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Suprathermal Electrons in Laser-Heated Plasmas**

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THEORETICAL RESULTS RELATING EXPERIMENTAL
X-RAY SPECTRA TO THE NUMBER AND TOTAL ENERGY OF
SUPRATHERMAL ELECTRONS IN LASER-HEATED PLASMAS

by

Michael A. Strosio

ABSTRACT

Formulae are derived which contain corrections to previous work and relate the x-ray spectrum produced from a laser-heated plasma to the number and total energy of suprathermal electrons in the plasma. These formulae, together with experimental x-ray spectra, have been used to determine the characteristics of the suprathermal electron distributions produced under differing experimental conditions. Specifically, the number and total energy of suprathermal electrons as well as the suprathermal electron temperature, akT_e , are related to the experimental x-ray data. It is found that the suprathermal electron temperature is nearly independent of wavelength when the incident laser energy is constant.

It is well known that the number and energy distribution of suprathermal electrons is of extreme importance in laser fusion. Therefore, it is highly desirable to have reliable experimental techniques to measure the suprathermal electron distributions. Considerable experimental progress has been made in measuring^{1,2,3} the x-ray spectra produced by laser-heated plasmas. Herein, it is shown that a knowledge of the experimental x-ray spectra (coupled with the theoretical relations derived in this paper) allows one to determine the character of suprathermal electrons.

Brueckner⁴ has derived formulae relating experimental x-ray spectra and the number and energy of suprathermal electrons based on the assumption that collisional drag between the thermal electrons and the suprathermal electrons uniformly dominates other energy losses. With this assumption, Ref. 4 shows that the energy radiated by a single electron

with initial energy ϵ_0 is *

$$\frac{d\epsilon_{\text{rad}}}{d(h\nu)} = \frac{\lambda_z}{2\pi c^2} \int_{h\nu}^{\epsilon_0} \ln \frac{\epsilon^{1/2} + (\epsilon - h\nu)^{1/2}}{\epsilon^{1/2} - (\epsilon - h\nu)^{1/2}} d\epsilon, \quad (1)$$

$$\text{where } \lambda_z = \frac{4}{3\pi} \frac{e^2}{\hbar c} \frac{\langle Z^2 \rangle}{\langle Z \rangle \ln \left(\left(\frac{3}{2e} \right) \left(\frac{kT_e^3}{\pi n_e} \right)^{1/2} \right)}$$

In Eq.(1), T_e is the temperature of the thermal electrons that accompany the suprathermal electrons.

* It has been noted by Donald F. DuBois that the Coulomb logarithm in λ_z contains the classical distance of closest approach (e^2/kT) as the impact parameter and that at high temperatures, the correct impact parameter is the thermal de Broglie wavelength $\lambda = h/(2mkT)^{1/2}$; thus, the argument of the logarithm $\lambda_D/(e^2/kT)$ should be replaced by λ_D/λ when $kT \gg \hbar \omega$.

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n_e is the density of electrons and Z is the nuclear charge of the material in question. In Ref. 4, the integrand of Eq. (1) is approximated by the constant 2.0 and it is shown that

$$\frac{d\epsilon_{\text{rad}}}{d(h\nu)} = \frac{1}{2} \frac{Z^2}{mc^2} (\epsilon_0 - h\nu). \quad (1a)$$

This approximation raises some concern since the integrand varies from zero to infinity over the range of integration as $h\nu/\epsilon_0$ goes to zero. Henderson and Strosio⁵ have performed the integration in Eq. (1) analytically. Upon changing variables to $x^2 = (1 - h\nu/\epsilon_0)$ and integrating by parts one finds the exact result.

$$\frac{d\epsilon_{\text{rad}}}{d(h\nu)} = \frac{1}{2} \frac{Z^2}{mc^2} x \left[\left(\epsilon_0 - \frac{h\nu}{2} \right) \ln \left[\frac{1 + (1 - \frac{h\nu}{\epsilon_0})^{1/2}}{1 - (1 - \frac{h\nu}{\epsilon_0})^{1/2}} \right] + \epsilon_0 (1 - \frac{h\nu}{\epsilon_0})^{1/2} \right] \quad (1b)$$

which behaves quite differently from Eq. (1a) as a function of $h\nu/\epsilon_0$. In particular, Eq. (1b) exhibits the normal logarithmic behavior as $h\nu/\epsilon_0 \rightarrow 0$, while Eq. (1a) approaches a constant.

The total energy radiated from a distribution of electrons $n(\epsilon_0)$ is

$$\frac{d\epsilon_{\text{rad}}}{d(h\nu)} = \int_{h\nu}^{\infty} n(\epsilon_0) \frac{d\epsilon_{\text{rad}}}{d(h\nu)} d\epsilon_0. \quad (2)$$

In Ref. 4 it is shown that substitution of Eq. (1a) into Eq. (2) allows one to derive a simple connection between the x-ray spectrum and the number and energy distribution of suprathermal electrons in a laser-heated plasma. These relations are derived by taking derivatives of Eq. (2) with respect to $h\nu$ when Eq. (1a) is taken as the kernel of Eq. (2). For the complicated kernel given by Eq. (1b), it is impossible to carry out this ingenious process of taking derivatives to obtain the simple relations referred to above.

To circumvent the difficulties involved in using the kernel given by Eq. (1b), one is led to search for approximate kernels which (1) are amenable to Brueckner's ingenious differentiation techniques and (2) contain the logarithmic term of Eq. (1b) which is of clear importance. In fact, one can show that any kernel containing only linear and logarithmic terms in $h\nu/\epsilon_0$ satisfies both of these conditions. A particularly simple kernel of this type, which approximates Eq. (1b) quite well (see Fig. 1), is given by

$$\frac{d\epsilon_{\text{rad}}}{d(h\nu)} = \frac{1}{2} \frac{Z^2}{mc^2} x \left[\frac{1}{5} (4 - h\nu/\epsilon_0) + \frac{1}{2} \left(\ln \left(\frac{4\epsilon_0}{h\nu} \right) + \ln(10) - 5 \right) \right] \quad (1c)$$

Fig. 1. Exact and approximate energy spectra.

Upon substituting Eq. (1c) into Eq. (2) and applying Brueckner's differentiation techniques, one finds

$$\frac{d^2 E_{\text{rad}}}{d(h\nu)^2} \bigg|_{\epsilon_b} = -\frac{\lambda^2}{8\pi c^2} N(\epsilon > \epsilon_b) - \frac{\lambda^2}{2\pi c^2 \epsilon_b} E(\epsilon > \epsilon_b), \quad (3a)$$

and

$$\frac{d^3 E_{\text{rad}}}{d(h\nu)^3} \bigg|_{\epsilon_b} = -\frac{\lambda^2}{\pi c^2} \left(\frac{7}{10} \right) n(\epsilon_b) + \frac{\lambda^2}{2\pi c^2 (\epsilon_b)^2} E(\epsilon > \epsilon_b), \quad (3b)$$

where

$$N(\epsilon > \epsilon_b) = \int_{\epsilon_b}^{\infty} n(\epsilon) d\epsilon, \quad (4a)$$

and

$$E(\epsilon > \epsilon_b) = \int_{\epsilon_b}^{\infty} \epsilon n(\epsilon) d\epsilon. \quad (4b)$$

Equations (4a) and (4b) represent, of course, the number and total energy of suprathermal electrons (with energies above ϵ_b), respectively. Equations (3a) and (3b) relate the experimentally measured x-ray spectrum $dE_{\text{rad}}/d(h\nu)$ to the suprathermal electron distribution $n(\epsilon)$, the number $N(\epsilon > \epsilon_b)$, and the total energy $E(\epsilon > \epsilon_b)$ of suprathermal electrons. By comparing Eqs. (3a) and (3b) with the analogous equations of Ref. 4, it is seen that the second term on the right-hand sides of both of these equations results from the logarithmic behavior of Eq. (1c) (which exactly represents Eq. (1b) as $h\nu/\epsilon_0 \rightarrow 0$).

As an application of these results, the suprathermal electron temperature will now be determined for a particular model of the suprathermal distribution. The exact distribution of suprathermal electrons is not well understood and is

likely to be very complex; however, a particularly simple model follows⁶:

1. The fraction f of the laser energy is absorbed into a suprathermal tail characterized by an energy distribution of the form

$$n(\epsilon) = B e^{-\epsilon/akT_e}, \quad (5)$$

where ϵ is the electron energy, a and B are constants, and kT_e is the thermal electron temperature.

2. The remaining fraction $1-f$ of the absorbed laser energy produces a nearly Maxwellian distribution.

The validity of this model can be questioned; indeed, the relations discussed herein provide a means of testing this model.

For the suprathermal electron distribution of Eq. (5), one finds that Eq. (3a) reduces to

$$\frac{d^2 E_{\text{rad}}}{d(h\nu)^2} \bigg|_{\epsilon_b} = -\frac{\lambda^2}{8\pi c^2} akT_e B e^{-\epsilon_b/akT_e} \left[\frac{7}{10} + \frac{akT_e}{2\epsilon_b} \right]. \quad (6)$$

From experimental values^{1,2,3} of $dE_{\text{rad}}/d(h\nu)$ as a function of $h\nu$, one may determine $d^2 E_{\text{rad}}/d\epsilon_a^2$ and $d^2 E_{\text{rad}}/d\epsilon_b^2$ for any two energies ϵ_a and ϵ_b in the experimentally covered range of energy. Thus, with Eq. (6) one finds,

$$\frac{(d^2 E_{\text{rad}}/d\epsilon_a^2)}{(d^2 E_{\text{rad}}/d\epsilon_b^2)} = e^{-(\epsilon_a - \epsilon_b)/akT_e} \frac{(\frac{7}{10} \epsilon_a + akT_e)\epsilon_b}{(\frac{7}{10} \epsilon_b + akT_e)\epsilon_a}, \quad (7)$$

which is a transcendental equation in akT_e — the suprathermal electron temperature.

TABLE I
CALCULATED VALUES OF THE SUPRATHERMAL ELECTRON TEMPERATURE
FOR SEVERAL DIFFERENT EXPERIMENTALLY MEASURED X-RAY SPECTRA

Wavelength (μm)	Peak Intensity (W/cm^2)	Pulse Width	Pulse Energy (J)	Source of Data	Calculated αkT_e (keV)	Average αkT_e (keV)
10.6 (CO_2)	$\sim 10^{14}$	1.2-1.5 ns	7.4	Reference 1	7.0	7.5
10.6	$\sim 10^{14}$	1.2-1.5 ns	7.8	Reference 1	9.0	
10.6	$\sim 10^{14}$	1.2-1.5 ns	8.5	Reference 1	10.0	
10.6	$\sim 10^{14}$	~ 1.2 ns	9.2	Reference 3	5.0	
10.6	$\sim 10^{14}$	~ 1.2 ns	9.2	Reference 3	8.5	
10.6	$\sim 10^{14}$	~ 1.2 ns	9.0	Reference 3	5.5	
1.06 (Nd)	$\sim 10^{16}$	~ 25 ps	9.2	Reference 1	6.5	6.6
1.06	$\sim 10^{16}$	~ 25 ps	10.4	Reference 1	4.2	
1.06	$> 10^{16}$	21 ± 2 ps	~ 10.0	Reference 2	9.0	

Equation (7) has been solved numerically for several different experimental measurements^{1,2,3} of $dE_{\text{rad}}/d(h\nu)$. These solutions give the values of the suprathermal electron temperature αkT_e for each experiment. Table I summarizes the various suprathermal temperatures αkT_e obtained for the x-ray spectra of Refs. 1-3. From the indicated values, it is apparent that the suprathermal temperatures for 10.6- μm and 1.06- μm lasers are nearly the same for pulses with total energies of approximately 9.0 joules. This result is not surprising in view of recent theoretical progress.⁷

To elicit the various values of α from these suprathermal temperatures, it is necessary to estimate the thermal temperatures; thus, a numerical simulation of each experiment was performed using the laser-absorption hydrodynamics code LASNEX.⁸ The simulations, without suprathermal electrons present, suggest that $(kT_e)_{10.6\mu\text{m}} \approx 0.1$ keV and $(kT_e)_{1.06\mu\text{m}} \approx 0.2$ keV near the region of suprathermal production for the experiments in question. From these estimates and the suprathermal temperatures of Table I, it appears that $\alpha_{10.6} \approx 75$ and $\alpha_{1.06} \approx 33$; however, the fluctuation in the values of αkT_e for each laser and the uncertainty associat-

ed with the numerical simulations of the thermal temperatures indicate that these values of α are correct only within a factor of 2.0.

It is clear that Eqs. (3a) and (3b) yield general information about suprathermal electrons when used in conjunction with measured x-ray spectra. The particular applications given here were to determine the suprathermal temperatures and the values of α ; however, one may also use these equations to gain information related to other quantities. For example, Eq. (3b), in conjunction with experimentally measured x-ray spectra and measurements of $E(\epsilon > \epsilon_b)$, yields the suprathermal electron distribution $n(\epsilon_b)$ for all values of ϵ_b at which the experiments are performed. In summary, three results are presented herein:

- (1) general formulae relating x-ray spectra to the suprathermal electron distribution,
- (2) the suprathermal electron temperatures for 10.6 μm and 1.06 μm at a nearly constant laser energy and
- (3) approximate values of α for 10.6- μm and 1.06- μm lasers. While the last two results are subject to differing interpretations depending on the model used for the suprathermal electron distribution, it is clear that the first result is of general applicability.

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