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WITH TWO-ION COMPONENTS

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**PLASMA PHYSICS
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Ion Cyclotron Heating of Plasmas With Two-Ion Components

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Abstract

The explanation of observed ion cyclotron heating in PLT was tried using cold wave theory. The theory allowing for mode conversion and/or tunneling effects explains the observed plasma heating.

1. Introduction

Ion cyclotron waves were coupled to two-component plasmas in the C-stellarator¹ and recently in PLT.^{2,3} In the former case, slow Alfvén waves were excited in the region where the ion cyclotron frequency is higher than the excited frequency. The waves were led to the weaker field region (magnetic beach) where the ion cyclotron resonance takes place. Because of the limitation in experimental time, only heating of the minority heavy ion component was tried. Minority light component ion heating was, to the memory of the author, not tried.

The dispersion relation of the two-component ion plasma may be expressed as

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$$0 = n^4 \cos^2 \theta - \left(\frac{\gamma_1 \Omega_1^2}{\Omega_1^2 - \omega^2} + \frac{\gamma_2 \Omega_2^2}{\Omega_2^2 - \omega^2} \right) (1 + \cos^2 \theta) n^2 - \frac{\Omega_1^2 \gamma^2 (\omega^2 - \Omega_h^2)}{(\Omega_1^2 - \omega^2) (\Omega_2^2 - \omega^2)} . \quad (1)$$

To avoid complication, we treat D and H plasmas. In the above equation

$$n^2 = \frac{k^2 c^2}{\omega^2}, \quad n^2 \cos^2 \theta = \frac{k_n^2 c^2}{\omega^2}, \quad \gamma_j = \frac{n_j M_j e^2}{B^2 / \mu_0} \quad (j = 1, 2),$$

$$\Omega_1^2 \gamma^2 = \frac{e^2 n^2}{B^2 \epsilon_0^2}, \quad n = n_1 + n_2, \quad \gamma \equiv \frac{n M_1 e^2}{B^2 / \mu_0},$$

here $\Omega_i = \Omega_1$, $n_j/n = v_j$ ($j = 1, 2$), $v_1 + v_2 = 1$, $x \equiv \omega/\Omega_1$, $\Omega_2/\Omega_1 = M_1/M_2 = 2$, $\Omega_h = (2v_1 + v_2)\Omega_1 = (1 + v_1)\Omega_1$.

Then

$$0 = n^2 n_n^2 - \gamma \left(\frac{v_1}{1 - x^2} + \frac{2v_2}{4 - x^2} \right) (n^2 + n_n^2) - \frac{\gamma^2 (x^2 - \Omega_h^2/\Omega_1^2)}{(1 - x^2) (4 - x^2)} . \quad (2)$$

Alfvén velocity v_A is defined as

$$v_A^2 = \frac{c^2}{\gamma} = \frac{B^2 / \mu_0}{n M_1} .$$

We define $N^2 = k^2 v_A^2 / \omega^2$, etc. Then Eq. (2) could be expressed as

$$0 = N^2 N_n^2 - \left(\frac{v_1}{1-x^2} + \frac{2v_2}{4-x^2} \right) (N^2 + N_n^2) - \frac{(x^2 - \Omega_H^2 / \Omega_L^2)}{(1-x^2)(4-x^2)} \quad (4)$$

If we rearrange the above equation in terms of N_1^2 and N_n^2 ,

$$0 = \left(N_n^2 - \frac{x+1+v_1}{(1+x)(4+x)} \right) \left(N_n^2 + \frac{x-(1+v_1)}{(1-x)(4-x)} \right) + N_1^2 \left[N_n^2 - \left(\frac{v_1}{1-x^2} + \frac{2v_2}{4-x^2} \right) \right] \quad (5)$$

If $N_1^2 = 0$, N_n^2 has a double root at $x = (1 + 3v_1)^{1/2}$.

The characteristic dispersion relation of heavy minority ions is shown in Fig. 1. The wave vector k_n increases as the magnetic field decreases to the resonance (magnetic beach). When $k_n v_1 = \Omega_1 \sim \omega$ there is resonance between ions and waves and net ion heating takes place.

On the other hand, for minority light ions (so that the cyclotron frequency is higher than that of the bulk ions), the dispersion relation looks like Fig. 2. Phase velocity ω/k_n is almost equal to fast Alfvén wave velocity and the (Stix) coil system of Ref. 1 cannot couple to the plasma unless the field is very close to the resonance field: then heating of plasmas under the coil could occur.

In PLT, the plasma size is 40-45 cm in minor radius, as compared to 5-7.5 cm in the C-stellarator. Also in PLT, there exists no straight section. The wave is coupled by two half-turn coils

located on the outer side of the plasma separated by approximately 50 cm. For simplicity we let $\Omega_2 = 2\Omega_1$ (such as D and H). As seen from Figs. 2-5, the fast wave launched at $\omega/\Omega_2 > 1$ eventually reaches the $\omega/\Omega_2 < 1$ region either by propagation across the magnetic field⁴ or along the magnetic field lines. As the wave moves perpendicular to the magnetic field; however, $k_{\perp}^2$ decreases, if $k_{\parallel}^2$ is kept constant. So the perpendicular group velocity decreases. Eventually the wave starts to move along the magnetic field line. The magnetic field strength changes along the field line due to rotational transform. As they approach the cross points in the diagrams, the waves could be mode converted into the branch which has resonance at $\omega = \Omega_2$. It is clear that the mode conversion does not depend on the amount of minority light ions, as mode conversion is expected to take place at the cross points which are similar for different concentrations. The waves in turn propagate along the magnetic field because of rotational transform and heat ions by magnetic beach heating, not unlike the C-stellarator.

The heating of minority heavy ions is difficult. The mode conversion is only to slow waves which have resonance at $\omega = \Omega_2$ and no resonance at $\omega = \Omega_1$. Thus the ion heating is expected only via thermal effect on waves near the hybrid resonance. We note that if the minority ions are 5%, there is heating, but not at 10% (see Fig. 4 of Ref. 2); whereas the minority light ions could be heated even for 50/50 D-H mixtures (see Section 2.3 of Ref. 2). This could be explained by the mode conversion such as Fig. 5 of this paper. (See appendix.)

Electron Landau damping⁵ is also expected, especially where minority light ions are more than 10 to 20%. Then before the condition $|k_{\parallel} v_1| = |\Omega_2 - \omega|$ is satisfied, the bulk electron velocity could become comparable to the parallel phase velocity. But since the damping rate is not too large⁵ if the wave can still reach near the resonant point, ion heating is also expected.

References

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Appendix

In the case of PLT, rotational transform is roughly π at the halfway point of the plasma. Here the aspect ratio is approximately 5. If we look at Fig. 3, the gap between the fast wave branch which ultimately ends at hybrid resonance and another branch reaching light ion resonance is approximately $\Delta\omega/\Omega_1 = 0.02$. Since the figure could be considered as the $1/B$ plot with ω fixed, 0.02 difference at 1.85 is roughly 1.08% change in field. This difference in PLT with major radius 135 cm, amounts to 1.5 cm in the radial difference. Thus if this cross over point is located at the center (i.e., $\rho = 135$ cm), the gap between the two branches parallel to magnetic field line amounts to only 15 cm $[1.5 \times (2\pi/\pi) \times 5]$. We also note that in this case the resonance occurs at $\rho = 146$ cm or 11 cm from the center of the plasma.

Experimentally it could be predicted that both the crossover point $x = (1 + 3v_1)^{1/2}$ and the resonance point must be within the plasma. The latter could be slightly outside because the thermal effect will give rise to heating.

Figure 1(a). Dispersion relation for minority heavy ions 5% ($0.5 < \omega / \Omega_i < 2.0$). $N_{\perp}^2 = k_{\perp}^2 v_A^2 / \omega^2 = 0.1$. (PPPL 806439)

Figure 1(b). Same as 1(a) but $0.95 < \omega / \Omega_i < 1.15$. (PPPL 806440)

Figure 2. Dispersion relation for minority light ions 5%
 $N_i = k_{||}^2 v_A^2 / \omega^2 = 0.1$ and 0. (PPPL 806438)

Figure 3. Dispersion relation for minority light ions 20%.
 $N_i^2 = 0.1$. (PPPL 806443)

Figure 4. Dispersion relation for minority light ions 30%.
 $N_1^2 = 0.1$. (PPPL 806441)

Figure 3. Dispersion relation for 50/50 H and D mixture.
 $N_{\perp}^2 = 0.1$. (PPPL 806442)