified according to their time of occurrence. There was a significant difference between daytime and nighttime, and between the weekday and the weekend occurrence of accidents.

In the analysis of the impact characteristics of truck accidents, data were obtained from the United States Bureau of Public Roads and the Interstate Commerce Commission. The relative frequencies of the different types of accidents were found to depend on the type of highway where the accident occurred. This is especially true of the direction of impact in motor vehicle collisions. However, the type of vehicle struck in such collisions was found to be independent of the type of highway or the direction of impact. The weight characteristics of the various types of trucks were studied and a common weight distribution was estimated. Fire was found to occur in all types of collision accidents about one percent of the time, but in overturn accidents fires occur with twice that frequency.

Because of the difficulty in obtaining reliable data, there is very little literature on the subject of accident speeds, and the best data of this type, known to be available, were acquired from the BPR. With these data it was possible to approximate the distribution of speed in various types of accidents by a compound density function, consisting of a normal, or bell-shaped, pattern in the upper speed range, and a rectangular pattern in the lower range. This was found to be in general agreement with the observations obtained from various speed studies of congested and free-flowing traffic. In this way, the speed of trucks and automobiles in collision with trucks were studied for possible differences. In two-vehicle collisions the two speeds were found to be statistically independent, and the net impact or collision velocity was taken to be the vector sum of the two speeds. Estimates were made of the distribution of impact velocities for various types of collisions, and at various points of impact on the critical vehicle.

From the standpoint of control, further study of truck accidents under these and other conditions may justify the use of special precautionary measures specifically designed to meet the needs of shipments of radioactive materials. In general, however, the above analysis makes it possible to consider three sets of alternatives which can be employed to reduce the frequency of accidents. Shipments which are normally made during periods of the year with a relatively high accident rate, could be deferred to other periods with lower rates. Secondly, shipping could be suspended during those hours of the day when highway congestion is greatest and accidents are more frequent. Finally, routes can be chosen so that trucks bypass highway sections with road characteristics which are unfavorable with respect to increased accident expectancy.

In a similar manner, the severity of accidents in terms of the mass, speed and direction of impact could be influenced by the avoidance of unfavorable highway sections, by controlling the speed of the critical vehicle, or by using special vehicles and containers which are designed to withstand the impacts experienced in highway accidents. At the present time, a study is being made of the potential effectiveness of these measures in reducing the risk in the transportation of radioactive materials, in terms of the total cost to a transportation system. Other methods of control might be considered, but in order to measure their effectiveness, the accident experience of trucks operating under these controls will have to be obtained by direct observation or experimental simulation. In any event, it is in the best interests of those responsible for the shipment of radioactive materials to document their accident and accident-free experience in a manner which will permit continuing analysis and inference.

2.0. Accident Frequency for Large Motor Carriers of Property with Reference to Typical Carriers of Radioactive Materials

2.1. Discussion

Accident involvement data for large motor carriers of property throughout the United States were obtained from the Bureau of Motor Carriers, Interstate Commerce Commission. Under the safety regulations of the Commission, all accidents must be reported in which there results an injury or death, or property damage to the apparent extent of $100 or more.[1/] In addition, the carriers furnish estimates of their total intercity vehicle miles of operation, which are the basis for computing accident rates. These data are summarized by the I.C.C. quarterly by type of carrier and by geographic area for carriers with annual operating revenues of $200,000 or more.

These data were analyzed for significant differences in accident frequency by quarter, year, and geographic region of the United States. The analysis of variance of quarterly accident rates by year indicated the presence of a seasonal pattern repeated each year. The technique of serial correlation was used to study the periodicity of the rate over four years, which was further analyzed by means of a Fourier series. The significance of the harmonic terms was evaluated by individual degree of freedom comparisons in the analysis of variance.

Significant geographic differences were also found to exist among the accident rates for carriers in twelve I.C.C. motor carrier districts.

1/ Interstate Commerce Commission, "Motor Carrier Safety Regulations," Revision of 1952, Washington, D. C., p. 48. A rule change which became effective on January 1, 1960, relieved carriers from reporting "property damage only" accidents in which the amount of damage was less than $250.

These differences could be explained largely by grouping the districts into an eastern and a western region of the United States, in which the interstate highway characteristics were also very different. It was not possible to explain satisfactorily within-region rate differences in terms of road characteristics. The I.C.C. rates were found to be comparable to those developed in a special study of similar vehicles on the New Jersey Turnpike.

The analysis of variance of quarterly accident rates for carriers classified by type of cargo or service showed that significant differences were present. The mean rates of the seven other carrier classes were compared to that of carriers of explosives, radioactive materials, and other dangerous articles; and four were judged to have a similar accident rate, which differed from the overall rate by the same amount in each quarter. The linear regression of accidents on mileage for this group of carriers by quarter was in good agreement with the data and the theory that accidents follow the Poisson distribution, with the expected number of accidents proportional to exposure.

Maximum likelihood estimates were made of the accident rate for all carriers by quarter and region, using the assumption of a Poisson accident frequency. These rates were adjusted to serve as estimates of the rate for typical carriers of radioactive materials. Because of the large number of miles in the estimates the variances of the estimates are quite small, such that all of the estimates are theoretically accurate to within $\pm$ 1 accident per 10 million vehicle miles. The estimated accident rates for typical carriers of radioactive materials are summarized in Table 2.1.

Table 2.1. Summary of Final Estimates of the Accident Rate per Million Vehicle Miles for Typical Carriers of Radioactive Materials.

Regions of U.S.	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	Annual
Eastern	5.271	3.774	3.920	4.174	4.273
Western	3.003	2.357	2.522	2.735	2.655
All Regions	4.344	3.208	3.360	3.610	3.626

2.2. Analysis of Accident Rates by Year and Quarter

The accidents, miles, and rates for 10 million vehicle miles (10 MVM) reported to the I.C.C. by all large motor carriers of property in each quarter of the years 1956 through 1959, are shown in Table 2.2. In making these figures available, the I.C.C. has noted that the data are not exactly comparable since the mileage represents intercity travel in all years but the accidents reported in 1956 and 1957 occurred largely in interstate commerce only. As of January 1, 1958, both accidents and miles were required on the intercity basis.[1/]

In examining the effect of this inconsistency, the years 1956-7 and 1958-9 were considered as separate periods, denoted by $i = 1, 2$ with two years, $j = 1, 2$ in each, which are cross-classified with quarters, $k = 1, 2, 3, 4$. Assuming that the mean effects of these factors are additive, the accident rate r_{ijk} for each year-quarter can be expressed as follows:

(2.1) $r_{ijk} = r_o + p_i + y_{ij} + q_k + e_{ijk}$

r_o = overall mean rate,

p_i = mean difference of period $i = 1, 2$

y_{ij} = mean difference of year $j = 1, 2$ in period i,

q_k = mean difference of quarter $k = 1, 2, 3, 4$,

e_{ijk} = measurement error, assumed to be approximately normal with mean zero and variance σ^2 common to all observations.

1/ H. O. McCoy, "Motor Carriers of Property-Accident Data, First Quarter, 1958," Interstate Commerce Commission, Washington, D. C., January 28, 1959.

The significance of each difference is tested in the analysis of variance of Table 2.2, where the mean sum of squares associated with each factor is computed and their ratio with the residual mean square is compared with the corresponding critical value of the F distribution.[1] Thus, the differences between years within periods is found to be insignificant; but both the period and quarter differences are very significant.

The analysis of variance for quarterly rates indicates the presence of significant quarterly rate variations which repeat themselves each year within 1956-7 and 1958-9. Such stationary time series can be represented mathematically as the sum of a series of cyclic terms in a Fourier series, i.e.

$$(2.2)\quad r(t) = a_0 + b_1 \cos \theta + b_3 \cos 2\theta + \ldots + b_{k-1} \cos \frac{k-1}{2}\theta$$
$$+ b_2 \sin \theta + b_4 \sin 2\theta + \ldots + b_k \sin \frac{k-1}{2}\theta$$

where $r(t)$ denotes the rate at time $t = 0, 1, 2, \ldots$ time units, and $\theta = 2\pi t/k$ for a period of length k time units.
The series repeats itself in the time intervals 0 to k, k to 2k, etc. The period length k, and the coefficients a and b are to be estimated.

One method of detecting periodicity in the data is to compute the product-moment correlation coefficient for the observations, r_t, at time t with those at time $t + 1$, $t + 2$, ...; where the serial correlation coefficient of order k is given by[2]

1/ A. J. Duncan, "Quality Control and Industrial Statistics," Chapters XXIX and XXX, Homewood, Illinois, R. D. Irwin, 1959.
2/ M. G. Kendall, "The Advanced Theory of Statistics," Vol. II, 1946, Griffin, London, p. 404.

$$(2.3)\quad R = \frac{\operatorname{cov}(r_t, r_{t+k})}{\left[\operatorname{var}(r_t)\operatorname{var}(r_{t+k})\right]^{1/2}}$$

Serial correlation coefficients were calculated for the data of table 2.2. The rates are plotted in Figure 2.1 and the serial correlation coefficients R_k are plotted against k in Figure 2.2. The plot indicates the presence of an undamped cycle with a period length of four quarters.

Model (2.2) with a period of four quarterly time units becomes

$$(2.4)\quad r(t) = a + b_1 \cos t\pi/2 + b_2 \sin t\pi/2 + b_3 \cos t\pi$$

which reduces to:

$$(2.5)\quad r_1 = a + b_1(1) + b_2(0) + b_3(1)$$

$$r_2 = a + b_1(0) + b_2(1) + b_3(-1)$$

$$r_3 = a + b_1(-1) + b_2(0) + b_3(1)$$

$$r_4 = a + b_1(0) + b_2(-1) + b_3(-1)$$

$$r_i = r_{i+4} = r_{i+8} = \ldots$$

When this model is fitted to the observed rates by the method of least squares, the normal equations give the following estimates for the values of a and b.[1/]

$$(2.6)\quad a = \bar{r}$$

$$b_1 = (\bar{r}_2 - \bar{r}_3)/2$$

$$b_2 = (\bar{r}_2 - \bar{r}_4)/2$$

$$b_3 = (\bar{r}_1 - \bar{r}_2 + \bar{r}_3 - \bar{r}_4)/4$$

1/ E. T. Whitaker and G. Robinson, "The Calculus of Observations," 4th Edition, 1944, Blackie, London, p. 267.

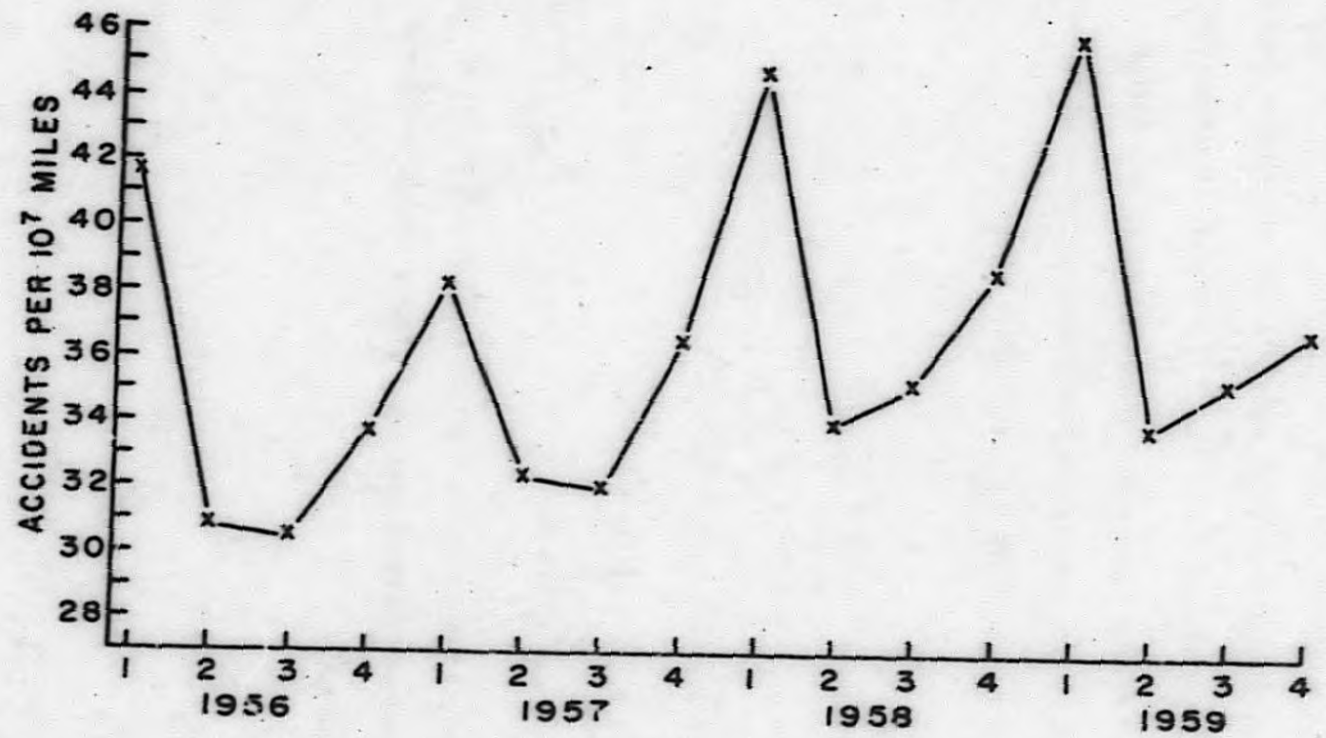

FIG. 2.1. ACCIDENT RATES BY QUARTER FOR LARGE ICC CARRIERS FROM 1956 THROUGH 1959

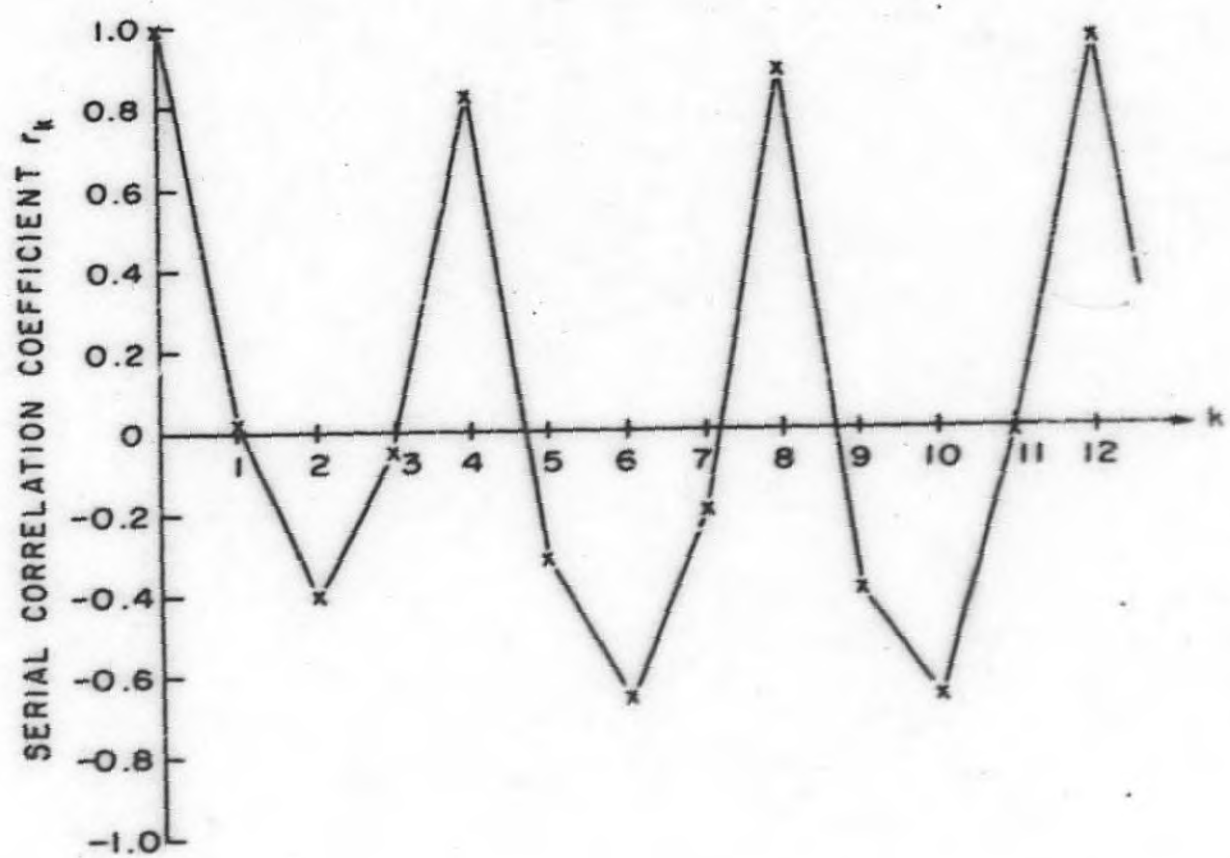

FIG. 2.2. SERIAL CORRELATION OF ACCIDENT RATES FOR PERIODS OF LENGTH K IN THREE MONTH OR QUARTERLY UNITS

Table 2.2. Accidents, Vehicle Miles, and Rates by Year and Quarter for Large I.C.C. Motor Carriers of Property

	1st Quarter			2nd Quarter			3rd Quarter			4th Quarter			Annual Total		
Year	Acc.	MVM*	Rate**	Acc.	MVM	Rate	Acc.	MVM	Rate	Acc.	MVM	Rate	Acc.	MVM	Rate
1956	6875	1649.3	41.7	5301	1719.1	30.8	5152	1690.4	30.5	5894	1755.3	33.7	23222	6814.1	34.1
1957	6885	1802.0	38.2	5859	1816.5	32.3	5936	1852.8	32.0	6470	1771.7	36.5	25150	7243.0	34.7
1958	8117	1814.0	44.7	6189	1816.7	34.1	6736	1911.8	35.2	8255	2137.5	38.6	29297	7680.0	38.1
1959	9800	2136.1	45.9	7758	2295.8	33.8	7677	2178.3	35.2	8216	2333.6	36.8	33451	8843.9	37.8
Total	31677	7401.4	42.8	25107	7648.0	32.8	25501	7633.3	33.4	28835	7898.2	36.5	111120	30580.9	36.3

*MVM, millions of vehicle miles. **Rates are given as accidents per 10 MVM.

Analysis of Variance

Source of Variation		Sum of Squares	d.f.	Mean Square	Ratio	$F_{0.05}$
Quarter		248.255	3	82.752	38.12	3.86
Years	1956-7 vs. 1958-9	51.123	1	51.123	23.60	5.12
	Within periods	0.762	2	0.381	0.18	4.26
	Total	51.885	3	17.295	7.97	3.86
Residual		19.540	9	2.171		

The simplicity of these results, which is due to the orthogonality of equation (2.5), makes it possible to test the significance of the harmonic terms, i.e. whether the "b" coefficients differ from zero, by means of individual degrees of freedom in the analysis of variance. The tests for b_1, b_2, and b_3 are equivalent to comparisons of mean differences between quarters: 1 vs. 3, 2 vs. 4, and 1, 3 vs. 2, 4. The analysis of variance of Table 2.2 is extended in Table 2.3 to include tests of significance for the ratio of mean square to residual for each of the harmonic terms. All three terms are found to be significant, with coefficients different from zero.

Table 2.3. Individual Comparisons for Harmonic Components of the Quarter Differences of Table 2.2.

Source of Variation	Sum of Squares	d.f.	Mean Square	Ratio	$F_{0.05}$
Quarters:					
1st cosine term, b_1	176.720	1	176.720	81.40	5.12
1st sine term, b_2	26.645	1	26.645	12.27	5.12
2nd cosine term, b_3	44.890	1	44.890	20.68	5.12
Total for Quarters	248.255	3	82.752	38.12	3.86
Residual	19.540	9	2.171		

The seasonal cycle in accident rates represented by the Fourier series (2.2) was evaluated for the accident data in Table 2.2 with the estimators of (2.6). The estimate is given by

(2.7) $r_j = a_j + 4.696 \cos \theta - 1.754 \sin \theta + 1.758 \cos 2\theta$

$a_1 = 34.667$ for 1956-7

$a_2 = 37.974$ for 1958-9

Equations (2.7) are plotted in Figure 2.3, where each quarterly rate is interpreted as applying to the mid-point of the quarter, and θ is expressed as a function of the fraction T, of the year that has elapsed since January 1st, i.e.

(2.8) $\theta = 2\Pi(T - 0.125)$

The observed rates are also plotted in Figure 2.3, and the difference in the periods 1956-7 and 1958-9 is quite apparent.

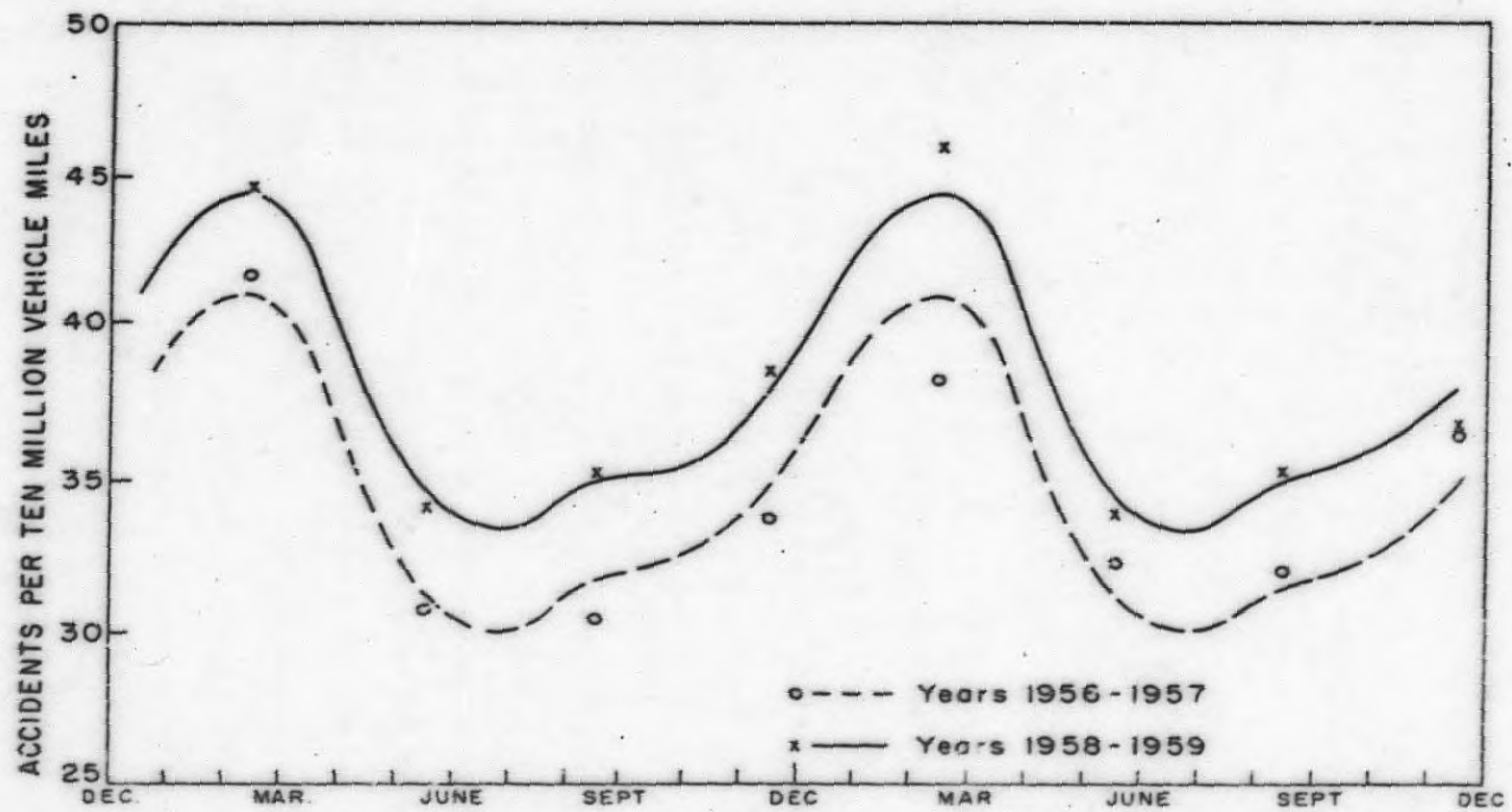

FIG. 2.3. OBSERVED VS. THRORETICAL ACCIDENT RATES FOR ICC CARRIERS BASED ON A FOURIER SERIES OF SEASONAL RATE VARIATIONS

2.3 Analysis of Geographic Differences in Accident Rates

The 1958-59 accident rates for large carriers located in the various I.C.C. motor carrier districts are given in Table 2.4. The districts are identified on the map in Figure 2.4.[1/] The mean differences in accident rates by district, quarter, and year were examined by means of the following analysis of variance model.

(2.9) $r_{ijk} = r_o + d_i + q_j + y_k + dq_{ij} + dy_{ik} + qy_{jk} + e_{ijk}$

r_o = overall mean rate

d_i = mean difference for district i = 1, 2, 3, 4 .....12.

q_j = mean difference for quarter j = 1, 2, 3, 4

y_k = mean difference for year y = 1, 2

dq_{ij} = mean interaction of district i with quarter j

dy_{ik} = mean interaction of district i with year k

qy_{jk} = mean interaction of quarter j with year k

e_{ijk} = measurement error, assumed approximately normal with mean zero and variance σ^2 common to all observations.

In the analysis of variance for Table 2.4, all of the interaction terms are not significant at the 5% level of probable error, as is the difference between years. However, both the mean differences among districts and quarters are significant.

In Table 2.5 the mean district rates are ranked in descending order, and compared with the probable range of mean rates when measured from both the highest and lowest values. The 95% probable range (known as the "Studentized" range) is given by[2/]

1/ I.C.C. districts 15 and 16 were combined into a single district, designated 14, because of the small number of vehicle miles reported in district 15.

2/ A. J. Duncan, op. cit., p. 601

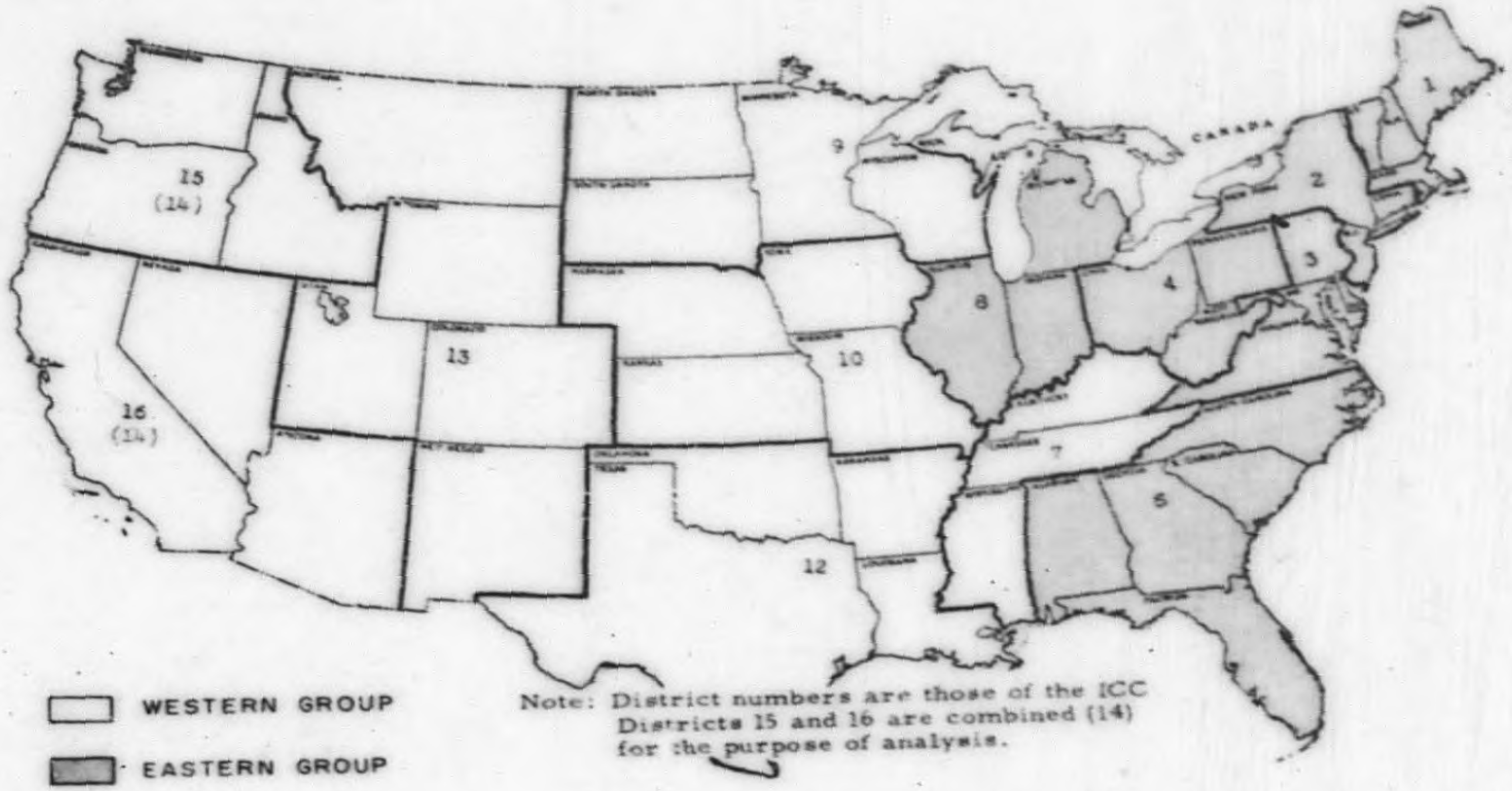

FIG. 2.4. ICC MOTOR CARRIER DISTRICTS

Table 2.4. Accident Rates per 10 MVM for Various Motor Carrier Districts as Reported by the I.C.C.

Motor Carrier Districts	1958 Quarters 1st	2nd	3rd	4th	1959 Quarters 1st	2nd	3rd	4th
1. Me.,N.H.,Vt.,Mass.,R.I.	52	35	33	35	53	30	32	39
2. N.Y.,Conn.,N.J.	63	42	40	40	48	35	36	36
3. Md.,Del.,Va.,E.Pa.	57	41	44	51	55	40	46	52
4. Ohio,W.Va.,W.Pa.	60	42	45	52	59	41	39	43
6. N.C.,S.C.,Ga.,Ala.,Fla.	45	34	35	38	44	40	39	42
7. Ky.,Tenn.,Miss.	33	26	28	30	41	31	30	25
8. Ill.,Ind.,Mich.	54	43	44	46	59	41	44	43
9. N.D.,S.D.,Minn.,Wis.	30	25	26	30	40	23	26	30
10. Neb.,Ka.,Io.,Mo.	30	26	27	28	39	25	26	28
12. Tex.,Ok.,La.	35	30	33	39	35	33	38	37
13. Mont.,Wy.,Col.,Ut., N.M.	28	28	26	28	33	19	24	31
14. Wash.,Or.,Cal.,Nev.,Ariz.	27	19	20	22	26	20	21	22

Analysis of Variance

Source of Variation	Sum of Squares	d.f.	Mean Square	Ratio	$F_{0.05}$
Quarters	1907.865	3	635.955	60.99	2.89
Districts	6938.365	11	620.760	60.49	2.09
Years	0.094	1	0.094	0.01	4.14
Interactions					
District-Quarter	591.760	33	17.932	1.72	1.83
District-Year	0.000	3	0.000	0.00	2.89
Quarter-Year	210.781	11	19.162	1.84	2.09
Residual	344.125	33	10.428		

(2.10) $R = q(12.84)s_{\bar{r}} = 4.78(36.365/8)^{1/2} = 10.18$

q = range factor for 12 means and 84 degrees of freedom

$s^2_{\bar{r}}$ = estimated variance of the mean rates = $s^2_r/8$

s^2_r = estimated variance of rates within districts assumed common to all districts.

The mean rate for the first six districts of Table 2.5 fall within the upper range, indicating that there may be no significant rate differences among them. The same reasoning applies to the last five districts of Table 2.5. This grouping of the districts corresponds to a division of the United States into two regions, east and west. Although the remaining district falls outside of the range of either group, it is geographically contained in the western region.

This grouping of the districts into two regions was further examined by extending the analysis of variance in Table 2.4 to permit the analysis of geographic differences and interactions both between the regions and within regions. This is done in Table 2.5, where the between-region differences and interactions appear to be significant. The within region differences are still significant, but without apparent interaction. Although this treatment fails to completely explain geographic differences, it provides a basis for gaining more precision in estimating accident rates.

Some explanation of the differences between accident rates in the eastern and western portions of the United States can be obtained from the differences in highway characteristics for these regions. Some characteristics for U. S. Interstate Highway System in these two regions are shown in Table 2.6. In general, higher accident rates appear to be associated with highways having denser traffic and closer

Table 2.5. Comparisons of Mean Accident Rates for Districts of Table 2.3

Ranking of Districts by Rate	District No.	Mean Rate	Difference* from Highest	Difference* from Lowest
Eastern	3	48.250	0.00	26.13
	4	47.625	0.63	25.50
Group or	8	46.750	1.50	24.63
	2	42.500	5.75	20.38
Region	6	39.025	8.53	17.50
	1	38.625	9.53	16.50
Western	12	35.000	13.25*	12.88*
	7	30.375	17.88	8.25
Group or	9	28.750	19.50	6.63
	10	28.625	19.63	6.50
Region	13	27.125	21.13	5.00
	14	22.125	26.13	0.00

*Critical Value is 10.18

Analysis of Variance - Extension of Table 2.4

Source of Variation	Sum of Squares	d.f.	Mean Squares	Ratio	$F_{0.05}$
Among Districts					
East vs. West	5505.510	1	5505.51	527.92	4.14
Within Regions	1432.855	10	143.29	13.74	2.13
District-Quarter Interaction					
East vs. West	277.198	3	92.40	8.86	2.89
Within Regions	314.562	30	10.49	1.01	1.81
District-Year Interaction					
East vs. West	46.760	1	46.76	4.48	4.14
Within Regions	164.021	10	16.40	1.57	2.13
Residual Error	344.125	33	10.428		

urban communities. An attempt was made to correlate these characteristics with individual district accident rates, but the results were not conclusive.

In a recent study[1/] of accidents on the New Jersey Turnpike over a period of six years (1952-57), large property carrying vehicles experienced 1698 accidents for a total of 565.3 million vehicle miles of travel. This yields an accident rate of 30.04 accidents per 10 MVM, which is slightly higher than the overall rate of 28.55 reported by all I.C.C. carriers in the western grouping of districts; and considerably lower than the overall rate of 44.73 for the eastern districts. Assuming no important measurement difference, this would indicate that turnpikes effectively reduce the accident rate normally experienced on highways in the same area.

Table 2.6. Some Characteristics of Interstate Highways in Different Regions*

Characteristics	Eastern	Western	U. S. Total
Urban Miles	3,171 (19%)	2,678 (11%)	5,849 (14%)
Rural Miles	13,134 (81%)	21,234 (89%)	34,368 (86%)
Divided Hwy.-Rural	26.74%	12.89%	20.27%
Avg. Daily Traffic-Rural	6226	4133	4920 (vehs./day)
Towns of 5000 or more	643	537	1180
Miles between Towns	20.4	39.5	29.1 (mi./town)
I.C.C. Accident Rate	43.9	28.7	36.3 (acc./10 MVM)

*Source: Bureau of Public Roads, "Highway Statistics, 1958," p. 134-137, 1960, U. S. Government Printing Office, Washington, D. C.

1/ John R. Crosby, "Accident Experience on the New Jersey Turnpike," pp. 2 4, New Jersey Turnpike Authority, New Brunswick, N. J., 1958. (unpublished)

2.4. Analysis of Accident Rates for Different Kinds of Carriers

Accident rates per 10 million vehicle miles are shown in Table 2.7 for eight types of I.C.C. carriers classified by the kind of cargo carried or service rendered. The significance of rate differences among classes was evaluated in the analysis of variance of Table 2.7, which follows the following linear expression for the rate.

$$(2.11)\quad r_{ijk} = r_o + c_i + q_j + y_k + cq_{ij} + cy_{ik} + qy_{jk} + e_{ijk}$$

r_o = overall mean rate

c_i = mean difference for carrier class i = 1, 2, ..., 8.

q_j = mean difference for quarter j = 1, 2, 3, 4.

y_k = mean difference for year k = 1, 2.

cq_{ij} = mean interaction of carrier i with quarter j.

cy_{ik} = mean interaction of carrier i with year k.

qy_{jk} = mean interaction of quarter j with year k.

e_{ijk} = measurement error, assumed approximately normal with mean zero and variance σ^2 common to all observations.

The analysis indicates that carrier differences are as highly significant as the quarter differences which do not interact, i.e. the carriers respond to seasonal factors uniformly. There also appears to be a consistent difference in rates for the two years.

Since the accident experience of carriers of radioactive materials is included with that of carriers of explosives and other dangerous articles, the accident rate for that class is of particular interest, as well as its differences from the rates of other classes. Under the assumption that there is no significant difference between the rate for a certain class and that of the explosives class, the mean rate $\bar{r}$

Table 2.7. Accident Rates for Various Types of Interstate Carriers

Class of Carrier Service or Cargo (Accidents per ten million miles)	1958 Quarters 1st	2nd	3rd	4th	1959 Quarters 1st	2nd	3rd	4th	Mean Rate
1. Explosives and Dangerous Articles	60	37	37	30	37	21	39	23	35.50
2. Petroleum Products	40	25	28	34	45	29	33	34	33.50
3. General Freight Carriers	44	33	33	36	45	32	32	34	36.13
4. Heavy Machinery and Large Units	48	32	28	51	32	39	32	32	36.75
5. Motor Vehicle Transportation	44	36	39	41	48	32	36	35	38.88
6. Other Carriers not Specified	46	37	38	44	49	39	39	45	42.13
7. Refrigerated Products	57	45	40	43	55	39	39	42	45.00
8. Household Goods	60	54	54	54	60	51	54	51	55.00

Analysis of Variance

Source of Variation	Sum of Squares	d.f.	Mean Square	Ratio	$F_{0.05}$
Carriers	2742.609	7	391.801	19.23	2.18
Years	92.641	1	92.641	4.55	4.02
Quarters	1400.922	3	466.974	22.92	2.78
Interaction					
Carrier-Year	283.734	7	40.533	2.00	2.18
Carrier-Quarter	532.703	21	25.367	1.25	1.76
Year-Quarter	70.297	3	23.433	1.15	2.78
Residual	427.828	21	20.373		
Extension of Carrier Comparison (see text)					
Classes 1-5, vs. 6-8	1890.009	1	1890.009	92.77	4.02
Among Classes 1-5	121.850	4	30.613	1.50	2.54
Among Classes 6-8	730.750	2	365.375	17.93	3.17

for that class is expected to lie within the following interval 90% of the time.[1/]

(2.12) $\bar{r} = \bar{r}_e \pm t_{0.05}(2s_{\bar{r}}^2)^{1/2}$

$$= 35.50 \pm 1.687(2\cdot 50.145/8)^{1/2} = 35.50 \pm 5.97$$

Here, $\bar{r}_e = 35.50$ is the mean accident rate for the explosives class from Table 2.7; $s_{\bar{r}}^2 = s_r^2/8 = 50.145/8$ where s_r^2 is the estimate of within-class variance of the rate assumed common to all classes; and $t_{0.05}$ is based on the 56 degrees of freedom for the estimate s_r^2. Thus, the mean rate should range from 29.5 to 41.5, which is not the case for carrier classes 6, 7, and 8 of Table 2.7. The resulting argument that carrier classes 1 through 5 have a common accident rate, which differs from that of the remaining classes, was further examined by extending the analysis of variance of Table 2.7. The individual comparisons between the two groupings and among the classes of each group supports the argument.

The common accident rate for classes 1-5 differs from the overall accident rate of all I.C.C. carriers by an amount d_i in each quarter i, which can be expressed as follows.

(2.13) $d_i = \bar{r}_i' - \bar{r} \qquad i = 1, 2, 3, 4$

$$= \bar{r}_i' - (5\bar{r}_i' + 3\bar{r}_i'')/8 = 3(\bar{r}_i' - \bar{r}_i'')8$$

Here, $\bar{r}_i$ denotes the overall mean for quarter i as a weighted average of $\bar{r}_i'$, the mean rate for the first 5 classes, and $\bar{r}_i''$, the mean rate for the remaining 3 classes. Since $\bar{r}_i'$ and $\bar{r}_i''$ are independent, the variance of d_i is given by

1/ A. J. Duncan, op. cit., p. 474.

$$(2.14)\ \mathrm{var}(d_i) = (3/8)^2\ \mathrm{var}(\bar{r}_i' - \bar{r}_i'')$$
$$= (3/8)^2(1/10 + 1/6)\ \mathrm{var}(r)$$

where r is assumed to have a common variance in all classes and quarters and $\bar{r}_i'$ is based on 10 observations (5 classes in 2 years) and r_i'' is based on 6 observations in Table 2.7. The variance of d_i is also independent of quarters. The estimated values are

$$(2.15)\ \bar{d}_1 = -3.95 \qquad s_{d_i}^2 = (9/64)(4/15)s_r^2$$
$$\bar{d}_2 = -4.71 \qquad = (3/80)50.145$$
$$\bar{d}_3 = -4.46 \qquad = 1.8804$$
$$\bar{d}_4 = -4.31 \qquad s_{d_i} = 1.37$$

Here s_r^2 is the estimated within-class variance having 56 degrees of freedom. The studentized range for the four d_i is given by $2(4.56)s_{d_i}$ = 5.43 while the actual range is relatively very small. Therefore, the four d_i can be considered as samples of the same d and independent of quarters, with estimates: $\bar{d} = -4.21$ and $s_{\bar{d}}^2 = s_{d_i}^2/4 = 0.34$. Carrier classes 1-5 have approximately 4 less accidents per 10 million vehicle miles than the mean rate of all I.C.C. carriers.

The Poisson frequency distribution is often used to describe the pattern of accident observations,[1/] where the number of accidents x occurring in the duration of m vehicle miles, with a rate r, has the following frequency.

$$(2.16)\ f(x) = e^{-rm}(rm)^x/x! \qquad x = 0, 1, 2, \ldots$$

Both the mean or expected number of accidents $E(x)$ and the variance are equal to rm, i.e. directly proportional to the mileage. When rm

1/ W. Feller, "An Introduction to Probability Theory and Its Applications," p. 147, John Wiley & Sons, New York, 1957.

is sufficiently large, 25 or more,[1/] the Poisson variate is approximately normal in distribution. The proportionality property suggests the applicability of a linear regression of accidents on miles, as shown in Figures 2.5, 2.6, 2.7 and 2.8 for carrier classes 1 through 5 with the data of Table 2.8 plotted separately for each quarter.

The regression model for accidents x_{ij} on millions of vehicle miles m_{ij} for carrier j in quarter i is given by

(2.17) $x_{ij} = a_i + b_i m_{ij}$ $\quad i = 1, 2, 3, 4; \; j = 1, 2, 3, 4, 5.$

Estimates of the regression values for each quarter are shown at the bottom of Table 2.8. The agreement of the data with the linear model is very good over an extremely large range of mileage exposures, as reflected in the correlation measures $R^2_{x/m}$ being very close to 1. Of particular interest are the constant terms a_i, since they should all equal zero under the Poisson assumption. All of the a_i estimates are well within the confidence limits for the hypothesis. On the other hand, the b values are significantly different from zero, falling well outside the confidence limits for such a hypothesis. These values are the regression estimates for the quarterly accident rates per million vehicle miles.

The differences between the four quarterly regressions are evaluated by the analysis of covariance in Table 2.9. At first the differences in slope are tested by determining the significance of the mean sum of squares associated with separate slopes as compared with a single common slope; and secondly, the significance of the separate

1/ D. A. S. Fraser, "Statistics, An Introduction," p. 125, John Wiley & Sons, New York, 1958.

mean regression levels is evaluated. The F test ratio uses the within-quarter variance as the denominator.[1/] Both F values are very significant which justifies the use of separate regressions.

1/ G. W. Snedecor, "Statistical Methods," p. 401, 1956, The Iowa State College Press, Ames, Iowa.

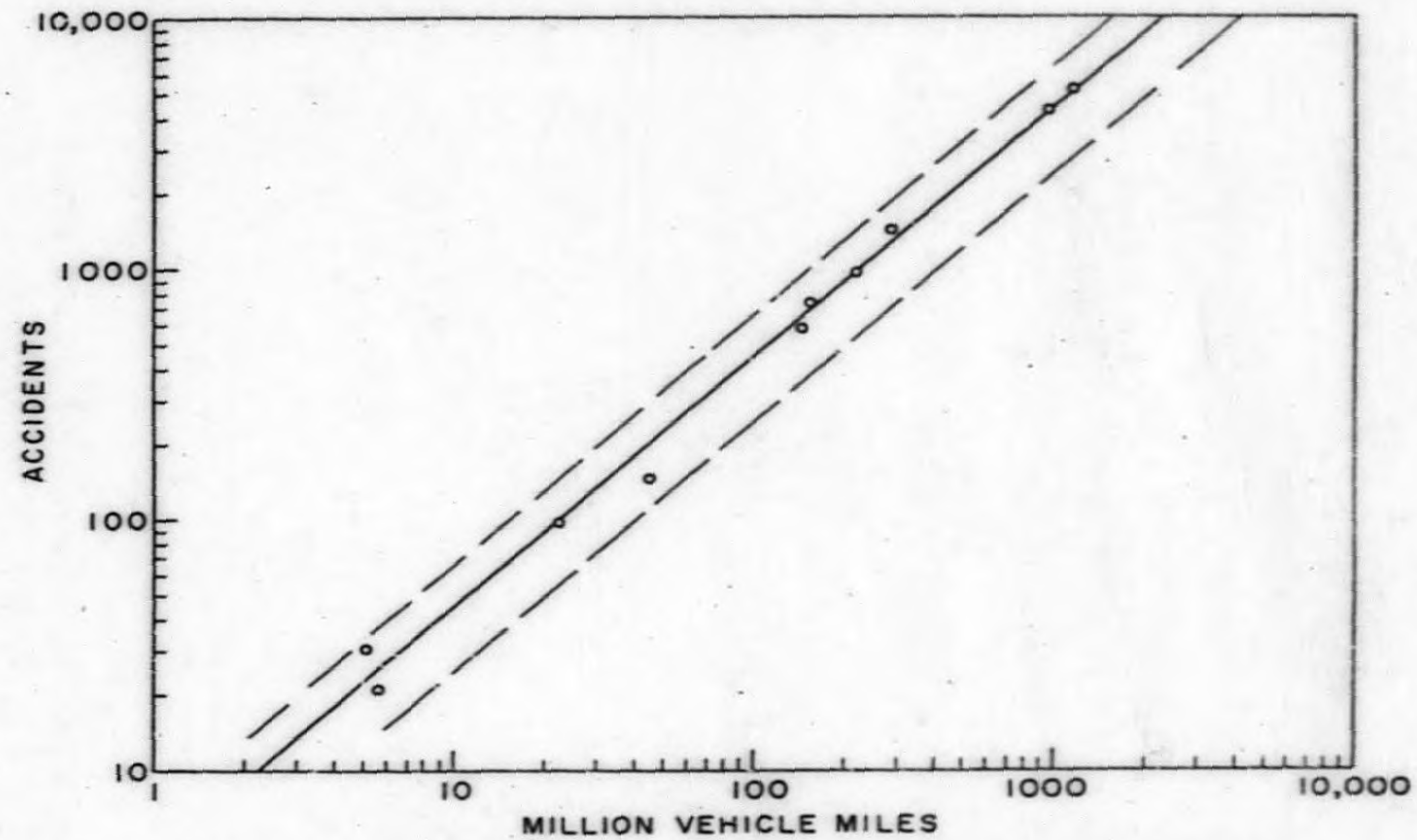

FIG. 2.5. REGRESSION OF ACCIDENTS ON VEHICLE MILES FOR FIVE CLASSES OF ICC CARRIERS IN THE FIRST QUARTERS OF 1958 AND 1959

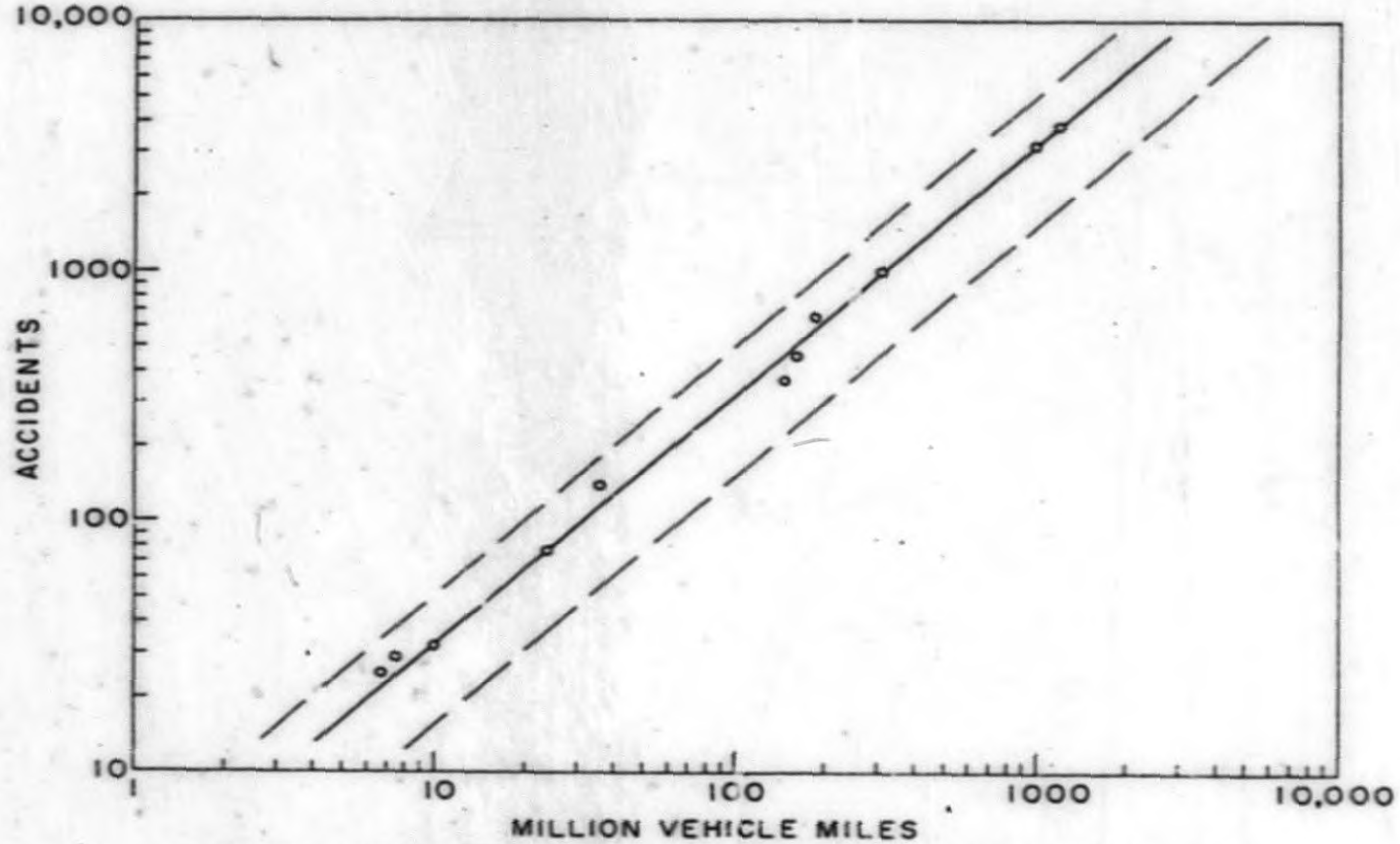

FIG. 2.6 REGRESSION OF ACCIDENTS ON VEHICLE MILES FOR FIVE CLASSES OF ICC CARRIERS IN THE SECOND QUARTERS OF 1958 AND 1959

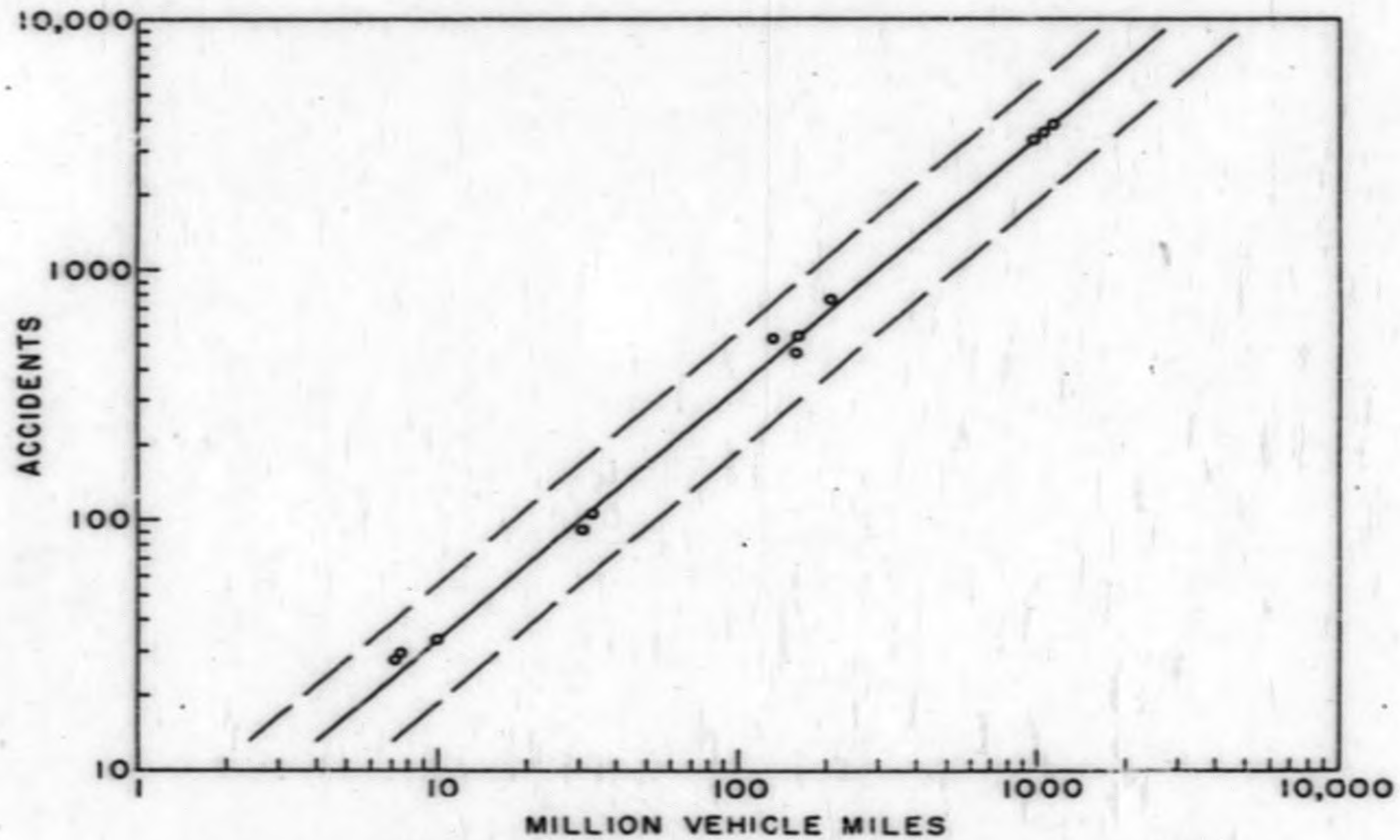

FIG. 2.7. REGRESSION OF ACCIDENTS ON VEHICLE MILES FOR FIVE CLASSES OF ICC CARRIERS IN THE THIRD QUARTERS OF 1958 AND 1959

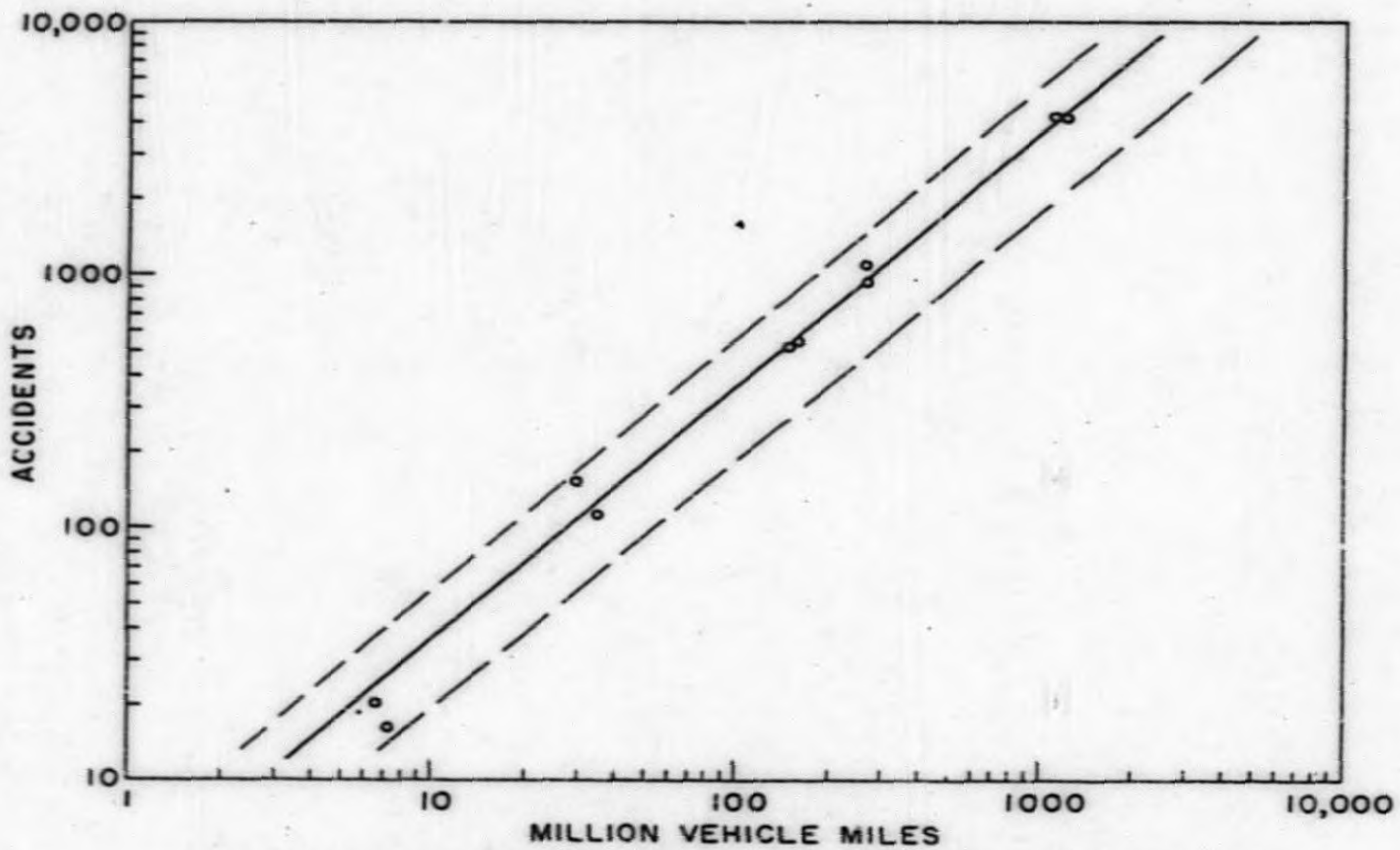

FIG. 2.8. REGRESSION OF ACCIDENTS ON VEHICLE MILES FOR FIVE CLASSES OF ICC CARRIERS IN THE FOURTH QUARTERS OF 1958 AND 1959

Table 2.8. Accidents and Mileage (in millions) for Five I.C.C. Carrier Classes by Quarter for 1958 and 1959

Class of Carrier	Yr.	1st Quarter Acc.	1st Quarter MVM	2nd Quarter Acc.	2nd Quarter MVM	3rd Quarter Acc.	3rd Quarter MVM	4th Quarter Acc.	4th Quarter MVM	Year Total Acc.	Year Total MVM
Explosives	'8	31	5.2	25	6.8	27	7.3	20	6.7	103	26.0
et al.	'9	21	5.7	15	7.3	29	7.5	16	7.1	81	27.6
Petroleum	'8	582	146.5	372	147.4	463	163.7	550	161.0	1967	618.6
Products	'9	703	154.9	459	159.2	542	165.0	533	154.8	2237	633.9
General	'8	4319	989.6	3323	1014.6	3514	1055.5	4273	1179.4	15429	4239.1
Freight	'9	5075	1139.3	3910	1215.9	3835	1181.9	4185	1217.3	17005	4754.4
Large	'8	98	20.3	77	24.1	90	31.7	153	30.0	418	106.1
Units	'9	144	45.4	140	36.0	107	33.6	112	35.0	503	150.0
Motor	'8	984	222.5	674	186.5	528	134.7	1083	264.5	3269	808.2
Vehicles	'9	1423	294.2	1004	310.2	746	207.3	928	267.4	4101	1079.1
Total		13380	3023.6	9999	3108.0	9881	2988.0	11853	3323.2	45113	12442.7

Regression Estimates	1st Q	2nd Q	3rd Q	4th Q
Estimate a	-3.0865	-0.9109	9.001	15.014
Limits (a' = 0)	±41.55	±35.19	±37.61	±56.07
Estimate b	4.4308	3.2465	3.2768	3.5216
Limits (b' = 0)	±0.106	±0.085	±0.090	±0.125
$R^2_{x/m}$	0.999	0.999	0.999	0.998
$s^2_{x/m}$	3,245.676	2,329.436	2,661.168	5,699.442
$s_{x/m}$	56.97	48.26	51.59	75.50

Table 2.9. Analysis of Covariance for the Regression of Accidents on Miles for the Data of Table 2.8

Source of Variation	Sum of Squares	d.f.	Mean Square	Ratio	$F_{0.05}$
Individual Quarters					
Within Quarter 1	25,965.410	8	3,245.676		
" " 2	18,635.489	8	2,329.436		
" " 3	21,289.343	8	2,661.168		
" " 4	45,595.535	8	5,699.442		
Total within Quarters	111,485.777	32	3,483.931	Residual	
Differences in Slope Coefficients	1,481,853.108	3	493,951.036	141.780	2.90
Regression with Common Slope	1,593,338.885	35	45,523.968		
Differences in Means Adjusted for Slope	834,072.594	3	278,024.198	79.802	2.90
Regression with Common Mean and Slope	2,427,411.479	38	63,879.250		

2.5. Maximum Likelihood Estimates of Accident Rates for Carriers of Radioactive Materials.

In the preceding analysis the accident rate was estimated by both a mean accident rate and by the slope of the regression line of accidents on miles. Although both methods provide unbiased estimates, a more desirable estimate can be obtained from the method of maximum likelihood. In particular, the latter estimate is normally distributed for large samples with minimum variance. The likelihood expression for accident observations x_i, following the Poisson distribution with a common rate r' for the observed mileages m_i, is given by

(2.17)
$$L = \prod_{i=1}^{n} e^{-r'm_i} (r'm_i)^{x_i}/x_i! \; .$$

which is maximized with the estimator

(2.18) $r = \sum x_i / \sum m_i$ when $d \log L/dr' = 0$.

For large samples, the estimator r is normally distributed with variance

(2.19) $\text{var } r = r/\sum m_i$

Maximum likelihood estimates of the accident rates for all large I.C.C. carriers are shown in Table 2.10 for each of the quarter-region classifications, which were found to be significant in the previous analysis. Because the data represent the accumulated experience of all types of carriers, an adjustment must be made to get the rate for typical carriers of radioactive materials. In the analysis of variance for carriers, it was shown that this adjustment differential is of the same magnitude for all quarters. It will now be necessary to assume it to be of the same magnitude for all region-quarters, for want of evidence to the contrary.

Using maximum likelihood estimates for the data of Table 2.2, the difference between the rate estimates for the typical carriers r_1 and that of all carriers r in all quarters and regions is given by

$$(2.20)\quad d = r_1 - r = x_1/m_1 - x/m$$
$$= 36.2566 - 38.2562 = -1.9996$$

Expanding the ratio x/m, d becomes

$$(2.21)\quad d = x_1/m_1 - (x_1 + x_2)/(m_1 + m_2)$$
$$= x_1/m_1 - (m_1/m)(x_1/m) - (m_2/m)(x_2/m_2)$$
$$= (1 - m_1/m)(x_1/m_1) - (m_2/m)(x_2/m_2)$$
$$= (m_2/m)(r_1 - r_2) = p_2(r_1 - r_2)$$

where $p_2 = m_2/m = m_2/(m_1 + m_2) = 1 - p_1$

The variance of d as estimated in (2.20) is given by

$$(2.22)\quad \text{var } d = p_2^2(\text{var } r_1 + \text{var } r_2)$$
$$= (0.25)^2(0.0291 + 0.1073)$$
$$= 0.0085$$

In order to obtain an unbiased estimate of the accident rate for typical carriers in a particular region-quarter, several assumptions are required. When the estimated accident rate for all carriers in a particular region-quarter r'' (denoted by the double prime) is adjusted by the factor d from a different set of data, the expected value is given by

$$(2.23)\quad E\,(r'' + d) = E\left(\frac{x_1'' + y_2''}{m_1'' + m_2''}\right) + p_2E(r_1 - r_2)$$
$$= p_1''E(r_1'') + p_2E(r_2'') + p_2E(r_1 - r_2)$$
$$= p_1''E(r_1'') + p_2E(r_1) + p_2''E(r_2'') - p_2E(r_2)$$

In order for this to yield an unbiased estimate of $E(r_1'')$, two assumptions are required: (a) $p_1 = p_1''$, i.e. the mileage distribution between the typical and non-typical carriers must be the same in both sets of data; and (b) $E(d) = E(d'')$ or $E(r_1 - r_2) = E(r_1'' - r_2'')$ as was indicated from the analysis of variance. Then (2.23) reduces to

$$(2.24)\ E(r'' + d) = (p_1 + p_2)E(r_1'') + (p_2 - p_2)E(r_2)$$
$$= E(r_1'')$$

as desired, with variance.

$$(2.25)\ var(r'' + d) = var(r'') + var(d)$$

where r" and d are independent estimates. However, with the data available, d is actually estimated from the sum total of observations for all of the eight region-quarter classifications used to estimate each r". Therefore (2.25) tends to overestimate the variance. The adjusted estimates of the accident rates for carriers of radioactive materials are shown in Table 2.10.

Table 2.10. Maximum Likelihood Estimates of Accident Rates for Carriers of Radioactive Materials

Observation		Number of	Millions of	Rate per	Estimate	Adjusted for Carriers of RA Materials		
Quarter	Region	Accidents	Vehicle Miles	10 MVM	Variance	Rate	Variance	Standard Deviation
1	East	12,922	2,361.8	54.7125	0.2317	52.7119	0.2402	0.490
	West	5,231	1,633.2	32.0291	0.1961	30.0285	0.2046	0.452
	Both	18,153	3,995.0	45.4393	0.1137	43.4387	0.1222	0.350
2	East	9,820	2,471.1	39.7394	0.1608	37.7388	0.1693	0.412
	West	4,197	1,641.3	25.5712	0.1558	23.5706	0.1643	0.405
	Both	14,017	4,112.4	34.0847	0.0829	32.0841	0.0914	0.302
3	East	10,104	2,452.5	41.1988	0.1680	39.1982	0.1765	0.420
	West	4,458	1,637.5	27.2244	0.1663	25.2238	0.1748	0.417
	Both	14,562	4,090.0	35.6039	0.0871	33.6033	0.0956	0.309
4	East	11,624	2,657.4	43.7420	0.1646	41.7418	0.1731	0.416
	West	5,030	1,714.0	29.3466	0.1712	27.3460	0.1797	0.424
	Both	16,654	4,371.4	38.0976	0.0872	36.0970	0.0957	0.309
All	East	44,470	9,942.8	44.7258	0.0450	42.7252	0.0535	0.231
	West	18,916	6,626.0	28.5481	0.0431	26.5475	0.0516	0.227
	Both	63,386	16,568.8	38.2562	0.0231	36.2556	0.0316	0.178
Carrier Classes								
1-5	1-5	45,113	12,442.7	36.2566	0.0291			
	Other	18,273	4,126.1	44.2864	0.1073			
	1-5 Bias			-1.9997	0.0085			

2.6. Accident Occurrence by Time of Day and Day of Week.

In its report of truck accident data for the fourth quarter of 1959, I.C.C. provided a detailed classification of the accidents by the time of day and the day of the week when they occurred. These are plotted in Figure 2.9 and summarized in Table 2.11 with the 24 hours of the day divided into four six hour periods. In the same table an analysis of variance was made to determine the significance of the observed differences. There is a highly significant difference between daytime and nightime, but no significant variation within these periods. There is also a very pronounced difference between weekends and the weekdays of Monday through Friday, with no apparent differences among or within these latter five days. There is a difference between Saturday and Sunday of the weekend.

Unfortunately, it was not possible to obtain a corresponding classification of the vehicle miles of operation by the I.C.C. carriers. This would have made it possible to determine and compare the accident rates for hours of the day and days of the week. However, there is evidence from other sources which indicate such variations in rate do occur. In a recent study by the Bureau of Public Roads[1/] it was found that the accident rate for large trucks (6 tons or more) was considerably less at night than during the day on representative sections of main rural highways in the United States. In the data collected for that study, the nighttime vehicle mileage reported for truck combinations was slightly greater than the daytime mileage.

1/ L. L. Strauss, "The Federal Role in Highway Safety," House Document 93, 86th Congress, Washington, D. C., 1959.

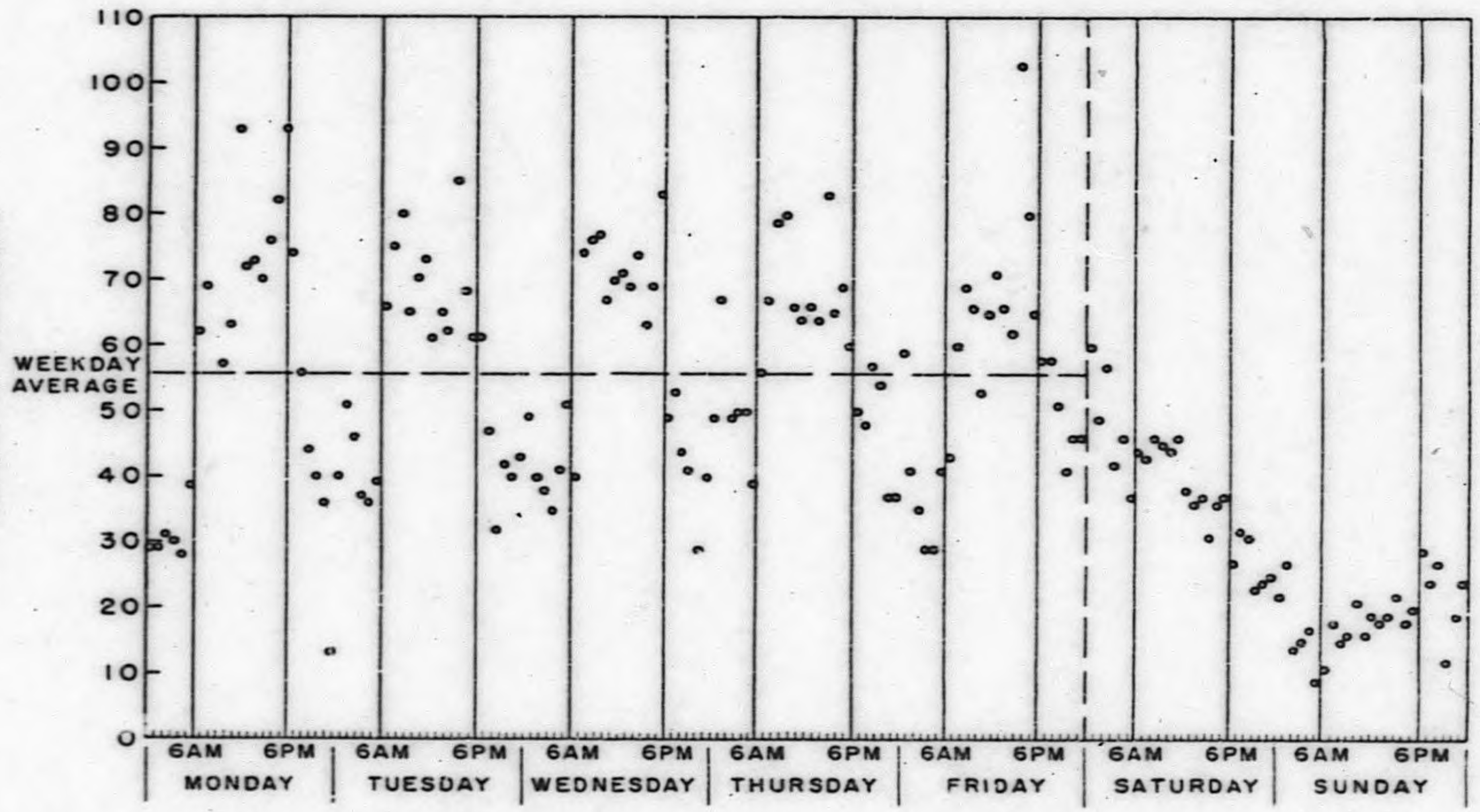

FIG. 2.9. THE NUMBER OF ACCIDENTS REPORTED BY LARGE ICC CARRIERS IN THE FOURTH QUARTER 1959 BY DAY OF WEEK AND HOUR OF DAY

Table 2.11. Number of Accidents Reported by Time of Day and Day of Week.

Accidents Reported by: Weekday	6 a.m. to 12 Noon	12 Noon to 6 p.m.	6 p.m. to 12 Midnight	12 Midnight to 6 a.m.	Total
Monday	415	467	254	186	1322
Tuesday	429	402	265	247	1343
Wednesday	404	429	256	254	1343
Thursday	412	407	283	304	1406
Friday	356	447	300	234	1337
Subtotal	2016	2152	1358	1225	6751
Saturday	268	215	162	291	936
Sunday	97	116	135	104	452
Total	2381	2483	1655	1620	8139

Source of Variation	Sum of Squares	d.f.	Mean Square	Variance Ratio	Critical Value
Time of Day					
Day vs. Night	90,175.7500	1	90,175.75	25.92	4.41
Within Day	743.1429	1	743.14	0.21	4.41
Within Night	87.5000	1	87.50	0.03	4.41
Subtotal	91,006.3929	3	30,335.46	8.72	3.16
Day of Week					
Weekday vs. Weekend	153,785.1571	1	153,785.16	44.20	4.41
Within Weekdays	1,154.8125	4	288.70	0.08	2.93
Within Weekend	29,282.0000	1	29,282.00	8.42	4.41
Subtotal	184,221.9696	6	30,703.66	8.82	2.66
Residual	62,631.9018	18	3,479.55		

Assuming no change in the mileage for night and day operations of trucks engaged in intercity commerce, the ratio of the accident rate for these two periods is equal to the ratio of the number of accidents which occur, i.e.

$$(2.26)\quad \frac{r_1}{r_2} = \frac{x_1}{m_1} \cdot \frac{m_2}{x_2} = \frac{m_2}{m_1}\frac{x_1}{x_2}$$

where the subscripts 1 and 2 denote day and night for accident rates r, accidents x, and miles, m. With m_2/m_1 equal to one, the accident rate ratio for the data of Table 2.11 is

$$(2.27)\quad \frac{r_2}{r_1} = \frac{x_2}{x_1} = \frac{3275}{4864} = 0.67.$$

With a mean accident rate of 3.626 accidents per MVM and the relationship:

$$(2.28)\quad 2\bar{r} = 72.52 = r_1 + r_2$$

these equations can be solved simultaneously to give the following value for r_1 and r_2:

$$(2.29)\quad r_1 = 4.343 \text{ acc.}/10^6 \text{ miles by day}$$

$$r_2 = 2.901 \text{ acc.}/10^6 \text{ miles at night,}$$

or a mean difference in accident rate of $\pm$ 0.721 from the mean rate. This difference in accident rate is quite similar to that observed for highways in different regions of the United States. In both comparisons the same underlying road factor can be cited, i.e. reduced traffic density. This conclusion is consistent with those found in other studies of highway accidents. In studies by Versace 1/ and Woo 2/ , average

1/ J. Versace, "Factor Analysis of Roadway and Accident Data," Highway Research Board Bulletin 240, Washington, D. C., 1960.

2/ J. C. H. Woo, "Correlation of Accident Rates and Roadway Factors," Joint Highway Research Project, Purdue University, 1957.

daily traffic was found to be the variable most highly related to accident occurrence. In an extensive study of interstate highway accidents, Roff reports: "In most cases the average daily traffic has a considerable effect on the accident rate on tangent highway sections. The common pattern is for the accident rate to increase as the volume increases."[1]

1/ M. S. Raff, "The Interstate Highway Accident Study," Public Roads, 27, 170-186, 1953.

3.0. Impact Characteristics of Truck Accidents

3.1. Introduction

Accidents can be divided into collision and non-collision types with the latter category consisting principally of overturns on and off the roadway. The collision accidents can be further divided into those with motor vehicles and those with fixed objects, railroad trains, animals, pedestrians, etc. Collisions with motor vehicles are the most frequent type and these can be classified according to the kind of vehicle struck and the manner of collision. From the standpoint of the severity of motor vehicle accidents the more important characteristics are the weight and structure of the opposing vehicles, their speed and direction at the time of impact, and the occurrence of such subsequent events as fires and secondary impacts.

Data for a large number of accidents involving large commercial trucks were obtained from the U. S. Bureau of Public Roads and the Interstate Commerce Commission. These data were used to develop estimates of the relative frequencies for the different types of accidents and how these frequencies change for different types of highways. For motor vehicle collisions, the type of vehicle struck was found to be statistically independent of both the direction of impact and the highway characteristics. However, the direction of impact is influenced by the highway, in that headon collisions are virtually eliminated on divided highways.

The vehicle struck was found to be closely related to the composition of the traffic stream on the highway. The weight characteristics of the various types of vehicles were studied, and the distributions of weight were estimated. The occurrence of fire was found to have a similar frequency in the various types of collision accidents, with approximately 1% of such accidents resulting in fires. However, in overturn accidents, fires occur with twice that frequency.

Estimates of the speed of the vehicles involved in various types of accidents were approximated with compound distributions having both a bell-shaped high speed component and a rectangular low speed component, which occurs when the vehicle is unable to maintain a normal highway speed pattern. The relative proportions of the two components differ for the various accident types. Unfortunately, no analytic studies of accident speeds were found in the literature, but the results of studies of vehicle speeds under various highway conditions appear to be in substantial agreement with these results.

The velocity of impact for motor vehicle collisions was taken to be the vector sum of the speeds of the colliding vehicles, e.g. the sum and difference in head-on and rear-end accidents respectively. Since the two speeds appear to be distributed independently, their joint distribution was defined as their product, and the distributions of the sum and difference in speed were evaluated from the compound marginal densities. An analytic evaluation was used for the bivariate normal and bivariate rectangular components of resulting distributions, but the normal-rectangular components were evaluated numerically in 5 mph intervals. Rather than use the vector sums in angle accidents, the speed of the opposing vehicle striking the critical vehicle was taken to be the impact velocity.

Those collisions in which the impact is received at the same point on the critical vehicle were combined into a single impact category with a single velocity distribution. Thus, rear-end accidents in which the critical vehicle strikes the rear or side of another vehicle were combined with the front-end impacts received in head-on collisions. Estimates of both the frequency of their occurrence and the distribution of velocity in the various impact categories are shown in Table 3.1. These estimates are based on the experience of large trucks operating in intercity travel throughout the United States. For particular types of highways the estimates may be different.

Table 3.1. Summary of the Estimated Distribution of Net Impact Velocity in the Types of Tractor Semitrailer Accidents.

Accident: Point of Impact:	Automobile Collisions				Truck Collisions				Overturns and Other Collisions
	Front	Rear	Side	Total	Front	Rear	Side	Total	
Relative Frequency:	0.316	0.204	0.043	0.563	0.094	0.052	0.011	0.157	0.090 & 0.190
Percentage of Accidents Exceeding the Stated Velocity in Each Collision Type									
Velocity									
10 mph	.850	.722	.888	.807	.786	.639	.885	.744	.980
20	.688	.460	.739	.610	.621	.376	.730	.548	.952
30	.574	.275	.587	.467	.517	.228	.575	.426	.924
40	.486	.121	.416	.348	.421	.103	.385	.313	.795
50	.400	.019	.195	.246	.319	.016	.085	.202	.234
60	.333	.000	.060	.192	.267	.000	.002	.160	.006
70	.287		.011	.162	.231		.000	.138	.000
80	.241		.001	.135	.184			.110	
90	.173		.000	.097	.095			.057	
100	.088			.049	.019			.011	
110	.027			.015	.001			.001	
120	.005			.003	.000			.000	

3.2. The Classification and Frequency of Collision and Non-Collision Accidents.

Motor vehicle accidents are usually classified according to the first event of the accident[1/] which is included in one of three groups: collision with another motor vehicle, collision with an object other than a motor vehicle, and non-collision such as overturning and running off the road. In Table 3.2, the number of accidents in each group is given for data from two sources. The I.C.C. data represent all accidents for large carriers of property for the fourth quarter of 1959.[2/] The B.P.R. data represent the accident involvements for tractor trailer type vehicles over several years on 35 representative highway sections in 11 states.[3/] The relative frequencies of the accident types were compared by means of a chi square test, and the result, shown in Table 3.2, indicates that there are no significant differences among the two sets of accident data.

In Table 3.2, the accidents reported in the B.P.R. study are further classified according to the kind of highway on which they occurred, a distinction being made between two and four lane highways, with a further distinction between the two lane roads having an average daily traffic (ADT) of more or less than 5000. The chi square comparisons indicate considerable differences exist in the relative frequency of non-collision and collision type accidents for these roads. The proportion of non-collision accidents tends to decrease

1/ Department of Health, Education, and Welfare, "Uniform Definitions of Motor Vehicle Accidents," Washington, D. C., 1956.

2/ Interstate Commerce Commission," Motor Carriers of Property, Accident Data for Fourth Quarter, 1959," September 16, 1960, Washington, D. C.

3/ "The Federal Role in Highway Safety," 86th Congress, 1st Session, House Document No. 93, U.S. Gov't Printing Office, pp. 71-84. 1959

Table 3.2. Collision and Non-Collision Accidents for Tractor Trailer Type Vehicles from Various Sources

Source or Location of Accidents	Collision Accidents: Motor Vehicle	Other	Total	Non-Collision Accidents	All Accidents
B.P.R. Highway Study, 1956-59					
2 lanes under 5000 vehs/day	133	30	163	60	223
2 lanes over 5000 vehs/day	210	42	252	45	297
4 lanes, divided roads	215	53	268	14	282
Total for B.P.R. Study	558	125	683	119	802
I.C.C. Carriers, 4th Quarter, 1959, Intercity Travel	5897	1141	7038	1151	8189

Comparison of Accident Types	Chi Square	d.f.	5% Critical Value
Between I.C.C. and B.P.R. Total	2.342	2	5.99
Among Highways in B.P.R. Study			
Between motor vehicle and other collisions	0.766	2	5.99
Between collision and non-collision	47.634	2	5.99

as traffic density and/or highway control decreases. However, no differences were found between the two types of collision, i.e. motor vehicle vs. other objects, for the different roads.

A more detailed analysis of motor vehicle collisions was made of the two-vehicle accidents reported in the B.P.R. study, in which the vehicle struck and the direction of impact were identified. The data are shown in Table 3.3 for each of the three types of highways. Chi square tests were used to compare the marginal frequencies of vehicle struck and direction of impact for the road types, i.e. road interaction with each characteristic. In addition the interaction of vehicle struck with direction was tested within each road type.

The results indicate that the vehicle struck is independent of both the road type and the direction of impact.[1/] However, the direction of impact is dependent on whether the highway is a two or four lane (divided) road; in particular, head-on collisions are rarely experienced on divided highways. The traffic density does not appear to have a significant influence on the types of accidents occurring on the two lane roads.

The more detailed classification of accidents reported by I.C.C. carriers is shown in Table 3.4 for both the fourth quarter of 1959 and the first quarter of 1960. (More recent data were not available at this time.) The two quarters are not exactly comparable, since the I.C.C. made a rule change in the beginning of 1960, which relieved carriers

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1/ A comparison of the vehicle struck in the I.C.C. 4th Quarter 1959 data with the B.P.R. data showed that there were no significant differences.

Table 3.3. Vehicle Struck and Direction of Impact in Two Vehicle Collisions of the B.P.R. Study

Direction of Impact	2 Lanes, under 5000 ADT			2 Lanes, over 5000 ADT			4 Lane Divided Highways		
	Auto	Truck	Total	Auto	Truck	Total	Auto	Truck	Total
Rearend (or Sideswipe in the Same Direction)	47	15	62	72	26	98	115	47	162
Head-on (or Sidwswipe in the Opposite Direction)	29	4	33	49	9	58	3	1	4
Angle Collisions	14	3	17	18	2	20	28	3	31
Total Two Vehicle Collisions	90	22	112	139	37	176	146	51	197

Tests for Interaction	Chi Square	d.f.	5% Critical Value
Vehicle Struck vs. Road Type	2.022	2	5.99
Direction of Impact vs. Road Type			
Between the 2 Lane Roads	1.045	2	5.99
Between 2 and 4 Lane Roads	67.497	2	5.99
Vehicle Struck vs. Direction of Impact			
For 2 Lanes under 5000 ADT	4.314	2	5.99
For 2 Lanes over 5000 ADT	2.348	2	5.99
For 4 Lane, Divided Roads	5.069	2	5.99
Total for All Roads	11.731	6	12.60

from reporting those accidents which result in property damage only between \$100 and \$250. The chi square tests, shown in Table 3.4, indicate that significant differences exist in the accident proportions both among the three major groupings and within the motor vehicle collision group. Attributing these differences to the procedural change would indicate that the largest proportion of the minor damage accidents omitted are with automobiles. The tests also show that there are no significant differences in the proportions within the other collision and non-collision groups.

Approximately 60% of the non-collision accidents reported by the I.C.C. occur when the truck "ran off roadway." In order to obtain some estimate of the secondary, damage-producing events in such accidents, additional data were obtained from the National Safety Council's directional analyses of motor vehicle accidents which appeared in the Council's annual reports for the years 1955-58.[1/] Vehicles leaving the roadway, subsequently overturned 52.8%, 62.5%, 59.9% and 59.1% of the time in the respective years. The average of 58.6% overturns appears to be a representative estimate. The remaining 41.4% of the off-roadway accidents terminated in collisions with fixed objects.

The above results were used to estimate the frequency of accidents by manner of accident and direction of impact for tractor trailer type vehicles operating on various types of intercity highways. For want of evidence to the contrary unidentified accidents in any one group

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1/ National Safety Council, "Accident Facts," 1956-1959 editions, Chicago, Illinois. These data apparently represent both auto and truck accidents. Similar data for large trucks only were not available.

Table 3.4. Types of Accidents Reported to the I.C.C. by Large Motor Carriers of Property

Accident Types by First Event	4th Quarter 1959 Accidents	4th Quarter 1959 Pct.	1st Quarter 1960 Accidents	1st Quarter 1960 Pct.
1. Motor Vehicle Collisions				
Passenger Auto	4564	0.5573	3604	0.495
Property-Carrier	1215	.1484	1206	.165
Motor Bus	51	.0062	38	.005
Other Motor Vehicle	67	.0082	78	.011
Sub-total	5897	0.7201	4926	0.676
2. Other Type Collisions				
Pedestrian, Animal, etc.	146	.0178	109	.015
Railroad, Streetcar	66	.0081	78	.011
Fixed Object	819	.1000	663	.091
Other Object	110	.0134	72	.010
Sub-total	1141	0.1393	922	0.127
3. Non-Collision Accidents				
Overturn on Road	149	.0182	166	.023
Ran off Road	681	.0832	898	.123
Other Non-Collision	321	.0392	373	.051
Sub-total	1151	0.1406	1437	0.197
Total	8189	1.000	7285	1.000

Comparison of Quarters	Chi Square	d.f.	Critical Value
Among the 3 Group Sub-totals	89.47	2	5.99
Within Each Group			
Within motor vehicle collisions	28.71	3	7.82
Within other collisions	7.58	3	7.82
With non-collision group	3.07	2	5.99

were distributed proportionally among the identified accidents of the group. These estimates are developed in Table 3.5.

Table 3.5. Estimated Frequency of Accidents by Manner of Collision and Direction of Impact

Accident Types		Two Lane Roads Under 5000 ADT	Two Lane Roads Over 5000 ADT	Two Lane Roads All ADT	Four Lanes (Divided)	All Roads (Intercity)
Motor Vehicle Collisions						
Head-on -	Auto	0.154	0.197	0.179	0.012	0.110
	Truck	0.021	0.036	0.030	0.004	0.031
	Total	0.175	0.233	0.209	0.016	0.141
Rearend -	Auto	0.250	0.290	0.273	0.464	0.374
	Truck	0.080	0.104	0.094	0.142	0.104
	Total	0.330	0.394	0.367	0.606	0.478
Angle -	Auto	0.075	0.072	0.073	0.113	0.079
	Truck	0.016	0.008	0.011	0.027	0.022
	Total	0.091	0.080	0.084	0.140	0.101
Total Auto Collisions		0.479	0.559	0.525	0.589	0.563
Total Truck Collisions		0.117	0.148	0.135	0.173	0.157
Total Vehicle Collisions		0.596	0.707	0.650	0.762	0.720
Other Type Collisions		0.232	0.196	0.211	0.206	0.190
Overturn		0.172	0.097	0.129	0.032	0.090
Total All Accidents		1.000	1.000	1.000	1.000	1.000

3.3. Further Analysis of Motor Vehicle Accidents by Type and Weight of Vehicle Struck and the Occurrence of Fire.

A further analysis was made of the type and weight of the vehicles struck by tractor trailers in motor vehicle collisions. The percentage distribution of travel by vehicle types on main rural roads of the United States in the Summer of 1955 is shown in Table 3.6 from a study by Dimmick.[1/] The estimated percentage of automobile travel, 0.785, compares favorably with the estimated frequency of automobile accidents, 0.783 from Table 3.4, i.e. 4564 (automobile) out of 5830 identified I.C.C. motor vehicle collisions. The remaining breakdown of commercial vehicle travel provides an estimate of the types of vehicles and their frequency of occurrence in motor vehicle collisions.

In the study cited, Dimmick reports on the weight characteristics of approximately 135,000 trucks observed at weighing stations in 44 states. These findings are shown in Table 3.7. In another study[2/], Samson reports on the registration weight of trucks throughout the United States. The distribution of registered weight by vehicle type is shown in Table 3.8. Also shown in this table is a composite weight distribution for "large" trucks, i.e. trucks with two axles and six tires or more than two axles. The frequencies for the included vehicles were weighted according to their percentage of travel (Table 3.6), and summed.

The weight distributions are plotted in Figure 3.1 with logarithmic-probability coordinates. The straight-line approximation to the plotted data correspond to lognormal distribution functions.

1/ T. B. Dimmick, "Traffic and Travel Trends, 1955," Public Roads, Vol. 29, No. 5, December 1956.

2/ E. Samson, "State Highway User Taxes," Public Roads, Vol. 29, No. 12, February 1958.

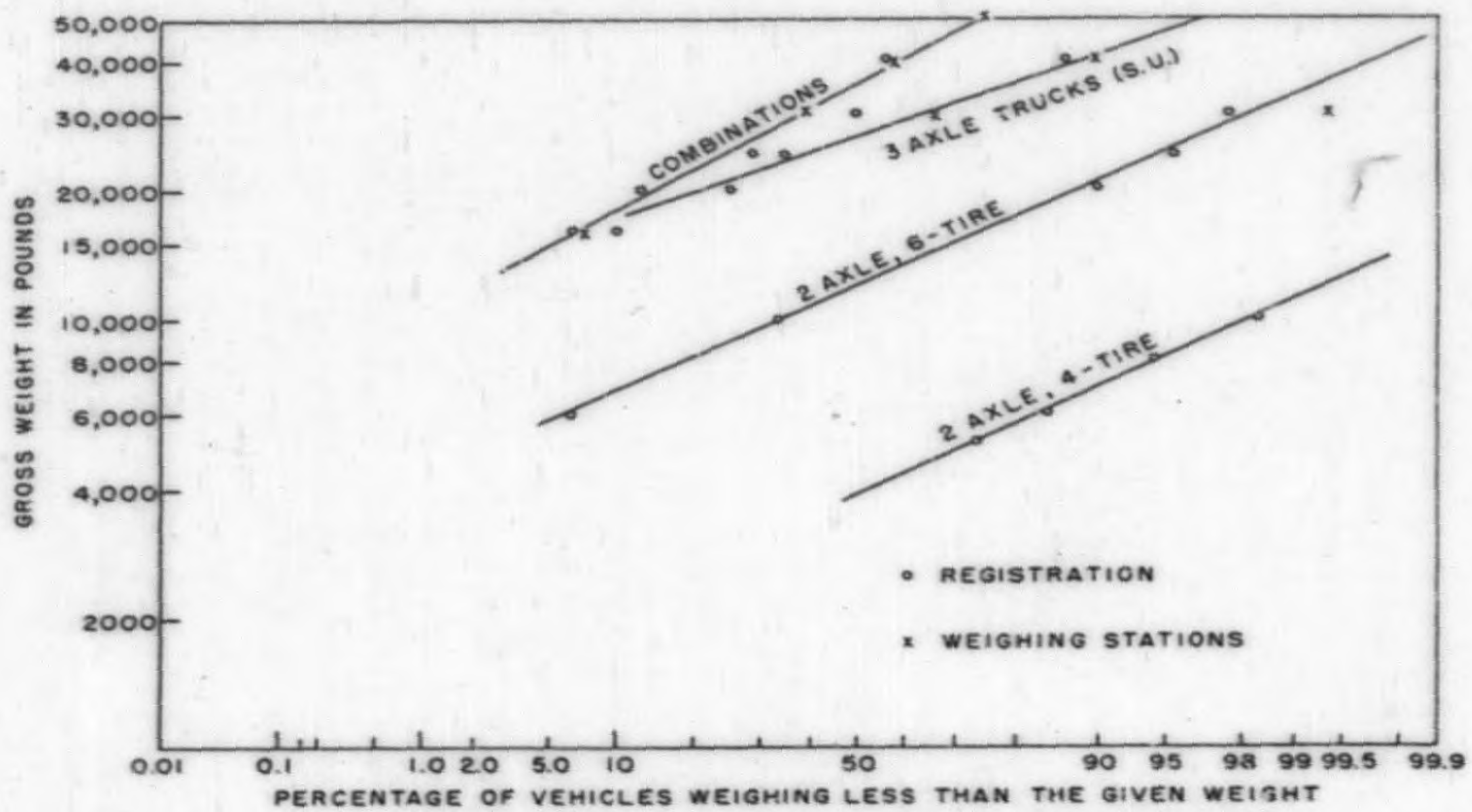

FIG. 3.1. THE DISTRIBUTION OF GROSS VEHICLE WEIGHT BASED ON TRUCK REGISTRATIONS AND OBSERVATIONS AT TRUCK WEIGHING STATIONS

The lognormal parameters were estimated by evaluating two standardized normal deviates, z_1 and z_2, which correspond to the observed cumulative frequency for weights w_1 and w_2 at each end of the weight range. The relationships,

$$(3.1)\quad z_i = (\log w_i - m_{\log})/\sigma_{\log} \qquad i = 1, 2$$

can be solved simultaneously for estimators of the mean $m_{\log}$ and standard deviation $\sigma_{\log}$, namely:

$$(3.2)\quad m_{\log} = \log w_i - z_i \sigma_{\log} \qquad i = 1, 2$$

$$\sigma_{\log} = (\log w_2 - \log w_1)/(z_2 - z_1)$$

With values for $m_{\log}$ and $\sigma_{\log}$, estimates can be made for the parameters of the skewed, weight distribution from the following relationships:[1/]

$$(3.3)\quad \text{Mean } (w) = \exp\ (m_{\log} + 1/2\sigma^2_{\log})$$

$$\text{Median } (w) = \exp(m_{\log})$$

$$\text{Mode } (w) = \exp(m_{\log} - \sigma^2_{\log})$$

$$\text{Variance } (w) = \exp(2m_{\log} + 2\sigma^2_{\log}) - \text{Mean}^2\ (w)$$

In Table 3.8 the estimated parameters are shown for the various weight distributions. It is interesting to note that the mean registration weight is less than the average weight for the loaded trucks in Table 3.7.

Among recent changes in the reporting of accidents by the I.C.C. was the identification of the number of accidents in which fire occurs. In Table 3.9, the percentage of accidents resulting in fire is shown for the major types of accidents in the fourth quarter of 1959, and the first and second quarters of 1960. A comparison of these percentages was made with likelihood ratio tests for samples from a Poisson dis-

1/ J. Aitcheson and J. A. C. Brown, "The Lognormal Distribution," Cambridge University Press, 1957.

Table 3.6. Percentage Distribution of Travel on U. S. Rural Roads, 1955.

Vehicle Type	Eastern U. S.	Central	Mountain	Pacific	U. S. Average
Passenger Cars	.793	.771	.758	.839	.785
Buses	.007	.006	.006	.009	.007
2-axle, 4 tire	.083	.079	.115	.077	.083
2-axle, 6 tire	.055	.059	.061	.019	.053
3-axle, s u	.007	.004	.006	.006	.005
Combination	.055	.081	.054	.050	.067
Total	1.000	1.000	1.000	1.000	1.000

Table 3.7. Observed Truck Weights on U. S. Rural Roads, 1955

Observed Weight Characteristics	Single Unit Trucks 2 axle 4-tire	2 axle 6-tire	3 axle all	Truck Combinations	All Truck Types
Percentage Less than:					
15 tons	1.000	0.994	0.688	0.391	0.795
20 tons		1.000	0.898	0.581	0.872
25 tons			0.990	0.751	0.925
Mean Weight (tons)					
Loaded Trucks	2.71	7.34	15.24	22.9	12.17
Empty Trucks	2.09	4.29	7.38	10.9	4.71
All Trucks	2.32	6.15	12.10	18.54	8.80
Percentage					
Loaded	0.375	0.612	0.601	0.682	0.682
Empty	0.625	0.388	0.389	0.318	0.318

Table 3.8. Cumulative Distribution of Registered Truck Gross Weights.

Registered Gross Weight	Single Unit Trucks 2-axle, 4-tire	2-axle, 6-tire	3-axle	Vehicle Combinations	Heavy Trucks (Composite)
3 tons or less	0.845	0.064			0.027
3-4	0.944	0.179			0.076
4-5	0.984	0.339			0.144
5-6	1.000	0.506			0.215
6-8	(Panel vehicles	0.754	0.101	0.065	0.358
8-10	not included	0.900	0.250	0.127	0.460
10-12	with heavy	0.954	0.351	0.289	0.573
12-15	trucks)	0.977	0.500	0.382	0.639
15-20		1.000	0.867	0.558	0.758
% Travel (Heavy Trucks)		(0.424)	(0.040)	(0.536)	(1.000)

Estimated Lognormal Parameters for Registered Truck Weights

Lognormal Parameters	Single Unit Trucks 2-axle, 4-tire	2-axle, 6-tire	3-axle	Truck Combination	Heavy Vehicles
Mean (log w)	0.278	0.783	1.114	1.266	1.062
Var. (log w)	0.039	0.039	0.028	0.057	0.269
Mean (w) tons	1.98	7.99	13.45	19.72	17.7
Median (w) tons	1.90	6.07	12.99	18.45	11.5
Mode (w) tons	1.74	5.55	12.17	16.17	6.2
Var. (w)	0.37	5.94	12.26	54.95	21.24

Table 3.9. The Occurrence of Fire in Major Types of Accidents

Accident Type	4th Quarter, 1959			1st Quarter, 1960			2nd Quarter, 1960		
	Acc.	Fires	Pct.	Acc.	Fires	Pct.	Acc.	Fires	Pct.
Collision with									
Auto	4564	42	.0092	3604	35	.0097	2526	18	.0071
Truck	1215	15	.0123	1206	19	.0158	787	16	.0203
Fixed Object	819	9	.0110	663	6	.0090	482	4	.0083
Collision Total	6598	66	.0100	5473	60	.0110	3795	38	.0100
Non-collision	1151	75	.0652	1437	98	.0682	883	98	.1110
Accident Total	7749	141	.0182	6910	158	.0229	4678	136	.0291

Comparisons	Chi Square	d.f.	Critical Value (5% Risk)
Among Collision Accidents for: 4th Quarter, 1959	0.921	2	5.991
1st Quarter, 1960	2.549	2	5.991
2nd Quarter, 1960	8.836	2	5.991
Total	12.306	6	12.592
Among Quarters for Total Collisions Only	0.872	2	5.991
Between Collision and Non-Collision for: 4th Quarter, 1959	56.180	1	3.841
1st Quarter, 1960	62.953	1	3.841
2nd Quarter, 1960	90.612	1	3.841
	209.745	3	7.815
Among Quarters for Non-Collision Accidents Only	7.227	2	5.991

tribution, under the assumptions that the small probability of fire permits a Poisson approximation and that the likelihood ratio is distributed approximately as a chi square variate. The results shown in Table 3.9 indicated that there is no significant difference in the frequency of fires for collision accidents, but a considerable difference between non-collision and collision accidents.

The data for the second quarter of 1960 appear to differ from those of the other two quarters. However, this is probably due to the change in the I.C.C.'s reporting procedures which occurred at the beginning of 1960. At that time, carriers were relieved of the responsibility of reporting accidents resulting in property damages only between $100 and $250. The reduction in the number of accidents reported under the new rule is apparent in Table 3.9. That there is a greater reduction in the second quarter than in the first quarter may indicate a time lag in this change over, although a seasonal reduction in accidents is to be expected for this period.

For the purpose of the present study, accident frequencies based on the $100 limit have been used, and therefore the probability of fire in such accidents is best estimated from the data of the fourth quarter of 1959, in which 1% of the collision accidents and 6.5% of the non-collision accidents resulted in fires. In addition to overturns, the latter class of accidents also includes such incidents as tire and cargo fires without prior collision or overturn.

In the fourth quarter of 1959, the I.C.C. reported 3 fires for 149 overturning accidents on the roadway, or a frequency of 0.020. In the first two quarters of 1960, the 8 fires in 285 overturns raised the frequency to 0.0281, which may be due to the change in reporting procedures. An estimate of 0.02 for fires in overturns appears to be reasonable for accidents of $100 damage or more. This is twice the fire rate experienced in collision accidents.

3.4. Estimated Vehicle Speeds in Highway Accidents.

The estimated speed of vehicles involved in accidents was included in the study of vehicle characteristics made by the Bureau of Public Roads.[1/] Provision was made for both the prior travel speed and the impact speed, but in most cases the impact speed was unknown or reported to be the same as the travel speed. The reported travel speeds in various accidents are shown in Table 3.10 for 705 tractor trailer type vehicles and 390 automobiles involved in tractor trailer accidents, as well as the speed for all automobile involvements. The resulting speed distributions appear to follow a bell-shaped pattern in the upper speed range; but because of the large and unequal intervals used, no clearly discernable pattern is apparent for the vehicles with accident speeds less than 32 mph. There appear to be considerable differences in the proportions of slow speed vehicles for the different types of accidents.

In analyzing the accident speed distributions of Table 3.10, the observations in the upper range were treated as a truncated sample from a normal population. Maximum likelihood estimates of the mean and variance for each accident type were obtained, using the methods of A. C. Cohen;[2/] and these were compared by use of the maximum likelihood ratio. The ratio L_0/L_1, where L_0 is the sample likelihood with common

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1/ "Although reasonably accurate estimates of vehicle speed just prior to an accident can be made by experienced investigators, it was recognized that not all accidents used in the study were investigated and that involved drivers, especially those at fault, often underestimate their speed." Secretary of Commerce, "Federal Role in Highway Safety," p. 73, House Document 93, 86th Congress, U. S. Government Printing Office, Washington, D. C.

2/ Cohen, A. C., Jr., "Estimating the mean and variance of normal, populations from singly truncated and doubly truncated samples," *Ann. Math. Statist.*, Vol. 21 (1950), pp. 557-69.

mean and variance and L_1 is the likelihood with different means and variances, for truncated samples from a normal population, is given by:

(3.4)

$$\frac{L_0}{L_1} = \frac{(f_o s_o)^{-N} \exp. \sum_{i=1}^{k} \sum_{j=1}^{n_i} -(x_{ij} - m_o)^2/2s_o^2}{\prod_{i=1}^{k} (f_i s_i)^{-n_i} \exp. \sum_{i=1}^{k} \sum_{j=1}^{n_i} -(x_{ij} - m_i)^2/2s_i^2}$$

This yields the test statistic:

(3.5)

$$-2 \log L_0/L_1 = N(2 \log f_o s_o - b_o/s_i^2 + 1) - \sum_{i=1}^{k} n_i(2 \log f_i s_i - b_i/s_i^2 + 1)$$

which is approximately a chi square variate with 2K - 2 degrees of freedom. Here, the following notation is used:

x_{ij} = speed observation $j = 1, 2, \ldots, n_i$ for accident type i

x_t = point of truncation, and x_{ij} is greater than x_t

$N = n_1 + n_2 + \ldots + n_k$ = total number of observations

f_i = estimated probability that x_i exceeds x_t

m_i = estimate of mean for accident class i

s_i^2 = estimate of variance

$b_i = (\bar{x}_i - x_t)^2 \theta_i(1 - \theta_i), \theta_i = (\bar{x}_i - m_i)/(\bar{x}_i - x_t)$

f_o, m_o, s_o, b_o = estimates for common population

Estimates of the mean and variance of the higher speeds, when assumed normal, are shown in Table 3.11 for the sample data truncated at various points. The hypothesis that the distributions are the same in all truck accident samples was tested with the likelihood ratio test, and the value $-2 \log L_1/L_0$ was found to be well within the 5%

critical value for a chi square variate with 10 degrees of freedom, which supports the hypothesis.

In searching for a suitable distributional form to include all speeds greater than zero, the estimated normal component was subtracted from the sample, and the remaining distribution suggested a simple rectangular pattern. Therefore it was postulated that the data had the following compound form:

(3.6) $f(x) = p/b + (1 - p)g(x; m, \sigma^2)$

$E(x) = pb/2 + (1 - p)m$

where p is the proportion having a rectangular distribution in the range 0 to b mph, and (1 - p) is the proportion in the normal component with mean m and variance σ^2 estimated as (44.06, 30.785) and (50.33, 101.59) for trucks and autos respectively.

Using the "method of moments" to estimate b, the sample means were set equal to the expected values in equation (3.6) with the estimates for m and σ^2 from the truncated samples and the observed values for p. In solving for b, it was found to be very close to m in every sample; i.e. the upper limit of the rectangular component is the mean of the normal component. Thus equation (3.6) reduces to:

(3.7) $f(x) = p/m + (1 - p)g(x; m, \sigma^2)$

$E(x) = pm/2 + (1 - p)m = m - pm/2$

Again using the estimates for m and σ^2 from the truncated data, the proportion p can now be estimated by the method of moments, setting the expectation in (3.7) equal to the sample mean, which yields

(3.8) $p = 2(m - \bar{x})/m$

The resulting estimates are shown in Table 3.11.

The estimated frequencies for each accident type were computed and are shown in Table 3.10. These were compared with the reported frequencies by use of the chi square test for goodness of fit. Since the parameters m and σ were estimated in common for trucks and autos, the comparison should be made on the basis of all accidents for each vehicle type rather than for each accident type independently. The total chi square for each vehicle type is within the 5% critical value. The largest deviations of observed from expected frequencies occurred in the angle accidents at low speeds and particularly with automobiles, where the unusually large number of vehicle speeds less than 23 mph made it necessary to omit this interval from the estimation and comparison of the theoretical distribution.

Considerable effort was expended in trying to improve and verify the form and dimensions of the speed distributions estimated above. The speed data used are admittedly questionable since, usually, the only observers are those involved and this is likely to bias the data.[1/] However, the size of the samples is quite large and the error effects may be self-cancelling in part. Unfortunately, it was not possible to obtain additional data. Furthermore, a search of the literature failed to uncover any analytic studies of the frequency distributions of accident speeds. Some empirical data on the distribution of vehicle travel speeds have been published and were compared with the results obtained above.

1/ It is of interest to note that the I.C.C. relieved carriers from reporting speeds on their accident forms, because of the questionableness of this data.

In a summary report of 835 speed studies in 28 states during 1958[1/], the measured speed frequencies were given in 5 mph intervals for autos and trucks. These measurements were made at highway locations and times such that most drivers could travel at their desired speeds. Thus, virtually no vehicles were observed traveling below 30 mph, and the observed speeds follow an approximately normal pattern similar to the fast speed component developed in the analyses above. The measured speed distributions and the estimated fast speed components for trucks and autos are plotted in Figure 3.2 on normal probability paper. It is interesting to observe a mean bias of approximately 5 mph between the observed and estimated distributions. This could be due to observer bias in accidents and/or the effect of differences in traffic and highway conditions for the measured speed locations.

In a study by Schwender[2/] speed characteristics were obtained for an instrumented tractor-trailer operating on the West Virginia Turnpike and on alternate routes which are characterized by "bad alignment, steep grades, inadequate sight distance, and roadside development." Average speeds were 40.6 mph for the turnpike and 31 mph for alternate routes, as compared with the mean accident speed of 34.8 mph including standing vehicles and 37.1 mph excluding them for the data of Table 3.10. Of particular interest is the distribution of trip time for the ten gear ratios of the truck tractor, which are shown in Figure 3.3. The

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1/ Bureau of Public Roads, "Traffic Speed Trends," Department of Commerce, March 1959, Washington, D. C.

2/ H. C. Schwender, "Vehicle Operating Characteristics on the West Virginia Turnpike and Alternate Routes," Vol. 36, p. 539, Highway Research Board Proceedings, 1957, Washington, D. C.

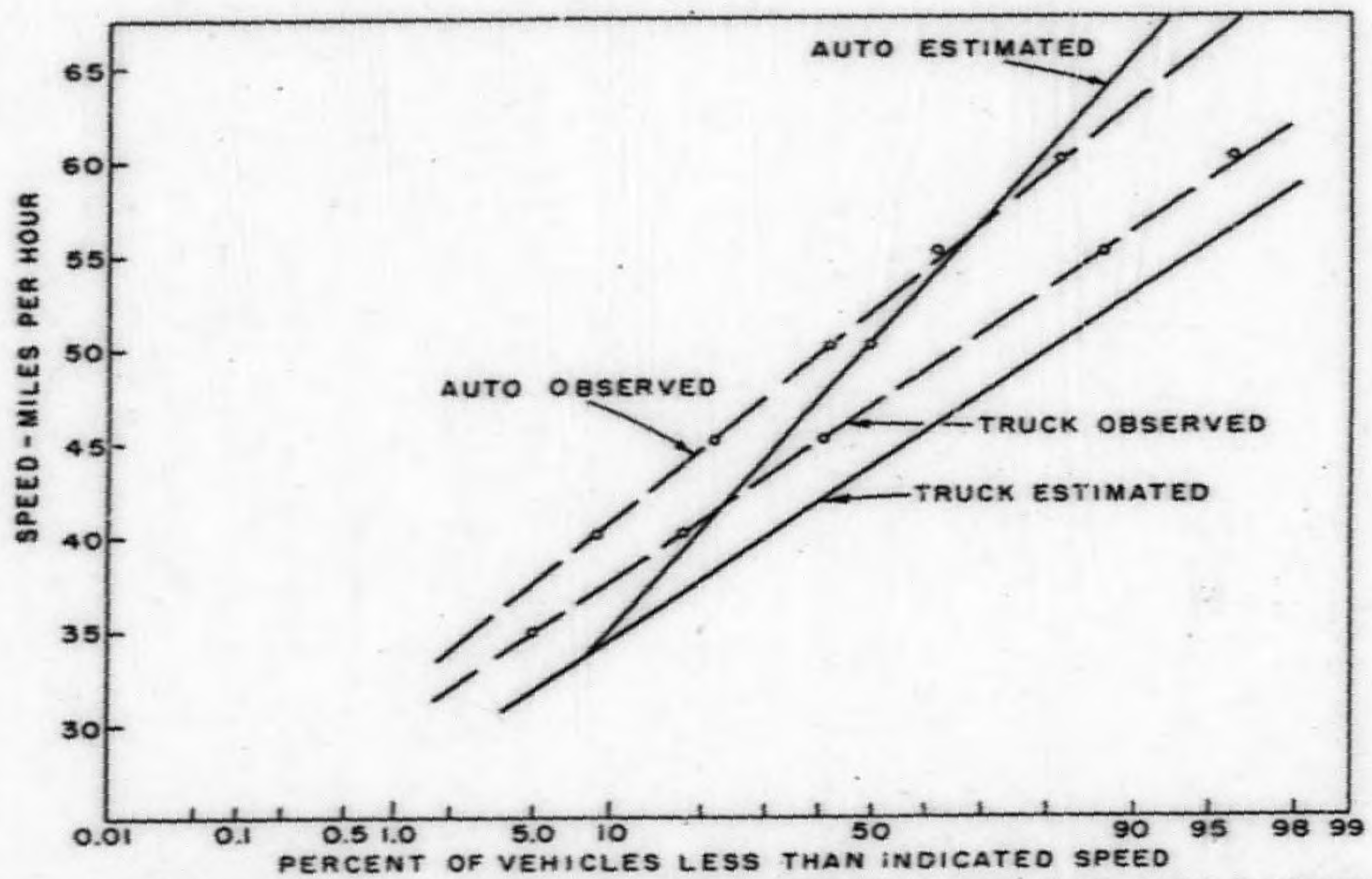

FIG. 3.2. COMPARISON OF OBSERVED SPEED MEASUREMENTS FOR FREE-FLOWING TRAFFIC AND ESTIMATED ACCIDENT SPEED FOR SIMILAR VEHICLES

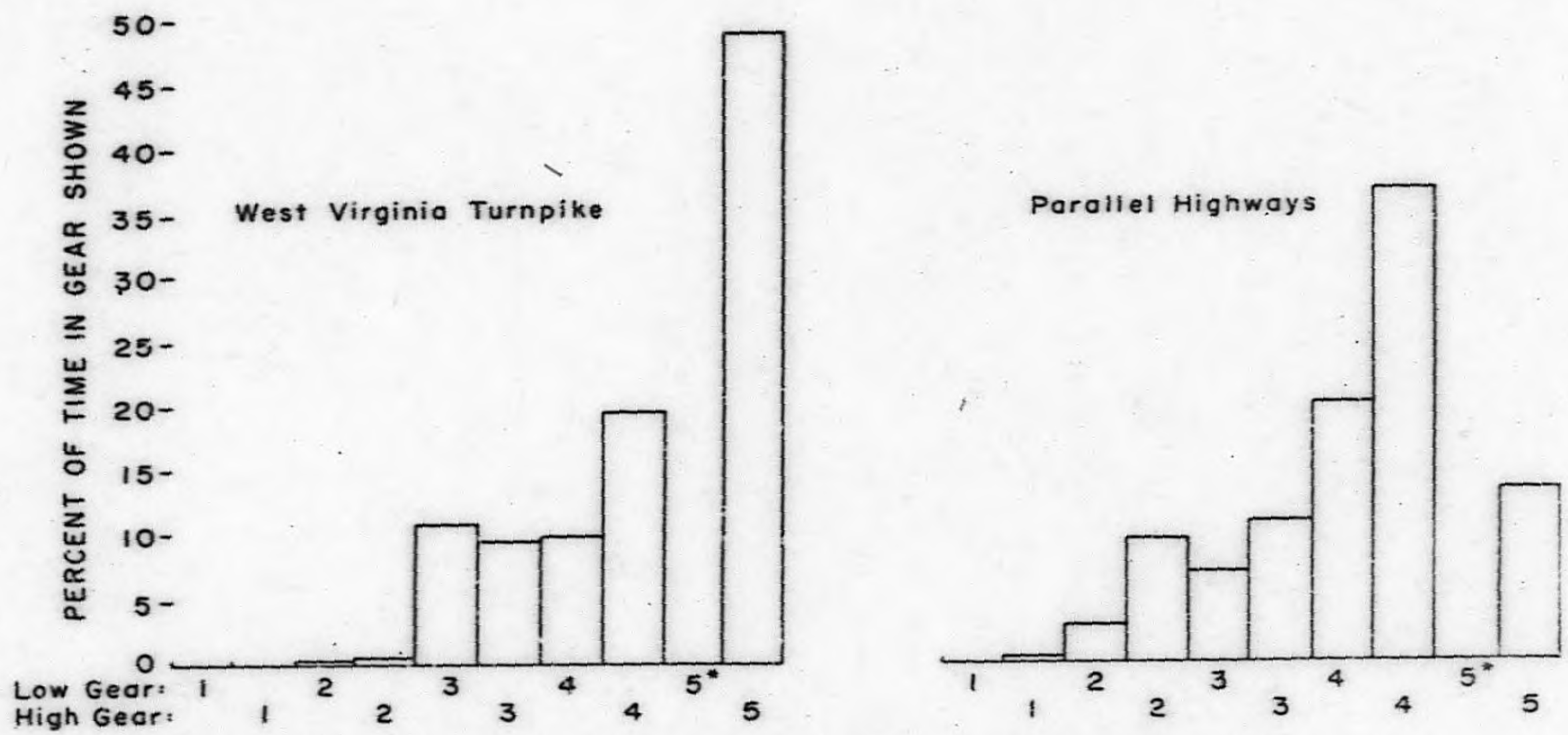

*Plate On Truck To The Effect That The 5th Gear Low Range Is Not To Be Used.

FIG. 3.3. Percent of Trip Time Spent in Various Gears for a Tractor Semi-Trailer on the W. Va. Turnpike and Parallel Highways
(From: H. C. Schwender, "Vehicle Operating Characteristics on the West Virginia Turnpike and Alternate Routes," Proc. Highway Research Board. Vol. 36, 1957, p. 557)

distributions provide an explanation for the compound speed distribution observed in accidents, in that speed in each gear ratio may follow unimodal patterns about the mean speed for that ratio. Thus, the overall speed density is a linear combination of the densities in each ratio weighted with the relative frequencies of Figure 3.3. In the lower speed range this would generate a pattern approaching a rounded, rectangular form, while in the upper range the high proportion of travel in the low gear ratios produces a prominent bell shaped pattern.

In the Schwender study the observed speed distributions for the truck are not shown; however, the speed distributions for an automobile over 30 miles of turnpike and 38 miles of an alternate road are unimodal with a prominent low speed tail. Similar distributions for an instrumented automobile operating in city traffic are reported by Stonex,[1/] with a notable difference in a second mode in the lowest speed interval. However, since zero speed observations are included in this interval, they may be the cause of this phenomenon.

1/ K. A. Stonex, "Survey of Los Angeles Traffic Characteristics," Vol. 36, p. 509, Highway Research Board Proceedings, 1957, Washington, D. C.

Table 3.10a. Reported and Estimated Speeds of Tractor Trailers in Truck Accidents

Reported Vehicle Speed	Rearend Collision 2 Lane	Rearend Collision 4 Lane	Head-On Collision	Angle Collision	Other Collision	Non-Collision	Total
1-22 mph	36 (33.00)	25 (29.15)	12 (14.50)	21 (21.86)	7 (5.45)	7 (5.81)	108
23-32	14 (14.61)	21 (14.80)	7 (7.69)	8 (10.11)	4 (3.94)	3 (4.19)	57
33-37	17 (16.65)	23 (16.35)	10 (9.87)	5 (6.97)	8 (9.33)	10 (9.94)	73
38-42	26 (32.60)	26 (33.24)	27 (21.25)	7 (10.59)	18 (23.22)	24 (24.37)	128
43-47	35 (34.35)	34 (35.99)	29 (23.87)	16 (8.29)	28 (28.31)	30 (30.15)	172
48-52	22 (18.99)	22 (20.12)	10 (13.55)	5 (4.31)	24 (16.54)	20 (17.62)	103
53-57	7	2	0	2	2	3	16
58-62	1	3	0	0	1	0	5
63-72	0 (6.00)	0 (6.35)	0 (4.28)	0 (1.36)	0 (5.22)	1 (5.56)	1
72-over	—	—	—	—	—	—	—
Total	158	156	95	64	92	98	663
Chi Square	5.25	8.04	8.36	9.83	6.87	1.93	46.35 (28 d.f.)

3.10b. Reported and Estimated Speeds of Automobiles in Accidents with Tractor Trailers

Reported Vehicle Speed	Rearend Collision	Headon Collision	Angle Collision	Total	All Auto Accidents
1 22 mph	51 (47.54	8 (7.87)	27 (omit)	86	1129
23-32	28 (24.70)	4 (5.81)	2 (1.44)	34	396
33-37	21 (17.18)	3 (6.03)	1 (1.87)	25	413
38-42	22 (22.34)	11 (9.37)	2 (3.12)	35	662
43-47	23 (29.42)	15 (13.96)	5 (4.82)	43	844
48-52	24 (26.92)	15 (14.56)	5 (5.20)	44	1319
53-57	19 (19.83)	18 (12.19)	6 (4.53)	43	1049
58-62	13 (13.25)	10 (8.88)	2 (3.19)	25	658
63-72	8	2	2	12	457
72-over	3 (11.82)	0 (7.65)	2 (2.83)	5	135
Total	212	86	54	352	7062
Chi Square	3.33	9.65	2.45	15.43 (18 d.f.)	

Table 3.11. Estimated Parameters for Compound Speed Distributions

Accident Class and Vehicle	Standing Vehicles Obs.	Standing Vehicles Pct.	Moving Vehicles No. Vehs.	Moving Vehicles Mean Speed	Proportion Slow	Proportion Fast	Fast Speed Component Pt. of Trunc.	Fast Speed Component Mean	Fast Speed Component Std. Dev.
Tractor Trailers:									
Rearend, 2 Lane	26	.165	132	35.15	.409	.591	32	43.35	6.50
							37	44.49	5.74
Rearend, 4 Lane	12	.077	144	36.09	.366	.634	32	42.05	7.12
							37	43.93	6.03
Headon Collision	2	.021	93	37.53	.299	.701	32	42.38	4.62
							37	42.33	4.58
Angle Collision	2	.031	62	29.48	.669	.331	32	43.55	5.54
							37	44.80	4.55
Other Collision	0	0	92	41.53	.116	.884	32	44.62	5.51
							37	45.28	4.95
Non Collision	0	0	98	41.36	.124	.876	32	43.71	6.42
							37	46.65	6.83
Total	42	.063	621	37.13	.315	.685	32	43.26	6.11
							37	44.06	5.55
							42	44.79	5.20
Automobiles with Tractor Trailers:									
Rearend	34	.138	212	36.79	.500	.500	32	44.27	12.96
Headon	2	.023	86	45.30	.200	.800	32	49.63	7.85
Angle	2	.036	54	31.36	.754	.246	32	52.01	11.43
Total	38	.097	352	37.90	.277	.723	32	47.48	11.07
Automobiles in All Accidents:									
Total	546	.072	7062	44.88	.185	.815	32	50.33	10.08
							37	50.30	10.29
							42	50.35	10.24

3.5. Analysis of the Impact Velocities in Accidents.

The impact velocity in a collision is a function of the speeds of the colliding vehicles and their direction of impact. The sum of speeds for both vehicles in a headon collision and the difference in speeds for a rearend collision may be taken as equivalent to the speed of a moving vehicle striking a stationary one. The probability distribution of speed sums and differences is a function of the joint distribution of the two speeds. This is the product of the marginal speed distributions of the two vehicles when their speeds are independent.

In Table 3.12 the joint speed distribution is shown for the tractor trailer accidents reported in the Bureau of Public Roads Study and in which the speeds of both vehicles were given. A chi square test for independence was made by comparing the observed frequency in each cell with that given by the product of the two marginal frequencies for that cell. The resulting chi square measure is within the 5% critical value and supports the hypothesis that the speeds are independent.

The marginal density $f_i(x_i)$ of speed x for the ith vehicle is given by the compound distribution of equation (3.7), and the joint density is given by their product, i.e.

$$(3.9) \quad f(x_1, x_2) = f_1(x_1)f_2(x_2) = \left[p_1/m_1 + (1 - p_1)g_1\right]\left[p_2/m_2 + (1 - p_2)g_2\right]$$

$$= p_1p_2/m_1m_2 + p_1(1 - p_2)g_2/m_1 + p_2(1-p_1)g_1/m_2$$

$$+ (1 - p_1)(1 - p_2)g_1g_2$$

where g_i is the normal density function with mean m_i and variance σ_i^2, $(1 - p_i)$ is the proportion of speeds in the normal component g_i, and p_i is the proportion in the rectangular component with density $1/m_i$.

Table 3.12. Joint Speeds in Two Vehicle Accidents

Speed of Other Vehicle	1 - 22	23 - 37	38 - 42	43 - 47	Over 47 mph	Total
1-22 mph	7 (12.16)	18 (12.63)	7 (10.99)	18 (14.03)	8 (8.19)	58
22-37	8 (7.34)	6 (7.62)	10 (6.63)	4 (8.47)	7 (4.94)	35
38-47	12 (10.90)	11 (11.32)	8 (9.86)	10 (12.58)	11 (7.34)	52
48-57	15 (14.68)	14 (15.24)	13 (13.27)	22 (16.94)	6 (9.88)	70
Over 57	10 (6.92)	5 (7.19)	9 (6.25)	6 (7.98)	3 (4.66)	33
Total	52	54	47	60	35	248

Test for Independence	Chi Square	d.f.	5% Critical Value
Excluding accidents with standing vehicles (Expected values shown in parentheses)	22.68	16	26.3
Including standing vehicle accidents (Not shown in above table)	20.76	16	26.3

The joint density is a linear combination of four bivariate terms: a bivariate rectangular, a bivariate normal, and two bivariate normal-rectangular terms. In deriving the distribution of the sum or difference of the two speeds, the densities of the sum or difference in each term was evaluated individually and combined using the weighting factor for the terms. For the bivariate normal term, the sum or difference is normal with mean $m_1 \pm m_2$ and variance $\sigma_1^2 + \sigma_2^2$, and the weight is $(1 - p_1)(1 - p_2)$.

The distribution of the sum $S = x_1 + x_2$ of two rectangular variates is discontinuous over its range $(0, m_1 + m_2)$ at $S = 0, m_1, m_2$, and $m_1 + m_2$. When S is evaluated in the intervals $(0, m_1)$, (m_1, m_2), and $(m_2, m_1 + m_2)$, the following results are obtained:

(3.10) $F(S) = S^2/2m_1m_2,\ f(S) = S/m_1m_2,\ (0 < S < m_1 < m_2)$

$F(S) = (2S - m_1)/2m_2,\ f(S) = 1/m_2,\ (m_1 < S < m_2)$

$F(S) = 1 - (m_1 + m_2 - S)^2/2m_1m_2,\ f(S) = (m_1 + m_2 - S)/m_1m_2,$

$(m_2 < S < m_1 + m_2)$

In the event that $m_1 = m_2$, equations (3.10) describe a triangular distribution; and if they are not equal, the distribution is trapezoidal. The distribution of the difference, $D = x_2 - x_1$, has the same form as the sum with $F(D) = F(s - m_1)$.

The probability density functions for the sum and difference of a rectangularly and a normally distributed speed were evaluated numerically rather than analytically. Using the midpoint values for 5 mph intervals the frequency distributions were convoluted to give approximate interval frequencies for the sum and difference of the two marginal midpoint values. The frequencies for the same midpoint values in 5 mph intervals were evaluated for both the bivariate normal and the bivariate rectangular components; and the weighted sum of the frequencies was obtained in the manner of equation (3.9). In this way, the distributions of equivalent velocities were obtained for headon and rearend collisions between two moving tractor trailers and a tractor trailer and automobile. The estimated distributions are shown in Tables 3.13 and 3.14.

In rearend accidents the range of the distribution of the difference in vehicle speeds includes both negative and positive values, where negative differences occur when the vehicle of interest is struck from behind. This type of collision is expected to occur in 50% of the rearend accidents for two tractor trailers and in approximately 54% of the truck-auto rearend accidents. Furthermore, since a considerable number of rearend accidents occur between a moving vehicle and a standing tractor trailer, they should be included with this class of accidents. Of the 262 identified rearend involvements for tractor trailers in the BPR study, the distribution between moving and standing accidents is as follows:

Rearend Collision with:	Truck	Auto	Total	%
Tractor trailer standing	5	8	13	.0496
Other vehicle standing	9	16	25	.0954
Both vehicles moving	83	141	224	.8550
Accident total	97	165	262	1.0000

The chi square value is only 0.02 with 2 degrees of freedom for a test of difference in frequency between vehicle type and accident type.

Table 3.13. Estimated Distribution of the Sum of Speeds in Headon Collisions

Speed	Two Tractor Trailers					Tractor Trailer and Automobile					
mph	$r_t r_t$	$2r_t n_t$	$n_t n_t$	Total	Cum.	$r_t r_a$	$r_t n_a$	$n_a n_t$	$n_t n_a$	Total	Cum.
2.5	14	($\times 10^{-5}$)		14	14	8	($\times 10^{-5}$)			8	8
7.5	111			111	125	67				67	75
12.5	222			222	347	133				133	208
17.5	333			333	680	200				200	408
22.5	444			444	1124	267	1			268	676
27.5	556	1		557	1681	333	3	1		337	1013
32.5	667	25		692	2373	400	16	7		423	1436
37.5	778	231		1009	3382	467	58	69		594	2030
42.5	889	1071		1960	5342	533	171	321		1025	3055
47.5	972	2648		3620	8962	592	407	795		1794	4849
52.5	889	4013		4902	13864	592	805	1204	6	2607	7456
57.5	778	4558		5336	19200	592	1298	1367	34	3232	10688
62.5	667	4658	25	5350	24550	592	1809	1397	112	3785	14473
67.5	556	4666	172	5394	29944	533	2116	1400	386	4402	18875
72.5	444	4665	897	6006	35950	467	2469	1400	1047	5249	24124
77.5	333	4642	3126	8101	44051	400	2582	1400	2447	6696	30820
82.5	222	4436	7286	11944	55995	333	2589	1393	4390	8572	39392
87.5	111	3595	11432	15138	71133	267	2491	1331	6933	10888	50280
92.5	14	2018	12020	14052	85185	200	2259	1079	8971	12376	62656
97.5		654	8462	9116	94301	133	1862	606	9654	12130	74786
102.5		109	4003	4112	98413	67	1368	196	8579	10143	84929
107.5		9	1264	1273	99686	8	858	33	6345	7236	92165
112.5			274	274	99960		450	3	3892	4345	96510
117.5			34	34	99994		194		1966	2160	98670
122.5			1	1	99995		68		840	908	99578
127.5							20		286	306	99884
132.5							4		90	94	99978
137.5							1		17	18	99996
Total	.09	.42	.49	1.00		.06	.24	.14	.56	1.00	

Table 3.14. Estimated Distribution of the Difference in Speeds for Rearend Collisions

Speed* mph	Two Tractor Trailers r_tr_t	r_tn_t	n_tr_t	n_tn_t	Total	Cum.	Tractor Trailer and Automobile r_rr_a	r_tn_a	n_ar_t	n_an_t	Total	Cum.
-87.5	($\times 10^{-5}$)							1	($\times 10^{-5}$)		1	1
-82.5								4			4	5
-77.5								16			16	21
-72.5								57			57	78
-67.5								162			162	240
-62.5								375			375	615
-57.5		5			5	5		715			715	1330
-52.5		62			62	67		1140			1140	2470
-47.5		374			374	441	28	1552		3	1583	4053
-42.5	25	1153			1178	1619	222	1882		12	2116	6169
-37.5	198	2054			2252	3871	444	2076		54	2574	8743
-32.5	395	2535			2930	6801	667	2157		180	3004	11747
-27.5	593	2653		7	3253	10054	889	2152		501	3542	15289
-22.5	790	2666		61	3517	13571	1111	2058	1	1164	4334	19623
-17.5	988	2666	1	371	4026	17597	1333	1847	16	2229	5425	25048
-12.5	1185	2662	14	1508	5369	22966	1556	1508	149	3528	6741	31789
-7.5	1383	2605	132	4097	8217	31183	1778	1082	689	4743	8292	40081
-2.5	1580	2293	612	7423	11908	43901	1972	671	1703	5172	9518	49599
+2.5	1728	1513	1513	9065	13819	56910	1972	339	2580	4743	9634	59233
7.5	1580	612	2293	7423	11908	68818	1778	143	2930	3528	8379	67612
12.5	1383	132	2605	4097	8217	77035	1556	48	2994	2229	6827	74439
17.5	1185	14	2662	1508	5369	82404	1333	13	3000	1164	5510	79949
22.5	988	1	2666	371	4026	86430	1111	3	3000	501	4615	84564
27.5	790		2666	61	3517	89947	889		2999	180	4068	88632
32.5	593		2653	7	3253	93200	667		2984	54	3705	92337
37.5	395		2535		2930	96130	444		2852	12	3308	95645
42.5	198		2054		2252	98382	222		2311	3	2536	98181
47.5	25		1153		1178	99560	28		1298		1326	99507
52.5			374		374	99934			420		420	99927
57.5			62		62	99996			70		70	99997
62.5			5		5				6		6	
Total	.16	.24	.24	.36	1.00		.2	.2	.3	.3	.3	

*Minus speed indicates the tractor trailer was struck in the rear by the second vehicle.

Therefore the common estimates can be used for both auto and truck accidents. Tractor trailers apparently strike standing vehicles in the rear twice as often as they are struck in the rear. The distributions of the speed difference when tractor trailers are struck in the rear by autos and trucks are estimated in Table 3.16 by combining the negative speed densities of Table 3.14 with the speed densities for standing accidents in the proportions estimated above.

In angle collisions, as in the rearend accidents, the tractor trailer is assumed to be the slower of the two vehicles in approximately 50% of the truck accidents and 54% of the automobile accidents. By designating the faster vehicle as the striking vehicle and the slower one as that struck in the side, the fraction of such impacts and the speed of the striking vehicle can be estimated as shown in Table 3.15 1/.

Because of their similarity in the point of impact, the following accidents were combined into a single class of front end impacts: all headon collisions and those rearend and angle accidents when the tractor trailer is the faster of the two vehicles. The weights used in combining their frequencies were obtained by first taking the estimated frequency of each accident type from Table 3.5 and then distributing them among the impact categories in the proportions shown in Table 3.15. The weighted distributions of velocity are summed for each impact category as shown in Tables 3.16 and 3.17, to furnish an estimate of

1/ Although there is insufficient evidence to specify the true speed measure in angle accidents, the resultant speed vector, $(x_1^2 + x_2^2)^{1/2}$, is a logical alternative. However, it is difficult to estimate the distribution of this function.

the distribution of velocity among the various types of impacts a tractor semi-trailer is subjected to in an accident. In Table 3.1 the marginal distributions of impact velocities are evaluated for auto, truck, fixed object, and overturn accidents. These are also shown in Figure 3.4.

Table 3.15. Estimated Frequency of Impact Types.
(Based on intercity mileage for large carriers.)

Point of Impact	Opposing Object	Type of Accident			Est. Frequency	
		Headon	Rearend	Angle	Total	Subtotal
Front	Auto	0.110	0.170	0.036	0.316	
	Truck	0.031	0.052	0.011	0.094	
	Other	0.190			0.190	0.600
Rear	Auto		0.204		0.204	
	Truck		0.052		0.052	0.256
Side	Auto			0.043	0.043	
	Truck			0.011	0.011	0.054
Overturn				0.090	0.090	0.090
Total		0.331	0.478	0.191		1.000

Table 3.16. Estimated Distribution of Velocity among Different Types of Front End Impacts

Velocity (midpoint) × 10⁻⁵	Impact in Front with Automobile				Impact in Front with Truck			
	Headon	Rearend	Angle	Total	Headon	Rearend	Angle	Total
2.5 mph	1	1451	134	1586	1	689	42	732
7.5	7	2868	268	3143	3	1189	85	1277
12.5	15	2504	268	2787	7	824	85	916
17.5	22	2044	269	2335	10	542	85	637
22.5	29	1653	271	1953	14	410	85	509
27.5	37	1333	276	1646	17	360	85	462
32.5	47	1105	292	1444	21	335	90	446
37.5	65	961	325	1351	31	316	120	467
42.5	113	855	374	1342	61	277	181	519
47.5	197	742	422	1361	112	174	149	435
52.5	287	576	328	1191	152	69	72	293
57.5	356	413	159	928	165	15	20	200
62.5	416	265	112	793	166	1	2	169
67.5	484	141	63	688	167			167
72.5	577	61	27	665	186			186
77.5	737	21	9	767	251			251
82.5	943	6	3	952	370			370
87.5	1198	2	1	1201	469			469
92.5	1361			1361	436			436
97.5	1334			1334	283			283
102.5	1116			1116	127			127
107.5	796			796	39			39
112.5	478			478	8			8
117.5	238			238	1			1
122.5	100			100				
127.5	34			34				
132.5	10			10				
137.5	2			2				
Total	11000	17001	3601	31602	3097	5201	1101	9399

$\times 10^{-5}$

Table 3.17. Estimated Distribution of Velocity among the Impact Categories of Accidents

Velocity (midpoint)	Automobile Striking				Truck Striking				Striking Other Object	Overturn
	Front x 10^{-5}	Rear	Side	Total	Front	Rear	Side	Total		
2.5 mph	1586	2070	160	3816	732	689	42	1463	129	61
7.5	3143	3607	320	7070	1277	1189	85	2551	259	123
12.5	2787	2947	320	6054	916	824	85	1825	259	123
17.5	2335	2387	321	5043	637	542	85	1264	259	123
22.5	1953	2007	324	4284	509	410	85	1004	259	123
27.5	1646	1775	330	3751	402	360	85	907	281	133
32.5	1444	1630	349	3423	446	335	90	871	535	253
37.5	1351	1513	388	3252	462	316	120	903	1908	904
42.5	1342	1294	446	3082	519	277	181	977	4804	2275
47.5	1361	795	504	2660	435	174	149	758	5860	2776
52.5	1191	306	392	1889	293	69	72	434	3404	1612
57.5	928	65	190	1183	200	15	19	234	923	437
62.5	795	7	134	934	169	1	2	172	114	54
67.5	688		75	763	167			167	7	3
72.5	665		32	697	186			186		
77.5	767		11	778	251			251		
82.5	952		4	956	370			370		
87.5	1201		1	1202	469			469		
92.5	1361			1361	436			436		
97.5	1334			1334	283			283		
102.5	1116			1116	127			127		
107.5	796			796	39			39		
112.5	478			478	8			8		
117.5	238			238	1			1		
122.5	100			100						
127.5	34			34						
132.5	10			10						
137.5	2			2						
	31602	20403	4301	56306	9399	5201	1100	15700	19001	9000

x 10^{-5}

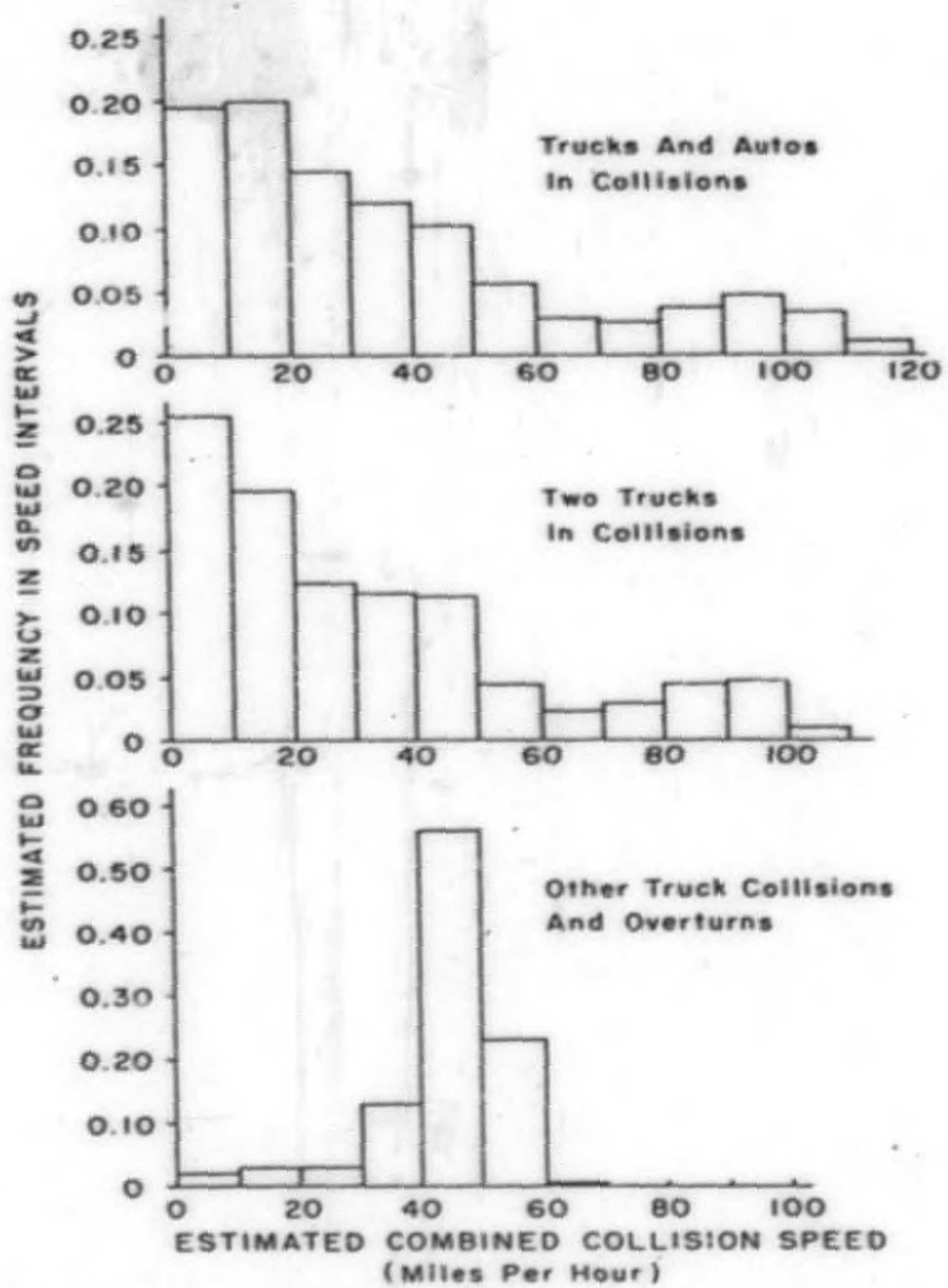

FIG. 3.4. ESTIMATED DISTRIBUTION OF THE NET IMPACT VELOCITY FOR VARIOUS COLLISIONS INVOLVING LARGE TRUCKS

III. ANALYSIS OF THE SEVERITY OF TRUCK ACCIDENTS IN TERMS OF THEIR VEHICLE AND CARGO DAMAGES.

1.0. Introduction.

In order to discuss analytically the severity of truck accidents, a quantitative measure, or "response," yielding information directly related to the degree of severity of a tractor-semitrailer accident involvement was required. Consistent with measuring the severity of highway accidents over a meaningful continuum is the notion of considering the extent of damages sustained in accidents. The conversion of hazard consequences into damages leads directly to the contemplation of the economic costs of damages evaluated as a direct result of an accident.

The desirability of equating severity to dollar costs has been manifested by the Bureau of Public Roads in the following context:

> Placing a dollar value on losses resulting from traffic accidents in no way minimizes the personal tragedy of either traffic fatilities or serious injuries; it is simply a means of identifying and measuring financicl losses that in turn can be used as a tool in the planning of both highway safety and highway improvement programs.[1/]

Total dollar damages sustained by the tractor and semitrailer as well as economic costs of damages to certain types of cargos supplied the required quantitative responses, so that the investigation

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1/ "A Symposium on Traffic Accident Costs," Public Roads, Vol. 31, No. 2, June 1960, p. 33.

considered highway accident severity in terms of vehicle damage and cargo damage costs.

The primary purpose of the analysis was to generate statistical models describing the costs of tractor-semitrailer highway accidents in terms of frequency of occurrence and in relation to different accident characteristics. In order to understand the mechanism through which accident costs arise statistical techniques including distribution theory, regression and variance analysis were employed. The procedure consisted in gathering meaningful damage cost data and deriving probability distributions of damages for different accident types in an attempt to describe the expected frequencies of certain amounts of damage. Attention then centered on explaining the variability in costs or degree of severity as a function of particular characteristics relating to the objects involved in the collision. Variables such as vehicle speeds, weights, object struck, direction of impact and the energy released in impact were related to damage costs in an attempt to account for cost variation.

The specific form that the analysis took can be stated in an outline of the mathematical expressions that were deirved. Thus, the development was conducted in the manner described below.

A. Vehicle Damage Analysis

1. Derivation of probability distribution of the form:

$$\text{prob.}\left[g(C) > g(C_0)\right] = l\left[g(C);\ \theta_i\right]$$

θ = parameters associated with the specific density function.

where: $g(C)$ = a transformed variate of the original cost.

θ = parameters associated with the specific density function.

2. Relationship of damages and accident variables:

$$g(C) = f(A_i)$$

where: A_i = accident variables.

3. Tests for significant effects of accident characteristics on costs.

B. Cargo Damages

1. Derivation of probability distributions of the form:

$$\text{prob}\left[h(d) > h(d_0)\right] = k\left[h(d); \lambda_i\right]$$

where: $h(d)$ = some function of observed cargo damage

λ_i = parameters to be estimated.

2. Relationship of cargo damages, vehicle damages and accident variables:

$$h(d) = m\left[g(C); A_i\right]$$

3. Return period approach to cargo damage:

$$t\left[h(d)\right] = \frac{1}{1 - k\left[h(d); \lambda_i\right]} = \text{return period.}$$

The development of the aforementioned models suggested the estimation of threshold probabilities for cargo damages: The probabilistic approach to the point at which damage to a shipment originates led to inferences as to the probability of occurrence of container damage in shipments of radioactive materials.

1.1. Definitions.

The analysis of the severity of truck highway accidents was confined to accident experience of tractor semitrailers. The data employed were results of samples of reports of accidents, involving a tractor semitrailer, prepared by and for the Interstate Commerce Commission. Thus, all reported responses refer to a particular tractor semitrailer involved in a certain type of accident. The accidents were motor vehicle traffic accidents which are defined as any accident involving a motor vehicle in motion occurring on a trafficway and resulting in death, injury or property damage.[1/] The population of accident reports included all accidents resulting in damages to all vehicles and cargos involved equalling and exceeding $100.

Damage costs are defined as the money value of damages to the vehicle and cargo involved in the accident. Thus, vehicle costs are the dollar damages sustained by the tractor and semitrailer and cargo damage costs are the dollar damages or losses sustained by the truck's cargo.

The accident type was classified by the object struck and the direction of impact ascertained from the I.C.C. accident report and complied with the manual on "Uniform Definitions of Motor Vehicle Accidents."[2/] Thus, the involvements were categorized into automobile, truck, fixed object and non-collision accidents indicating that the responding tractor semitrailer was involved in the particular kind

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1/ Uniform Definitions of Motor Vehicle Accidents, U. S. Dept. of Health, Education, and Welfare, U. S. Gov't Printing Office, 1956.
2/ Ibid.

of accident. Fixed objects referred to any stationary object, not a motor vehicle, such as bridge railings, utility poles, buildings, etc. Non-collision accidents were primarily characterized by the truck overturning, on or off the roadway, without striking another vehicle or fixed object. The direction of impact was determined by the manner in which the vehicle collided and classified as head-on, angle and rear-end accidents.

The weight of the tractor semitrailer, representing one of the mass components contributing to accident severity, was defined as the gross vehicle weight consisting of the vehicle and cargo weights. Velocities were defined as the reported estimates of vehicle speeds prior to impact.

The foregoing definitions pertain to those more general components of the analyses of cargo and vehicle damages in tractor semitrailer accidents. Other relevant definitions arising in specific areas are presented in their context.

2.0. Analysis of Vehicle Damage

2.1. Theoretical Distribution of Vehicle Damage Costs in Accidents.

In attempting to measure and describe accident severity as a component of the expected hazards involved in transportation, it is necessary to consider the frequency with which the severity measure assumes different values. Thus total costs sustained by the vehicle in an accident may be treated as a random variable, taking on different values which possess an inherent order in their nature, for different events. If the probability of the variable taking on values between any given values along the entire possible range of numbers is known, then the probability distribution of the random variable is also known. Thus, if the probability distribution for total dollar damages were known, it would be possible to completely describe the total cost severity measure in terms of its distributional form.

When working with probability distributions, it is usually desirable to classify or describe them by means of statistical measures characterizing the location, variability and degree of symmetry. It is well known that many distributions encountered in the collection of statistical data in many diverse fields tend to form skewed or asymmetric distributions. Indeed, probability distribution arising in such fields as biology, psychology and economics are frequently skewed. The observed distributions of vehicle damages for all types of objects struck exhibited high positively skewed tendencies. Fig. 2.1.1 shows the histograms of total vehicle costs for a tractor-semitrailer striking automobiles, trucks, fixed objects and non-collision

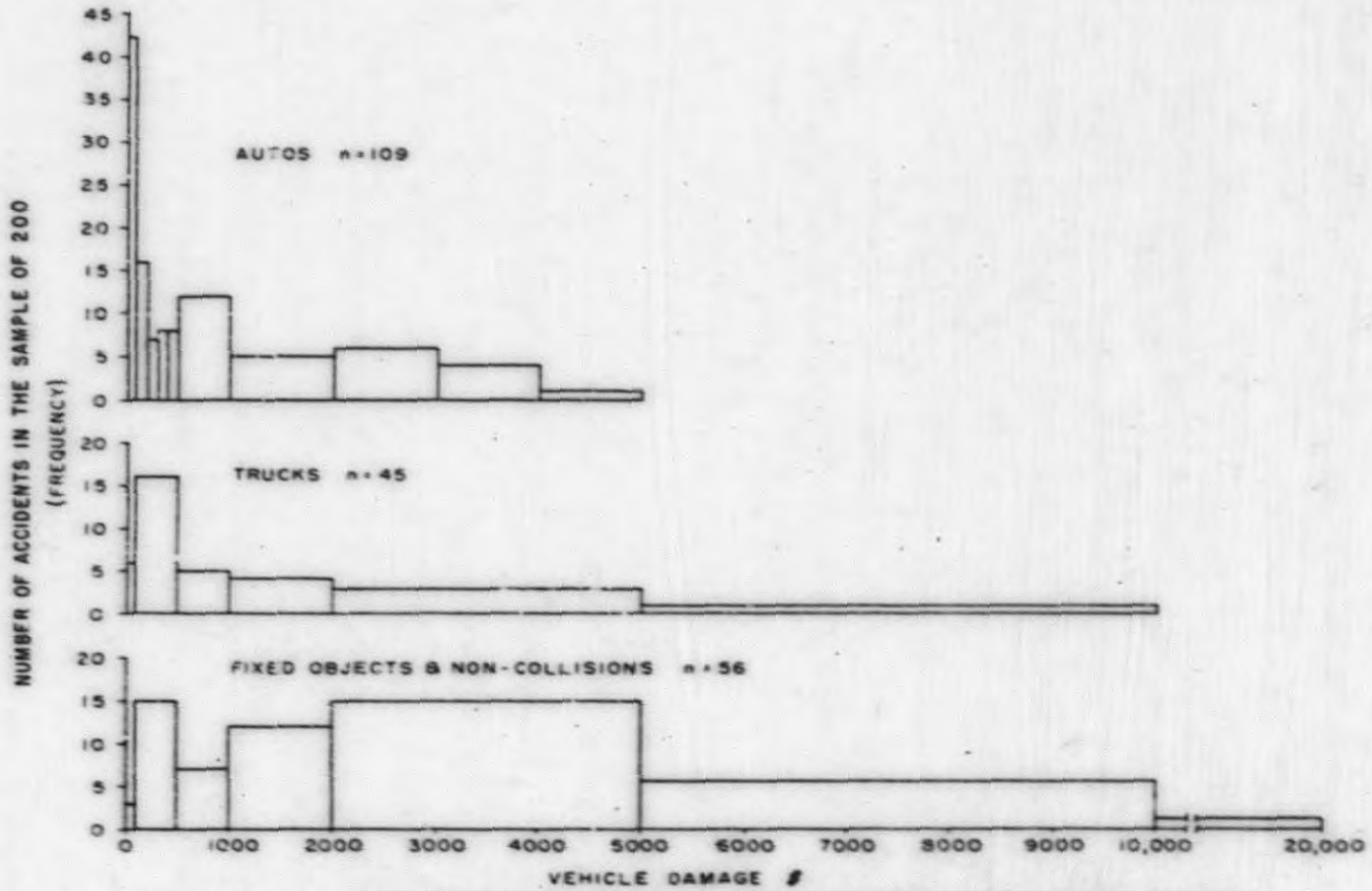

FIG. 2.1.1. HISTOGRAMS OF VEHICLE DAMAGES FOR AUTO, TRUCK, FIXED OBJECT AND NON-COLLISION ACCIDENTS

accidents. The data are results of a random sample of 200 vehicle damage producing accidents taken from the files of the Interstate Commerce Commission. The accidents occurred over the two year period 1958-1959 and the number selected for each accident type was determined by the total accident population's distribution of automobile, tractor-semitrailer, fixed object and non-collision accidents.

The rationale for considering the logarithmic normal distribution as the underlying probability distribution for the total costs of vehicle damages was actually two-fold. The first reason was that in statistical analysis of quantitative information it is often desirable and at times necessary to consider the normal or Gaussian distribution as characterizing a variable. Thus, ability to transform a skewed distribution into a normalized form is highly desirable. One such transformation is where the logarithm of the variable is considered, and if it is normally distributed, the original variable is said to possess a logarithmic normal distribution. It has been stated that the usefulness of lognormal theory lies in the fact that with its aid a numerous class of skewed distributions in a number of fields are brought within the domain of normal test statistics.[1/] The second important reason for introducing log-normal theory, was not in considering a transformed distribution in order to satisfy certain prerequisites for applied statistical analysis, but the fact that the physical interpretation of the theory offered a suggestion as to generalizing about the degree

1/ Aitchison, J., and Brown, J. A. C., "The Lognormal Distribution," Cambridge, Cambridge University Press. 1957.

of accident severity. This approach arose since the lognormal distribution may also be derived by considering the position of a result or event in the range of the associated variable. If the position is affected by a number of independent influences, not acting additively as in the normal case, but dependent upon the importance or strength of the influence, according to the central limit theorem the variable should be lognormally distributed as the number of factors increases.[1/] Thus, if total damage costs were to follow a lognormal distribution, the phenomena could be explained as arising from a multitude of causes operating simultaneously in determining the amount of damages. Furthermore, it is not implausible to suppose that total damages in any one collision might be determined by the product rather than the sum of a great many different random factors.

Mathematically, it can be said that if the logarithm of a variable, log x is normally distributed with mean m and variance σ^2, then the probability density function of x is

$$(2.1)\quad f(x;\, m;\sigma) = \frac{1}{x\sigma\sqrt{2\pi}}\, e^{-1/2\left(\frac{\log x\, -\, m}{\sigma}\right)^2},\ \text{for } x > 0.$$

The positive skewness of the distribution is emphasized by the positions of the mode, median and mean of x, since

$$\text{mode} = e^{m - \sigma^2}$$

$$\text{median} = e^{m}$$

$$\text{mean} = e^{m + 1/2\sigma^2}$$

and the greater the variance the greater is the skewness.

2/ Cramer, H., Mathematical Methods of Statistics, Princeton, 1946.

Table 2.1.1. Distributions of Vehicle Damages for 200 Accidents by Object Struck

Upper Limit of Vehicle Damages	Collisions with Automobiles	Upper Limit of Vehicle Damages	Collisions with Trucks	Upper Limit of Vehicle Damages	Collisions with Fixed Objects and Non-Collision
$ -100	42	$ -100	6	$ -100	3
-200	16	-500	16	-500	15
-300	7	-1,000	5	-1,000	7
-400	8	-2,000	4	-2,000	12
-500	8	-5,000	3	-5,000	15
-1,000	12	-10,000	1	-10,000	3
-2,000	5		35	20,000	1
-3,000	6				56
-5,000	5				
	109				

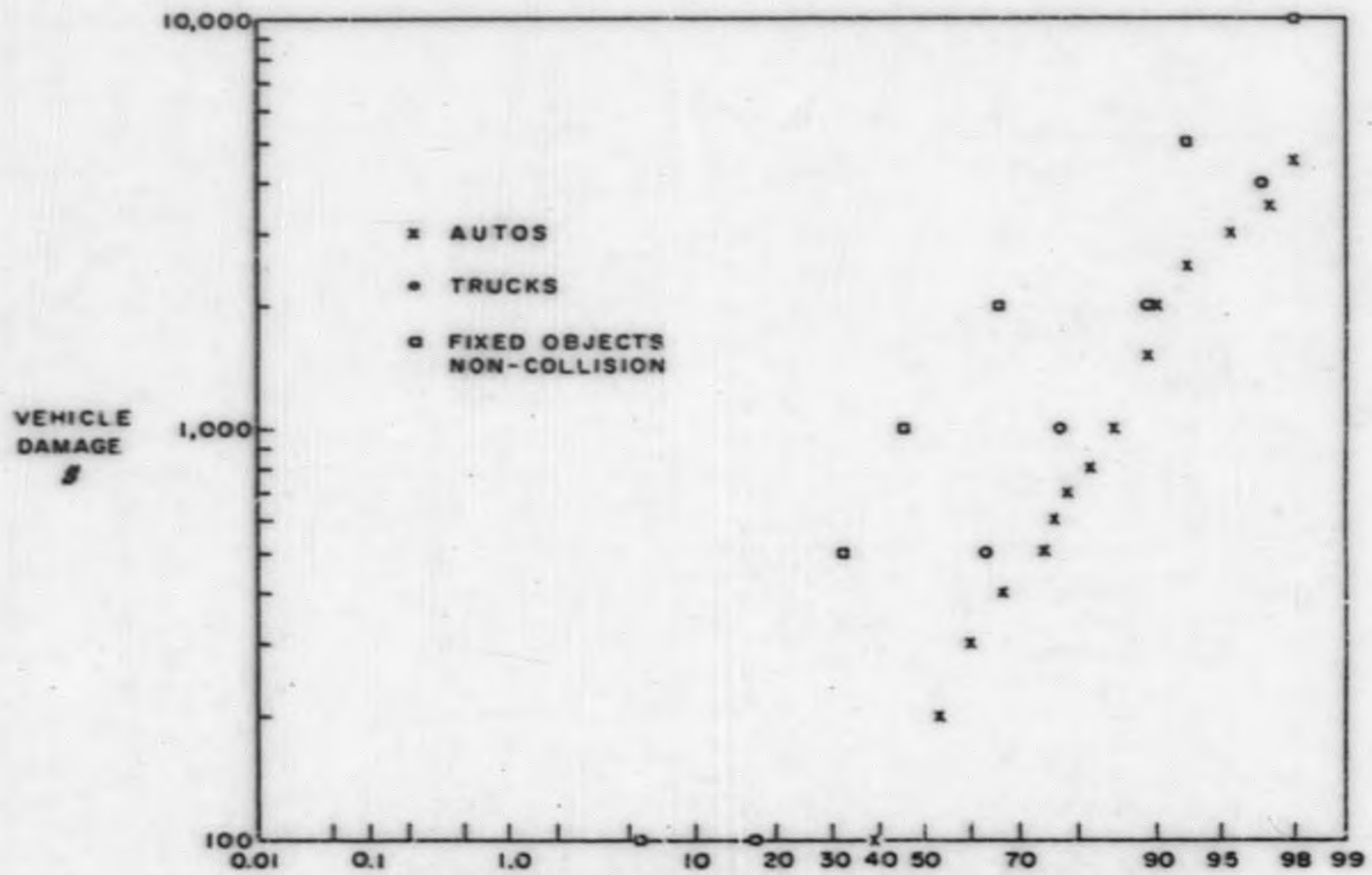

FIG. 2.1.2. LOG PROBABILITY PLOTS OF VEHICLE DAMAGES FOR AUTO, TRUCK, FIXED OBJECT AND NON-COLLISION ACCIDENTS

One of the inherent advantages in using the lognormal distribution is that it allows for the subjection of data to a simple graphical analysis as a preliminary to more detailed work. By plotting the cumulative frequencies on logarithmic probability paper the tenacity of the lognormal assumption can be quickly and easily observed. The tendency of the points to form a straight line provides an approximation to judging the feasibility of the underlying lognormal population. An obvious advantage of the graphical approach is that no transformation of the values is necessary. The data of Table 2.1.1 are shown plotted on a logarithmic probability scale in Figure 2.1.2. The lognormal nature of the population of total costs to the tractor-semitrailers involved in automobile, truck and fixed object collision and non-collision appeared to be reflected in the sample observations. Before actually fitting theoretical distribution to the observations, tests of significance were performed on the sample means and variances. It was hoped that a common variance could be applied to the groups, which would reflect, as might be expected, a common degree of skewness associated with the distributions. It was also desired to justify combining the fixed object and non-collision accidents, based on a priori reasons as well as to defer applying theoretical frequencies to only sixteen non-collision accidents.

Bartlett's test for homogeneity of variances[1/] was made on the variances of the four accident type distributions. Assuming the logarithms to be normally distributed, the sample variances of the

1/ Bartlett, S., "Properties of Sufficiency and Statistical Tests," Proceedings of Royal Society of London, A.160 (1937).

distributions of the logarithm, in common log terms, were computed and the hypothesis of equal variances was tested. Table 2.1.2 shows the necessary information to apply the test.

Table 2.1.2. Test of Homogeneity of Variances of Log Cost Distribution Four Accident Types.

Collision with:	Sums of Squares $\sum x_i^2$	Degrees of Freedom $n - 1$	S_i^2
Automobile	54.6317	108	.5012
Truck	12.1515	34	.3500
Fixed Object	8.7687	39	.2192
Non-Collision	6.6655	15	.4166

$L = 3.1041$ $\qquad\qquad$ $X^2_{.05} = 7.83$

The test statistic is

$$(2.2)\quad L = \frac{2.3026}{C}\left[\log S^2 \sum_{i=1}^{4} (n_i - 1) - \sum_{i=1}^{4} (n_i - 1) \log S_i^2\right],$$

where:

$$2.3026 = \log_e 10$$

$$S^2 = \frac{\sum_i \left(\sum x_i^2\right)}{\sum_{i=1}^{4} (n_i - 1)}$$, which is an unbiased estimate of the common σ^2 if the variances are equal.

$$C = 1 + \left[\sum \frac{1}{n_i - 1} \quad \frac{1}{\sum (n_i - 1)}\right] / 3\left(\sum n_i - 1\right).$$

Under the hypothesis of equal variances, L is distributed as χ^2 with K - 1 degrees of freedom, where K = 4 groups. The computed value of L was found to be 3.1041, which does not reject the hypothesis of equal variances among accident types. Thus, the common variance of .42 may be considered as representative for the four distributions.

Analysis of variance methods were employed in testing for the significance of the sample means of vehicle damage costs. The model under consideration was of the form

(2.3) $x_{ij} = m + \theta_j + e_{ij}$. $i = 1, 2, \ldots n_i$; $j = 1, 2, \ldots 4$

where: x_{ij} = logarithm of total vehicle costs

m = common mean value

θ_j = effect of accident classification

e_{ij} = random error; assumed normally distributed with mean zero and variance σ_e^2.

The hypothesis to be tested was that the mean costs for the different accident types were equal. The analysis of variance in Table 2.1.3 showed significant differences among total vehicle damages.

Table 2.1.3. Analysis of Variance of Logarithms of Vehicle Damages for Automobile, Tractor-Semitrailer, Fixed Object, and Non-Collision Accidents.

Collision with	Mean of Log Cost
Auto	2.2846
Truck	2.6295
Fixed Object	3.0366
Non-Collision	3.0463

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F
Object Struck	22.7229	3	7.5743	18.38
Residual	82.2174	196	.4195	
Total	104.9403		$F_{.05}(3,196) \cong 2.70$	

The significant F ratio suggests that the cost sustained by a tractor semitrailer involved in a collision varies for different objects struck. Accident severity for the tractor semitrailer appeared to be greater in fixed object and non-collision accidents and descending from truck accidents to automobile accidents.

In order to determine significant differences, if they existed, within the mass classification, individual comparisons were made. The three interesting comparisons, one for each degree of freedom, were between automobiles and trucks, fixed objects and non-collisions, and autos and trucks versus fixed objects and non-collisions. Generally, when the number of responses in each group is equal, the subdivision of the sum of squares yielding as many planned orthogonal comparisons as there are degrees of freedom, proceeds simply since the necessary orthogonal coefficients are usually obvious. However, certain adjustments must be made when the samples are of unequal size, in order to account for the unbalanced nature of the data. The comparisons made, among the sums of the logarithms of vehicle damage, were set up so that the following relationships were satisfied.[1/]

1. $C_i = \sum l_j S_j$ is a comparison if $\sum n_j l_j = 0$, $j = 1, 2, .. 4$,
$i = 1, 2, 3$

2. $\dfrac{C_i^2}{\sum n_j l_j^2}$ = sum of squares for the comparison

3. C_1 and C_2 are two orthogonal comparisons if $\sum n_j l_{1j} l_{2j} = 0$,

where l_j = orthogonal coefficients

S_j = sums making up the comparisons

Thus, the coefficients found to satisfy the restrictions were set up in the following form:

Comparison	S_1	S_2	S_3	S_4
Auto vs. Truck	1	-3.11	0	0
Fixed Object vs. Non-Collision	0	0	1	-2.50
Auto, Truck vs. Fixed Object, Non-Collision	1	1	-2.57	-2.57
n_j	109	35	40	16

The analysis of the mean differences within the accident class is in Table 2.1.4.

1/ Snedecor, G. W., "Statistical Methods," Iowa State, 1956

Table 2.1.4. Comparisons between Costs of Damages for Different Objects Struck.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F
Auto vs. Truck	4.6754	1	4.6754	11.1452
Non-Collision vs. Fixed Object	.0011	1	.0011	.0026
Auto, Truck vs. Non-Collision, Fixed Object	18.0464	1	18.0464	43.0188
Total	22.7229	3		
Residual	82.2174	196	.4195	

The only comparison showing no significant difference was between fixed object and non-collision accidents. Significant differences were evidenced for damages sustained in auto and truck accidents as well as for accidents between other vehicles and accidents only involving the responding vehicle, i.e. fixed object collisions and non-collisions.

Logarithmic normal distributions, with two parameters, were fitted to the observed distributions for the four accident types, combining fixed object and non-collisions on the assumption of equal means and variances. Figure 2.1.3 shows the theoretical distributions, compared with the actual data, of the logarithms of costs while Figure 2.1.4 presents the lognormal distributions on a log probability scale. The method of fitting consisted of fitting normal distributions to the distribution of logarithms. Table 2.1.5 illustrates the method, with automobile accidents. Since a one to one relationship exists in the logarithmic transformation, goodness of fit tests were applied to the normal distributions. The X^2 values failed to disprove the

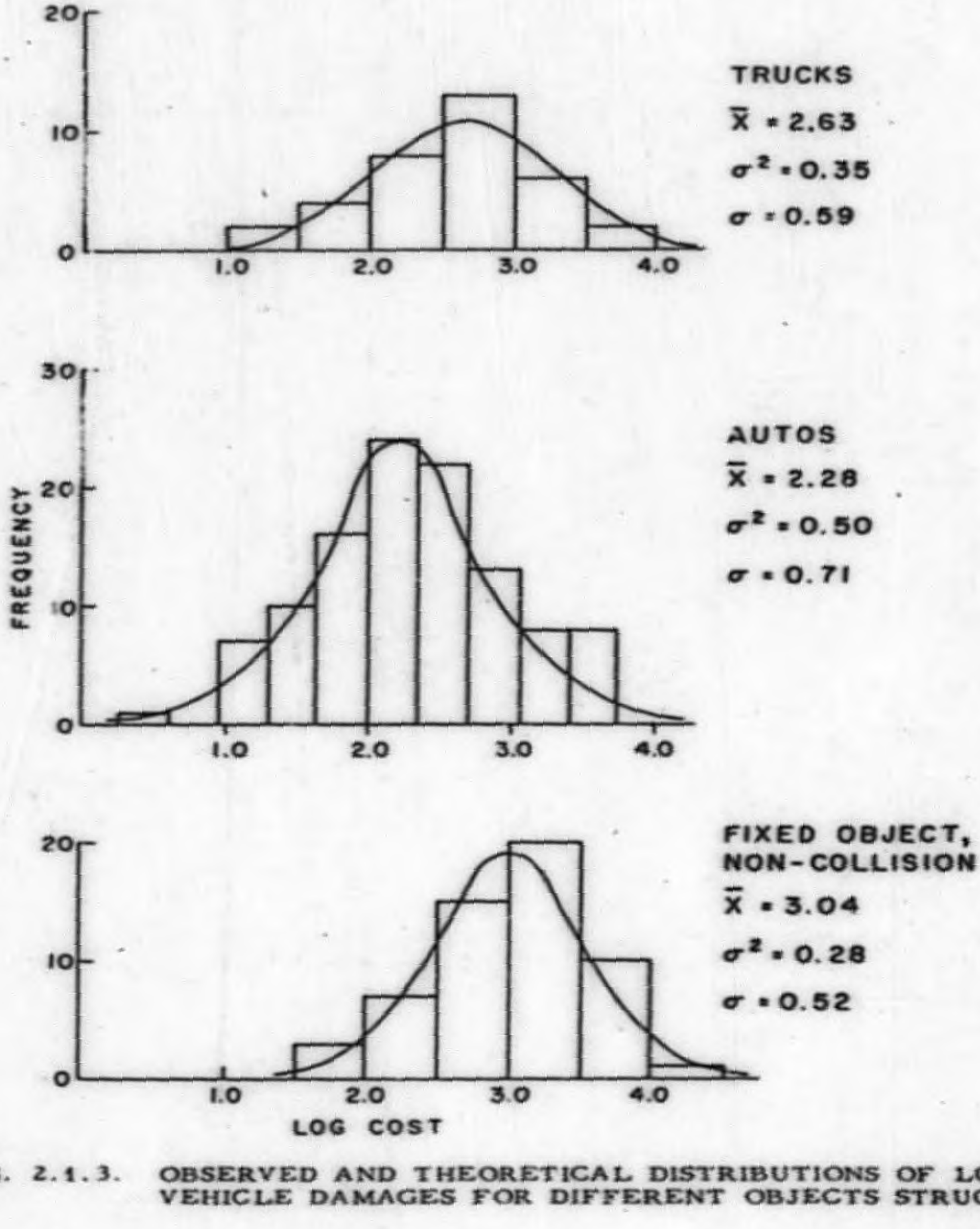

FIG. 2.1.3. OBSERVED AND THEORETICAL DISTRIBUTIONS OF LOG VEHICLE DAMAGES FOR DIFFERENT OBJECTS STRUCK

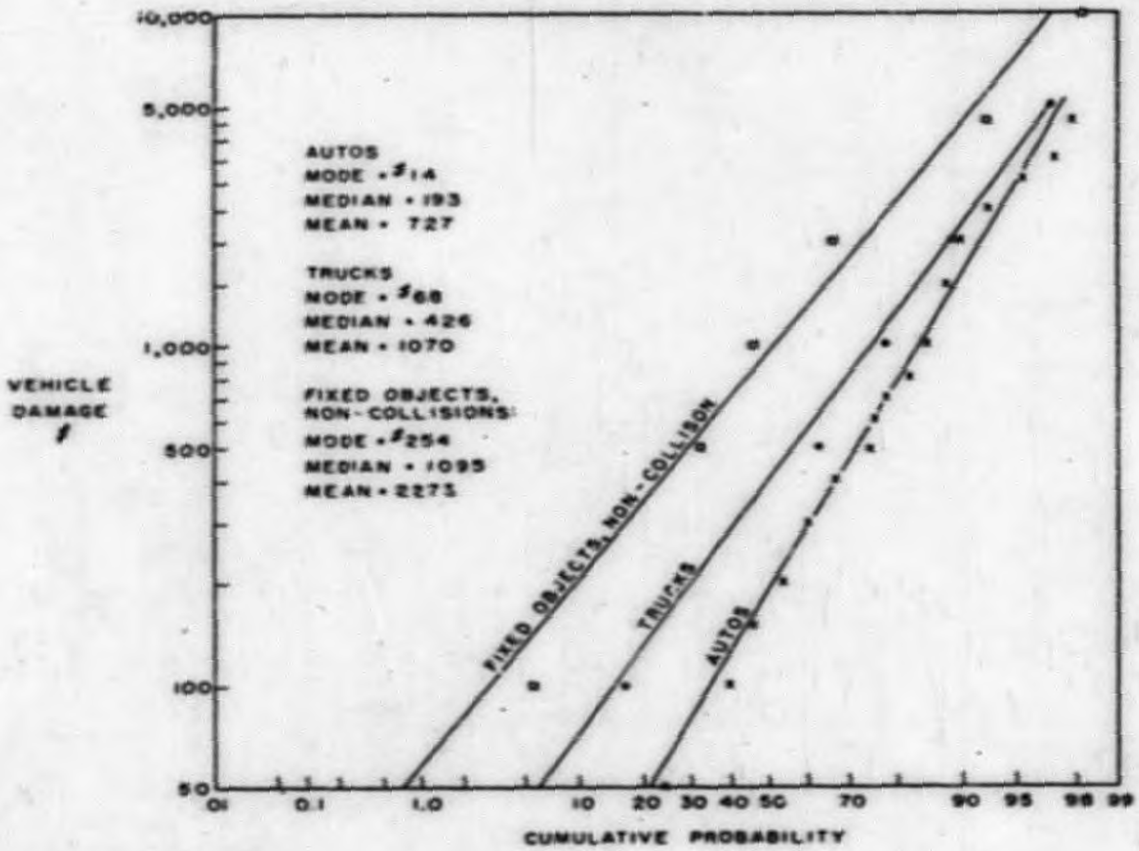

FIG. 2.1.4. THEORETICAL LOGNORMAL DISTRIBUTIONS OF VEHICLE DAMAGES FOR DIFFERENT OBJECTS STRUCK

assumption of normally distributed logarithms, thereby justifying the assumption of the population of vehicle damages following a lognormal distribution. The mean, median and mode were computed, using the following equations.

$$\log \text{mode} = \overline{\log \text{cost}} - 2.3026\sigma^2$$

$$\log \text{median} = \overline{\log \text{cost}}$$

$$\log \text{mean} = \overline{\log \text{cost}} + 1.1513\sigma^2$$

The results are summarized in Table 2.1.6.

Table 2.1.6. Summary of Results of Lognormally Distributed Damages.

Accident Type	Modal Cost ($)	Median Cost ($)	Mean Cost ($)	$S^2_{\log}$	$\bar{X}_{\log}$	$V_{\log} = S/\bar{X}$	χ^2	Critical χ^2
Autos	14	193	727	.50	2.28	.31	8.67	15.50
Trucks	68	426	1070	.35	2.63	.22	1.71	7.82
Fixed Object, Non-Collision	254	1095	2273	.28	3.04	.17	3.01	7.82

($.42$ = common S^2)

In summarizing then, the vehicle damages tend to follow a lognormal distribution for each accident class, with a common variance and different costs between automobile and truck collisions and collisions with other motor vehicles as opposed to non-motor vehicle accidents.

The apparent goodness of fitting lognormal distributions to vehicle damages suggested that perhaps a law of accident cost might be brought forth. In the same manner as income distributions[1/], organisms' growth[2/], number of words in sentences[3/] and other natural

1/ Aitchison, J., and Brown, J. A. C., "The Lognormal Distribution," Cambridge, Cambridge University Press, 1957.
2/ Cramer, H., Mathematical Methods of Statistics, Princeton, 1946.
3/ Williams, C. B., "Biometrica," Vol. 31, p. 356, 1940.

Table 2.1.5. Fitting Normal Distributions to Logarithms of Vehicle Damage, Auto Accidents

(1) Upper Limit of Class Interval (Log Damage)	(2) Limits in Standard Units	(3) Theoretical Cumulative Frequency	(4) Theoretical Relative Frequency	(5) Absolute Theoretical Frequency		(6) Absolute Observed Frequency	(7) Goodness of Fit: (6) - (5)	(8) Goodness of Fit: $\left[\frac{(6) - (5)}{(5)}\right]^2$	(9) Upper Limit of Class Interval (Actual Damage)	(10) Theoretical Cumulative Frequency	(11) Observed Cumulative Frequency
.595	-2.38	.0087	.0087	0.95)		1					
.949	-1.89	.0294	.0207	2.26)	8.98	0	-0.98	0.960	$ 50	22.16	27
1.299	-1.39	.0823	.0529	5.77)		7			100	37.56	42
1.649	-0.90	.1611	.0788	8.59		10	1.41	0.231	200	55.37	58
1.999	-0.40	.3121	.1510	16.46		16	-0.46	0.013	300	66.10	65
2.349	0.09	.5359	.2238	24.39		24	-0.39	0.006	500	78.37	81
2.699	0.59	.7224	.1865	20.33		22	1.67	0.137	1,000	91.97	93
3.049	1.08	.8599	.1375	14.99		13	-1.99	0.264	2,000	100.84	98
3.399	1.57	.9418	.0819	8.93		8	-0.93	0.097	3,000	103.93	104
3 749	2.07	.9808	.0390	4.25)	6.34	8	-1.66	0.435	5,000	106.51	109
∞	∞	1.0000	.0192	2.09)		0			10,000	108.52	109
				109.01		109					

Log Damage = 2.285

σ_{Log} = 0.501

χ^2 = 2.143

$\chi^2_{.05}$ (5) = 11.1

phenomena have been explained by the law of proportionate effect, so might accident costs. Thus, the model could be interpreted as suggesting that the distribution of damage costs arises from a multitude of causes operating simultaneously. Hence, the number of causes generating different costs in a collision take the form of speeds, directions of impact, road conditions, weather conditions, driver characteristics and most probably an infinity of other causes. Thus, the plausibility of lognormal theory being applied to accident costs can not only be justified by the goodness of fit of the empirical results, but also from a general consideration of what is happening in a collision to generate such cost distributions. It shall be shown at a later point that this concept may be applied to disasters in general as well as to direct costs of all automobile accidents.

2.2. Analysis of Vehicle Damages with Respect to Accident Types.

The qualitative accident characteristics directly related to the collision are the direction of impact and the object struck. In the preceding section attention was centered on the object struck, while the present analysis investigates costs with respect to both the object struck and the type of impact. Table 2.2.1 represents the average costs sustained by the tractor-semitrailer and the average common logarithms for all combinations of direction and object struck, utilizing the random sample of 200 accidents discussed previously.

Table 2.2.1. Average Costs and Logarithms of Costs to Vehicle for Direction and Object Struck in 200 Accidents.

Collision with:	Direction of Impact	Average Cost	Range of Costs	Average Log_{10} Cost
Automobile	Tractor-Trailer Strikes in Rear	$ 332	$ 3-3,700	2.014
	Tractor-Trailer Struck in rear	353	10-1,590	2.121
	Angle	223	45-500	2.237
	Headon	1,810	150-5,000	3.029
Trucks	Tractor-Trailer Strikes in Rear	958	100-3,000	2.804
	Tractor-Trailer Struck in Rear	565	25-2,575	2.456
	Angle	88	50-125	1.898
	Headon	2,980	450-8,000	3.190
Fixed Object	Headon*	1,699	100-5,000	3.037
Non-Collision	Headon*	2,816	110-11,000	3.046

*Non-Collision and Fixed Object accidents were considered as Headon accidents.

The logarithmically transformed costs were used to test for significant differences between objects struck and direction as well as for any interaction effect among the two characteristics. Since object differences were explored previously, fixed object and non-collision involvements were not treated, as both only occurred in headon type accidents. Essentially then the information analyzed constituted a four by two matrix as in Table 2.2.2.

Table 2.2.2. Means of Logarithms of Vehicle Damages for Collisions with Other Vehicles and Directions.

	Collision with				
	Truck		Auto		
Direction of Impact	n_T	Log Cost	n_A	Log Cost	n_{T+A}
Striking in Rear	10	2.808	47	2.014	57
Struck in Rear	18	2.456	22	2.121	40
Angle	2	1.898	17	2.237	19
Headon	5	3.190	23	3.029	28
n	35		109		144

It is to be noted that the number of replicates in each cell not only differed from cell to cell, but was also disproportionate to the totals for each effect. This necessitated altering the usual analysis of variance procedures for testing the effects, in order to account for the inherent non-orthogonality of the data due to disproportionate subclasses. The method of weighted squares of means furnished an exact test for interaction as well as an exact test for main effects.[1/] The hypothesis was represented by the model,

$$\text{Log Cost}_{ijk} = m + d_i + o_j + e_{ijk} \qquad i = 1, \ldots 4, \quad j = 1, 2, \quad k = 1, \ldots, n_{ij}$$

1/ Gates, F., "The Analysis of Multiple Classifications with Unequal Numbers in the Different Classes," J.A.S.A., Vol. 29, No. 185, 1934.

where:

m = overall mean

d_i = effect of the ith direction

o_j = effect of the jth object

e_{ijk} = random error; assumed normally distributed with mean zero and variance σ^2.

The exact test for the assumption of negligible interaction is provided by weighing the squared differences between means for each direction by the ratio $n_T n_A/(n_T + n_A)$. The interaction term proved not to be significant and the completed analysis of variance was performed in Table 2.2.3.

Table 2.2.3. Analysis of Variance on Logrithmic Costs of Vehicle Damages for Object and Direction.

Source of Variation	Unadjusted Sum of Squares	Adjusted Sum of Squares*	Degrees of Freedom	Mean Squares	$F_{.05}$	Critical $F_{.05}$
Vehicles	3.1496	4.0903	1	4.0903	11.989	3.9
Direction	16.8611	17.8018	3	5.9339	17.381	2.7
Vehicle & Direction	2.5347	2.5347	3	.8449	2.475	2.7
Error	46.4315	46.4315	136	.3414		

*Correction for Disproportionality = -.9407, applied to main effect sum of squares.

The analysis rejected the hypothesis of no difference in costs when striking autos and trucks and also showed differences to exist for direction of impact. Thus, while cost differentials between objects struck and direction existed, they were consistent.

2.3. Analysis of Costs with Respect to Quantitative Accident Characteristics.

Thus far, the relationship between vehicle damage costs, impact direction and object struck has been investigated. Generally, highway involvements are discussed in terms of severity characteristics such as vehicle weight and vehicle age[1/]. Accident types are also considered. The present section attempts to relate certain quantitative accident characteristics to the severity response. It was hoped that the variables, vehicle weight and speeds of the involved vehicles prior to impact, might aid in explaining the variations of the severity of tractor semitrailer accidents.

Primarily, attention was centered upon speeds and existing accident records were searched in an effort to attain speed information. The data employed in the previous analyses, contained speed information of questionable merit due to the reporting method involved, since speed data were not specifically asked for and was only volunteered by the reporting party if he so desired. Thus, resort was made to another area of the Interstate Commerce Commission's motor vehicle acitvities. The study was based on specific accidents investigated by the ICC. These investigable accidents are selected from the entire accident population by the ICC on the bases of degree of damage involved, amount of personal injury sustained, and/or evidence of negligence. From this substantial group of accident reports a selective sample of 108 accidents was taken. Essentially the selection criteria were comprehensiveness of report, and specifically reliable estimates

1/ McCartny, J. F., "The Economic Cost of Traffic Accidents in Relation to the Vehicle," Public Roads, Vol. 31, No. 2, June 1960.

of speeds and weights. It should be noted that reliability is mentioned not in the statistical terminology, but refers to the adequacy of the report itself as determined by the source. Accidents where fires or explosions occurred were not included in the sample. These investigated accidents were the only source where adequate speed information coupled with a competent accident severity response could be discovered. Thus, in order to arrive at meaningful conclusions over a range of costs with a sufficient number of cases for each variable, a selective sample was necessary. The above observation does not mean to imply a limit to the generalizations from the derived models, since the purpose was to study the interrelationships of the variables and not necessarily to predict the cost response in future involvements.

The typical ICC investigated report contains the following information relating to the accident: date, location, carrier, type of impact, type of highway, time of day, weather conditions, description of tractor and trailer (make, type, axle arrangement), description of vehicle or object struck, cargo description, vehicle weights (tractor, trailer, cargo), speeds of both vehicles, description of accident and subsequent movements, dollar damages to the vehicle and cargo, injury extent, and fire or explosive incidence.

The objective of the analysis was to study the mutual relationships between vehicle damage costs and the accident characteristics by deriving a series of functional equations. Regression and variance techniques were employed in an effort to explain accident costs.

Throughout the regression analysis six independent or causation variables were analyzed as to their degree of association with accident costs. The variables are presented below:

1. Weight of responding vehicle (W_1): Gross vehicle weight, composed of the tractor, trailer and cargo; measured in tons.
2. Object struck (W_2): Qualitatively defined (as previously used) as commercial motor vehicle, automobile, fixed object, and non-collision.
3. Direction of impact (D): Qualitatively defined (as previously used) as headon, angle, responding vehicle struck in rear (reflecting damage to the trailer), responding vehicle struck second vehicle in rear (reflecting tractor damages).
4. Speed of responding vehicle (V_1): Speed prior to impact, measured in M.P.H.
5. Speed of struck vehicle (V_2): Speed prior to impact, measured in M.P.H.
6. Combined speeds of two vehicles ($V_1 \pm V_2$): Speeds were combined in the following manner:

   headon collision: $V_1 + V_2$

   rearend collision: $V_1 - V_2$ or $V_2 - V_1$, depending upon the larger speed

   angle collision: $1/2(V_1 + V_2)$ 1/

The reports were initially classified by object struck and direction as in Table 2.3.1, where the representative notation for

1/ Since the angle of impact was unknown, the average combined speed was utilized.

each factor is shown as well as the number of observations in the sample. As an illustration of the notation to be employed, Ah designates vehicle 1 or the responding tractor semitrailer colliding headon with an automobile.

Table 2.3.1. Number of Investigated Accidents by Object Struck and Direction of Impact.

Object Struck (W_2)	Direction (D)	n	
Commercial Motor Vehicle (T)	Headon (h)	20	
	Striking in rear (r_1)	15	
	Struck in rear (r_2)	14	
	Angle (a)	2	
Total			51
*Automobile (A)	Headon (h)	24	
	Struck in rear (r_2)	6	
	Angle (a)	8	
Total			38
Fixed Objects (F)	Headon (h)		12
Non-Collision (N)	Headon (h)		7
Total Accidents			108

*No rearend accidents were observed where vehicle 1 struck autos in the rear.

A cursory survey of the average vehicle damage costs for the nine combinations of object and direction revealed increasing costs and dispersion, indicated by the range of damages, as accident type varied from auto-angle to truck-headon as in Figure 2.3.1.

The regression analysis consisted in a progression of analytic

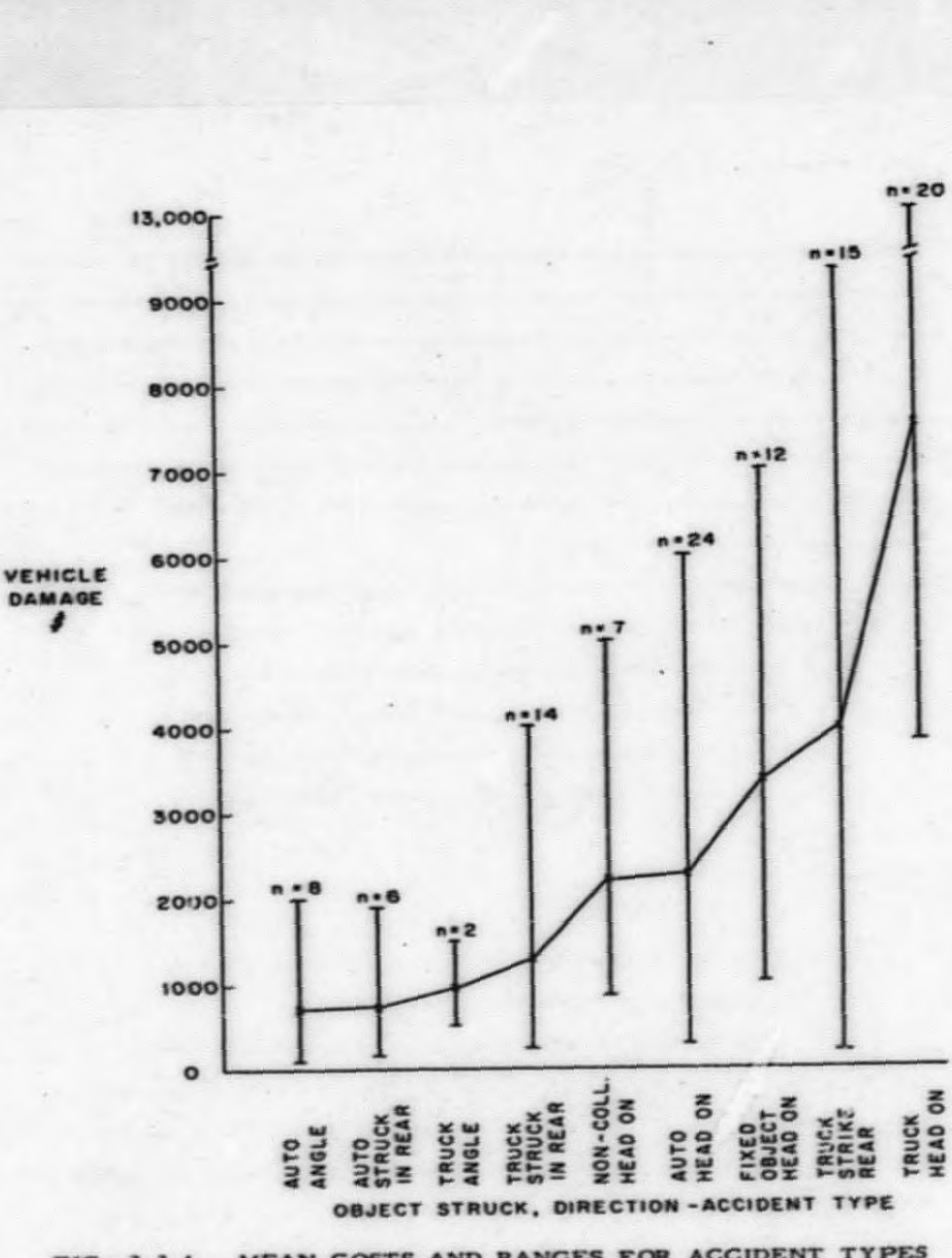

FIG. 2.3.1. MEAN COSTS AND RANGES FOR ACCIDENT TYPES

models attempting to explain more and more of the variability in vehicle damage costs by directly fitting successive equations to the observations. Since the logarithm of dollar damages was seen to be a normally distributed random variable, the dependent variable was transformed from actual damages to the logarithm of damages. The transformation also facilitated the regression analysis by reducing the absolute range of the dependent variable. In general, the regression models were of the form:

$$\text{Log (Cost)} = a + b_i f(x_i)$$

where: a and the b_i's are parameters to be estimated and the x_i are the independent variables with the functional notation accounting for higher order polynomials in x_i. (a) represents the mean log cost value corresponding to equating the independent variables to zero, while b_i represents the change in log costs per unit change in $f(x_i)$.

The initial model was:

Model I: $\text{Log(Cost)} = a + b_1(V_1) + b_2(V_2) + b_3(W_1)$

where: V_1 and V_2 are the speeds of the vehicles and W_1 is the weight of the responding vehicle.

In attempting to develop the most meaningful relationships, seven equations were derived by combining certain classes of observations. Thus, within automobile collisions the eight angle accidents and the six rearend involvements were combined in order to increase the number of observations employed. Fixed object and non-collision accidents were also combined on the basis of the previous analysis of the two groups. Table 2.3.2 records the regression equations, coefficients of multiple correlations (R) and the standard errors of estimate, for the various struck objects and directions. The values of R^2, representing the amount of variation in log cost explained by the two speeds and weight, are also included.

An examination of equations (1) - (3), which are at a constant W_2, i.e. all truck accidents with varying directions, suggested an increasing mean cost level, as direction of impact moved from r_2 to r_1 to h. Within automobile accidents, (4) and (5), the mean level appeared to remain the same. Equation (7) which combined fixed object and truck-headon collisions showed a greater amount of explanation in variability than either group considered separately. Generally, a mean cost displacement was indicated among auto, truck, and fixed object and non-collision accidents.

Of particular interest in the first model were the signs of the coefficients of the variables. The sign reflects the direction in which the dependent variable changes given an increase in the independent variable. With respect to W_1, the weight of the responding vehicle, it was felt that gross weight was actually fairly constant in the sampled accidents, in that there would be no significant variability within and between classifications. In order to show that gross vehicle weight was not a significant contributor to vehicle damages within the accident classifications, tests of significance were performed on the W_1 coefficients, b_3 for the seven equations. Merely examining the regression equations revealed a changing sign attached

Table 2.3.2. Regression Equations of Model I

$\text{Log (Cost)} = a + b_1(V_1) + b_2(V_2) + b_3(W_1)$

$\left[Y = \log(\text{Cost})\right]$

Object Struck (W_2)	Direction (D)	Regression Equations	Multiple Correlation Coeff. (R)	R^2	Standard Error of Estimate
Truck	Head-on	(1) $y = 3.55 + .0050V_1 + .0070V_2 - .0066W_1$	.49	.24	.152
	Striking in Rear	(2) $y = 2.26 + .0180V_1 - .0061V_2 + .0260W_1$	.74	.55	.361
	Struck in Rear	(3) $y = 2.11 + .0006V_1 + .0200V_2 + .0021W_1$	.46	.21	.392
Auto	Head-on	(4) $y = 2.56 + .0110V_1 + .0099V_2 - .0110W_1$	.59	.35	.373
	Struck in Rear and Angle	(5) $y = 2.60 - .0024V_1 + .0072V_2 - .0038W_1$	.50	.25	.420
Fixed Object and Non-Collision	Head-on	(6) $y = 3.31 + .0012V_1 + .0120V_2^* - .0061W_1$	.30	.09	.336
Fixed Object and Truck	Head-on	(7) $y = 3.18 + .0054V_1 + .0090V_2 + .0047W_2$	.70	.49	.277

*2 accidents where the tractor trailer struck slow moving railroad trains were classified as fixed object involvements.

to b_3, i.e. it was negative in four equations. The W_1 coefficient was found to be significantly greater than zero only in equations (2) and (5), both rearend accidents. A control chart analysis was performed on the mean weights among the groups. Essentially, the mean weights for the seven equations were used to determine whether or not the deviations in mean weights were due to non-random variation. As is generally done in industrial analysis to control current processes, the mean, represented by the control line in Fig. 2.3.2, was determined from an external source. The means were compared to the mean loaded weight of a study by Dimmick[1/], in which 135,000 trucks were observed at weighing stations in 44 states. An estimate of the expected variance in truck weights was obtained from Sampson's report on gross weight of trucks.[2/] Figure 2.3.2 shows the observed means and the upper and lower control limits for each accident group. Since each mean was based on a different sample size, the limits, calculated from:

$$UCL = \overline{W}_1' + 3\frac{\sigma'}{\sqrt{n}},$$

$$LCL = \overline{W}_1' - \frac{3\sigma'}{\sqrt{n}}, \text{ where: } \overline{W}_1' = \text{expected mean}$$

σ' = expected standard deviation

varied for each group. No evidence appeared to suggest that the means were out of statistical control. The above analyses supported the hypothesis that variability in weights was not a significant contributor to the variability in vehicle damages and that changing sign of b_3

1/ Dimmick, T. B., "Traffic and Travel Trends, 1955," Public Roads, Vol. 29, No. 5, December 1956.

2/ Sampson, E., "State Highway User Taxes," Public Roads, Vol. 29, No. 12, February 1958.

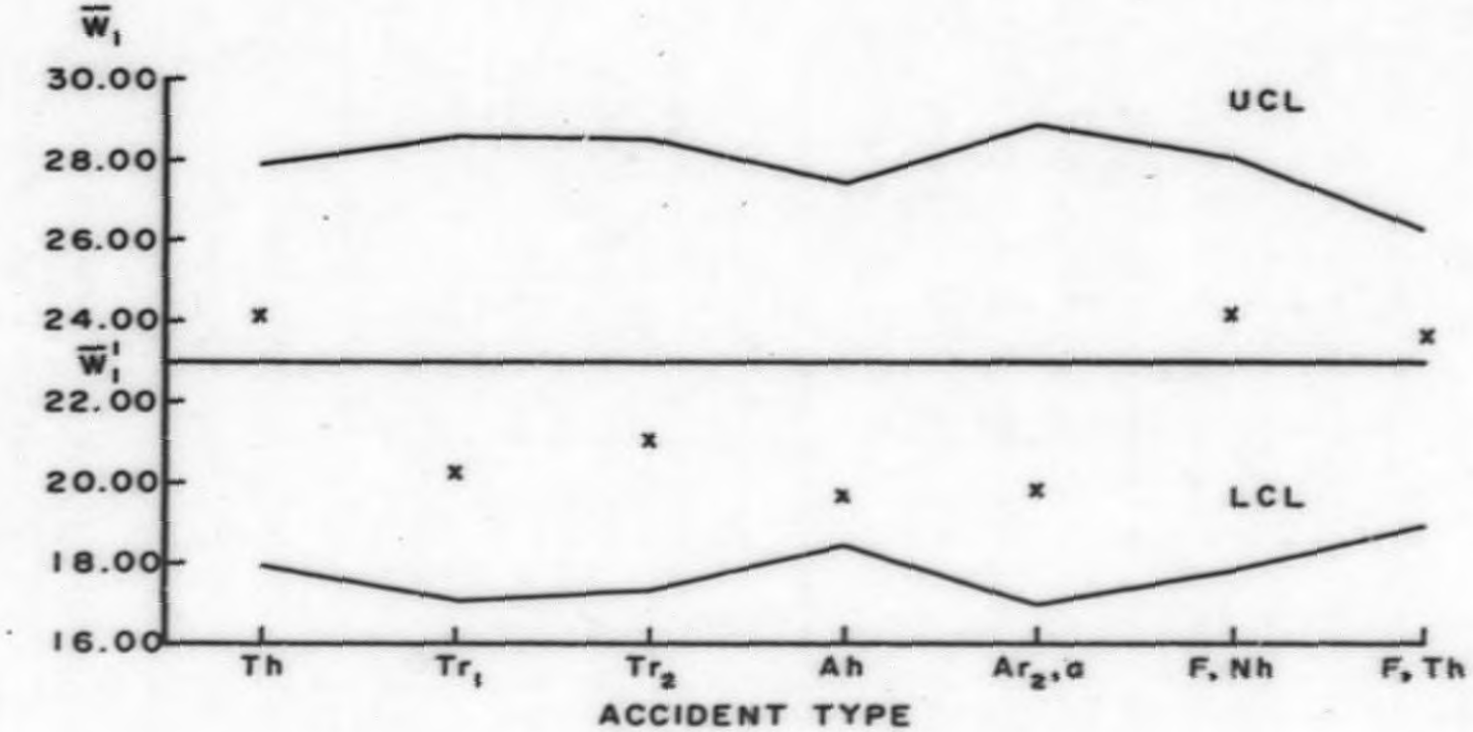

FIG. 2.3.2. CONTROL CHART FOR MEAN LOADED WEIGHTS FOR ACCIDENT TYPES: $\overline{W}_1' = 22.90$, $\sigma'_{w_1} = 7.41$

in the equations was merely a chance occurrence.

The signs of b_1 and b_2 seemed to offer more meaning than the weight coefficient. The signs of the speed coefficients lent insight for investigating the hypotheses that a) speeds may be added in headon accidents, i.e. $V_1 + V_2$ and b) lower speeds may be subtracted in rearend accidents, i.e. $V_2 - V_1$ or $V_1 - V_2$. All of the headon collisions showed positive signs for both coefficients, supporting the idea of an additive combined speed for this type of accident. Equation (2) where V_1 is the greater speed in the rearend accident and equation (5), where V_2 is the larger speed, the theory of subtraction seemed to be substantiated. Only in equation (3) where a negative sign was expected for V_1's coefficient was there disparity. However, the coefficient was found not to be significantly greater than zero, implying that the sign might be either negative or positive.

Model II, of the form: $\text{Log(Cost)} = a + b(V_1 \pm V_2)$, omitted the weight of the responding vehicle and concentrated on simple linear regressions of log cost versus the two speeds, combined according to the prior assumption. As only two variables were considered in order to simplify the model, the original classifications were used since the number of observations was considered sufficient for the simpler regression model. Table 2.3.3 presents equations relating combined speeds to log costs for truck-headon and both forms of rearend accidents; automobile-headon, rearend and angle; fixed objects and non-collision involvements.

Table 2.3.3. Regression Equations for Model II

$$\text{Log(Cost)} = a + b(V_1 \pm V_2)$$

$$\left[y = \text{Log(Cost)}\right]$$

W_2	D		Regression Equations	r	r^2	Std. Error
T	h	(8)	$y = 3.65 - .0026(V_1 + V_2)$	.22	.05	.161
	r_1	(9)	$y = 2.71 + .0264(V_1 - V_2)$	.73	.53	.293
	r_2	(10)	$y = 3.06 + .0004(V_2 - V_1)$	.00	.00	.127
A	h	(11)	$y = 2.33 + .0105(V_1 + V_2)$	.55	.30	.371
	a	(12)	$y = 2.04 + .0200\left[1/2(V_1+V_2)\right]$	.34	.15	.452
	r_2	(13)	$y = 2.02 + .0159(V_2 - V_1)$	.85	.72	.250
F	h	(14)	$y = 3.12 + .0101(V_1 + V_2)$	.32	.10	:311
N	h	(15)	$y = 3.38 - .0037(V_1 + V_2)$	-.10	.01	.353

In order to learn if the linear regressions of combined speed on log cost was the same for trucks, automobiles and fixed objects in headon accidents and also for trucks and autos in rearend accidents, analyses of covariance were performed. The primary purpose was to ascertain if the linear regression of equations (8), (11), (14) were the same for the three objects struck. Equal slopes with elevation differentials would support the concept of different mass levels, representing the type of object struck, contributing to accident severity differences. Figure 2.3.3 shows graphs of the three linear equations. The covariance model was of the form:

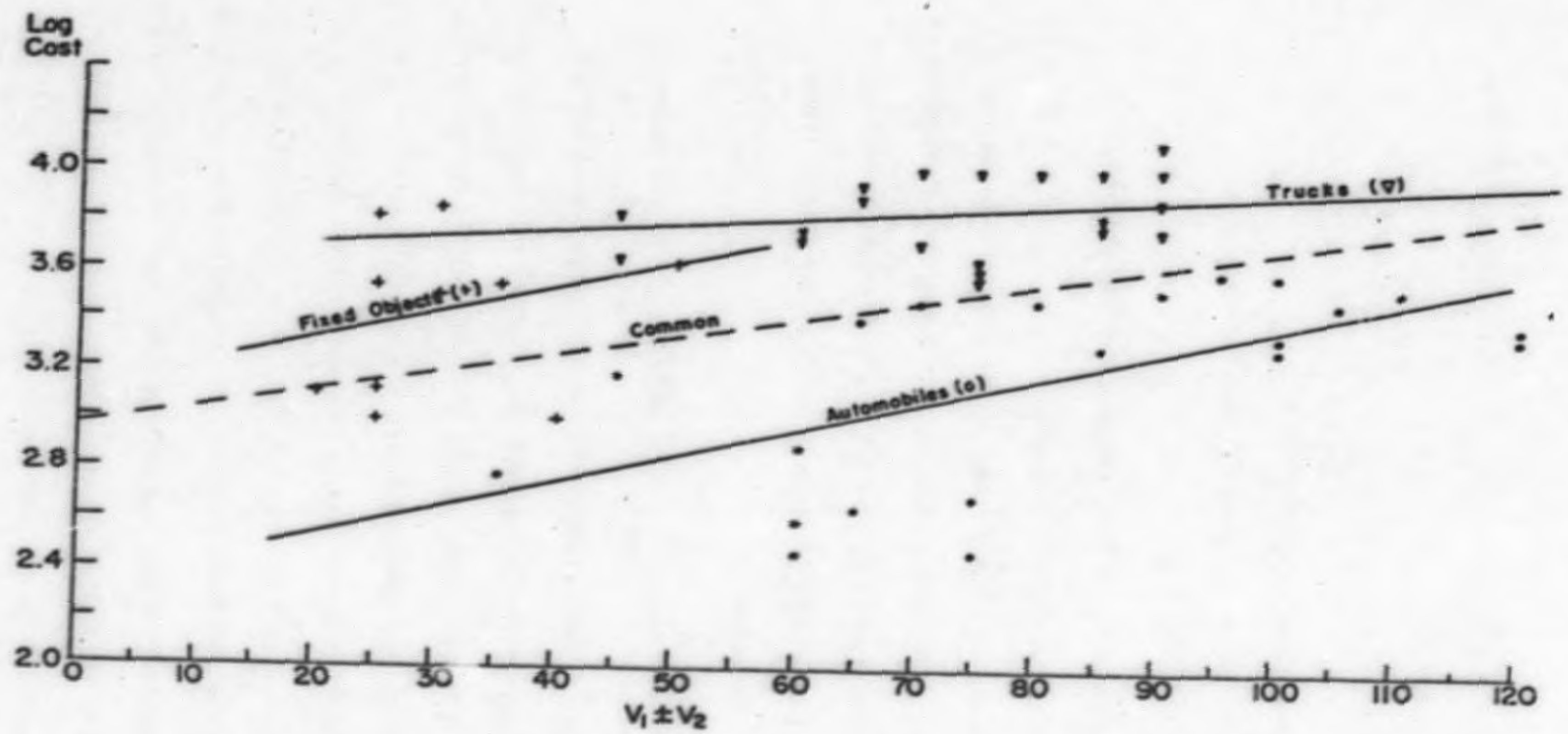

FIG. 2.3.3. LINEAR AND COMMON REGRESSION OF LOG COST ON COMBINED SPEED-HEAD ON ACCIDENTS

$$\text{Log(Cost)} = m + W_{2_j} + Bx_{ij} + e_{ij};\ i = 1, 2, \ldots n_j,\ j = 1, 2, 3$$

where: m = overall mean

W_{2_j} = effect of the three levels of W_2; $\sum W_{2_j} = 0$

B = assumed common regression slope

x_{ij} = deviation of any $(v_1 + v_2)$ from the total mean, $\overline{(v_1 + v_2)} = 68.4$

e_{ij} = random error; assumed normally distributed with mean zero and variance σ^2.

Table 2.3.4 illustrates a typical analysis of covariance arrangement. As seen from the graphs of the equations, the assumption of statistically equal slopes is not obvious so that an F test comparing the mean square for regression coefficients to the mean square within objects struck was performed. The result

$$F = \frac{\text{MS Regression}}{\text{MS within}} = \frac{.090}{.089} = 1.01,$$ failed to reject the assumption of slope equality so that a common slope was indicated. When the mean square for adjusted means was compared to the common mean square a significant F ratio was found, implying that although the regressions had a common slope, the equations differed in elevation levels which was partly ascribed to differences in the type of object struck. Furthermore, the common regression was found to be significant, when the hypothesis that $B = 0$ was rejected. The conclusion from the analysis might be alternatively stated that mean cost levels in headon accidents would be equal if the mean effect were constant when different types of objects were struck by a tractor semitrailer.

Table 2.3.4. Analysis of Covariance of Log(Cost) and Added Speeds for Headon Accidents and Object Struck (Trucks, Autos, Fixed Objects).

Source of Variation	Degrees of Freedom	Sums of Squares of Deviations			Regression Coefficient	Deviations from Regression		
		x^2	xy	y^2		d.f.	$\sum v^2$	Mean Squares
Fixed Objects	11	1072.89	10.78	1.07	.0101	10	.97	.097
Trucks	19	3693.75	9.46	.49	.0026	18	.47	.026
Automobiles	23	11483.20	121.08	4.30	.0105	22	3.02	.137
Within W_2						50	4.46	.089
Reg. Coefficient						2	.18	.090
Common Regression	53	16249.84	141.32	5.86	.0087	52	4.63	.089
Adjusted Means						2	5.50	2.750
Total	55	36155.03	109.15	10.46		54	10.13	

A similar analysis was conducted for rear-end accidents where the responding vehicle struck autos and trucks in the rear,[1/] i.e. equations (10) and (13). The analysis of covariance found slopes as well as cost elevation levels to differ significantly, most probably due to the wide scatter of the truck observations, manifested by the zero correlation coefficient. As will be shown later, when these types of accidents are grouped within their respective classification of object struck, they seemed to represent the lower cost level of damages within a mass category.

The data seemed to reveal that dollar damages received by a tractor semitrailer, and hence, accident severity, approaches an upper limit dependent upon the object struck and combined speeds. It was felt that the introduction of curvature to the regression models would not merely explain more of the variability in damages, but would also lead to an analytic inference as to the area of the upper limit. Emphasis was lent to the above argument, particularly the curvilinear aspect, when combined speeds were plotted for all directions within a given object struck level. Figure 2.3.4, showing all of the observed points for truck accidents, offers empirical reasoning for the introduction of curvature, and the necessity for combining all accidents for a particular object struck. In the framework of response surface language, in two-dimensional space, i.e. log costs and combined speeds, when attention is relegated to a particular region of interest a linear fit is often acknowledged. Previously, the regions of interest associated with rearend and headon accidents for a particular object

1/ There were no r_1 type accidents reported for truck semitrailer auto collisions.

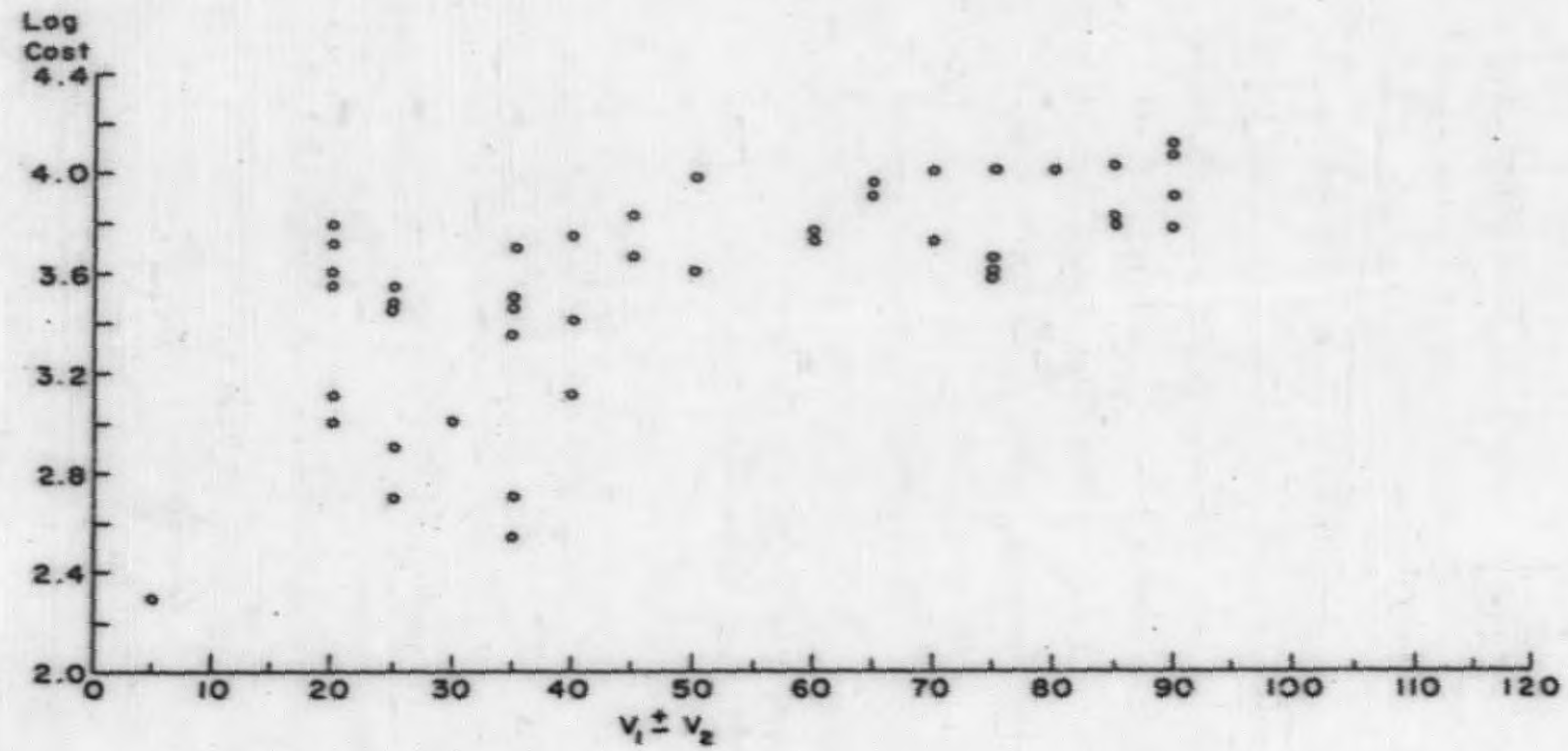

FIG. 2.3.4. OBSERVATIONS OF LOG COST AND COMBINED SPEEDS FOR ALL TRUCK ACCIDENTS

were considered and linear approximations were found adequate. However, when the entire region of operability is considered, a curve of higher degree is usually required in order to satisfactorily represent the surface. Indeed, curvature appears to occur in Figure 2.3.4 when the range of combined speeds, including headon accidents occurring at greater speeds, are incorporated with the lower speed rearend accidents. Thus, aside from improving the goodness of fit and explaining more variability in the dependent variable through curvature as well as offering the upper limit area, an empirical rationale for curvature was developed.

Since some headon accidents themselves occurred at low speeds, curvilinear equations of the second degree were fitted to headon accidents as well as all accidents for a given mass level, in Table 2.3.5, where Model III was of the form:

Log($\text{Log(Cost)} = a + b(V_1 \pm V_2) + c(V_1 \pm V_2)^2$

The consistently high correlation indexes, I in Table 2.3.5 reflect the high degrees of association between combined speeds and dollar damages for all types of collisions given a specific object struck. In other words, the grouping of direction components increased the ability of combined speeds to explain variability in damage costs. In order to ascertain the appropriateness of parabolic curves to describe the cost response significance tests were performed between linear regressions and the curvilinear regressions for the groups. Figures 2.3.5-2.3.7 and Table 2.3.6 point out the lack of a significance difference between any of the curvilinear and linear regressions.

Table 2.3.5. Regression Equations for Model III

y = Log(Cost)

Direction	Object Struck	n	Regression Equations	Index of Correlation (I)	I^2	Standard Error
All	Truck	49	(16) $y = 2.744 + .0213\ (V_1 \pm V_2) - .00009(V_1 \pm V_2)^2$	.65	.43	.344
All	Automobile	38	(17) $y = 2.131 + .0166(V_1 \pm V_2) - .00004(V_1 \pm V_2)^2$	.69	.47	.362
Headon All	Fixed Object) Truck)	61	(18) $y = 2.797 + .0205(V_1 \pm V_2) - .00009(V_1 \pm V_2)^2$	.62	.39	.334
Headon	Auto	24	(19) $y = 2.246 + .0128(V_1 \pm V_2) - .00001(V_1 \pm V_2)^2$	.55	.30	.379

Table 2.3.6. r^2 and I^2 Values for Linear and Curvilinear Regressions: All Types of Accidents.

Direction	Object Struck	Linear (r^2)	Curvilinear (I^2)
All	Truck	.42	.43
All	Auto	.47	.47
Headon	Auto	.30	.30
Headon	Fixed Object)		
	)	.38	.39
All	Truck)		

Significance tests similar to that in Table 2.3.7 were performed and a straight line was found to be as applicable as a second degree curve.

Table 2.3.7. Test of Significance of Curvilinear Regressions: All Truck Accidents.

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square
Curvilinearity of Regression	1	.088	.088
Deviations from Curv. Regression	46	5.458	.119
Deviations from Linear Regression	47	5.546	

$$F = \frac{.088}{.119} = .739, \text{ not significant}$$

A covariance analysis was performed on the linear equations shown in Figures 2.3.5-2.3.7, in order to determine the effectiveness of a single equation depicting the relationship between costs and speeds for all accidents. The model under consideration was:

$$\text{Log(Cost)} = m + W_{2_j} + Bx_{ij} + e_{ij};\ i = 1_1 \ldots n_j,\ j = 1, 2, 3$$

where: m = overall mean

W_{2_j} = effect of jth object struck

j = 1, all truck accidents

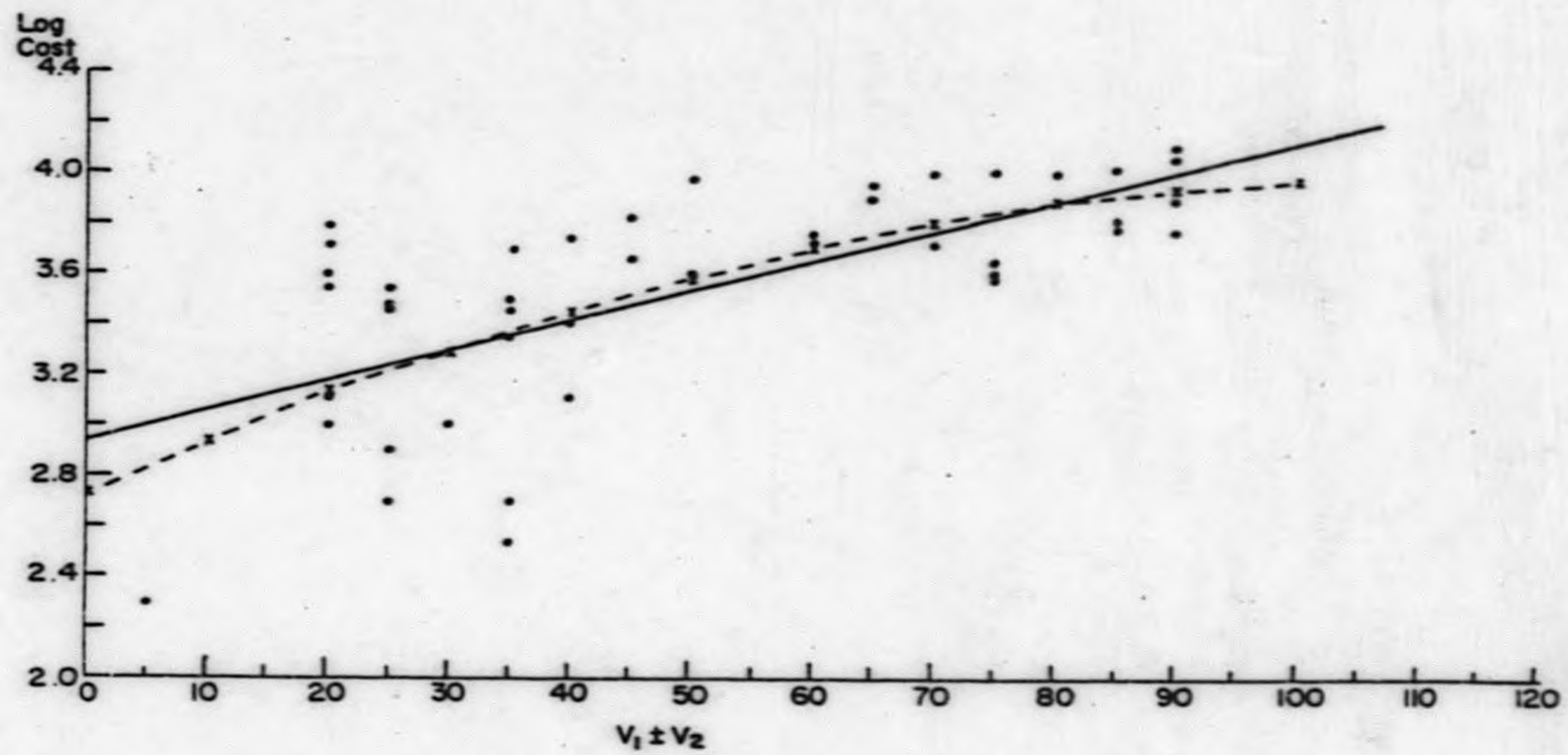

FIG. 2.3.5. LINEAR AND CURVILINEAR REGRESSION OF LOG COST ON COMBINED SPEEDS FOR ALL TRUCK ACCIDENTS

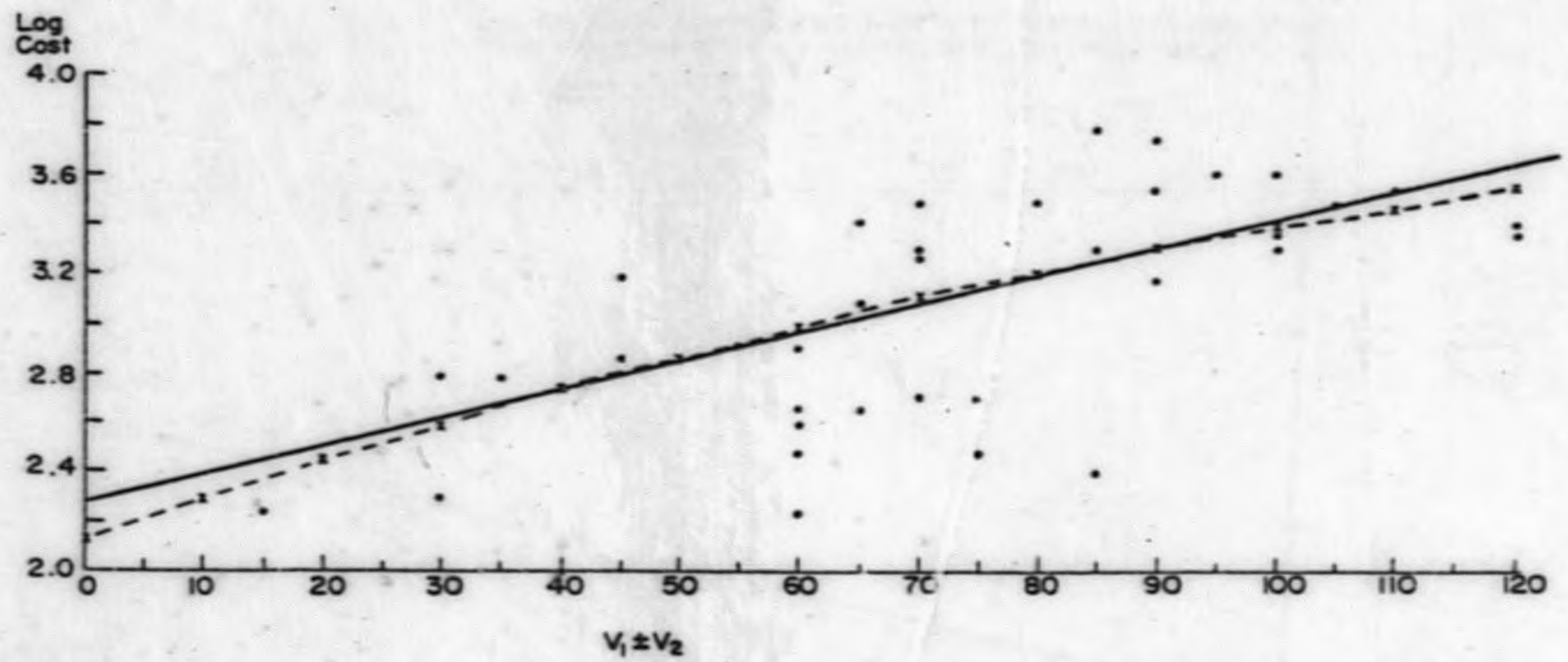

FIG. 2.3.6. LINEAR AND CURVILINEAR REGRESSION OF LOG COST ON COMBINED SPEEDS FOR ALL AUTO ACCIDENTS

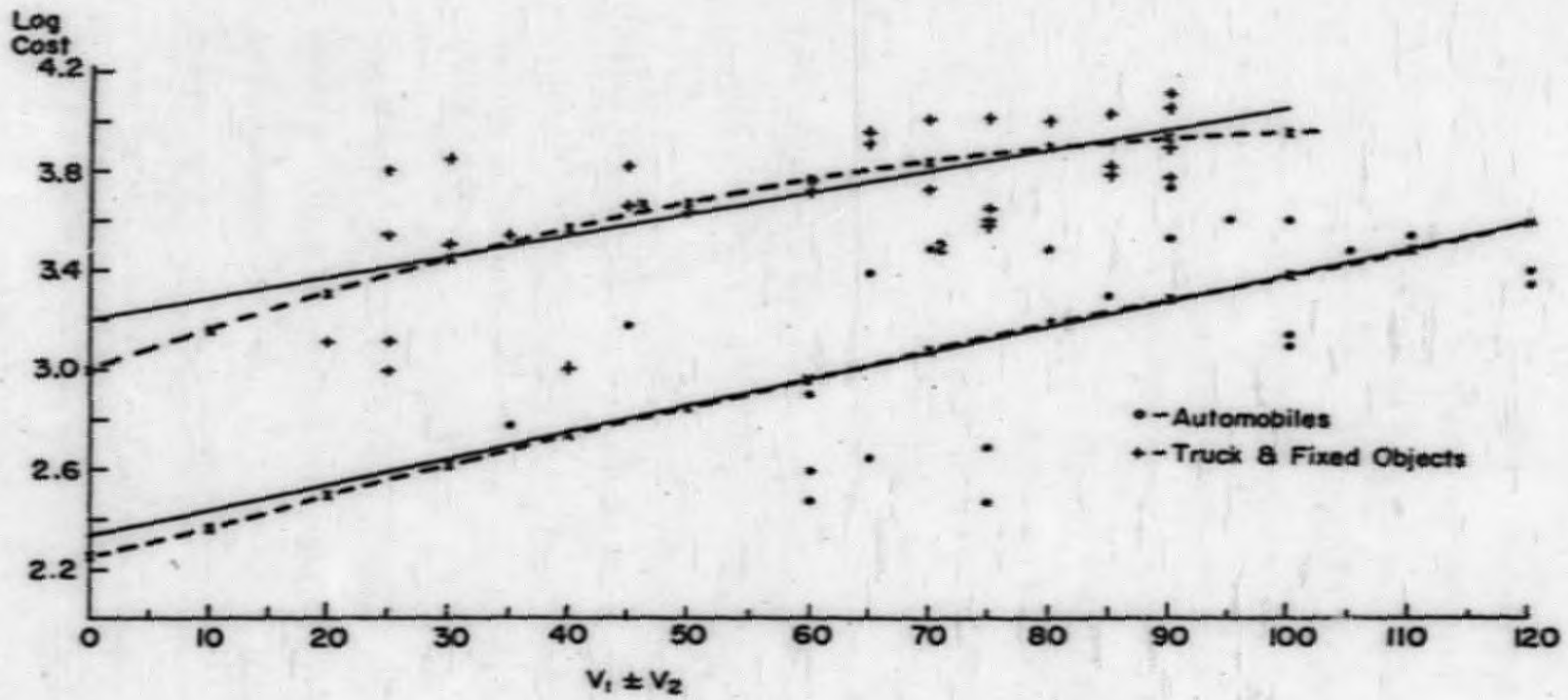

FIG. 2.3.7. LINEAR AND CURVILINEAR REGRESSION OF LOG COSTS ON COMBINED SPEEDS FOR ALL HEAD ON ACCIDENTS

$j = 2$, all automobile accidents

$j = 3$, all fixed object accidents

x_{ij} = deviation of any $(V_1 \pm V_2)$ from the overall mean, $\overline{(V_1 \pm V_2)} = 52.9$

e_{ij} = random error; assumed normally distributed with mean zero and variance σ^2.

Table 2.3.8 shows the analysis of covariance table. The results showed the three equations

$j = 1$, $\text{Log(Cost)} = 2.94 + .0119(V_1 \pm V_2)$

$j = 2$, $\text{Log(Cost)} = 2.28 + .0113(V_1 \pm V_2)$

$j = 3$, $\text{Log(Cost)} = 3.12 + .0101(V_1 \pm V_2)$

did not have different slopes, so that a common regression coefficient could be applied to a representative equation for all accidents. The resulting common equation was: $\text{Log(Cost)} = 2.70 + .0115(V_1 \pm V_2)$. However, the high variance ratio for the adjusted mean indicated differential elements in cost elevation, after adjusting for different objects struck. Thus, although a common slope could be introduced, there remained significant discrepancies in the cost levels, ascribed to the type of object struck. Figure 2.3.8 transforms the common equation in log costs to the exponential form of actual dollar damages sustained by the tractor semitrailer as a function of combined speeds.

The parabolic curves, though not significantly different from lineality in describing the cost-speed relationship, offered, for each accident group, a maximum cost area that a particular combined speed might generate. Table 2.3.9 shows the upper cost levels, obtained by

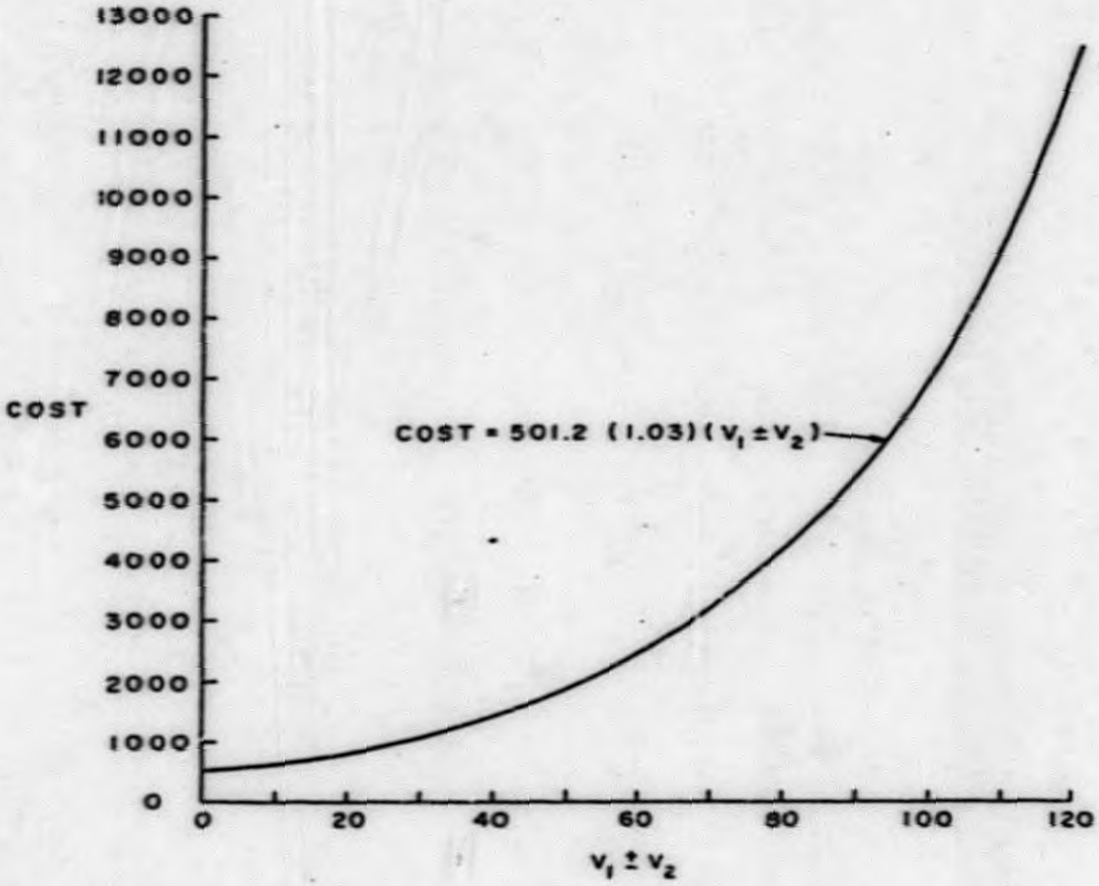

FIG. 2. 3. 8. ACTUAL DAMAGE COSTS AND COMBINED SPEEDS; TRANSFORMATION OF COMMON EQUATION

Table 2.3.8. Analysis of Covariance of Log(Cost) and Combined Speeds for All Directions and Objects Struck (Truck, Auto, Fixed Object)

Sources of Variation	Degrees of Freedom	$\sum x^2$	$\sum xy$	$\sum y^2$	Reg. Coeff.	d.f.	$\sum v^2$	Mean Square	F	$F_{.05}$
Fixed Objects	11	1072.8903	10.7846	1.0736	.0101	10	.9651	.0965		
Trucks	48	28998.0000	344.6246	9.6417	.0119	47	5.5460	.1180		
Automobile	37	31928.8948	359.3072	8.6799	.0113	36	4.6377	.1288		
Within W_2						93	11.1488	.1199		
Reg. Coefficient						2	.0071	.0036	.0300	3.1
Common Regression	96	61999.7851	714.7164	19.3952	.0115	95	11.1559	.1174		
Adjusted Mean						2	10.2004	5.1002	43.443	3.1
Total	98	74163.4862	491.646	24.6182		97	21.3563			

equating the first derivative of the second degree polynomial to zero, and solving for $V_1 \pm V_2$.

Table 2.3.9. Upper Cost Levels for All Accident Groups

Object Struck	Log(Dollars)	Actual Dollars	Required ($V_1 \pm V_2$)
Truck	4.01	10,200	115
Automobile	3.85	7,080	205
Fixed Object)) Non-Collision)	3.43	2,700	40

The value of combined speed represented the point where the maximum cost level might be expected to occur, as a result of a certain level of speeds. The figures in Table 2.3.9 indicated, for all tractor-semitrailer involvements with other trucks, a combined speed of 115 miles per hour was necessary before dollar damages to the vehicle reached $10,200. In contrast, automobile involvements require speeds exceeding 200 m.p.h. before a level of $7,000, less than that for trucks, was reached. Similarly, fixed object and non-collision accidents, essentially including only the speed of the responding vehicle, required a speed of 40 m.p.h. to reach a relatively low cost level of $2,700. The reason for this last conclusion probably arises from the confounding of fixed objects such as trees, utility poles and fences with objects with considerably greater mass such as bridges, buildings and railroad trains, while non-collision accidents may run the gamut from a slight damage-producing skid to a severe overturn as a result of jackknifing.

2.4. Vehicle Damage and Mechanical Energy

Whereas the foregoing analysis attempted empirically to derive theoretical models relating the vehicle damage cost of a collision to accident characteristics, the subsequent analysis utilizes the concepts of energy and momentum, commonly studied in connection with impact problems. A theoretical model representing the mechanical energy of the system was developed and the resulting energy equation was then considered as a dependent variable in fitting empirical equations for damage costs. This mechanism yielded a general equation for all types of accidents relating dollar damages to the concept of energy released or considered available for damage.

The general model represents the conditions of two bodies approaching one another, colliding, and at some instant after impact, separating. Because of the laws of conservation of energy, the total energy remains constant throughout the impact. From the principle of the conservation of momentum, the total momentum of the colliding bodies is also unaltered by the collision. Assuming perfect inelasticity, where the colliding bodies remain together after the collision and move with the same velocities, the maximum work done under impact can be determined. Generally, for only an instant do the bodies have equal velocity before separation occurs.

If a collision between two bodies is perfectly inelastic, the equations

(2.4) $1/2m_1v_1^2 + 1/2m_2v_2^2 = 1/2(m_1 + m_2)v_c^2 + E$

and (2.5) $m_1v_1 + m_2v_2 = (m_1 + m_2)v_c$

must be satisfied. Equation (2.4) represents the conservation of energy and (2.5) the conservation of momentum, where: v_1 and v_2 are the velocities of the colliding bodies before impact, m_1 and m_2 are the respective masses, $v_c = v_1 \pm v_2$, is the common velocity after impact and E is the maximum amount of energy available for damage. Solving the two equations simultaneously yields

(2.6) $E = 1/2\frac{m_1 m_2}{m_1 + m_2}(v_1 \pm v_2)^2.$

Combining the units of the terms of (2.6) reveals the dimensions of E. Since m_1 and m_2 are in $lb \cdot sec^2/ft.$ and v_1 and v_2 are in ft/sec, the dimensional analysis yielded

$$\frac{(lb \cdot sec^2/ft)^2}{lb \cdot sec^2/ft} \cdot ft^2/sec^2 = ft \cdot lb,$$ as the unit of

available energy. Equation (2.6) readily becomes

(2.7) $E = 1/2\ mv^2 = 1/2\frac{w}{g}v^2$

where: $\frac{w}{g} = m = m_1 m_2/(m_1 + m_2)$, $v = v_1 \pm v_2$ and g is the acceleration of gravity ($32.17\ ft/sec^2$). (2.7) represents the theoretical model for the maximum energy available for damage due to an impact of two colliding vehicles.

Anticipating the following section where energy was considered as a dependent variable, it became necessary to correct for the fact that velocity was recorded in miles per hour (V) and weight in tons (W). Thus, if $E = WV^2$, a dimensional transformation yields

$$K = \frac{1}{64.34\ ft/sec^2} \times \frac{2000\ lb}{ton}\ \frac{(5280)^2\ ft^2/min^2}{(3600)^2 sec^2/hr^2}$$

or $K = 66.89\ \frac{lb}{ton} \times \frac{hr^2}{min^2} \times ft$. Therefore, the working model for energy was

(2.8) $E = 66.89\ WV^2$ in foot-pounds.

The previous regression models actually derived equations for the logarithm of costs resulting from a collision as a function of the energy components, mass, velocity and direction (reflected in the manner of combining speeds). These equations enabled statistical analysis to be performed in analyzing the components. Thus, it remained to fit a generalized equation to log costs and energy as theoretically derived. The points of Figure 2.4.1 indicated that a hyperbolic function of the form:

$$(2.9) \quad \text{Log(Cost)} = \frac{E}{a + bE}$$

might best describe the relationship between damage and energy. The two branches of the hyperbola are asymptotic to the lines $E = \frac{-a}{b}$ and $\text{Log Cost} = \frac{1}{b}$.

By algebraic manipulation, (2.9) is easily transferred into the form of a straight line for a specific function of log cost and energy. Thus,

$$(2.10) \quad \frac{E}{\text{Log Cost}} = a + bE,$$ which is equivalent to (2.9)

is linear in E and E/Log Cost. To facilitate the estimation of the two parameters, a and b, the data were grouped in a bivariate frequency table, as presented in Table 2.4.1. The resulting equation:

$$\text{Log Cost} = \frac{E}{(.02) \cdot 10^{6} + .242E}$$

yielded a flexible curve to describe energy and damage.

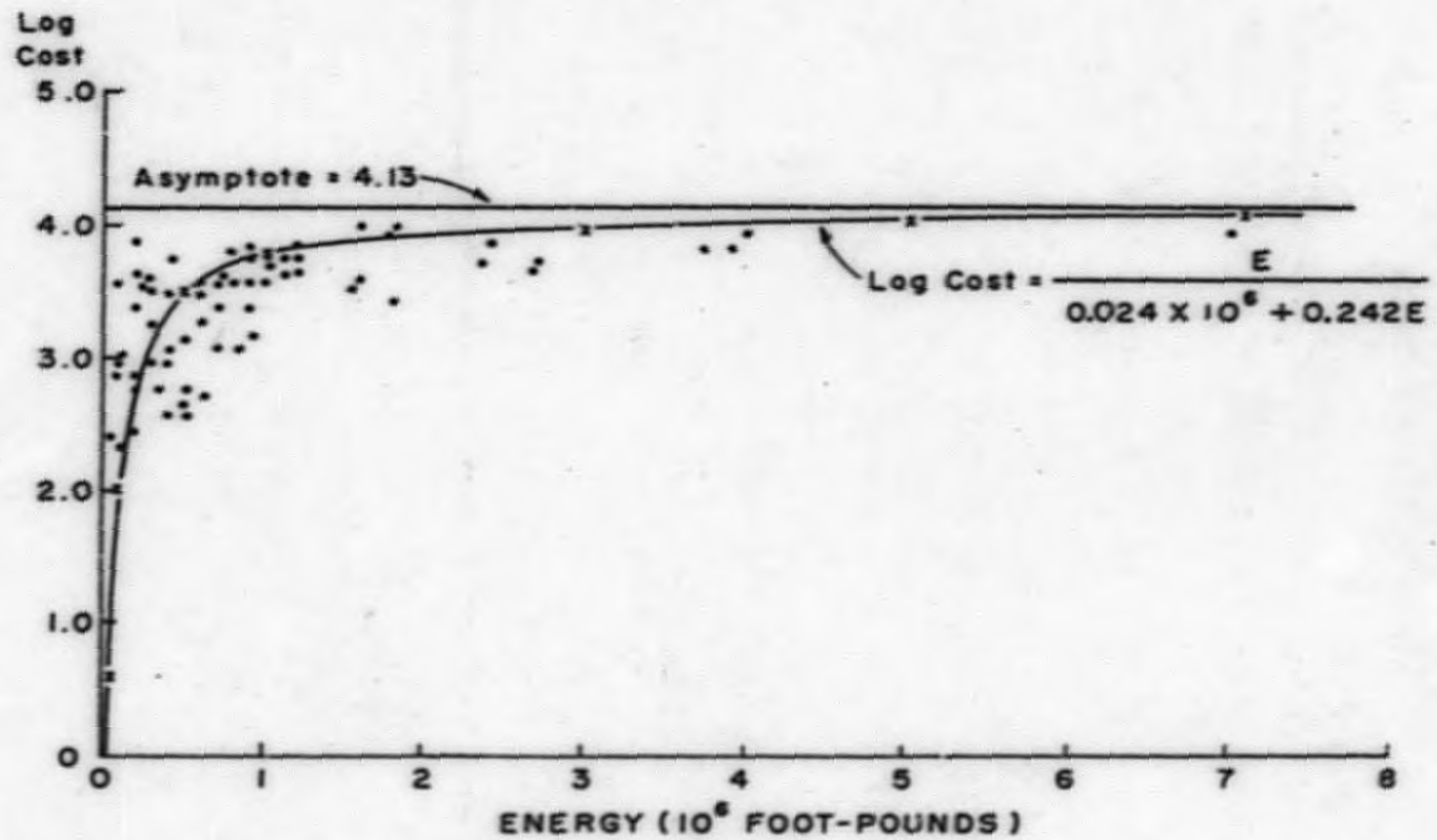

FIG. 2.4.1. RELATION BETWEEN MECHANICAL ENERGY AND VEHICLE DAMAGE

Table 2.4.1. Bivariate Frequency Table of Energy/Log Cost and Energy

Energy / Log Cost in 10^5 ft-lbs	Energy in 10^6 Foot-lbs. 0.0-.9	1.0-1.9	2.0-2.9	3.0-3.9	4.0-4.9	5.0-5.9	6.0-6.9	7.0-7.9
17.0-17.9								4
16.0-16.9								
15.0-15.9								
14.0-14.9								
13.0-13.9								
12.0-12.9						1		
11.0-11.9					3			
10.0-10.9				2	3			
9.0-9.9		1		1				
8.0-8.9								
7.0-7.9		1		2				
6.0-6.9			2					
5.0-5.9		3						
4.0-4.9		9	1					
3.0-3.9	3	9						
2.0-2.9	7	8						
1.0-1.9	18							
0.0-0.9	21							

The upper asymptote emphasized the approach of a maximum cost level as a function of energy available for damage as a result of impact between two vehicles. Indeed, if the upper asymptote of the curve is considered to represent the point at which total loss of the vehicle occurs, total loss probabilities may be derived from the probability distribution of vehicle damages. By inferring that total loss of the vehicle occurs at the upper asymptotic level of log cost, the probability of a tractor-semitrailer sustaining this level of damage in an accident may be obtained by considering the normally distributed variable.

$$z = \frac{T - C}{S}$$

where: z = standard normal deviate

T = total loss level, 1/b in $\frac{E}{a + bE}$

C = mean of logarithm of cost, from cost distribution in 2.1

S = standard deviation of distribution of log cost.

Table 2.4.2 summarizes the results and yeilds the probability of total loss when different types of objects are struck.

Table 2.4.2. Deviation of Total Loss Probabilities for Objects Struck, Total Loss Level - 4.1322 (T).

Object Struck	Mean Log Cost (C)	Standard Deviation (S)	$\frac{T - C}{S} = z$	Probability of Log Cost $\geq$ T
Auto	2.28	.71	2.61	.0045
Truck	2.63	.59	2.55	.0054
Fixed, Non-Collision	3.04	.52	2.09	.0188

It is evident that the probability of vehicle total loss occurring increases, as the general mass of the object struck increases. Thus, the probability of a total loss is greatest when fixed objects or non-collision accidents occur and least when automobiles are involved. Figure 2.4.2 exhibits the rate at which the total loss level is approached for different objects struck. The three curves indicate the concept that as the total loss level decreases, the probabilities of attaining total loss showed greater differences among accident groups. Whereas the differences in probabilities are small at the high level derived from the energy curve, a reduction in this level would indicate sharp contrast in attaining total loss for different mass levels of objects struck.

Returning to the cost-energy relationship, the function substantiated the feeling that beyond a certain level of damage producing energy, dollar damage increases in smaller and smaller amount as total loss is approached. Alternatively, there is a rapid rise in damages generated by the initially released energy, until a point is reached where energy can no longer contribute significantly to the damage sustained by the vehicle.

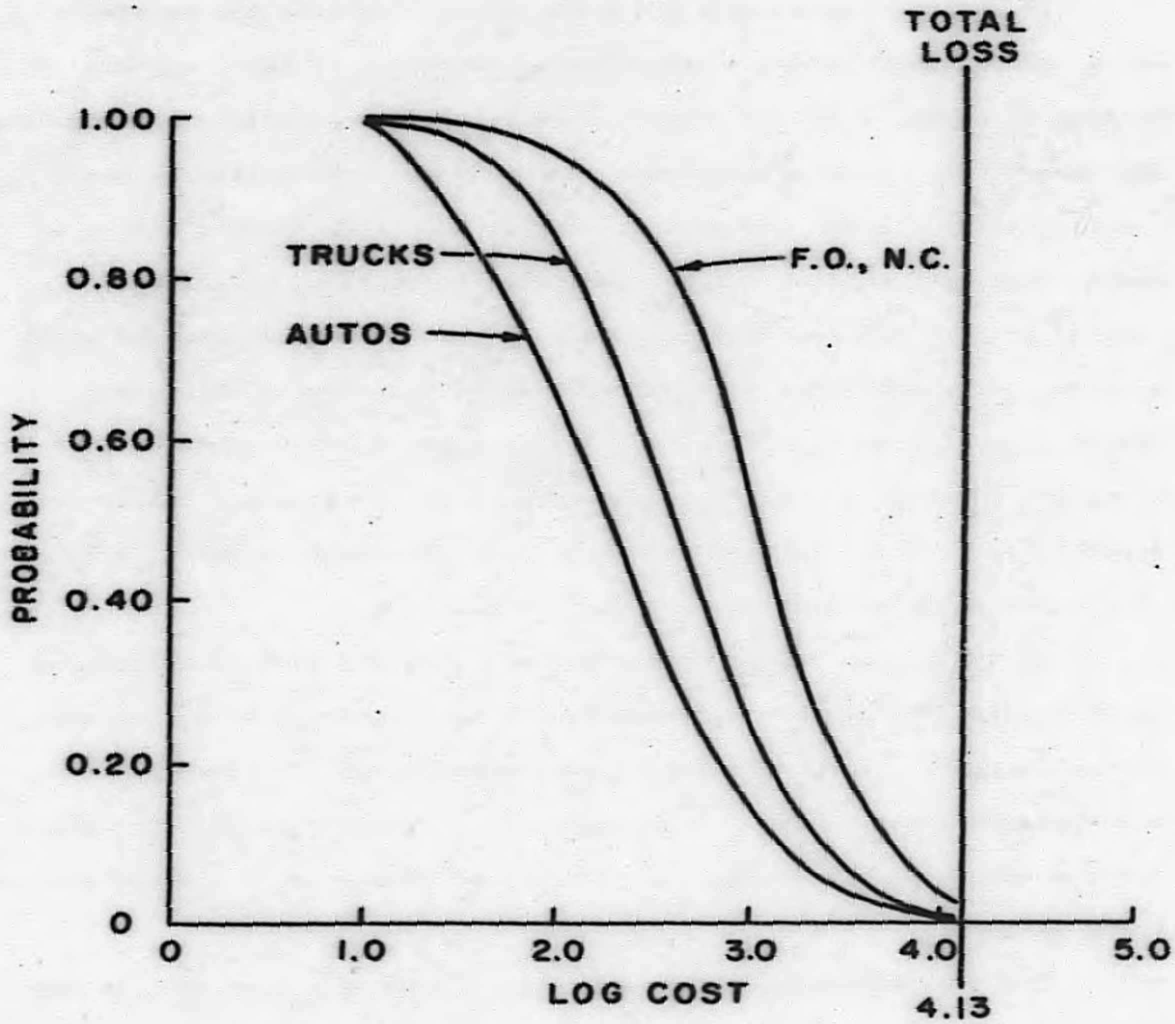

FIG. 2.4.2. MANNER IN WHICH TOTAL LOSS IS APPROACHED IN ACCIDENT TYPES

3.0. Analysis of Cargo Damages Resulting from Accidents.

In order to understand and meaningfully describe the manner in which tractor-semitrailers, loaded with commercial freight, sustain damages to cargo, different types of cargos must be properly distinguished and identified. Since a multitude of cargos are transported by our transportation system, the general analysis of cargo damages has to be partitioned into sets of individual analyses pertinent to a specific cargo. So that inferences analogous to those for vehicle damages might be drawn, it was necessary to gather information about a sufficient number of accidents involving cargo damages for a homogeneous load of freight. Furthermore, in order to investigate the accident characteristic effects, those damages resulting from extraneous effects, such as fires, had to be eliminated.

At the present time, this somewhat Herculean task of collecting cargo damage data has been satisfactorily performed for only one cargo, new automobiles. The three predominant reasons for fruitless searches elsewhere have been lack of sufficient cargo damage records, inadequate systems of recording information, and the predominance of non-homogeneous cargo shipments.

The following discussion presents the analysis performed on the automobile carrier accident experience and suggests the type of analysis that might be employed on other forms of cargo.

3.1. Damages Sustained by New Automobile Carriers.

Prior experience suggested that automobile carriers travelled a relatively large number of vehicle miles and were involved in a sizeable number of accidents. This assumption was substantiated by the Interstate Commerce Commission report of property-accident data for the fourth quarter of 1958, which showed motor vehicle carriers travelling 264,524 thousand vehicle miles (12% of all carriers), ranking third behind general freight and miscellaneous commodity carriers. The same relative position was held for total number of accidents reported (13% of all accidents). Automobiles certainly satisfied the homogeneity criterion and new automobile damage could be evaluated fairly easily at the occurrence of an accident, thereby easing the reporting of cargo damages in the ICC report form.

A list of ICC licensed auto carriers was procured and a random sample of the ICC accident files for the years 1958 and 1959 was selected. Only those accidents resulting in both vehicle damage and cargo damage were included in the sample. A total of 189 observations was gathered. Table 3.1.1 shows the breakdown by type of accident, i.e. object struck.

Table 3.1.1. Sample of Accidents of New Automobile Carriers by Object Struck, 1959-1960

Object Struck	Number in Sample
Automobile	43
Commercial Vehicle	50
Fixed Object	29
Non-Collision*	67
	189

*Most non-collision accidents were characterized by the tractor-semitrailer jacknifing and overturning.

Since the carriers included in the sample were carrying from one to five new passenger cars at the time of collision, the percentage of total cargo that was damaged was considered as the measure descriptive of cargo damage. In order to approximate the original cargo value, the average wholesale value of passenger cars for the years involved, as reported by the Automobile Manufacturers' Association[1/], was employed. This figure of $1,880 for both 1958 and 1959 was used as the average value of the transported automobile. The average value, multiplied by the number of cars being carried, afforded a base upon which the percentage of cargo damaged was computed. Table 3.1.2 presents the positively skewed frequency distributions of percentage of cargo damaged and type of object struck.

It seemed obvious that the distribution of cargo damages would differ significantly between collision and non-collision type accidents. However, within collision type accidents differences were not obvious. A chi-square test was performed for the homogeneity of the distributions of percentage cargo damaged for auto, truck, and fixed object accidents. The test for homogeneity is presented in Table 3.1.3. The computed value of chi-square was 10.96, less than the 0.05 critical value of 18.34 with 10 degrees of freedom. This indicates that the distributions of percentage of cargo damaged for automobile cargos did not differ among objects struck in collision accidents.

1/ "Automobile Facts and Figures," Automobile Manufacturers Asso., 1959-1960.

Table 3.1.2. Frequency Distributions of Percentage of Cargo Damaged and Object Struck, Automobile Carriers.

% of Cargo Damaged	Object Struck: Trucks	Autos	Fixed-Objects	Non-Collision
-10.0	35	34	19	7
10.1 - 20.0	7	5	5	14
20.1 - 30.0	3	1	1	12
30.1 - 40.0	1	1	1	10
40.1 - 50.0	1	0	1	10
50.1 - 60.0	1	1	1	4
60.1 - 70.0	1	0	0	5
70.1 - 80.0	0	0	0	3
80.1 - 90.0	1	0	0	1
90.1 -100.0	0	1	1	1
Total	50	43	29	67
Mean % Damage	12.80	10.58	14.66	34.25
Variance of % Damage	2.77	2.57	3.90	4.64

Table 3.1.3. Observed and Theoretical Frequencies of Percentage Cargo Damaged for Collision-Type Accidents.

% of Cargo Damaged	Collision with: Autos	Trucks	Fixed Objects	Total
- 1.00	12 (9.87)	12 (11.48)	4 (6.65)	28
1.01 - 5.00	19 (16.57)	20 (19.26)	8 (11.17)	47
5.01 - 10.00	3 (4.58)	3 (5.33)	7 (3.09)	13
10.01 - 20.00	5 (5.99)	7 (6.97)	5 (4.04)	17
20.01 - 40.00	2 (2.82)	4 (3.28)	2 (1.90)	8
Greater than 40.00	2 (3.17)	4 (3.69)	3 (2.14)	9
Total	43	50	29	122
Chi-Square	2.20	1.26	7.50	10.96
			$Chi\text{-}Square_{0.05}$ (10)	18.34

Theoretical probability functions were derived for percentage of cargo damaged for collision and non-collision type accidents. Since the variable, percent damage, was restricted to the positive range between zero and one, a theoretical distribution was fitted which satisfied this restraint and was of a form thought to be descriptive of the observed distribution. The density function

$$f(p;\, a,\, b) = \frac{(a + b + 1)!}{a!\; b!}\, p^{a}(1 - p)^{b}, \quad 0 \leq p \leq 1,\ a,\ b > -1$$

known as the beta or Pearson type I distribution was considered. The method of moments provided estimates of the two parameters, a and b. By simultaneously solving the two equations representing the expected value, E(p) and variance, Var(p), the distribution was specified. Thus,

$$\bar{p} = E(p) \frac{a + b}{a + b + 2}$$

$$Var(p) = \frac{(a + 1)(b + 1)}{(a + b + 2)^2(a + b + 3)}$$

yields

$$a = \frac{\bar{p}(b + 2) - 1}{1 - \bar{p}} \quad \text{and } b = \frac{\bar{p}\left[(1 + \bar{p})^2 + Var(p)\right] - 2\,Var(p)}{Var(p)}$$

Table 3.1.4 summarizes the results of fitting beta distributions to the percentage of cargo damaged for pooled collision and for non-collision accidents, as well as the X^2 test for goodness of fit. The distributions were found to adequately describe the frequencies with which cargo damages occurred. Figure 3.1.1 shows graphs of the theoretical distributions and serves to indicate the flexibility of the type I curve, since the exponential form for collision accidents arises when $-1 < a \leq 0$ and $b > 0$. The more general form is that for non-collision accidents.

Table 3.1.4. Observed and Theoretical Frequencies, Based on Beta Distribution Assumptions, for Percentage of Automobile Cargos Damaged in Collision and Non-Collision Accidents.

Proportion of Cargo Damaged	Collision			Non-Collision		
	Observed (f_o)	Theoretical (f_t)	$\frac{(f_o-f_t)^2}{(f_t)}$	Observed (f_o)	Theoretical (f_t)	$\frac{(f_o-f_t)^2}{(f_t)}$
- .10	88	78.7	1.099	7	9.6	.610
.11 - .20	17	16.3	.030	14	11.7	.451
.21 - .30	5	9.8	2.351	12	11.3	.043
.31 - .40	3	6.2	1.651	10	11.1	.109
.41 - .50	2	4.4	1.309	10	8.5	.265
.51 - .60	3	3.0	0.000	4	6.7	1.088
.61 - .70	1	1.9	0.426	5	4.8	.008
.71 - .80	0	1.1		3	2.9	
.81 - .90	1	.5	0.994	1	.3	.876
.91 -1.00	2	.1		1	.1	
	122		$x^2 = 7.656$	67		$x^2 = 3.450$

$x^2_{.05}(5) = 7.82$ $\qquad$ $x^2_{.05}(5) = 11.07$

Collision	Non-Collision
$\bar{p} = 12$	$\bar{p} = .34$
$Var(p) = .03$	$Var(p) = .05$
$a = -.67$	$a = .32$
$b = 1.32$	$b = 1.53$

$$^{*}f(p) = \frac{(1.65)!}{(-.67)!(1.32)!} \quad p^{-.67}(1-p)^{1.32}$$

$$f(p) = \frac{(2.85)!}{(.32)!(1.53)!} \, p^{.32}(1-p)^{1.53}$$

*The factorials were evaluated from Davis, H. T., "Tables of the Higher Mathematical Functions," Vol. I, Colorado Springs, Col., 1933.

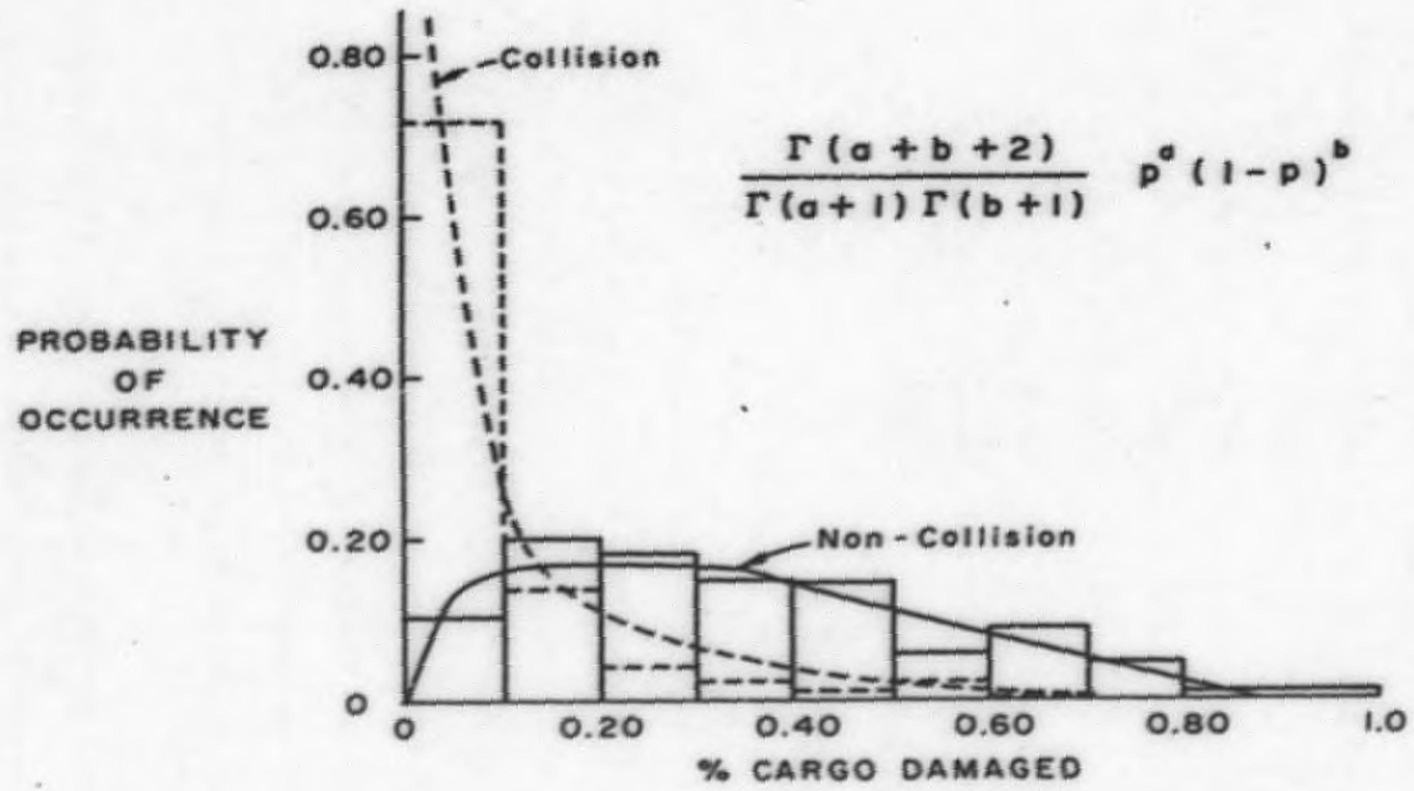

FIG. 3.1.1. BETA DISTRIBUTION OF PERCENTAGE OF CARGO DAMAGED FOR AUTOMOBILE CARGO IN COLLISION AND NON-COLLISION ACCIDENTS

The apparent differences between the two curves indicated a greater degree of cargo damage in non-collision accidents, which might have been anticipated especially for carriers of a product as exposed and susceptible to damages as automobiles.

Another method of viewing the distributions of per cent damage is the return period, discussed by Gumbel[1/], and commonly employed in the analysis of floods. Under the assumption of an underlying beta distribution, the theoretical cumulative distribution, F(p), can be determined. 1 - F(p) represents the probability of p equalling or exceeding a certain value of p. The reciprocal of 1 - F(p), denoted by

$$T(p) = \frac{1}{1 - F(p)}$$

is called the return period. T(p) represents the number of accidents such that, on the average, there is one observation equalling or exceeding p. Thus, the return period of the distribution function indicates the number of accidents such that a certain amount of cargo damage might be expected to occur. The return period distribution corresponding to the cumulative distribution function of the densities of Figure 3.1.1 are shown in Figure 3.1.2, plotted on a logarithmic scale and return periods for specific values of p are shown in Table 3.1.5.

The obvious differences between the two return period distributions emphasize the severity differences between collision and non-collision accidents. Thus, on the average, twenty non-collision accidents are

1/ Gumbel, E. J., Statistics of Extremes, Columbia University Press, New York, 1958.

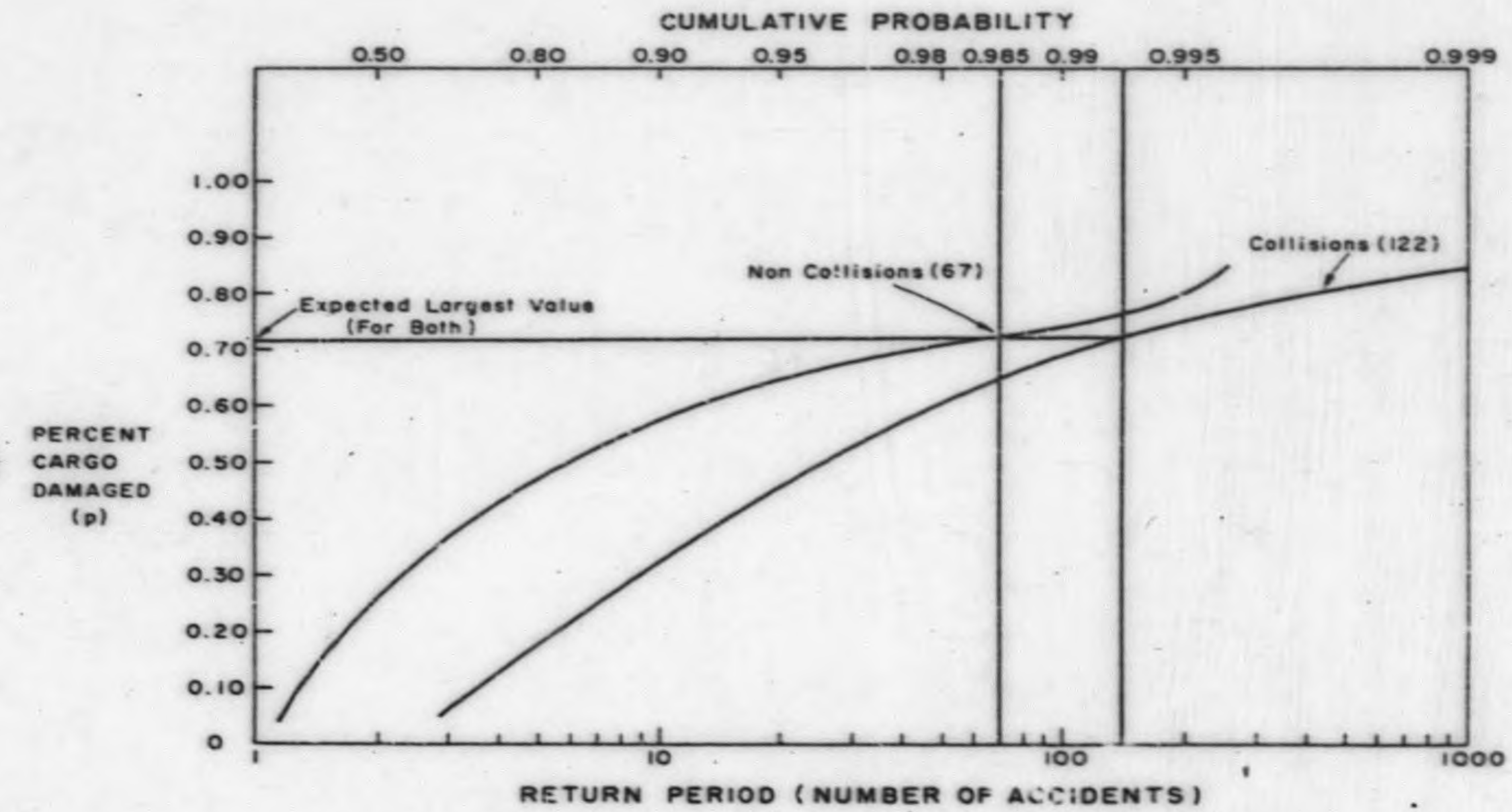

FIG. 3.1.2. RETURN PERIOD DISTRIBUTION OF CARGO DAMAGES FOR AUTOMOBILE CARGO

required before p = .65 is expected to be equalled or exceeded, whereas seventy-seven collision accidents are required.

Table 3.1.5. Return Periods of Cargo Damages in Collision and Non-Collision Accidents for Automobile Carriers

Percentage of Cargo Damaged (p)	Return Period: Number of Accidents — Accident Type: Collision	Non-Collision
.05	2.8	1.2
.15	4.5	1.5
.25	7.1	1.9
.35	11.1	2.9
.45	18.5	4.5
.55	34.5	8.1
.65	76.9	19.6
.75	250.0	125.0
.85	1000.0	250.0
.95	1340.2	333.3

3.2. Cargo Damages and Vehicle Damages.

A logical relation between cargo damage and vehicle damage may be expressed and expected to maintain for all types of cargo. It seems natural to expect cargo damage to increase as vehicle damage increases, not that the relationship is strictly causal, but the two forms of damage result from forces being applied at impact. In most accidents cargo damages do not occur when the vehicle sustains no damage, although the vehicle may be damaged with no destruction of cargo. In most tractor-semitrailer involvements the latter result arises. The relationship between the two types of damages emerges from a consideration of what happens to the cargo as vehicle damage varies, in accidents where some cargo damage does occur. As vehicle damage increases, cargo damages would be expected to increase at a continually increasing rate as vehicle damage approaches its maximum. The function form, C(Cargo Damage) = V(Vehicle Damage), where C(Cargo Damage) might be the proportion of total cargo destroyed, should supposedly be concave from above, and originating where the level of vehicle damage generates no cargo damages.

Thus, a parabolic function would not only satisfy the assumed relationship, but what is more interesting, the parameters of the curve enable a simple interpretation as to the rates of increase of cargo damages and would lend meaningful comparisons for different cargos. In this manner, relative susceptibility of different cargos to damages could be identified as a function of the rate of increasing cargo damage as vehicle damage increases. In order to illustrate the observation, Figure 3.2.1 presents two hypothetical curves depicting cargo damages versus vehicle damages.

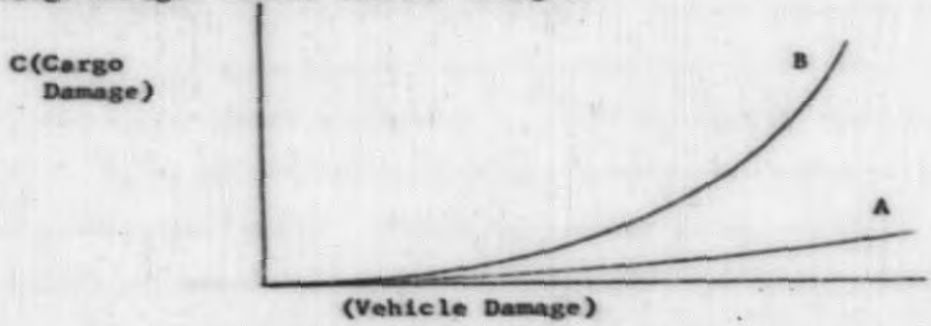

Figure 3.2.1. Hypothetical Curves of Cargo Damage

Curve A reflects a nearly indestructible cargo, whereas curve B shows a cargo that is easily damaged at a relatively lower level of vehicle damage.

In order to observe and measure the important relationship between cargo damages, as a proportion of total cargo value, and vehicle damage, sets of regression equations were devised. When percent cargo damage was plotted against the logarithm of vehicle damage the exponential tendency was exhibited as describing the growth of damage to cargo. The regression model was of the form:

$$\% \text{ Cargo Damage} = a + b \text{ Log Vehicle Damage} + c(\text{Log Vehicle Damage})^2.$$

Table 3.2.1 shows the derived equations for non-collision, fixed object, truck and auto accidents, along with the correlation indexes.

Table 3.2.1. Regression Equations of % Cargo Damage (y), Log Vehicle Damage (X), Automobile Cargo

Object Struck	Equations	Index of Correlation (I)	(I^2)
Truck	$y = 1.06 - 1.08X + .27X^2$	.78	.60
Auto	$y = .86 - .77X + .17X^2$	.50	.25
Fixed Object	$y = 1.04 - 1.00X + .24X^2$	.67	.45
Non-Collision	$y = -.23 + .18X + .01X^2$	.50	.25

Though no evidence was found in the previous analysis to indicate significant differences between objects struck in collision accidents when the distributional form was considered, per cent cargo damage as a function of vehicle damage seemed to behave differently within the group, as in Figure 3.2.2. The ability of the functional relationship to elucidate small differences suggested different rates of approaching total cargo loss, with truck and fixed object accidents resulting in higher cargo damages at fixed levels of vehicle damage.

With respect to non-collision accidents, the wide scattering of points called attention to the lack of a clear relationship between cargo and vehicle damage, most probably due to the nature of the cargo and the great range that damages can assume when the tractor-trailer overturns.

In general, the curves, with the solid portion emanating from the minimum, suggested a form of exponential growth of percentage cargo damaged, approaching total loss, asymptotic to the maximum possible vehicle damage.

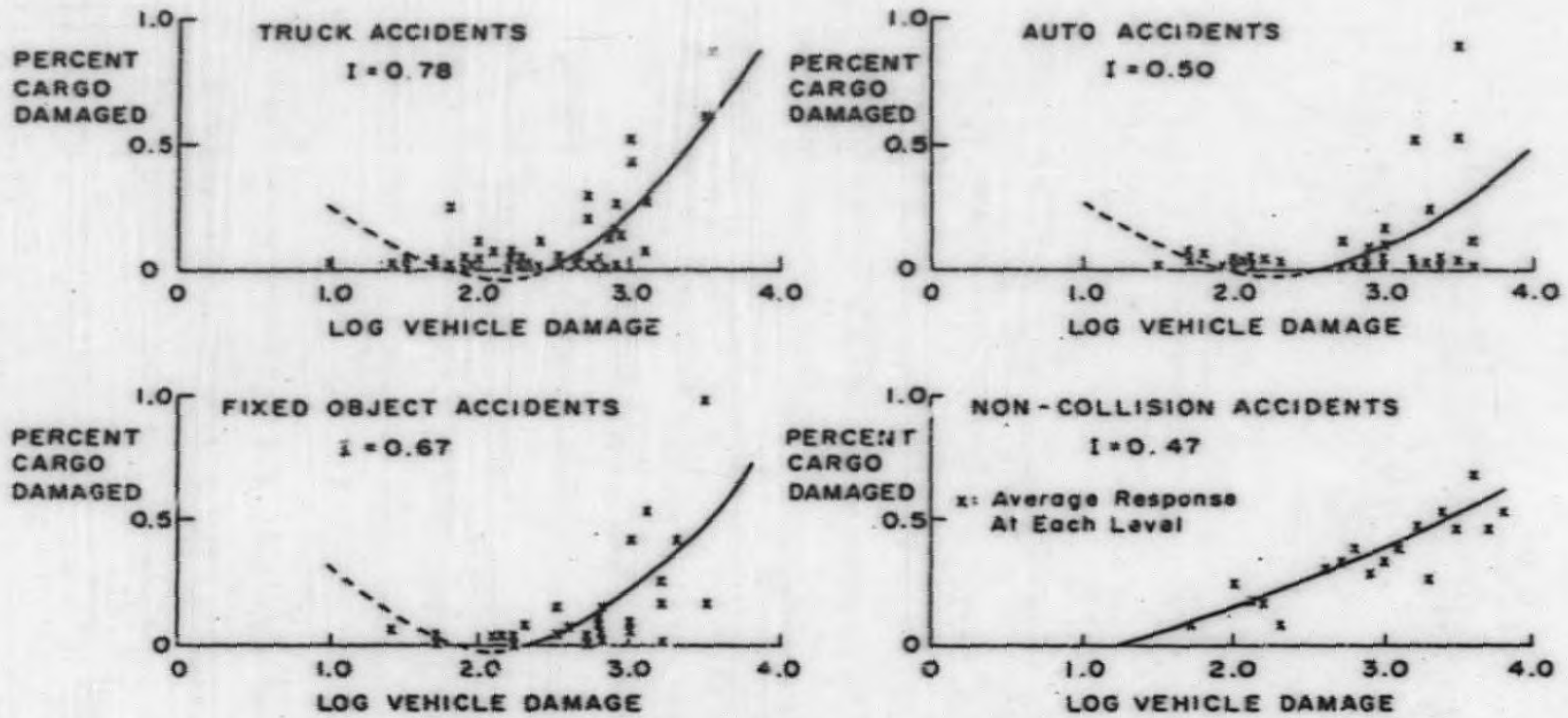

FIG. 3.2.2. REGRESSIONS OF PERCENTAGE OF CARGO DAMAGED AND LOG VEHICLE DAMAGE IN TRUCK, AUTO, FIXED OBJECT AND NON-COLLISION ACCIDENTS; NEW AUTOMOBILE CARGO

In addition to the automobile carriers' reports procured from the Interstate Commerce Commission, some data were made available by a large transporter of petroleum products. Collision type accidents involving the oil carrier and automobiles and other trucks from 1955-1959 were studied and the cargo damage and vehicle damage relation for this type of cargo derived. Figures 3.2.3 and 3.2.4 present the scatter diagram and the curves for both types of objects struck. The potential development of a family of cargo damage curves as a function of vehicle damage is exhibited in Figures 3.2.5 and 3.2.6 where the curves for auto carriers and oil carriers are shown for involvements with trucks and automobiles. In accidents involving automobiles and other trucks, the levels of the curves for oil and auto carriers indicated displacements due to the cargo carried. Hence, it might be inferred that automobiles have a lower damage susceptibility than petroleum and have a smaller degree of potential destruction than the liquid cargo. This, of course, might be due to oil leakage from a puncture in the trailer which can easily occur in accidents. However, of particular interest is the fact that given a certain level of vehicle damage, differences in the expected amount of cargo damage may be evaluated for different types of cargo.

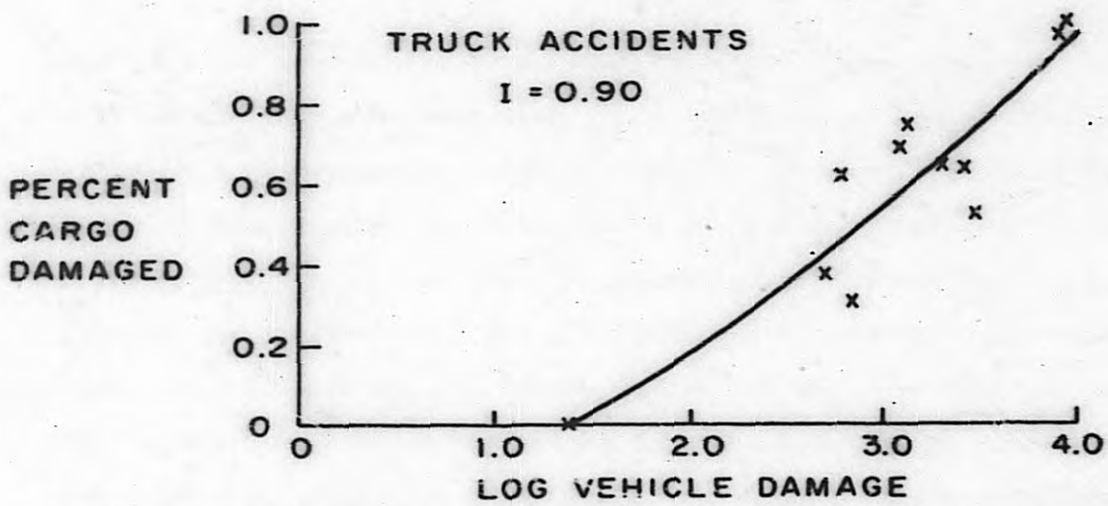

FIG. 3.2.3. REGRESSION OF PERCENTAGE OF CARGO DAMAGED AND LOG VEHICLE DAMAGE IN TRUCK ACCIDENTS; OIL CARGO

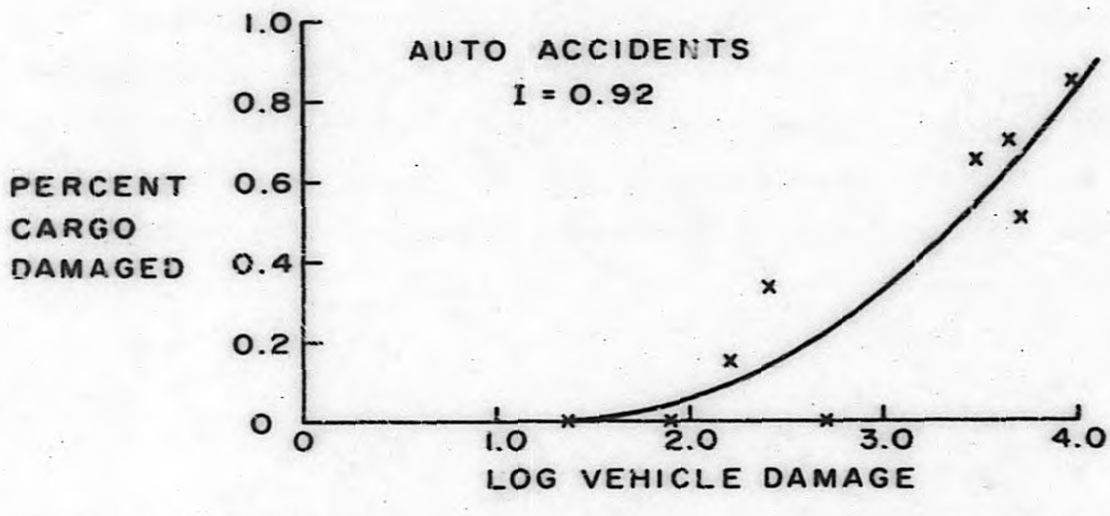

FIG. 3.2.4. REGRESSION OF PERCENTAGE OF CARGO DAMAGED AND LOG VEHICLE DAMAGE IN AUTO ACCIDENTS; OIL CARGO

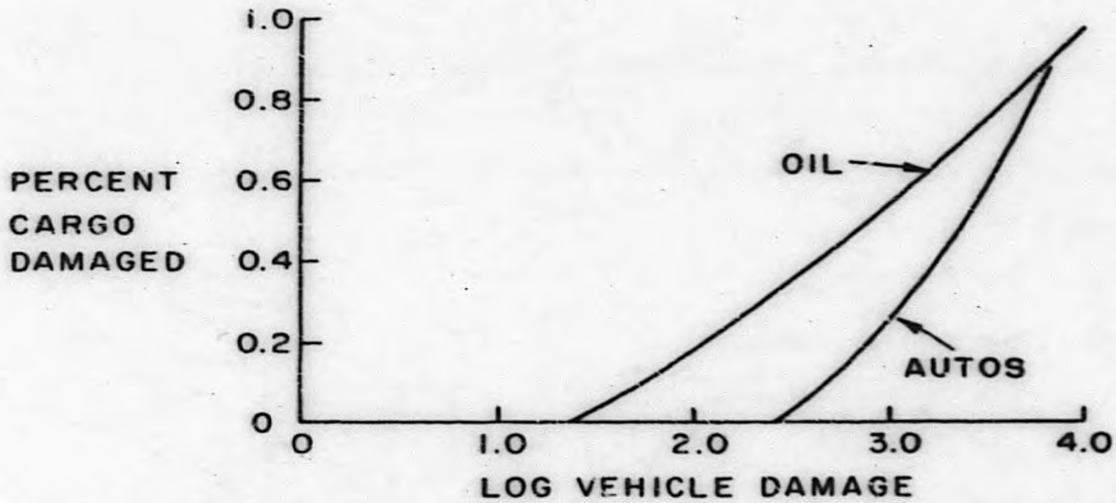

FIG. 3.2.5. PERCENTAGE OF CARGO DAMAGED IN AUTOMOBILE ACCIDENTS; NEW AUTOMOBILE CARGO AND OIL CARGO

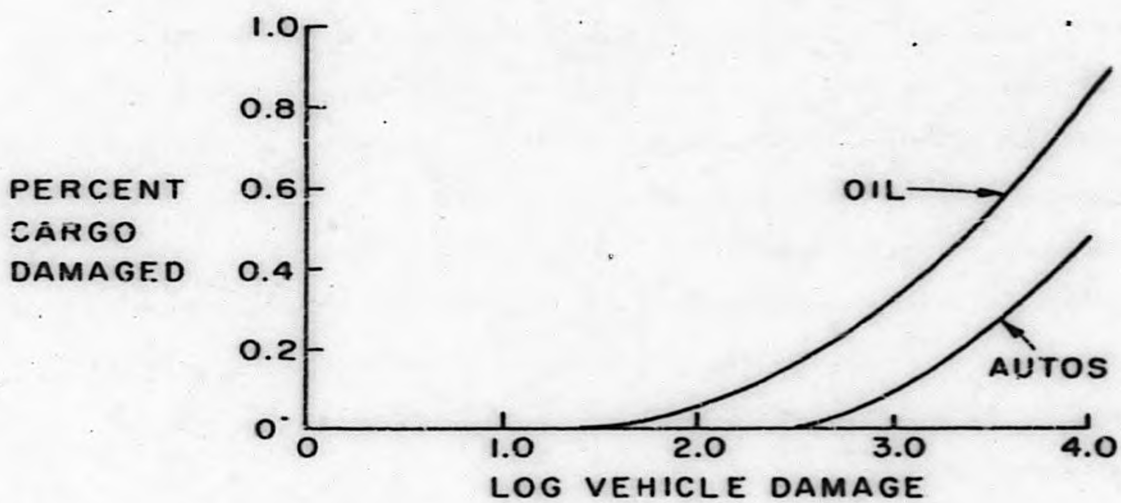

FIG. 3.2.6. PERCENTAGE OF CARGO DAMAGED IN TRUCK ACCIDENTS; NEW AUTOMOBILE CARGO AND OIL CARGO

3.3. Cargo Damage, Vehicle Damage and Mechanical Energy.

In equation (2.9) and in Figure 2.4.1, vehicle damage was found to be related to the energy released in a collision and available for damage by

(3.1) $\text{Log } V = \frac{E}{a + bE}$

where: V = Vehicle damage

E = Energy available for damage

$a = .02 \times 10^6$

$b = .24.$

A general parabolic relation has also been applied to vehicle damage and cargo damage, taking the form

(3.2) $p = a_0 + b_1 \text{Log } V + b_2 (\text{Log } V)^2$

where: p = Proportion of total cargo damaged

a_0, b_1, b_2 = Functions of cargo type and object struck.

Equations (3.1) and (3.2) adequately described the respective relationships in the regions where observation from accident reports occurred. As mentioned previously, it was impossible to observe directly a scatter diagram for cargo damages and energy, due to the lack of speed information in the reporting system when cargo damages occurred. However, since indications were obtained of vehicle damages as a function of energy and cargo damage as a function of vehicle damage, it was possible to infer the functional form relating cargo damages and mechanical energy.

Substituting Log V = $\frac{E}{a + bE}$ in (3.2) yields

(3.3) $p = a_0 + b_1E/(a + bE) + b_2E^2/(a + bE)^2$, which by algebraic manipulation becomes

(3.4) $$p = \frac{k_0 + k_1E + k_2E^2}{k_3 + k_4E + k_5E^2}$$

where: $k_0 = a^2a_0$

$k_1 = a(2a_0b + b_1)$

$k_2 = b(a_0b + b_1) + b_2$

$k_3 = a^2$

$k_4 = 2ab$

$k_5 = b^2$

Thus, p equals the ratio of two quadratic equations in E.

When (3.4) was plotted for the damage relationships for automobile cargos over the positive energy region, the curves of Figure 3.3.1 resulted for auto and truck collisions. The graph, which is read as a nomograph starting in the first quadrant, indicates the manner in which energy released from impact may generate damages to the tractor-semitrailer's cargo, similar to the form taken by vehicle damages. Similarly, relative potential destructability of cargos may be considered as a function of the velocities and direction of impact as well as mass reflected in the energy variable and it is hoped that a continuum of potential cargo destruction in collisions might be developed.

As a matter of fact, extensive efforts were made in attempting to gather pertinent cargo damage information for numerous types of cargo. Except for the two cargos discussed, these efforts have proven

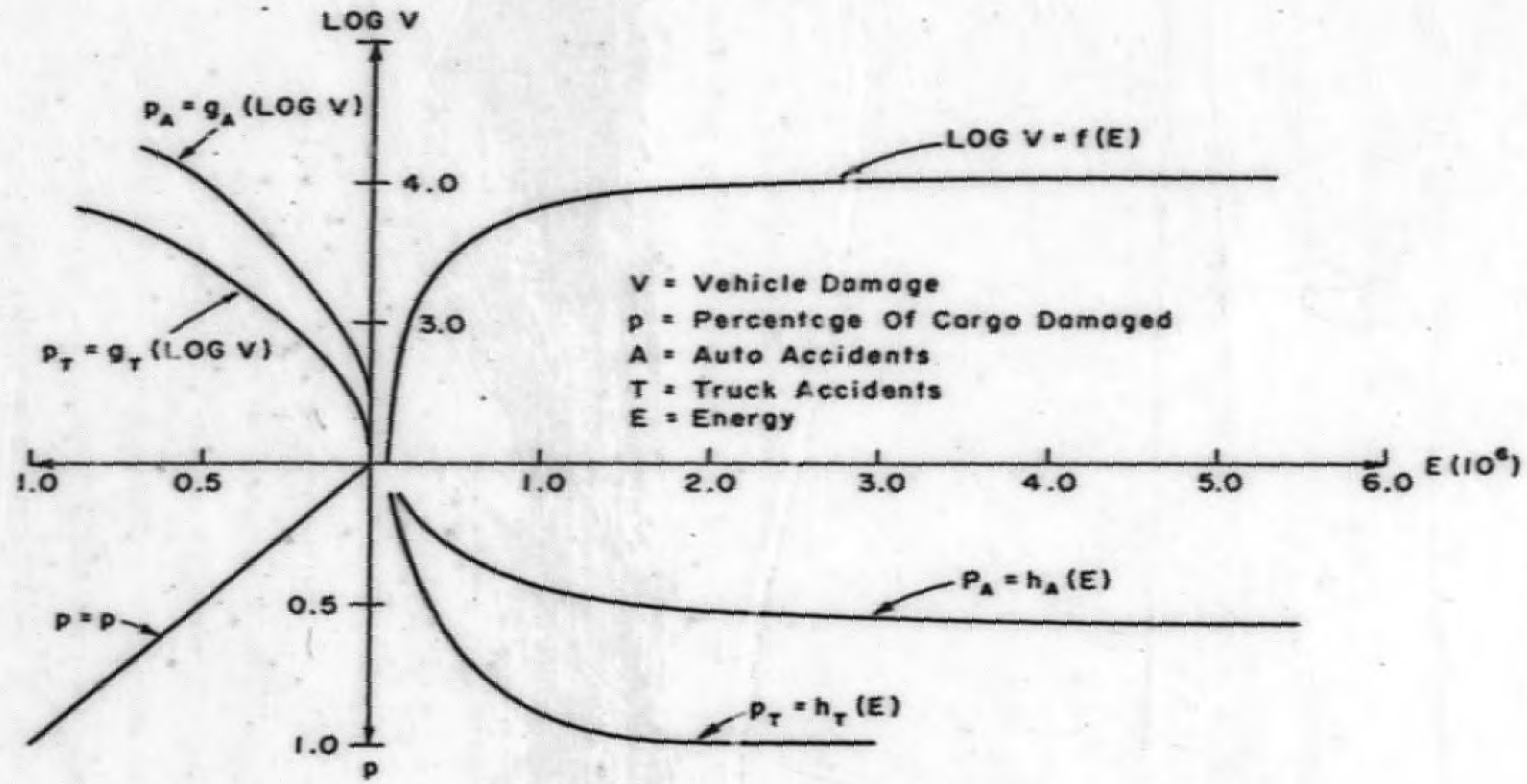

FIG. 3.3.1. DERIVED THEORETICAL RELATIONS BETWEEN VEHICLE DAMAGE, CARGO DAMAGE AND ENERGY AVAILABLE FOR DAMAGE

fruitless. Discussions with shippers as well as searches of company records emphasized the lack of adequate information with respect to monetary losses of cargo in highway accidents. At the present time a survey of large private truck-fleet owners is under consideration. Unfortunately, extenuating circumstances have so far prevented the initiation of the study. However, it is believed that the availability of adequate cargo damage information of the form discussed would yield the results necessary to describe sufficiently a severity continuum of cargo damages.

3.4. Thresholds of Cargo Damages.

The concept of a threshold for cargo damages may be defined in terms of the probability of cargo damages occurring in an accident. By utilizing the equations relating cargo and vehicle damage and probability distributions of vehicle damages, the probability of damage to cargo can be determined.

The curves relating cargo and vehicle damages, for carriers of new automobiles, in accidents involving autos, trucks, fixed objects and non-collisions were:

Trucks: $p = 1.06 - 1.08X + .27X^2$

Autos: $p = .86 - .77X + .17X^2$

Fixed Objects: $p = 1.04 - 1.00X + .24X^2$

Non-Collisions: $p = -.23 + .18X + .01X^2$

where: p = percentage of cargo damaged

X = logarithm of vehicle damages.

Equating p to zero in each relationship and solving for X yields the value of log vehicle damage where cargo damages may be expected to originate. From the lognormal density functions of vehicle damages, the probability was determined of log vehicle damage being less than the value of X satisfying the equation at $p = 0$. As an illustration, if the truck equation for cargo damage is set equal to zero, $X_0 = 2.167$. The amount of vehicle damages corresponding to $X_0 = 2.167$ is $187; this implies that no cargo damage will result unless vehicle damage exceeds $187 in truck collisions. Since log vehicle damage in truck accidents is distributed normal (mean = 2.63, variance = .35), the probability of vehicle damage being less than

\$187 is .27. Thus, the inference is that in twenty-seven percent of all truck accidents, no cargo damage will occur. Table 3.4.1 summarizes the threshold probabilities for different accident types.

Table 3.4.1. Probabilities of Cargo Damage Occurrence for New Automobile Carriers.

Accident Types	Minimum Vehicle Damage Necessary	Probability of No Cargo Damage
Truck	187	0.27
Auto	338	0.63
Fixed-Object	147	0.05
Non-Collision	16	0.00

The probabilities of Table 3.4.1 may be interpreted as conditional probabilities denoting the probability of no cargo damage given a certain accident type, and subtracting each from one yields the probability of cargo damage occurring. Of particular interest is the probability of the simultaneous occurrence of the two events, accident type and cargo damages. In probabilistic notation, this probability is defined as:

$$p\left[CA_i\right] = p\left[A_i\right]p\left[C \mid A_i\right]$$

where: C = Cargo damage

A_i = accident type; i = truck, auto, fixed object, non-collision

and $p\left[CA_i\right]$ = probability of cargo damage and a particular accident type occurring

$p\left[A_i\right]$ = probability of a particular accident type occurring;

$$\sum p\left[A_i\right] = 1$$

$p\left[C \mid A_i\right]$ = conditional probability of cargo damage occurrence given a particular accident type.

From the analysis of impact characteristics of truck accidents discussed in section three, the probabilities of the four accident types were derived. Thus, the total event space was described as in Table 3.4.2.

Table 3.4.2. Probabilities of Cargo Damage and Accident Type.

Accident Type $[A_i]$	$P[A_i]$	$P[C \mid A_i]$	$P[CA_i]$
Trucks	.157	.73	.115
Autos	.563	.37	.208
Fixed Objects	.190	.95	.181
Non-Collision	.090	1.00	.090
	1.000		.594

The last column of Table 3.4.2 indicates the probability of cargo damage and a specific accident type occurring, while the sum yields the probability of cargo damages occurring in all types of accidents.

The relevance of this analysis to carriers of radioactive materials is that it offers pessimistic conditional threshold probabilities and total event probabilities for expected cargo damage. Since the container and cargo in shipments of radioactive materials may be considered less susceptible to damage in accidents than are new automobiles, the resulting probabilities would be high or pessimistic for these shippers. In order to reduce $P[CI_i]$, i.e. the probability of some container damage and a particular accident type's occurrence, it is only necessary to reduce the threshold probabilities. The threshold probabilities can be reduced by design of containers such that a greater amount of vehicle damage is required before cargo damage results. Thus, if controls are applied to increase the threshold vehicle damage in non-collision accidents, the most severe type for automobile cargo damage, and if it may be assumed further that if cargo can better withstand a certain increase in vehicle damage for these more severe accident types, it will be able to withstand at least the same percentage increment in other accident types, then a reduction in the overall probability of cargo damage in all accidents may be realized.

As an illustration, consider the minimum amount of vehicle damage required in non-collision accidents before damages to new automobiles are expected to occur. It was seen that only sixteen dollars of vehicle damage was required and that virtually all non-collision accidents would generate this much damage and hence cargo damage was almost certain to result. If design were such that it would take five times this amount, or eighty dollars of vehicle damage to generate container damage in non-collision accidents, due to the lognormal vehicle damage distributions the probability of exceeding this amount, or of container damage resulting, would be .985 or a 1.5% reduction in the threshold probability. However, assuming that the required vehicle damage level in the other accident types increased at least five times that for auto carriers, the resulting threshold probabilities would be reduced by 61.0%, 75.7% and 61.5% for fixed-object, automobile, and truck collisions respectively, due to skewness of the vehicle damage probability distributions. The assumption is equivalent to a constant shift in the log vehicle damage and percentage cargo damage curves. Table 3.4.3 summarizes the results of increasing the resistance of cargo damage, by a factor of five, in non-collision accidents. The probabilities may now be considered more optimistic than the original thresholds. Furthermore, the overall probability of cargo damage in all accident types has been reduced from .594 to .304, or by 48.8%.

Thus, the container damage threshold represents the first stage of the accident mechanism approaching release of radioactive cargo. The threshold probability refers to the probability of container damage, a prerequisite for release, analogous to exceeding the injury threshold related to fatality occurrence of "human cargo."

Table 3.4.3. Probabilities of Cargo Damage and Accident Types, Based on New Automobile Carrier Thresholds and a Design Factor of Five.

Accident Type, (A_i)	Automobile Carrier Experience				Five Fold Increase of Vehicle Damage Threshold			
	$P[A_i]$	Minimum Vehicle Damage Required, V_0	Threshold Probability $(P[C\|A_i])$	$P[CA_i]$	$5V_0$	Threshold Probability $(P[C\|A_i])$	Reduction in Threshold Probability	$P[CA_i]$
Trucks	.157	$ 187	.73	.115	$ 935	.281	61.51%	.044
Autos	.563	338	.37	.208	1,690	.090	75.65	.051
Fixed Objects	.190	147	.95	.181	735	.629	33.79	.120
Non-Collision	.090	16	1.00	.090	80	.985	1.50	.089
	1.000			.594				.304