REFERENCE WIND SPEED DISTRIBUTIONS AND HEIGHT PROFILES
FOR WIND TURBINE DESIGN AND PERFORMANCE
EVALUATION APPLICATIONS

Technical Report

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August 1976

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MASTER

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Georgia Institute of Technology
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ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION
Division of Solar Energy

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Abstract

The purpose of this report is to provide a set of reference or standard values of wind profiles, wind speed distributions and their effects on wind turbine performance for engineering design applications. Based on measured Weibull distribution parameters, representative average, low, and high variance data are given for height profiles of mean, 25 percentile, and 75 percentile wind speeds; and for wind speed probability density (velocity frequency) functions and cumulative probability (velocity duration) functions at selected heights. Results of a sensitivity analysis of the dependence of wind turbine performance parameters on cut-in speed, and rated speed for various mean wind and wind variance regimes are also presented. Wind turbine performance is expressed in terms of capacity factor (ratio of mean power output to rated power) and recovery factor (ratio of mean energy output to energy theoretically available in the wind). The representative high, mean, and low variance cases were determined from calculated Weibull distributions at 140 sites across the Continental U.S., and all of the representative functions are evaluated at mean wind speeds of 4, 5, 6, 7, and 8 m/s at standard 10 m level. As background to these representative design functions, the basic characteristics of the Weibull distribution are summarized, and methodologies are given for height projection from a measured level to a desired height (in relatively flat terrain). The methodologies are also described whereby wind turbine performance for input values other than given here can be calculated by design engineers.

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1. INTRODUCTION

For several years the Weibull distribution has been used to represent wind speed distributions for application in wind loads studies (Davenport, 1963). Recently the Weibull distribution has also been found useful and appropriate for wind energy applications (Justus, 1974; Baynes, 1974; Zimmer et al, 1975; Wentink, 1975; Justus et al, 1976; Hennessey, 1976). In Appendix A and B, this report gives a description of the basic properties and applications of the Weibull distribution, including a methodology for its use in evaluation of wind turbine performance. Appendix C gives seasonal and annual Weibull distribution parameters determined (Zimmer et al, 1975) at 140 sites across the continental U.S. In Section 2, the Weibull parameters from Appendix C are used to construct average, high variance, and low variance reference wind speed distributions for various mean 10 m reference level wind speeds. For these representative high, average, and low variance cases, and for average 10 m speeds of 4 through 8 m/s, tables are presented in Sections 3-5 of: 1) average, 25 percentile, and 75 percentile wind speed at 5 m height intervals 5-60 m, 2) wind speed probability density (velocity frequency) and cumulative probability (velocity duration) at heights 10, 30, 45, and 60 m, 3) wind turbine capacity factor (ratio of actual average power output to rated power) and energy recovery factor (ratio of actual energy output to theoretically available energy) at 10, 30, 45, and 60 m heights, and the dependence of these factors on rated speed and cut-in speed of the wind turbine.

The purpose of these reference data is to provide wind turbine design engineers with standard data on height profiles of wind speed, wind speed probability distributions, and the effects of these on wind turbine performance as functions of the design values of cut-in and rated speed of the wind turbine. Valuable additional discussion of the application of the Weibull dis-

tribution to this problem, as well as to other engineering problems, is given by Baynes (1974).

2. DEFINITION OF THE WIND REGIME

In Appendix C, seasonal and annual measured values of Weibull distribution scale factor c and shape factor k (see Appendix A for definitions) are given for 140 sites across the continental United States. These Weibull c and k values were determined (Zimmer et al, 1975) from wind speed summary data obtained from the National Climatic Center at Asheville for National Weather Service airport sites which had constant anemometer height (as listed in Appendix C) over a five year or longer summary period. In order to define reference wind regimes of high, average, and low wind speed variance for various mean wind speeds, the Weibull c and k values were converted to a standard reference level of 10 m by the application of equations (A-14) through (A-16) in Appendix A. By analysis of the frequency of occurrence of 10 m level c and k values, representative c and k values for high, average, and low variance for 10 m mean speeds of 4, 5, 6, 7 and 8 m/s were then computed as described in this section.

Figure 1 shows a scatter diagram of the computed c and k value combinations at standard height 10 m for Winter (Dec, Jan, Feb), as determined by conversion of c and k values in Appendix C to the 10 m reference height (by equations (A-14) through (A-16)). Sets of values of c and k corresponding to mean 10 m wind speeds of 3, 4, 5, and 6 m/s were then computed from equation (A-5), i.e. for each $\bar{V}$ value a set of k values were selected (for $\bar{V} = 3$ m/s, $k = 1.0, 1.1, \dots, 2.0$ were used) and for each of these k and $\bar{V}$ values the corresponding C value was evaluated from equation (A-5). Next, the number of occurrences of c and k values within ± 0.25 m/s in c value and ± 0.05 in k value (e.g. the lattice of areas in Figure 1) was determined for each season by counting occurrences from the seasonal scatter plots (Figure 1 and analogous ones for the other seasons). These occurrence frequencies were combined for all seasons, and cumulative probabilities of occurrence of k values, for each 10 m level mean wind speed $\bar{V}$, were then plotted, as exemplified by Figure 2, for 10 m $\bar{V} = 5$ m/s. From these cumulative

Figure 1. Scatterplot of Weibull c and k values from Appendix C, converted to 10 m height, for Winter (Dec, Jan, Feb).

Figure 2. Cumulative probability of Weibull k values for a 10 m mean wind speed of 5 m/s.

probabilities of k , the representative low and high (10 percentile, and 90 percentile) k values were determined, along with the average k value. Corresponding c values for each k were then calculated from equation (A-5). For example, the 90 percentile k values for 10 m level mean speed $\bar{V} = 5$ m/s is, from Figure 2, $k_{90} = 2.33$. For $\bar{V} = 5$ m/s, the corresponding c value from equation (A-5) would be

$$c_{90} = 5/\Gamma (1 + 1/2.33) = 5.64 \text{ m/s}$$

Note that low k values are associated with high variance of wind speeds about the mean (c.f. Figure A-3) and vice versa. Therefore the low (10 percentile) k values correspond to high (90 percentile) variances and the high (90 percentile) k values correspond to low (10 percentile) variances.

Sufficient data from seasonal plots such as Figure 1 were available to determine high, average, and low variance c and k values for 10 m mean speed values of 3, 4, 5, and 6 m/s. For 10 m mean speeds above 6 m/s, an extrapolation shown in Figure 3 was used to estimate high, average, and low variance k values for 10 m reference mean speeds of 7 and 8 m/s. Table 1 gives the results for the 10 m reference values of Weibull c and k values for 10 m mean speed $\bar{V} = 4 - 8$ m/s for high (90%), average, and low (10%) variance cases. Considerable variation in k values is seen from high to low variance cases.

With reference values of high, average, and low variance for Weibull distribution c and k specified for the 10 m reference height, the complete height profile of wind speed distributions can be computed by the methods described in Appendix A. Section 3 gives the results of the 10 m c and k reference values and height profiles of the mean wind speed and 25 percentile and 75 percentile wind speeds determined by the Appendix A methodology. Section 4 gives calculated wind speed probability density distribution (velocity frequency curve) and cumulative probability (velocity duration curve) at 10, 30, 45 and 60 m heights, using the Appendix A methodology and the 10 m reference c and k values. Section

Figure 3. Variation of low (10%), average, and high (90%) Weibull k values with 10 m mean wind speed.

Table 1. Representative Weibull c and k values for high (90%), average, and low (10%) variance for various 10 m mean wind speeds. Weibull c values are in m/s; k values are dimensionless.

$\bar{V} = 4 \text{ m/s}$		$\bar{V} = 5 \text{ m/s}$		$\bar{V} = 6 \text{ m/s}$		$\bar{V} = 7 \text{ m/s}$		$\bar{V} = 8 \text{ m/s}$		
c	k	c	k	c	k	c	k	c	k	
4.42	1.47	5.59	1.64	6.75	1.79	7.89	1.94	9.03	2.07	High Variance
4.51	1.87	5.65	2.10	6.77	2.30	7.89	2.49	9.00	2.67	Avg. Variance
4.52	2.10	5.64	2.33	6.76	2.55	7.87	2.74	8.97	2.92	Low Variance

5 gives results of sensitivity studies of wind turbine performance (capacity factor and energy recovery factor) versus wind turbine cut-in and rated speed, for the wind speed distributions (Section 4 results) at 10, 30, 45, and 60 m, using methodologies described in Appendix B.

It has been proposed by Wentink (1975) and Hennessey (1976) that the Rayleigh distribution (or chi-square with two degrees of freedom) provides an adequate description of the wind speed distribution. The Rayleigh distribution is a sub-set of the Weibull with $k = 2$. The formulas in Appendix A would be simplified by an assumption of $k = 2$, and the distribution would be specified entirely in terms of the mean wind $\bar{V}$ (where $c = 2\bar{V}/\sqrt{\pi}$ for $k = 2$). Note that $k = 2$ is within the range of k values between 10 percentile and 90 percentile for all mean wind speeds in Table 1 except $\bar{V} = 8$ m/s. The data presented in the subsequent sections and in Appendix A will allow the engineer or meteorologist to determine if the simplification of assuming $k = 2$ will produce sufficiently accurate results for his specific requirements, or conversely, under what conditions he must use the more general k values to achieve some prescribed desired accuracy.

3. REFERENCE HEIGHT PROFILES OF WIND

From the Weibull c and k values at reference height 10 m for high, average, and low variance, listed in Table 1, profiles of 25 percentile, average, and 75 percentile wind speed at heights 5m to 60 m were evaluated and are shown in Tables 2 through 4, for 10 m reference height mean wind speeds $\bar{V}$ of 4, 5, 6, 7, and 8 m/s. These computations were made in the following manner: 1) For a given 10 m level $\bar{V}$, c and k from Table 1, the appropriate c and k for the desired height were found by height projection equations (A-14) through (A-16), 2) From the c and k value at the desired height the 25% and 75% cumulative probability speed values were computed by inversion of the cumulative probability equation (A-2) $p(V \leq V_x) = 1 - \exp[-(V_x/c)^k]$, hence

$$\begin{aligned} V_x &= c\{-\ln[1 - p(V \leq V_x)]\}^{1/k} \\ V_{0.25} &= c[-\ln(0.75)]^{1/k} \\ V_{0.75} &= c[-\ln(0.25)]^{1/k} \end{aligned} \quad (1)$$

3) From the c and k value at the desired height, the average value $\bar{V}(z)$ was computed from equation (A-5).

The height profiles of 25 percentile, average, and 75 percentile wind speed given in Tables 2 through 4 provide a rough description of the wind speed distribution at fine height resolution (5 m intervals). Note that, for the range of 10 m $\bar{V}$ reference speeds from 4 to 8 m/s, the height profile of mean wind speed $\bar{V}(z)$ is pretty much independent of the variance (or k value), but there is an observable effect on the height profile of 25 percentile and 75 percentile speed values.

Table 2. Height profile of 25 percentile, average, and 75 percentile wind speeds for various 10 m mean wind speeds, high (90%) variance.

SPEED DISTRIBUTIONS VERSUS 10 M VBAR

HEIGHT	VBAR=4.0M/S			VBAR=5.0M/S			VBAR=6.0M/S			VBAR=7.0M/S			VBAR=8.0M/S		
	25%	AVG	75%	25%	AVG	75%	25%	AVG	75%	25%	AVG	75%	25%	AVG	75%
5.0M	1.5	3.4	4.7	2.1	4.3	5.9	2.8	5.2	7.1	3.5	6.1	8.3	4.2	7.1	9.4
10.0M	1.9	4.0	5.5	2.6	5.0	6.8	3.4	6.0	8.1	4.1	7.0	9.3	4.9	8.0	10.6
15.0M	2.1	4.4	6.0	2.9	5.4	7.4	3.7	6.5	8.7	4.6	7.5	10.0	5.4	8.6	11.3
20.0M	2.3	4.7	6.4	3.2	5.8	7.8	4.0	6.9	9.2	4.9	8.0	10.5	5.8	9.0	11.8
25.0M	2.5	4.9	6.7	3.4	6.1	8.2	4.3	7.2	9.6	5.2	8.3	10.9	6.1	9.4	12.3
30.0M	2.7	5.1	7.0	3.6	6.3	8.5	4.5	7.5	9.9	5.4	8.6	11.3	6.4	9.7	12.6
35.0M	2.8	5.3	7.3	3.7	6.5	8.8	4.7	7.7	10.2	5.6	8.8	11.6	6.6	10.0	12.9
40.0M	2.9	5.5	7.5	3.9	6.7	9.0	4.8	7.9	10.5	5.8	9.1	11.9	6.8	10.2	13.2
45.0M	3.0	5.6	7.7	4.0	6.9	9.2	5.0	8.1	10.7	6.0	9.3	12.1	7.0	10.4	13.5
50.0M	3.1	5.8	7.8	4.1	7.0	9.4	5.1	8.3	10.9	6.1	9.4	12.3	7.1	10.6	13.7
55.0M	3.2	5.9	8.0	4.2	7.2	9.6	5.3	8.4	11.1	6.3	9.6	12.5	7.3	10.8	13.9
60.0M	3.3	6.1	8.2	4.4	7.3	9.8	5.4	8.6	11.3	6.4	9.8	12.7	7.4	11.0	14.1

Table 3. Height profile of 25 percentile, average, and 75 percentile wind speeds
for various 10 m mean wind speeds, average variance.

SPEED DISTRIBUTIONS VERSUS 10 M VBAR															
HEIGHT	VBAR=4.0M/S			VBAR=5.0M/S			VBAR=6.0M/S			VBAR=7.0M/S			VBAR=8.0M/S		
	25%	AVG	75%	25%	AVG	75%	25%	AVG	75%	25%	AVG	75%	25%	AVG	75%
5.0M	1.9	3.4	4.6	2.6	4.3	5.7	3.3	5.2	6.8	4.1	6.1	8.0	4.8	7.1	9.1
10.0M	2.3	4.0	5.4	3.1	5.0	6.6	3.9	6.0	7.8	4.8	7.0	9.0	5.6	8.0	10.2
15.0M	2.6	4.4	5.9	3.5	5.5	7.2	4.4	6.5	8.4	5.2	7.6	9.7	6.2	8.6	10.9
20.0M	2.8	4.7	6.3	3.8	5.8	7.6	4.7	6.9	8.9	5.6	8.0	10.2	6.6	9.1	11.4
25.0M	3.0	5.0	6.6	4.0	6.1	7.9	4.9	7.2	9.3	5.9	8.3	10.6	6.9	9.4	11.8
30.0M	3.2	5.2	6.8	4.2	6.3	8.2	5.2	7.5	9.6	6.2	8.6	10.9	7.2	9.7	12.2
35.0M	3.4	5.4	7.1	4.4	6.6	8.5	5.4	7.7	9.9	6.4	8.9	11.2	7.4	10.0	12.5
40.0M	3.5	5.5	7.3	4.5	6.8	8.7	5.6	7.9	10.1	6.6	9.1	11.5	7.6	10.3	12.8
45.0M	3.6	5.7	7.5	4.7	6.9	9.0	5.7	8.1	10.4	6.8	9.3	11.7	7.8	10.5	13.0
50.0M	3.7	5.8	7.7	4.8	7.1	9.2	5.9	8.3	10.6	6.9	9.5	11.9	8.0	10.7	13.3
55.0M	3.8	6.0	7.8	4.9	7.3	9.3	6.0	8.5	10.8	7.1	9.7	12.1	8.2	10.9	13.5
60.0M	3.9	6.1	8.0	5.1	7.4	9.5	6.1	8.6	10.9	7.2	9.9	12.3	8.3	11.0	13.7

Table 4. Height profile of 25 percentile, average, and 75 percentile wind speeds for various 10 m mean wind speeds, low (10%) variance.

SPEED DISTRIBUTIONS VERSUS 10 M VBAR

HEIGHT	VBAR=4.0M/S			VBAR=5.0M/S			VBAR=6.0M/S			VBAR=7.0M/S			VBAR=8.0M/S		
	25%	AVG	75%	25%	AVG	75%	25%	AVG	75%	25%	AVG	75%	25%	AVG	75%
5.0M	2.0	3.4	4.5	2.7	4.3	5.6	3.5	5.2	6.7	4.3	6.1	7.8	5.0	7.1	8.9
10.0M	2.5	4.0	5.3	3.3	5.0	6.5	4.1	6.0	7.7	5.0	7.0	8.9	5.8	8.0	10.0
15.0M	2.8	4.4	5.8	3.7	5.5	7.0	4.6	6.5	8.3	5.5	7.6	9.5	6.4	8.6	10.7
20.0M	3.0	4.7	6.2	4.0	5.8	7.5	4.9	6.9	8.8	5.8	8.0	10.0	6.8	9.1	11.3
25.0M	3.3	5.0	6.5	4.2	6.1	7.8	5.2	7.2	9.1	6.1	8.3	10.4	7.1	9.4	11.7
30.0M	3.4	5.2	6.7	4.4	6.4	8.1	5.4	7.5	9.5	6.4	8.6	10.8	7.4	9.8	12.0
35.0M	3.6	5.4	7.0	4.6	6.6	8.4	5.6	7.7	9.7	6.6	8.9	11.1	7.6	10.0	12.4
40.0M	3.7	5.6	7.2	4.8	6.8	8.6	5.8	8.0	10.0	6.8	9.1	11.3	7.9	10.3	12.6
45.0M	3.9	5.7	7.4	4.9	6.9	8.8	6.0	8.2	10.2	7.0	9.3	11.6	8.1	10.5	12.9
50.0M	4.0	5.9	7.6	5.1	7.1	9.0	6.1	8.3	10.4	7.2	9.5	11.8	8.3	10.7	13.1
55.0M	4.1	6.0	7.7	5.2	7.3	9.2	6.3	8.5	10.6	7.4	9.7	12.0	8.4	10.9	13.3
60.0M	4.2	6.1	7.9	5.3	7.4	9.4	6.4	8.7	10.8	7.5	9.9	12.2	8.6	11.1	13.5

4. REFERENCE WIND SPEED DISTRIBUTIONS

From the Weibull c and k values for high, average, and low variance at reference height 10 m, listed in Table 1, the complete wind speed probability density function (velocity frequency curve) and cumulative probability (velocity duration curve) can be evaluated. These results, for heights 10, 30, 45, and 60 m and for 10 m reference mean speeds of 4, 5, 6, 7, and 8 m/s are shown in Tables 5-16. The probability distributions in these tables were computed in the following manner: 1) The Weibull c and k from Table 1 for a given 10 m mean speed and variance condition were projected to the desired height by equations (A-14) through (A-16). 2) The height projected Weibull c and k values were used in equations (A-1) and (A-2) to compute the wind speed probability density and cumulative probability at 1 m/s intervals.

Probability density functions (velocity frequency curves) are listed in Tables 5 through 16 in percent per m/s. To convert to speeds in mph, multiply the listed values by 0.447 m/s per mph to get percent per mph. Cumulative probabilities are in percent and require no conversion. If tabulated probability density functions from Tables 5 through 16 are to be used to compute approximate probabilities of speeds over a finite speed interval

$$p(V_a \leq V \leq V_b) = \int_{V_a}^{V_b} p(V) dV \quad (2)$$

numerical approximations (trapezoidal rule, Simpson's rule, etc.) can be used with values of $p(V)$ as tabulated, with the speed interval Δv of 1 m/s and V_a and V_b in m/s. If the speed units are mph, conversion is required to account for the 1 m/s Δv being equivalent to $\Delta v = 2.23$ mph.

If velocity frequency curve values in annual hours per m/s are desired, multiply the probability density values listed in Tables 5 through 16 by 8760 hours/year. If velocity duration curves in hours per year for which $V \geq V_x$ i.e.

Table 5. Wind speed probability density (velocity frequency) and cumulative probability (velocity duration) for various 10 m mean wind speeds, for high variance and at 10 m height.

SPEED M/S	HEIGHT 10.0M HIGH VARIANCE									
	VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB
1.0	14.8	10.7	9.2	5.8	5.7	3.2	3.5	1.8	2.2	1.1
2.0	16.8	26.8	12.6	17.0	9.1	10.7	6.3	6.8	4.4	4.3
3.0	15.8	43.3	13.8	30.3	11.1	20.9	8.5	14.2	6.4	9.7
4.0	13.4	57.9	13.3	44.0	11.9	32.5	9.9	23.5	8.0	17.0
5.0	10.6	69.9	11.9	56.6	11.7	44.3	10.6	33.9	9.1	25.5
6.0	8.0	79.2	10.0	67.5	10.7	55.6	10.6	44.5	9.6	35.0
7.0	5.8	86.0	8.0	76.5	9.4	65.7	9.9	54.8	9.7	44.7
8.0	4.0	90.9	6.1	83.5	7.8	74.3	8.9	64.3	9.2	54.2
9.0	2.7	94.2	4.5	88.8	6.2	81.3	7.6	72.6	8.5	63.0
10.0	1.8	96.4	3.2	92.6	4.8	86.8	6.3	79.5	7.4	71.0
11.0	1.1	97.8	2.2	95.2	3.5	90.9	5.0	85.2	6.3	77.9
12.0	.7	98.7	1.4	97.0	2.5	94.0	3.8	89.6	5.1	83.5
13.0	.4	99.2	.9	98.2	1.7	96.1	2.8	92.9	4.0	88.1
14.0	.2	99.6	.6	98.9	1.2	97.5	2.0	95.2	3.1	91.6
15.0	.1	99.8	.4	99.4	.8	98.5	1.4	96.9	2.3	94.3
16.0	.1	99.9	.2	99.6	.5	99.1	.9	98.1	1.6	96.2
17.0	.0	99.9	.1	99.8	.3	99.5	.6	98.8	1.1	97.6
18.0	.0	100.0	.1	99.9	.2	99.7	.4	99.3	.7	98.5
19.0	.0	100.0	.0	99.9	.1	99.8	.2	99.6	.5	99.1
20.0	.0	100.0	.0	100.0	.1	99.9	.1	99.8	.3	99.4

Table 6. As in Table 5 for average variance, 10 m height.

		HEIGHT		10.0M		AVG VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
SPEED	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	
M/S	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	
1.0	10.6	5.8	5.4	2.6	2.8	1.2	1.5	.6	.8	.3	
2.0	16.5	19.7	10.6	10.7	6.6	5.9	4.0	3.2	2.4	1.8	
3.0	18.3	37.4	14.2	23.3	10.1	14.3	6.8	8.6	4.5	5.2	
4.0	16.8	55.1	15.7	38.5	12.7	25.9	9.6	15.9	6.8	10.9	
5.0	13.5	70.4	15.0	54.0	13.9	39.3	11.6	27.5	9.0	18.9	
6.0	9.6	81.9	12.8	67.9	13.6	53.2	12.7	39.8	10.8	28.8	
7.0	6.2	89.8	9.8	79.2	12.0	66.1	12.6	52.5	11.7	40.1	
8.0	3.7	94.6	6.8	87.5	9.7	77.0	11.4	64.6	11.7	51.9	
9.0	2.0	97.4	4.3	93.0	7.2	85.5	9.6	75.1	10.9	63.3	
10.0	1.0	98.8	2.5	96.4	4.8	91.4	7.4	83.6	9.4	73.5	
11.0	.4	99.5	1.3	98.3	3.0	95.3	5.2	89.9	7.5	82.0	
12.0	.2	99.8	.7	99.2	1.7	97.6	3.4	94.2	5.5	88.5	
13.0	.1	99.9	.3	99.7	.9	98.9	2.1	95.9	3.8	93.1	
14.0	.0	100.0	.1	99.9	.4	99.5	1.1	98.5	2.4	96.2	
15.0	.0	100.0	.0	100.0	.2	99.8	.6	99.3	1.4	98.0	
16.0	.0	100.0	.0	100.0	.1	99.9	.3	99.7	.7	99.0	
17.0	.0	100.0	.0	100.0	.0	100.0	.1	99.9	.4	99.6	
18.0	.0	100.0	.0	100.0	.0	100.0	.0	100.0	.2	99.8	
19.0	.0	100.0	.0	100.0	.0	100.0	.0	100.0	.1	99.9	

Table 7. As in Table 5 for low variance, 10 m height.

		HEIGHT		10.0M		LOW VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
SPEED	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	
M/S	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	
1.0	8.5	4.1	4.1	1.8	1.9	.8	1.0	.4	.5	.2	
2.0	15.9	16.6	9.5	8.6	5.5	4.4	3.1	2.3	1.8	1.2	
3.0	19.4	34.6	14.2	20.6	9.5	11.9	6.1	6.9	3.8	4.0	
4.0	18.7	54.0	16.7	36.3	12.9	23.2	9.2	14.5	6.3	9.1	
5.0	15.1	71.0	16.5	53.1	14.9	37.2	11.9	25.1	8.9	16.7	
6.0	10.3	83.8	14.1	68.6	15.0	52.3	13.5	37.9	11.1	26.7	
7.0	6.1	91.9	10.5	81.0	13.3	66.6	13.8	51.7	12.5	38.5	
8.0	3.1	96.4	6.9	89.6	10.5	78.6	12.6	65.0	12.8	51.2	
9.0	1.4	98.6	3.9	94.9	7.4	87.5	10.4	76.5	11.9	63.7	
10.0	.6	99.5	2.0	97.8	4.6	93.4	7.7	85.5	10.1	74.8	
11.0	.2	99.8	.9	99.1	2.5	96.9	5.1	91.9	7.8	83.8	
12.0	.1	100.0	.3	99.7	1.2	98.7	3.0	95.9	5.5	90.4	
13.0	.0	100.0	.1	99.9	.5	99.5	1.6	98.1	3.4	94.8	
14.0	.0	100.0	.0	100.0	.2	99.8	.7	99.2	1.9	97.5	
15.0	.0	100.0	.0	100.0	.1	100.0	.3	99.7	1.0	98.9	
16.0	.0	100.0	.0	100.0	.0	100.0	.1	99.9	.4	99.6	
17.0	.0	100.0	.0	100.0	.0	100.0	.0	100.0	.2	99.8	

Table 8. As in Table 5 for high variance, 30 m height.

SPEED M/S	HEIGHT 30.0M HIGH VARIANCE									
	VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB
1.0	8.9	5.7	5.0	2.8	2.9	1.5	1.6	.8	.9	.4
2.0	12.2	16.5	8.2	9.6	5.4	5.6	3.5	3.3	2.3	2.0
3.0	13.3	29.4	10.3	18.9	7.5	12.2	5.3	7.8	3.7	5.0
4.0	13.0	42.6	11.3	29.8	9.0	20.5	6.9	13.9	5.2	9.5
5.0	11.7	55.0	11.3	41.1	9.9	30.0	8.1	21.5	6.4	15.3
6.0	9.9	65.9	10.7	52.2	10.1	40.0	8.9	30.0	7.5	22.3
7.0	8.1	74.9	9.5	62.3	9.8	50.1	9.3	39.2	8.2	30.2
8.0	6.3	82.0	8.1	71.1	9.1	59.5	9.2	48.4	8.6	38.6
9.0	4.7	87.5	6.7	78.5	8.0	68.1	8.7	57.4	8.6	47.2
10.0	3.4	91.5	5.2	84.5	6.8	75.5	7.9	65.7	8.3	55.6
11.0	2.4	94.4	4.0	89.1	5.6	81.7	6.9	73.1	7.7	63.6
12.0	1.6	96.4	2.9	92.5	4.4	86.7	5.8	79.4	6.9	70.9
13.0	1.1	97.7	2.1	95.0	3.4	90.6	4.7	84.7	5.9	77.3
14.0	.7	98.6	1.4	96.8	2.5	93.5	3.7	89.0	5.0	82.7
15.0	.4	99.2	1.0	98.0	1.8	95.7	2.8	92.2	4.0	87.2
16.0	.3	99.5	.6	98.7	1.2	97.2	2.1	94.7	3.1	90.8
17.0	.2	99.7	.4	99.2	.8	98.2	1.5	96.5	2.4	93.5
18.0	.1	99.8	.2	99.6	.5	98.9	1.0	97.7	1.7	95.6
19.0	.1	99.9	.1	99.7	.3	99.3	.7	98.6	1.2	97.1
20.0	.0	100.0	.1	99.9	.2	99.6	.4	99.1	.9	98.1

Table 9. As in Table 5 for average variance, 30 m height.

SPEED M/S	HEIGHT 30.0M									
	VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
	PRCB DENS	CUMM PROB	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB
1.0	5.2	2.6	2.4	1.0	1.1	.4	.5	.2	.3	.1
2.0	10.1	10.3	5.7	5.0	3.2	2.5	1.8	1.3	1.0	.7
3.0	13.5	22.2	9.0	12.4	5.7	6.9	3.5	3.9	2.1	2.2
4.0	15.0	36.6	11.6	22.7	8.2	13.9	5.5	8.4	3.6	5.0
5.0	14.5	51.5	13.1	35.2	10.3	23.2	7.6	14.9	5.3	9.5
6.0	12.7	65.2	13.2	48.4	11.7	34.3	9.4	23.4	7.1	15.7
7.0	10.0	76.6	12.2	61.2	12.1	46.3	10.7	33.5	8.7	23.6
8.0	7.3	85.2	10.3	72.5	11.6	58.2	11.3	44.5	9.9	32.9
9.0	4.9	91.3	8.0	81.7	10.3	69.2	11.0	55.8	10.5	43.2
10.0	3.0	95.2	5.8	88.6	8.4	78.6	10.1	66.4	10.5	53.8
11.0	1.7	97.5	3.8	93.3	6.4	86.0	8.6	75.8	9.9	64.1
12.0	.9	98.8	2.3	96.4	4.5	91.4	6.8	83.5	8.7	73.4
13.0	.4	99.5	1.3	98.2	2.9	95.0	5.0	89.4	7.1	81.3
14.0	.2	99.8	.7	99.1	1.8	97.3	3.4	93.6	5.5	87.6
15.0	.1	99.9	.3	99.6	1.0	98.7	2.2	96.4	3.9	92.3
16.0	.0	100.0	.1	99.8	.5	99.4	1.3	98.1	2.6	95.5
17.0	.0	100.0	.1	99.9	.2	99.7	.7	99.1	1.6	97.5
18.0	.0	100.0	.0	100.0	.1	99.9	.3	99.6	.9	98.8
19.0	.0	100.0	.0	100.0	.0	100.0	.2	99.8	.5	99.4
20.0	.0	100.0	.0	100.0	.0	100.0	.1	99.9	.2	99.7

Table 10. As in Table 5 for low variance, 30 m height.

		HEIGHT		30.0M		LOW VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
SPEED	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	
M/S	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	
1.0	3.8	1.6	1.6	.6	.7	.2	.3	.1	.1	.0	
2.0	8.8	7.9	4.6	3.7	2.4	1.7	1.3	.8	.7	.4	
3.0	13.2	19.0	8.2	10.1	4.8	5.3	2.8	2.8	1.6	1.5	
4.0	15.9	33.8	11.5	20.0	7.6	11.5	4.9	6.7	3.1	3.9	
5.0	16.1	50.0	13.8	32.8	10.3	20.5	7.2	12.7	4.8	7.8	
6.0	14.2	65.3	14.5	47.0	12.3	31.8	9.4	21.0	6.8	13.6	
7.0	11.1	78.0	13.5	61.1	13.2	44.7	11.2	31.3	8.7	21.4	
8.0	7.6	87.3	11.3	73.6	12.8	57.8	12.2	43.1	10.3	30.9	
9.0	4.6	93.4	8.5	83.6	11.3	70.0	12.1	55.3	11.3	41.8	
10.0	2.5	96.9	5.7	90.7	9.0	80.2	11.1	67.0	11.5	53.3	
11.0	1.2	98.7	3.4	95.2	6.5	88.0	9.3	77.2	10.8	64.5	
12.0	.5	99.5	1.8	97.7	4.2	93.4	7.1	85.4	9.4	74.6	
13.0	.2	99.8	.9	99.1	2.5	96.7	4.9	91.4	7.5	83.1	
14.0	.1	99.9	.4	99.6	1.3	98.5	3.1	95.4	5.5	89.5	
15.0	.0	100.0	.1	99.9	.6	99.4	1.7	97.7	3.6	94.0	
16.0	.0	100.0	.0	100.0	.2	99.8	.9	99.0	2.2	96.9	
17.0	.0	100.0	.0	100.0	.1	99.9	.4	99.6	1.2	98.5	
18.0	.0	100.0	.0	100.0	.0	100.0	.2	99.9	.6	99.4	
19.0	.0	100.0	.0	100.0	.0	100.0	.1	100.0	.2	99.8	
20.0	.0	100.0	.0	100.0	.0	100.0	.0	100.0	.1	99.9	

Table 11. As in Table 5 for high variance, 45 m height.

		HEIGHT		45.0M		HIGH VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
SPEED	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB
M/S	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	DENS
1.0	7.1	4.3	3.9	2.1	2.1	1.0	1.2	.5	.7	.3	
2.0	10.5	13.3	6.8	7.4	4.3	4.3	2.7	2.4	1.7	1.5	
3.0	12.0	24.6	8.9	15.3	6.3	9.6	4.3	5.9	2.9	3.8	
4.0	12.3	36.9	10.2	24.9	7.8	16.7	5.8	11.0	4.2	7.4	
5.0	11.6	48.9	10.7	35.4	8.9	25.1	7.1	17.5	5.4	12.2	
6.0	10.3	59.9	10.5	46.0	9.5	34.3	8.1	25.1	6.5	18.2	
7.0	8.8	69.5	9.8	56.1	9.6	43.9	8.7	33.5	7.4	25.2	
8.0	7.1	77.4	8.7	65.4	9.2	53.3	8.9	42.3	8.0	32.9	
9.0	5.6	83.8	7.4	73.4	8.4	62.1	8.7	51.1	8.3	41.1	
10.0	4.2	88.6	6.1	80.1	7.4	70.0	8.2	59.5	8.2	49.3	
11.0	3.1	92.2	4.8	85.6	6.3	76.9	7.4	67.4	7.9	57.4	
12.0	2.2	94.8	3.7	89.8	5.2	82.7	6.5	74.3	7.3	65.0	
13.0	1.5	96.6	2.7	93.0	4.1	87.4	5.5	80.3	6.5	71.9	
14.0	1.0	97.9	1.9	95.3	3.2	91.0	4.5	85.3	5.7	78.0	
15.0	.7	98.7	1.4	96.9	2.4	93.8	3.6	89.3	4.8	83.3	
16.0	.4	99.2	.9	98.0	1.7	95.8	2.7	92.5	3.9	87.6	
17.0	.3	99.5	.6	98.8	1.2	97.3	2.0	94.8	3.0	91.0	
18.0	.2	99.7	.4	99.3	.8	98.3	1.4	96.5	2.3	93.7	
19.0	.1	99.8	.2	99.6	.5	98.9	1.0	97.8	1.7	95.7	
20.0	.1	99.9	.1	99.7	.3	99.3	.7	98.6	1.2	97.1	

Table 12. As in Table 5 for average variance, 45 m height.

SPEED M/S	HEIGHT 45.0M AVG VARIANCE									
	VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB	PROB DENS	CUMM PROB
1.0	3.8	1.8	1.7	.7	.7	.3	.3	.1	.2	.1
2.0	8.6	7.7	4.3	3.6	2.3	1.8	1.2	.9	.7	.4
3.0	11.4	17.6	7.2	9.4	4.4	5.1	2.6	2.7	1.5	1.5
4.0	13.5	30.1	9.8	17.9	6.6	10.5	4.3	6.1	2.7	3.6
5.0	14.6	44.6	11.7	28.7	8.7	18.2	6.1	11.3	4.2	7.0
6.0	13.1	57.7	12.5	40.9	10.4	27.8	7.9	18.4	5.7	12.0
7.0	11.1	69.8	12.3	53.4	11.4	38.8	9.4	27.1	7.3	18.5
8.0	8.7	79.8	11.1	65.2	11.5	50.3	10.5	37.1	8.7	26.5
9.0	6.3	87.2	9.3	75.4	10.6	61.5	10.8	47.8	9.7	35.8
10.0	4.2	92.5	7.2	83.6	9.5	71.7	10.5	58.5	10.2	45.8
11.0	2.6	95.8	5.1	89.8	7.7	80.3	9.5	68.5	10.1	56.0
12.0	1.5	97.8	3.4	94.0	5.8	87.1	8.0	77.3	9.4	65.8
13.0	.8	98.9	2.1	96.7	4.1	92.0	6.4	84.5	8.2	74.7
14.0	.4	99.5	1.2	98.3	2.7	95.4	4.7	90.1	6.8	82.2
15.0	.2	99.8	.6	99.2	1.6	97.5	3.2	94.0	5.2	88.2
16.0	.1	99.9	.3	99.6	.9	98.8	2.1	96.6	3.7	92.6
17.0	.0	100.0	.1	99.9	.5	99.4	1.2	98.2	2.5	95.7
18.0	.0	100.0	.1	99.9	.2	99.8	.7	99.1	1.5	97.6
19.0	.0	100.0	.0	100.0	.1	99.9	.3	99.6	.9	98.8
20.0	.0	100.0	.0	100.0	.0	100.0	.1	99.8	.5	99.4

Table 13. As in Table 5 for low variance, 45 m height.

		HEIGHT		45.0M		LOW VARIANCE						
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.		
		-----		-----		-----		-----		-----		
SPEED	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM
M/S	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB
1.0	2.6	1.1	1.1	.4	.4	.1	.2	.1	.1	.1	.0	
2.0	6.7	5.7	3.4	2.5	1.7	1.1	.9	.5	.4	.3		
3.0	10.8	14.5	6.3	7.4	3.6	3.7	2.0	1.9	1.1	1.0		
4.0	13.9	27.0	9.4	15.3	5.9	8.4	3.6	4.7	2.2	2.7		
5.0	15.2	41.7	12.0	26.0	8.4	15.6	5.6	9.3	3.6	5.6		
6.0	14.6	56.8	13.5	38.9	10.6	25.2	7.7	16.0	5.3	10.1		
7.0	12.5	70.5	13.6	52.5	12.1	36.6	9.6	24.7	7.2	16.3		
8.0	9.5	81.4	12.3	65.5	12.6	49.1	11.1	35.1	8.9	24.4		
9.0	6.4	89.4	10.1	76.8	12.0	61.5	11.7	46.6	10.2	34.0		
10.0	3.9	94.4	7.5	85.6	10.4	72.7	11.5	58.3	11.0	44.6		
11.0	2.1	97.4	5.0	91.8	8.2	82.1	10.4	69.3	11.0	55.7		
12.0	1.0	98.9	3.0	95.8	5.9	89.2	8.6	78.9	10.3	66.4		
13.0	.4	99.6	1.6	98.0	3.8	94.0	6.6	86.5	8.9	76.1		
14.0	.2	99.9	.8	99.2	2.2	97.0	4.5	92.0	7.0	84.0		
15.0	.1	100.0	.3	99.7	1.2	98.6	2.9	95.7	5.1	90.1		
16.0	.0	100.0	.1	99.9	.5	99.4	1.6	97.9	3.4	94.4		
17.0	.0	100.0	.0	100.0	.2	99.8	.8	99.1	2.1	97.1		
18.0	.0	100.0	.0	100.0	.1	99.9	.4	99.6	1.1	98.6		
19.0	.0	100.0	.0	100.0	.0	100.0	.1	99.9	.5	99.4		
20.0	.0	100.0	.0	100.0	.0	100.0	.1	100.0	.2	99.8		

Table 14. As in Table 5 for high variance, 60 m height.

		HEIGHT		60.0M		HIGH VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
SPEED	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB
M/S	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	PROB
1.0	6.0	3.5	3.1	1.6	1.7	.8	.9	.4	.5	.2	
2.0	9.2	11.2	5.8	6.1	3.6	3.4	2.2	1.9	1.4	1.1	
3.0	11.0	21.4	7.9	13.0	5.4	8.0	3.6	4.9	2.4	3.0	
4.0	11.7	32.9	9.3	21.6	7.0	14.2	5.0	9.2	3.6	6.1	
5.0	11.4	44.5	10.1	31.4	8.2	21.8	6.3	14.9	4.8	10.2	
6.0	10.5	55.5	10.2	41.6	8.9	30.4	7.4	21.8	5.8	15.6	
7.0	9.2	65.3	9.8	51.6	9.2	39.5	8.1	29.5	6.8	21.9	
8.0	7.7	73.7	8.9	60.9	9.1	48.7	8.5	37.9	7.5	29.0	
9.0	6.2	80.6	7.8	69.3	8.6	57.5	8.6	46.5	7.9	36.7	
10.0	4.8	86.1	6.6	76.6	7.8	65.8	8.3	54.9	8.1	44.7	
11.0	3.6	90.3	5.4	82.6	6.8	73.1	7.7	62.9	7.9	52.7	
12.0	2.6	93.4	4.2	87.4	5.8	79.4	6.9	70.2	7.5	60.5	
13.0	1.9	95.6	3.2	91.1	4.7	84.6	6.0	76.7	6.9	67.7	
14.0	1.3	97.1	2.4	93.9	3.7	88.8	5.0	82.2	6.1	74.2	
15.0	.9	98.2	1.7	95.9	2.8	92.1	4.1	86.8	5.3	79.9	
16.0	.6	98.9	1.2	97.3	2.1	94.5	3.2	90.5	4.4	84.8	
17.0	.4	99.3	.8	98.3	1.5	96.3	2.5	93.3	3.6	88.7	
18.0	.2	99.6	.5	99.0	1.1	97.6	1.8	95.4	2.8	91.9	
19.0	.1	99.8	.3	99.4	.7	98.5	1.3	97.0	2.1	94.3	
20.0	.1	99.9	.2	99.6	.5	99.1	.9	98.0	1.5	96.1	

Table 15. As in Table 5 for average variance, 60 m height.

		HEIGHT		60.0M		AVG VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
SPEED	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	
M/S	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	
1.0	3.0	1.4	1.3	.5	.6	.2	.2	.1	.1	.0	
2.0	6.7	6.2	3.5	2.8	1.8	1.3	.9	.6	.5	.3	
3.0	10.0	14.6	6.0	7.5	3.5	4.0	2.1	2.1	1.2	1.1	
4.0	12.3	25.8	8.5	14.8	5.6	8.5	3.5	4.9	2.2	2.8	
5.0	13.3	38.8	10.5	24.4	7.6	15.1	5.2	9.2	3.4	5.6	
6.0	13.0	52.1	11.8	35.7	9.4	23.6	6.9	15.2	4.9	9.7	
7.0	11.7	64.5	12.1	47.7	10.6	33.6	8.5	22.9	6.4	15.3	
8.0	9.6	75.2	11.4	59.5	11.2	44.6	9.7	32.0	7.8	22.4	
9.0	7.3	83.6	10.0	70.2	10.9	55.7	10.4	42.1	9.0	30.8	
10.0	5.2	89.8	8.1	79.3	10.0	66.2	10.5	52.6	9.7	40.2	
11.0	3.4	94.1	6.1	86.4	8.5	75.5	9.9	62.8	10.0	50.1	
12.0	2.1	96.7	4.3	91.6	6.8	83.2	8.8	72.2	9.7	60.0	
13.0	1.2	98.3	2.8	95.2	5.1	89.2	7.3	80.2	8.8	69.3	
14.0	.6	99.2	1.7	97.4	3.5	93.4	5.7	86.7	7.6	77.5	
15.0	.3	99.6	.9	98.7	2.2	96.2	4.1	91.6	6.1	84.4	
16.0	.1	99.8	.5	99.4	1.3	98.0	2.8	95.0	4.6	89.8	
17.0	.1	99.9	.2	99.7	.7	99.0	1.7	97.2	3.2	93.7	
18.0	.0	100.0	.1	99.9	.4	99.5	1.0	98.6	2.1	96.4	
19.0	.0	100.0	.0	100.0	.2	99.8	.5	99.3	1.3	98.0	
20.0	.0	100.0	.0	100.0	.1	99.9	.3	99.7	.7	99.0	

Table 16. As in Table 5 for low variance, 60 m height.

		HEIGHT		60.0M		LOW VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
SPEED	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	PROB	CUMM	
M/S	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	DENS	PROB	
1.0	2.0	.8	.8	.3	.3	.1	.1	.0	.1	.0	
2.0	5.4	4.5	2.6	1.9	1.3	.8	.6	.4	.3	.2	
3.0	9.2	11.8	5.2	5.8	2.8	2.8	1.6	1.4	.9	.7	
4.0	12.4	22.6	8.0	12.4	4.9	6.6	2.9	3.6	1.7	2.0	
5.0	14.3	36.1	10.6	21.7	7.1	12.6	4.6	7.4	2.9	4.3	
6.0	14.5	50.6	12.4	33.3	9.3	20.9	6.5	12.9	4.4	8.0	
7.0	13.1	64.5	13.2	46.2	11.1	31.2	8.5	20.5	6.1	13.2	
8.0	10.6	76.4	12.6	59.2	12.1	42.9	10.1	29.8	7.8	20.2	
9.0	7.7	85.6	11.0	71.1	12.1	55.0	11.2	40.4	9.3	28.7	
10.0	5.1	91.9	8.7	81.0	11.1	66.7	11.4	51.8	10.4	38.6	
11.0	3.0	95.9	5.2	88.5	9.3	76.9	10.9	63.0	10.8	49.3	
12.0	1.6	98.1	4.0	93.6	7.1	85.2	9.6	73.3	10.6	60.1	
13.0	.7	99.2	2.4	96.8	5.0	91.2	7.7	82.0	9.6	70.2	
14.0	.3	99.7	1.2	98.5	3.1	95.2	5.7	88.7	8.1	79.1	
15.0	.1	99.9	.6	99.4	1.8	97.6	3.9	93.5	6.3	86.3	
16.0	.0	100.0	.2	99.8	.9	99.0	2.4	96.5	4.5	91.7	
17.0	.0	100.0	.1	99.9	.4	99.6	1.3	98.3	2.9	95.4	
18.0	.0	100.0	.0	100.0	.2	99.9	.6	99.3	1.7	97.6	
19.0	.0	100.0	.0	100.0	.1	100.0	.3	99.7	.9	98.9	
20.0	.0	100.0	.0	100.0	.0	100.0	.1	99.9	.4	99.5	

$H(V \geq V_x)$ are desired, convert tabulated cumulative probability values $p(V \leq V_x)$ to $H(V \geq V_x)$ by

$$H(V \geq V_x) = 8760 [1 - p(V \leq V_x)] \quad (3)$$

5. WIND TURBINE PERFORMANCE SENSITIVITY TO WIND DISTRIBUTION

Weibull c and k values from Table 1 for each 10 m level wind speed and variance combination were used to compute wind turbine performance parameters of capacity factor (average output power relative to rated power) and recovery factor (average energy output relative to average available energy in the wind). These results are shown in Table 17 through 28 for heights 10, 30, 45, and 60 m and 10 m mean speed values of 4, 5, 6, 7, and 8 m/s for high, average, and low variance cases. Tables 17 through 28 also show the sensitivity of wind turbine capacity factor and recovery factor to the wind turbine rated speed and cut-in speed. These data were computed as follows: 1) The Weibull c and k values were projected to the desired height by the use of equations (A-14) through (A-16), 2) For a specified wind turbine rated speed V_r , a cut-in speed V_i of $0.4V_r$, or $.45V_r$, or $0.5V_r$ was used and the wind turbine power output curve parameters A , B , and C were computed from equations (B-4) through (B-7). 3) The Weibull c and k values determined in step 1 and A , B , and C from step 2 were used in equation (B-10) to compute capacity factor, i.e.

$$C_f = A(e^{-x_i} - e^{-x_r}) + Bc \int_{x_i}^{x_r} x^{1/k} e^{-x} dx + Cc^2 \int_{x_i}^{x_r} x^{2/k} e^{-x} dx + e^{-x_r} \quad (4)$$

where $x_i = (V_i/c)^k$, $x_r = (V_r/c)^k$ and the cut-out speed is taken to be effectively infinite (i.e. capacity factor is assumed to be independent of cut-out speed), a reasonable assumption for sufficiently large actual cut-out speeds. The integrals in (4) were evaluated by the series expansion method equation (B-13).

Recovery factors were evaluated, with an assumed power coefficient C_p of 1.0, from equation (B-17). For values of power coefficient $C_p = P_r / (0.5 \rho V_r^3 A_s)$ other than 1.0, multiply the Table 17 through 28 recovery factor values by the actual C_p . (Note that the Betz limit of 0.593 would imply that actual recovery

Table 17. Wind turbine capacity factor (CF) and recovery factor (RF) for various 10 m mean wind speeds and various wind turbine rated speeds (with cut-in speeds of 0.4, 0.45, and 0.5 times rated speed), for high variance and 10 m height. Recovery factors tabulated are for $C_p = 1.0$, use actual $C_p = Pr/(0.5\rho V_r^3 A_s)$ for true RF values, where by the Betz limit C_p would be ≤ 0.593 .

		HEIGHT		10.0M		HIGH VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
CUT-IN	RATED	CF	RF	CF	RF	CF	RF	CF	RF	CF	RF
SPEED	SPEED										
M/S	M/S	%	%	%	%	%	%	%	%	%	%
3.2	8.0	22.0	63.1	33.0	56.5	44.2	48.7	54.6	41.5	63.8	34.6
3.6	8.0	21.4	61.4	32.4	55.5	43.7	48.1	54.2	41.2	63.4	34.4
4.0	8.0	20.7	59.3	31.7	54.2	43.0	47.3	53.5	40.7	62.9	34.2
3.6	9.0	17.0	69.3	26.6	64.8	37.0	58.1	47.2	51.1	56.7	43.8
4.1	9.0	16.4	66.9	26.0	63.3	36.4	57.1	46.7	50.5	56.2	43.5
4.5	9.0	15.7	64.1	25.2	61.4	35.6	55.9	46.0	49.8	55.6	43.0
4.0	10.0	13.1	73.1	21.3	71.0	30.7	66.0	40.3	59.9	49.8	52.8
4.5	10.0	12.5	69.9	20.6	68.8	30.0	64.6	39.7	59.0	49.2	52.2
5.0	10.0	11.8	66.1	19.8	66.2	29.2	62.8	38.9	57.8	48.5	51.5
4.4	11.0	10.0	74.5	16.9	75.0	25.2	72.1	34.1	67.4	43.3	61.1
5.0	11.0	9.5	70.4	16.2	72.1	24.5	70.1	33.4	66.1	42.7	60.2
5.5	11.0	8.8	65.3	15.4	68.6	23.6	67.7	32.6	64.4	41.9	59.1
4.8	12.0	7.6	73.9	13.3	76.9	20.5	76.3	28.6	73.4	37.2	68.2
5.4	12.0	7.1	69.0	12.7	73.2	19.8	73.7	27.9	71.5	36.6	67.0
6.0	12.0	6.6	63.5	11.9	68.8	18.9	70.4	27.0	69.2	35.7	65.4
5.2	13.0	5.8	71.7	10.5	77.0	16.6	78.6	23.8	77.6	31.8	74.0
5.9	13.0	5.4	68.0	9.9	72.4	15.9	75.2	23.0	75.2	31.0	72.4
6.5	13.0	4.9	59.7	9.1	67.1	15.1	71.2	22.1	72.2	30.1	70.2
5.6	14.0	4.5	68.4	8.2	75.4	13.4	79.2	19.7	80.1	26.9	78.3
6.3	14.0	4.0	61.9	7.6	70.0	12.7	75.1	18.9	77.0	26.1	76.1
7.0	14.0	3.6	55.0	7.0	64.0	11.9	70.1	18.0	73.2	25.2	73.3
6.0	15.0	3.4	64.2	6.4	72.7	10.8	78.4	16.2	81.1	22.7	81.1
6.8	15.0	3.0	57.1	5.9	66.5	10.1	73.4	15.4	77.2	21.9	78.3
7.5	15.0	2.6	49.7	5.3	59.7	9.3	67.6	14.5	72.6	20.9	74.7

Table 18. As in Table 17 for average variance and 10 m height.

		HEIGHT		10.0M		AVG VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
CUT-IN	RATED	CF	RF	CF	RF	CF	RF	CF	RF	CF	RF
SPEED	SPEED										
M/S	M/S	%	%	%	%	%	%	%	%	%	%
3.2	8.0	20.1	73.5	32.8	73.8	46.1	65.2	58.5	55.3	68.9	45.9
3.6	8.0	19.4	75.6	32.1	72.2	45.5	64.3	58.0	54.9	68.6	45.6
4.0	8.0	18.5	72.1	31.2	70.2	44.7	63.2	57.4	54.3	68.1	45.3
3.6	9.0	14.4	80.3	25.1	80.3	37.2	74.9	49.5	66.6	60.6	57.4
4.1	9.0	13.7	76.1	24.3	77.8	36.5	73.5	48.9	65.9	60.2	57.0
4.5	9.0	12.9	71.2	23.3	74.6	35.5	71.6	48.1	64.8	59.6	56.4
4.0	10.0	10.3	73.3	18.8	82.8	29.4	81.4	41.0	75.8	52.3	68.0
4.5	10.0	9.6	72.9	18.0	79.2	28.7	79.2	40.3	74.5	51.8	67.2
5.0	10.0	8.7	66.6	17.0	74.8	27.6	76.4	39.4	72.8	51.0	66.3
4.4	11.0	7.3	75.8	14.0	82.1	23.0	84.5	33.4	82.2	44.3	76.6
5.0	11.0	6.6	67.1	13.2	77.2	22.1	81.4	32.6	80.3	43.6	75.5
5.5	11.0	5.9	59.6	12.2	71.4	21.0	77.4	31.6	77.7	42.7	73.9
4.8	12.0	5.1	67.6	10.4	78.8	17.7	84.7	26.8	85.7	36.9	82.9
5.4	12.0	4.5	59.8	9.6	72.7	16.8	80.4	26.0	83.0	36.1	81.2
6.0	12.0	3.9	51.3	8.6	65.5	15.7	75.2	24.9	79.4	35.1	78.9
5.2	13.0	3.6	62.6	7.6	73.7	13.6	82.5	21.3	86.6	30.3	86.6
5.9	13.0	3.1	51.8	6.9	66.4	12.7	77.0	20.4	82.9	29.5	84.2
6.5	13.0	2.8	42.8	6.0	58.1	11.6	70.5	19.2	78.1	28.4	81.0
5.6	14.0	2.5	53.2	5.6	67.6	10.4	78.5	16.8	85.3	24.7	88.0
6.3	14.0	2.1	43.9	4.9	59.2	9.5	71.9	15.9	80.4	23.8	84.7
7.0	14.0	1.7	34.8	4.1	50.0	8.4	64.0	14.7	74.4	22.6	80.4
6.0	15.0	1.8	45.9	4.1	60.9	7.9	73.4	13.2	82.3	19.9	87.4
6.8	15.0	1.4	36.4	3.5	51.6	7.0	65.6	12.2	76.3	18.9	83.1
7.5	15.0	1.1	27.5	2.8	42.0	6.1	56.7	11.1	69.0	17.7	77.6

Table 19. As in Table 17 for low variance and 10 m height.

		HEIGHT		10.0M		LOW VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
CUT-IN	RATED	CF	RF	CF	RF	CF	RF	CF	RF	CF	RF
SPEED	SPEED	%	%	%	%	%	%	%	%	%	%
M/S	M/S										
3.2	8.9	18.8	32.8	32.3	79.9	46.7	71.4	59.8	60.4	70.7	49.7
3.6	8.0	18.0	79.2	31.5	78.0	46.1	70.4	59.4	59.9	70.4	49.5
4.0	8.0	17.0	74.8	30.5	75.5	45.2	69.1	58.7	59.3	69.9	49.1
3.6	9.0	13.0	81.4	24.0	84.6	37.0	80.6	50.2	72.1	61.9	62.0
4.1	9.0	12.2	76.3	23.2	81.6	36.3	79.0	49.6	71.2	61.5	61.6
4.5	9.0	11.2	70.2	22.1	77.8	35.3	76.8	48.8	70.1	60.9	61.0
4.0	10.0	8.9	76.4	17.6	84.8	28.7	85.8	41.0	80.9	53.0	72.8
4.5	10.0	8.1	69.7	16.7	80.5	27.9	83.2	40.3	79.5	52.5	72.1
5.0	10.0	7.2	62.0	15.6	75.1	26.8	79.9	39.4	77.6	51.7	71.0
4.4	11.0	6.1	69.2	12.7	81.7	21.9	87.1	32.9	86.3	44.4	81.2
5.0	11.0	5.3	61.0	11.8	75.8	21.0	83.4	32.1	84.2	43.8	80.0
5.5	11.0	4.6	52.1	10.7	68.9	19.8	78.7	31.0	81.3	42.8	78.3
4.8	12.0	4.1	60.9	9.1	76.2	16.5	95.3	26.0	88.5	36.6	86.8
5.4	12.0	3.5	51.6	8.3	68.9	15.6	80.3	25.0	85.3	35.8	84.9
6.0	12.0	2.8	41.9	7.2	60.4	14.4	74.0	23.8	81.2	34.7	82.3
5.2	13.0	2.8	52.3	6.5	69.4	12.4	81.3	20.3	87.8	29.6	89.4
5.9	13.0	2.2	42.3	5.7	60.7	11.4	74.8	19.3	83.5	28.7	86.7
6.5	13.0	1.7	32.5	4.8	51.1	10.2	67.0	18.0	78.0	27.5	83.0
5.6	14.0	1.9	43.9	4.7	61.8	9.2	75.7	15.7	85.0	23.8	89.5
6.3	14.0	1.4	33.7	3.9	52.0	8.3	67.9	14.7	79.4	22.8	85.8
7.0	14.0	1.0	24.3	3.2	41.8	7.2	58.7	13.4	72.4	21.5	80.9
6.0	15.0	1.2	36.0	3.3	54.0	6.9	69.3	12.1	80.6	18.9	87.7
6.8	15.0	.9	26.1	2.7	43.5	6.0	60.1	11.1	73.7	17.9	82.8
7.5	15.0	.6	17.5	2.0	33.0	5.0	49.9	9.8	65.2	16.5	76.5

Table 20. As in Table 17 for high variance and 30 m height.

		HEIGHT		30.0M		HIGH VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
CUT-IN	RATED	CF	RF	CF	RF	CF	RF	CF	RF	CF	RF
SPEED	SPEED	%	%	%	%	%	%	%	%	%	%
M/S	M/S										
3.2	8.0	34.6	53.8	47.4	45.6	58.9	37.7	68.5	31.1	76.2	25.3
3.6	8.0	34.0	52.9	46.9	45.1	58.5	37.4	68.2	30.9	75.9	25.2
4.0	8.0	33.3	51.8	46.3	44.5	57.9	37.0	67.8	30.7	75.6	25.1
3.6	9.0	28.2	62.3	40.2	55.1	51.6	47.0	61.8	39.9	70.2	33.3
4.1	9.0	27.5	60.9	39.6	54.2	51.1	46.6	61.4	39.6	69.9	33.1
4.5	9.0	26.8	59.2	38.9	53.2	50.5	46.0	60.8	39.3	69.5	32.9
4.0	10.0	22.7	69.0	33.7	63.3	44.8	55.9	55.0	48.3	64.1	41.6
4.5	10.0	22.1	67.0	33.1	62.1	44.2	55.2	54.6	48.3	63.7	41.4
5.0	10.0	21.3	64.6	32.2	60.6	43.4	54.3	53.9	47.8	63.2	41.0
4.4	11.0	18.2	73.6	28.0	70.0	38.4	63.8	48.5	57.2	57.8	50.0
5.0	11.0	17.6	70.9	27.3	68.3	37.8	62.8	48.0	56.6	57.4	49.6
5.5	11.0	16.8	67.8	26.4	66.1	36.9	61.4	47.2	55.7	56.7	49.1
4.8	12.0	14.5	76.2	23.1	74.9	32.6	70.5	42.3	64.8	51.7	58.0
5.4	12.0	13.9	72.8	22.4	72.6	32.0	69.0	41.7	63.9	51.2	57.4
6.0	12.0	13.1	68.8	21.5	69.8	31.1	67.1	40.9	62.6	50.4	56.6
5.2	13.0	11.5	77.0	18.9	78.0	27.5	75.6	36.6	71.3	45.8	65.3
5.9	13.0	10.9	72.8	18.2	75.0	26.8	73.6	35.9	70.0	45.2	64.4
6.5	13.0	10.2	67.9	17.3	71.4	25.9	71.0	35.1	68.2	44.4	63.3
5.6	14.0	9.1	76.2	15.4	79.4	23.1	79.1	31.4	76.4	40.2	71.6
6.3	14.0	8.5	71.2	14.7	75.7	22.3	76.5	30.7	74.6	39.5	70.4
7.0	14.0	7.9	65.5	13.8	71.3	21.4	73.2	29.7	72.3	38.6	68.8
6.0	15.0	7.2	74.1	12.5	79.4	19.2	81.0	26.8	80.1	35.0	76.7
6.8	15.0	6.7	68.3	11.8	74.9	18.4	77.8	26.0	77.8	34.3	75.1
7.5	15.0	6.0	61.9	11.0	69.6	17.5	73.7	25.0	74.8	33.3	73.1

Table 21. As in Table 17 for average variance and 30 m height.

		HEIGHT		30.0M		AVG VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
CUT-IN	RATED	CF	RF	CF	RF	CF	RF	CF	RF	CF	RF
SPEED	SPEED										
M/S	M/S	%	%	%	%	%	%	%	%	%	%
3.2	8.0	35.3	70.4	50.5	60.6	63.7	49.8	74.3	40.2	82.2	32.1
3.6	8.0	34.6	69.0	50.0	59.9	63.3	49.5	74.0	40.0	82.0	32.1
4.0	8.0	33.7	67.3	49.2	59.0	62.8	49.1	73.6	39.8	81.7	32.0
3.6	9.0	27.4	77.9	41.6	71.0	55.1	61.3	66.7	51.4	76.0	42.3
4.1	9.0	26.7	75.8	40.9	69.9	54.6	60.8	66.4	51.1	75.7	42.2
4.5	9.0	25.7	73.1	40.1	68.4	53.9	60.0	65.8	50.7	75.4	42.0
4.0	10.0	21.0	81.8	33.6	78.7	46.7	71.3	58.8	62.1	69.1	52.8
4.5	10.0	20.2	78.7	32.9	77.0	46.0	70.3	58.4	61.6	68.8	52.6
5.0	10.0	19.2	74.9	31.9	74.7	45.2	69.1	57.7	60.9	68.3	52.2
4.4	11.0	15.9	82.5	26.8	83.4	38.9	79.0	50.9	71.6	61.9	62.9
5.0	11.0	15.1	78.3	25.9	80.8	38.1	77.5	50.4	70.8	61.4	62.5
5.5	11.0	14.1	73.1	24.9	77.5	37.2	75.6	49.6	69.7	60.8	61.9
4.8	12.0	11.9	80.5	21.0	85.1	31.9	84.1	43.4	79.2	54.5	71.9
5.4	12.0	11.1	75.1	20.2	81.6	31.1	82.0	42.7	77.9	54.0	71.2
6.0	12.0	10.2	68.7	19.1	77.1	30.0	79.2	41.8	76.3	53.2	70.3
5.2	13.0	8.9	76.6	16.4	84.4	25.8	86.7	36.5	84.6	47.3	79.4
5.9	13.0	8.2	69.9	15.5	79.7	24.9	83.7	35.7	82.3	46.6	78.3
6.5	13.0	7.3	62.3	14.4	74.0	23.6	79.8	34.6	80.3	45.8	76.8
5.6	14.0	6.7	71.3	12.7	81.7	20.7	86.9	30.3	87.7	40.5	84.8
6.3	14.0	5.9	63.6	11.8	75.9	19.8	83.0	29.4	85.1	39.7	83.3
7.0	14.0	5.1	55.0	10.7	68.9	18.6	78.0	28.2	81.8	38.7	81.2
6.0	15.0	5.0	65.3	9.8	77.6	16.5	85.3	24.9	88.7	34.2	88.3
6.8	15.0	4.3	56.5	8.9	70.6	15.6	80.3	23.9	85.3	33.4	86.2
7.5	15.0	3.6	47.3	7.9	62.4	14.4	74.0	22.7	80.9	32.3	83.2

Table 22. As in Table 17 for low variance and 30 m height.

		HEIGHT		30.0M		LCW VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
CUT-IN	RATED	CF	RF	CF	RF	CF	RF	CF	RF	CF	RF
SPEED	SPEED	%	%	%	%	%	%	%	%	%	%
M/S	M/S										
3.2	8.0	35.2	77.3	51.5	66.6	65.6	54.6	76.4	43.5	84.1	34.4
3.6	8.0	34.5	75.7	50.9	65.9	65.3	54.3	76.1	43.3	83.9	34.3
4.0	8.0	33.5	73.6	50.2	64.9	64.7	53.8	75.8	43.1	83.7	34.2
3.6	9.0	26.7	83.4	41.8	77.0	56.4	66.8	68.5	55.5	77.9	45.3
4.1	9.0	25.9	80.8	41.1	75.8	55.9	66.2	68.2	55.2	77.7	45.2
4.5	9.0	24.8	77.5	40.2	74.1	55.2	65.4	67.7	54.8	77.3	45.0
4.0	10.0	19.9	85.2	33.2	83.9	47.3	76.8	60.2	66.9	70.8	56.5
4.5	10.0	19.0	81.4	32.4	81.8	46.7	75.8	59.7	66.3	70.6	56.3
5.0	10.0	17.9	76.6	31.3	79.2	45.8	74.4	59.1	65.6	70.0	55.9
4.4	11.0	14.6	83.3	25.9	87.0	38.8	84.3	51.7	76.5	63.2	67.1
5.0	11.0	13.7	78.2	25.0	84.0	38.1	82.4	51.1	75.6	62.8	66.7
5.5	11.0	12.6	71.9	23.8	80.1	37.1	80.2	50.3	74.4	62.2	66.1
4.8	12.0	10.7	79.0	19.9	86.9	31.3	88.0	43.6	83.7	55.3	76.3
5.4	12.0	9.8	72.4	19.0	82.8	30.5	85.6	42.9	82.4	54.8	75.6
6.0	12.0	8.7	64.6	17.7	77.5	29.3	82.3	41.9	80.5	54.1	74.6
5.2	13.0	7.7	73.0	15.2	84.3	25.0	89.1	36.2	88.3	47.6	83.5
5.9	13.0	6.9	65.0	14.2	78.8	24.0	85.7	35.4	86.3	47.0	82.4
6.5	13.0	5.9	55.9	13.0	72.1	22.6	81.2	34.2	83.6	46.1	80.8
5.6	14.0	5.6	66.0	11.5	80.0	19.7	87.9	29.6	90.3	40.3	88.4
6.3	14.0	4.8	56.8	10.5	73.1	19.7	83.3	28.7	87.5	39.6	86.7
7.0	14.0	4.0	46.9	9.4	64.9	17.4	77.4	27.5	83.7	38.5	84.4
6.0	15.0	4.1	58.6	8.7	74.4	15.5	84.8	24.0	90.1	33.8	91.0
6.8	15.0	3.4	48.5	7.8	66.2	14.4	79.0	23.0	86.3	32.9	88.7
7.5	15.0	2.6	38.1	6.7	56.7	13.1	71.7	21.7	81.3	31.7	85.4

Table 23. As in Table 17 for high variance and 45 m height.

		HEIGHT		45.0M		HIGH VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
CUT-IN	RATED	CF	RF	CF	RF	CF	RF	CF	RF	CF	RF
SPEED	SPEED	%	%	%	%	%	%	%	%	%	%
M/S	M/S										
3.2	8.0	40.2	49.7	53.3	41.3	64.4	33.6	73.4	27.4	80.3	22.2
3.6	8.0	39.6	49.0	52.8	40.9	64.0	33.4	73.1	27.3	80.1	22.2
4.0	8.0	38.9	48.1	52.2	40.4	63.5	33.1	72.7	27.2	79.8	22.1
3.6	9.0	33.3	58.6	46.0	50.7	57.4	42.6	67.1	35.7	74.9	29.5
4.1	9.0	32.7	57.6	45.4	50.1	57.0	42.3	66.8	35.5	74.7	29.4
4.5	9.0	31.9	56.2	44.7	49.3	56.4	41.9	66.3	35.3	74.3	29.3
4.0	10.0	27.4	66.1	39.2	59.4	50.6	51.5	60.7	44.3	69.2	37.4
4.5	10.0	26.7	64.6	38.6	58.4	50.1	51.0	60.3	44.0	68.9	37.2
5.0	10.0	25.9	62.6	37.8	57.3	49.4	50.3	59.7	43.6	68.5	37.0
4.4	11.0	22.3	71.8	33.1	66.8	44.1	59.8	54.3	52.8	63.3	45.6
5.0	11.0	21.7	69.6	32.5	65.4	43.5	59.0	53.8	52.3	62.9	45.3
5.5	11.0	20.9	67.0	31.6	63.7	42.7	57.9	53.1	51.7	62.4	44.9
4.8	12.0	18.1	75.6	27.8	72.7	38.1	67.0	48.1	60.7	57.4	53.6
5.4	12.0	17.4	72.8	27.1	70.8	37.4	65.9	47.5	60.0	56.9	53.2
6.0	12.0	16.6	69.3	26.2	68.5	36.6	64.4	46.8	59.0	56.3	52.6
5.2	13.0	14.6	77.5	23.1	76.9	32.6	73.0	42.2	67.7	51.5	61.2
5.9	13.0	13.9	74.0	22.4	74.4	31.9	71.4	41.6	66.7	51.0	60.5
6.5	13.0	13.1	69.7	21.5	71.4	31.0	69.4	40.8	65.4	50.2	59.6
5.6	14.0	11.8	77.9	19.1	79.4	27.7	77.5	36.8	73.7	45.8	68.0
6.3	14.0	11.1	73.5	18.4	76.3	27.0	75.4	36.1	72.3	45.2	67.1
7.0	14.0	10.3	68.5	17.5	72.6	26.0	72.7	35.1	70.4	44.4	65.9
6.0	15.0	9.4	76.3	15.8	80.5	23.4	80.5	31.8	78.3	40.5	73.8
6.8	15.0	8.8	71.7	15.0	76.6	22.6	77.8	31.0	76.4	39.8	72.6
7.5	15.0	8.1	65.9	14.1	72.0	21.7	74.5	30.0	74.0	38.9	70.9

Table 24. As in Table 17 for average variance and 45 m height.

		HEIGHT		45.0M		AVG VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
CUT-IN	RATED	CF	RF	CF	RF	CF	RF	CF	RF	CF	RF
SPEED	SPEED										
M/S	M/S	%	%	%	%	%	%	%	%	%	%
3.2	8.0	42.1	65.4	57.5	54.5	70.0	43.9	79.4	34.8	86.1	27.6
3.6	8.0	41.5	64.4	57.0	54.1	69.6	43.7	79.2	34.7	86.0	27.6
4.0	8.0	40.6	63.1	56.4	53.4	69.2	43.4	78.9	34.6	85.8	27.5
3.6	9.0	33.7	74.5	48.7	65.7	61.9	55.3	72.7	45.4	80.9	37.8
4.1	9.0	33.0	72.9	48.1	64.9	61.5	54.9	72.4	45.2	80.7	36.9
4.5	9.0	32.0	70.9	47.3	63.8	60.9	54.4	72.0	45.0	80.4	36.7
4.0	10.0	26.5	80.5	40.4	74.8	53.7	65.8	65.4	56.1	74.9	46.9
4.5	10.0	25.7	78.1	39.7	73.5	53.2	65.1	65.0	55.7	74.6	46.8
5.0	10.0	24.7	75.1	38.8	71.8	52.5	64.3	64.5	55.3	74.2	46.5
4.4	11.0	20.6	83.2	33.0	81.2	45.8	74.7	57.9	66.0	68.2	56.9
5.0	11.0	19.8	79.9	32.2	79.3	45.2	73.6	57.4	65.5	67.9	56.6
5.5	11.0	18.7	75.7	31.2	76.8	44.3	72.2	56.7	64.7	67.4	56.2
4.8	12.0	15.9	83.2	26.5	84.9	38.5	81.4	50.4	74.6	61.3	66.4
5.4	12.0	15.0	78.7	25.7	82.2	37.7	79.8	49.8	73.8	60.8	65.9
6.0	12.0	14.0	73.3	24.6	78.7	36.7	77.7	49.0	72.6	60.2	65.2
5.2	13.0	12.1	80.9	21.2	86.0	31.8	85.7	43.2	81.4	54.2	74.7
5.9	13.0	11.3	75.3	20.2	82.3	31.0	83.5	42.5	80.1	53.7	73.9
6.5	13.0	10.3	68.6	19.1	77.7	29.9	80.5	41.6	78.3	52.9	72.9
5.6	14.0	9.2	76.9	16.7	85.0	26.1	87.7	36.6	86.1	47.3	81.4
6.3	14.0	8.4	70.1	15.8	81.2	25.2	84.6	35.8	84.2	46.7	80.3
7.0	14.0	7.5	62.4	14.6	74.4	24.0	80.7	34.8	81.7	45.8	78.8
6.0	15.0	7.0	71.8	13.2	82.2	21.2	87.6	30.7	88.7	40.8	86.3
6.8	15.0	6.2	64.0	12.2	76.3	20.2	83.7	29.8	86.2	40.0	84.7
7.5	15.0	5.4	55.2	11.1	69.3	19.0	78.6	28.6	82.8	39.0	82.6

Table 25. As in Table 17 for low variance and 45 m height.

		HEIGHT		45.0M		LOW VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
CUT-IN	RATED	CF	RF	CF	RF	CF	RF	CF	RF	CF	RF
SPEED	SPEED										
M/S	M/S	%	%	%	%	%	%	%	%	%	%
3.2	8.0	42.7	72.4	59.0	60.0	72.2	47.9	81.6	37.5	87.9	29.4
3.6	8.0	42.1	71.3	58.6	59.5	71.9	47.7	81.4	37.4	87.8	29.3
4.0	8.0	41.2	69.8	57.9	58.9	71.5	47.4	81.1	37.3	87.6	29.3
3.6	9.0	33.5	80.8	49.5	71.6	63.7	60.1	74.7	48.9	82.8	39.4
4.1	9.0	32.7	79.0	48.9	70.7	63.3	59.7	74.5	48.7	82.6	39.3
4.5	9.0	31.7	76.5	48.1	69.6	62.7	59.2	74.1	48.5	82.4	39.2
4.0	10.0	25.7	85.2	40.5	80.4	54.9	71.1	67.1	60.3	76.7	50.1
4.5	10.0	24.9	82.4	39.8	78.9	54.4	70.4	66.8	60.0	76.5	49.9
5.0	10.0	23.8	78.7	38.8	77.0	53.6	69.5	66.3	59.5	76.1	49.7
4.4	11.0	19.5	85.9	32.5	85.8	46.3	79.9	59.1	70.7	69.9	60.7
5.0	11.0	18.6	81.8	31.7	83.6	45.7	78.8	58.7	70.1	69.5	60.4
5.5	11.0	17.4	76.8	30.6	80.7	44.8	77.3	58.0	69.3	69.1	60.0
4.8	12.0	14.6	83.6	25.7	88.0	38.4	85.9	51.1	79.3	62.5	70.5
5.4	12.0	13.7	78.2	24.8	84.9	37.6	84.2	50.5	78.4	62.1	70.0
6.0	12.0	12.5	71.7	23.6	80.8	36.6	81.9	49.7	77.1	61.5	69.3
5.2	13.0	10.9	79.2	20.1	87.5	31.3	89.2	43.4	85.6	55.0	78.9
5.9	13.0	10.0	72.4	19.1	83.2	30.5	86.7	42.7	84.2	54.5	78.1
6.5	13.0	8.9	64.4	17.8	77.7	29.3	83.3	41.7	82.3	53.7	77.0
5.6	14.0	8.1	73.4	15.6	84.8	25.3	89.8	36.3	89.5	47.6	85.3
6.3	14.0	7.2	65.3	14.5	79.2	24.3	86.4	35.5	87.5	47.0	84.1
7.0	14.0	6.2	56.1	13.3	72.3	23.0	81.8	34.4	84.7	46.1	82.5
6.0	15.0	6.0	56.8	12.0	80.6	20.2	88.4	30.1	91.1	40.7	89.6
6.8	15.0	5.2	57.6	11.0	73.7	19.2	83.9	29.1	88.3	39.9	87.9
7.5	15.0	4.2	47.4	9.8	65.4	17.8	78.0	27.9	84.5	38.9	85.6

Table 26. As in Table 17 for high variance and 60 m height.

		HEIGHT		60.0M		HIGH VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
CUT-IN	RATED	CF	RF	CF	RF	CF	RF	CF	RF	CF	RF
SPEED	SPEED										
M/S	M/S	%	%	%	%	%	%	%	%	%	%
3.2	8.0	44.4	46.5	57.5	38.2	68.2	30.8	76.7	25.0	83.0	20.1
3.6	8.0	43.8	46.0	57.0	37.9	67.9	30.6	76.5	24.9	82.8	20.1
4.0	8.0	43.1	45.2	56.5	37.5	67.5	30.4	76.1	24.8	82.6	20.0
3.6	9.0	37.3	53.7	50.2	47.5	61.5	39.5	70.8	32.8	78.1	27.0
4.1	9.0	36.7	54.8	49.7	47.0	61.1	39.2	70.5	32.7	77.9	26.9
4.5	9.0	36.0	53.7	49.1	46.4	60.6	38.9	70.1	32.5	77.5	26.8
4.0	10.0	31.1	63.7	43.4	56.3	54.8	48.3	64.7	41.1	72.8	34.5
4.5	10.0	30.4	62.4	42.8	55.6	54.3	47.8	64.3	40.9	72.5	34.4
5.0	10.0	29.6	60.7	42.1	54.6	53.7	47.3	63.8	40.5	72.1	34.2
4.4	11.0	25.7	70.0	37.1	64.1	48.3	56.7	58.5	49.5	67.2	42.4
5.0	11.0	25.0	68.2	36.5	63.0	47.8	56.0	58.0	49.1	66.8	42.2
5.5	11.0	24.2	65.9	35.7	61.6	47.1	55.1	57.4	48.5	66.4	41.9
4.8	12.0	21.1	74.6	31.5	70.6	42.2	64.2	52.4	57.5	61.5	50.3
5.4	12.0	20.4	72.2	30.8	69.0	41.6	63.3	51.8	56.9	61.0	50.0
6.0	12.0	19.5	69.2	29.9	67.1	40.8	62.1	51.1	56.2	60.4	49.5
5.2	13.0	17.2	77.4	26.5	75.5	36.6	70.7	46.4	64.9	55.7	58.0
5.9	13.0	16.5	74.3	25.8	73.5	35.9	69.4	45.8	64.0	55.2	57.5
6.5	13.0	15.7	70.5	24.9	70.9	35.0	67.7	45.0	62.9	54.5	56.8
5.6	14.0	14.0	78.6	22.2	78.9	31.4	75.9	40.8	71.2	50.0	65.1
6.3	14.0	13.3	74.7	21.4	76.2	30.7	74.1	40.2	70.1	49.5	64.3
7.0	14.0	12.5	70.1	20.5	72.9	29.7	71.8	39.3	68.6	48.7	63.3
6.0	15.0	11.3	78.3	18.4	80.7	26.8	79.6	35.6	76.5	44.6	71.3
6.8	15.0	10.7	75.6	17.7	77.4	26.0	77.3	34.9	74.9	43.9	70.3
7.5	15.0	9.9	68.2	16.7	73.3	25.0	74.4	34.0	72.9	43.1	68.9

Table 27. As in Table 17 for average variance and 60 m height.

		HEIGHT		60.0M		AVG VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
CUT-IN	RATED	CF	RF	CF	RF	CF	RF	CF	RF	CF	RF
SPEED	SPEED										
M/S	M/S	%	%	%	%	%	%	%	%	%	%
3.2	8.0	47.2	61.3	62.4	50.1	74.2	39.7	82.7	31.3	88.6	24.7
3.6	8.0	46.6	60.6	62.0	49.7	73.9	39.6	82.5	31.2	88.5	24.7
4.0	8.0	45.9	59.6	61.5	49.3	73.5	39.4	82.3	31.1	88.3	24.6
3.6	9.0	38.6	71.3	53.8	61.4	66.6	50.8	76.7	41.3	84.0	33.3
4.1	9.0	37.9	70.1	53.3	60.8	66.3	50.5	76.4	41.1	83.8	33.3
4.5	9.0	37.0	68.4	52.6	60.0	65.7	50.1	76.1	40.9	83.6	33.2
4.0	10.0	31.0	78.6	45.5	71.2	58.8	61.5	69.9	51.6	78.6	42.8
4.5	10.0	30.2	76.6	44.9	70.3	58.3	61.0	69.6	51.4	78.4	42.7
5.0	10.0	29.2	74.2	44.0	68.9	57.7	60.3	69.2	51.1	78.1	42.5
4.4	11.0	24.5	82.8	37.8	78.8	51.0	71.0	62.8	61.7	72.6	52.6
5.0	11.0	23.7	80.0	37.1	77.3	50.4	70.2	62.4	61.3	72.3	52.4
5.5	11.0	22.7	76.5	36.1	75.3	49.6	69.1	61.8	60.7	71.8	52.1
4.8	12.0	19.2	84.2	31.0	83.8	43.5	78.6	55.5	70.8	66.0	62.1
5.4	12.0	18.3	80.4	30.2	81.6	42.8	77.4	55.0	70.1	65.6	61.7
6.0	12.0	17.3	75.7	29.1	78.7	41.9	75.7	54.3	69.2	65.1	61.2
5.2	13.0	14.9	83.1	25.1	86.3	36.6	84.1	48.3	78.4	59.2	70.8
5.9	13.0	14.0	78.3	24.2	83.2	35.8	82.3	47.7	77.4	58.7	70.3
6.5	13.0	13.0	72.4	23.1	79.3	34.8	79.9	46.8	76.0	58.1	69.5
5.6	14.0	11.5	80.3	20.1	86.5	30.4	87.3	41.5	84.1	52.4	78.2
6.3	14.0	10.7	74.3	19.2	82.5	29.6	84.8	40.8	82.6	51.8	77.4
7.0	14.0	9.7	67.2	18.0	77.4	28.4	81.6	39.8	80.6	51.0	76.2
6.0	15.0	8.9	76.0	16.1	84.8	25.1	88.5	35.3	87.9	45.8	84.1
6.8	15.0	8.0	68.9	15.1	79.8	24.1	85.2	34.4	85.8	45.1	82.8
7.5	15.0	7.1	60.8	13.9	73.5	22.9	80.9	33.3	83.1	44.2	81.1

Table 28. As in Table 17 for low variance and 60 m height.

		HEIGHT		60.0M		LOW VARIANCE					
		VBAR= 4.		VBAR= 5.		VBAR= 6.		VBAR= 7.		VBAR= 8.	
		-----		-----		-----		-----		-----	
CUT-IN	RATED	CF	RF	CF	RF	CF	RF	CF	RF	CF	RF
SPEED	SPEED	%	%	%	%	%	%	%	%	%	%
M/S	M/S										
3.2	8.0	48.3	68.1	64.3	55.0	76.5	43.2	84.8	33.5	90.3	26.1
3.6	8.0	47.7	67.2	63.9	54.7	76.2	43.0	84.6	33.4	90.2	26.1
4.0	8.0	46.9	66.1	63.3	54.2	75.9	42.8	84.4	33.3	90.0	26.1
3.6	9.0	38.8	77.9	55.0	67.1	68.7	55.2	78.8	44.3	85.9	35.4
4.1	9.0	38.1	76.5	54.5	66.4	68.3	54.9	78.5	44.2	85.7	35.3
4.5	9.0	37.2	74.6	53.8	65.6	67.8	54.5	78.2	44.0	85.5	35.2
4.0	10.0	30.5	84.1	46.0	76.9	60.3	66.5	71.9	55.4	80.5	45.5
4.5	10.0	29.7	81.8	45.4	75.8	59.9	66.0	71.6	55.2	80.3	45.4
5.0	10.0	28.7	78.9	44.5	74.4	59.2	65.3	71.1	54.9	80.0	45.2
4.4	11.0	23.6	86.5	37.7	83.9	51.9	76.2	64.3	66.0	74.3	55.9
5.0	11.0	22.7	83.2	36.9	82.2	51.3	75.3	63.9	65.6	74.0	55.7
5.5	11.0	21.6	79.0	35.9	79.9	50.5	74.2	63.4	65.1	73.7	55.4
4.8	12.0	18.1	85.9	30.4	87.7	43.8	83.5	56.6	75.4	67.5	65.9
5.4	12.0	17.1	81.4	29.5	85.2	43.1	82.2	56.0	74.7	67.1	65.6
6.0	12.0	15.9	75.7	28.4	81.9	42.2	80.3	55.3	73.7	66.6	65.1
5.2	13.0	13.7	82.8	24.2	88.7	36.4	88.1	48.9	82.8	60.3	74.9
5.9	13.0	12.7	77.0	23.2	85.2	35.6	86.2	48.2	81.7	59.8	74.3
6.5	13.0	11.6	69.9	22.0	80.6	34.5	83.5	47.4	80.3	59.2	73.5
5.6	14.0	10.3	78.1	19.1	87.4	29.8	90.2	41.6	87.9	53.0	82.2
6.3	14.0	9.4	70.9	18.0	82.7	28.9	87.5	40.8	86.4	52.5	81.4
7.0	14.0	8.3	62.5	16.7	76.8	27.7	83.7	39.8	84.2	51.7	80.1
6.0	15.0	7.8	72.3	14.9	84.2	24.2	90.1	34.9	90.8	46.0	87.7
6.8	15.0	6.9	63.8	13.9	78.3	23.2	86.4	34.1	88.6	45.3	86.4
7.5	15.0	5.8	54.2	12.6	71.0	21.9	81.4	32.9	85.6	44.3	84.6

factors, with the true C_p , would not be more than 0.593 times the tabulated values which used $C_p = 1.0$ for reference.) The mean cube of speed $\langle V^3 \rangle$ in (B-17) is computed from (A-4)

$$\langle V^3 \rangle = c^3 \Gamma(1 + 3/k) \quad (5)$$

Figures 4 through 12 display graphically the sensitivity of capacity factor CF and recovery factor RF to rated speed, and cut-in speed, height, mean wind speed, and wind variance. Figures 4 through 6 show CF versus wind turbine rated speed and cut-in speed for the various heights, for low, average and high variance, for a 10 m mean wind speed of 6 m/s. Figures 7 through 9 are an alternate display which plots CF versus 10 m mean wind speed for various cut-in and rated speeds and for low, average and high variance. The simple linear relationship for capacity factor proposed by Justus (1976), and expressed mathematically as

$$CF = (\bar{V} - 0.69 V_i) / (1.27 V_r - 0.69 V_i) \quad (6)$$

where $\bar{V}$ is mean hub height wind speed, V_i is cut-in and V_r is rated speed, is shown as the dashed lines in Figure 8. This simple relationship is seen to reproduce fairly well the approximate features of the more complete curves for CF. The error bars in Figure 8 show the limits of variability of CF values with wind speed variance at given mean speed conditions. At high 10 m mean wind speeds and low rated speeds CF takes on its largest absolute variation with variance ($\Delta CF \approx 9\%$) but at low 10 m mean wind speeds and high rated speeds the proportional variation in CF is largest ($\Delta CF/CF \approx 50\%$). Thus the effects of variance (Weibull k value) are most noticeable on the capacity factor (in terms of fractional effect) at low mean speeds and high rated speeds.

Figures 10 through 12 show the recovery factor RF versus wind turbine rated speed and cut-in speed for various 10 m mean wind speeds and variance conditions (with C_p taken to be unity; multiply by C_p at rated speed for realistic recovery

Figure 4. Wind turbine capacity factor versus rated wind speed at various heights for a 10 m mean wind speed of 6 m/s and high variance.

Figure 5. Wind turbine capacity factor versus rated wind speed at various heights for a 10 m mean wind speed of 6 m/s and average variance.

Figure 6. Wind turbine capacity factor versus rated wind speed at various heights for a 10 m mean wind speed of 6 m/s and low variance.

Figure 7. Wind turbine capacity factor versus 10 m mean wind speed for various wind turbine rated speeds and cut-in speeds at a height of 45 m for high wind speed variance.

Figure 8. As in Figure 7 for average variances. Dashed curves are equation (6) versus 45 m (hub height) mean wind speed (alternate abscissa).

Figure 9. As in Figure 7 for low variance.

Figure 10. Wind turbine energy recovery factor (ratio of average energy output to that available in the wind) versus wind turbine rated speed and cut-in speed and 10 m mean wind speed, for a hub height of 45 m and high wind speed variance. $C_p = 1.0$ is assumed in the plot, for actual recovery factor, multiply by the true C_p value (by the Betz limit, $C_p = P_r / (0.5 \rho V_r^3 A_s)$ would be ≤ 0.593).

Figure 11. As in Figure 10 for average variance.

Figure 12. As in Figure 10 for low variance.

factor values, where, by the Betz limit, $C_p = P_r / 10.5 \rho V_r^3 A_s$) would be ≤ 0.593). While it is interesting to note what effects the various parameters have on the recovery factor, keep in mind that for optimum performance it is the capacity factor which should be optimized, not the recovery factor. This is due to the fact that the energy production costs in ¢/kW hr for a given wind turbine are inversely proportional to the capacity factor. This relationship is described in the following equation

$$\left(\frac{\text{annual cost}}{\text{kW hr}} \right) = \left(\frac{\text{capital cost}}{\text{rated kW}} \right) \left(\frac{\text{rated kW}}{\text{average kW}} \right) (\text{amortization factor}) \quad (7)$$

where the annual cost per kW hr is the average annual cost in ¢/kW hr for energy output, the capital cost per rated kW is the total capital cost divided by the rated power of the wind turbine, the rated kW per average kW is the reciprocal of the capacity factor, and the amortization factor depends on economic assumptions regarding interest rate, and assumptions on maintenance rate, etc.

Since capacity factor is intimately related to the cost of output power (through equation 7), it seems that most design applications would involve optimization of capacity factor, rather than recovery factor. However, Reed (1976) has used an optimization method which finds the point of equal marginal (incremental) capital costs and marginal (incremental) recovery factor.

It should be noted that most parameters affect the capacity factor and recovery factor oppositely, for example Figures 4 through 12 show decreasing capacity factor with wind turbine rated speed while Figures 10 through 12 show increasing recovery factor with rated speed. The essentially inverse relationship between capacity factor and recovery factor seems somewhat unusual, since both are measured of wind turbine performance (especially since equation (B-17) seems to show them proportional to each other). However, this relationship has a straight-forward explanation. For a high recovery factor the wind turbine parameters and wind speed distribution parameters must be such that most of the time the wind turbine would be operating in the

strongly increasing portion of its output curve, i.e. between cut-in and rated speed (and very little of the time on the flat portion of the curve, above rated speed). For a high capacity factor the wind turbine parameters and wind speed distribution parameters must be such that a large portion of the time the machine is operating at or near rated power (i.e. near or above rated speed). These two situations are somewhat mutually exclusive and would mean that a given combination of wind turbine parameters and wind speed distribution parameters could produce either high capacity factor or high recovery, but not both. Mathematically, this effect is explained in the fact that an increase in C_f is accompanied by a larger increase in $\langle V^3 \rangle$ which, from equation (B-17) would lead to a decrease in R_f , and vice versa.

APPENDIX A SOME ASPECTS OF THE WEIBULL DISTRIBUTION

Definitions

The Weibull distribution for wind speed V can be expressed in terms of the probability density function (velocity frequency curve)

$$p(V)dV = (k/c)(V/c)^{k-1} \exp[-(V/c)^k] dV, \quad (A-1)$$

where c is called the scale factor and k is called the shape factor, or in terms of the cumulative probability function (velocity duration curve)

$$p(V \leq V_x) = \int_0^{V_x} p(V) dV = 1 - \exp[-(V_x/c)^k]. \quad (A-2)$$

The Weibull distribution has successfully been applied to the description of wind speed distributions (at least above relevant threshold speeds) by Davenport (1963), Justus (1974) Baynes (1974) Zimmer et al (1975), Wentink (1975), and by Justus et al (1976). In addition to the characteristics discussed here, other Weibull characteristics are discussed by Hennessey (1976).

The probability density function $p(V)$ in (A-1) is the probability per unit speed interval at speed V [i.e. $p(V) dV$ is the probability of finding a speed between V and $V + dV$]. Hence, $p(V)$ has units of probability per unit speed, and a conversion factor is necessary to change from english to metric units or vice-versa. Thus if $p(V)$ has a certain value in, say, percent per m/s, then to get % per mph, one has to multiply by the conversion factor 0.447 m/s per mph. Figure A-1 shows a plot of the Weibull distribution $p(x)$, where x is the nondimensional speed $x = V/c$ [and $p(x)$ is the probability of finding V/c between x and $x + dx$].

The cumulative probability $p(V \leq V_x)$ in (A-2) is dimensionless probability but, of course, the speed value V_x has units of m/s or mph associated with it.

Figure A-1. Standard Weibull probability distribution (velocity frequency) curves for various Weibull k values.

The wind speed distribution is often expressed as a "velocity duration curve" in hours per year for which $V \geq V_x$, i.e. $H(V \geq V_x) = 8760 [1 - p(V \leq V_x)]$. Thus, the Weibull velocity duration curve would be expressed mathematically as

$$H(V \geq V_x) = 8760 \exp[-(V_x/c)^k]. \quad (A-3)$$

Curves of $H(V \geq V_x)$ for various k values are plotted in Figure A-2, corresponding to the probability density functions (velocity frequency curves) of Figure A-1.

Mean and Standard Deviation Related to Weibull Parameters

Various moments $\mu_n = \langle V^n \rangle$ of the Weibull distribution can be evaluated from integration of equation (A-1) by

$$\mu_n = \langle V^n \rangle = \int_0^{\infty} V^n p(V) dV = c^n \Gamma(1 + n/k) \quad (A-4)$$

where Γ is the usual gamma function. The mean wind speed $\bar{V}$ is just the first moment $\langle V \rangle$. Hence, the mean wind speed $\bar{V}$ and the Weibull scale factor c are related by

$$\bar{V} = c \Gamma(1 + 1/k). \quad (A-5)$$

Table A-1 gives values of the dimensionless ratio $c/\bar{V}$ for various values of the Weibull shape factor k , and $c/\bar{V}$ is presented graphically in Figure A-3. Note that c and $\bar{V}$ are very nearly proportional since for $1.5 < k < 3.2$ the $c/\bar{V}$ ratio varies only between 1.11 and 1.13.

The standard deviation σ of the wind speed (about the mean) is related to first and second moments by $\sigma^2 = \mu_2 - (\mu_1)^2$. Hence the standard deviation σ and the Weibull c and k factors are related by

$$\sigma^2 = c^2 [\Gamma(1 + 2/k) - \Gamma^2(1 + 1/k)], \quad (A-6)$$

or, in terms of the mean wind speed $\bar{V}$ by

Figure A-2. Cumulative probability (velocity duration) curves for various Weibull k values.

Table A-1. Values of the dimensionless ratios $c/\bar{V}$, $\sigma/\bar{V}$, and $\langle V^3 \rangle / \bar{V}^3$ for various values of Weibull shape parameter k . $\bar{V}$ is the mean wind speed, σ is the standard deviation of the wind speed about its mean value, c is the Weibull scale factor, and $\langle V^3 \rangle$ is the mean of the cube of the wind speed.

k	$c/\bar{V}$	$\sigma/\bar{V}$	$\langle V^3 \rangle / \bar{V}^3$
1.3	1.083	.776	3.43
1.4	1.097	.724	3.03
1.5	1.108	.673	2.72
1.6	1.115	.640	2.48
1.7	1.121	.605	2.29
1.8	1.124	.575	2.14
1.9	1.127	.547	2.01
2.0	1.128	.523	1.91
2.1	1.129	.500	1.82
2.2	1.129	.480	1.75
2.3	1.129	.461	1.68
2.4	1.128	.444	1.63
2.5	1.127	.428	1.58
2.6	1.126	.413	1.53
2.7	1.125	.399	1.50
2.8	1.123	.387	1.46
2.9	1.121	.375	1.43
3.0	1.120	.363	1.40
3.1	1.118	.353	1.38
3.2	1.117	.343	1.36

Figure A-3. Ratio of Weibull c to mean wind speed versus Weibull k value.

$$\sigma^2 = \bar{V}^2 \{ [\Gamma(1 + 2/k) / \Gamma^2(1 + 1/k)] - 1 \} \quad (A-7)$$

Table A-1 also gives values of the dimensionless ratio $\sigma/\bar{V}$ for various values of k ; and $\sigma/\bar{V}$ is presented graphically in Figure A-4.

Since the wind energy flux (power) per unit area is given by $P = 0.5 \rho V^3$, the average wind power per unit area is $\bar{P} = 0.5 \bar{\rho} \langle V^3 \rangle$ (provided there is no correlation between density variations and speed variations). Thus the ratio of average power $\bar{P}$ to the power in the mean wind $P_0 = 0.5 \bar{\rho} \bar{V}^3$, is the same as the ratio of $\langle V^3 \rangle$ to $\bar{V}^3$, i.e., from (A-1)

$$\bar{P}/P_0 = \langle V^3 \rangle / \bar{V}^3 = \Gamma(1 + 3/k) / \Gamma^3(1 + 1/k) \quad (A-8)$$

Table A-1 also gives values of this dimensionless ratio $\langle V^3 \rangle / \bar{V}^3$ for various values of k , and $\langle V^3 \rangle / \bar{V}^3$ is presented graphically in Figure A-5.

An example of a simple method, based on Table A-1 results, for estimating Weibull c and k parameters and mean wind power $\bar{P}$ for a site is given by the following:

Step 1 - Determine $\bar{V}$ (e.g. annual average, seasonal average, or monthly average wind speed) and the standard deviation σ . The National Climatic Center publishes monthly mean speeds for a number of US cities in their "Local Climatological Data" (LCD) series. Values for σ are not listed in LCD data, but could be computed monthly by computing rms deviation of 3 hourly (1 minute average) wind speeds from the monthly mean. (Values for σ could also be estimated by computing rms deviations of the average speeds by time of day from the monthly mean and dividing by $\sqrt{n}$ where n is the number of days per month, or by computing rms deviations of the daily mean speeds from the monthly mean and dividing by $\sqrt{m}$ where m is the number of observations per day (each of these is only approximate, since the factors $1/\sqrt{n}$ or $1/\sqrt{m}$ would apply only to completely independent data, daily means or monthly means by time of day, and these statistics are not strictly independent).

$$\sigma/\bar{V} = \{[\Gamma(1 + 2/k)/\Gamma^2(1 + 1/k)] - 1\}^{1/2}$$

Figure A-4. Ratio of wind speed standard deviation about the mean to the mean speed versus Weibull k value.

$$\langle V^3 \rangle / \bar{V}^3 = r(1 + 3/k) / r^3(1 + 1/k)$$

Figure A-5. Ratio of mean cube of speed to cube of mean speed for various Weibull k values.

Step 2 - Enter Table A-1 with the value of $\sigma/\bar{V}$, determined in Step 1, and, by interpolation, find the corresponding value of the Weibull shape factor k .

Step 3 - Interpolate between the same two lines of data as used in Step 2 to find the corresponding value of the ratio $c/\bar{V}$. Multiply by $\bar{V}$ to find the value of the Weibull scale factor c . To determine wind turbine performance (as in Appendix B), use this value of c and the k value determined in Step 2 as the Weibull distribution parameters.

Step 4 - Interpolate between the same two lines of data as used in Step 2 to find the corresponding value of ratio $\langle V^3 \rangle / \bar{V}^3$. To find the mean wind power per unit area power $\bar{P} = 0.5 \bar{\rho} \langle V^3 \rangle$, multiply the determined ratio value by $0.5 \bar{\rho} \bar{V}^3$.

For the above steps, it does not matter what units are used for V or c , since all of the parameters in Table A-1 are expressed as dimensionless ratios. If V is in m/s and ρ is in kg/m^3 , the power $\bar{P}$ will automatically be in watts per square meter. If other wind speed and density units are used, a conversion factor on the power will be required.

Figures A-6 through A-8 give examples of observed values of the ratios $c/\bar{V}$, $\sigma/\bar{V}$, and $\langle V^3 \rangle / \bar{V}^3$, compared with the theoretically expected values from the Weibull distribution equations (A-5), (A-7), and (A-8) or Figures A-3 through A-5. The observed data points come from five years of monthly wind distributions in the New England and Central US areas (Justus, 1976). Figure A-6 shows the observed $c/\bar{V}$ ratio values to average consistently about 2% higher than those expected from equation (A-5). The $\sigma/\bar{V}$ observed data points in Figure A-7 range from about 3% (high k region) to 5% (low k region) above the theoretical values, and the $\langle V^3 \rangle / \bar{V}^3$ observed values in Figure A-8 range from about 6% (high k region) to 9% (low k region) above the theoretical values.

If it is desired, the empirically observed data results of Figures A-6

Figure A-6. Observed c/V ratios from 5 years of monthly wind data in New England and Central U.S. (Justus, 1976), compared to theoretical Weibull c/V factor from equation (A-5).

Figure A-7. As in Figure A-6 for observed $\sigma/\bar{V}$ and Weibull theoretical ratio from Equation (A-7).

Figure A-8. As in Figure A-6 for observed $\langle V^3 \rangle / \bar{V}^3$ and Weibull theoretical ratio from Equation (A-8).

through A-8 can be used instead of the theoretical results of Figure A-3 through A-5 or Table A-1 in the above outlined methodology for estimating Weibull c and k values and mean power output $\bar{P}$ from observed $\bar{V}$ and σ statistics.

Determination of Weibull Parameters from Observed Wind Distributions

Wind speed distribution summaries are often available (or can easily be calculated) from observed wind speeds. These are expressed as number (or frequency) of calms and number (or frequency) of wind speeds within several ranges of wind speed values. If the calms are considered to be occurrences of wind speed below the anemometer threshold V_0 and have a frequency of occurrence f_0 , and if the frequencies of each successive wind speed interval $V_0 - V_1$, $V_1 - V_2$, $V_2 - V_3$, etc. have frequencies of occurrence f_1 , f_2 , f_3 , etc., respectively, then the Weibull c and k values can be accurately estimated by the following procedure:

Step 1 - Compute the set of n cumulative probabilities corresponding to the above wind speed intervals, $p_1 = f_0 + f_1$, $p_2 = f_0 + f_1 + f_2$, ... $p_n = p_{n-1} + f_n$.

Step 2 - Transform the wind speed upper limit and cumulative probability data pairs (V_i, p_i) to new variables (X_i, Y_i) by the transformations

$$\begin{aligned} X_i &= \ln V_i \\ Y_i &= \ln[-\ln(1 - p_i)] \end{aligned} \quad (A-9)$$

where $\ln$ is the natural logarithm. Note that, by transformation of (A-2), X and Y variables should be linearly related.

Step 3 - Find the best least squares fit coefficients a and b to $Y_i = a + b X_i$.

These can be found by evaluation of the following relations:

$$\begin{aligned} a &= (\sum Y_i \sum X_i^2 - \sum X_i Y_i \sum X_i) / D \\ b &= (n \sum X_i Y_i - \sum X_i \sum Y_i) / D, \end{aligned} \quad (A-10)$$

where D is given by

$$D = n \sum X_i^2 - (\sum X_i)^2, \quad (A-11)$$

and $\sum$ denotes a summation over i from 1 to n .

Step 4 - Evaluate the Weibull c and k values from the a and b coefficients, determined in Step 3, by

$$\begin{aligned} c &= \exp(-a/b) \\ k &= -b \end{aligned} \quad (A-12)$$

which result from the double $\ln$ transformation applied to equation (A-2). To determine wind turbine performance (as in Appendix B), use these c and k values as the Weibull distribution parameters. Figure A-9 shows results of the least squares process described in Steps 1 - 4 applied to two sets of wind summary data for Atlanta, together with the c and k values determined from (A-12).

Step 5 - To find mean wind power per unit area $P = 0.5 \bar{\rho} \langle V^3 \rangle$, find the appropriate value of the dimensionless ratio $\langle V^3 \rangle / \bar{V}^3$ from Table A-1, by interpolation to the k value determined in Step 4, then multiply this ratio value by $0.5 \bar{\rho} \bar{V}^3$, where $\bar{V}$ can either be the directly calculated mean wind, or the mean wind estimated from the Weibull parameters c and k by $\bar{V} = c \Gamma(1 + 1/k)$.

Height Variation of Weibull Distribution Parameters

The height variation of Weibull statistics has been discussed by Zimmer et al (1975), Justus et al (1976), and by Justus and Mikhail (1976). That the Weibull statistics vary significantly with height is evident from the two Weibull distributions shown in Figure A-5. In the work reported in Zimmer et al (1975), and Justus et al (1976), observed seasonal and annual Weibull wind distribution parameters were studied. The height variation of Weibull distribution c and k

Figure A-9. Average distribution of wind speed in Atlanta, Ga. from anemometer at 72 ft. (1951-1960) and at 20 ft. (1967-1971). The straight lines represent Weibull Distributions.

values were evaluated from four sets of meteorological tower data: Kennedy Space Flight Center, Florida; Wallops Island, Virginia; Hanford, Washington (Stone et al, 1972); and the WKY-TV tower in Oklahoma City, Oklahoma (Crawford and Hudson, 1970).

The Weibull scale factors c at all four tower sites were found to vary as a power law with height

$$c/c_0 = (z/z_0)^n \quad (A-13)$$

where the exponent n averaged 0.23 for all sites and seasons combined. A logarithmic height variation for c would also have done well up to about 100 m, but the power law worked better when the entire height range (up to 400 m) was considered. Figure A-10 shows the observed 3 year average annual and seasonal height profiles of Weibull c values at Kennedy SFC, with the straight line fits to each set of data representing a power law with exponents ranging from 0.17 (Spring) to 0.23 (Winter). Separate year data from Kennedy showed similar power law variation, as did the data from the other tower sites. The data points of Figure A-11 are the seasonal or annual exponent values for power law c variation with height, plotted versus c_{10} the value of c at 10 m reference height.

Weibull distribution k values determined at different tower levels were also found to vary with season and between towers. However, when normalized by a k value at a given level, such as in Figure A-12, where the k profiles are normalized by a subjectively determined $k_{\max}$ near 70 m, all of the profiles follow a similar curve.

In Justus and Mikhail (1976) a methodology was considered whereby height variation of "instantaneous" wind profiles and height variation of Weibull c and k values could be analytically evaluated by a self-consistent methodology. It was determined that up to a height of 100 m (where k begins to decrease notice-

Figure A-10. Observed power law height variation of the Weibull scale factor c . Numbers beside each curve are power law exponent values.

Figure A-11 - Velocity or scale factor power law exponent n , versus velocity or scale factor at 10m reference level.

Figure A-12. The "universal" curve for $k/k_{\max}$ versus height, where $k_{\max}$ is the subjectively determined maximum k value (near 70 m).

ably) a consistent methodology, based on the power profile, is as follows:

1) If Weibull parameters c_1 and k_1 are known at anemometer height z_1 and values c_2 and k_2 are desired at height z_2 ($z_2 < 100$ m), then c_2 and k_2 can be evaluated by

$$c_2 = c_1 (z_2/z_1)^n \quad (A-14)$$

$$k_2 = k_1 [1 - 0.088 \ln (z_1/10)] / [1 - 0.088 \ln (z_2/10)] \quad (A-15)$$

where z_1 and z_2 are in meters, and the power law exponent n is given by

$$n = [0.37 - 0.088 \ln c_1] / [1 - 0.088 \ln (z_1/10)] \quad (A-16)$$

The height variation of k as given by equation (A-15) is illustrated as the solid line through the data points in Figure A-13. Equations (A-14) through (A-16) are valid for height projection of c and k provided the terrain is relatively flat, but is applicable over a fairly wide range of surface roughnesses for relatively flat terrain.

2) If "instantaneous" (e.g. one minute average) wind speed V_1 is known at anemometer height z_1 and the wind speed value V_2 is desired at height z_2 ($z_2 < 100$ m), then V_2 can be evaluated by

$$V_2 = V_1 (z_2/z_1)^n \quad (A-17)$$

when n is given by

$$n = [0.37 - 0.088 \ln V_1] / [1 - 0.088 \ln (z_1/10)] \quad (A-18)$$

which is analogous to (A-16) for the c variation.

3) Because c and $\bar{V}$ are very nearly proportional (c.f. Table A-1 and Figure A-2), the power law can also be used, at least approximately, if $\bar{V}_1$ is known at ane-

Figure A-13 - Observed k_{10}/k Ratios Versus Height with least Squares Fit (solid curve) and Reed β Function (dashed curve).

anemometer height z_1 and $\bar{V}_2$ is desired at height z_2 , i.e.

$$\bar{V}_2 = \bar{V}_1 (z_2/z_1)^n \quad (\text{A-19})$$

where n is given by

$$n = [0.37 - 0.088 \ln \bar{V}_1] / [1 - 0.088 \ln(z_1/10)] \quad (\text{A-20})$$

If the anemometer is at a standard height of 10 m, then the exponent n versus c_1 (A-16), versus V_1 (A-18), or versus $\bar{V}_1$ (A-20) is given as the solid line in Figure A-11.

The power law variations above are based on an assumed velocity transformation $V_2 = \alpha V_1^\beta$ where V_1 and V_2 are at heights z_1 and z_2 . This transformation on instantaneous velocity, plus the assumption that each V_1 has a unique V_2 associated with it (as would be the case if α and β dependence on atmospheric stability is ignored) allow the transformation of wind speed distributions to be evaluated, thus if

$$V_2 = \sigma V_1^\beta \quad (\text{A-21})$$

and the cumulative probability $p(V \leq V_1)$ is

$$p(V \leq V_1) = 1 - \exp[-(V_1/c_1)^{k_1}] \quad (\text{A-22})$$

then at height z_2 $p(V \leq V_2)$ is equal to $p(V \leq V_1)$ at height z_1 and

$$\begin{aligned} p(V \leq V_2) &= 1 - \exp[-(V_2/c_2)^{k_2}] \\ &= 1 - \exp[-(\alpha V_1^\beta/c_2)^{k_2}] \\ &= p(V \leq V_1) = 1 - \exp[-(V_1/c_1)^{k_1}] \end{aligned} \quad (\text{A-23})$$

provided

$$\begin{aligned} c_2 &= \alpha c_1^\beta \\ k_2 &= k_1/\beta. \end{aligned} \tag{A-24}$$

If effects of atmospheric stability were to be included, considerable complication would result in converting from velocity transformation (A-21), with α and β dependent on stability, to wind distribution height variation. For example if stability dependent α and β , $\alpha = \alpha_i(s_i)$ and $\beta = \beta_i(s_i)$ are used for each of $i = 1$ to n stability categories, then a conversion from (A-21) to (A-24) could be made separately for each of the n stability categories, but to reconstruct the total wind speed distribution function, a weighted average of each of the n stability category results would have to be made, where the weighting would depend on the relative frequency of each of the stability classes. Therefore, in view of the added complexity to account for stability effect, the forms (A-14) through (A-20), dependent only on speed, are recommended for design purposes in wind energy calculations.

The logarithmic height profile, which is more firmly based on boundary layer theory and measurement than the power law, might be adapted for use instead of the power law, however, even more complexity would be involved. The simple logarithmic law relating V_1 and V_2 at heights z_1 and z_2

$$V_2/V_1 = \ln(z_2/z_0)/\ln(z_1/z_0) \tag{A-25}$$

applies only under neutral (high wind speed) stability conditions. For winds at medium speed and other than neutral conditions, the boundary layer logarithmic profile

$$V(z) = (u_*/K)[\ln z/z_0 - \psi(z/L)] \tag{A-26}$$

must be used where u_* is friction velocity, K is von Karman's constant, and

$\psi(z/L)$ is a universal function (Panofsky, 1963) of the Monin-Obukov stability parameter z/L . Alternately, an empirical form of (A-25) could be used if z_0 is allowed to be a function of speed (u_*) and stability (z/L). The problem with either of these is in evaluating L (which cannot be directly evaluated from routine National Weather Service data which do not have vertical temperature gradient data), and additional complexity in relating the velocity transformation (A-25) or (A-26) to wind distribution parameter variations. Therefore, again because of reasons of simplicity, the relations (A-14) through (A-20) are recommended for design purposes in wind energy calculations.

Alternate Weibull Description Based on Median Wind Speed

Recently Ljungstrom (1976) reported on wind distribution studies in Sweden which indicate that a cumulative probability

$$p(V \geq V_x) = 2^{-(V/V_m)^\beta} \quad (A-27)$$

adequately describes the observed distributions, where V_m is the median (50% probability) wind speed, and β is an empirical exponent found, by best fit to the observed distributions, to vary between 1.7 and 2.3. From equation (A-2) the Weibull cumulative probability $p(V \geq V_x)$ is given by

$$p(V \geq V_x) = \exp[-(V/c)^k]. \quad (A-28)$$

By comparison, it is seen that equations (A-27) and (A-28) are identical provided that

$$\begin{aligned} \beta &= k \\ V_m &= c(\ln 2)^{1/\beta} \end{aligned} \quad (A-29)$$

which is always true since from (A-28) $1/2 = \exp[-(V_m/c)^k]$ reduces to the second member of equation (A-29).

If the 25 percentile, median (50 percentile), and 75 percentile wind speeds $V_{0.25}$, V_m , and $V_{0.75}$ are known from an observed wind speed distribution, then an easy way of estimating the Weibull c and k values is, from equation (1)

$$\begin{aligned} k &= \ln[\ln(0.25)/\ln(0.75)]/\ln(V_{0.75}/V_{0.25}) \\ &= 1.573/\ln(V_{0.75}/V_{0.25}) \end{aligned} \quad (A-30)$$

and, from equation (A-29),

$$c = V_m/(\ln 2)^{1/k} \quad (A-31)$$

Note that, as in equation (1) $V_{0.25}$ and $V_{0.75}$ are velocities corresponding to cumulative probabilities $p(V \leq V_{0.25}) = 0.25$ and $p(V \leq V_{0.75}) = 0.75$ respectively.

APPENDIX B WIND TURBINE PERFORMANCE AND THE WEIBULL DISTRIBUTION

Representation of the Wind Turbine Power Output Curve

For a known wind speed probability distribution $p(V)$, such as the Weibull distribution, the average output power $\bar{P}$ from a wind powered generator can be evaluated by

$$\bar{P} = \int_0^{\infty} P(V) p(V) dV \quad (B-1)$$

where $P(V)$ is the output power of the generator as a function of speed. The power output function $P(V)$ can be described analytically in terms of a cut-in speed V_i , a rated speed V_r , and a cut-out speed V_o (all at hub height) by

$$P(V) = \begin{cases} 0 & V \leq V_i \\ P_r(A + BV + CV^2) & V_i < V \leq V_r \\ P_r & V_r < V \leq V_o \\ 0 & V > V_o \end{cases} \quad (B-2)$$

where V is the wind speed at the wind turbine hub height, P_r is the rated power of the wind turbine, and A , B , and C are coefficients determined by the conditions

$$\begin{aligned} P(V_i) &= A + BV_i + CV_i^2 = 0 \\ P(V_r)/P_r &= A + BV_r + CV_r^2 = 1 \\ P(V_m)/P_r &= A + BV_m + CV_m^2 = (V_m/V_r)^3 \end{aligned} \quad (B-3)$$

where V_m is defined as $V_m = (V_i + V_r)/2$.

This analytical form (B-2) for $P(V)$ as described by conditions (B-3) has been found to describe adequately actual or estimated power output curves from a wide range of sizes of wind turbine (e.g. from few kW small commercial

Figure B-1. Output Curve for the GE 500 kW Rated Power Wind Turbine.

size units up to 1500 kW Mod 1 ERDA design units). Figure B-1 shows an example power curve $P(V)$ evaluated by this method. The solutions for A, B, and C from (B-3) are:

$$B = [(V_m^2 - V_i^2) - (V_m/V_r)^3(V_r^2 - V_i^2)]/D \quad (B-4)$$

$$C = [(V_m/V_r)^3(V_r - V_i) - (V_m - V_i)]/D \quad (B-5)$$

$$A = -BV_i - CV_i^2 \quad (B-6)$$

where D is given by

$$D = (V_r - V_i)(V_m^2 - V_i^2) - (V_m - V_i)(V_r^2 - V_i^2). \quad (B-7)$$

Wind Turbine Capacity Factor

One measure of wind turbine performance is the ratio of actual average power output to the rated power of the wind turbine. This is called the capacity factor $C_f = \bar{P}/P_r$. The annual energy output $\bar{E}$ and annual capacity factor C_f are related by $\bar{E} = 8760 C_f P_r$. From (B-1), the capacity factor is given by

$$C_f = \int_0^{\infty} [P(V)/P_r] p(V) dV, \quad (B-8)$$

which, from (B-2) reduces to

$$C_f = V_i \int_{V_i}^{V_r} (A + BV + CV^2) p(V) dV + V_r \int_{V_r}^{V_o} p(V) dV. \quad (B-9)$$

If the Weibull distribution (A-1) is used for $p(V)$, and substitutions $x = (V/c)^k$, $x_i = (V_i/c)^k$, $x_r = (V_r/c)^k$, and $x_o = (V_o/c)^k$ are made, then (B-9) becomes

$$C_f = A p_{ir} + Bc \int_{x_i}^{x_r} x^{1/k} e^{-x} dx + Cc^2 \int_{x_i}^{x_r} x^{2/k} e^{-x} dx + p_{ro} \quad (B-10)$$

where p_{ir} and p_{ro} are the cumulative probabilities

$$p_{ir} = \int_{V_i}^{V_r} p(V) dV = e^{-x_i} - e^{-x_r} \quad (B-11)$$

$$p_{ro} = \int_{V_r}^{V_o} p(V) dV = e^{-x_r} - e^{-x_o} \quad (B-12)$$

The integrals in (B-10) can be evaluated by series approximation

$$\int_{x_i}^{x_r} x^{n/k} e^{-x} dx = \left\{ x^{n/k} [x (1 + n/k)^{-1}/0! - x^2 (2 + n/k)^{-1}/1! + x^3 (3 + n/k)^{-1}/2! - x^4 (4 + n/k)^{-1}/3! + \dots] \right\} \Big|_{x_i}^{x_r} \quad (B-13)$$

where n takes on the values 1 or 2 for the integrals in (B-10). The series integration (B-13) is easily programmed onto a small computer or programmable calculator. Alternate methods, such as Simpson's rule numerical integration could also be employed in digital computer programs.

Wind Turbine Energy Recovery Factor

Another measure of wind turbine performance is the ratio of the actual total energy output (e.g. over the year) to the total energy that was available in the wind. This ratio is called the recovery factor R_f . It incorporates both the power (energy flux) available in the wind and the probability with which that power is available. Analytically the recovery factor is given by

$$R_f = \frac{\int_0^{\infty} P(V) p(V) dV}{\int_0^{\infty} (0.5 \rho V^3 A_s) p(V) dV} \quad (B-14)$$

where ρ is the air density and A_s is the rotor swept area of the wind turbine (the theoretical power or energy flux is $0.5 \rho V^3 A_s$). The numerator in (B-14) is just $P_r C_f$ and the denominator is $0.5 \rho A_s \langle V^3 \rangle$, where $\langle V^3 \rangle$ is the mean of the speed cubed (see Appendix A). Thus (B-14) becomes

$$R_f = P_r C_f / (0.5 \rho A_s \langle V^3 \rangle). \quad (B-15)$$

The rated power P_r can be expressed in terms of the rated speed V_r and the power coefficient C_p (at that speed), by

$$P_r = C_p (0.5 \rho V_r^3 A_s). \quad (B-16)$$

Hence, R_f becomes

$$R_f = C_p C_f V_r^3 / \langle V^3 \rangle. \quad (B-17)$$

Thus R_f is directly proportional to the power coefficient C_p , to the capacity factor C_f and to the ratio $V_r^3 / \langle V^3 \rangle$. For sensitivity studies of the effects of cut-in speed, rated speed, and wind distribution ($\langle V^3 \rangle$) on the recovery factor, such as presented in this report, it is convenient to express the recovery factor for an assumed value of $C_p = 1$ at $V = V_r$) thus R_f becomes $C_f V_r^3 / \langle V^3 \rangle$. For actual wind turbine designs, the actual R_f can then be evaluated through multiplication by the actual power coefficient $C_p = P_r / (0.5 \rho V_r^3 A_s)$, which, by the Betz limit would have values ≤ 0.593 .

APPENDIX C
NATIONWIDE OBSERVED WEIBULL DISTRIBUTION PARAMETERS

Table C-1 shows computed seasonal and annual values of Weibull distribution scale factors (c , m/s) and shape factors (k , dimensionless) for various sites in the U.S., arranged alphabetically by state and city. Codes are the three letter airport codes identifying the airport at which the data were taken. All data had constant anemometer heights (as listed) over the period of record. All data sets contained five or more years of wind data except the Kennedy SFC (1967-1969) and WKY-TV (1966-1967) tower data. Table C-1 gives c and k values at the indicated anemometer heights, as calculated from least squares fit procedures (equations (A-9) through (A-12) over the range of wind speeds above 3.6 m/s (8 mph). Weibull c and k values at any other desired height can be computed from equations (A-14) through (A-16) where c_1 and k_1 at z_1 are the c and k values in Table C-1 at the given anemometer height z_1 , and c_2 and k_2 are the desired c and k values at desired height z_2 (where z_1 and z_2 must be used in meters for equation (A-14) through (A-16)). Seasonal definitions for Table C-1 are Winter = Dec, Jan, Feb; Spring = Mar, Apr, May; Summer = Jun, Jul, Aug; Fall = Sep, Oct, Nov.

TABLE C-1. WEIBULL c AND k VALUES AT "SURFACE" ANEMOMETER HEIGHTS

CITY	CODE	ANEMOMETER	WINTER		SPRING		SUMMER		FALL		ANNUAL		
		HEIGHT	<u>c</u> m/s	<u>k</u>	<u>c</u> m/s	<u>k</u>	<u>c</u> m/s	<u>k</u>	<u>c</u> m/s	<u>k</u>	<u>c</u> m/s	<u>k</u>	
		m(ft.)											
<u>Alabama</u>													
Birmingham	BHM	19.2(63)	4.74	1.91	4.59	1.82	2.97	1.52	3.90	1.73	4.09	1.71	
Mobile	MOB	17.1(56)	5.47	1.95	5.21	2.06	3.37	1.54	4.42	1.81	4.64	1.81	
	MOB	6.7(22)	4.59	2.14	4.50	1.95	2.43	1.31	3.56	1.77	3.89	1.79	
<u>Arizona</u>													
Phoenix	PHX	5.5(18)	1.95	1.09	2.91	1.41	2.76	1.32	2.62	1.35	2.58	1.29	
Tucson	TUS	6.1(20)	3.43	1.31	4.11	1.53	3.87	1.57	3.73	1.42	3.80	1.46	
Yuma	YUM	6.1(20)	3.23	1.48	3.86	1.80	3.98	1.79	3.14	1.47	3.58	1.64	
<u>Arkansas</u>													
Ft. Smith	FSM	7.0(23)	4.00	1.93	4.11	2.09	-	-	-	-	3.62	1.86	
Little Rock	LIT	6.1(20)	4.13	2.01	4.23	2.07	-	-	-	-	3.71	1.88	
<u>California</u>													
Bakersfield	BFL	61.(20)	1.93	1.11	2.91	1.42	-	-	2.06	1.13	2.67	1.38	
Fresno	FAT	12.8(42)	2.30	1.27	3.46	1.72	3.04	1.72	2.30	1.46	2.68	1.42	
Los Angeles	LAX	6.1(20)	3.09	1.52	4.26	1.92	-	-	3.73	1.90	3.92	1.92	
Oakland	OAK	14.9(49)	2.75	1.12	4.54	1.91	4.53	2.17	3.15	1.45	3.85	1.63	
Sacramento	SMF	6.1(20)	3.22	1.35	4.04	1.79	3.82	1.93	3.18	1.48	3.57	1.60	
<u>Colorado</u>													
Colorado Springs	COS	6.7(22)	3.97	1.27	5.15	1.51	4.15	1.52	4.14	1.39	4.35	1.40	
Denver	DEN	21.9(72)	4.80	1.71	5.11	1.62	4.46	1.75	4.25	1.67	4.63	1.65	
Pueblo	PUB	6.7(22)	3.09	1.03	4.57	1.30	4.28	1.43	3.20	1.11	3.82	1.21	

TABLE C-1 (cont.)

CITY	CODE	ANEMOMETER	WINTER		SPRING		SUMMER		FALL		ANNUAL	
		HEIGHT										
		m(≡t.)	c	k	c	k	c	k	c	k	c	k
			m/s		m/s	m/s	m/s	m/s	m/s	m/s		
<u>Connecticut</u>												
Bridgeport	BDR	25.6(84)	6.87	2.04	6.24	2.06	4.95	2.21	5.83	1.93	5.92	1.94
Hartford	BDL	30.2(99)	5.32	1.96	5.46	2.11	4.26	2.03	4.58	2.01	4.89	2.01
<u>Florida</u>												
Kennedy SC	XMR	60.0(197)	6.57	2.50	6.69	2.92	4.82	2.26	6.45	2.42	6.14	2.41
Dayton	DAB	7.0(23)	4.29	1.98	4.83	2.27	-	-	4.39	2.14	4.32	2.03
Jacksonville	JAX	19.2(63)	4.15	1.71	4.56	2.04	3.98	2.01	4.30	1.89	4.08	1.62
Miami	MIA	7.0(23)	4.66	2.10	5.02	2.34	3.71	1.86	4.37	2.09	4.47	2.08
Orlando	ORL	16.2(53)	4.63	1.83	4.65	1.94	3.97	1.76	4.19	1.78	4.27	1.78
Tallahassee	TLH	16.5(54)	3.50	1.91	3.83	2.32	3.61	1.78	3.97	1.76	4.19	1.78
	TLH	7.6(25)	3.63	1.88	-	-	2.18	1.24	-	-	3.18	1.70
Tampa	TPA	6.7(22)	3.87	1.75	4.41	2.26	2.58	1.25	3.57	1.70	3.69	1.73
<u>Georgia</u>												
Atlanta	ATL	21.9(72)	5.51	2.05	5.11	1.94	3.47	1.66	4.43	1.41	4.65	1.82
	ATL	6.1(20)	4.22	2.14	3.83	1.94	-	-	-	-	3.55	1.86
Augusta	AGS	7.6(25)	3.55	1.62	3.46	1.51	3.00	1.78	2.69	1.39	3.09	1.48
	AGS	6.1(20)	3.58	1.80	3.40	1.56	-	-	2.36	1.38	2.94	1.50
Columbus	CSG	6.1(20)	3.83	2.02	3.83	1.97	-	-	-	-	3.52	1.90
Macon	MCN	7.0(23)	3.74	1.85	3.83	1.86	-	-	-	-	3.41	1.77
Savannah	SAV	18.0(59)	4.30	1.80	4.20	1.84	3.21	1.66	3.83	1.87	3.79	1.65
	SAV	6.1(20)	3.68	1.67	4.07	1.89	-	-	-	-	3.45	1.69
<u>Idaho</u>												
Pocatello	PIH	10.4(34)	5.48	1.41	5.32	1.42	4.39	1.47	4.17	1.32	4.80	1.38

TABLE C-1 (cont.)

CITY	CODE	ANEMOMETER	WINTER		'SPRING		SUMMER		FALL		ANNUAL	
		HEIGHT m(ft.)	$\frac{c}{m/s}$	$\frac{k}{m/s}$	$\frac{c}{m/s}$	$\frac{k}{m/s}$	$\frac{c}{m/s}$	$\frac{k}{m/s}$	$\frac{c}{m/s}$	$\frac{k}{m/s}$	$\frac{c}{m/s}$	$\frac{k}{m/s}$
Illinois												
Chicago	MDW	6.1(20)	5.47	2.05	5.65	2.14	4.21	1.96	4.80	1.96	5.02	1.96
	ORD	6.1(20)	4.66	1.86	4.87	1.91	-	-	3.94	1.92	4.10	1.72
Rockford	RFD	6.1(20)	5.30	2.29	5.49	2.31	4.32	2.20	4.80	2.13	4.95	2.16
Springfield	SPI	14.9(49)	6.70	2.29	6.66	2.11	4.36	1.83	5.64	1.92	5.85	1.94
	SPI	6.1(20)	5.96	2.10	6.09	2.30	4.07	1.93	4.96	2.07	5.26	1.98
Indiana												
Evansville	EVV	19.5(64)	5.29	2.17	5.27	1.93	3.36	1.60	4.36	1.85	4.63	1.88
	EVV	6.1(20)	4.54	2.04	4.49	2.09	-	-	3.82	1.86	3.97	1.87
Indianapolis	IND	6.1(20)	4.52	1.84	4.51	1.93	-	-	3.52	1.69	3.90	1.73
South Bend	SBN	17.7(58)	5.88	2.63	5.95	2.27	4.47	2.12	5.25	2.10	5.39	2.20
Iowa												
Burlington	BRL	10.1(33)	5.60	2.14	5.60	2.14	3.51	1.60	4.77	1.99	4.92	1.91
Des Moines	DSM	20.1(66)	5.98	2.07	6.55	2.15	4.80	2.05	5.55	1.88	5.71	1.97
Kansas												
Wichita	ICT	8.8(29)	6.58	2.08	7.26	2.19	6.29	2.27	6.49	2.28	6.64	2.18
Kentucky												
Lexington	LEX	18.6(61)	5.61	2.20	5.31	2.02	3.88	1.94	4.33	1.73	4.77	1.87
Louisville	SDF	21.6(71)	4.99	2.14	4.93	1.93	3.44	1.77	4.13	1.87	4.35	1.84
	SDF	6.1(20)	4.48	2.12	4.44	2.12	-	-	-	-	3.98	1.97

TABLE C-1 (cont.)

CITY	CODE	ANEMOMETER	WINTER		SPRING		SUMMER		FALL		ANNUAL		
		HEIGHT	c	k	c	k	c	k	c	k	c	k	
		m(ft.)											m/s
Louisiana													
New Orleans	MSY	16.2(53)	5.08	1.84	4.87	1.87	3.57	1.99	4.50	1.85	4.43	1.73	
	MSY	6.1(20)	4.49	2.05	4.51	1.97	2.63	1.40	3.62	1.73	3.91	1.79	
Shreveport	SHV	16.8(55)	5.15	2.06	5.06	2.00	3.72	1.79	4.17	1.84	4.53	1.88	
	SHV	6.1(20)	3.89	1.77	4.04	1.98	2.52	1.42	3.19	1.63	3.48	1.70	
Maine													
Portland	PWM	16.8(55)	4.83	1.71	5.14	1.92	3.96	1.83	4.27	1.66	4.53	1.73	
Maryland													
Baltimore	BAL	40.5(133)	5.08	1.64	5.54	2.08	4.09	1.84	4.45	1.75	4.77	1.71	
Massachusetts													
Boston	BOS	6.7(22)	6.86	2.31	6.71	2.41	5.53	2.71	5.75	2.34	6.19	2.30	
Michigan													
Detroit	DET	24.7(81)	5.60	2.20	5.21	2.08	3.95	1.78	4.89	1.93	4.97	2.03	
Flint	FNT	6.4(21)	5.41	2.18	5.19	2.11	3.94	1.92	4.77	2.07	4.83	2.01	
Grand Rapids	GRR	19.5(64)	5.31	2.19	5.53	2.11	4.05	1.92	4.75	1.90	4.92	2.00	
Lansing	LAN	15.8(52)	7.11	2.54	6.91	2.27	4.53	1.93	6.02	2.28	6.16	2.13	
Muskegon	MKG	6.1(20)	5.95	2.20	5.54	2.21	4.47	2.18	5.18	2.13	5.26	2.08	
Minnesota													
Duluth	DLH	6.4(21)	5.29	2.17	5.38	2.15	4.42	1.91	4.68	2.01	4.95	2.04	
Minneapolis	MSP	6.4(21)	4.92	2.11	5.44	2.22	4.46	2.02	4.83	2.10	4.91	2.09	

TABLE C-1 (cont.)

CITY	CODE	ANEMOMETER	WINTER		SPRING		SUMMER		FALL		ANNUAL	
		HEIGHT										
		m(ft.)	<u>c</u>	<u>k</u>	<u>c</u>	<u>k</u>	<u>c</u>	<u>k</u>	<u>c</u>	<u>k</u>	<u>c</u>	<u>k</u>
			m/s		m/s		m/s		m/s		m/s	
<u>Mississippi</u>												
Jackson	JAN	6.1(20)	4.28	1.91	4.12	2.03	2.27	1.33	3.22	1.60	3.59	1.72
<u>Missouri</u>												
Columbia	COU	14.6(48)	3.57	1.77	3.75	1.78	-	-	-	-	3.16	1.62
Springfield	SGF	18.0(59)	6.76	2.36	6.93	2.28	5.29	2.21	5.92	2.19	6.21	2.17
	SGF	6.1(20)	5.13	2.33	5.16	2.24	-	-	4.35	2.07	4.60	2.08
St. Louis	STL	25.0(82)	4.91	2.09	5.18	2.00	3.50	1.89	4.14	1.93	4.38	1.83
	STL	6.1(20)	4.93	1.92	4.85	1.95	3.29	1.70	4.03	1.88	4.27	1.78
<u>Montanna</u>												
Billings	BIL	11.9(39)	6.11	2.15	5.52	1.66	4.56	1.53	5.15	1.73	5.37	1.75
	BIL	7.6(25)	6.00	2.14	5.11	1.96	3.97	1.69	5.45	2.14	5.16	1.94
Glasgow	GGW	6.1(20)	5.13	1.73	5.80	1.82	5.07	1.78	5.12	1.78	5.27	1.77
Havre	HVR	6.1(20)	4.93	1.56	5.48	1.76	4.74	1.73	5.07	1.68	5.05	1.67
Helena	HLN	13.4(44)	3.24	1.12	4.15	1.54	3.54	1.37	3.41	1.50	3.57	1.39
Kalispell	FCA	6.1(20)	3.20	1.15	3.90	1.67	3.25	1.57	2.88	1.36	3.25	1.36
Miles City	MLS	12.2(40)	4.47	1.65	5.23	1.74	4.12	1.61	4.45	1.47	4.56	1.59
<u>Nebraska</u>												
Scottsbluff	BFF	6.1(20)	5.38	1.70	5.60	1.79	3.96	1.66	4.39	1.52	4.81	1.61
<u>Nevada</u>												
Ely	ELY	6.1(20)	4.58	1.58	4.44	1.52	3.99	1.69	4.23	1.65	4.27	1.57

TABLE C-1 (cont.)

CITY	CODE	ANEMOMETER	WINTER		SPRING		SUMMER		FALL		ANNUAL		
		HEIGHT	<u>c</u> m/s	<u>k</u>	<u>c</u> m/s	<u>k</u>	<u>c</u> m/s	<u>k</u>	<u>c</u> m/s	<u>k</u>	<u>c</u> m/s	<u>k</u>	
		m(ft.)											
<u>New Jersey</u>													
Atlantic City	ACY	6.1(20)	5.62	1.71	5.68	2.00	4.17	1.90	4.36	1.77	4.91	1.73	
Newark	EWR	6.1(20)	5.57	2.14	5.39	2.07	4.20	2.13	4.52	1.96	4.93	1.96	
<u>New Mexico</u>													
Albuquerque	ABQ	14.6(48)	3.09	1.10	4.72	1.48	3.77	1.39	3.30	1.19	3.76	1.28	
<u>New York</u>													
Albany	ALB	12.2(40)	4.94	1.89	4.99	2.01	3.97	1.97	4.25	1.92	4.50	1.87	
	ALB	6.1(20)	5.74	1.96	4.02	1.71	-	-	-	-	3.61	1.65	
Buffalo	BUF	6.1(20)	6.11	1.92	5.43	1.86	4.46	1.75	4.72	1.79	5.15	1.77	
New York City	JFK	6.1(20)	6.22	2.08	5.86	2.14	4.76	2.21	5.12	1.89	5.45	1.97	
	LGA	6.1(20)	6.32	2.39	5.70	2.37	4.57	2.33	5.08	2.18	5.42	2.13	
Syracuse	SYR	21.9(72)	5.14	1.87	5.10	1.89	4.16	1.82	4.71	1.88	4.77	1.86	
<u>North Carolina</u>													
Asheville	AVL	28.0(92)	3.95	1.57	4.23	1.85	2.46	1.47	3.00	1.37	2.85	1.20	
	AVL	6.1(20)	4.84	1.70	4.09	1.57	2.44	1.41	3.43	1.48	3.69	1.45	
Cape Hatteras	HAT	9.8(32)	6.33	2.22	5.82	2.32	4.99	2.37	5.61	2.20	5.67	2.19	
Charlotte	CLT	6.1(20)	3.27	1.57	3.58	1.73	-	-	-	-	2.97	1.54	
Greensboro	GSO	17.1(56)	4.04	1.65	4.33	1.81	2.96	1.46	3.60	1.62	3.73	1.60	
	GSO	6.1(20)	3.68	1.74	3.69	1.87	-	-	-	-	3.16	1.64	
Raleigh	RDU	6.1(20)	4.35	1.85	4.32	1.78	-	-	-	-	3.76	1.68	
<u>North Dakota</u>													
Bismark	BIS	13.1(43)	5.03	1.68	6.26	1.95	5.44	1.98	5.58	1.67	5.58	1.80	
	BIS	6.1(20)	4.96	1.64	5.58	1.82	4.79	1.91	4.95	1.67	5.06	1.73	
Williston	ISN	6.1(20)	4.61	1.72	5.15	1.84	4.41	1.76	4.76	1.77	4.73	1.76	

TABLE C-1 (cont.)

CITY	CODE	ANEMOMETER HEIGHT m(ft.)	WINTER		SPRING		SUMMER		FALL		ANNUAL		
			<u>c</u>	<u>k</u>	<u>c</u>	<u>k</u>	<u>c</u>	<u>k</u>	<u>c</u>	<u>k</u>	<u>c</u>	<u>k</u>	
			m/s		m/s		m/s		m/s		m/s		
<u>Ohio</u>													
Akron	CAK	11.3(37)	5.88	2.35	5.44	2.07	3.76	1.79	4.74	1.94	4.98	1.96	
Cincinnati	CVG	19.5(64)	5.21	2.11	5.16	2.02	3.53	1.77	4.50	2.00	4.59	1.89	
Cleveland	CLE	6.1(20)	5.79	2.23	4.97	2.01	-	-	4.41	1.90	4.71	1.90	
Dayton	DAY	16.8(55)	5.83	2.16	5.38	1.88	3.58	1.66	4.43	1.67	4.82	1.75	
	DAY	6.1(20)	5.14	2.14	4.86	2.05	3.59	1.81	4.35	2.02	4.49	1.95	
Toledo	TOL	14.3(47)	6.14	2.20	6.23	2.09	4.05	1.69	5.02	1.88	5.38	1.89	
Youngstown	YNG	18.9(62)	5.85	2.38	5.19	1.95	3.74	1.77	4.74	1.93	4.91	1.94	
	YNG	6.1(20)	5.50	2.44	4.91	2.13	3.21	1.74	4.26	2.10	4.53	2.03	
<u>Oklahoma</u>													
Tulsa	TUL	7.0(23)	5.02	1.89	5.80	2.11	4.57	2.01	4.71	1.90	5.02	1.93	
Okla. City TV Tower	WKY	44.5(146)	-	-	-	-	-	-	-	-	6.57	2.07	
<u>Oregon</u>													
Eugene	EUG	6.1(20)	3.68	1.57	3.34	1.51	-	-	3.16	1.48	3.35	1.54	
Portland	PDX	6.1(20)	4.66	1.78	3.17	1.44	-	-	3.40	1.45	3.49	1.49	
Salem	SLE	6.1(20)	4.27	1.53	3.44	1.52	-	-	3.65	1.64	3.49	1.47	
<u>Pennsylvania</u>													
Allentown	ABE	6.1(20)	5.18	1.58	4.96	1.76	3.39	1.58	3.94	1.51	4.31	1.52	
Harrisburg	HAR	14.0(46)	4.16	1.54	4.33	1.77	2.55	1.37	3.15	1.45	3.53	1.46	
	HAR	6.7(22)	4.52	1.51	4.11	1.60	2.70	1.52	3.23	1.39	3.58	1.40	
Philadelphia	PHL	6.1(20)	5.24	1.97	5.31	2.15	3.51	1.77	4.22	1.88	4.58	1.85	
Pittsburgh	PIT	6.1(20)	5.14	2.06	4.69	1.85	-	-	4.20	1.81	4.37	1.81	
Scranton	AVP	28.0(92)	4.74	2.17	4.57	2.09	3.70	2.05	4.09	2.05	4.15	2.01	

TABLE C-1 (cont.)

CITY	CODE	ANEMOMETER	WINTER		SPRING		SUMMER		FALL		ANNUAL	
		HEIGHT	c	k	c	k	c	k	c	k		
		m(ft.)									m/s	m/s
Rhode Island												
Providence	PVD	6.1(20)	5.46	1.78	5.48	2.05	4.16	2.15	4.14	1.76	4.73	1.76
South Carolina												
Charleston	CHS	6.1(20)	3.98	1.74	4.30	1.92	-	-	3.43	1.77	3.79	1.77
Columbia	CAE	11.0(36)	3.81	1.69	4.09	1.70	3.09	1.63	3.17	1.61	3.41	1.51
	CAE	6.1(20)	3.61	1.78	3.81	1.76	-	-	2.69	1.45	3.13	1.61
Greenville	GSP	7.0(23)	-	-	3.62	1.83	-	-	-	-	-	-
South Dakota												
Huron	HON	12.5(41)	5.56	1.91	6.47	2.08	5.38	2.03	5.84	1.91	5.81	1.95
Rapid City	RAP	9.8(32)	4.80	1.29	5.82	1.70	5.04	1.79	5.38	1.47	5.27	1.54
	RAP	6.4(21)	4.81	1.22	5.97	1.47	4.53	1.63	5.09	1.33	5.06	1.36
Sioux Falls	FSD	5.2(17)	5.51	1.97	6.30	2.36	5.11	2.28	5.40	2.16	5.57	2.14
Tennessee												
Bristol	TRI	6.1(20)	3.28	1.62	3.46	1.76	-	-	-	-	2.78	1.49
Chattanooga	CHA	17.4(57)	3.93	1.81	3.95	1.76	2.34	1.33	3.35	1.83	3.38	1.60
	CHA	6.1(20)	3.43	1.75	3.70	1.88	1.86	1.18	-	-	3.16	1.69
Knoxville	TYS	6.7(22)	3.14	1.50	3.44	1.56	-	-	-	-	2.83	1.43
Memphis	MEM	6.7(22)	4.74	1.93	4.84	2.27	3.16	1.67	3.93	1.93	4.18	1.87
Nashville	BNA	7.6(25)	4.55	1.88	4.68	2.01	-	-	3.61	1.68	3.99	1.75
Texas												
Abilene	ABI	6.1(20)	5.50	2.09	6.05	2.42	5.08	2.37	5.27	2.14	5.47	2.20
Amarillo	AMA	10.1(33)	6.12	1.96	6.96	2.24	5.85	2.34	5.88	2.14	6.20	2.12
Austin	AUS	9.8(32)	5.03	2.06	5.41	2.27	4.57	2.27	4.17	1.74	4.81	2.04
	AUS	6.1(20)	4.73	1.84	4.99	2.07	-	-	3.96	1.70	4.39	1.84

CITY	CODE	ANEMOMETER	WINTER		SPRING		SUMMER		FALL		ANNUAL		
		HEIGHT	c	k	c	k	c	k	c	k			
		m(ft.)									m/s	m/s	m/s
Texas (continued)													
Brownsville	BRO	17.1(56)	6.10	2.12	7.12	2.40	6.09	2.59	5.33	2.09	6.21	2.13	
Dallas	DAL	6.1(20)	5.20	2.15	5.81	2.58	4.49	2.26	4.71	2.08	5.06	2.21	
El Paso	ELP	25.9(85)	5.06	1.42	6.32	1.70	5.05	1.81	4.70	1.64	5.25	1.60	
Fort Worth	FTW	25.9(85)	6.21	2.27	6.71	2.35	5.67	2.44	5.64	2.14	6.05	2.29	
	FTW	6.7(22)	4.83	1.78	5.22	1.93	3.78	1.73	9.17	1.77	4.48	1.74	
Galveston	GLS	43.6(143)	6.33	2.09	6.25	2.48	5.47	2.51	5.45	1.86	5.86	2.18	
Houston	HOU	26.5(87)	6.14	2.23	6.55	2.21	4.64	1.96	5.21	2.12	5.60	2.09	
	HOU	6.1(20)	5.40	2.18	5.68	2.34	4.11	1.96	4.23	1.81	4.88	2.02	
Lubbock	LBB	20.7(68)	6.65	2.03	7.49	2.34	6.06	2.35	6.03	2.16	6.54	2.15	
Port Arthur	BPT	6.1(20)	5.21	2.22	5.66	2.58	4.24	2.23	4.29	1.92	4.87	2.16	
Waco	ACT	23.8(78)	6.16	2.24	6.58	2.41	5.87	2.59	5.39	2.06	6.01	2.28	
Wichita Falls	SPS	9.4(31)	4.94	2.12	5.66	2.28	4.72	2.29	4.65	2.00	5.00	2.14	
Utah													
Salt Lake City	SLC	6.1(20)	2.86	1.08	3.64	1.33	3.33	1.45	3.13	1.25	3.23	1.26	
Vermont													
Burlington	BTV	6.1(20)	4.44	2.02	4.38	1.95	-	-	4.17	1.88	4.18	1.92	
Virginia													
Lynchburg	LYH	6.1(20)	3.67	1.74	4.09	1.96	-	-	-	-	3.38	1.71	
Richmond	RIC	20.4(67)	3.76	1.85	4.06	1.98	2.66	1.46	3.08	1.57	3.31	1.59	
Roanoke	ROA	9.1(30)	4.58	1.55	4.46	1.74	-	-	3.35	1.45	3.75	1.46	
Wallops	WAL	61.0(200)	8.03	2.06	8.28	2.33	6.49	2.14	7.41	2.31	7.57	2.17	

TABLE C-1 (cont.)

CITY	CODE	ANEMOMETER	WINTER		SPRING		SUMMER		FALL		ANNUAL	
		HEIGHT m(ft.)	<u>c</u> m/s	<u>k</u>	<u>c</u> m/s	<u>k</u>	<u>c</u> m/s	<u>k</u>	<u>c</u> m/s	<u>k</u>	<u>c</u> m/s	<u>k</u>
<u>Washington</u>												
Seattle	SEA	6.1(20)	4.44	1.78	3.69	1.64	-	-	3.59	1.62	3.36	1.60
Spokane	SFF	6.1(20)	4.36	1.44	4.42	1.75	3.92	1.80	4.00	1.55	4.12	1.57
Hanford	HAN	61.0(200)	4.28	1.27	5.58	1.55	5.47	1.53	4.43	1.35	4.95	1.41
<u>West Virginia</u>												
Charleston	CRW	9.8(32)	3.92	2.19	3.38	1.64	2.13	1.31	2.92	1.69	2.98	1.54
Huntington	HTS	6.1(20)	3.32	1.53	3.14	1.53	-	-	-	-	2.71	1.41
<u>Wisconsin</u>												
Green Bay	GRB	14.3(47)	5.89	2.21	5.81	2.09	4.49	1.83	5.65	1.92	5.47	1.97
	GRB	6.1(20)	5.29	2.10	5.17	2.17	3.94	1.76	4.66	1.97	4.78	1.98
La Crosse	LSE	6.4(21)	4.21	1.85	4.46	1.94	3.72	1.88	4.11	1.90	4.11	1.86
Madison	MSN	6.4(21)	5.01	2.07	5.06	2.06	3.99	2.03	4.60	2.07	4.62	1.98
Milwaukee	MKE	6.1(20)	5.89	2.32	5.52	2.06	4.54	2.04	5.20	2.22	5.78	2.10
	MWC	6.1(20)	5.29	2.25	5.18	2.36	4.21	2.11	4.81	2.25	4.87	2.20
<u>Wyoming</u>												
Casper	CPR	6.1(20)	7.54	1.95	6.06	1.89	4.27	1.54	5.90	1.80	5.93	1.70
Cheyenne	CYS	12.2(40)	7.69	1.63	6.76	1.64	4.88	1.74	5.60	1.64	6.12	1.55
	CYS	10.1(33)	7.28	1.76	6.48	1.84	4.76	1.85	5.94	1.81	6.05	1.71

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