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THE COMPUTATIONAL PHYSICS PROGRAM OF THE NATIONAL MFE COMPUTER CENTER

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June 1977

MASTER

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I. INTRODUCTION

Since June 1974 the MFE Computer Center has been engaged in a substantial computational physics research effort. This effort has involved the development and application of advanced numerical techniques for modeling plasma behavior in fusion devices such as PLT, PDX, TFTR, Doublet III, MFTF, and TMX.

The computational physics group, led by J. Killeen, currently consists of five PhD computational physicists - A. A. Mirin, K. D. Marx, M. G. McCoy, D. E. Shumaker and A. I. Shestakov. In addition, several students in the Department of Applied Science at U.C. Davis/Livermore are doing fusion research under the guidance of senior group members.

One of the principal objectives of the computational physics group is to provide support for experimental and theoretical work within the MFE community. At present, this support falls into the following areas:

- 1) We are performing Fokker-Planck/Transport and guiding center simulation calculations for the TFTR, PLT and PDX experiments at Princeton in collaboration with R. C. Grimm, D. L. Jassby and D. E. Post, and with J. G. Cordey of Culham Laboratory on Fokker-Planck/Transport applications to DITE.
- 2) We are collaborating with M Division (Lawrence Livermore Laboratory) personnel - namely M. E. Rensink, T. A. Cutler, B. G. Logan, J. K. Boyd, L. S. Hall, A. H. Futch, R. W. Moir and T. K. Fowler - on the continuing development and application of Fokker-Planck codes to the TMX, MFTF, 2XII and direct vector.

A summary of our program follows.

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II. COMPUTATIONAL STUDIES

A. Fokker-Planck Calculations

Beginning in late 1974 our one-and two-dimensional multispecies Fokker-Planck codes were generalized to deal with both toroidal and open-ended configurations. Since then many physical effects have been incorporated into these codes, and a variety of physical problems have been studied. They include:

1. a model for the Tandem Mirror Experiment (TMX), in which Fokker-Planck equations are solved separately in the plug and the main trap.
2. fusion source and loss terms for a complete D-D burning cycle, in which deuterium, tritium, helium-3, protons and alpha particles are treated as separate species.
3. the behavior of distribution functions in mirror devices employing a direct convertor. Comparisons are made with a Monte Carlo code.
4. a complete five-dimensional $\overline{\sigma v}$ calculation, taking into account the anisotropy of the ion species.
5. the addition of radial dependence to model finite Larmor radius effects.
6. applications to the Counterstreaming Ion Torus (CIT) and the Two Component Torus (TCT), including fast major radius compression and energy clamping.
7. Q calculations for the toroidally linked mirror (TLM).
8. a study of the reversed field mirror.

9. the effects of plasma rotation on the ion loss-cone boundary and resulting $n\tau$, in collaboration with V. Volosov of the Institute of Nuclear Physics in Novosibirsk.
10. a self-consistent treatment of thermal and Frank-Condon neutrals in mirror systems.
11. the calculation of $n\tau$, the axial density profile and maximum stable β for FERF plasmas.
12. the study of runaway electrons and ions.
13. the effect of off-angle injection in mirror systems.
14. a study of the electrostatically plugged cusp with T. Dolan of the University of Missouri - Rolla.
15. applications to the fission/fusion hybrid reactor.
16. bounce averaging for mirror devices.

Papers

1. M. E. Rensink, T. K. Fowler, R. P. Freis, J. Killeen, A. A. Mirin, R. W. Moir, L. D. Pearlstein, R. F. Post, C. G. Tull, L. S. Hall, B. McNamara, J. K. Boyd, C. H. Finan, III, D. Fuss, and C. A. Wilgus, "Theoretical Studies of Plasma Confinement in Magnetic Mirrors", Fifth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Tokyo (1974), IAEA, Vienna, 311-320.
2. H. L. Berk, H. P. Furth, D. L. Jassby, R. M. Kulsrud, C. S. Liu, M. N. Rosenbluth, P. H. Rutherford, F. H. Tenney, T. Johnson, J. Killeen, A. A. Mirin and M. E. Rensink, "Two Energy Component Toroidal Fusion Devices", Fifth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Tokyo (1974), IAEA, Vienna, 569-580.

3. J. A. Byers, R. P. Freis, J. Killeen, E. Lee, B. McNamara, A. A. Mirin and M. E. Rensink, "Computational Studies of Two Component Mirror Reactors", Fifth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Tokyo (1974), IAEA, Vienna, 551-556.
4. J. Killeen, K. D. Marx, A. A. Mirin, and M. E. Rensink, "Energy Multiplication and Transport in a Two-Component Torus," Seventh European Conference on Controlled Fusion and Plasma Physics, Lausanne (1975), 22.
5. K. D. Marx, A. A. Mirin, M. G. McCoy, M. E. Rensink and J. Killeen, "Calculation of $(\overline{\sigma v})_{DT}$ for Anisotropic Mirror and Toroidal Distributions," Nucl. Fusion 16, No. 4 (1976), 702-706.
6. J. G. Cordey, K. D. Marx, M. G. McCoy, A. A. Mirin, M. E. Rensink and J. Killeen, "A New Expansion Method for Computing $\overline{\sigma v}$ for Reactant Distribution Functions," submitted to J. Comp. Physics (1977).
7. A. A. Mirin, J. Killeen and C. J. H. Watson, "Some Preliminary Q Calculations for Toroidally Linked Mirror Reactors," Nucl. Fusion 17, No. 1 (1977), 47-51.
8. J. Killeen, A. A. Mirin and M. E. Rensink, "The Solution of the Kinetic Equations for a Multispecies Plasma," in Methods of Computational Physics 16 (1976), Academic Press, New York, 389-432.
9. J. Killeen and A. A. Mirin, "Numerical Solution of the Fokker-Planck Equations for a Multispecies Plasma", College in Theoretical and Computational Plasma Physics, Trieste (1977), IAEA, Vienna.

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1. A. A. Mirin, "Isotions, A One-Dimensional Multispecies Fokker-Planck Computer Code", Lawrence Livermore Laboratory Report UCRL 51616 (1974).

2. A. A. Mirin, "Hybrid I, a Two-Dimensional Fokker-Planck Code", Lawrence Livermore Laboratory Report UCRL 51598 (1974).
3. A. A. Mirin, "Hybrid II, A Two-Dimensional Multispecies Fokker-Planck Computer Code", Lawrence Livermore Laboratory Report UCRL 51615, Rev. 1 (1975).
4. K. D. Marx, M. G. McCoy, A. A. Mirin, M. E. Rensink and J. Killeen, "A Legendre Expansion Method for Computing $\overline{\sigma v}$ for Reactant Distribution Functions," Lawrence Livermore Laboratory Report UCRL 78362 (1976).

Abstracts

1. J. Killeen, A. A. Mirin and M. E. Rensink, "Energy Multiplication Studies of the Two Component Torus with Major Radius Compression", Bull. Am. Phys. Soc. 19 (1974), 974 (Albuquerque).
2. A. A. Mirin, J. Killeen and M. E. Rensink, "Multi-Species Fokker Planck Calculations for the Two Component Torus", Bull. Am. Phys. Soc. 19 (1974), 974 (Albuquerque).
3. M. E. Rensink, J. Killeen and A. A. Mirin, "Computational Studies of Mirror Confinement", Bull. Am. Phys. Soc. 19 (1974), 920 (Albuquerque).
4. T. Cutler, J. A. Byers, D. Freeman, B. McNamara, A. A. Mirin and M. E. Rensink, "Buildup of Two Component Mirror Plasmas", Bull. Am. Phys. Soc. 19 (1974), 921 (Albuquerque).
5. A. A. Mirin, J. Killeen, K. D. Marx and M. E. Rensink, "Energy Multiplication Studies for a Pulsed Two Component Torus", Annual Meeting on Theoretical Aspects of Controlled Thermonuclear Research, Rosslyn (1975), 68
6. K. D. Marx, J. Killeen, A. A. Mirin and M. E. Rensink, "Energy Multiplication Studies for a Steady State Two Component Torus", Annual Meeting on Theoretical Aspects of Controlled Thermonuclear Research, Rosslyn (1975), 64

7. M. E. Rensink, A. A. Mirin and J. Killeen, "Computational Studies on Mirror Confined Plasma", Annual Meeting on Theoretical Aspects of Controlled Thermonuclear Research, Rosslyn (1975), 80.
8. A. A. Mirin, M. E. Rensink, and D. L. Jassby, "Fokker-Planck Studies of the Counterstreaming Ion Torus," Annual Meeting on Theoretical Aspects of Controlled Thermonuclear Research, Madison (1976), 2A-15.
9. A. A. Mirin, M. E. Rensink and D. L. Jassby, "Fokker-Planck Studies of the C.I.T.," Bull. Am. Phys. Soc. 21 (1976), 1064 (San Francisco).
10. K. D. Marx, W. L. Barr, R. W. Moir and C. E. McDowell, "A Monte Carlo Calculation of Collisional Effects in a Mirror-Confined Plasma," Annual Meeting on Theoretical Aspects of Controlled Thermonuclear Research, Madison (1976), 1C-11.
11. M. G. McCoy, J. Killeen, K. D. Marx, A. A. Mirin and M. E. Rensink, "Calculations of $(\overline{\sigma v})_{DT}$ for Anisotropic Mirror and Toroidal Distributions," Annual Meeting on Theoretical Aspects of Controlled Thermonuclear Research, Madison (1976), 2B-11.
12. A. H. Futch, R. P. Freis and A. A. Mirin, "Plasma Build-up Calculations of Neutral Beam Injection with a Radially Dependent Fokker-Planck Code," Bull. Am. Phys. Soc. 21 (1976), 1182 (San Francisco).
13. T. A. Cutler, A. A. Mirin, L. D. Pearlstein and M. E. Rensink, "Bounce Average Code Mirror Plasma Studies," Bull. Am. Phys. Soc. 21 (1976), 1187 (San Francisco).
14. K. D. Marx, T. J. Dolan, R. W. Moir and C. E. McDowell, "Trapping Rates and Loss Rates for Electrons in an Electrostatically Plugged Cusp," Bull. Am. Phys. Soc. 21 (1976), 1044 (San Francisco).

15. B. G. Logan, T. K. Fowler, A. H. Futch, A. A. Mirin and M. E. Rensink, "The Tandem Mirror Concept," 18th Annual Meeting of the Division of Plasma Physics of the Amer. Phys. Soc., San Francisco (1976) (post-deadline paper).
16. M. E. Rensink, R. H. Cohen, T. A. Cutler and A. A. Mirin, "Particle Confinement by Ambipolar Potentials in Mirror Machines", Annual Controlled Fusion Theory Conference, San Diego (1977), B16.
17. R. H. Cohen, M. E. Rensink and A. A. Mirin, "Tandem Mirror Confinement Studies," Annual Controlled Fusion Theory Conference, San Diego (1977), I4.

B. Transport Studies

1. Radial Transport

A multispecies Tokamak radial transport code has been written in which the ion densities, the ion temperature, the electron temperature and the poloidal magnetic field are calculated. A combination of neoclassical models of Connor and of Rosenbluth, Hazeltine and Hinton is used to describe the plasma transport.

2. Noncircular Transport

A variational principle has been used to compute neoclassical transport coefficients for noncircular cross-sectional toroidal plasmas. The model is essentially one-dimensional, in that the transport coefficients are obtained through integrations over surfaces of constant magnetic stream function ψ . It is assumed that the ion and electron temperatures are equal. These transport coefficients are being incorporated into a Tokamak transport code which uses adiabatic invariants (mass, entropy and toroidal magnetic flux) as dependent variables, thereby eliminating the need to compute the toroidal electric field. A fast Poisson solver is used to compute new equilibria.

3. Two Dimensional Transport

A code has been written to solve a system of transport equations for a plasma in two dimensions. Since no theoretical model exists for two-dimensional neoclassical transport, the classical transport coefficients are being used. It is intended that this code be used to study the PDX.

Papers

1. D. E. Shumaker and I. B. Bernstein, "Improved Variational Method for Axisymmetric Plasmas," *Phys. Fluids* 18 (1975), 1487-1488.
2. A. A. Mirin, J. Killeen, K. D. Marx and M. E. Rensink, "A Radial Transport/Fokker-Planck Model for a Tokamak Plasma," *J. Comp. Physics*, 23, No. 1 (1977), 23-41.
3. M. L. Watkins, M. H. Hughes, K. V. Roberts, P. M. Keeping and J. Killeen, "ICARUS-A One-Dimensional Plasma Diffusion Code," in *Methods of Computational Physics* 16 (1976), Academic Press, New York, 166-209.

Reports

1. D. E. Shumaker, "Calculation of Transport Coefficients in an Axisymmetric Torus," University of California Laboratory Report UCRL 52218 (1977).

Abstracts

1. K. D. Marx, B. Kelly, J. Killeen, A. A. Mirin and M. E. Rensink, "A Two-Dimensional Transport Code for Toroidal Plasmas," Seventh Conference on Numerical Simulation of Plasmas, New York (1975), 19-22.

C. Fokker-Planck/Transport Studies

These studies employ a Fokker-Planck/Transport code, in which there is a warm Maxwellian background plasma described by a set of macroscopic radial transport equations and several energetic species described by two-dimensional Fokker-Planck equations. The model is applicable to circular cross-section toroidal systems in which there is beam injection, and is currently being generalized to handle non-circular devices such as Doublet III. A Monte Carlo neutrals package developed at Princeton Plasma Physics Laboratory is used to self-consistently compute the neutral density profile. Energy multiplication studies for the CIT have been carried out in which the diffusion constants, source parameters, neutral densities and impurity content have been varied. Applications to PDX and DITE are in progress.

Papers

1. J. Killeen, K. D. Marx, A. A. Mirin, and M. E. Rensink, "Energy Multiplication and Transport in a Two-Component Torus," Seventh European Conference on Controlled Fusion and Plasma Physics, Lausanne (1975), 22.
2. A. A. Mirin, J. Killeen, K. D. Marx and M. E. Rensink, "A Radial Transport/Fokker-Planck Model for a Tokamak Plasma," J. Comp. Physics 23, No. 1 (1977), 23-41.
3. D. L. Jassby, R. M. Kulsrud, F. W. Perkins, J. Killeen, K. D. Marx, M. G. McCoy, A. A. Mirin, M. E. Rensink and C. G. Tull, "Counterstreaming-Ion Tokamak Fusion Reactors," Sixth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, FRG (1976), IAEA-CN-35/B12-2.

Abstracts

1. A. A. Mirin, J. Killeen, K. D. Marx and M. E. Rensink, "A Radial Transport/Fokker-Planck Model for a Tokamak Plasma", Seventh Conference on Numerical Simulation of Plasmas, New York (1975), 15-18.
2. A. A. Mirin, J. Killeen, K. D. Marx, and M. E. Rensink, "Computational Study of the Toroidal Fusion Test Reactor," Bull. Am. Phys. Soc. 20 (1975), 1257 (St. Petersburg).
3. M. G. McCoy, A. A. Mirin, M. E. Rensink, D. L. Jassby, J. Killeen and K. D. Marx, "Computational Study of the Counterstreaming Ion Torus," Bull. Am. Phys. Soc. 21 (1976), 1064 (San Francisco).
4. A. A. Mirin, M. G. McCoy, D. L. Jassby and J. Killeen, "Fokker-Planck/Transport Simulations of Counterstreaming Ion Plasmas in the PDX Experiment", Annual Controlled Fusion Theory Conference, San Diego (1977), 815.

D. MHD Studies

A two-dimensional nonlinear MHD code written for general orthogonal Eulerian coordinates has been used to study the evolution and saturation of magnetic islands in both Cartesian (sheet pinch) and cylindrical (diffuse) geometries. Both the linear and non-linear behavior of the double tearing instability have been studied. Cases involving helical symmetry are currently being examined. Further studies utilizing previously written one-dimensional linear codes are underway.

A three-dimensional generalization of the above model containing the full MHD equations has been written. Investigations of the tearing mode, internal kink modes and stability studies for the flux conserving Tokamak (FCT) are being carried out.

Papers

1. J. A. Dibiase, J. Killeen, D. C. Robinson and D. Schnack, "Linear and Non-Linear Calculation of the Tearing Mode in Reversed Field Pinches," in Pulsed High Beta Plasmas (1976), Pergamon Press, Oxford, 283-289.
2. D. Schnack and J. Killeen, "Linear and Non-Linear Calculations of the Tearing Mode," Third Int'l (Kiev) Conf. on Plasma Theory, Trieste (1977).
3. J. A. Dibiase and J. Killeen, "A Numerical Model for Resistive Magnetohydrodynamic Instabilities," J. Comp. Phys. 24 (1977).

Abstracts

1. D. Schnack, J. Killeen, and C. Finan, "Nonlinear Evolution of the Tearing Mode in a Sheet Pinch," Bull. Am. Phys. Soc. 20 (1975), 1311 (St. Petersburg).
2. D. D. Schnack, Jr. and J. Killeen, "The Non-Linear Tearing Mode," Bull. Am. Phys. Soc. 21 (1976), 1133 (San Francisco).

3. D. D. Schnack and J. Killeen, "Linear and Non-Linear Numerical Studies of Resistive Instabilities in Cylindrical Geometry", Annual Controlled Fusion Theory Conf., San Diego (1977), A17.
4. C. H. Finan and J. Killeen, "3D Non-Linear Evolution of the Internal Kink Mode in a Resistive Plasma," Annual Controlled Fusion Theory Conf., San Diego (1977), A6.

E. Vlasov and Particle Simulations

1. Guiding Center Toroidal

A self-consistent guiding center particle model coupled with a fluid equilibrium code has been used to study ion beam motion in a toroidal geometry. The model includes Fokker-Planck slowing down, and a Monte Carlo procedure is being developed to treat collisional angular scattering. Physical applications include:

- a) studies of equilibrium and neutral beam injection for the Toroidal Fusion Test Reactor.
- b) studies of collisionless beam behavior for scenarios involving strong counterstreaming beams, both with and without a background plasma.
- c) steady state modeling of the proposed CIT reactor, in which the steady state beam currents, safety factor and energy reinjection rates necessary to maintain a steady state are computed.

2. Mirror

A study of self-consistent magnetic fields in a high beta TCM reactor was completed in Fiscal Year 1975.

The modified Superlayer code was used for this purpose. This work was in collaboration with M Division of LLL.

3. Vlasov

A one-dimensional (z - v_z) Vlasov finite difference code for ions and electrons was written to study the formation of a plasma sheath and to compute the steady state distribution functions along with a self-consistent plasma potential.

Papers

1. D. L. Jassby, R. M. Kulsrud, F. W. Perkins, J. Killeen, K. D. Marx, M. G. McCoy, A. A. Mirin, M. E. Rensink and C. G. Tull, "Counter-streaming-Ion Tokamak Fusion Reactors," Sixth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, FRG (1976), IAEA-CN-35/B12-2.
2. T. H. Johnson, J. Killeen, O. A. Anderson and M. E. Rensink, "Guiding-Center Simulation of Toroidal Plasmas", J. Comp. Physics 23, No. 3 (1977), 219-241.
3. H. L. Berk, H. P. Furth, D. L. Jassby, R. M. Kulsrud, C. S. Liu, M. N. Rosenbluth, P. H. Rutherford, F. H. Tenney, T. Johnson, J. Killeen, A. A. Mirin and M. E. Rensink, "Two Energy Component Toroidal Fusion Devices," Fifth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Tokyo (1974), IAEA, Vienna, 569-580.

Abstracts

1. T. Johnson, J. Killeen and M. E. Rensink, "Guiding Center Simulation of the Two Component Torus", Bull. Am. Phys. Soc. 19 (1974), 974 (Albuquerque).
2. J. A. Byers and A. A. Mirin, "Formation of Reversed Field Equilibria during Large Current Neutral Injection into Magnetic Mirror Target Plasma", Bull. Am. Phys. Soc. 19 (1974), 921 (Albuquerque).
3. C. G. Tull, T. H. Johnson, M. E. Rensink and J. Killeen, "Guiding Center Particle and Plasma Equilibrium Simulation in Toroidal Systems with Azimuthal Symmetry", Seventh Conference on Numerical Simulation of Plasmas, New York (1975), 131-134
4. C. G. Tull, D. L. Jassby, T. H. Johnson, J. Killeen and M. E. Rensink, "Steady State Simulations of the Counterstreaming Ion Torus (CIT) Fusion Reactor," Bull. Am. Phys. Soc. 21 (1976), 1064 (San Francisco).

5. T. H. Johnson, J. Killeen, M. E. Rensink and C. G. Tull, "Diversification of Guiding Center Simulation Methods," Bull. Am. Phys. Soc. 21 (1976), 1038 (San Francisco).
6. C. G. Tull, J. Killeen and M. E. Rensink, "A Steady State Guiding Center Model for the Counterstreaming Ion Torus (CIT)", Annual Controlled Fusion Theory Conf., San Diego (1977), 113
7. J. A. Byers, A. A. Mirin and D. R. Schnabel, "Simulation of Self-Consistent Magnetic Fields in 2XIIB", Annual Meeting on Theoretical Aspects of Controlled Thermonuclear Research, Rosslyn (1975), 12.

F. Equilibrium Studies

1. Tormac

A code has been developed to solve non-linear elliptic equations on domains of very general shape. Its main application is to solve the equilibrium fluid equations proposed for the TORMAC device. Equilibria including the effects of a rotating plasma will be investigated.

Reports

1. J. C. Taylor and J. V. Taylor, "The Elipti Code-Part 1", Lawrence Livermore Laboratory Report UCRL 79328 (1977).

Abstracts

1. J. C. Taylor and J. V. Taylor, "ELIPTI-TORMAC: A Code for the Solution of General Non-linear Elliptic Problems over 2-D Regions of Arbitrary Shape", Proc. 2nd IMACS Int'l Symposium on Computer Methods for Partial Differential Equations, Bethlehem (1977).

G. Convergent Neutral Beam Studies

A study has been made involving the creation of a very dense plasma by injecting convergent neutral beams into spherical or cylindrical chambers. Calculations of particle distribution functions, densities, ionization rate parameters and ionization probabilities have been carried out for both geometries.

Papers

1. R. F. Post, K. D. Marx, and C. J. Eggens, "Creation of Transient High-Density Plasmas by Convergent Neutral Beams," Nucl. Fusion 15, No. 4 (1975), 701-703.
2. K. D. Marx, C. J. Eggens, and R. F. Post, "Particle Distributions and Early Phases of Ionization in Proposed Convergent Neutral Beam Experiments," Journal of Applied Physics 48 (1977).

Abstracts

1. R. F. Post, K. D. Marx and C. J. Eggens, "The Use of Convergent Neutral Beams to Create Transient High Density Plasmas", Bull. Am. Phys. Soc. 19 (1974), 912 (Albuquerque).
2. K. D. Marx, C. J. Hartman, and R. F. Post, "Approximate Analytical Formulas for Systems which Employ Convergent Neutral Beams," Bull. Am. Phys. Soc. 20 (1975), 1364 (St. Petersburg).
3. C. J. Hartman, K. D. Marx, and R. F. Post, "Numerical Calculations of Processes in Convergent Neutral Beams," Bull. Am. Phys. Soc. 20 (1975), 1365 (St. Petersburg).

H. General Numerical Methods

A general study of the Incomplete Cholesky Conjugate Gradient Method (ICCG) of matrix "inversion" is being carried out. This new technique has been shown to be considerably faster than other traditional linear solvers such as ADI, SIP and SOR. An ICCG solver applicable to general nine point two-dimensional difference operators has been written, and modifications to existing codes are being considered where beneficial.

III. APPENDIX - MISCELLANEOUS REFERENCES

A. Review Publications

Papers

1. J. Killeen, "Computer Applications in Controlled Fusion Research," Nucl. Tech. 27 (1975), 107-113.
2. J. Killeen, "Computational Plasma Physics," 3rd National Computational Physics Conference on Partial Differential Equations in Physics, Glasgow (1975), and Computational Methods in Classical and Quantum Physics (1976), Advance Publications Ltd., London, 83-120.
3. John Killeen, "Computer Models of Magnetically Confined Plasmas," Nucl. Fusion 16, No. 5 (1976), 841-864.

Abstracts

1. J. Killeen, "National CTR Computer Center", Seventh Conference on Numerical Simulation of Plasmas, New York (1975).
2. J. Killeen, "Computer Models of Magnetically Confined Plasmas," Bull. Am. Phys. Soc. 20 (1975), 1358. (St. Petersburg).

B. Other publications of group members

Papers

1. J. A. Byers, J. P. Holdren, J. Killeen, A. B. Langdon, A. A. Mirin, M. E. Rensink and C. G. Tull, "Computer Simulation of Pulse Trapping and Pulse Stacking of Relativistic Electron Layers in Astron", Phys. Fluids 17, No. 11 (1974), 2061-2080.
2. R. P. Freis, C. W. Hartman, J. Killeen, A. A. Mirin and M. F. Uman, "Calculations of Combined Stellerator-Multipole Toroidal Magnetic Field Configurations," Nucl. Fusion 17, No. 2 (1977), 281-289.

Reports

1. C. H. Finan, III, "SIGHDT-Atomic and Molecular Cross Sections for Hydrogen Isotopes," Lawrence Livermore Laboratory Report UCRL 51805 (1975).
2. C. H. Finan, III, and B. McNamara, "MAFJ-Some Simple Additions to MAFCO", Lawrence Livermore Laboratory Report UCRL 51806 (1975).
3. C. H. Finan, III, "MAFC076-Magnetic Field Code", Lawrence Livermore Laboratory Report UCRL 51804 (1975).
4. R. P. Freis, C. W. Hartman, J. Killeen, A. A. Mirin and M. F. Uman, "Numerical Studies of Toroidal Magnetic Field Configurations", Lawrence Livermore Laboratory Report UCRL 78041 (1976).
5. A. A. Mirin, M. F. Uman, C. W. Hartman and J. Killeen, "An Analytic Representation of Fields Resulting from Currents on a Torus," Lawrence Livermore Laboratory Report UCRL 52069 (1976).
6. D. V. Anderson, J. Breazeal, C. H. Finan and B. M. Johnston, "ABCXYZ: Vector Potential (A) and Magnetic Field (B) Code (C) for Cartesian (XYZ) Geometry using General Current Elements", Lawrence Livermore Laboratory Report UCRL 52029 (1976).

Abstracts

1. C. G. Tull, D. V. Anderson and J. H. Foote, "Equilibrium Adiabaticity and Confinement Time Studies for Minimum-B Mirror Devices", Bull. Am. Phys. Soc. 19 (1974), 921 (Albuquerque).
2. R. P. Freis, M. E. Rensink, J. Killeen and A. A. Mirin, "Mulflu, a 1-D Multifluid Code", Bull. Am. Phys. Soc. 19 (1974), 922 (Albuquerque).

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101-125	5.50	426-450	11.75
126-150	6.00	451-475	12.00
151-175	6.75	476-500	12.50
176-200	7.50	501-525	12.75
201-225	7.75	526-550	13.00
226-250	8.00	551-575	13.50
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276-300	9.25	601-up	*
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*Add \$2.50 for each additional 100 page increment from 601 to 1,000 pages;
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