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WATER RESOURCES RESEARCH PROGRAM

SURFACE THERMAL PLUMES: EVALUATION OF MATHEMATICAL MODELS FOR THE NEAR AND COMPLETE FIELD

by

William E. Dunn, Anthony J. Policastro,
and Robert A. Paddock

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Energy and Environmental Systems Division
Great Lakes Project

August 1975

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FOREWORD

A major objective of the Great Lakes Research Program for the past few years has been to determine which mathematical models, if any, could be used to predict with reasonable reliability the characteristics of thermal plumes from surface outfalls. The present report is the final one in a series on the subject and summarizes our attempts to test the usefulness of existing mathematical models by comparing model predictions with laboratory and field data. In conducting these comparisons, we had to examine many aspects of hydrothermal prediction. These included an in-depth study of the formulation of the models being tested and a close examination of the physical processes being modeled. Comparisons of the model predictions to experimental data were the major ingredient in our evaluation of a given model. At the same time, however, our evaluation of the formulation of each model was an integral part of our judgment as to its competitiveness. The present effort to evaluate models updates previously published works in this research area, such as those of Policastro and Tokar (1972); and Benedict, Anderson, and Yandell (1974). Furthermore, we feel that this report presents a viewpoint not available in other review documents, such as those of Brady, Edinger, and Geyer (1974); and Jirka, Abraham, and Harleman (1975). This report advances the state of the art by its evaluation of existing models, its discussion of important modeling issues, and in its suggestions for further modeling improvements.

Because all the models described here were developed by persons outside Argonne National Laboratory, frequent consultation with model authors was necessary to resolve questions. We are grateful to all authors who gave generously of their time and knowledge through conversations and personal correspondence. Without their cooperation a report such as this would have been impossible. Among those researchers who provided valuable assistance were:

1. Dr. Barry Benedict, now at Tulane University, on the Motz-Benedict Model (in Sec. II.B).
2. Dr. Keith Stolzenbach and Mr. Eric Adams of the Massachusetts Institute of Technology on the Stolzenbach-Harleman Model (in Secs. II.C and IV.D).
3. Dr. Edmund Prych now with the U. S. Geological Survey on his own model (in Sec. II.D).
4. Dr. Mostafa Shirazi of the Pacific Northwest Environmental Research Laboratory of the U. S. EPA and Dr. Lorin Davis of Oregon State University on the Prych and Shirazi-Davis Models (in Sec. II.E) and Shirazi's data analysis (Sec. IV.B).
5. Dr. Donald Pritchard of The Johns Hopkins University on the two Pritchard Models (in Secs. II.F and II.G).
6. Dr. John Edinger of Edinger Associates Inc. on the Brady-Geyer Model (in Sec. II.H).

7. Dr. John Paul, now at the U. S. EPA Grosse Ile Laboratory, and Dr. Wilbert Lick of Case Western Reserve University on the Paul-Lick Model (in Sec. II.K).

8. Dr. William Waldrop (now with the Tennessee Valley Authority) and Dr. Richard Farmer of Louisiana State University on the Waldrop-Farmer Model (in Sec. II.L).

9. Mr. James Taft, Dr. Walt England (both now at Science Applications, Inc.), and Dr. John Pritchett of Systems, Science, and Software on the Pritchett-England-Taft Model (in Sec. II.I).

10. Dr. Henry Till of the Electric Power Research Institute on his own model (in Sec. II.J).

11. Dr. Frank Fang of the Virginia Institute of Marine Science on the Surry Plant field surveys (Sec. IV.B).

12. Dr. James Weil now with the Coppola Co. of San Francisco on his work verifying the Prych model at Oskarshamnsverket (in Chapter IV, Sec. E).

13. Mr. Jim Davis of the Boston Edison Company and Mr. Dan Frye of EG&G Environmental Consultants for their discussions with us on the Pilgrim Plant studies (in Sec. IV.D).

14. Dr. Heinz Stefan of the St. Anthony Falls Hydraulic Laboratory for his helpful review comments on Chapter III.

The fact that the model authors mentioned above reviewed our discussion of their work before publication should not be construed as agreement by them with the opinions expressed nor as an endorsement of the presentation.

The length of the present report necessitated its publication in two parts. The titles of the chapters contained in each part are as follows:

Part One

- I. Physical Aspects and Mathematical Modeling of Surface Thermal Plumes
- II. Summary and Evaluation of Individual Models

Part Two

- III. Discussion of Near- and Complete-field Modeling Issues
- IV. Verification Efforts for Surface-discharge Models
- V. Observations on the State of the Art

Appendixes

The two parts of the report do not stand alone, and the reader is urged to acquire both parts because they are often cross-referenced.

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WATER RESOURCES RESEARCH PROGRAM
SURFACE THERMAL PLUMES: EVALUATION OF
MATHEMATICAL MODELS FOR THE NEAR AND COMPLETE FIELD

by

William E. Dunn, Anthony J. Policastro,
and Robert A. Paddock

ABSTRACT

This report evaluates mathematical models that may be used to predict the flow and temperature distributions resulting from heated surface discharges from power-plant outfalls. Part One discusses the basic physics of surface-plume dispersion and provides a critical review of 11 of the most popular and promising plume models developed to predict the near- and complete-field plume. Part Two compares predictions from the models to prototype data, laboratory data, or both. Part Two also provides a generic discussion of the issues surrounding near- and complete-field modeling.

The principal conclusion of the report is that the available models, in their present stage of development, may be used to give only general estimates of plume characteristics; precise predictions are not currently possible. The Shirazi-Davis and Pritchard (No. 1) models appear superior to the others tested and are capable of correctly predicting general plume characteristics. (The predictions show roughly factor-of-two accuracy in centerline distance to a given isotherm, factor-of-two accuracy in plume width, and factor-of-five accuracy in isotherm areas.).

The state of the art can best be improved by pursuing basic laboratory studies of plume dispersion along with further development of numerical-modeling techniques.

III. CRITICAL DISCUSSION OF NEAR- AND COMPLETE-FIELD ISSUES

A. Introduction

This chapter considers many diverse issues relating to the modeling of surface discharges in the near and complete fields. Many of the issues are now, or have been, points of controversy or confusion in the literature. This discussion brings out potential areas of concern to the thermal-plume investigator and illustrates the often overly simplified viewpoint adopted by model developers. The issues are grouped into four sections. In Sec. III.B, we discuss the philosophy of near-field modeling and the often-implicit assumptions associated with the near-field approximation. Section III.C is a general discussion of plume physics which attempts to show the large body of experimental observation that often cannot be accounted for within the narrow conceptualizations of the plume commonly advanced. Sections III.D and E treat the integral and differential-numerical methods, respectively. In these last two sections, we identify important difficulties with each of these modeling techniques. This chapter covers most of the areas of current interest and identifies which of these areas merit further study.

B. The Near-field Approximation versus the Complete-field Analysis

During the past several years, the focus of attention has been on near-field modeling. This emphasis is the result of several factors. One is that, because plume velocities and temperatures deviate the most from ambient in the near field, it is the region of greatest biological concern. Another factor is the widely held belief that near-field mixing largely determines the size and character of the measurable plume, almost independently of the environment. Yet a third consideration is the relatively greater ease with which the near field can be studied in the laboratory. Far-field processes such as turbulent diffusion and surface heat loss are not easily scaled. Nevertheless, the study of prototype cases has revealed that the use of near-field models alone is probably inadequate. Prototype data taken under apparently identical near-field conditions often show order-of-magnitude differences in plume characteristics. Unfortunately, the present state of plume modeling does not permit an adequate treatment of the complete-field problem.

Our discussion of this important issue begins with a brief look at two of the basic assumptions upon which near-field models are based. This is followed by a general description of far- and complete-field modeling techniques. The value of far-field models lies solely in their combination with near-field models for a complete-field analysis, since purely far-field models alone are too crude an approximation to be of much use in treating far- or complete-field thermal-plume dispersion without some consideration of near-field mixing.

First, two minor points warrant comment. One is the difference in length and time scales between the near and far fields. On the one hand, the near field is characterized by the small length and time scales associated with jet turbulence, entrainment, and advection. On the other hand, the far field is characterized by large scales corresponding to surface heat loss, buoyant spreading, ambient turbulence, and gross circulations. In a sense, the governing equations are stiff in the spatial variables. One seeks computational leverage from this stiffness. The difference in scales can be handled naturally in a numerical model, through the use of a variable grid spacing. A fine horizontal grid is used in the near field, where detail is important, and a coarser grid is used in the far field, where greater lateral coverage is needed. In contrast, separate treatments for the near and far fields are generally required for the integral and phenomenological approaches, due to the restrictive idealizations involved with these models. In some cases, the separation of near and far fields gets muddled. For example, in a near-field phenomenological model, certain far-field effects may be present in the near-field data used for model development if prototype data are used. If, on the other hand, laboratory data alone are used, the model will probably not reflect any far-field influences.

The second point to be made here is that there is a somewhat arbitrary distinction between near- and complete-field models. The precise location at

which a near-field model is no longer applicable is always unknown, and an arbitrary choice must be made. Numerous near-field models are often used improperly in the far field far beyond the point where the assumptions of the models have ceased to be valid. A similar problem relates to the use of numerical models. These models are, in principle, complete-field models, if their formulation contains sufficient far-field physics. Yet, in practice, the completeness of many numerical models suffers in their application. If the computational grid is too small, if turbulent diffusivities are improperly selected in the far field, if boundary conditions are misapplied, or if surface heat loss is inadequately handled, then the model cannot perform well in the far field. Thus, although the labels near and complete field are useful for descriptive purposes, they are not hard-and-fast distinctions.

1. The Near-field Approximation

The use of near-field models for thermal-plume problems (integral and phenomenological) is based on a number of often-implicit assumptions, of which many are open to question. One is that heat is removed from the near field at the same rate at which it enters and, thus, a steady plume is possible. Moreover, heat that leaves the near field is assumed to be absorbed by the far field and never returned. This is not always true in prototype applications.

Removal of heat from the near field is a key consideration in using steady, near-field models, since a steady plume cannot exist unless heat leaves the system at the same rate at which it enters. In the picture commonly advanced, heat is mechanically diluted in the near field by jet turbulence and entrainment, advected out of the near field by the jet flow, and finally either lost to the atmosphere through surface heat transfer or carried away by the ambient current. Our experience indicates that such a picture is grossly oversimplified and is, perhaps, a significant reason why existing models have fared so poorly in prototype testing. Confining geometry, low surface heat loss and a stagnant ambient can prevent heat from being dissipated in the far field. Heat accumulating there may return to the near field through buoyant spreading, through circulation set up by the entrainment demands of the near-field plume, or through a current reversal.

A prime example of bottom and lateral boundary interference is provided by the once-through Diablo Canyon Plant discharge into Diablo Cove off the California coast. Entrainment of ambient water is sufficiently cut off by the geometry of the cove to invalidate the application of the usual near-field models. Tidal variations further complicate the situation. Problems involved in applying existing analytical models to the Diablo Canyon case led eventually to the use of a hydraulic model.

Highly stagnant ambients can be found at a number of sites, and, even at many sites where an ambient current exists, reversals are common.

When a reversal occurs, dilution is significantly reduced, since entrainment water is at a higher-than-ambient temperature. In his report on the development of a transient, two-dimensional, far-field model that can be used to predict the effects of current reversals, Eraslan¹ concludes that, for discharges into large ambient receiving bodies,

"a near-field model by itself is definitely inadequate and unreliable for the assessment of the thermal impact of a power plant unless it is preceded or supplemented by an analysis which can account for the effect of the discharge on the ambient receiving body of water."

Thus, there are several cases of practical interest where heat either remains in or returns to the near field, invalidating the near-field approximation.

An important difference exists between the mechanisms of heat removal in a laboratory model and the prototype situation. In a flow-through hydraulic model, most of the heat flows completely out of the system through the end or side walls of the tank. In a stagnant tank, on the other hand, heat builds up in the tank until the entire tank temperature approaches the discharge temperature. Surface heat loss is typically much smaller in laboratory models than in the field.

A second key assumption closely related to the first is that plume characteristics are solely determined by discharge parameters such as the initial densimetric Froude number and other factors such as a steady, uniform crossflow. This assumption is often violated in prototype plumes. The study of prototype discharges has shown that outfall parameters do not alone control the size and character of the discernible plume. Not surprisingly, models that do not consider key environmental effects have been generally inadequate. An excellent example, here, is the Pilgrim Nuclear Power Station. The initial densimetric Froude number varies from less than 2 to more than 10 as the tide ebbs, yet no statistically significant difference in high- and low-tide plumes has been observed. This probably is because wind- and tidally-induced shear currents and bottom interference are controlling factors at the site. Popular models that do not consider such effects cannot be expected to perform well, and, in fact, they have not. At most sites where model/data comparisons have been made, environmental conditions appear to affect plume behavior as much as, if not more than, discharge parameters. The Pilgrim site and its attendant difficulties are fully discussed in Sec. IV.D.

A subtle yet important consequence of the near-field approximation is a change in the mathematical character of the problem. The governing equations, in their most general form, are spatially elliptic. Implementation of the near-field approximation (especially in the integral approach) makes the problem spatially parabolic, i.e., changes it to an initial-value problem, beginning at the outfall and extending outward into the receiving body. An

initial-value problem is easier to solve, but the gain in mathematical tractability comes at the expense of lost plume physics. In this parabolic form, the equations can never take into account changes in the ambient and the consequent feedback on the plume. A parabolic model determines downstream conditions solely from previously calculated upstream parameters with no reentrainment or feedback effects possible. Of the various modeling techniques, the integral approach relies the most heavily on the validity of the near-field approximation. Although fix-ups can be used to account for far-field effects in the same way other physical mechanisms (buoyancy and ambient currents) are treated, the basic elliptic nature of the problem can never be handled by the integral technique. The phenomenological and numerical models often make use of the near-field approximation in some form, although it is conceptually possible to retain the elliptic nature of the problem.

2. The Complete-field Analysis

a. Far-field Modeling Techniques

For the purpose of discussion, let us consider the far-field as being semi-infinite in the offshore direction, infinite in the shore-parallel direction, and of constant depth H . Further, let us define a Cartesian coordinate system for the far field in which x represents the offshore coordinate, y the shore-parallel coordinate, and z the vertical coordinate. Let positive z be downward and positive y be such that x, y, z is a right-handed system. Placement of the origin is arbitrary, but let us suppose it to be at some convenient point representing the location of the discharge.

The three basic physical processes generally acknowledged as governing dispersion in the far field are advection by ambient currents, turbulent diffusion, and surface heat exchange. These processes can be embodied into a single expression for θ , the local temperature excess, and the attendant boundary conditions. Thus,

$$\begin{aligned} \frac{\partial \theta}{\partial t} + \frac{\partial}{\partial x}(u\theta) + \frac{\partial}{\partial y}(v\theta) + \frac{\partial}{\partial z}(w\theta) = & \frac{\partial}{\partial x}\left(D_x \frac{\partial \theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial \theta}{\partial y}\right) \\ & + \frac{\partial}{\partial z}\left(D_z \frac{\partial \theta}{\partial z}\right) + S(x, y, z, t), \end{aligned} \quad (1)$$

$$D_z \frac{\partial \theta}{\partial z} = \frac{K}{\rho c_p} \theta \text{ at } z = 0, \quad (2)$$

$$D_z \frac{\partial \theta}{\partial z} = 0 \text{ at } z = H, \quad (3)$$

$$\theta; \frac{\partial \theta}{\partial x} \rightarrow 0 \text{ as } x \rightarrow \infty, \quad (4)$$

$$\theta; \frac{\partial \theta}{\partial y} \rightarrow 0 \text{ as } y \rightarrow \pm \infty, \quad (5)$$

and

$$\theta = \theta_0(x, y, z) \text{ at } t = 0, \quad (6)$$

where S is a known heat source and the other variables assume their customary definitions.

The same equation is applicable to the dispersion of any passive tracer with one minor addition. If the tracer decays exponentially with time (as is the case with some fluorescent dyes), then θ/τ (where τ is the decay time) must be subtracted from the right-hand side of Eq. 1.

This single linear equation is considerably easier to solve than the set of several linear and nonlinear equations typical of near-field models. Of course, the proper choice of coefficients (e.g., u , D_x , etc.) for the far-field model is a key difficulty in obtaining adequate predictions. To reduce complexity even further, most investigators eliminate at least one spatial dimension and, in some instances, time. For example, the two-dimensional, depth-averaged, far-field dispersion equation can be shown to be

$$\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial}{\partial x}(\bar{u}\bar{\theta}) + \frac{\partial}{\partial y}(\bar{v}\bar{\theta}) = \frac{\partial}{\partial x}\left(\bar{D}_x \frac{\partial \bar{\theta}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\bar{D}_y \frac{\partial \bar{\theta}}{\partial y}\right) - \frac{K}{\rho c_p H} \theta_s + \bar{S}, \quad (7)$$

where

$$\bar{\theta} = \frac{1}{H} \int_0^H \theta \, dz, \quad (8)$$

$$\bar{u} = \frac{1}{\bar{\theta}} \int_0^H u \theta \, dz, \quad (9)$$

etc.

Since the dispersion equation is linear, a solution for an arbitrary source can be written using a Green's function. As a simple example, consider the depth-averaged case and suppose that the coefficients are constant and that θ_s is proportional to $\bar{\theta}$. With these assumptions, Eq. 7 becomes

$$\frac{\partial \bar{\theta}}{\partial t} + \bar{u} \frac{\partial \bar{\theta}}{\partial x} + \bar{v} \frac{\partial \bar{\theta}}{\partial y} = \bar{D}_x \frac{\partial^2 \bar{\theta}}{\partial x^2} + \bar{D}_y \frac{\partial^2 \bar{\theta}}{\partial y^2} - k \bar{\theta} + \bar{S}. \quad (10)$$

With the introduction of transformed variables

$$\alpha = \exp(kt)\bar{\theta}, \quad (11)$$

$$X = (x - \bar{u}t)/\bar{D}_x^{1/2}, \quad (12)$$

and

$$Y = (y - \bar{v}t)/\bar{D}_y^{1/2}, \quad (13)$$

Eq. 10 becomes

$$\frac{\partial \alpha}{\partial t} = \frac{\partial^2 \alpha}{\partial X^2} + \frac{\partial^2 \alpha}{\partial Y^2} + S. \quad (14)$$

This is the heat-conduction equation for which we have the well-known Green's function

$$G(X, Y, t; X_0, Y_0, t_0) = \frac{1}{4\pi(t - t_0)} \exp\left[-\frac{(X - X_0)^2 + (Y - Y_0)^2}{4(t - t_0)}\right]. \quad (15)$$

The solution for $\bar{\theta}$ can now be written as

$$\bar{\theta}(x, y, t) = \exp(-kt) \int_0^t \int_0^\infty \int_{-\infty}^\infty G(x, y, t; x_0, y_0, t_0) S(x_0, y_0, t_0) dx_0 dy_0 dt_0. \quad (16)$$

Examples of far-field models based on the analytic solution of the advection/diffusion heat equation are those of Van Dam,^{2,3} Christodoulou et al.,⁴ Koh and Fan,⁵ Koh and Chang,⁶ Ahn and Smith,⁷ and Adams et al.⁸

In the general case, an analytic solution is too cumbersome, so some form of numerical solution must be used. Any of the techniques applicable to linear equations is suitable. For example, Leendertse,⁹ Kolesar and Sonnichsen,¹⁰ Ahn and Smith,⁹ Abbott et al.,¹¹ Tsai and Chang,¹² Giles et al.,¹³ Wu and Gallagher,¹⁴ and Boericke and Hall¹⁵ have developed finite-difference solutions for the far field.

Eraslan¹ formulated a far-field model using the so-called discrete-clement technique. Finite-element solutions of the far-field heat-transport equation are given by Leimkuhler¹⁶ and Loziuk et al.^{17,18} Ahn and Smith⁷ describe a Monte Carlo solution of the two-dimensional, depth-averaged heat-dispersion equation. With this method, far-field dispersion is modeled through a computer-simulated release of heat-containing particles. Time is first divided into discrete increments. Then at each time step, a large number of particles, each containing a small fraction of the total heat produced during the increment, are released to simulate the real instantaneous heat

source. Each particle is tracked by the computer until it leaves the finite solution domain. Particle movements at each time step are determined as follows: First, the particle is displaced deterministically according to the local velocity assumed for the ambient. Turbulent diffusion is then included by adding a second displacement that is a Gaussian random variable of zero mean, and variance determined by the local diffusivity. Finally, the "excess temperature" of the particle is reduced to account for surface heat loss. Once all the particles have been advanced to their new positions and temperatures, the local excess temperature is calculated by averaging particle excess temperature over a suitable sample volume. Policastro and Tokar¹⁹ discussed several individual far-field models in detail. Other techniques applicable to linear equations, such as response matrix and synthesis models, are potentially usable, but, to our knowledge, no model does so.

Our opinion is that far-field models that lack a proper simulation of near-field mixing are of little or no value for environmental-impact assessment. The primary value of these models lies in their ability to predict the temperature of near-field entrainment water and thus improve near-field calculations. Major difficulties with this approach arise in devising an imaginative scheme for coupling two largely different types of models. The techniques for and difficulties associated with coupling near- and far-field models are the subject of Sec. b. below.

b. Complete-field Modeling Techniques

This section considers only those models with different treatments for the near and far fields. Furthermore, this discussion is kept on a fairly superficial level to avoid the detail of each individual model, each with its separate parts. Single complete-field models, as are conceptually possible with the recently developed numerical schemes, are not discussed in this section. The primary difficulties in applying these models to the complete field are grid resolution and the proper selection of diffusion coefficients for both near and far fields. These matters are addressed in Sec. III.E.

When separate near- and far-field models are used, the coupling may be carried out iteratively. Here, the near-field model is applied first, assuming an unperturbed ambient. Then the far-field model is applied using the predictions of the near-field model as a source term in the advection/diffusion heat equation. The far-field model provides an estimate of the temperature of the near-field entrainment water that is used in the next calculation of the near-field model. Ostensibly, this procedure is continued until convergence is achieved, but in actuality, only one or two iterative steps are made. With the current state of model development, the need for converging the iteration is rather unimportant compared with the difficulties in obtaining adequate near- and far-field models and a satisfactory coupling scheme.

Pritchard²⁰ uses a background temperature excess to include far-field influences in his phenomenological models. This background temperature presumably accounts for the circulation of far-field plume water into the near field. No specific method is given by Pritchard to calculate this re-entrainment temperature, however. (Reentrainment is discussed further in Sec. III.C.)

Sundaram et al.²¹ developed a closed-form integral model in which they match near- and far-field solutions at the point where the eddy diffusivity associated with near-field (jet) turbulence becomes equal to that associated with far-field (ambient) turbulence. They then fit the model to data from the Milliken Station on Lake Cayuga in New York. Giles et al.¹³ combined a closed-form integral model for the near-field with a three-dimensional, finite-difference solution of the advection/diffusion heat equation in the far field. Loziuk et al.¹⁷ used a two-dimensional integral model to obtain the flow field and then executed a two-dimensional, finite-element solution of the heat-energy equation. Wu and Gallagher¹⁴ added a three-dimensional, finite-difference solution of the heat-energy equation to the Prych model in an effort to assess far-field influences. Recently, Adams et al.⁸ coupled a three-dimensional, integral model (similar to that of Stolzenbach and Harleman) to a two-dimensional solution of the heat-energy equation based on the Green's-function method. None of these models have been sufficiently tested for us to comment on their adequacy for complete-field analysis.

Koh and Fan⁵ made a significant effort to model the interaction of the near and far fields for heated surface discharges. They coupled their integral models for the near field to an analytical-solution-based model of the far field. This analysis considers only longitudinal and vertical coordinates, making it useless for impact assessment. Yet the models do give insight into the nature of far-field backwash on the near field and the associated possible breakdown of the jet structure. Laboratory studies of this phenomenon are described in Sec. III.C.

Koh and Fan designate four separate flow regimes within the plume: Zone I, an initial region of flow development; Zone II, a jet regime; Zone III, a transition region between Zones II and IV; and Zone IV, a stratified flow regime. The four zones are shown schematically in Fig. III.B.1. The typical discharge is visualized as follows: The plume is initially a relatively high densimetric Froude number jet with a region of flow establishment (Zone I) and a region of established flow (Zone II). As a result of vertical entrainment, momentum is diluted more rapidly than heat, decreasing the

Fig. III.B.1. Schematic View of the Four Flow Regimes Considered by the Koh-Fan Model. (Adapted from Koh and Fan⁵.)

local densimetric Froude number along the jet centerline, until a critical value is reached. A hydraulic jump (Zone III) then occurs, and the flow, thereafter, resembles a stratified two-layer flow (Zone IV). Large surface heat loss and ambient mixing can destroy Zones III and IV. Conversely, if the initial densimetric Froude number is low and surface heat loss and ambient mixing are sufficiently small, then Zone IV may inundate the outfall.

Table III.B.1 summarizes the essential features of the Koh-Fan model development. An intuitive understanding of the model can be gotten from Fig. III.B.2. Here, consider the flow as it enters the receiving system. To overcome interfacial shear and sustain forward flow, the interface must maintain a slight positive slope; therefore it intersects the surface at some location X. As the volume of water in the plume grows due to continued efflux at the outfall, the maximum plume thickness and X will increase as long as vertical entrainment can produce such an increase. When the limit of vertical entrainment is reached, a hydraulic jump must occur (according to this conceptualization). The jump (Zone III) divides the plume into a jet (Zone II) and a stratified flow (Zone IV). If low surface heat loss and poor ambient mixing allow heat to accumulate in Zone IV, then the jump can be forced all the way back to the outfall, resulting in an inundated source.

The key nondimensional parameters in this analysis are:

Initial Densimetric Froude Number

$$F_o = \rho_o U_o^2 / (2\alpha_1 \Delta \rho_o h_o) \quad (17)$$

Reynolds Number

$$R = h_o U_o / (\epsilon \alpha_2) \quad (18)$$

Heat-loss Parameter

$$k = \alpha_2 K / (\rho_o c_p U_o) \quad (19)$$

where

ρ_o = discharge density,

U_o = discharge velocity,

h_o = discharge depth,

$\Delta \rho_o$ = discharge density disparity,

ϵ = effective viscosity,

$\alpha, \alpha_1, \alpha_2$ = constants dependent on the similarity profile used,

K = surface-heat-loss coefficient,

and

c_p = specific heat.

TABLE III.B.1. Koh-Fan Model for Two-dimensional Surface Warm Jets

Basic Assumptions

- a) Velocity and density profiles in the vertical direction are similar.
 b) Steady-state solution exists.
 c) Boussinesq approximation may be made: density differences are only important in the gravity term of the vertical equation of motion.
 d) Flow is primarily horizontal (boundary-layer assumption).

Conservation Equations

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

continuity

$$\frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial z}(uw) = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial z}$$

x momentum

$$0 = \frac{\partial p}{\partial z} + \rho g$$

z momentum

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial z}(\rho w) = \frac{\partial}{\partial z}\left(D \frac{\partial \rho}{\partial z}\right)$$

density deficiency

where

x, z = horizontal and vertical coordinates

u, w = velocity components in the horizontal and vertical directions

p = pressure

 τ = kinematic shear stress (shear stress/density) ρ, ρ_0 = density, ambient density

D = eddy thermal diffusivity in vertical direction

Further Conditions and Assumptions

- a) Similarity of velocity and density distribution

$$u(x, z) = U(x) f\left(\frac{z - \eta}{h}\right)$$

$$\rho(x, z) = \rho_c - T(x) f\left(\frac{z - \eta}{h}\right)$$

where

 $\eta(x)$ = free-surface elevation $h(x)$ = characteristic thickness of the warm-water layer

$$f(\xi) = \text{shape function} = \begin{cases} 1 & |\xi| < 1 \\ 0 & |\xi| > 1 \end{cases}$$

Further Conditions and Assumptions (contd.)

- b) Kinematic surface boundary condition

$$\frac{D}{Dt}(z - \eta) = 0 \text{ at } z = \eta(x)$$

- c) Vertical-entrainment assumption

$$w(-\infty) = eU$$

where

$$e = e(Ri) = \begin{cases} e_0 \left[\frac{2}{1 + \frac{Ri}{(Ri)_{cr}}} - 1 \right]^n, & Ri < (Ri)_{cr} \\ 0 & , Ri > (Ri)_{cr} \end{cases}$$

for

$$Ri = \text{Richardson number} = \frac{gTh}{\rho_0 U^2}$$

 $(Ri)_{cr}$ = critical Richardson number (≈ 0.85), beyond which $e = 0$ n = exponent ($\approx 7/4$) e_0 = constant (≈ 0.075)

- d) Assumptions on shear

$$T_i \text{ (interfacial)} = \epsilon \frac{U}{h}, (\epsilon, \text{constant})$$

$$T_s \text{ (surface)} = 0$$

- e) Surface heat loss =
- $-KT$

(K, constant)

- f) Initial conditions:
- $U = U_0, h = h_0, T = T_0$
- at
- $x = 0$
- .

Fig. III.B.2. Jet Structure as Conceptualized by the Koh-Fan Model. (Adapted from Koh and Fan.⁵)

Note that these parameters deviate somewhat from the customary definitions. First, the constants are usually excluded, and second, the densimetric Froude number is the square of the usual one. Despite these differences, the physical interpretation is roughly the same:

F_0 is a measure of inertial to buoyant terms, R indicates the importance of interfacial shear, and k measures the rate of surface heat loss.

Koh and Fan discovered that the type of solution predicted by their analysis depends on F_0 and the product of k and R . The solution types expected for various combinations of these parameters are shown in Fig. III.B.3. These results are based on empirical observations made by Koh and Fan from many different computer outputs. Interestingly, the same results can be deduced from the equations themselves. For example, it can be shown that the equation for the line separating pure jet flow from jet flow with a hydraulic jump is

$$kR = 2 \frac{2F_{CR} + 1}{2F_0 + 1} \left(\frac{F_0}{F_{CR}} \right)^{1/3} \approx \frac{6.35F_0^{1/3}}{2F_0 + 1}, \quad (20)$$

where $F_{CR} = 1/0.85$ is the critical densimetric Froude number. These results indicate that backwash effects decrease with (1) increasing initial densimetric Froude number, (2) decreasing interfacial shear, and (3) increasing surface heat loss. These trends are certainly plausible.

Fig. III.B.3

Types of Solutions Predicted by the Koh-Fan Model. (Adapted from Koh and Fan.⁵)

In our opinion, the problem of the complete-field analysis is important and should be given priority consideration in future work. None of

the models mentioned in this section have been adequately tested. Consequently, no endorsements or specific indictments are given. The following observations are warranted, however.

(1) Complete-field models that use closed-form or integral-type models for the near field can be expected to suffer from many of the same difficulties as do their near-field models alone.

(2) In general, no existing model adequately treats the question of the intermediate field. The intermediate-field question would most adequately be handled either by a phenomenological model that is based upon complete-field prototype data or better by a numerical model that had its turbulence parameters "properly" evaluated in both near and far fields.

(3) At the present stage of development, no method for coupling near- and far-field models has been shown clearly superior to any other. Purely far-field models are of little or no value without a proper treatment of near-field mixing.

(4) Sophisticated numerical schemes offer the potential of a single complete-field model and thus avoid the difficulties in coupling separate near- and far-field models.

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C. Physical Aspects of Thermal Plume Dispersion

This section discusses several physical aspects of surface thermal plumes that are important to analytical models. Some are well known and widely studied. Others are more obscure, but their impact on modeling may be as profound as the more generally acknowledged physical aspects. The order of discussion is arbitrary; no ranking of importance is implied.

1. Ambient Current and Wind Effects

All natural bodies of water possess a current structure that depends on many meteorological, hydrological, and topographical factors. Near-shore currents in lakes are generally believed to be wind-induced. Wind-induced currents depend not only on the immediate wind characteristics, but also on the wind history of the water body. The effects of storms, for example, are known to linger for several days.

The most commonly used formula for calculating wind stress is

$$\tau_s = \frac{1}{2} C_f \rho_{\text{air}} U_w^2, \quad (1)$$

where

τ_s = shear stress at the water surface,

C_f = friction coefficient,

ρ_{air} = air density,

and

U_w = wind velocity at some short distance above the water surface.

The friction coefficient, C_f , is actually a function of the vertical wind-velocity distribution, which in turn is dependent upon local land topography (for a shoreline discharge), the type of lake-surface disturbances (waves, ripples, etc.), and the fetch of the wind. Some investigations drew an analogy to uniform flow over a flat plate and use the Blasius skin friction coefficient. The above wind-stress formula and analogy assume the lake is smooth and the wind velocity is steady.

The prevailing current in lakes is generally shore-parallel, although periods of upwelling or downwelling can be brought on by strong off-shore or onshore winds, respectively. Nearshore currents in lakes are complex, even when upwelling and downwelling are absent. Variations in all spatial directions and time are possible. The bottom topography can strongly influence nearshore currents.

In coastal areas, the current is generally made up of a combination of wind-induced surface currents and tidally controlled subsurface currents.

Large shear currents can result. Bottom topography can also be an important factor in determining nearshore currents in coastal waters. Coastal currents such as those of lakes can exhibit large changes with time.

The current in rivers is largely controlled by gravity and seasonal flow patterns. It is far more steady and usually much greater than the current in lakes or coastal areas. Wind is typically a small effect because of the domination by gravity and the shorter fetch of the wind over the river. Again, bottom topography can be an important factor.

Thus, the size and type of current encountered depend on the nature of the receiving body of water. Currents can range from large, rather steady currents, such as are generally found in rivers, to highly transient shear currents, as can exist in lakes and coastal waterways. Each type of current presents unique problems to the modeler.

The most obvious effect of the ambient current is the bending of the jet trajectory. In numerical models, bending is handled as a part of the total flow, entering the problem naturally as a boundary condition on the up-current, open-water boundary of the solution domain. In the integral models, jet bending is generally singled out for special consideration. Two mechanisms of bending are usually postulated. One is the entrainment of ambient momentum. The other is form drag on the jet, occurring as a result of the ambient current moving under and around the jet. The first mechanism is most often the primary one. The treatment of ambient current by the integral and numerical models is discussed more fully in Secs. III.D and E, respectively. Knowledge of the most probable plume trajectories is important to the architects who must locate the outfall and intake so as to minimize recirculation.

In addition to bending the jet trajectory, the ambient current may produce:

- a. A shorebound plume and the cutoff of lee side entrainment (discussed in Sec. III.C.2).
- b. Deformation of the plume cross section, resulting in plumes having sharp-upcurrent edges and diffuse-downcurrent edges (discussed in Sec. III.C.3).
- c. Internal circulations in the jet due to interaction between the ambient current and more slowly moving water near the edges of the plume (discussed in Sec. III.C.4).

A rather subtle question concerns whether wind directly affects plume trajectory or whether its effects are indirectly manifested through the ambient currents it induces. It is generally believed that the ambient current is a global effect of the wind working in combination with tidal forces and bottom friction. The question really is: "Are there any significant local effects not already accounted for by the ambient current?" There is some

evidence (mostly visual observations of surface disturbances) that indicates that wind affects the surface of the plume differently than that of the adjoining ambient waters. The minute elevation of plume surface and a possible dependency of surface shear on water temperature are sometimes offered as explanations for these observations. Although some local wind effects are apparent, the meaning of these effects is usually unimportant in light of the very simplistic conceptualizations of the wind and ambient current used in existing models. Better models of nearshore currents are needed before significant progress can be made in improving the treatment of wind and ambient-current effects.

Wind may also indirectly affect plume behavior by modifying the ambient-turbulence structure and by generating surface waves. Various empirical relationships are available in the literature to relate ambient eddy diffusivity to wind speed. These are mostly of the type in which eddy diffusivity is proportional to the n th power of the wind speed, where n depends on the properties of the water surface. For strong winds and an unstable water surface, the height of surface waves can be related to surface shear, which in turn can be related to wind speed through empirical relationships existing in the literature on oceanography.

Wind is also a controlling factor in evaporative heat loss at the water surface. Evaporative heat loss is significant in the far field, but is generally thought to be of little importance to near-field dilution. Several empirical formulas in the literature relate evaporative heat loss to wind speed. The formulas are generally acceptable for near-field modeling, but care should be exercised when applying them to other situations, e.g., the analysis of fully mixed rivers.

2. Cutoff of Lee-side Entrainment for a Shorebound Plume

Most integral models overlook the possibility that inshore entrainment can be cut off for a shoreline jet in a moderate-to-strong ambient cross-current. If such shore interaction occurs, the usual assumption made in the integral models of a uniform and unperturbed lateral entrainment is incorrect. The problem is often exacerbated by the presence of a shallow bottom, where the ambient crossflow is blocked from moving under the jet and forces the plume water up against the shore. A crosscurrent produces dynamic pressures in the lee of the jet that increase the tendency for reentrainment of partly diluted plume water. This situation, occasionally referred to as the Coanda effect, is important because the cutoff of entrainment water from the lee side of the jet will reduce dilution and is possibly more critical to design engineers than the stagnant-lake case.

The most thorough attempt at studying the shore-bound plume was carried out experimentally and analytically by Adams, Stolzenbach, and Harleman.¹ Other useful experimental results were obtained by Carter;²

Carter, Schiemer, and Regier;³ and Koester.⁴ For a three-dimensional jet in the presence of a crossflow, with or without bottom attachment, there is entrainment demand that cannot be completely fulfilled on the lee side of the jet.

Carter² noticed, in his hydraulic experiments, that the fluid being entrained on the near-shore side comes from a large eddy, which possesses little momentum in the shore-parallel direction. Water in that eddy was found to travel upstream along the wall of the tank, and then follow the lee side of the jet downstream, creating a circulation pattern. Dropping a large number of small pieces of paper or a small dye spot in this eddy revealed a large-scale circulation of a relatively long time scale.

Fig. III.C.1

Typical Nearshore Velocity and Temperature Profiles. (Adapted from Carter.²) ANL Neg. No. 190-1177.

Carter measured the velocity and temperature profiles across this highly bent jet. Typical profiles are shown in Fig. III.C.1. Temperature decays approximately like a Gaussian profile from the jet centerline to a very large distance offshore. On the lee side, temperature falls to a value that remains fairly constant all the way to the wall. Velocity is zero at the wall, is negative near the wall, indicating significant return flow, is maximum at the jet centerline, and finally decays to the large ambient value offshore. Carter observed that velocity and temperature profiles were neither

symmetric nor similar along the jet centerline. These observations throw further doubt on the ability of the integral models to treat jets discharged into large ambient currents.

Adams et al.¹ analyzed their own data as well as those of Carter et al.³ and Koester⁴ and came up with the following conclusions on reentrainment due to crossflow.

a. A crossflow increases the dilution of a detached jet. The exact relationship between the crossflow velocity and the amount of entrainment is uncertain.

b. Reentrainment due to a crossflow can significantly decrease the dilution of a jet that is attached to the bottom. The data of Carter et al.³ revealed that, for two-dimensional jets (discharge depth and bottom depth constant) with high F_0 and under moderate currents, all the lee-side entrainment water was partly diluted plume water, sharply affecting dilution.

c. The predominant mechanism leading to crossflow reentrainment on the lee side for detached jets is a density-driven counter current that causes lateral reentrainment. For detached jets, with or without crossflows, the ratio of lateral to vertical entrainment is small; consequently, the contribution of the lateral reentrainment is small.

d. Drag forces play an important role in the deflection of the trajectory of a surface jet in a crossflow. If a jet is completely attached, the drag force is apparently significant and needs to be included in a model. If the jet is unattached, a formulation without drag should adequately predict the trajectory; a small drag force should give an improvement, however.

An interesting case is the limiting situation of an extremely large current that completely cuts off inshore entrainment. Although not common, the centerline temperature in this case is maintained at a significant level for large distances downstream. Figures III.C.2-5 show the shorebound plume;⁵ water at least 2 C° above ambient extends more than 2.5 km southeastward along the shore. Due to rough lake conditions, the strong ambient current was not measured. The vertical plume sections in Figs. III.C.2-5 show that the plume changed very little with depth. This fact indicates that this plume (in contrast to those observed under weaker-crosscurrent situations) was not dominated by buoyant forces. The isotherms in the vertical cross-section drawings in Fig. III.C.6 are almost all vertical. In spite of the slow decay of centerline temperature, plume surface areas for this case were smaller than for plumes directed offshore, because the plume was much deeper. The case of a jet in a very large crossflow appears to be beyond the integral approach. Extension of the integral approach to handle jets with boundary effects (bottom and shoreline) as done by Adams et al. needs careful verification.

3. Sharp Upwind Edge of Buoyant Surface Plumes

Surface thermal plumes are often observed to have a sharp upwind edge and a diffuse lee edge; i.e., surface isotherm maps often show isotherms that are well defined and closely spaced on the upwind side of the plume with isotherms drawn out, sometimes with separated pockets of warm water downwind. This phenomenon is especially dramatic from aerial remote sensing.

One possible cause of these sharp-edged plumes is discussed by Sundaram and Wu⁶ in their paper on the effects of a wind-induced shear current on a thermal plume. They conclude that the surface layers of the plume will show greater deflection due to a higher surface current. Figure III.C.7 illustrates this point. A typical vertical current profile is shown in Fig. III.C.7a. The contour ABDC is the distorted jet cross section that results from the vertical gradient in the ambient current. Since the lower layers are less strongly deflected, region ACD will underlie cooler ambient water. Because of buoyancy, thermal-convection currents are set up that increase the nominal width of the jet from AB to EB. Sundaram and Wu therefore conclude that the interaction between the buoyancy of the jet and the shear current leads to a

Fig. III.C.2. Thermal Plume at Surface for Point Beach Power Plant (Unit 1): June 25, 1971, 1000-1215 Hours. (Adapted from Frigo and Frye.⁵) ANL Neg. No. 190-340.

Fig. III.C.3. Thermal Plume at 1.0-m Depth for Point Beach Power Plant (Unit 1): June 25, 1971, 1000-1215 Hours. (Adapted from Frigo and Frye.⁵) ANL Neg. No. 190-341.

Fig. III.C.4. Thermal Plume at 2.0-m Depth for Point Beach Power Plant (Unit 1): June 25, 1971, 1000-1215 Hours. (Adapted from Frigo and Frye.⁵) ANL Neg. No. 190-342.

Fig. III.C.5. Thermal Plume at 3.0-m Depth for Point Beach Power Plant (Unit 1): June 25, 1971, 1000-1215 Hours. (Adapted from Frigo and Frye.⁵) ANL Neg. No. 190-343.

Fig. III.C.6. Vertical Cross Sections at Point Beach (Unit 1): June 25, 1971, 1000-1215 Hours.
(Adapted from Frigo and Frye.⁵) ANL Neg. No. 190-397.

Fig. III.C.7. Effect of a Vertical Gradient in the Crosscurrent on the "Bending Over" of a Heated Surface Jet. (Adapted from Sundaram and Wu.⁶) ANL Neg. No. 190-1205.

widening of the surface jet and a sharply defined upwind edge. They also state that the downwind edge of the plume will be more diffuse, since the upper layers are sheared over onto colder underlying water.

Based upon experimental data, Sundaram and Wu estimate the increase δb in jet width that results from the difference in velocity δu from the surface to the bottom of the jet as

$$\frac{\delta b}{b} \approx -3.0 \frac{\delta u}{u_s}, \quad (2)$$

where

b = jet width at any centerline cross section

and

u_s = surface drift current.

From their empirical relationship, Sundaram and Wu note that, since $\delta u/u_s$ is usually quite large even for fairly shallow jets, an appreciable increase in jet width (over a uniform current) can be caused by wind-induced shear currents.

No integral model, in its present form, treats shear currents. In light of Gaussian-like distributions usually assumed for velocity and temperature, no sharp upwind edge can be predicted by these models. Sundaram and Wu note that, quite often, field data on thermal discharges are used to deduce horizontal entrainment coefficients by fitting model predictions to data. This approach could lead to unrealistically high values for the horizontal entrainment coefficients because the effects of shear currents are not explicitly considered. Sundaram and Wu suggest this scenario as a partial cause of the wide disparity in entrainment coefficients deduced by various authors from field data taken under apparently comparable conditions.

The differential-numerical models should conceptually be able to predict a sharp upwind edge due to the more general nature of their formulation, although no model has done so yet. This is perhaps because of problems of inadequate resolution in the finite-difference grid or is because of a poor choice of dispersion coefficients. Other more profound difficulties may be at the root of the problem, however. Although the phenomenon of the sharp upwind edge has been seen in both laboratory and prototype plumes, it may not be significant to overall plume characteristics.

A second theory that could explain the sharp edge of a surface plume is presented by Tulin and Schwartz.⁷ They attempt to explain observations that show the presence of sharp edges at the lateral interfaces of a plume spreading under stagnant ambient conditions. These investigators draw an analogy between the spread of oil films and the lateral spread of a buoyant plume on the surface of a stagnant receiving body or in a coflowing ambient current. From the work of Benjamin,⁸ they conclude that the type of head flow that occurs in spreading oil slicks also occurs in buoyant plumes, both being the result of density differences between the slick or plume and underlying ambient water. Due to the retarding effects of lateral shear and to the hydrostatic pressure force acting on the buoyant plume, a bulge or "head" develops at the edge (or head) of the laterally spreading mass.

Tulin and Schwartz develop a simple theory which includes gravitational, inertial, and certain drag forces acting on the spreading mass. Entrainment of mass and momentum is assumed to arise from three sources:

- (a) Breaking of the head or bulge that develops at the edge of the spreading plume.
- (b) Turbulent diffusion resulting from the residual turbulent energy of the discharge; this turbulence is assumed to decay with time.
- (c) Turbulence that is continuously generated in the spreading plume due to the action of wind shear.

Tulin and Schwartz formulate a simple one-dimensional, time-dependent integral model for the late stages of spreading. The model estimates the influence of initial conditions, mixing, and dissipation on lateral velocity, layer depth, plume-density difference (above ambient), and turbulence intensity. Time-dependent conservation equations are written for mass, volume, lateral momentum, and turbulent energy. These equations include four empirical coefficients: an entrainment coefficient associated with the turbulence in the plume, a second entrainment coefficient associated with breaking and dissipation at the head, a drag coefficient for head drag, and a drag coefficient associated with the decay of turbulence. In the case of wind-sustained turbulence, the decay equation is not needed, nor is the associated drag coefficient, as the turbulence intensity is assumed constant. The solution of these equations under certain special assumptions gives information about the asymptotic (in time) spreading and dilution that occurs naturally under a variety of initial conditions.

Tulin and Shwartz conclude that, in the late stages of mixing in a stagnant environment, after the initial plume turbulence has decayed, the transverse spreading of the plume is governed only by gravity forces, and "head" flow at constant Froude number results. This head flow is hypothesized to cause a sharp edge of separation between the spreading plume and the ambient fluid. Such a sharp plume interface has indeed been often observed under calm ambient conditions, especially for highly buoyant discharges under zero wind conditions.

4. Vortex Phenomena within Thermal Plumes

It has been experimentally observed that an internal vortex circulation pattern is set up for a large-initial-densimetric-Froude-number heated surface jet in the presence of an ambient crossflow. Drag forces acting at the boundary of the jet are generally believed to be responsible. These circulations are similar to the ones that are present in a vertical round jet in the presence of a horizontal ambient crosscurrent. In the submerged case, two counter-rotating vortices are observed in the plume cross section (normal to the jet centerline); the surface jet in a current shows only one such circulation pattern. Circulations in the submerged jet are a significant cause of increased entrainment.

Low-densimetric-Froude-number discharges do not have such circulations, since buoyancy opposes their formation. Stefan et al.⁹ set tentative bounds for the initiation of these circulations based on the initial densimetric Froude number and radius of curvature:

$$F_L > 6.3 \quad (3)$$

and

$$R/b < 100 \quad (4)$$

(larger values of R/b acceptable as F_L decreases),

where

F_L = local densimetric Froude number,

R = local jet radius of curvature in the horizontal plane,

and

b = standard deviation of velocity distribution at water surface.

For F_L and R in the above range, circulations can occur, voiding the assumption of Gaussian-like distributions of temperature and velocity made by the integral models. Stefan et al.⁹ speculate that a more uniform temperature distribution will result from these internal circulations. In spite of the above

remarks, Stefan et al. state that experiments in the laboratory and in the field paradoxically support the symmetric Gaussian-like distributions for large-initial-densimetric-Froude-number discharges. The proper choice of entrainment coefficient for use when these circulations occur is difficult. Present integral models have ignored the impact of internal circulations on horizontal entrainment and similarity profiles.

5. Bottom Interference

Boundary interference is a serious modeling problem. Unfortunately, it is often completely ignored or inadequately treated by existing near-field plume models. Even for numerical models where boundaries enter naturally, a correct simulation is hampered by an inadequate consideration of bottom friction, by an irregular or unknown bottom and shoreline topography, or by too coarse a horizontal and vertical grid spacing.

It was noted in Sec. III. B that boundary interference may void the use of near-field models altogether. This section focuses on those instances where the plume cannot dilute freely because of a shallow bottom, but where such interaction does not totally void the near-field approximation. This case is one of great practical interest, particularly in view of the trend towards high-velocity discharges.

Assuming the jet conceptualization to be correct, the principal effect of a shallow bottom is the cutoff of vertical entrainment. This reduces dilution, causing larger plume temperatures and perhaps greater lateral spreading, although the relationship between a shallow bottom and increased spreading is presently uncertain. Secondary effects include shaping of the jet trajectory by an irregular bottom topography and distortion of the jet cross section. Observations of both laboratory and prototype plumes indicate that a plume may be attached to the bottom at its centerline and may be simultaneously spreading laterally at its edges.

Bottom interaction has been studied in the laboratory by Wiegel et al.¹⁰ as early as 1964; by Stolzenbach and Harleman,¹¹ by Carter, Schiemer and Regier,³ and by Carter and Regier.¹² The most complete measurements of bottom interaction have been made by Koester⁴ and Adams et al.¹ The results of Adams et al., which include data for both a flat and a sloping bottom, provide some sorely needed information on bottom interference.

Figures III.C.8-10 present the data of Koester for a circular surface discharge of diameter D_0 into a stagnant tank for three different initial densimetric Froude numbers. These tests were made in a flat-bottom tank of depth H . The data indicate the effects of decreasing water depth on centerline temperature decay and isotherm areas. Here, θ_c denotes the temperature at the surface at given distances along the jet centerline, θ denotes a particular surface isotherm, and θ_0 denotes the temperature of the discharge--all values referenced to ambient. These plots show that dilution is reduced by a shallow

bottom. Quantitatively interpreting these results for prototype discharges is difficult, however, because of the many complicating factors present at prototype sites.

Fig. III.C.8. Laboratory Data Showing the Effect of a Shallow Bottom on a Circular Surface Jet for Initial Densimetric Froude Number of 14. (Adapted from Koester.⁴)

Fig. III.C.9. Laboratory Data Showing the Effect of a Shallow Bottom on a Circular Surface Jet for Initial Densimetric Froude Number of 9. (Adapted from Koester.⁴)

Fig. III.C.10. Laboratory Data Showing the Effect of a Shallow Bottom on a Circular Surface Jet for Initial Densimetric Froude Number of 5. (Adapted from Koester.⁴)

While bottom interaction is important for most existing discharges, analytical tools for its analysis are few. Most attempts at treating bottom interaction have been simpleminded at best. Generally, the bottom is taken to be smooth with constant slope. Bottom friction has often been neglected.

Attempts to include bottom effects in existing integral and phenomenological models have so far proved unsuccessful. Stolzenbach and Harleman tried to include bottom effects in the original form of their three-dimensional integral model, but generally found their treatment inadequate. Pritchard¹³ attempted to model bottom effects in developing his second phenomenological model, but this analysis has not yet been shown satisfactory. Adams developed an integral model that includes bottom effects. The model involves some modifications of the normal integral approach and is compared with the hydraulic-model results noted above. Hopefully, this new model will provide an improved treatment of bottom interaction, an important prototype effect.

The numerical models conceptually handle bottom effects, but due to the complexities of these models, the treatment is often grossly simplified. Paul and Lick¹⁴ have attempted to treat the irregular bottom topography at Point Beach by transforming the vertical coordinate to yield an equivalent flat receiving basin. This procedure is discussed in Sec. II.K and Appendix B. More work is needed, however, before the important question of bottom interference can be considered resolved.

6. Breakdown of the Jet Structure

Adams, Stolzenbach, and Harleman¹ discuss the breakdown of the basic structure of the jet after a long period in a stagnant ambient receiving body. This breakdown occurs at some large longitudinal distance from the outfall, is followed by a buildup of heat in the far field, and eventually results in a return of the heat into the near field due to buoyancy-driven currents and the need for the near-field jet to fulfill its entrainment requirements. Adams et al. note that the problem is exacerbated by a low initial densimetric Froude number and a shallow receiving waterbody. They also state that an analysis of such a breakdown is extremely difficult because it involves the study of an instability and probably depends greatly on both discharge and receiving-body geometries. The density spreading and the uncertain method of heat removal from the stagnation area complicate it further. Note that the available integral and phenomenological models for the near field ignore the possibility of re-entrainment and may, in some cases, give significantly incorrect predictions.

This section discusses some of the experimental work that bears on this type of near/far-field interaction. Most studies have been made in two-dimensional (longitudinal and vertical) laboratory models. The phenomenon is more obvious in such two-dimensional experiments, because of limited surface area available to remove heat accumulated in the far field.

This type of near/far-field interaction actually stems from the fact that a heated surface jet exhibits characteristics of both a turbulent incompressible jet in the near field and a stratified flow in the far field. The effects of downstream stratification have been studied experimentally in two dimensions by Stefan and Schiebe.¹⁵ To describe the effects that down-current stratification can have on near-field flow patterns, we first consider the non-dimensional groups basic to jet flow. The dynamics of a heated plume are largely determined by the densimetric Froude number, IF , defined in Chapter I. For homogeneous flow in which there is no temperature difference, the densimetric Froude number is infinite, indicating that dynamic forces completely govern the flow and buoyancy is absent. On the other end of the scale is the case of zero densimetric Froude number in which the flow is hydrostatic, governed entirely by buoyancy (zero dynamic forces). The initial densimetric Froude numbers of prototype surface discharges vary over a broad range, indicating that real flow patterns are a combination of both homogeneous flow and stratified flow. The Reynolds number is a second important parameter in determining flow patterns, but for most prototype discharges, the dependency on Reynolds number is assumed to be weak.

In estimating the relative influence of outlet and downstream parameters, one needs to consider both the initial densimetric Froude number, IF_0 , and the downstream or asymptotic densimetric Froude number, IF_1 . However, it is more expedient to characterize downstream flow by its asymptotic depth, d_1 , rather than the downstream densimetric Froude number, since velocities there are small. Invoking conservation of mass flux, Stefan and Schiebe¹⁵ derive a simple relation between outfall and downstream conditions in two dimensions:

$$\frac{IF_1}{IF_0} = \left(\frac{d_0}{d_1} \right)^{3/2}, \quad (5)$$

where d_0 is the discharge depth. The parameters IF_0 and d_0/d_1 are then taken as the outfall and downstream control parameters, respectively.

If the initial densimetric Froude number is larger than about 1.2, a well-defined jet will exist. As one progresses out along the jet centerline from the outfall, the local densimetric Froude number decreases. Depending on the initial densimetric Froude number and the rate of surface heat loss, the local densimetric Froude number may become so small that a transition to stratified flow must occur. This transition may take the form of a submerged internal hydraulic jump, as shown schematically in Fig. III.C.11. The rapid

Fig. III.C.11

Idealization of an Internal Hydraulic Jump. ANL Neg. No. 190-1170.

deceleration of fluid from jet to stratified flow requires a correspondingly rapid increase in plume thickness to ensure conservation of momentum.

If the initial densimetric Froude number is less than about 1, stratified flow starts at the outfall and no jet regime exists. The source may be inundated with heated effluent discharged into a layer at essentially its own temperature. A full analysis of downstream conditions is required. Modeling of outfall conditions alone is inadequate. The integral models for the near field presently available in the literature are not applicable to this type of flow.

Steady flow conditions cannot exist in the thermal plume, unless heat is removed at a rate equal to the source influx. The mechanism of heat removal, combined with such other factors as interfacial shear, bottom friction, and turbulent diffusion, determines the asymptotic depth of stratified flow in the far field. This mechanism is markedly different for hydraulic models and prototype situations. For physical hydraulic models, surface heat loss is small, and the depth of stratified flow will be determined by the overflow weir and the sidewalls of the tank. In prototype situations in which the receiving body is large, surface heat loss will determine the stratification pattern, unless ambient mixing is so great as to destroy far-field stratification.

Stefan and Schiebe suggest that the relative importance of outfall and downstream factors may be assessed by considering the propagation of interfacial waves. They state that interfacial long waves will travel with the jet velocity in outlet-controlled flow and will travel counter to the jet velocity in downstream-controlled flow. Based on these traveling-wave arguments, Stefan and Schiebe divide the parameter plane consisting of Fr_0 and d_1/d_0 into several regions of flow as indicated in Fig. III.C.12. If the initial densimetric Froude number is less than 1, then one of the three types of flow shown in Fig. III.C.13 results, depending on the relative value of d_1/d_0 . Under suitable conditions, given in Fig. III.C.12, a cold-water wedge can intrude into the outfall as shown in profile A-1 of Fig. III.C.13. If stratification is unable to force its way into the outfall, but the asymptotic plume depth is less than the outfall depth, then the flow pattern shown in A-2 of Fig. III.C.13 will be obtained. If d_1/d_0 is greater than 1, then the flow pattern shown as A-3 in Fig. III.C.13 may be expected.

If the initial densimetric Froude number is greater than 1, then one of the three profiles shown in Fig. III.C.14 results. The differences among profiles B-1, -2, and -3 lie in their stability to interfacial short waves. Flow B-1 possesses neutral stability to interfacial waves, the pattern illustrated in B-2 is unstable, and pattern B-3 is stable against interfacial waves. The regions of the parameter plane corresponding to stability and instability are indicated in Fig. III.C.12.

As previously noted, a submerged internal hydraulic jump may result if the flow passes from the jet regime to a regime of stratified flow. Figure III.C.15 shows the types of internal hydraulic jumps considered by Stefan and Schiebe. Profiles Q-1, -2, and -3 represent increasing densimetric Froude numbers with increasing amounts of interfacial mixing.

Fig. III.C.12

Regimes of Flow. (Adapted from Stefan and Schiebe.¹⁵) ANL Neg. No. 190-1210.

Fig. III.C.13

Scheme of Interfacial Profiles That are Tailgate-controlled ($F_0 < 1$); Type-A Flow: Stable, Smooth Interface. (Adapted from Stefan and Schiebe.¹⁵) ANL Neg. No. 190-1195.

Fig. III.C.14

Scheme of Interfacial Profiles That are Tailgate-controlled ($F_0 < 1$); Type-B Flow: Periodic Interfacial Waves. (Adapted from Stefan and Schiebe.¹⁵) ANL Neg. No. 190-1194 Rev. 1.

Fig. III.C.15

Scheme of Interfacial Profiles That are Partially or Entirely Outlet-controlled. Top diagram: submerged internal jump; middle diagram: internal hydraulic jump; bottom diagram: two-dimensional jet. (Adapted from Stefan and Schiebe.¹⁵) ANL Neg. No. 190-1193.

The above classifications of flow are based on traveling-wave stability considerations of the plume/ambient-water interface treated in two dimensions. These results may not be generally applicable in three-dimensional prototype situations, due to the effects of lateral mixing and to the general diffuseness of the interface. However, Stefan and Schiebe¹⁶ support these theoretical results with observations from three-dimensional hydraulic studies. We consider a few of their results.

Figures III.C.16 and 17 taken from Stefan and Schiebe,¹⁶ show, respectively, longitudinal and lateral cross sections of the thermal plume

Fig. III.C.16

Isotherms and Isovels in Vertical Plane along Centerline of Tank, $F_0 = 0.62$. (Adapted from Stefan and Schiebe,¹⁶) ANL Neg. No. 190-1211.

Fig. III.C.17. Isotherms in a Series of Vertical Planes Normal to the Centerline of the Tank, $F_0 = 0.62$. (Adapted from Stefan and Schiebe,¹⁶) ANL Neg. No. 190-1214.

resulting from a discharge of very low initial densimetric Froude number ($IF_0 = 0.62$). The longitudinal profile given in Fig. III.C.16 reveals stratification over a significant portion of the plume. In this low-initial-densimetric-Froude-number case, stratification has projected its influence all the way back to the point of discharge and a cold-water wedge has penetrated into the outlet channel. Such behavior is expected only when buoyant forces are strong.

Of interest are the lateral wave patterns illustrated in Fig. III.C.17. Again, this phenomenon is associated only with very low-densimetric-Froude-number discharges. At $x = 3.0$ ft, the plume has divided itself laterally into three pools of warm water with slightly cooler water in between. At $x = 8.0$ ft, the stratification has become almost uniform laterally.

Figures III.C.18 and 19, also from Stefan and Schiebe,¹⁶ give the longitudinal and lateral cross sections, respectively, of a thermal plume with

Fig. III.C.18

Isotherms and Isovels in Vertical Plane along Centerline of Tank, $F_0 = 3.73$. (Adapted from Stefan and Schiebe.¹⁶) ANL Neg. No. 190-1213.

Fig. III.C.19. Isotherms in a Series of Vertical Planes Normal to the Centerline of the Tank, $F_0 = 3.73$. (Adapted from Stefan and Schiebe.¹⁶) ANL Neg. No. 190-1216.

an initial densimetric Froude number of 3.73. As in the previous case, stratification is apparent over a significant portion of the plume. Stefan and Schiebe, however, note that this stratification is partially controlled by the far end and both sides of the hydraulic tank. The influence of geometry was less severe in the case of strong buoyancy. Referring to Fig. III.C.18, there appears to be a well-defined jet initially that is separated from the stratified flow by some form of an internal hydraulic jump. A stationary-wave pattern is apparent in the stratified region of the plume. Stefan and Schiebe attribute the asymmetry of the lateral profiles of Fig. III.C.19 to the limited width of the tank and to the diverging boundary.

Apparently, a hydraulic jump can occur under special conditions in a three-dimensional flow during the breakdown of the jet structure. This is not an often-expected phenomenon, since three directions for heat dispersion provide a more effective means for the disposal of the heat. The jet breakdown is more likely and undoubtedly occurs more often in two dimensions. Yet, the above study of the two-dimensional jet does provide some physical insight into the three-dimensional jet.

7. Transient Environmental and Discharge Conditions

Existing thermal-plume models are largely designed to handle steady-state problems, although the recently developed numerical models have made possible first attempts at treating unsteady cases. The next section describes the thermal-front phenomenon, which is an internal transient effect beyond the capabilities of any existing model. Here, we concentrate on the gross transient behavior of the plume associated with changing environmental and discharge conditions. An excellent example of a prototype situation in which transient effects are obviously present is the Pilgrim Nuclear Power Station described in Sec. IV.D. At this site, the thermal effluent is discharged into a tidally influenced bay. Environmental and discharge conditions (through the level of the tide) are continuously varying. Even at lake sites, environmental parameters are usually transient. Field data taken at the Point Beach Nuclear Power Station⁵ show that thermal-plume maps acquired in close succession are surprisingly different. Rivers are, perhaps, the most steady of all. The flow rate in rivers is normally slowly changing, and other environmental parameters are usually dominated by the large gravity-controlled flow.

No truly adequate way of including transient effects into available integral and phenomenological models presently exists. One common practice is to assume that the plume is quasi-steady, i.e., that the relaxation time for the plume to reach steady state under truly steady environmental and discharge conditions is much shorter than the characteristic time during which these conditions are changing in the prototype. If such an assumption is used, then one can calculate the transient plume by inputting the transient environmental conditions to the steady-state model in a step-wise fashion. It is doubtful that such an approach is valid for thermal discharges, because of the significant

amount of heat (typically, 3-8 hr of heat from the discharge) contained in the measurable plume. An alternative approach, with even less justification, is to average environmental and discharge conditions over some period of time and then input these time-averaged quantities to the steady-state models. Both of these approaches were tried at the Pilgrim plant.¹⁷ In one attempt, the Stolzenbach-Harleman model was run for conditions existing at various tidal stages and the results time-averaged to give a mean plume. In the second attempt, time-averaged input data were used. Very little difference was observed in the predictions of the two methods. The investigators found both approaches inadequate, since the steady-state model could not account for the significantly reduced dilution that occurs during midtide when discharge and environmental conditions are rapidly varying.

In environmental-impact studies, the usual approach is to seek out a worst case. The difficulty here is that one must often speculate as to the worst steady-state case and, even if one is successful in finding such a worst case, the truly worst case may still be an unsteady one.

The problems associated with transient environmental and discharge conditions are indeed formidable. More experimental and analytical studies of this matter are sorely needed.

8. The Thermal-front Phenomenon

The thermal-front phenomenon refers to the often rapid internal temperature variations that have been noticed within the thermal plume, even when discharge parameters and environment conditions are relatively steady. Scarpace and Green¹⁸ have found, from thermal images, that prototype plumes often possess a series of concentric thermal fronts that spread radially outward from the outfall. The periods of these thermal fronts ranged from 5 to 8 min. Most of these measurements were made at the Point Beach Power Station on Lake Michigan. Boat measurements at the same site showed changes of 4 C° in 1 or 2 sec at 400 ft from the outfall, with an approximate period of 10-30 sec. Figures III.C.20 and 21 are infrared images of the thermal fronts observed at Point Beach. Superimposed on the thermal fronts is a series of remarkably regular secondary waves. The wavelength of the secondary waves is about 70 ft with a period of about 40 sec. These secondary waves are only barely visible in the photographs. Figure III.C.22 best illustrates the secondary-wave phenomenon.

Scarpace and Green have determined that horizontal temperature gradients across the thermal fronts can be as high as 0.5 C°/ft, with that intensity persisting for the first 3/4 mile of their propagation outward through the plume. High waves and rough weather tend to destroy both the fronts and the secondary waves. In very calm weather, on the other hand, plumes in lakes appear dominated by sharp concentric thermal fronts with a strong appearance of secondary waves. These spreading fronts raise questions on the

Fig. III.C.20. The Thermal-front Phenomenon Observed at Point Beach on September 17, 1971; Flying Height is 1600 ft. The wind direction is indicated by the arrow. (Adapted from Scarpace and Green.¹⁸)

Fig. III.C.21. The Thermal-front Phenomenon Observed at Point Beach on September 15, 1971; Flying Height is 1600 ft; 5 min between Images. (Adapted from Scarpace and Green.¹⁸)

Fig. III.C.22. Secondary Thermal Waves Observed at Point Beach on September 15, 1971;
Flying Height is 1600 ft. (Adapted from Scarpace and Green.¹⁸)

applicability of steady-state models and on the adequacy of nonsynoptic boat-based measurements of plume temperatures. These significant local temperature oscillations will probably go unnoticed by a passing boat monitoring the plume. The instantaneous plume may be quite different from the "mean" plume determined by boat measurements and predicted by analytical models. More specifically, traversing boat measurements of the plume may lead to erroneous pictures of the mean plume by masking the large temperature gradients that can exist in the instantaneous plume.

In addition to the above study of thermal-plume images, boat-based measurements of thermal fronts have been recently completed by Frigo et al.¹⁹ Observations made on four dates during May, June, and August 1973 revealed systematic oscillations of temperature and current when measured at a single point about 250 ft from the lakeside end of the discharge canal. A thermistor boom was used to sense water temperatures simultaneously at 0.5-m intervals beginning at the water surface and extending to a depth of 3 m. A full set of seven temperatures was sampled every 2 sec. The local plume velocity was also measured and recorded.

Results of a study conducted on June 7, 1973, are shown in Fig. III.C.23, where all seven temperature histories have been superimposed on a single plot. Several aspects of the transient behavior are immediately apparent. First, there is a coherent oscillation of all seven temperature records appearing as slightly oblique slashes with a period of about 140 sec.

Fig. III.C.23. Temperatures at 0-, 0.5-, 1.0-, 1.5-, 2.0-, 2.5-, and 2.7-m Depths and Current Speed at Depths of 2.5 m (between triangle marks) and 1.5 m, at 67-m from the Point Beach Nuclear Power Plant Unit 2 Outfall: June 7, 1973, 1529-1645 Hours. (Adapted from Frigo et al.¹⁹)

The character of these oscillations is as follows: The temperature of all seven thermistors drops 3-5 C° in about 60 sec, followed by a scattered recovery during the rest of the period. A somewhat less regular oscillation, with a period of about 1200 sec, is discernible in the lower edge of the envelope of data points. These lower-frequency oscillations appear to coincide with current oscillations, as can be seen from the plot of current data in Fig. III.C.23.

To verify the period of these oscillations and confirm the cross correlation of temperature and velocity fluctuations, the power spectra of the disturbance were numerically determined by fast-Fourier transform methods. The results are shown in Figs. III.C.24 and 25. Several interesting features are revealed. First, the power spectrum of temperature variations appears to be dominated by a reciprocal frequency behavior, with a peak clearly evident at a period of about 140 sec. The current speed spectrum also reveals a peak near 140 sec, indicating strong coherence with the temperature oscillation.

Fig. III.C.24

Power Spectrum of Plume-water Temperatures, Point Beach Unit 2, June 7, 1973. Data points are average of spectral estimates for the seven simultaneous measurements in the previous figure. Error bars are one standard deviation of the seven estimates. Each of the seven estimates was computed with seven degrees of freedom. (Adapted from Frigo et al.¹⁹)

Fig. III.C.25

Power Spectrum of Current Speed in the Plume, Point Beach Unit 2, June 7, 1973: Eleven degrees of Freedom. (Adapted from Frigo et al.¹⁹)

Temperature and velocity measurements were made in the discharge channel to determine if fluctuations existed there and what influence the discharge channel might have on the period of the fluctuations. Both current speed and direction showed a fluctuation having a period of about 140 sec, although no temperature variations were detected. No evidence of the slower oscillation was found. The fundamental longitudinal resonant oscillation of the channel was calculated to be about 50 sec. The difference between the natural frequency of the channel and the observed frequency of the disturbance, coupled with the fact that a transverse component of the oscillation was observed, suggests that the channel is responding to and not generating the disturbances seen in the jet. Presumably, the temperature in the flume is determined by the plant-discharge temperature, whereas velocity fluctuations are the result of momentum feedback from points farther out in the jet.

The correlation between current fluctuations and the lower-frequency temperature oscillations supports the contention that these variations are the result of plume meandering. Recent and yet unpublished dye studies conducted by Argonne National Laboratory confirm this statement. In these studies, both red and green dyes were used. A timewise continuous line source of red dye was placed at the south lakeside end of the north flume at Point Beach, while a line source of green dye was located on the north lakeside end, both sources being in the indigenous ambient water. A photograph of the resulting dye patterns is shown in Fig. III.C.26. Pools of red and green dye are apparent, each on their respective sides. In addition, regions of alternately red and green dye are apparent in the jet portion of the plume. The wavelength of these dye patterns is about 50 m, which corresponds fairly well to the 140-sec-period disturbance previously detected. The existence of these patterns has not yet been explained.

Similar types of wavelike phenomena have been observed in other experimental situations, e.g., Stefan and Schiebe,¹⁶ Thorpe,^{20,21} and Kuhlman and Prahl.²² Stefan and Schiebe investigated the waves with alumina powder, lycopodium powder, and fluorescein dye. Particles floating at the water surface were observed to accumulate in streaks or fronts. The investigators state that the streaks or fronts were caused by an interfacial wave pattern that began at the bottom of the outlet channel. The internal waves were observed to break after only a few wavelengths.

Thorpe²⁰ studied the propagation of interfacial waves in a long, closed horizontal tube containing a highly stratified fluid. In these experiments, the end of the tube was raised (to angles of 5-30°) to produce a laminar, accelerating flow. To aid in visualization, the heavier fluid at the bottom was colored with potassium permanganate crystals. Thorpe noticed developing interfacial waves fairly sinusoidal in form that later (in time) distorted into spiral configurations. Most of the heavier fluid "rolled up" into a series of evenly spaced spirals along the tube. Rows of dyed and undyed fluid were seen as line vortices in the top view of the tube. These line vortices (appearing as a long row of spirals in the side view of the tube) then moved downward due to

Fig. III.C.26. Spread of Dye Placed at Edges of Point Beach Unit 1 Flume. ANL Neg. No. 190-1540K.

the gravitational force on them. At the latest stage of motion, the onset and growth of transverse instability leading eventually to three-dimensional turbulent motion were observed. A type of roll-up phenomenon was similarly observed in the studies of Kuhlman and Prah1 discussed below.

Thorpe²¹ compared his experimental measurements to a theory developed for small-amplitude disturbances in an accelerating free-shear layer at the interface of two fluids of different density. The study involved the early stages in the life of interfacial instability. Thorpe's work suggests that the instabilities that developed at the interface between the two fluids is of the so-called Kelvin-Helmholtz type. As noted above, the waves observed by Thorpe are similar to the ones seen in the laboratory experiments of Stefan and Schiebe and of Kuhlman and Prah1 for surface thermal discharges.

Kuhlman and Prah1²² noted a series of transverse line vortices in their laboratory simulation of low-initial-densimetric-Froude-number discharges. These vortices, apparently generated at the plume thermocline, were observed to rise toward the surface. Kuhlman and Prah1 explain this phenomenon, which was only noticeable for initial densimetric Froude numbers less than 0.52, as a Kelvin-Helmholtz instability. For the same range of initial densimetric Froude numbers, a cold-water wedge was observed to penetrate the discharge channel. Also, small sinusoidal disturbances were noted in the channel thermocline. These small disturbances apparently form line vortices transverse to the plume centerline, once the plume has entered the ambient crossflow. Kuhlman and Prah1 surmise that the small waves seeding the instability were the result of internal-gravity waves formed at the plume thermocline. The instability led to cold and warm water being rolled up side by side in the vortex. The vortices rose to the surface due to rotation and mixing. Colder water in each vortex was thus brought to the surface, creating definite bands of warm and cold fluid at the surface. Dye injected at the plume thermocline was first drawn into the line vortices, then convected to the surface, and finally appeared as bands of dye on the surface. The spacing of the vortices was approximately equal to the channel dimensions. For initial densimetric Froude numbers above 0.52, a regular pattern of vortices was not observed. Vertical mixing, however, was even more significant indicating a random, turbulent process. The regular pattern of vortices was found to break down as an initial densimetric Froude number of 0.52 was approached. The vortices become less and less regular and finally break down to a more random turbulence at a point that becomes nearer and nearer the outfall as initial densimetric Froude numbers increase. Note that thermal fronts have been seen in the field at initial densimetric Froude numbers significantly larger than 0.52.

Many questions remain unanswered on the nature and origin of the thermal fronts, although the existence of these fronts is fairly well documented. There is still controversy as to the mechanisms that produce the thermal fronts as well as the real connection between the fronts seen in the field and

the apparently similar wavelike phenomena observed in the laboratory. The possibility of mathematically modeling the thermal fronts has not been thoroughly explored. It is generally agreed that conventional integral models cannot do the job, but uncertainty exists as to whether differential-numerical models can simulate thermal fronts. Our general feeling is that present differential-numerical models will be inadequate. The models that treat turbulence as a diffusion process can only handle transient effects that are of a scale much longer than the typical mixing times of the plume. The thermal fronts, however, occur on a time scale only slightly larger than that of the turbulent mixing.

Some investigators argue that, although intriguing, the thermal-front phenomenon has little impact on overall plume estimation. They claim that a "mean" plume can be clearly identified and that it is the "mean" plume that should be modeled. We feel, however, that the significance of the thermal-front phenomenon lies in its commentary on the very grossly oversimplified view that is commonly held of a heated plume. The relationship of the thermal fronts to the key jet-mixing processes such as entrainment is presently unclear. Our inability to explain these fronts underscores the backward nature of our understanding of plume physics and the art of plume modeling.

9. Ambient Stratification

During winter and early spring, most temperature lakes exhibit a nearly uniform temperature distribution. As spring continues, the lake experiences longer and more direct exposure to solar heating. Initially, this absorbed solar heat is mixed throughout the top and deeper layers of the lake by wind-induced currents and turbulent mixing. As the rate of heating increases, the rate of removal of heat to the deeper layers lags behind, producing a warm, well-mixed surface layer above a second, well-mixed, but cooler, layer. The plane of maximum temperature gradient separating the two layers is known as the thermocline.

Ambient stratification inhibits vertical mixing of a surface plume into the deeper layers of the lake, while at the same time producing an almost uniformly mixed surface layer. This circumstance is a boon to two-dimensional models that ignore vertical mixing. Conversely, a situation of no ambient stratification benefits three-dimensional models in which vertical entrainment is allowed.

Another problem occurs during and after periods of natural upwelling or downwelling. In large lakes, such as Lake Michigan, for example, upwelling often produces a nonuniform temperature distribution in the upper layer. The temperature distribution tends to be a function only of the distance from shore, with temperature increasing as one moves away from shore. Upwelling results when the wind remains offshore for a considerable length of time. Numerical models, once sufficiently refined, may handle ambient

stratification as boundary conditions on temperature. For integral models, the treatment of ambient stratification requires more complicated assumptions.

10. Recirculation and Reentrainment

Recirculation refers to partly diluted plume water that directly enters the plant intake. Recirculation produces an outfall temperature greater than that calculated by using the ambient temperature and need be considered only when the discharge temperature is not measured. In contrast, reentrainment occurs when partly diluted plume water is circulated back into the vicinity of the outfall and eventually becomes entrainment water. Reentrainment must be considered by any model that fixes the temperature of the entrainment water. These definitions, although not universally used in the literature, are used throughout this report.

Recirculation is particularly important for preparing environmental-impact statements, since direct measurements of discharge temperature are usually impossible. Normally, the calculated discharge-temperature excess θ_o is multiplied by a factor that depends on the fractional amount of recirculation r that is assumed to occur. The value of r is usually guessed, although values of 10-15% are typical worst-case estimates for a properly designed discharge. Two typical formulas used to find the corrected discharge temperature excess θ_{or} are

$$\theta_{or} = \theta_o(1 + r) \quad (6)$$

and

$$\theta_{or} = \theta_o(1 - r)^{-1}. \quad (7)$$

Reentrainment is an important prototype effect, yet the tools for including it in the popular models are lacking. It is conceptually simple to include reentrainment in the integral models by simply raising the temperature assumed for the entrainment water. Adams et al.¹ use a far-field model to calculate the "ambient" temperature for the near-field plume. Pritchard²³ includes reentrainment in his integral/phenomenological analysis by postulating a fictitious background temperature that is used to correct calculated temperatures by

$$\theta' = \theta + (1 - \theta/\theta_o)\varphi, \quad (8)$$

where

θ' is the corrected temperature,

θ is the uncorrected temperature,

and

θ_o is the discharge temperature.

The value of ϕ is determined by certain undesigned, site-specific factors; ϕ serves to improve predictions and explain the discrepancies between data taken under apparently identical conditions.

Numerical models implicitly handle reentrainment, provided that the far-field processes responsible for returning plume water to the near field are simulated. If, as is usually the case, the computational grid is too small to contain a significant part of the far field, then the reentrainment mechanisms are lost. The difficulties of setting up a numerical scheme are more fully addressed in Sec. III. E.

11. Surface Heat Exchange

Surface heat exchange is an important far-field and lakewide physical process, but is usually negligible in its direct effects on the near field. Therefore, far- and complete-field models are forced to consider surface heat transfer, but near-field models generally are not. Surprisingly, surface heat exchange is included in most near-field models, since addition is usually trivial.

As was pointed out in Chapter I, proportionality is normally assumed between the surface heat-transfer rate and the difference between the local surface temperature and the equilibrium temperature. The equilibrium temperature is defined as that temperature at which no net heat exchange occurs. The coefficient of proportionality, usually referred to as the surface-heat-exchange coefficient, depends on many factors (including the equilibrium and surface temperatures). Brady et al.²⁴ made extensive studies of surface heat transfer. Theirs is a typical analysis. Yet, there is some evidence that values based on this or one of the similar studies are grossly incorrect, perhaps by an order of magnitude. Even such a large error is forgivable in the near-field model, but more accurate values are required for complete- and far-field models.

Surface heat exchange can control the type of stratification in the far field. Stefan and Schiebe²⁵ noted that there are two general categories of stratification in the far field, depending on the relative values of the outfall temperature T_0 , the ambient temperature T_a , and the equilibrium temperature, E . These types, illustrated in Fig. III.C.27, distinctly influence downstream flow

Fig. III.C.27

Types of Stratification. (Adapted from Stefan and Schiebe,²⁵ ANL Neg. No. 190-1185.

and the ultimate fate of the thermal discharge. For a lake in the heating part of its cycle ($E < T_a$), cooling ends when the plume reaches the equilibrium temperature and the discharge forms a continuous surface layer. For a lake in the cooling part of its cycle ($E < T_a$), stratification terminates when the plume reaches ambient temperature and the plume mixes into the deeper layers of the impoundment.

These flow patterns are valid only when all temperatures are greater than 4°C, the temperature at which the density of water is greatest. When temperatures above and below 4°C are involved (e.g., a discharge into a freezing lake), a sinking plume can result, and the above flow patterns will no longer apply. The sinking-plume problem is discussed in the next section.

Stefan and Schiebe suggest that two parameters are useful in determining the type of downstream stratification. The first is defined by

$$\alpha_1 = \frac{E - T_a}{T_a} \quad (9)$$

Positive values of α_1 indicate that a continuous surface layer will form; negative values indicate a plume that will mix into the deeper lake. Stefan defines the second parameter as

$$\alpha_2 = \frac{T_o - E}{T_a}, \quad (10)$$

which is interpreted as "degree of stratification loss" or "buoyancy loss" as the heated water cools to equilibrium temperature. Large values of α_2 indicate that buoyant stratification will be governed by surface cooling. The two stratification types of Fig. III.C.27 may be destroyed by ambient turbulence, surface waves, or currents.

12. The Sinking-plume Phenomenon

A problem important in thermal plume research is the prediction of the winter sinking plume. As noted earlier, a sinking plume can occur if the ambient temperature is less than 4°C, the densest point of water. As an example, consider a freezing lake at 0°C; then plume water between 0 and 8.2°C will be denser than ambient water. Therefore, the plume will sink to a position of neutral buoyancy or until it contacts the lake bottom, where it will spread laterally by negative buoyancy. A sinking plume can also occur when water drawn from the hypolimnion of a deep lake or reservoir is not heated sufficiently to provide a buoyant plume at its level of discharge.

The principal biological effect of the sinking plume concerns the incubation period of fish eggs. This effect has been studied experimentally by Hoglund and Spigarelli²⁶ at the Point Beach Nuclear Power Station. They found that the sinking plume could advance the emergence of lake herring by about seven days.

The negatively buoyant jet is observed in many other problems of pollutant discharge. Any chemical-waste discharge that is heavier than the receiving water will sink. The dispersion and settling of ocean-barge wastes also involves sinking jets.

Available integral and phenomenological models for buoyant surface discharges are unable to treat sinking-plume dispersion. An implicit assumption made in the development of these models is that the buoyant force is always directed upward. Moreover, the data used in model development come from buoyant surface discharges. Any of the differential-numerical models can, however, handle the sinking-plume problem directly, since no algebraic sign is presumed for the buoyancy term in the vertical momentum equation before the equations are solved. The variable direction of buoyant forces in the plume is naturally accounted for in numerical models. The Paul-Lick¹⁴ model has already been applied to sinking-plume problems. Other three-dimensional numerical models can also treat this problem. The proper assignment of turbulence parameters for the sinking plume (buoyant as well as sinking portions) remains a problem, however.

We consider the problem of the proper modeling of the sinking plume as one of the most difficult problems facing plume modelers. A paucity of laboratory and field data on the phenomenon is perhaps the major difficulty in any such modeling effort. Some sparse field data on the winter sinking plume appear in Pipes, Pritchard, and Beer.²⁷ There are several laboratory and analytical studies for single round- or slot-port submerged discharges into a lighter receiving environment. These studies should be of some value for an investigation of the sinking-plume phenomenon, especially if an integral-type model were being developed.

First, Anderson, Parker, and Benedict²⁸ carried out 24 experimental (laboratory) investigations of a submerged round-port sinking-plume discharge and then modified and calibrated the Fan²⁹ and Abraham³⁰ integral models for heated submerged discharges to handle sinking jets. Jain and Pena³¹ modified the Fan-Brooks³² integral models for single round- and slot-port submerged discharges in a stagnant environment to treat the submerged sinking plume. Koh and Chang³³ developed an integral model for a sinking plume discharged from an ocean barge. The effects of an ambient horizontal current, density stratification, and diffusion (in later stages of dispersion) were simulated. A region of bottom interaction was considered, and the possibility of a collapse phase preceding the region of bottom interaction was allowed. These submerged-sinking-plume studies provided a starting point for sinking-plume model development. The acquisition of more experimental data should be a top priority, however.

13. Natural Temperature Fluctuations in a Thermal Plume

Data taken at the Piacenza Plant on the Po River in Italy show significant temperature oscillations in specific regions of the jet. Rota and

Zavatarelli³⁴ measured these temperature oscillations by recording ~30 temperature measurements at intervals of 2 sec at numerous points of lateral plume cross sections. They determined the average value and standard deviation of measurements at each point and compared the average temperatures (see Figs. III.C.28 and 29) while varying lateral distance from shore (Y direction) and keeping the longitudinal (X) and vertical (Z) distances constant. In

Fig. III.C.28

Variation in Excess Temperature $\Delta\bar{T}$ as a Function of Lateral Distance Y for Longitudinal Distances $X = 16, 38$, and 64 m and Depth $Z = 0.60 \text{ m}$. (Adapted from Rota and Zavatarelli.³⁴)

Fig. III.C.29

Example of Variation of Excess Temperature $\Delta\bar{T}$ as a Function of Y for a Fixed Value of X and Z with a Band of Temperature Oscillation. (Adapted from Rota and Zavatarelli.³⁴)

Fig. III.C.28, standard deviations of each experimental point are represented by vertical segments of height 2δ having their center coincident with the average value of the point referred. In Fig. III.C.29, the shaded area represents the noted temperature oscillations in a lateral cross section.

The largest δ corresponds to the points of strongest oscillations, which are found in the zone of largest thermal gradients. This zone coincided with the region of greatest interaction of plume and ambient water. Rota and Zavatarelli found δ to be a good parameter for determining the edge of the jet. Significantly, they also found that, for points in which more than 30 temperature measurements were made, the average temperature and standard deviation varied very little, i.e., only a few percent with the increase in the number of measurements. The value of the two parameters stabilized to within $\pm 1\%$ for sampling of more than 60 measurements. Rota and Zavatarelli do not explain these oscillations, i.e., whether they are random or systematic, or whether they are thermal fronts. They are continuing their work in this area.

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D. Integral Models

The integral model has been the most widely used technique for the study of the near field of heated surface jets. Probable reasons for this are its greater simplicity relative to the numerical methods and its well-known success in treating classical nonbuoyant jets. Yet, the comparisons with field and laboratory data made to date have been largely disappointing. The "add-on" method of including key phenomena into the theory appears to be at least partly responsible. Here, an important prototype effect is treated as causing only a small perturbation of the dominant jetlike character of discharge. Certainly, a second factor in the poor performance of existing models is their failure to include all important prototype phenomena. However, certain phenomena conflict with the basic jet idealization and therefore can never be handled by an integral model. Thus, the integral model is somewhat limited in its applicability and should be restricted to those situations closely matching its idealizations. There has been an apparently irresistible tendency over the past several years to extend integral models to situations for which they were not designed. In our opinion, the integral approach has been extended to its limit and, in some instances, well beyond its limit of applicability.

Our discussion of the integral model begins with a review of certain results from classical jet theory. The common methods of extending the classical theory to the study of heated surface jets are then described in an effort to show how the integral model has evolved into its current state of development.

1. Equations Governing a Classical Jet

The submerged nonbuoyant jet is a classic type of free turbulent flow. For this reason, it has been widely studied both analytically and experimentally. Related flows are the jet boundary, the free turbulent wake, and the release of fluid into a coflowing ambient of greater velocity. The subject of free turbulent flows is covered in several standard texts, the treatise by Schlichting¹ being probably the best known.

A classical nonbuoyant jet results when fluid is discharged through either a round or slot orifice into an infinite body of ambient fluid at rest and of the same temperature as the discharge. Due to the ideally discontinuous velocity (infinite shear) at the edge of an issuing jet, the flow passes through an initial zone of flow establishment where the lateral jet-velocity profile attains dynamic equilibrium and the flow becomes established. The zone of flow establishment is discussed in Sec. III.D.6. We now restrict our attention to the zone of established flow and, for convenience, to two-dimensional, steady jets.

Assuming incompressibility, the general equations governing two-dimensional, steady jets are

$$\left. \begin{array}{l} \nabla \cdot \vec{V} = 0 \\ \text{and} \\ \rho \vec{V} \cdot \nabla \vec{V} = -\nabla p + \nabla \cdot \bar{\tau} \end{array} \right\} \quad (1)$$

where

$\vec{V}$ = vector velocity,

ρ = density,

p = pressure,

and

$\bar{\tau}$ = stress tensor.

The same set of equations applies to any steady, incompressible, isothermal flow. An analytic solution is not possible for the general case. Fortunately, for a classical jet, certain simplifications can be made that allow an approximate solution to be derived analytically. This follows the approach commonly used in the study of boundary layers developing over plates, wedges, wings, etc., and is, for that reason, usually referred to as the boundary-layer approximation. The boundary-layer equations are typically derived by scaling the various terms in Eqs. 1 and 2 and then eliminating those of small magnitude. The details of these arguments are not considered here, since they are given in a number of standard texts on fluid mechanics. The essence of the boundary-layer approximation is as follows.

a. Longitudinal gradients of stress are assumed to be negligible, and momentum transport along the jet centerline is assumed to be dominated by advection. Thus, upstream conditions can and will influence downstream flow, but downstream conditions cannot influence upstream flow. In this sense, the boundary-layer approximation makes the problem spatially parabolic, i.e., an initial-value problem.

b. Pressure is taken as approximately constant throughout the entire solution domain. In treatment of the true boundary layer, due to friction losses at the boundary (wall), longitudinal pressure gradients must be considered. In free turbulent flow, such losses are absent, and no accounting is made of pressure.

c. With the elimination of pressure as a dependent variable, the set of equations is overspecified. Therefore, the y-momentum equation is not considered in the solution.

If the above assumptions are applied, Eqs. 1 and 2 become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

and

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \tau_{xy}, \quad (4)$$

where x and u are, respectively, the spatial coordinate and velocity along the jet axis; y and v are, respectively, the spatial coordinate and velocity perpendicular to the jet axis; and τ_{xy} represents the xy component of the stress tensor. For the classical jet, the appropriate boundary conditions are:

Ambient at Rest

$$u; \tau_{xy} \rightarrow 0 \quad \text{as } y \rightarrow \pm\infty. \quad (5)$$

Symmetry

$$v; \frac{\partial u}{\partial y} = 0 \quad \text{at } y = 0. \quad (6)$$

Alternative forms of the x -momentum equation are easily derived. One alternative form of Eq. 4 obtained through the use of continuity (Eq. 3) is

$$\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \tau_{xy}. \quad (7)$$

This so-called conservative form is sometimes preferred in finite-difference solutions of the boundary-layer equations. Also, an integral form of the x -momentum equation can be derived by integrating Eq. 7 over $y = -\infty$ to $y = +\infty$ and applying the boundary conditions. Thus,

$$\frac{d}{dx} \int_{-\infty}^{\infty} \rho u^2 dy = 0. \quad (8)$$

The intuitive interpretation of Eq. 8 is that the total integrated longitudinal-momentum flux through any jet cross section is constant. This must be true, since no mechanism for addition or loss of momentum is considered.

To solve the governing equations, we must relate stress to the local flow field. This is commonly done through the use of an eddy diffusivity as follows:

$$\tau_{xy} = \epsilon(x) \frac{\partial u}{\partial y}. \quad (9)$$

Here, we have made the customary assumption that the eddy diffusivity is constant across each jet cross section. Substituting this expression into Eq. 4, we have for the simplified equations governing the classical jet

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (10)$$

and

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \epsilon \frac{\partial^2 u}{\partial y^2} \quad (11)$$

It is well known that boundary-layer equations of this type admit similarity solutions, i.e., solutions of the form

$$u(x, y) = U(x)f[y/b(x)], \quad (12)$$

where U is the local centerline velocity and b is a local characteristic jet width. If a similarity solution exists, then the problem of describing the flow reduces to that of finding the one-variable functions U , b , and f .

For the simple case of the classical two-dimensional jet, equations can be derived for these functions by substituting the similarity form into the governing equations and separating variables. However, for heated surface jets where more terms are present in the governing equations, the usual approach is to assume forms for f and b and then derive an equation for U based on the application of the law of conservation of momentum. We refer to the former technique as a similarity method and the latter as an integral method. These labels are used in lieu of better terminology by which to distinguish the two approaches. Section 2 below briefly describes the application of the similarity method to the classical two-dimensional jet in an effort to illustrate several important modeling concepts.

2. Similarity Solutions for Plane and Axisymmetric Jets

The similarity solutions for the plane and axisymmetric jets are outlined in Tables III.D.1 and 2, respectively. The results here are very much like those of the true boundary layer. A third-order ordinary differential equation is developed for the similarity profile. (Unlike many boundary-layer problems--the Falkner-Skan Problem, for instance--the ordinary equation may be integrated in closed form.) Then a pair of first-order equations are developed for the characteristic velocity and width. Once a form of the eddy diffusivity is specified, these last two equations can be solved as an initial-value problem. The choice of diffusivity traditionally made in the treatment of the classical jet follows Prandtl's second hypothesis that

$$\epsilon = \kappa Ub, \quad (13)$$

where κ is a constant.

TABLE III.D.1. Obtaining a Similarity Solution
for a Plane Nonbuoyant Jet

1. Begin the solution by stating the governing equations and boundary conditions.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (A)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \epsilon(x) \frac{\partial^2 u}{\partial y^2} \quad (B)$$

Ambient at Rest

$$u \rightarrow 0 \text{ at } y \rightarrow \pm\infty \quad (C)$$

Symmetry

$$v = 0 \text{ at } y = 0 \quad (D)$$

$$\partial u / \partial y = 0 \text{ at } y = 0 \quad (E)$$

2. Transform variables to simplify notation.

$$\eta = y/b(x) \quad (F)$$

$$F = \int f(\eta) d\eta. \quad (G)$$

Continuity becomes

$$\frac{\partial u}{\partial x} + \frac{1}{b} \frac{\partial v}{\partial \eta} = 0. \quad (H)$$

Conservation of x momentum becomes

$$u \frac{\partial u}{\partial x} + \frac{v}{b} \frac{\partial u}{\partial \eta} = \frac{\epsilon}{b^2} \frac{\partial^2 u}{\partial \eta^2}. \quad (I)$$

The similarity form becomes

$$u(x, y) = U(x)F'(\eta). \quad (J)$$

3. Deduce from the conservation of integral momentum flux ($U^2 b = \text{constant}$) that

$$U'b = -\frac{1}{2} Ub'. \quad (K)$$

TABLE III.D.1 (Contd.)

4. Integrate continuity to obtain the similarity form for v .

$$v = Ub'(\eta F' - \frac{1}{2}F). \quad (L)$$

5. Substitute the similarity forms into the conservation of momentum equation to derive the basic equation for F .

$$(F')^2 + FF'' + \frac{1}{2}\gamma^2 F''' = 0, \quad (M)$$

where

$$\gamma = \sqrt{\frac{4\epsilon}{Ubb'}}. \quad (N)$$

For similarity to hold, γ must be constant so that the basic equations governing U and b are

$$\frac{db}{dx} = \frac{1}{\gamma^2} \frac{4\epsilon}{Ub} \quad (O)$$

and

$$\frac{dU}{dx} = -\frac{1}{\gamma^2} \frac{2\epsilon}{b^2}. \quad (P)$$

6. Determine boundary conditions for F , U , and b .

$$\text{From } u \rightarrow 0 \text{ at } y \rightarrow \pm\infty$$

$$F \rightarrow 0 \text{ as } \eta \rightarrow \infty. \quad (Q)$$

$$\text{From } v = 0 \text{ at } y = 0$$

$$F = 0 \text{ at } \eta = 0. \quad (R)$$

$$\text{From } \partial u / \partial y = 0 \text{ at } y = 0$$

$$F'' = 0 \text{ at } \eta = 0. \quad (S)$$

Initial Conditions on Region of Established Flow

$$U = U_0 \text{ at } x = x_0 \quad (T)$$

and

$$b = b_0 \text{ at } x = x_0. \quad (U)$$

TABLE III.D.1 (Contd.)

7. Integrate Eq. M three times to obtain

$$F(\eta) = \gamma \tanh^2(\eta/\gamma) \quad (V)$$

and

$$F'(\eta) = \operatorname{sech}^2(\eta/\gamma). \quad (W)$$

8. Evaluate γ , by defining b such that

$$\frac{u(x, b)}{U(x)} = \frac{1}{2}. \quad (X)$$

Then

$$\gamma = 1.125, \quad (Y)$$

$$\frac{db}{dx} = 3.16 \frac{\epsilon}{Ub}, \quad (Z)$$

and

$$\frac{du}{dx} = -1.58 \frac{\epsilon}{b^2}. \quad (AA)$$

9. Obtain the solution.

$$u(x, y) = U(x) \operatorname{sech}^2\left(0.889 \frac{y}{b}\right) \quad (AB)$$

$$v(x, y) = U(x)b'(x)\left[(y/b)\operatorname{sech}^2(0.889 y/b) - 0.563 \tanh^2(0.889 y/b)\right], \quad (AC)$$

where u and b are the solutions of Eqs. AA and Z, respectively.

TABLE III.D.2. Obtaining a Similarity Solution
for an Axisymmetric Nonbuoyant Jet

1. Begin the solution by stating the governing equations and boundary conditions.

$$\frac{\partial u}{\partial x} + \frac{1}{y} \frac{\partial}{\partial y}(yv) = 0 \quad (A)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\epsilon(x)}{y} \frac{\partial}{\partial y}\left(y \frac{\partial u}{\partial y}\right). \quad (B)$$

TABLE III.D.2 (Contd.)

Ambient at Rest

$$u \rightarrow 0 \text{ at } y \rightarrow \pm\infty. \quad (C)$$

Symmetry

$$v = 0 \text{ at } y = 0 \quad (D)$$

$$\partial u / \partial y = 0 \text{ at } y = 0. \quad (E)$$

2. Transform variables to simplify notation

$$\eta = y/b \quad (F)$$

$$F = \int \eta f(\eta) d\eta. \quad (G)$$

Continuity becomes

$$\frac{\partial u}{\partial x} + \frac{1}{b} \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta v) = 0. \quad (H)$$

Conservation of x momentum becomes

$$u \frac{\partial u}{\partial x} + \frac{v}{b} \frac{\partial u}{\partial \eta} = \frac{\epsilon}{b^2} \frac{1}{\eta} \frac{\partial}{\partial \eta} \left(\eta \frac{\partial u}{\partial \eta} \right). \quad (I)$$

The similarity form becomes

$$u(x, y) = U(x) F'(\eta) / \eta. \quad (J)$$

3. Deduce from the conservation of integral momentum flux ($U_b = \text{constant}$) that

$$U' b = -U b' s. \quad (K)$$

4. Integrate continuity to obtain the similarity form for v.

$$v = U b' (F' - F/\eta). \quad (L)$$

5. Substitute the similarity forms into the conservation of momentum equation to derive the basic equation for F.

$$F F'' / \eta + (F')^2 / \eta - F F' / \eta^2 = \beta (F' / \eta - F''), \quad (M)$$

TABLE III.D.2 (Contd.)

where

$$\beta = \frac{\epsilon}{U b b'}. \quad (N)$$

For similarity to hold, β must be constant so that the basic equations governing U and b are

$$\frac{db}{dx} = \frac{1}{\beta} \frac{\epsilon}{U b} \quad (O)$$

and

$$\frac{dU}{dx} = -\frac{1}{\beta} \frac{\epsilon}{b^2}. \quad (P)$$

6. Determine boundary condition for F , U and b .

From $u \rightarrow 0$ at $y \rightarrow \pm\infty$

F' finite as $\eta \rightarrow \infty$. (Q)

From $u \rightarrow U$ as $y \rightarrow 0$

$F'/\eta = 1$ as $\eta \rightarrow 0$. (R)

From $v = 0$ at $y = 0$

$F/\eta \rightarrow 0$ as $\eta \rightarrow 0$. (S)

Initial Conditions on Region of Established Flow

$U = U_0$ at $x = x_0$ (T)

and

$b = b_0$ at $x = x_0$. (U)

7. Integrate Eq. M three times to obtain

$$F(\eta) = \frac{1}{2} \frac{\eta^2}{1 + \eta^2/(8\beta)} \quad (V)$$

and

$$F'(\eta)/\eta = \frac{1}{1 + \eta^2/(8\beta)^2}. \quad (W)$$

TABLE III.D.2 (Contd.)

8. Evaluate β , by defining b such that

$$\frac{u(x, b)}{U(x)} = \frac{1}{2} \quad (X)$$

Then

$$\beta = 0.302, \quad (Y)$$

$$\frac{db}{dx} = 3.31 \frac{\epsilon}{Ub}, \quad (Z)$$

and

$$\frac{dU}{dx} = -3.31 \frac{\epsilon}{b^2}. \quad (AA)$$

9. The solution.

$$u(x, y) = U(x)[1 + 0.414(y/b)^2]^{-2} \quad (AB)$$

$$v(x, y) = U(x)b'(x)\frac{y}{2b}[1 + 0.414(y/b)^2]^{-2}[1 - 0.414(y/b)^2], \quad (AC)$$

where U and b are the solutions of Eqs. AA and Z, respectively.

From the results of Tables III.D.1 and 2, it is readily observed that such a hypothesis leads to a constant spreading rate given by

$$\frac{db}{dx} = 3.16\kappa \quad (14)$$

for the plane jet and by

$$\frac{db}{dx} = 3.31\kappa \quad (15)$$

for the axisymmetric jet. Experimental data² have shown that jets do indeed spread linearly with

$$\frac{db}{dx} = 0.116 \text{ and } \kappa = 0.0366 \quad (16)$$

for the plane jet and

$$\frac{db}{dx} = 0.0847 \text{ and } \kappa = 0.0256 \quad (17)$$

for the axisymmetric jet. The similarity solution further shows that

$$U \sim x^{-1/2} \quad (18)$$

for the plane jet and

$$U \sim x^{-1} \quad (19)$$

for the axisymmetric jet. Moreover, the experimental data² show that, for $y/b < 1.5$, the shape of the velocity profile is accurately given by f . Note that use of a constant eddy diffusivity for a plane jet leads to the erroneous conclusion that $b \sim x^{2/3}$.

An interesting consequence of the similarity solution is that the longitudinal mass flow rate increases along the jet centerline (as $x^{1/2}$ for the plane jet and as x for the axisymmetric jet). This growth in mass flux with its corresponding diluting effects on momentum is the result of entrainment, i.e., fluid drawn from the ambient into the jet.

Entrainment is often treated with an entrainment velocity and an associated entrainment coefficient as follows. The jet is conceived as having a boundary at some point on the lateral profile ($y = b$ is a convenient choice). The lateral velocity at that point is then the entrainment velocity, and the entrainment velocity normalized by the centerline velocity is called the entrainment coefficient. To illustrate this point with a simple example, consider the boundary of the jet to be $y = b$. Then for the plane jet, the entrainment coefficient α is given by

$$\begin{aligned} \alpha(x) &= b'[F'(1) - \frac{1}{2}F(1)] \\ &= 0.219b' \\ &= 0.0254, \end{aligned} \quad (20)$$

and for the axisymmetric jet, α is given by

$$\begin{aligned} \alpha(x) &= b'[F'(1) - F(1)] \\ &= 0.146b' \\ &= 0.0124. \end{aligned} \quad (21)$$

Thus, for both the plane and axisymmetric jets, the entrainment coefficient is constant. The concept of the entrainment coefficient is returned to in Sec. III.D.4b.

3. The Integral Approach for Nonbuoyant Jets

A somewhat easier method of obtaining the basic jet characteristics than the similarity method described in Sec. 2 above involves the use of the conservation laws in integral form. For classical nonbuoyant jets, the only such law not trivially satisfied is the integral form of the conservation of momentum. Equation 8 gives the appropriate mathematical statement for two dimensions. For the more general case of three dimensions, we have

$$\frac{d}{dx} \int_A \rho u^2 dA = 0. \quad (22)$$

where A is a plane region containing the jet cross section. As before, no sources or sinks of momentum are considered. With the integral method, one assumes a similarity form for the velocity such as

$$u(x, y, z) = U(x)F[y/b(x)]G[z/h(x)], \quad (23)$$

where the functions F and G are assumed to be known, usually based on experimental data. Upon substituting Eq. 23 into Eq. 22 and performing the indicated integrations, one obtains

$$\frac{d}{dx} [U^2(x)b(x)h(x)] = 0. \quad (24)$$

The similarity forms F and G have disappeared from the equation, indicating that the exact functions chosen are relatively unimportant. This is also true for the more general case if one is careful in defining macroscopic jet parameters.

Unfortunately, there are now three dependent variables-- U , b , and h --and only one equation, so that further assumption is necessary. Any two additional relations may be specified, but, in general, selection is made from among the following three alternatives:

- a. Specify a form for db/dx , the rate of lateral jet growth.
- b. Specify a form for dh/dx , the rate of vertical jet growth.
- c. Specify an entrainment function for use in an equation of dilution. The equation of dilution is obtained by setting the rate of growth in longitudinal mass flux equal to the rate of mass entrainment through the jet boundary. Thus,

$$\frac{d}{dx} \int_A \rho u dA = \int_C \rho V_e dC, \quad (25)$$

where C is the circumference of the jet and V_e is the local entrainment velocity. Typically, the entrainment velocity is related to the local jet centerline velocity through an entrainment coefficient $\alpha(x, y, z)$ as before:

$$V_e(x, y, z) = \alpha(x, y, z)U(x). \quad (26)$$

The two-dimensional integral models implicitly assume $dh/dx = 0$ and then use either a specified spreading rate (e.g., Pritchard³ Model No. 1) or an entrainment coefficient (e.g., Motz-Benedict⁴ model). For three-dimensional models, the customary approach is to specify a spreading rate db/dx and an entrainment coefficient α . This procedure is the result of the way buoyancy is included in these models (increased lateral spreading and reduced vertical entrainment). We shall return to this point in Sec. III.D.4.b.

Once the appropriate functional forms are decided upon, one or more ordinary differential equations are derived. These equations are solved either analytically (usually done for only the simplest cases) or with one of the standard numerical schemes such as a Runge-Kutta or a Predictor-Corrector method.

4. Extending the Integral Formalism to the Study of Heated Surface Jets

Sections 2 and 3 above have outlined, in the most simplistic terms, the formulation of an integral model for the classical free completely turbulent jet. This section treats the more practically important question of how such simple ideas are extended to buoyant surface jets. Here, however, the treatment departs from the experimentally based and universally accepted success of the nonbuoyant theory. The treatment often becomes highly intuitive, particularly when treating the prototype case, since it is usually necessary to schematize many potentially important effects so that an integral model may be applied. Indeed, the highly intuitive analysis discussed below and the attendant need for much schematizing of the prototype problem are serious shortcomings of the integral approach.

a. Adding Temperature to the Model

The first step in extending nonbuoyant jet theory to buoyant surface jets is adding temperature as an independent variable and simultaneously developing a conservation-of-heat equation. Again, starting with the basic integral conservation law, we have

$$\frac{dH}{dx} = Q, \quad (27)$$

where H is the integral heat flux given by

$$H = \int_A \rho C_p u T \, dA \quad (28)$$

C_p = specific heat,

T = local temperature, (29)

$$Q = \int_C q \, dC,$$

and

q = rate of heat influx across the boundary C .

Since the heat-energy equation is linear, it is permissible and usually more convenient to write it in terms of the temperature excess ΔT given by

$$\Delta T = T - T_E, \quad (30)$$

where T_E is a suitably chosen reference temperature.

If one is particularly fortunate, the temperature profiles will be similar to those of velocity. If one is even more fortunate, the presence of temperature gradients and the associated buoyancy forces will not destroy the velocity similarity. This latter circumstance, of course, voids the applicability of an integral model. Obviously, in the limit of very small temperature differences, indicated by very large initial densimetric Froude numbers, the similarity of velocity will not be appreciably affected. Also, in the limit of a highly buoyant discharge, where advection does not dominate, it is doubtful that jetlike flow really exists. In the middle, where most prototype plumes fall, there is no definite answer; there is no universally accepted criterion with which to judge if a jet exists.

Yet another complicating factor is the change in density caused by the elevated temperature. This is normally disregarded, however, in accordance with the Boussinesq approximation. Assuming the best of all possible worlds (no change in the similarity of velocity, similar temperature profiles, and negligible density effects on the advection terms), one can derive an ordinary differential equation governing temperature decay, as was previously done for velocity.

First, a suitable similarity form must be chosen for the temperature profile across the jet. The mixing-length hypothesis tells us that the nondimensional temperature and velocity profiles should be identical. Taylor's Vorticity Theory, which gives results identical to the mixing-length hypothesis for momentum transport, indicates that the similarity form for temperature should be the square root of that of velocity, and there should therefore be a more rapid lateral transport of heat as compared with momentum. Experimental evidence indicates that Taylor's Theory is the more nearly

correct and that temperature does, in fact, have a wider profile than velocity. Some integral models (e.g., Motz-Benedict model) fail to recognize this fact and thus give plume isotherms that are far too narrow.

If temperature behaved as a passive tracer, then with the addition of temperature as a dependent variable and an equation of heat conservation, the model of the heated jet would be complete. Unfortunately, density gradients caused by temperature differences, although insignificant in the evaluation of the various integrals, do create buoyancy forces that can be quite significant. It is therefore necessary to alter the treatment of momentum transport previously advanced. This is the subject of Sec. b below.

b. Buoyancy

Buoyancy is generally thought to inhibit, or at least lessen, vertical mixing and to induce horizontal spreading through longitudinal and lateral gradients of the hydrostatic pressure. Not all integral models explicitly handle these dynamic effects, however. For example, the two-dimensional integral models such as that of Motz and Benedict assume that buoyancy, or perhaps a shallow bottom, restricts vertical mixing completely. Some inclusion of buoyant lateral spreading is undoubtedly made in calibrating the model. The three-dimensional integral models generally treat buoyancy differently in the three coordinate directions of the jet. In the vertical direction, the entrainment coefficient is reduced, usually in accordance with the experimental results of Ellison and Turner.⁵ Increased lateral spreading is included by adding an additional term to the usual nonbuoyant spreading rate. Longitudinally, a pressure-gradient term is added to the conservation equation of integral momentum. There is some variation in approach from model to model (e.g., the model of Stefan, not reviewed, does not include a longitudinal pressure force), but the ideas are roughly the same.

Before discussing each of these items in more detail, let us consider a unique exception offered by the quasi-two-dimensional, quasi-integral model developed by Pritchard³ (No. 1). This model is basically two-dimensional, nonbuoyant jet theory with a fixup for vertical entrainment. This fixup is based on Pritchard's observation that lake plumes tend to be about 10 ft deep asymptotically, and that, if not constrained by the bottom, the plume will indeed mix vertically until an asymptotic plume depth of 10 ft is reached. The principal shortcoming of this approach is that the asymptotic plume depth is not universally 10 ft, but rather depends on many factors. Pritchard⁵ tried to improve this formulation in developing his second model, although it is not yet clear how successfully this was done.

Experiments with application to the vertical entrainment of heated surface jets have been carried out by Ellison and Turner,⁶ Lean and Whillock,⁷ Stefan and Schiebe,⁸⁻⁹ Kato and Phillips,¹⁰ and Wada.¹¹ Ellison and Turner found that the entrainment coefficient e defined by

$$e = \frac{1}{U} \frac{d}{dx}(Uh), \quad (31)$$

where

U = mean velocity of the jet

and

h = average depth of the jet,

is a monotonically decreasing function of the local Richardson number Ri , defined in Chapter I. Their experimental results, also given in Chapter I, may be approximated by any of several expressions. One example has been noted. Others are

$$e = \begin{cases} 0.075[2/(1 + Ri/0.85) - 1] & 0 < Ri \leq 0.85 \\ 0 & \text{otherwise,} \end{cases} \quad (32)$$

and

$$e = \begin{cases} 0.075 \frac{\exp(-5Ri) - \exp(-4)}{1 - \exp(-4)} & 0 \leq Ri \leq 0.85 \\ 0 & \text{otherwise.} \end{cases} \quad (33)$$

These results show that vertical entrainment is negligible for Ri exceeding about 0.85.

Present three-dimensional integral models typically use one of these formulas to account for the effects of buoyancy on vertical entrainment. Not accounted for are the other factors affecting vertical entrainment. For example, Wada¹¹ discovered that vertical entrainment depends not only on the local Richardson number, but also on the slope of the jet/ambient interface. He observed that entrainment increases as the interfacial slope increases for a fixed Richardson number and that no vertical entrainment occurs for interfacial slopes less than 0.04.

Stefan¹² therefore questions whether the entrainment concept as developed by Ellison and Turner for nearly uniform flow along a sloping boundary (and used by the popular three-dimensional integral models) can be applied to rapidly varying flows such as often occur in heated surface jets. Stefan quotes Charles and Lilleleht:¹³ "This theory (Ellison and Turner) predicts that in most practical cases the layer will rapidly attain an equilibrium state in which the Richardson number does not vary with distance downstream and the gravitational force on the layer is just balanced by the drag due to

entrainment together with friction on the floor or roof." Stefan concludes that the flow from a horizontal channel into a receiving body of water is essentially a nonuniform flow, making inappropriate the use of a local entrainment coefficient as described above.

Most of the popular integral models do not consider the effects of buoyancy on lateral entrainment. Yet, the investigations carried out by Abraham,¹⁴ Hoult and Weil,¹⁵ and Ooms¹⁶ have revealed that choosing two lateral entrainment coefficients, one for use in the momentum-dominated region of the plume and the other for the buoyancy-dominated region, yielded better comparisons with data. This approach was thought to eliminate the need to fit the entrainment coefficient with environmental and discharge parameters, as done by Motz and Benedict⁴ and Fan.¹⁷

The second major macroscopic effect of buoyancy generally recognized by the popular three-dimensional integral models is that of increased lateral spreading. The models of Stolzenbach and Harleman,¹⁸ Prych,¹⁹ Stefan,²⁰ and Shirazi and Davis²¹ all treat lateral spreading in a similar fashion. The rate of growth in plume width db/ds is written as the sum of a nonbuoyant and a buoyant spreading rate. Thus,

$$\frac{db}{ds} = \left(\frac{db}{ds}\right)_{nb} + \left(\frac{db}{ds}\right)_b, \quad (34)$$

where the subscripts nb and b are used, respectively, to denote the nonbuoyant and the buoyant contributions. This simple summation neglects the nonlinear interaction possible between buoyancy and entrainment when both processes are effective at the same order of magnitude.

Except for the Stolzenbach-Harleman model, the above integral models write the local buoyant spreading rate as a function of the local densimetric Froude number and, in the case of Shirazi and Davis, the local jet aspect ratio. The formulations are:

Stefan:

$$\left(\frac{db}{ds}\right)_b = \frac{c}{F} \quad (35)$$

Prych:

$$\left(\frac{db}{ds}\right)_b = \frac{c}{\sqrt{F^2 - 1}} \quad (36)$$

Shirazi and Davis:

$$\left(\frac{db}{ds}\right)_b = \frac{c}{\sqrt{(b/h)F^2 - 1}}, \quad (37)$$

where

F = local densimetric Froude number,

b = local width,

h = local depth,

and

c = a constant.

The form used by Stefan is based on an analysis of a spreading density wedge. In adding this result to his integral jet model, Stefan assumes that spreading takes place perpendicular to the jet axis. Prych uses roughly the same analysis, but assumes that spreading occurs perpendicular to the jet boundary. The attendant geometric considerations lead to a slightly increased lateral spreading rate as reflected in the above form.

Shirazi and Davis modified the Prych formulation in an attempt to account for lateral shear. Their approach is loosely linked to an analysis given by Koh and Fan.²² The net effect of the Shirazi-Davis modification is a reduction in lateral spreading. An interesting point is that the Prych and Shirazi-Davis formulations predict an infinite spreading rate at certain low densimetric Froude numbers, although in the Shirazi-Davis model the discontinuity is seldom important.

Stolzenbach and Harleman¹⁸ include buoyant spreading through the use of an integrated y-momentum equation rather than through an explicit expression such as those given above. The analysis proceeds as follows:

(1) Based on an order-of-magnitude analysis of the y-momentum equation in its general form, Stolzenbach and Harleman obtain

$$0 = -g \frac{\partial \eta}{\partial y} - \frac{1}{\rho_a} \frac{\partial p_d}{\partial y} - \frac{\partial v'^2}{\partial y} - \frac{\partial v'w'}{\partial z} \quad (38)$$

for the nonbuoyant surface jet and

$$\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = \frac{\beta g}{\rho_a} \int_z^{-\infty} \frac{\partial \Delta T}{\partial y} dz - \frac{1}{\rho_a} \frac{\partial p_d}{\partial y} - \frac{\partial v'^2}{\partial y} - \frac{\partial v'w'}{\partial z} \quad (39)$$

for the buoyant surface jet, where the symbols take on their usual definitions. (See Sec. II.C for details.) Assuming that the overlapping terms are identical for the buoyant and nonbuoyant jets, Stolzenbach and Harleman subtract Eq. 38 from Eq. 39 to obtain

$$\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = \frac{\beta g}{\rho_a} \int_z^{-\infty} \frac{\partial \Delta T}{\partial y} dz. \quad (40)$$

(2) The integral form of the y-momentum equation is gotten by integrating Eq. 40 over half the jet cross section and applying the boundary conditions. Thus,

$$\frac{d}{dx} \int_{A_{1/2}} uv \, dA = \frac{\beta g}{\rho_a} \int_{A_{1/2}} \int_z^{-\infty} \frac{\partial \Delta T}{\partial y} dz, \quad (41)$$

where $A_{1/2}$ denotes half the jet cross section.

(3) A similarity form is then assumed for v :

$$v = \left(\frac{db}{ds} \right)_{nb} u_c \left(\frac{y - s}{b} \right)^{1/2}, \quad (42)$$

and the indicated integrations of Eq. 41 are carried out.

The validity of Eq. 42 is certainly open to dispute. Unfortunately, there is no method by which to test its validity independently of overall model performance.

Comparisons between model predictions and both laboratory and field data generally show that lateral spreading is greatly overpredicted by these formulations. Even with the substantial reductions incurred by Shirazi and Davis with the use of the local aspect ratio, these latter investigators still found it necessary to have a multiplicative factor by which to calibrate the model with existing data. In our opinion, the treatment of lateral spreading is a serious weakness of all these existing three-dimensional integral models.

The third and last effect of buoyancy is the longitudinal pressure force that should be included in the conservation-of-momentum equations. The longitudinal pressure is usually given by

$$P(x) = \int_A \int_{-\infty}^z \Delta \rho g \, dz' \, dA. \quad (43)$$

Stolzenbach and Harleman, Prych, and Shirazi and Davis all consider the force associated with the horizontal gradients of this hydrostatic pressure field, but the model by Stefan does not. From his study of various integral-model formulations, prior to the development of the Shirazi-Davis model, Shirazi²³ concluded that longitudinal gradients of the hydrostatic pressure can be important for low-initial-densimetric-Froude-number discharges and that the omission of it by Stefan is a serious shortcoming.

5. Jets in a Crossflow

In setting up an integral model, one usually idealizes the ambient current as steady, horizontally and vertically uniform, and shore-parallel, although off-shore variations are easily handled, since model equations are ordinarily integrated as an initial-value problem. It was noted in Sec. III.C that such an idealization of the current is grossly oversimplified. Furthermore, most integral models assume that the bending of the jet trajectory is the only effect of the ambient crossflow, although Motz and Benedict correlate their entrainment coefficient with the magnitude of the ambient current.

Bending is generally treated as the result of two physical effects: (1) entrainment of ambient momentum and (2) form drag associated with the ambient fluid being forced under and around the jet. The entrainment of ambient momentum refers to the fact that ambient water drawn into the jet during the dilution process possesses momentum in the direction of the current and thus can affect the momentum flux both parallel and perpendicular to the jet axis. This mechanism is by far the more significant of the two.

Form drag on the jet is typically treated by using the standard drag formula for a solid body in crossflow modified to account for the fact that some ambient fluid mixes into the jet rather than moving under and around the jet, as would be the case for a solid body. When a blunt, solid body is subjected to a turbulent crossflow of fluid, a net pressure force develops across the body in the direction of flow due to the deceleration of fluid on the side exposed to the flow and to separation and eddying (wake) on the rear side. The integral models assume (1) that a similar situation arises when a surface or submerged jet experiences a crosscurrent, but (2) that the increase in pressure is somewhat less than it would be at the wall of a solid body of the same form as the curved jet, due to the mixing of fluid into the jet. The contributions of dynamic pressure to the overall pressure field are essentially unknown, as no one, to our knowledge, has successfully measured total pressure either in a hydraulic model or a prototype situation to the precision required. The best direct pressure measurements available have been made for gas jets of fairly high velocity, for which the measurement of pressure is a fairly standard procedure.

In developing his integral model, Carter²⁴ defines the drag force across the jet by

$$\text{drag force per unit length of jet axis} = \frac{\rho_a C_D A u_a^2}{2}, \quad (44)$$

where

ρ_a = density of ambient fluid,

C_D = coefficient of drag,

A = cross-sectional area normal to the jet centerline,

and

u_a = ambient current.

Other forms used in integral models for both surface and submerged discharges replace u_a^2 by a multiple of $u_a^2 \sin \beta$ or $u_a^2 \sin^2 \beta$, where β is the angle between the jet and ambient current. Such forms for the drag are not equivalent. The use of form-drag formulas to account for jet bending represents only a very crude approximation to the actual situation. In fact, these formulas merely provide an additional coefficient, C_D , for fitting the models to data.

6. The Region of Flow Establishment

The region of flow establishment refers to the initial region of the jet beginning at the discharge port, in which the initial velocity (and temperature) profiles develop into self-similar profiles. For descriptive purposes, consider a classical, nonbuoyant axisymmetric jet. Intense mixing due to shear begins around the circumference of the discharge port and jet boundary and gradually eats its way inward toward the jet centerline. Simultaneously, longitudinal momentum is transported laterally due to self-induced jet turbulence. As a result of these two interactive phenomena, the jet boundary grows. At about 4-6 outfall diameters along the jet axis, the mixing reaches the centerline and the velocity profile becomes established. The so-called potential core, where mixing has not yet reached, is conical in shape, its base coinciding with the discharge port and its apex lying in the jet cross section that marks the end of flow establishment. The jet boundary has a megaphone shape, the small end coinciding with the discharge port and the large end forming the edge of the jet at the end of flow establishment. The region of the jet outside the potential core is commonly referred to as the shear region.

The meaning of flow establishment is complicated somewhat for heated surface jets, since the integral analysis applies only to the region in which both temperature and velocity are self-similar. Conceptually, one profile can become self-similar before the other, and consequently one decay can be observed along the centerline before the other. Moreover, with the presence of buoyancy, mixing acts differently in the vertical and horizontal directions. Therefore, profiles may be similar in one direction and not in the other. These ambiguities make the classic meaning of flow establishment somewhat confusing.

For prototype surface discharges of condenser cooling water, the situation is further exacerbated by the fact that velocity profiles at the outfall are not uniform, as uniform profiles are assumed in the classic visualization of flow establishment. For example, Fig. III.D.1 shows a typical outfall-velocity profile as measured at the Point Beach Nuclear Power Station and reported by Frigo and Frye.²⁵ Clearly, velocity is not uniform across the

Fig. III.D.1

Three-dimensional Representation of the Outfall-velocity Profile at the Point Beach Nuclear Power Station (Unit 1)

outfall. Other sites have similar problems. For example, as discussed in Sec. IV.E, a cold-water wedge intrudes into the Oskarshamnsverket discharge. The experimental observation that some form of fully developed open-channel flow exists in most discharge channels has led Stefan²⁶ to coin the term zone of flow "re-establishment." In contrast to velocity, temperature profiles are generally found to be uniform in the discharge channel.

Assuming that a region of flow establishment exists in the classical sense, there are two fundamental ways of including it in an integral model: experimentally and analytically. Experimental results with emphasis on the region of flow establishment are given by Motz and Benedict,⁴ Shirazi and Davis,¹⁸ and

Stefan et al.²⁶ Analytical treatments of flow establishment are given by Stolzenbach and Harleman¹⁸ and McLay et al.²⁷

Motz and Benedict developed phenomenological relationships for the length of the region of flow establishment s'_e and the angle between the jet and the ambient current at the end of flow establishment β . In laboratory tests, the ratio A of the ambient velocity to discharge velocity was varied from 0.18 to 0.73 and the discharge angle β'_0 was set successively at 90, 60, and 45°. Curves were then drawn for s'_e/b_0 and β_0/β'_0 as functions of A , where b_0 is the half-width of the jet at the end of flow establishment and is taken as 1.6 times the discharge half-width. These results were previously presented. (See Figs. II.B.4 and 5.)

Shirazi and Davis based their analysis of flow establishment on a few tests carried out in a stagnant tank at the EPA Laboratory in Corvallis, Oregon. The length of the region of flow establishment S_i was measured for several different aspect ratios A and initial densimetric Froude numbers F_0 . The data were found to be well correlated by

$$S_i = 5.4H_0(A^2/F_0)^{1/3}, \quad (45)$$

where H_0 is the depth of the outfall. It was noted in Sec. II.E that this form suffers in the limit as $F_0 \rightarrow \infty$ and $A \rightarrow \infty$.

By far the most complete study of the region of flow establishment was made by Stefan et al.²⁶ The length of the region of flow establishment, the volumetric flow rate along the jet axis, and the angle between the spreading jet boundaries were correlated with initial densimetric Froude number, channel aspect ratio, and the ratio of the ambient velocity to the discharge velocity. These correlations and their ranges of applicability are summarized in Table III.D.3. In addition, Stefan et al.²⁶ noted the following:

a. A significant deviation from the correlation was noted for a rectangular discharge with an aspect ratio of 2.4, but a semicircular discharge with an aspect ratio of 2 did not show such deviation. Since 2 is a common aspect ratio in outfall design, such behavior could be very significant.

b. Except for narrow, deep outfalls, vertical mixing appeared to dominate horizontal mixing in affecting surface temperatures apparently as the result of large eddies and breaking internal waves.

c. In the presence of a crossflow, large eddies dominate mixing.

d. Buoyancy effects were apparent in the region of flow establishment for initial densimetric Froude number up to about 15; moreover, the initial densimetric Froude number was found to be an important parameter in the experimental results.

TABLE III.D.3. Correlations for the Zone of Flow Establishment due to Stefan et al.²⁶

1. Length of Flow Establishment

Correlation:

$$\frac{S(T)}{h_o} = (16 - 12.8T)^{1.2} A^{0.85-0.44T} \cdot \exp(-0.9R) \left[1 + \frac{0.5F_o - 1.5}{\exp(0.4F_o)} \right]$$

Range of Applicability:

$$1.0 \leq A \leq 9.6 \quad (\text{see discussion for } A \approx 2)$$

$$0 \leq R \leq 0.41$$

$$2.0 \leq F_o \leq 15$$

$$0.8 \leq T \leq 0.98$$

2. Volumetric Flow Rates

Correlation:

$$\frac{Q(s)}{Q_o} = 1 + \left[0.087 \left(\frac{S}{2h_o} \right)^{(2.35+0.75A)/(0.90+A)} \right] \left(1 - \frac{0.52}{F_o^{0.41}} \right)$$

Range of Applicability:

$$1.0 \leq A \leq 9.6$$

$$R = 0$$

$$F_o \geq 1.8$$

$$S < 30h_o \quad (\text{varies with particular case})$$

TABLE III.D.3 (Contd.)

3. Angle of Jet Growth

$$\varphi = \frac{225 + 10F_o}{2.25 + F_o}$$

Range of Applicability:

$$F_o \geq 1.0$$

4. Symbols:

A = aspect ratio, b_o/h_o b_o = channel width F_o = initial densimetric Froude number h_o = channel depth

Q = volumetric flow rate

 Q_o = discharge flow rate

R = ratio of ambient velocity to discharge velocity

S = distance along jet centerline

T = excess temperature ratio

 φ = (defined in sketch)

Theoretical treatments of the region of flow establishment for use with integral models are few. Stolzenbach and Harleman use an integral technique in which four separate regions of the jet are considered, similarity profiles assumed for each, and the solution executed as an initial-value problem, just as in the region of established flow. This procedure and its attendant difficulties are discussed in Sec. II.C. Prych applies the same integral conservation laws to the region of flow establishment as are applied to the region of established flow. The Shirazi-Davis model follows Prych's analysis quite closely, except for some minor alterations. The Prych and Shirazi-Davis formulations are critically summarized in Secs. II.D and E, respectively. McLay et al.²⁷ use a multiple-layer, two-dimensional numerical treatment of flow establishment to obtain initial conditions for the Hoopes-Zeller-Rohlich integral model.²⁸ None of these theoretical treatments have been sufficiently tested to comment on their validity, beyond the statement that if the region of flow establishment is important, then existing experimental data or a hydraulic-model simulation are preferable.

7. Other Phenomena

Various integral-model developers have attempted to schematize potentially important prototype effects into their models. For example, Stolzenbach and Harleman¹⁸ tried to handle bottom interaction, Prych¹⁹ (and Shirazi and Davis²¹ following Prych) derived an expression for including ambient turbulence, and recently, Adams et al.²⁹ attempted to treat attached jets. The difficulty here is that these treatments are often so highly heuristic that they evade theoretical assessment. In some cases, there is practical merit to the treatment, as they can improve another aspect of the model. For example, the inclusion of ambient turbulence by Prych and by Shirazi and Davis tends to improve width predictions. Unfortunately, no amount of schematization or "adding in" of additional effects can ever remove the fundamental limitations of the integral method. Additional schematization can only provide more knobs to turn (calibration parameters) for tuning the model to laboratory and field data.

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E. Numerical Models

Solution of the Navier-Stokes and heat-energy equations by finite-difference techniques provides the most general and promising method for the proper modeling of surface thermal plumes. An excellent summary of numerical methods for the fluid-flow problem is given by Roache.¹ This section reviews the basic governing conservation equations, along with the numerical finite-difference formulations that are presently available in the literature on surface thermal discharges. A general discussion follows on methods of reducing the governing partial differential equations to finite-difference form, along with the practical problems of obtaining a computer solution.

1. Basic Assumptions in Translating Physical Reality into Mathematical Equations

a. Governing Equations

All finite-difference models for surface thermal plumes begin with the traditional form of the three-dimensional, time-dependent equations of turbulent, incompressible (in the Boussinesq sense; see Sec. III.E.1.b) flow and heat transport. Two generic forms of these equations are given in Tables III.E.1 and 2. The models account for the sub-grid-scale motions through specification of the eddy viscosities and eddy thermal diffusivities (see Sec. III.E.1.g). Note that the diffusion terms in Tables III.E.1 and 2 are different. The form given in Table III.E.2 enforces symmetry of the stress tensor. This symmetry is a result of Newton's law of motion, which states that the rate of change of total momentum of an infinitesimal fluid element is equal to the momentum of the external forces acting on the system. As a result of Newton's law of motion, the stress at a point in a continuous material is easily shown to be a symmetric tensor. The different forms of the diffusion terms given in the tables require different rules for the eddy viscosity coefficients, if diffusion is to be represented properly by both forms. Note also that Coriolis forces have been neglected in the two equations sets due to the small spatial scale of the plume. Several assumptions often made in the study of surface thermal plumes are reviewed below along with statements concerning their validity.

b. The Boussinesq Approximation

It was first noted by Boussinesq that if variations in water temperature, say over a range of 10 C°, are small, variations in density are at most 1%. In addition, variations in other coefficients dependent on these density variations are of the same order. For thermal-plume problems, temperature variations are within the above small range. Consequently, in the Boussinesq approximation, one neglects density variations in the inertial terms of the equations of motion, yet considers these variations in the gravity term

TABLE III.E.1. Generalized Governing Equations for Surface Thermal-plume Problems

Conservation of Mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Conservation of Longitudinal Momentum

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left(A_{H1} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(A_{H2} \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left(A_V \frac{\partial u}{\partial z} \right)$$

Conservation of Lateral Momentum

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \frac{\partial}{\partial x} \left(A_{H1} \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left(A_{H2} \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left(A_V \frac{\partial v}{\partial z} \right)$$

Conservation of Vertical Momentum

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + g + \frac{\partial}{\partial x} \left(A_{H1} \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left(A_{H2} \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial z} \left(A_V \frac{\partial w}{\partial z} \right)$$

Conservation of Heat Energy

$$\frac{\partial \Delta T}{\partial t} + u \frac{\partial \Delta T}{\partial x} + v \frac{\partial \Delta T}{\partial y} + w \frac{\partial \Delta T}{\partial z} = \frac{\partial}{\partial x} \left(B_{H1} \frac{\partial \Delta T}{\partial x} \right) + \frac{\partial}{\partial y} \left(B_{H2} \frac{\partial \Delta T}{\partial y} \right) + \frac{\partial}{\partial z} \left(B_V \frac{\partial \Delta T}{\partial z} \right)$$

Equation of State

$$\rho = f(S, T)$$

where

x, y Horizontal coordinates

z Vertical coordinate

u, v, w Components of velocity in x, y, and z directions, respectively

P Local pressure

A_{H1} , A_{H2} Longitudinal and lateral coefficients of eddy viscosity, respectively

A_V Vertical coefficient of eddy viscosity

B_{H1} , B_{H2} Longitudinal and lateral coefficients of eddy thermal diffusivity, respectively

B_V Vertical coefficient of eddy thermal diffusivity

f Function relating temperature and salinity to density in equation of state

g Acceleration due to gravity

S Local salt concentration

T Local temperature

ρ Local density

TABLE III.E.2. Alternative Set of Generalized Governing Equations for Surface Thermal-plume Problems

Conservation of Mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Conservation of Longitudinal Momentum

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + 2 \frac{\partial}{\partial x} \left(\epsilon_{xx} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left[\epsilon_{xy} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + \frac{\partial}{\partial z} \left[\epsilon_{xz} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \right]$$

Conservation of Lateral Momentum

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \frac{\partial}{\partial x} \left[\epsilon_{yx} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \right] + 2 \frac{\partial}{\partial y} \left(\epsilon_{yy} \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left[\epsilon_{yz} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \right]$$

Conservation of Vertical Momentum

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + g + \frac{\partial}{\partial x} \left[\epsilon_{zx} \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \right] + \frac{\partial}{\partial y} \left[\epsilon_{yz} \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) \right] + 2 \frac{\partial}{\partial z} \left(\epsilon_{zz} \frac{\partial w}{\partial z} \right)$$

Conservation of Heat Energy

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{\partial}{\partial x} \left(\alpha_P \epsilon_{xx} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\alpha_P \epsilon_{yy} \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\alpha_P \epsilon_{zz} \frac{\partial T}{\partial z} \right)$$

Equation of State

$$\rho = f(S, T)$$

where

x, y = Horizontal coordinates

z = Vertical coordinate

u, v, w = Components of velocity in x, y , and z directions, respectively

P = Local pressure

ϵ_{ij} = Molecular plus turbulent eddy viscosity coefficients

α_P = Turbulent Prandtl number

g = Acceleration due to gravity

S = Local salt concentration

T = Local temperature

f = Function relating temperature and salinity to density in equation of state

ρ = Local density

of the vertical equation of motion. This assumption implies that, since the density difference $\Delta\rho$ occurs in the inertial terms as

$$\left(1 - \frac{\Delta\rho}{\rho} \right) \frac{Du}{Dt}, \quad (1)$$

where ρ is the variable density, the inertial term may be simplified to Du/Dt , since $\Delta\rho/\rho$ is assumed to be $\ll 1$. However, the term $(\Delta\rho/\rho) \cdot g$ is not neglected

in the vertical equation of motion, since the acceleration resulting from $(\Delta\rho/\rho) \cdot g$ can be significant. In fact, the simulation of stratified layers within a heated plume as well as the often sharp interface between a thermal plume and the underlying cooler water depend on the presence of this term in the governing equations. Hence, in the Boussinesq approximation the density ρ is treated as a constant (some reference density ρ_r) in all places except in the gravitational term.

The Boussinesq approximation is regarded as quite accurate for water jets, since the compressibility effects it eliminates are vanishingly small. Thus, the fluid is treated as incompressible, but of variable density.

c. The Hydrostatic Approximation

The hydrostatic approximation involves the removal of the vertical advection and diffusion terms from the equation of vertical motion. The x- and y-momentum equations remain unaltered. Two different statements of the hydrostatic approximation appear in the literature. The stronger of the two is

$$P = P_0 - \rho_0 g z, \quad (2)$$

where ρ_0 is a constant reference density. This assumption eliminates the possibility of x and y dependency in the pressure and further requires a horizontal water surface. Neither of these conditions is satisfied in an actual jet. This relationship, applicable to fluid statics, is not used in differential-numerical models of surface thermal plumes.

A somewhat weaker statement of the hydrostatic approximation allows x and y pressure gradients and is expressed as

$$\frac{\partial P}{\partial z} = -\rho(x, y, z)g. \quad (3)$$

This form allows determination of pressure by vertical integration of the density field.

If the equations of motion for the jet are nondimensionalized, the pressure and gravity terms become, in the x-momentum equation,

$$-\frac{1}{F'^2} \frac{\partial \bar{P}}{\partial \bar{x}}; \quad (4)$$

in the y-momentum equation,

$$-\frac{1}{F'^2} \frac{\partial \bar{P}}{\partial \bar{y}}; \quad (5)$$

and in the z -momentum equation,

$$-\frac{1}{F'^2} \frac{\partial \bar{P}}{\partial \bar{z}} + \frac{\bar{p}}{F'^2}; \quad (6)$$

where

$$F' = \text{Froude number} = U_0(gd_0)^{-1/2},$$

$$\bar{P} = P/(\rho_0 g d_0),$$

$$\bar{x} = x/d_0,$$

$$\bar{y} = y/d_0,$$

$$\bar{z} = z/d_0,$$

and

$$\bar{\rho} = \rho/\rho_0.$$

If the stronger of the two hydrostatic approximations is applied to the above equations, then Eqs. 4 and 5 both become zero, while Eq. 6 reduces to

$$\frac{\bar{\rho} - 1}{F'^2} = \frac{1}{IF^2} \quad (7)$$

where IF is the densimetric Froude number. If, on the other hand, the weaker approximation is used, Eq. 6 becomes zero and Eqs. 4 and 5 are related to horizontal density gradients. Thus, despite the relatively minor difference between the two forms of the hydrostatic approximation, there is considerable difference in the corresponding effects on the governing equations. All differential-numerical models that employ the hydrostatic approximation use this weaker form. This form is also used in numerous models of coastal, ocean, and lake circulation. Note that this weaker form allows the coupling of hydrodynamics and heat transport through the terms represented by Eqs. 4 and 5.

A question arises as to the validity of the hydrostatic approximation in general surface thermal-plume problems. The strong form is not generally valid, as it ignores buoyant convective motions and decouples flow and heat transport. Based on the realization that the strong form of the hydrostatic approximation requires horizontal planes of constant density, the validity of that approximation may be ascertained by locating the surfaces of constant density and determining their deviation from horizontal.

The weak form of the hydrostatic approximation is valid only for situations in which the vertical scale of motion is small compared to the total vertical dimension of the fluid medium and in which the dynamic contribution to the total pressure field is small. As a check of the weak form of the

hydrostatic approximation, one can examine the scale of vertical motion. In most cases, the use of the weak form of the hydrostatic approximation can be justified by scaling arguments.

In general, we expect the hydrostatic approximation (weak form) to be valid for discharges with small and moderate initial densimetric Froude numbers. Higher-velocity discharges produce significant vertical entrainment near the outfall, involving significant transfer of momentum in the vertical direction and a rapid growth in jet thickness. In this situation, it is doubtful that the vertical advection and diffusion terms in the vertical-momentum equation are negligible compared to the pressure and gravity terms. The full vertical-momentum equation probably should be treated in such cases.

The opposite case of discharges with very low initial densimetric Froude numbers is interesting. The effluent here is very buoyant with significant vertical buoyant convective motions, a thinning of the jet, and augmented lateral spreading. In this situation, we expect that the hydrostatic approximation will be valid, because the vertical motions are induced by the vertical pressure gradient and by gravity (through density coupling and not by the inertial effects that were neglected by dropping the vertical and diffusion terms from the hydrostatic approximation). Based upon calculations made with their numerical model (using the hydrostatic approximation and applying the model to discharges with very low initial densimetric Froude numbers), Paul and Lick² state that vertical velocities in the problems treated are one to two orders of magnitude smaller than are necessary to invalidate the hydrostatic approximation.

Even when the hydrostatic approximation is not made, it is often convenient to define and use a "reduced" pressure, i.e., a pressure that is referenced to the hydrostatic pressure. This is useful because the dynamic components of the pressure are small compared with the hydrostatic part, resulting in reduced accuracy in the numerical calculation of the dynamic component. If the reduced pressure P_r is defined by

$$P_r = P - \rho_0 g z, \quad (8)$$

where P is the actual pressure, ρ_0 is a reference density, g is the acceleration of gravity, and z is the vertical distance measured from the free surface, then the pressure gradient and gravity terms in the equation of vertical motion can be written as

$$-\frac{\partial P}{\partial z} + \rho g = -\frac{\partial P_r}{\partial z} + (\rho - \rho_0)g. \quad (9)$$

The Brady-Geyer³ and Spraggs-Street⁴ models employ a reduced pressure in their formulations.

d. The Rigid-lid Approximation

A surface thermal plume is a free-surface problem. To accurately compute thermal-plume dispersion numerically, one must be able to calculate the location of the free surface. Free-surface variations within the plume are on the order of a few millimeters. Yet, the free-surface location must be calculated with good accuracy, since the slope of the free surface is directly related to local pressure gradients that generate the dispersion of flow.

A difficulty in solving the free-surface thermal-plume problem is the presence of free-surface gravity waves. These waves are physically real, yet their small magnitude makes them insignificant to the overall plume determination. The very small time steps that are needed for the stability of the solution when these free-surface gravity waves are included, can greatly reduce the efficiency of the numerical calculation.

An often-used method of circumventing this problem is to assume that a rigid frictionless lid has been placed at the undisturbed horizontal free surface of the ambient receiving body. No surface motions are allowed, and surface gravity waves and their corresponding small time scales are eliminated. Eliminating the surface gravity waves is expected to greatly reduce computation time. Paul and Lick² estimate that the rigid-lid approximation reduces computation time by one-hundredfold. In addition, the rigid-lid approximation allows the use of a fixed computational grid.

Since the surface level is fixed by the rigid-lid condition, a surface-pressure distribution is introduced to handle the slight elevation of the surface associated with even a steady plume. These surface-pressure variations may be translated into equivalent water levels above the surface plane. It is generally thought that the free-surface levels calculated using the rigid-lid approximation are only negligibly different (in terms of magnitude and its effect on the internal hydrodynamics) than the water levels calculated from a free-surface model. Any difference, in fact, implies a violation of the continuity equation, but this violation of continuity principles will be negligible whenever $\eta/H \ll 1$, where H is the average water depth.

Note that the rigid-lid condition does not eliminate the internal gravity waves that hopefully die out as the solution approaches steady state. The rigid-lid condition might, however, change characteristics of these waves by damping out the higher frequency components.

A more subtle effect⁴ of the rigid-lid approximation is that it changes the character of the pressure boundary condition at the surface. In a free-surface model, a Dirichlet boundary condition on pressure is applied at the surface (i.e., pressure is equal to atmospheric value) that allows a faster solution for the three-dimensional pressure calculation at each time

step. In contrast, the rigid-lid approximation introduces a Neumann (derivative) boundary condition at the surface; i.e., a vertical pressure gradient is calculated from the vertical-momentum equation that slows convergence in schemes iterating on the pressure.

The use of the rigid-lid approximation in conjunction with the hydrostatic pressure approximation appears reasonable. This effectively reduces the pressure determination to two dimensions and decreases the computation time accordingly. This is done by Paul and Lick.²

One may say, in general, that the rigid-lid approximation can be used whenever surface elevations are much less than the depth of the flow field, but cannot be used when free-surface variations are rapid in time (between time steps) or large in space such as in storm-surge problems. The disadvantage of the rigid-lid approximation in some problems is that the surface cannot move, for instance to account for new water storage in an impoundment. Overall, however, we expect the rigid-lid approximation to be a valid and useful approximation in surface thermal-plume problems, since surface-elevation changes are small and should not be rapid in time.

e. Equation of State

One may write the general form of the equation of state in the terms

$$\rho = \rho(P, S, T, C), \quad (10)$$

where ρ is the local density and P , S , T , and C are, respectively, the local pressure, salinity, temperature, and constituent concentrations (more than one, perhaps). If we assume that no contaminants are present that effect the density, this reduces to

$$\rho = \rho(P, S, T). \quad (11)$$

It turns out that pressure effects are also negligible, since we are dealing with rather shallow water on an oceanographic scale. A simple numerical example illustrates this point. The relative-expansion coefficients for water at 20°C and 1 atm of pressure are

$$\frac{1}{\rho} \frac{\partial \rho}{\partial T} = 2.7 \times 10^{-4}/^{\circ}\text{C} \quad (12)$$

and

$$\frac{1}{\rho} \frac{\partial \rho}{\partial P} = 4.5 \times 10^{-6}/(\text{meters of water head}). \quad (13)$$

Thus, a change of 1 C° in the temperature produces roughly the same change in density as does a change in depth of about 60 m. So in these shallow-water

plume problems, the effects of pressure may justifiably be neglected. Pressure effects in the evaluation of density are not always negligible and must be included in the equation of state in oceanic modeling.^{4,5}

Omitting pressure, the equation of state becomes

$$\rho = \rho(S, T). \quad (14)$$

An empirical equation of state has been developed by oceanographers and is useful in thermal-plume applications. In this form, density is calculated in terms of a sigma-t defined from

$$\sigma_t = (\rho - 1)1000. \quad (15)$$

Here σ_t has cgs units and measures the deviation in density from 1.0 g/ml. Now the equation of state in general use by oceanographers^{8,9} is written

$$\sigma_t = \Sigma_t + (\sigma_0 + 0.1324)[1 - A_t + B_t(\sigma_0 - 0.1324)], \quad (16)$$

where

$$\Sigma_t = \frac{(T - 3.98)^2}{503.370} \frac{T + 283}{T + 67.26}; \quad (17)$$

$$A_t = 10^{-3}T(4.7867 - 0.098185T + 0.0010843T^2), \quad (18)$$

$$B_t = 10^{-6}T(18.030 - 0.8164T + 0.01667T^2), \quad (19)$$

and

$$\sigma_0 = -0.093 + 0.8149S - 0.000482S^2 + 0.0000068S^3. \quad (20)$$

In the above equations, T is in degrees Celsius and salinity is in parts per thousand. The quantity σ_0 is the density of sea water in sigma-t units at zero pressure and temperature. For fresh water, $\sigma_0 = 0.1324$ and $\sigma_t = \Sigma_t$.

An alternative approach to the use of the above equations is used by Paul and Lick.² Dealing with fresh water only, they employ a linear least-squares curve fit over the specific range of temperature variation used in the problem they are simulating. They find that the data fit is fairly close over the range used. Paul and Lick² note that a quadratic least-squares fit would have to be used for much larger temperature variations or for temperature variations in the vicinity of the maximum density point (4°C). A generalization of the Paul-Lick procedure involves defining any density change as a bilinear function of any corresponding changes in temperature and salinity. In mathematical terms, this linearization of Eq. 14 yields

$$\rho - \rho_0 = -a(T - T_0) + b(S - S_0), \quad (21)$$

where a and b are constants (over a range of temperatures and salinities) and T_0 and S_0 are the temperature and salinity at a reference density ρ_0 . Equation 21 is often used in numerical models of thermal plumes, although the above formulation in terms of Eqs. 15-20 will be slightly more accurate.

f. The Simulation of Turbulence

The modeling of turbulence is very complex and has an extensive literature of its own. This section reviews the classifications of turbulence models, placing emphasis on those that are appropriate to the numerical simulation of thermal plumes.

The importance in developing a proper treatment of plume turbulence cannot be understated since the entire hydrodynamic and energy calculation may be thrown into serious doubt if a poor simulation of turbulence is used. Unfortunately, the simulation of turbulence is very difficult since interesting and important motions exist on many different scales (in both space and time). In particular, the larger the Reynolds number, the smaller the scales that need to be considered.

Turbulence in a thermal plume can be generated in two different ways. Near the outfall, turbulence is initiated by the internal motion of the effluent stream. The nature of this internal turbulence depends on the outlet geometry, velocity, and turbulence levels. Turbulence is also generated in the ambient due to wind stress at the surface of the receiving body (also the plume), wave action, and friction at the shoreline and bottom. A complicating factor is that the jet-induced and ambient turbulence interact nonlinearly. However, the self-induced jet turbulence is expected to predominate until most of the jet momentum is lost. The ambient contribution then becomes important. Thus, ambient turbulence is generally secondary to jet turbulence near the outfall, unless the ambient contribution is inordinately high, as might occur, for example, in rapid rivers. In any case, the ambient contribution dominates in the far field of the thermal plume.

The literature on turbulence modeling suggests four classifications of models. The first are the algebraic turbulent-viscosity models that use Boussinesq's suggestion¹⁰ that the stress-strain law for time-averaged turbulent flows be represented in the same form as that for a Newtonian fluid in laminar motion. Algebraic formulas involving only the properties of the mean velocity profile are then used to determine the equivalent turbulent viscosity, known as the eddy viscosity. The second type of turbulence model involves the solution of differential equations for one or two properties of the turbulent motion, but the goal still is to determine an equivalent turbulent viscosity to relate the turbulent stresses to the mean flow. A third type of turbulence model dismisses the use of effective turbulent transport properties and employs differential transport equations for the turbulent fluxes themselves. This third type is most ambitious and is beyond practical application

at present. A fourth type of turbulence simulation may be loosely referred to as the vortex-transport method. Here, individual vortices are constructed and moved through the flow field as discrete entities. This is usually done by tracking vortex movements on a computer using a combination of deterministic and stochastic displacements. This approach avoids some of the difficulties associated with the other techniques at high Reynolds number. It has had moderate success at calculating such things as form drag over rigid bodies, but has, to our knowledge, never been used for thermal-plume problems or free turbulent flows of any kind.

The ideas underlying each of the various types of models will now be briefly summarized.

The equations of incompressible flow may be written, after the variables are decomposed into mean and fluctuating components, as

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0, \quad (22)$$

$$\frac{\partial \bar{u}_j}{\partial t} + \frac{\partial}{\partial x_k} (\bar{u}_k \bar{u}_j) = -\frac{1}{\rho} \frac{\partial \bar{P}}{\partial x_j} + \bar{X}_j + \frac{\partial}{\partial x_k} (\bar{\tau}_{kj} - \overline{u'_k u'_j}), \quad (23)$$

$$\frac{\partial u'_i}{\partial x_i} = 0, \quad (24)$$

and

$$\frac{\partial u'_j}{\partial t} + \frac{\partial}{\partial x_k} (\bar{u}_k u'_j + \bar{u}_j u'_k) = -\frac{1}{\rho} \frac{\partial P'}{\partial x_j} + X'_j + \frac{\partial}{\partial x_k} (\tau'_{kj} - u'_k u'_j + \overline{u'_k u'_j}), \quad (25)$$

where

$$\bar{\tau}_{ij} \equiv \nu \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \quad \text{and} \quad \tau'_{ij} \equiv \nu \left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right). \quad (26)$$

These equations are in tensor form. Equation 23 represents the momentum equation for the mean velocity $\bar{u}_j$, and Eq. 25 represents the momentum equation for the fluctuating velocity u'_j .

The terms $\overline{u'_k u'_j}$ in Eqs. 23 and 25 are called Reynolds stresses after O. Reynolds, who first used the decomposition of velocity into mean and fluctuating components. Very little is gained by this manipulation, however, since these turbulent stresses are still undetermined and assumptions must be made regarding the form of these extra terms in order to solve the governing equations. The need for additional assumptions in order to calculate the Reynolds stresses is the so-called closure problem of turbulence theory.

The algebraic and differential-equation-type turbulence models use Boussinesq's analogy¹⁰ to laminar flow, in which it is assumed that

$$-\overline{u_i' u_j'} = \nu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right), \quad (27)$$

where ν_t is a scalar quantity that may be a function of space and time known as the kinematic eddy viscosity. Often a constant value is used, although this may be shown to be theoretically incorrect.

Although the eddy-viscosity concept is in popular use, it is grossly incorrect theoretically for the following reasons.¹¹

(1) The kinematic viscosity ν_t is, in actuality, a fourth-order tensor consisting of 64 independent scalar elements for three-dimensional flow, rather than a single scalar coefficient (zero-order tensor).

(2) Equation 27 implies that the length and time scales of turbulent motion are of the same order of magnitude as those of the mean flow. This is perhaps a good assumption for pure shear flows and is probably not bad near the outfall in a thermal plume, where shear dominates, but it must fail when buoyant forces become important, since these forces have their own characteristic length and time scales. Beyond the inertial regime of flow, buoyant scales play at least a coequal role with inertial scales in determining the turbulent motion.

(3) The eddy-viscosity model assumes that the turbulence adjusts immediately to changes in mean conditions and that a universal relationship exists between the turbulent stresses and the mean strain rates. No such universal relationship exists, since the history of the turbulence field and the effects of the mean velocity field and turbulence levels of nearby points are very important in establishing the local turbulence.

It is found¹¹ that Eq. 27 is satisfactory when applied to bounded flows if a single scale exists--for example, in pipes, where the basic scale is the nozzle diameter; in boundary layers, where the scale is the momentum thickness; in jets, where the scale is the nozzle diameter; and near wakes, where the scale is the blockage diameter. In such flows, the strong effect of local shear dominates the history and transport mechanisms. On the other hand, Eq. 27 is known¹¹ to fail completely in unbounded flows such as the far wake and free convection, because turbulent transport is dominant over shear in these cases and the characteristic length scales are determined by the turbulence itself, since no solid boundaries are present.

In thermal-plume applications, we expect Eq. 27 to be adequate near the outfall where the plume is shear-dominated. As the inertial forces wane and buoyancy effects begin to be of equal importance, Eq. 27 is probably only marginally adequate. In the region where buoyancy is predominant, the

eddy-viscosity concept should fail altogether. Pratt¹¹ notes that in a buoyancy-dominated region, $\overline{u_1' u_2'}$ is positive due to turbulence generated by buoyant convective motions, yet the gradient of the mean velocity approaches zero; an infinite eddy viscosity results.

In spite of the above-stated shortcomings of the eddy-viscosity approach, it is nevertheless the most widely used in modeling turbulence.

Turbulence results not only in the transport of momentum, but also in the transport of scalar quantities such as temperature or the concentration of a pollutant. These scalar quantities will themselves exhibit turbulent fluctuations. Thus, when the transport equations for these variables (e.g., the heat-energy equation for temperature) are time-averaged, additional terms, like the Reynolds stresses, arise. In analogy to the eddy-viscosity approach, an eddy conductivity or an eddy thermal diffusivity can be defined to treat the turbulent transport of heat. This parameter, like the eddy viscosity, can be space (and perhaps time)-dependent.

We next consider the two basic types of turbulence models that use the eddy-viscosity approach.

(1) Algebraic Models. Most turbulence models, especially those of the algebraic type, are developed with a specific flow in mind. The most widely studied is the two-dimensional (parallel to wall and normal to wall) boundary layer. Other widely studied flows include turbulent pipe flow and free turbulent flows such as jets and wakes. In all these cases, one is primarily interested in the lateral transport of longitudinal momentum due to turbulence. Consequently, the Reynolds stress of greatest interest is

$$\overline{u'v'} = \nu_t \frac{\partial u}{\partial y}, \quad (28)$$

where u is in the principal direction of flow and v and y lie in the normal direction. If the flow is nonisothermal, then the turbulent heat exchange

$$\overline{u'T'} = \epsilon_t \frac{\partial u}{\partial y} \quad (29)$$

must also be considered, where ϵ_t is an eddy thermal diffusivity.

A partial list of available algebraic turbulence models is given in Table III.E.3. A brief statement is included for selected models. Although there are other types of algebraic turbulence models, the most prevalent are ones based on some form of the mixing-length hypothesis. This concept, briefly considered in our discussion of integral models, has been supported by many experimental investigations. It has, in fact, been deduced from at least two different conceptualizations (von Karman¹³ and Taylor¹⁴) as well as by Prandtl.¹²

TABLE III.E.3. Partial List of Algebraic Turbulence Models for Calculation of Eddy Viscosity

1. Mixing-length Theory

- a. Prandtl
- ¹²
- suggests a mixing-length hypothesis in which

$$\nu_t = \ell^2 \left| \frac{\partial u}{\partial y} \right|,$$

where ℓ is the mixing length. Prandtl gives $\ell \approx y$ for boundary-layer flow, where y is distance to nearest wall.

- b. von Karman
- ¹³
- derives a mixing-length hypothesis using local similarity, but gets

$$\ell = \kappa |(\partial u / \partial y) / (\partial^2 u / \partial y^2)|,$$

where κ is a "universal constant" equal to about 0.4. This form agrees with experiment only near a wall, where $\ell = \kappa y$.

- c. Taylor
- ¹⁴
- derives a mixing-length hypothesis using vorticity-transport analogy.

- d. Rotta
- ¹⁵
- alters the von Karman form to give

$$\ell = \kappa y [1 - 6.8(\nu / u^* y)],$$

where u^* is the friction velocity given by $\sqrt{\tau_w / \rho}$, τ_w is the wall stress, and ν is the kinematic viscosity.

- e. van Driest
- ¹⁶
- alters the von Karman form to give

$$\ell = \kappa y [1 - \exp(\nu y^* / A)],$$

where $A \approx 26$. Others who have derived similar expressions are Patankar and Spalding,¹⁷ Launder and Jones,¹⁸ Cebeci et al.,¹⁹ Pletcher,²⁰ Kays, Moffat, and Thielbahr,²¹ and Powell and Strong.²²

- f. Escudier
- ²³
- alters the von Karman form to give

$$\ell = \begin{cases} \kappa y & y/\delta < \lambda/\kappa \\ \lambda \delta & y/\delta \geq \lambda/\kappa \end{cases},$$

where $\lambda \approx 0.09$ and δ is the thickness of the boundary layer.

- g. Nikuradse
- ²⁴
- deduces a mixing-length distribution for pipe flow to be

$$\ell = 0.14 - 0.08[1 - (y/R)]^2 - 0.06[1 - (y/R)]^4,$$

where R is the pipe radius. This form agrees with von Karman ($\ell = \kappa y$) fairly well for $y/R < 0.2$.

- h. Spalding
- ²⁵
- gives a "universal" velocity profile for pipe flow based on Nikuradse's measurements.

2. Other Eddy-viscosity Models

- a. Clauser
- ²⁶
- derives

$$\nu_t \approx U \delta^* f(y/\delta),$$

where U is the free-stream velocity, δ^* is the displacement thickness of the boundary layer, and f is a function of one variable. Mellor and Gibson²⁷ use this formula along with the velocity-defect law to calculate equilibrium wake profiles.

- b. Johnson and Sparrow
- ²⁸
- use a similar idea for pipe flow.

The crux of the mixing-length hypothesis is the simple relationship that

$$v_t = \ell^2 \left| \frac{\partial u}{\partial y} \right|, \quad (30)$$

where ℓ is a parameter with dimensions of length (the mixing length). Table III.E.3 gives several expressions for ℓ developed for boundary-layer and turbulent pipe flows. Free turbulent flows, such as a classical jet, typically assume that the mixing length is independent of y , the lateral coordinate. Moreover, it is found that

$$\ell \approx b, \quad (31)$$

where b is a characteristic half-width of the jet. The mean velocity gradient is further assumed to be of the form

$$\left| \partial u / \partial y \right| = (U - U_{\text{ext}}) / b, \quad (32)$$

where U is the jet centerline velocity and U_{ext} is the external free-stream velocity. Thus, the mixing-length hypothesis gives

$$v_t = C(U - U_{\text{ext}})b, \quad (33)$$

where C is a constant, for the classical jet. This results in the simple similarity solutions for the classical jet given in an earlier section.

For heated jets, the situation is much more complicated. If temperature variations are small (so that heat behaves as a passive tracer), then a mixing-length hypothesis can also be used for the eddy thermal diffusivity. Prandtl's original thinking gives

$$\epsilon_t = \ell^2 \left| \partial u / \partial y \right| = v_t. \quad (34)$$

On the other hand, Taylor's theory gives

$$\epsilon_t = 2v_t, \quad (35)$$

which has been found to be the more nearly correct of the two expressions.

Careful experimental investigations has revealed that none of the above relationships between turbulent transfer and mean gradients are valid exactly. The mixing-length hypothesis is nevertheless sound. For many problems in nonbuoyant jet theory, the above relationships do predict the gross characteristics of the jet reasonably well in the region where free shear is dominant. The limits of their applicability to buoyant surface discharges are not clear, however.

Launder and Spalding²⁹ point out that the mixing-length hypothesis may be used with good results when mean velocity gradients have reasonable magnitude and a mixing length can be easily defined, but breaks down when velocity gradients are very small and the mixing length has doubtful interpretation. There is almost no successful experience in predicting recirculating flows with the mixing-length hypothesis. In such cases, the relationships between the stresses and velocity gradients are too complicated.

The mixing-length hypothesis has two more disadvantages. First, it predicts that the effective viscosity vanishes where the velocity gradient is zero. In reality, turbulent fluctuations can be nonzero. Second, the hypothesis does not account for the convection, diffusion, production, or dissipation of turbulence.

The main advantage of the mixing-length hypothesis is that it can provide realistic predictions of the velocity and shear-stress distributions, provided that good choices are made for the mixing-length distributions or profiles. It has been successful in predicting the general behavior of boundary-layer flows. In addition, it is simple to use, requiring no solutions of differential equations, and much experience has been accumulated with its use.

(2) Differential-equation Models for the Calculation of Kinematic Eddy Viscosity. The second general class of turbulence models to be discussed also makes use of the eddy-viscosity concept, but calculates it by solving one or more differential equations. These models are based on a concept probably first suggested by Kolmogorov,³⁰ but usually attributed to Prandtl,³¹ that the eddy viscosity is given by

$$\nu_t = k^{1/2} \ell, \quad (36)$$

where again ℓ is a length scale characteristic of the turbulence (similar to the mixing length) and k is the turbulent kinetic energy given by

$$k = (\overline{u'^2} + \overline{v'^2} + \overline{w'^2})/2. \quad (37)$$

Most of the models in this category involve a differential transport equation for k that is derived as follows. First, a transport equation for $\overline{u_i' u_j'}$ is obtained by multiplying the equation for the fluctuating velocity u_j' (Eq. 25) by u_i' , reversing the subscripts, adding the two resulting equations, and averaging. This process yields

$$\begin{aligned} \frac{D}{Dt}(\overline{u_i' u_j'}) = & -(\overline{u_i' u_k'}) \frac{\partial \overline{u_j}}{\partial x_k} - (\overline{u_j' u_k'}) \frac{\partial \overline{u_i}}{\partial x_k} - \frac{\partial}{\partial x_k} (\overline{u_i' u_j' u_k'}) - \frac{1}{\rho} \left[\frac{\partial}{\partial x_j} (\overline{u_i' P'}) + \frac{\partial}{\partial x_i} (\overline{u_j' P'}) \right] \\ & + \frac{P'}{\rho} \left(\frac{\partial \overline{u_i'}}{\partial x_j} + \frac{\partial \overline{u_j'}}{\partial x_i} \right) + \nu \frac{\partial^2 (\overline{u_i' u_j'})}{\partial x_k^2} - 2\nu \frac{\partial \overline{u_i'}}{\partial x_k} \frac{\partial \overline{u_j}}{\partial x_k} + \overline{X_i' u_j'} + \overline{X_j' u_i'}. \end{aligned} \quad (38)$$

This transport equation gives the substantial derivative of the turbulent stress in terms of production, diffusion, redistribution, and dissipation of turbulent stress. Then the transport equation for k is obtained by adding the three scalar equations that correspond to Eq. 38 with i set equal to j and dividing by 2. Thus,

$$\frac{\partial k}{\partial t} + \frac{\partial}{\partial x_j} \left(\overline{u_j k} + \overline{u_j' k'} + \overline{u_j' p'} - \overline{u_i' \tau_{ji}'} \right) = \left(-\overline{u_j' u_i'} \right) \frac{\partial \overline{u_i}}{\partial x_j} - \overline{\tau_{ji}'} \frac{\partial \overline{u_i}}{\partial x_j} + \overline{X_i' u_i'} \quad (39)$$

The various terms in Eq. 39 are defined by Pratt¹¹ and Tennekes and Lumley,³² as follows:

- A. advection of k
- B. turbulent transport of k by velocity fluctuation
- C. pressure gradient work
- D. turbulent transport of k by viscous stresses
- E. production of k from the mean flow
- F. viscous dissipation of k
- G. production of k due to body forces.

Models of this category may be further subdivided on the basis of whether they solve some form of Eq. 39 alone and prescribe ℓ algebraically or whether they solve an additional differential equation for ℓ . For obvious reasons, the former class are called one-equation models, while the latter are called two-equation models.

A partial list of one-equation models appears in Table III.E.4. Regarding these one-equation models, Launder and Spalding²⁹ conclude that:

- (a) The expression $k^{1/2}$ is preferable to $\ell |\partial u / \partial y|$.
- (b) The Prandtl-Emmons-Glushko simulation of the unknown terms in Eq. 39 is the best available.
- (c) Under most circumstances, the one-equation model is only marginally better than the mixing-length hypothesis.

A partial list of two-equation models is given in Table III.E.5. Because of the complexity of these models, only the basic variables are indicated.

The two-equation turbulence models have come into prominence due to the often-occurring difficulty in prescribing the length-scale

TABLE III.E.4. Partial List of One-equation Turbulence Models

1. Prandtl³² suggests that

$$\nu_t = k^{1/2} \ell,$$

where k is the turbulent kinetic energy and ℓ is a length scale prescribed algebraically. The various unknown terms in the transport equation for k are specified by Prandtl, and later improvements are made by Glushko³³ and Emmons.³⁴ Wieghardt³⁵ gives $\ell = C_D^{1/4} \kappa y$ near a wall, where C_D is the drag coefficient, and Emmons gives $\ell = 0.04b$ for a free plane jet, where b is the total (to free stream) half-width of the jet.

2. Nee and Kovasznay³⁶ give a differential equation for ν_t that is similar to the transport equation for k . A length scale that is prescribed algebraically is used in the equation.

3. Gawain and Pritchett³⁷ develop heuristic one-equation turbulence model. This model was discussed in Sec. II.H.

TABLE III.E.5. Partial List of Two-equations Turbulence Models

Model	Dependent Variables
Kolmogorov ³⁰	$k, f = k^{1/2}/\ell^a$
Chou ³⁸	$k, \epsilon = k^{3/2}/\ell^b$
Davidov ³⁹	k, ϵ
Harlow and Nakayama ⁴⁰	k, ϵ
Jones and Launder ⁴¹	k, ϵ
Rotta ⁴²	k, ℓ
Spalding ⁴³	k, ℓ
Rotta ⁴⁴	$k, k\ell$
Rotta ⁴⁵	$k, k\ell$
Rodi and Spalding ⁴⁶	$k, k\ell$
Ng and Spalding ⁴⁷	$k, k\ell$
Spalding ⁴⁸	$k, W = k/\ell^{2c}$

^a f represents the characteristic frequency of the turbulence.

^b ϵ represents the kinematic turbulent dissipation rate.

^c $W = f^2$.

distribution throughout the flow field. There is uncertainty, even with reasonable guesses. Moreover, in flows governed by elliptic equations, the profiles of ℓ are too complex to guess. Convective transport of the length scale is important and difficult to simulate; furthermore, it is more difficult in elliptic recirculating-type flows (such as most thermal plumes) to determine the length-scale profile by measurement (as compared to boundary-layer flows).

The most popular second equation is for the turbulence-energy dissipation rate, ϵ . An equation for ϵ is usually used, rather than the length scale ℓ itself, because of the relative ease with which the exact equation for ϵ can be derived and partly because ϵ appears directly as an unknown in the equation for k .

The dissipation of k (defined as ϵ) is given by

$$\epsilon \equiv \overline{\tau'_{ji} \frac{\partial u'_i}{\partial x_j}} = \frac{\partial}{\partial x_j} (\overline{u'_i \tau'_{ji}}) - \overline{u'_i \frac{\partial \tau'_{ji}}{\partial x_j}}. \quad (40)$$

Recall that

$$\tau'_{ji} \equiv \nu \left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right). \quad (41)$$

Substitution yields

$$\epsilon \equiv \nu \overline{\left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right) \frac{\partial u'_i}{\partial x_j}} = \nu \overline{\left(\frac{\partial u'_i}{\partial x_j} \right)^2} + \nu \overline{\left(\frac{\partial u'_j}{\partial x_i} \frac{\partial u'_i}{\partial x_j} \right)}. \quad (42)$$

When the Reynolds number $Re \gg 1$, an order-of-magnitude analysis shows that the last term is negligible, from which

$$\epsilon \cong \nu \overline{\left(\frac{\partial u'_i}{\partial x_j} \right)^2} \text{ when } Re \gg 1. \quad (43)$$

The equation for ϵ is derived by taking the derivative of the fluctuating momentum equation for u'_j (Eq. 25 with approximate buoyant forces) with respect to x_i , multiplying by $\partial u'_j / \partial x_i$, and time-averaging. The result for steady flow is

$$\begin{aligned}
& \begin{array}{ccccc} \text{A} & & \text{B} & & \text{C} & & \text{D} & & \text{E} \end{array} \\
& \frac{\partial}{\partial x_j} (\bar{u}_j \epsilon) = \nu \frac{\partial^2 \epsilon}{\partial x_j^2} - 2\nu \left[\frac{\partial^2 u_j}{\partial x_i \partial x_k} \left(\overline{u'_k \frac{\partial u'_j}{\partial x_i}} \right) + \frac{\partial \bar{u}_j}{\partial x_k} \left(\overline{\frac{\partial u'_j}{\partial x_i} \frac{\partial u'_k}{\partial x_i}} \right) + \frac{\partial \bar{u}_k}{\partial x_i} \left(\overline{\frac{\partial u'_j}{\partial x_i} \frac{\partial u'_j}{\partial x_k}} \right) \right. \\
& \begin{array}{ccc} \text{F} & & \text{G} & & \text{H} \end{array} \\
& \left. + \left(\overline{\frac{\partial u'_j}{\partial x_i} \frac{\partial u'_k}{\partial x_i} \frac{\partial u'_j}{\partial x_k}} \right) + \frac{1}{2} \frac{\partial}{\partial x_k} \left(\overline{u'_k \frac{\partial u'_j}{\partial x_i} \frac{\partial u'_j}{\partial x_i}} \right) + \frac{\partial}{\partial x_j} \left(\overline{\frac{\partial u'_j}{\partial x_i} \frac{\partial p'}{\partial x_i}} \right) \right. \\
& \begin{array}{cc} \text{I} & \text{J} \end{array} \\
& \left. + \nu \left(\overline{\left(\frac{\partial^2 u'_j}{\partial x_k \partial x_i} \right)^2} \right) + 2\nu g \delta_{i3} \left(\overline{\frac{\partial u'_i}{\partial x_j}} \right) \frac{\partial}{\partial x_j} \left(\frac{\rho_r - \rho'}{\rho_r} \right) \right] \quad (44)
\end{aligned}$$

For $Re \gg 1$, all terms except F, G, I, and J may be neglected. Equation 44 then becomes

$$\begin{aligned}
& \begin{array}{cccc} \text{A} & & \text{F} & & \text{G} & & \text{I} \end{array} \\
& \frac{\partial}{\partial x_j} (\bar{u}_j \epsilon) \cong -2\nu \left(\overline{\frac{\partial u'_j}{\partial x_i} \frac{\partial u'_k}{\partial x_i} \frac{\partial u'_j}{\partial x_k}} \right) - \nu \frac{\partial}{\partial x_k} \overline{u'_k \left(\frac{\partial u'_j}{\partial x_i} \right)^2} - 2\nu^2 \left(\overline{\frac{\partial^2 u'_j}{\partial x_k \partial x_i}} \right)^2 \\
& \begin{array}{c} \text{J} \end{array} \\
& + 2\nu g \delta_{i3} \left(\overline{\frac{\partial u'_i}{\partial x_j}} \right) \frac{\partial}{\partial x_j} \left(\frac{\rho_r - \rho'}{\rho_r} \right), \quad Re \gg 1. \quad (45)
\end{aligned}$$

Each of the terms F, G, I, and J are then modeled from k and mean flow quantities. Pratt¹¹ does this for a thermal plume application with the result that

$$\begin{aligned}
& \begin{array}{ccccc} \text{A} & & \text{G} & & \text{F} & & \text{I} & & \text{J} \end{array} \\
& \frac{\partial}{\partial x_j} (\bar{u}_j \epsilon) = \frac{\partial}{\partial x_k} \left(\frac{\nu}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x_k} \right) - \frac{C_1 \epsilon^2}{k} + \frac{C_2 \rho p_s}{k} + \frac{\nu g \delta_{i3}}{\sigma_T} \left(\frac{\rho_r - \rho_o}{\rho_r} \right) \frac{\partial^2}{\partial x_j^2} \left(\nu \frac{\partial \Delta}{\partial x_i} \right), \quad (46)
\end{aligned}$$

where

$C_1, C_2 = \text{constants,}$

$p_s = \text{the production of } k \text{ from the mean flow,}$

$\sigma_\epsilon = \text{turbulent Prandtl number for transport of dissipation,}$

$\sigma_T = \text{turbulent Prandtl number for heat,}$

$$\Delta = \frac{\rho_r - \rho}{\rho_r - \rho_0}, \text{ nondimensional buoyancy parameter} \quad (47)$$

ρ_r = reference density (ambient value),

and

ρ_0 = outlet discharge density.

The reader is strongly urged to acquire a copy of Pratt's report and follow through his detailed derivation of equations for k and ϵ , with assumptions on closure for thermal-plume applications. The k, ϵ model is then included by Pratt in the Trent-Welty⁴⁹ numerical model for heated discharges from a submerged vertical outfall in shallow water. The Trent-Welty model is axisymmetric (two-dimensional); consequently, Pratt¹¹ reduces his general k, ϵ equations to two-dimensional form. Boundary conditions are also presented and discussed for the k, ϵ turbulence model. Pratt¹¹ and Launder and Spalding²⁹ note that the values of numerical constants in these differential-equation models, although recommendations are often made for them, may have to be carefully adjusted or fine-tuned to achieve best agreement with measurements.

The mean turbulent energy methods (two-equation type) are receiving considerable attention at present and are at the brink of becoming state of the art for the numerical treatment of thermal plumes.

A specific example of a two-equation turbulence formulation used in a surface thermal-plume model is given in Table III.E.6 as employed by McGuirk and Spalding.⁵⁰ This McGuirk-Spalding work and the work of Pratt on the Trent model are the only investigations we are aware of that use two-equation turbulence models for thermal-plume applications. The only one-equation turbulence simulation used in a plume model is that employed by Pritchett, England, and Taft in the STUFF model⁵¹ (see Sec. II.H).

The two-equation turbulence model should be a significant improvement over the mixing-length hypothesis for more complex flows, where the mixing-length hypothesis is not generally valid. Much guesswork is removed from the choice of mixing lengths, and the two-equation model is hopefully more universal. However, the two-equation model is not completely satisfactory in itself. First, the criticisms of the eddy-viscosity concept apply to two-equation models as much as they do to mixing-length models. Second, the same set of so-called "universal" constants does not apply for all applications. For instance,²⁹ the flow from an axisymmetric round jet cannot be predicted accurately with the same set of "constants" applicable to plane jets.

TABLE III.E.6. Two-equation Turbulence Model Used in the McGuirk-Spalding Thermal-plume Model⁵⁰Eddy Viscosity (momentum transport):

$$\mu_{\text{eff}} = C_{\mu} \rho k^2 / \epsilon \text{ (Prandtl-Kolmogorov formula),}$$

where

$$C_{\mu} = \text{constant (taken as 0.09),}$$

$$\rho = \text{local density,}$$

$$k = \text{turbulent kinetic energy,}$$

and

$$\epsilon = \text{energy-dissipation rate.}$$

Transport Equation for k:

$$\frac{\partial}{\partial x}(\rho u k) + \frac{\partial}{\partial y}(\rho v k) + \frac{\partial}{\partial z}(\rho w k) = \frac{\partial}{\partial y}\left(\Gamma_{\text{eff},k} \frac{\partial k}{\partial y}\right) + \frac{\partial}{\partial z}\left(\Gamma_{\text{eff},k} \frac{\partial k}{\partial z}\right) + S_k,$$

where

$$S_k = G - \rho \epsilon$$

and

$$G = \mu_{\text{eff}} \frac{\partial v_i}{\partial x_j} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right).$$

Here, G represents the generation rate of turbulence; $\rho \epsilon$ the disappearance of turbulence, and S_k the net source of turbulent energy; $\Gamma_{\text{eff},k}$ is an effective transport coefficient related to μ_{eff} as described below.

Transport Equation for ϵ :

$$\frac{\partial}{\partial x}(\rho u \epsilon) + \frac{\partial}{\partial y}(\rho v \epsilon) + \frac{\partial}{\partial z}(\rho w \epsilon) = \frac{\partial}{\partial y}\left(\Gamma_{\text{eff},\epsilon} \frac{\partial \epsilon}{\partial y}\right) + \frac{\partial}{\partial z}\left(\Gamma_{\text{eff},\epsilon} \frac{\partial \epsilon}{\partial z}\right) + S_{\epsilon}.$$

where

$$S_{\epsilon} = \frac{\epsilon}{k} (C_1 G - C_2 \rho \epsilon).$$

Here, S_{ϵ} is the net source of ϵ . The generation and dissipation rates of turbulent energy are amplified by constants C_1 and C_2 , respectively. Normally, C_1 is 1.44 and C_2 is 1.92. $\Gamma_{\text{eff},\epsilon}$ is an effective transport coefficient for ϵ related to μ_{eff} as described below.

Transport Coefficients

In addition to the two transport coefficients used in the turbulent energy and dissipation rate equation, an effective conductivity $\Gamma_{\text{eff},T}$ is used in the heat-energy equation. These three parameters are all related to μ_{eff} as follows:

$$\Gamma_{\text{eff},k} = \mu_{\text{eff}} / \sigma_k$$

$$\Gamma_{\text{eff},\epsilon} = \mu_{\text{eff}} / \sigma_{\epsilon}$$

$$\Gamma_{\text{eff},T} = \mu_{\text{eff}} / \sigma_T$$

The constants have the following values: $\sigma_k = 1$, $\sigma_{\epsilon} = 1.3$, and $\sigma_T = 0.7$.

Boundary Conditions

At bottom or shoreline boundaries, near-wall values are prescribed from simplified forms of the full differential equations applicable near walls. For planes of symmetry and the water surface, zero gradients for k , ϵ are assumed. The value of turbulence intensity in both jet and ambient is assumed to be 3% of the longitudinal velocity (for lack of better information). The specification of energy dissipation rate at the jet exit and ambient inlet is given by

$$\epsilon = \rho k^{3/2} / \ell,$$

which was derived from dimensional analysis. The length scale of turbulence in the jet and ambient is estimated and given the value ℓ . McGuirk and Spalding note that their jet-inlet conditions are idealized, since they neglect (1) the presence of boundary layers inside and outside the jet nozzle and (2) the small recirculation region that might be present just downstream of the nozzle edge. Close to the nozzle exit, these factors may be dominant. Consequently, McGuirk and Spalding modify ϵ in the region near the inlet to produce an effective viscosity value that results in the correct mixing just downstream of the discharge point.

(3) Stress-transport Models. The stress-transport models, also known as the Mean Reynolds Stress (MRS) closure method, rejects the use of the eddy-viscosity concept and instead solves equations for the Reynolds stresses such as Eq. 38. The simplest of such methods is one conceived by Bradshaw et al.,⁵² which is very much like a one-equation eddy-viscosity model. The main difference is that the Reynolds stress is taken proportional to k , the turbulent kinetic energy. Thus, the k -transport equation becomes that of stress transport, and the length scale appearing in that equation is specified algebraically.

Most of the other stress-transport models solve for individual correlations such as $\overline{u'v'}$ using several differential transport equations. The many transport equations involved are derived by considering higher- and higher-order correlations. For example, the transport equation for $\overline{u'v'}$ contains third-order (or triple) correlations such as $\overline{u'v'^2}$. To solve that equation, either some plausible assumption must be made about the nature of triple correlations or else a transport equation for the triple correlations must be derived. In the latter case, fourth-order correlations appear. The question of closure now becomes apparent. The problem is to decide at what level assumptions can be made concerning the unknown terms so as to retain the greatest generality of the model. Table III.E.7 gives a partial list of stress-transport models, along with a brief notation concerning the level of closure.

TABLE III.E.7. Mean Reynolds Stress Models for Turbulence

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1.	Chou: ³⁸ equations for mixing length, shear stress, normal stresses, and triple correlations for fluctuating velocities.
2.	Rotta: ⁴² equations for mixing length and for shear and normal stresses.
3.	Davidov: ³⁹ equations for mixing length, shear and normal stresses, triple correlations for fluctuating velocities, and the various components of length-scale diffusion.
4.	Donaldson: ⁵³ equations for shear and normal stresses only.
5.	Daly and Harlow: ⁵⁴ equations for mixing length and for shear and normal stresses.
6.	Hanjalic: ⁵⁵ equations for mixing length and shear stresses.
-

According to Launder and Spalding,²⁹ triple correlations are small in practical flows. They state that an engineer should never need to simulate them by separate transport equations. If this is true, the models of Chou³⁸ and Davidov³⁹ are more complex than necessary. Launder and Spalding also state that it is usually more important to provide an equation for the length scale than for the normal stresses. As a result, they do not regard Donaldson's⁵³ choice of differential-equation set as appropriate for the prediction of the majority of turbulent flows.

An important consideration is the number of additional equations that require solution in these MRS models. This is important, since

additional equations involve additional computer time and cost. Typical cases are the models of Rotta⁴² and Daly and Harlow.⁵⁴ For three-dimensional applications, three mean-flow equations must be solved along with seven turbulence equations (a total of 10 equations in all). The turbulence equations consist of six stress equations and a length-scale equation. In Chou's model,³⁸ 20 equations must be solved. In the turbulence equations of Rotta and Daly and Harlow, there are 10 triple correlations that require assumptions. The assumptions often use undetermined constants that must be optimized upon comparison with data. Each additional turbulence equation increases the task of optimizing the constants. This is a disadvantage in using higher- and higher-order models.

Launder and Spalding²⁹ note that extensive investigations have reached only the level of two-equation turbulence models. They recommend that, when the flow is too complex for faith to be placed in mixing-length models, the two-equation level probably provides the best starting level for engineering calculations. In our opinion, a top priority in future modeling efforts for thermal plumes should be to include the two-equation turbulence treatment in numerical model formulations. Such models should be tested to determine the level of improvement (if any) observed in model predictions over the mixing-length models.

(4) Vortex-transport Methods. A relatively recent and somewhat interesting treatment of turbulence is the vortex-transport method. Here, a large number of discrete vortices are artificially released into the flow field and individually tracked on a computer. The method described here is due to Chorin,^{56,57} although other investigators have used similar concepts. Chorin gives two advantages for his method. First, since the model does not solve any differential equations, it avoids the difficulties associated with grid refinement at high Reynolds number. Second, the model numerically simulates the physical processes of vorticity generation and dispersal in a way that makes intuitive understanding possible and consequently improves the physical nature of the treatment. Of course, the latter point could be debated.

In a typical application of this method, one considers vortex blobs moving in a potential flow field. The solution proceeds in a sequence of time steps just as in the explicit forward-difference solution of any time-dependent problem. At each time step, each vortex is displaced (1) deterministically according to the local velocity it experiences and then (2) stochastically to simulate diffusion. The calculation of the deterministic displacement includes the effects of both the potential flow and other vortices in the field. A vortex dies when it crosses a boundary; vortices are simultaneously created at the boundaries by the natural flow. This method has been used by Davari⁵⁸ to calculate form drag on a cylinder. Base⁵⁹ used a similar idea to simulate the behavior of a particle in turbulent flow. These are typical applications of the idea. To our knowledge, no one has gone beyond these basic problems and attempted to use this technique in a complex flow such as a thermal plume.

For thermal-plume applications, buoyancy effects become important as inertial (shear) effects diminish. A turbulence model for plume dispersion should account for the turbulence energy generated or removed by buoyancy. Shear forces enhance mixing and turbulence, while positive buoyancy tends to stabilize the destabilizing effects of shear. Buoyancy effects affect not only the level of turbulent energy in the plume but also the length scale. In any unstable density field, gravitational effects tend to increase the dimensions of an eddy, but in a stably stratified fluid, the opposite trend holds. Although buoyancy effects are easily incorporated into the turbulent-energy equation (as done above following Pratt¹¹), it is not possible to derive exactly the factor that amplifies the length scale in the equation for ℓ . Turbulence models of the differential and MRS categories that include buoyancy effects nonheuristically in the k and ℓ (or ϵ , etc.) equations are presently under development.

Concerning the algebraic models of turbulence, much work has been done in the field of oceanography to help determine the vertical diffusivity in the presence of stratification. The basic parameter used is the local Richardson number that relates the relative effects of shear and buoyancy on vertical diffusion:

$$R_i = \frac{\frac{g}{\rho} \frac{d\rho}{dz}}{\left(\frac{du}{dz}\right)^2} \quad (48)$$

If R_i is less than zero, then the flow is unstable and considerable local mixing should occur. Table III.E.8 lists the most prominent Richardson-number correlations for vertical diffusivity. At times, the formulas in the table are used for the vertical spread of heat, momentum, and mass interchangeably; occasionally a Prandtl-type number is used as a multiplicative factor to account for the different rates of spreading expected. Since any of the above correlations is rough at best, the effect of the Prandtl or Schmidt numbers should not be significant. Note that the formulas given in Table III.E.8 are derived by multiplying the diffusion coefficient under neutral conditions by a multiplicative factor containing the Richardson number.

A sample of formulas used for the vertical diffusivity under neutral conditions is given in Table III.E.9. Although the formulas in Tables III.E.8 and 9 are really for ambient stratification, some numerical-model investigators have used such formulas to simulate plume stratification since no better information is apparently available. Horizontal viscosities and diffusivities are generally taken to be constants and assumed to be the same value in the presence of lateral buoyant spreading.

TABLE III.E.8. Summary of Formulas on Correlation of Vertical Diffusion Coefficient K_z with Richardson Number R_i (or Density Gradient $\bar{\epsilon}$) (Adapted from Koh and Fan⁶⁰)^a

Investigators	Formula
Rossby and Montgomery (1935) ^b	$K_z = K_{zo}(1 + \beta R_i)^{-1}$
Rossby and Montgomery (1935) ^b	$K_z = K_{zo}(1 + \beta R_i)^{-2}$
Holzman (1943) ^b	$K_z = K_{zo}(1 - \beta R_i), \quad R_i \leq \frac{1}{\beta}$
Yamamoto (1959) ^b	$K_z = K_{zo}(1 - \beta R_i), \quad R_i \leq \frac{1}{\beta}$
Mamayev (1958) ^b	$K_z = K_{zo}e^{-\beta R_i}$
Munk and Andersen (1948) ^c	$K_z = K_{zo}(1 + \beta R_i)^{-3/2}$ $\beta = 3.33$ based upon data by Jacobsen (1913) and Taylor (1931)
Harremoes (1968)	$K_z = 5 \times 10^{-3} \times \bar{\epsilon}^{-2/3} \text{ cm}^2/\text{sec}$ Note: $\bar{\epsilon}$ in m^{-1} ; approximate experimental range: $5 \times 10^{-9} < \bar{\epsilon} < 15 \times 10^{-5} \text{ m}^{-1}$
Kolesnikov et al. (1961) ^d	$K_z = K_{z \min} + \beta/\bar{\epsilon}$ in cm^2/sec $K_{z \min}$ and β are empirically determined to be: $K_{z \min} = 12, \beta = 8.3 \times 10^{-5}$ (1958 and 1960 observations) $K_{z \min} = 2, \beta = 10.0 \times 10^{-5}$ (1959 observations)

^a K_{zo} defines K_z at $R_i = 0$, i.e., the neutral case; β is the proportionality constant and varies from case to case.

^bAs given by Okubo (1962).

^cAs given by Bowden (1962).

^dThe formulas presented in the translated version are apparently erroneous.

TABLE III.E.9. Selected Formulas for the Neutral Diffusion Coefficient K_{zo} (Adapted from Trent and Welty⁴⁹)

1. Kent and Pritchard:⁶¹ for wave-induced component from James River Estuary Study

$$K_{zo} = 0.01d \left(1 - \frac{d}{L} \right) \frac{H}{T} e^{-2\pi d/\ell},$$

where

d = distance from the surface,

L = depth of the water body,

H = wave height,

T = wave period,

and

ℓ = wavelength

2. Golubeva⁶² and Isayeva:⁶³ for well-mixed surface layer only

$$K_{zs} = 0.02 \frac{H^2}{T}.$$

3. Wada:⁶⁴ for tidal currents

$$K_{zo} = \frac{K^2 U_s z \sqrt{d}}{L \sqrt{L} \log \left\{ \frac{\sqrt{L} + \sqrt{L - L_0}}{\sqrt{L} - \sqrt{L - L_0}} \right\}}$$

where K is the Karman constant, U_s is the surface current, and L_0 is the scale of the bottom roughness. Where both tidal currents and wind waves are acting, Wada gives

$$K_{zo} = \frac{K^2(d + L_0)^2 z^2}{L^2} \frac{U_s \sqrt{L}}{z \sqrt{d + L_0}} + \frac{8H}{T} e^{-2\pi d/\ell}.$$

Trent⁶⁵ makes the point that estimates of K_z or K_{zo} (also horizontal values), when determined from sea measurements, are of questionable accuracy. This is due mainly to the nonsteady ambient conditions as well as the unforeseen spatial variability that often occurs as the measurements are taken. Also important is that these turbulence parameters are often deduced from concentration measurements and back-calculated through an analytical diffusion equation. Hence the values are strictly valid only for the diffusion equation used to calculate them in the first place. The appropriateness of such values is certainly open to question. The determination of ambient as well as plume diffusion coefficients requires much more extensive research.

2. Some Basic Concepts and Terminology

Much has been written about the study of the numerical solution of partial differential equations. Like many other sciences, it has acquired a jargon of its own. Since we assume that many readers are unfamiliar with some, if not all, of the concepts and terms used, we have included this section in an effort to make the subsequent discussion more lucid. This is, however, a very brief and basic treatment; the reader wishing to pursue the development or use of numerical thermal-plume models should consult the standard references in this area. The treatise by Roache is quite insightful and, even though several years old, is still very relevant.

The ultimate goal of the numerical model is to calculate an approximate solution using a computer. It is therefore necessary to discretize the governing equations, i.e., derive a set of algebraic equations that, when solved, can be used to reconstruct an approximate solution of the continuous system. Most often this is done using a finite-difference method.

A finite-difference scheme requires that a grid system be established. The most common method of defining a grid system is to first lay out on each coordinate axis a number of equally spaced points and then from each of these points draw grid lines parallel to the remaining coordinate axes. The points of intersection of these grid lines are called nodal points or simply nodes, and the many small regions bounded by the grid lines are called cells.

Since most plume problems are cast using a rectangular Cartesian-coordinate system, the grid pattern most often used is a rectangular array of nodes. In an effort to match the two distinct scales of the plume problem, the vertical separation of nodes is normally made small compared to the horizontal separation. In addition, some use has been made of variable horizontal and vertical nodal spacings in an effort to improve lateral coverage of the flow field and to accommodate the irregular bottom.

Several problems arise immediately from the choice of grid system. One is resolution. To cover a reasonably large lateral extent and hold computer storage requirements to a tractable level, nodal points must be

selected tens to hundreds of feet apart. With this type of grid structure, the numerical model fails to detect the fine detail of the plume that is necessary for a valid numerical solution. To cope with this problem, one must either increase the number of nodal points substantially, or draw the grid structure closer to the outfall. The former greatly increases the cost and complexity of the model without an assurance of success; the latter increases the influence of the far boundaries. This brings us to another of the problems produced by the finite grid structure.

Forcing boundary conditions on the flow field at the free extremities of the grid system is highly undesirable since it imposes unnecessary and incorrect constraints on the system. Since it is impossible to know beforehand what velocities and temperature exist at the free boundaries, any specification necessarily distorts model predictions.

Even at natural boundaries (shoreline, surface, and bottom), the specification of boundary conditions is tricky. The irregular nature of a typical bottom and the presence of surface waves make velocity and temperature specification at these boundaries difficult. Moreover, surface height variations are hard to accommodate within the framework of the rigid grid system.

Little has been done to study the effects that limited resolution or inappropriate boundary conditions have on differential-numerical model calculations. These issues need to be carefully studied as part of an effort to improve the predictive capabilities of these models.

Once the grid is established, the second step is to "define" the dependent variables at various locations within each cell. This means that one evaluates the dependent variable at one or more points in space and then associates these numerical values with the grid cell or node, as the case may be. Each of these so-called discretized variables becomes an algebraic unknown to be solved for by numerical manipulation.

Now, with the grid established and the locations of discretized dependent variables fixed, the third step is to derive a set of algebraic equations in the discretized variables that, in some sense, approximates the continuous system. One way of deriving such an algebraic system is to first represent the various partial derivatives in terms of the discretized variables and certain grid parameters, such as the nodal point separation, and then substitute these expressions into the governing equations. The two common methods of deriving approximations to individual derivatives are the Taylor-Series method and the Polynomial method. The essence of these methods is discussed below.

a. The Taylor-Series Method

In the Taylor-Series method, the derivative is approximated by truncating a Taylor-Series expansion of the dependent variable about its

point of definition. For example, let f be an arbitrary dependent variable and x a representative independent variable. Let x_i represent the point of definition and f_i the value of f at x_i . Assuming that the derivatives of f exist and are continuous, we can write

$$f_{i+1} = f_i + \frac{df}{dx}(x_i)(x_{i+1} - x_i) + \frac{1}{2} \frac{d^2f}{dx^2}(x_i)(x_{i+1} - x_i)^2 + \left(\begin{array}{c} \text{Higher-order} \\ \text{terms} \end{array} \right), \quad (49)$$

where f_{i+1} is the value of f at x_{i+1} , an "adjacent" point of definition. Solving for the first derivative gives

$$\frac{df}{dx}(x_i) = \frac{f_{i+1} - f_i}{\Delta x} + O(\Delta x), \quad (50)$$

where $\Delta x = x_{i+1} - x_i$ and $O(\Delta x)$ means terms of order Δx . If one neglects these terms on the premise that Δx is "small," then one has a simple expression for the first derivative. The truncation error is of order Δx , and thus the above expression is said to have "first-order" accuracy. This approximation is referred to as a forward difference since it is based on a Taylor-Series expansion in the direction of increasing x .

One can also find the "backward"-difference form by expanding f_{i-1} about x_i . The result is

$$\frac{df}{dx}(x_i) = \frac{f_i - f_{i-1}}{\Delta x} + O(\Delta x). \quad (51)$$

This expression also has first-order accuracy. The term "one-sided" differencing is sometimes used to denote either a forward- or backward-difference formula. One-sided differences using the value of the variable upstream of the point of interest are called an "upwind" difference.

A third finite-difference expression can be found by subtracting the forward and backward differences:

$$\frac{df}{dx}(x_i) = \frac{f_{i+1} - f_{i-1}}{2\Delta x} + O(\Delta x^2). \quad (52)$$

This expression, called the central-difference form (for obvious reasons), has second-order accuracy.

While the above discussion is concerned with only the first derivatives of single-variable functions, analogous expressions can be found for higher-order and mixed derivatives. Higher-order approximations (as opposed to higher-order derivatives) can be found for any derivative by using a Taylor-Series expansion about additional neighboring points.

b. Polynomial Method

An alternative method of generating finite-difference expressions for various derivatives is to interpolate between the discretized variables using a polynomial and then differentiate the resulting interpolation formula. Up to second-order derivatives can be satisfactorily handled in this fashion. Unfortunately, polynomial fits to three or more data points can be decidedly poor. This is true since the intermediate values of the polynomial are not constrained to lie between the two extrema. As a consequence, polynomial interpolation is not used beyond second order. (In fact, very seldom is any finite-difference scheme for flow simulation higher than second order.)

The major drawback of independently deriving finite-difference expressions for individual derivatives, and then substituting these into the governing equations, is that there is no guarantee that the resulting approximation will be physically meaningful or mathematically acceptable. Two additional methods that derive the finite-difference forms by considering the equations to be solved are the integral method and the control-volume approach. These are discussed below.

c. The Integral Method

In the integral method, finite-difference expressions are not used to replace individual derivatives. Instead, the whole partial differential equation is integrated over a mesh-cell volume and over a short time increment. In this way, the differential equation is satisfied in an integral rather than a differential sense. Typically, the order of multiple integrations is interchanged and the mean-value theorem is used to evaluate the integrals. This results in a finite-difference equation based on the original partial differential equation. In this method, there is also much arbitrariness in developing the finite-difference equations, depending upon where in the region of integration the integral is evaluated using the mean-value theorem. For example, the forward-time, centered-space differencing formulation may be recovered by carrying out the above time integration from t to Δt , the space integration from $x - \Delta x/2$ to $x + \Delta x/2$, and using the midpoint in the mean-value-theorem evaluation.

d. The Control-volume Approach

This approach is the most physical in its basis. A control volume is constructed in space. Within the control volume, each conservation law is assumed to hold; i.e., the total increase in the variable is set equal to the net flux into the volume by advection plus the net flux into the volume by diffusion. This verbal expression of the conservation law is then put in mathematical form by analyzing the fluxes across each face of the control volume. Even at this, one must make some assumptions regarding the distribution of the inflow through the surface; in the evaluation of the diffusion terms, some finite-difference expression must be used for the derivatives.

Thus, in the course of deriving the discretized equations, some arbitrary choices must be made. These choices are not unimportant, however, since many things can go wrong if a poor arbitrary choice is made. We next consider a few of these.

A finite-difference scheme should be simultaneously convergent, stable, transportive, and conservative. These four terms are widely used to describe finite-difference formulations. The first two pertain to generic qualities of the finite-difference scheme; the latter two are specialized to numerical simulations of fluid flow. Basically, a finite-difference formulation is said to be convergent if the solution of the algebraic equations approaches the exact solution of the partial differential equations as the nodal separation or element size tends to zero. A nonconvergent scheme may be either divergent (that is, fail to approach a constant set of values) or inconsistent (that is, approach a constant but misrepresentative set of values). For multidimensional problems, the shape of the cell as the limit is taken may be a factor in the convergence of the solution. It is normally tacitly assumed and not proven that the discretized equations are convergent if a reasonable numerical solution can be obtained.

A numerical scheme is generally called unstable if random truncation or round-off error is amplified rather than attenuated by successive steps of the solution procedure.

A good example of a "reasonable" approximation gone wrong is the use of centered differencing to express the time derivative in the equations of motion. This results in an unconditionally unstable finite-difference scheme. On first examination, such a formulation may appear to be advantageous due to its second-order accuracy.

The transportive and conservative properties of finite-difference methods are widely discussed in relation to the numerical simulation of fluid flow. A scheme is said to be transportive, if "events" move in the downstream direction only, in the absence of physical diffusion mechanisms (say, when the turbulent viscosity is set to zero). For example, the use of centered-space differencing of the advection terms can result in upstream transport, even if the eddy diffusivity is zero. Upwind differencing, on the other hand, is transportive, since events always move downstream in the absence of physical diffusion, although upwind differencing is not synonymous with the term "transportive." Unfortunately, upwind differences are subject to numerical dispersion, since the truncation error is of the form

$$\frac{1}{2} \frac{d^2 f}{dx^2} \Delta x. \quad (53)$$

The second-order derivative here makes the one-sided differences subject to numerical dispersion.

The conservative property refers to the way in which the advection terms are written. Specifically, the conservative form is

$$\frac{\partial}{\partial x}(u\varphi) + \frac{\partial}{\partial y}(v\varphi) + \frac{\partial}{\partial z}(w\varphi), \quad (54)$$

where u , v , and w are the components of the velocity in the x , y , and z directions, respectively, and φ stands for the transported quantity. Roache¹ explains, in some detail, why the name "conservative" is used for this form. Basically, however, the name stems from the fact that, when the above form is integrated over any volume element, the result is equivalent to integrating the product of φ and the normal velocity over the surface or the net inflow of φ through the surface. Thus, the advection terms retain their physical meaning in the finite-difference formulation.

An alternative approach to finite differencing is the finite-element discretization. The finite-element technique requires the derivation of a functional for the governing equations that is minimized by the correct solution. A functional is a mathematical transformation mapping functions into real numbers. Representing the functional by V , a variational principle is of the general form

$$V(u) \leq V(u'), \quad (55)$$

where u is the exact solution of the equation set and boundary conditions associated with V , and u' is any arbitrary function.

In the finite-element technique, the flow domain is divided into discrete finite elements of an arbitrary shape selected mainly for convenience. Within each element, an approximation to the solution, u^* , is written as a synthesis expansion of trial functions, f_i , as

$$u^* = \sum_{i=1}^N u_i f_i(x, y, z), \quad (56)$$

where u_i is an expansion coefficient. One can minimize $V(u^*)$ with respect to the coefficients u_i by setting the partial derivatives equal to zero as follows:

$$\frac{\partial}{\partial u_i} V(u^*) = 0. \quad (57)$$

This results in one algebraic equation for each of the coefficients u_i and thus yields a proper system. Once the coefficients have been numerically determined by solving the algebraic equations above, the approximate solution may be easily reconstructed.

Little use has thus far been made of the finite-element method for treating plume dispersion. This is due in large part to the somewhat more complicated procedure needed to produce the finite-element equations. The finite-element method derives from a variational principle, whereas the finite-difference equations are obtained by straightforward substitution into the governing partial differential equations.

The finite-element method outlined above is a member of a broader class of solution schemes known as synthesis methods. (The terminology is not standard. Other synthesis methods are sometimes called finite-element methods.) The name "synthesis methods" derives from use of several functions to synthesize the solution as in Eq. 56. A synthesis technique known as the method of weighted residuals obtains the discretized equations by substituting the synthesis expansion (Eq. 56) into the governing equations and then averaging over all space using an "appropriate" set of weighting functions $\{W_j\}$. These averages are set to zero as follows:

$$\int_{\text{All Space}} W_j Au^* dV = 0, \quad (58)$$

where Au^* represents the governing equations with the synthesis expansion substituted. The number of weighting functions must be equal to the number of expansion functions so that a proper set of algebraic equations results. This technique is formally similar to the solution of self-adjoint linear partial differential equations by eigenfunction expansion. In this case, the weighting functions are the eigenfunctions themselves. Due to the orthogonality of eigenfunctions, the matrix system is diagonal and a solution of the algebraic equations is easy.

After the governing equations have been discretized, the resulting algebraic equations must be solved. A separate algebraic equation is developed for each variable at each node. Thus, with several variables and several hundreds or even thousands of nodes, the number of equations to be simultaneously solved is enormous. This last step in the solution procedure consumes excessive amounts of computer time and thus affords the greatest potential for savings. Computer times must be reduced by at least an order of magnitude in order to make the differential-numerical models really practical for general use in predicting thermal plumes.

Two types of methods may be used to solve the algebraic equations arising from finite-element or finite-difference discretizations. The first class is generally referred to as direct methods. A direct method is a step-by-step procedure that results in an exact solution (except for numerical imprecision) after a predetermined number of steps. A familiar direct method is Gaussian elimination. Because the governing equations for plume problems are nonlinear, the discretized algebraic equations are also nonlinear. For this reason, direct methods are generally not applicable, except possibly to solve a subproblem as part of a larger solution scheme.

The second large class of solution procedures is called iterative methods. Briefly, the iterative approach is an algorithm to obtain successively better approximations of the solution. The iterative procedure is terminated when a simple test known as the convergence criterion shows the approximate solution to be acceptably close to the exact solution. Designing an iterative method that yields the most rapid approach to an acceptable solution is a trial-and-error proposition. Due to this empirical nature, a number of equally valid algorithms exist. Usually, selecting the most efficient procedure from among the feasible alternatives can only be done by first implementing the algorithm on the computer and then measuring its convergence rate. A number of different problems should be considered, since the convergence rate may vary with the problem.

In the above context, convergence of iterative solutions means that, in the limit of an infinite number of steps, the iterative solution approaches the exact solution of the algebraic equations. The fact that a solution that becomes constant has converged is typically taken for granted. This may or may not be true. An iterative solution can become constant enough to satisfy the convergence criterion without being very close to the exact solution.

The differential-numerical models reviewed in Chapter II use an iterative procedure known as the asymptotic-time technique. In this technique, the steady-state solution is sought by time-integrating the time-dependent governing equations beginning with some neutral situation and hopefully ending with the steady plume. Each successive approximation is ostensibly an instantaneous picture of the plume after time t has elapsed. Preliminary tests of the Paul-Lick² and Waldrop-Farmer^{66,67} differential-numerical models indicate that they predict plumes consistently two to ten times smaller than those actually observed. This had led to speculation that these calculations were not run to steady state. To test this possibility, Waldrop and Farmer ran an already converged problem for a few additional hours of computer time. They found that the plume grew in size, but at a slow rate (see Sec. II.L).

Being parabolic, the time-dependent Navier-Stokes equations can be solved by an explicit finite-difference scheme. An explicit scheme is one in which the values at each successive time step are calculated using only known values from the previous time step. This is contrasted with an implicit scheme, or one that requires the solving of a coupled set of algebraic equations at each time step. The explicit scheme is obviously much easier to program and requires less computation at each step. However, the size of the time increment may be limited for explicit schemes by stability and convergence considerations, thus requiring an extremely large number of tiny time steps. Linear diffusion equations (first order in time and second order in space) have been extensively studied in this regard. Usually, an explicit scheme is stable and convergent (as the nodal separation tends to zero) if the time increment satisfies a restriction of the form

$$\Delta t < c \frac{h^2}{D}, \quad (59)$$

where h is the characteristic spatial separation, D is the characteristic diffusivity, and c is a constant. This type of criterion can be heuristically justified as follows: In an explicit scheme, each node is directly connected only to adjacent nodes through the previous time step. Thus, an explicit algorithm restricts the rate at which a local disturbance can propagate from node to node. The time increment must be small enough to match the rate of diffusion dictated by the problem. The situation for the time-dependent Navier-Stokes equations is complicated by the nonlinear transport mechanism of advection, but in general one wishes to restrict the size of the time increment to correspond to the rate at which mass, momentum, and energy are transported through the grid system.

There are several possible schemes to accelerate the convergence to steady state. One of the most obvious is overrelaxation. In this technique, one tries to anticipate the correct solution by adding to each successive guess a certain fraction of the difference between it and the previous guess. One hopes to oscillate from side to side about the exact solution rather than approach monotonically. The fraction added, called the relaxation parameter, is usually determined empirically so as to give optimum convergence for one or more problems.

Other simple methods to accelerate convergence of iterative solutions exist. Recasting the equations in alternative variables or shuffling the order in which the equations are solved can occasionally improve algorithm efficiency. Often unnecessary calculational steps can be eliminated with a corresponding savings of computer time. However, these modifications will probably not result in the savings of computer time necessary to make differential-numerical models practical for general use.

Since many models include a Poisson-type equation as part of their formulation, it is worthwhile to note the various methods that can be used to solve such an equation. Owing to the linearity of the equation, direct methods such as Gaussian elimination or Cholesky decomposition are applicable, but direct methods are not widely used in fluid-flow simulations. A large class of iterative solutions exists; three of the most popular are the Gauss-Seidel method, the successive-overrelaxation (SOR) method, and the alternating-direction-implicit (ADI) method.

The Gauss-Seidel method is an iterative procedure in which each variable is explicitly computed using the present value of the other variables. The algorithm treats each variable in succession, replacing each old value with the new value as soon as it is calculated. The same process, except that all old values are retained until an entire set of new values is computed, is called the Jacobi method. This latter approach is commonly not used due to its greater storage requirements. The overrelaxed (in the sense of the above definition) Gauss-Seidel iteration is usually called the SOR method.

The ADI scheme is an iterative technique in which a direct (implicit) method is used in only one direction at a time. After an intermediate solution is calculated using implicit coupling in one direction, a second step is made using implicit coupling in a different direction. One-dimensional implicit coupling gives a tridiagonal matrix that can be quickly solved using a special procedure. It has been shown that, in rectangular domains, the ADI method is preferable to the SOR method.

Another matter to be discussed is the problem of numerical accuracy. Inaccuracy results from both the truncation error in the discretization of the continuous system and from the limited precision with which numbers can be stored in the computer. A certain level of inaccuracy can be tolerated in individual values. The concern is for the accumulation and propagation of numerical error. For example, consider pressure. The dynamic and density-induced components of the total pressure field are usually quite small compared to the hydrostatic pressure, and yet these small components are very significant. Small errors in the pressure field may result in large errors in the velocity and temperature fields. This problem is commonly overcome by defining and working with the reduced pressure, i.e., the pressure measured relative to a local hydrostatic pressure.

A final point that warrants general discussion is the applicability of spatial-marching techniques in thermal-plume models. Boundary-value problems are generally more difficult to solve than initial-value problems. This is particularly true of numerical simulations in which the solution can be advanced step-by-step using an explicit finite-difference scheme. Many models use explicit forward-marching techniques in time, since the governing equations are truly parabolic in time. In certain problems in which a principal direction of flow can be identified, it may be possible to solve an initial-value-problem approximation of the true boundary-value problem. Certain effects are necessarily sacrificed, but the gain in computer time can be substantial. The models by McGuirk and Spalding,⁵⁰ Till,⁶⁸ and Waldrop and Farmer^{66,67} all use a spatial-marching technique. The specifics of these models are discussed elsewhere in this report. The applicability of these methods is presently uncertain.

3. Formulations of the Governing Equations

a. Introduction

Numerous formulations of the governing equations of flow and heat transport are used in numerical thermal-plume models. Although each is unique unto itself, the various formulations fall into several broad classes. This section summarizes the types of formulations in present use, by first describing each formulation in general terms and then considering the various models that use that formulation. First, a few words are warranted concerning the motivation behind the formulations.

As one ponders the hydrodynamic equations in their standard textbook form, two features are immediately recognized: The momentum transport equations are nonlinear, and continuity is of a different mathematical type than the transport equations. The nonlinearity of the momentum-transport equations means that the corresponding algebraic equations must be solved iteratively, or at least with a technique that treats the nonlinear terms as known quantities and thus solves a linear system. The simplest technique possible is an explicit one in which the coefficient matrix of the linear system is diagonal and the solution can be computed by an explicit formula. The implicit methods, on the other hand, require some matrix reduction, but still solve only a linear system. Thus, a prime motive behind each formulation is the need to find a linear system of algebraic equations that can be solved for the approximate solution.

The most common iterative scheme for calculating a steady thermal plume is the asymptotic-time technique. Here, the steady solution is obtained by relaxing the time-dependent equations beginning with some arbitrary and presumably unimportant initial condition. Normally, the boundary conditions are held steady during the process, although it is only necessary that the boundary conditions eventually become steady. Surprisingly then, most of the successful numerical models for steady thermal plumes are built around the time-dependent governing equations.

The use of the time-dependent equations has a number of beneficial side effects. One is that a model can easily be extended to time-dependent problems such as plant startup. A second is that existence and uniqueness of the solution are somewhat less troublesome if the unsteady equations are used. Since the time-dependent equations ostensibly represent physical reality, it can be argued on physical grounds that a solution must exist. The steady-state equations, on the other hand, are based on the assumption that, in the limit of large times, the unsteady solution approaches the solution of the steady equations. There are actually four possible outcomes:

- (1) No steady-state solution exists.
- (2) One steady-state solution exists, and the unsteady solution approaches it as time goes to infinity.
- (3) Two or more solutions of the steady-state equations exist; one is the limit of the unsteady solution, while the other(s) are extraneous.
- (4) Two or more solutions of the steady-state equations exist; more than one is the limit of the unsteady solution, depending on the initial condition, while any others are extraneous.

The steady-state equations are valid only in the case of outcome No. 2 above.

The problems of existence and uniqueness are exacerbated as the amount of approximation or tampering increases. If absurd boundary

conditions are specified, then the existence and uniqueness of the solution are severely degraded. Moreover, since the discretized equations always have a different eigenspectrum than the analog equations, they have different existence and uniqueness characteristics.

Although most formulations are based on the time-dependent equations, there are methods for solving the steady-state equations directly. These methods can offer important practical advantages to the asymptotic-time approach. This is particularly true when the equations are stiff in time. Stability may necessitate a very small time step, whereas the steady solution may be reached only after many, many such steps. The time-dependent solution may grind on, time step after time step, without reaching steady state. Obtaining the most efficient numerical procedure is the second factor providing motivation for trying various formulations.

As noted earlier, the governing equations in their standard form are of a mixed type. The hydrodynamic equations are of the general transport form consisting of a time derivative, advections terms, source terms, and diffusion terms. The source terms contain pressure gradients that are, in combination with body forces, the driving potential for the flow. However, the pressure is unknown and must therefore be regarded as a dependent variable. Continuity closes the equation set and also does not contain pressure explicitly. In addition, in contrast to the momentum equation, continuity is a first-order relation among the components of the velocity. Thus, a third significant motivation behind various formulations is to resolve the problem presented by the mixed nature of the governing equations.

In summary, three of the prime motives behind the various formulations are:

- (1) The need to find a linear system of algebraic equations.
- (2) The quest for an efficient numerical procedure.
- (3) The desire to handle pressure and continuity in the solution procedure.

It is interesting to see how each formulation accomplishes these objectives.

b. Methods Using the Textbook Form of the Governing Equations
(Brady and Geyer;³ Waldrop and Farmer^{66,67})

The Brady-Geyer³ and Waldrop-Farmer^{66,67} thermal-plume models solve the full Navier-Stokes equations, yet do so by iterating directly on the basic governing equations in their standard time-dependent form. No auxiliary equations are used. Both models are completely explicit using previously calculated values to obtain by simple algebra the new values. Both models also make use of the asymptotic-time technique.

Brady and Geyer (see Sec. II.H) solve a set of equations consisting of the three equations of momentum conservation (no terms dropped); the three equations stating the conservation of mass, heat, and salt concentration; the equation of state; and a separate equation stating the hydrostatic approximation.

In solving the above basic equation set, Brady and Geyer employ central differences on all spatial derivatives and simple forward differences for all time derivatives, except during the early stages of execution when mixed differencing is used to empirically determine the largest time step that yields a stable solution. Forward differencing in time allows the completely explicit integration of the model equations. Brady and Geyer always use the latest value of the variable that has been calculated and consequently do not retain the entire solution from the previous time step. This intermediate updating process makes the stability of the solution dependent on the order in which the equations are integrated and may necessitate a smaller time step than if the entire solution from the previous time step were saved.

The solution procedure at each time step is listed by Brady and Geyer as follows:

- (1) Determine relative pressures at all fluid cells by integrating vertically downwards from the free surface.
- (2) Search for solid boundaries adjacent to fluid cells, and set pressures equal to adjacent fluid pressures.
- (3) Compute velocities for each fluid cell due to pressure gradients, advection, turbulent mixing, and wind stresses.
- (4) Consult boundary conditions on velocities, and adjust computed values where necessary to achieve conformity.
- (5) Apply the principle of continuity to each fluid cell, and distribute any surplus inflow by adjusting velocities of adjacent fluid cells.
- (6) Repeat steps 4 and 5 until all cells balance satisfactorily.
- (7) Compute salinity and temperature changes at each fluid cell due to advection, turbulent mixing, and surface heat transfer.
- (8) Consult boundary conditions on salinity and temperature, and adjust computed values where necessary to achieve conformity.
- (9) Compute surface-elevation changes from vertical velocities.

This approach is similar to the velocity-corrector-potential (VCP) method described in Sec. c below. The basic philosophy here is to first determine the velocity components by finite-differencing the time derivatives in the momentum equations (step 3) and then correct the computed velocities

to bring them into conformance with continuity (step 5). We shall return to this point when the VCP method is considered. Unlike the VCP method, however, Brady and Geyer use the results of continuity to evaluate pressure as follows. The vertical velocities calculated at the top of the grid are translated into surface-height elevations (step 9) that are, in turn, used to determine the three-dimensional pressure distribution by integrating the hydrostatic body force vertically downward (step 1).

The Waldrop-Farmer model^{66,67} (see Sec. II.L) is also a completely explicit treatment of the basic Navier-Stokes equations written in standard form. The model solves the continuity equation, three momentum equations, equations for salt and heat conservation, and an equation of state. The complete vertical momentum equation containing all the advection and diffusion terms may be used, or else that equation may be replaced by the hydrostatic-pressure approximation.

The device used by Waldrop and Farmer is the dropping of the acceleration term from the vertical momentum equation. This enables the model developers to write the complete equation set as a sequence of parabolic equations that can be solved as initial-value problems by explicit finite-difference formulas. The x-momentum, y-momentum, heat-energy, and salt equations are rewritten with the time derivatives on the left-hand side of the equation and the remaining terms on the right-hand side. Simple forward differencing is then used to approximate the time derivative. On the right-hand side, all variables are evaluated at the earlier time step with central differencing used to approximate the pressure gradient and diffusion terms, and upwind (one-sided) differencing to approximate the advection terms.

Instead of using the z-momentum equation to calculate the vertical velocity, as is usually done, Waldrop and Farmer integrate continuity from bottom to surface by explicit forward-differencing of the vertical derivative. At the surface, the typically nonzero vertical velocity is translated into a free-surface-height elevation. The z-momentum equation is then integrated vertically downward from the free surface to obtain the pressure field by forward-differencing the pressure gradient term. Thus, whereas in the Brady-Geyer method the vertical velocity is obtained from the vertical momentum equation and pressure is obtained indirectly from an application of continuity, Waldrop and Farmer reverse the role of these two equations, calculating pressure from the vertical momentum equation and vertical velocity from the continuity equation. The sequence of steps in the Waldrop-Farmer model may be summarized as follows:

- (1) The x-momentum equation is solved for the u-velocity component (x direction offshore, u velocity in the direction of the x axis).
- (2) The y-momentum equation is solved for the v-velocity component (y direction shore parallel, v velocity in the direction of the y axis).

(3) The conservation-of-salt and -energy equations are solved for the salt concentration and temperature at each node.

(4) The continuity equation (time-independent) is solved from the bottom to the surface for the w-velocity component (w velocity in the vertical or z direction).

(5) The free-surface elevation is computed (adjusted) by performing a mass balance on the topmost layer of grid cells.

(6) The z-momentum equation is solved in space (time-independent) for the pressure field. The integration is carried out vertically downward from the free surface. This integration and determination of pressure at each grid row does become simpler if the hydrostatic approximation is made.

A casualty of integrating the elliptic governing equations as a parabolic set using simple forwarding differences is that certain boundary conditions are never enforced. This result implies that the calculation procedure violates the basic elliptic nature of the problem and may therefore be expected to give an inadequate approximation of the true solution under certain conditions. Unfortunately, there is no way (not even an empirical one) to determine how severe this omission is in thermal-plume problems. Other boundary conditions and the attendant difficulties of the Waldrop-Farmer model are described in Sec. II.L. Briefly, no-slip conditions and zero normal gradients of salt and heat are assumed at fluid-solid boundaries. The kinematic surface boundary condition is simplified to a mass-conservation relationship for the topmost grid rows, and velocities are specified at the upcurrent boundary.

c. Velocity-corrector-potential (VCP) Method (Pritchett, England, and Taft;⁵¹ Macqueen;⁶⁹ McGuirk and Spalding⁵⁰)

A popular method of solving the hydrodynamic equations is a technique using the primitive variables described by Chorin⁷⁰ and Harlow and Amsden.⁷¹ The models by Macqueen,⁶⁹ Pritchett, England, and Taft,⁵¹ and McGuirk and Spalding⁵⁰ all use a variation of this technique.

For the purpose of discussion, we write the hydrodynamic equations as

$$\nabla \cdot \vec{u} = 0 \quad (60)$$

and

$$\partial \vec{u} / \partial t = A(\vec{u}) - \nabla p, \quad (61)$$

where $A(\vec{u})$ stands for the terms not written. The velocity can be explicitly computed from Eq. 61 by simple forward differencing. Thus,

$$\vec{U}_{n+1} = \vec{U}_n + [A(\vec{U}_n) - \nabla p]\Delta t, \quad (62)$$

where the subscripts denote the number of the time step and Δt is the size of the time step. The problem here is that velocities calculated in this way do not necessarily obey continuity, depending on what pressure distribution is used. The thrust of the VCP method is to first calculate the velocity with a judiciously chosen pressure distribution in the above way and then correct that velocity so that continuity is satisfied. To see how this is done, consider the following. Let q be a suitably selected pressure distribution (an ingenious choice is described below), and let $\vec{V}$ be calculated from

$$\vec{V} = \vec{U}_n + [A(\vec{U}_n) - \nabla q]\Delta t. \quad (63)$$

Subtracting Eq. 63 from Eq. 62 yields

$$\vec{U}_{n+1} - \vec{V} = -\nabla\varphi, \quad (64)$$

where

$$\varphi = (p - q)\Delta t.$$

Thus, $\vec{U}_{n+1}$ differs from $\vec{V}$ only by the gradient of scalar function φ , called the velocity-corrector potential. Since we want $\vec{U}_{n+1}$ to satisfy continuity, we must have (taking the divergence of Eq. 64) that

$$\nabla^2\varphi = \nabla \cdot \vec{V}. \quad (66)$$

A velocity field that satisfies continuity can be calculated by the following multistep process.

- (1) Choose a convenient pressure distribution q .
- (2) Calculate the first approximation to the velocity from

$$\vec{V} = \vec{U}_n + [A(\vec{U}_n) - \nabla q]\Delta t. \quad (67)$$

- (3) Solve the Poisson equation

$$\nabla^2\varphi = \nabla \cdot \vec{V} \quad (68)$$

for the velocity-corrector potential φ .

- (4) Calculate the velocity that satisfies continuity and the pressure corresponding to that velocity from

$$\vec{U}_{n+1} = \vec{V} - \nabla\varphi \quad (69)$$

and

$$p = q - \varphi/\Delta t. \quad (70)$$

Since the choice of q is theoretically arbitrary, it is desirable to select a distribution that minimizes the difficulties involved in solving for the velocity-corrector potential. The choice that seems to be most advantageous is the one that leads to totally homogeneous boundary conditions on φ .

Before we examine how such a choice is arranged, we need to review what boundary conditions are possible on the primitive variables. First, either pressure or normal velocity, but not both simultaneously, must be specified at every point on the boundary. The tangential component may be set to zero (no-slip condition), its normal gradient set to zero (free-slip condition), or its normal gradient set to a fixed value. The no-slip condition is generally used along rigid boundaries, although, unless the grid resolution is fairly good near the boundary, free slip is probably a better approximation. Let us denote the solution domain by F , that portion of the boundary on which the normal component of velocity is specified by U , and that portion on which pressure is given by P , and let

$$p = p_B \text{ on } P \quad (71)$$

and

$$\vec{u} \cdot \vec{n} = u_B \text{ on } U, \quad (72)$$

where $\vec{n}$ is the unit normal. Of course, if the normal velocity is specified on the entire boundary, then consistency requires that

$$\int_U u_B \, dS = \int_F \nabla \cdot \vec{u} \, dV = 0. \quad (73)$$

Given the conditions specified in Eqs. 70 and 71, we can choose q to satisfy the requirements that

$$q = p_B \text{ on } P \quad (74)$$

and

$$\vec{n} \cdot \nabla q = \frac{u_{B,n+1} - u_{B,n}}{\Delta t} - Au_{B,n} \text{ on } U. \quad (75)$$

With q selected in this way, the boundary conditions on φ become

$$\varphi = 0 \text{ on } P \quad (76)$$

and

$$\vec{n} \cdot \nabla \varphi = 0 \text{ on } U. \quad (77)$$

These homogeneous conditions greatly facilitate the solution procedure.

Macqueen⁶⁹ uses the method described above for two-dimensional plume transport with no vertical variations. A constant-depth receiving body is assumed. The region of flow is divided into a rectangular grid system. Pressure, temperature, and the velocity-corrector potential ϕ are defined at the center of each grid cell. The u velocity is located on the left and right faces of the cell, with the v velocity defined on the top and bottom. In Macqueen's model, the heat equation is solved after the flow equations as they are uncoupled from one another. The method of upwind differencing as developed by Gosman et al.⁷² is used.

The Pritchett-England-Taft model⁵¹ also uses the VCP technique. The details appear in Sec. II.I. The model is three-dimensional and couples the hydrodynamics to the flow. Free-slip conditions and zero normal velocity are assumed at all boundaries, including the free surface at which the rigid-lid approximation is made. Consequently, the three-dimensional equation for the velocity-corrector potential has all Neumann boundary conditions. The Pritchett-England-Taft model also solves the equations of fluid density, passive solute, concentration, and turbulent energy along with the flow, but using an Eulerian/LaGrangian technique for them.

McGuirk and Spalding also use the VCP method, but it is only a part of their three-dimensional model. Instead, they couple a marching procedure in the predominant direction of flow, with the VCP method used to solve for the two-dimensional flow in the plane normal to that direction. The flow- and energy-transport equations are thus solved in two dimensions at a succession of locations along the direction of flow. No-slip conditions are assumed at solid boundaries, and zero gradients of all variables are assumed at the free surface. The staggered grid setup described above is used. The McGuirk-Spalding algorithm called SIMPLE (semi-implicit method for pressure-linked equations) solves for the three velocity components, the pressure, temperature, and two turbulence quantities (turbulent energy and energy dissipation rate).

d. The Divergence of the Navier-Stokes Formulations (Spraggs and Street;⁴ Paul and Lick;² Castellano and Tozzi⁷³)

In this report, the DNS method refers to any method that uses a Poisson equation for pressure that is developed by taking the divergence of the Navier-Stokes equations. The most famous of the DNS methods is the MAC (Marker and Cell) technique described by Welch et al.⁷⁴ Numerous MAC-like methods have been developed in recent years. Among those developed for thermal-plume problems are the models of Spraggs and Street,⁴ Paul and Lick,² and Castellano and Tozzi.⁷³ Our discussion begins with a brief summary of the MAC method, followed by a brief look at the MAC-like thermal-plume models.

The MAC method is characterized by four features: the use of the DNS formulation with its auxiliary equation for pressure, the specific

differencing scheme employed, the finite-difference cell structure, and the use of massless marker particles to map out streaklines that aid in flow visualization. Each of these distinguishing features will be briefly described.

The MAC method is unique in that the divergence of the velocity is not set to zero as in the usual DNS formulation. This is because the numerical approximation of the velocity field computed by forward-differencing the time derivatives of the momentum equations may not satisfy continuity exactly. The derivation begins with the momentum equations in standard form

$$\rho \left[\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) \right] = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\epsilon \frac{\partial u_i}{\partial x_j} \right) + X_i. \quad (78)$$

Here, we have used the familiar indicial notation, and the variables take on their usual meaning. Taking the divergence of Eq. 78 gives

$$\rho \left[\frac{\partial}{\partial t} \frac{\partial u_i}{\partial x_i} + \frac{\partial^2}{\partial x_i \partial x_j} (u_i u_j) \right] = - \frac{\partial^2 p}{\partial x_i \partial x_i} + \frac{\partial}{\partial x_j} \left(\epsilon \frac{\partial}{\partial x_j} \frac{\partial u_i}{\partial x_i} \right) + \frac{\partial X_i}{\partial x_i}, \quad (79)$$

where we have dropped the derivatives of density and eddy diffusivity. Letting $\partial u_i / \partial x_i = D$ and simplifying, we get

$$\frac{\partial^2 p}{\partial x_i \partial x_i} = \frac{\partial}{\partial x_j} \left(\epsilon \frac{\partial}{\partial x_j} D \right) + \frac{\partial X_i}{\partial x_i} - \rho \frac{\partial D}{\partial t} - \rho \frac{\partial^2}{\partial x_i \partial x_j} (u_i u_j). \quad (80)$$

As noted in the preceding paragraph, the numerical approximation of D may be nonzero. In the MAC method, an attempt is made to force D to zero by setting $\partial D / \partial t = -D_n / \Delta t$. This is equivalent to approximating $\partial D / \partial t$ by $(D_{n+1} - D_n) / \Delta t$ and then setting $D_{n+1} = 0$. This trick improves the stability of the finite-difference equations and aids in minimizing the accumulation of round-off and truncation error. It is generally thought that the consideration of $\partial D / \partial t$ is more important than the inclusion of the second-order spatial derivatives of D .

In summary, the equations used by the MAC method are

$$\frac{\partial u_i}{\partial t} = - \frac{\partial}{\partial x_i} (u_j u_i) - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\epsilon \frac{\partial u_i}{\partial x_j} \right) + X_i \quad (81)$$

and

$$\frac{\partial^2 p}{\partial x_j \partial x_j} = \frac{\partial}{\partial x_j} \left(\epsilon \frac{\partial}{\partial x_j} D \right) + \frac{\partial X_i}{\partial x_i} - \rho \frac{\partial D}{\partial t} - \rho \frac{\partial^2}{\partial x_i \partial x_i} (u_i u_j). \quad (82)$$

Equation 81 is solved for the velocity components by integrating in time. Equation 82 is then solved for the pressure field, while simultaneously forcing the divergence of the velocity field to zero.

The boundary conditions used in the MAC method are fairly general. No-slip or free-slip conditions can be used along rigid walls, and either pressure or normal velocity can be specified. If normal velocity is given, then the boundary conditions on Eq. 82 are obtained by solving Eq. 81 for the pressure gradient and evaluating it at the boundary. Out-flow and free-surface boundary conditions are the most difficult to handle. A more detailed discussion of boundary conditions is given by Roache¹ and Welch et al.⁷⁴

Before going on to the other characteristics of the MAC method, let us consider the similarity between the MAC formulation as outlined above and the VCP method.

- (1) Both methods use the primitive variables, although the velocity-corrector potential does not necessarily involve pressure explicitly.
- (2) Both methods solve an auxiliary Poisson equation for a scalar potential.
- (3) The boundary conditions on the auxiliary equation are derived from the specification of pressure or normal velocity on the boundary. The former produces Dirichlet boundary conditions; the latter yields Neumann boundary conditions.

The primary difference between the two methods is that the VCP method corrects the velocity field so that it satisfies continuity, while the MAC method accounts for the divergence of velocity in solving for the pressure.

The second distinguishing feature of the MAC method is the use of FTCS (forward-time, centered-space) differencing. This scheme, in combination with the cell structure to be described below, produced exceptionally good results for the problems considered by Welch et al. Later testing has revealed that some instabilities can arise from this scheme, and excessive damping can be produced in problems with free surface waves.⁴ For example, this differencing scheme is unconditionally unstable for inviscid flows. The finite-difference scheme has been improved in more recent investigations of the method.

The third distinguishing characteristic of the MAC method is its cell structure. If the cell is viewed as a cube, then pressure is defined at the center of the cube, and each component of the velocity is defined on those faces of the cube that are normal to it. The cell structure is widely copied by other models since it produces a convenient finite-difference representation

of the equation. Additionally, this cell structure has a strong intuitive appeal since the velocities represent flow into and out of the cell through the faces. An extra layer of fictitious grid cells is placed along each boundary to aid in specifying boundary conditions.

The final characteristic of the MAC method is the use of massless marker particles. The particles are advected through the grid, producing a "streakline" picture of the flow. The massless particles also define the location of the free surface and are thus essential for application of the method to free-surface problems. The particles map out the slope and curvature of the surface from which the normal and tangential stress boundary conditions can be applied.

The MAC method was first described in 1966; since then, important improvements have been made. These are described by Roache.¹

Several MAC-like models are available for thermal-plume prediction. One is the three-dimensional model due to Spraggs and Street.⁴ The full Navier-Stokes and heat-energy equations are solved allowing for the existence of a free surface. This model is perhaps the most general and most complex of available thermal-plume models.

As in any MAC-like method, a separate equation is derived for pressure by taking the divergence of the Navier-Stokes equations. Spraggs and Street derive this equation in terms of the reduced pressure, i.e., the total pressure with a hydrostatic reference pressure subtracted away. As previously noted, the divergence of the velocity field is not canceled out of the pressure equation, but is retained and used to correct for the fact that the numerical approximation of the velocity field does not usually satisfy continuity. As presently formulated, the Spraggs-Street model handles a thermal discharge into an irregularly shaped closed impoundment. No open-water boundaries are presently considered, but this restriction is easily removed. Also, the present formulation uses a uniform grid system.

The model developers use forward differences in time, central differences for the diffusion terms, and second-order upwind differencing of the advection terms, except at the boundaries, where FTCS differencing is used. Second-order upwind differencing is both conservative and transportive and retains something of the second-order accuracy of centered-space differencing. However, given the large number of boundary cells, it is not clear if the approximation is actually second order. The use of upwind differencing of the advection terms is thought to be an improvement over the centered-space differencing scheme used in the original MAC formulation.

The pressure equation is solved in three dimensions, a very time-consuming process, using point successive overrelaxation with the relaxation parameter varying between 1.4 and 1.72. The solution of the pressure equation normally takes fewer than 50 iterations.

Spraggs and Street do not employ massless tracers as is done in the MAC method, but rather enforce the kinematic free-surface condition at each time step. The differential equation representing this free-surface constraint is solved numerically using a dual-sweep alternating-direction-implicit method.

Spraggs and Street use a slightly modified version of the MAC staggered grid system. The main difference is that free-surface pressure is actually defined at the free surface and not at the cell center as in the MAC method.

Boundary conditions are philosophically similar to those used by the MAC method, but do differ in some specifics. Briefly, no-slip conditions are imposed at walls and a wind-stress condition is used at the free surface. Temperature is constrained to have a zero normal gradient at walls (adiabatic-wall condition) and to be determined by atmospheric heat exchange at the surface. Pressure gradients on the boundaries are determined by solving the momentum equations, except at the free surface, where the pressure is taken as zero.

To date, the model has been applied to lakes with wind over the surface and with heated inflows and outflows. Predictions appear reasonable, but no verification has yet been made.

The Paul-Lick thermal-plume model² is also developed using the DNS approach. The details are summarized in Sec. II.K and Appendix B. A few characteristics are discussed here to emphasize the comparison with similar models.

Paul and Lick solve the full momentum equations in the two lateral directions, but use the hydrostatic form of the vertical momentum equation. The rigid-lid approximation replaces the true kinematic surface-boundary condition. These two assumptions work in tandem to reduce the Poisson equation for pressure to two dimensions. Mathematically, this is done by vertically integrating the three-dimensional version and introducing a constant of integration called the surface pressure. Once the surface pressure is calculated, the three-dimensional pressure field is determined by vertically integrating the hydrostatic pressure form of the vertical momentum equation beginning at the surface. Since the pressure equation is vertically integrated, the $\partial D / \partial t$ term discussed with the MAC method becomes $\partial w / \partial t$. Paul and Lick apply the rigid-lid condition and represent $\partial w / \partial t$ by a backward difference, as described, in an effort to force w to zero and thus satisfy the rigid-lid condition at the surface. This corrective procedure was devised by Hirt and Harlow.⁷⁵ Even though the pressure equation is only two-dimensional, its solution is still very time-consuming, especially since all Neumann boundary conditions are used. Paul and Lick use an alternating-direction-implicit scheme for this part of the solution.

An FTCS differencing scheme is used but the staggered grid system is somewhat different than that used by Spraggs and Street and in the MAC method (see Sec. II.K). The efficiency and stability of the Paul-Lick discretization are unknown.

Other aspects of this model are discussed in Sec. II.K and Appendix B.

Castellano and Tozzi⁷³ employ a MAC-like method for their two-dimensional model of the dispersion of heated effluent in shallow rivers. The physical situation represented is a shoreline surface discharge oriented at some angle to the prevailing river current. An auxiliary two-dimensional pressure equation is derived and used in a manner similar to that described in the earlier discussion of the MAC method.

An interesting feature of the Castellano-Tozzi model is that the heat-energy equation is completely uncoupled from the flow equations. In fact, once the steady flow field is found, a separate computer program is used to solve the steady-state, heat-energy equation. The longitudinal diffusion of heat is removed, and a uniformly distributed source is added to simulate surface heat loss. The omission of longitudinal diffusion is probably adequate for one-directional river flow, but is certainly unacceptable for lake situations.

Upstream of the discharge, the velocity is taken as shore-parallel with the lateral variation in magnitude known. At the far downstream boundary, the first derivatives of the two velocity components are assumed to be zero. Reflective boundary conditions are applied along the shore, except at the point of discharge, where the velocity components are specified. Neumann boundary conditions are derived for pressure by solving the momentum equation there for the pressure gradient. Finally, in solving the heat-energy equation, adiabatic-wall conditions are applied at the shoreline, except at the outfall, where the temperature is prescribed. Upstream temperatures are assumed to be known, but since longitudinal heat diffusion is eliminated, no downstream boundary conditions are needed.

The finite-difference approximation is similar to that used in the MAC method. Pressure and temperature are defined at the cell center, and velocity components on the faces normal to their direction. Centered-space differences are used for the diffusion terms, first-order upwind differences for the advection terms, and forward differences for the time derivatives.

This ends our discussion of the formulations that use only primitive variables, namely, the components of velocity and pressure. Next, we consider a method that uses the components of velocity and the components of vorticity (instead of pressure). Later, we consider methods involving completely derived variables, namely, variations of the stream-function/scalar-potential/vorticity formulation.

e. Velocity-Vorticity Formulation (Chien and Schetz⁷⁶)

The Chien-Schetz⁷⁶ model uses the velocity-vorticity formulation to solve the steady-state equations of flow. The vorticity is a vector quantity defined by

$$\vec{\omega} = \nabla \times \vec{u}, \quad (83)$$

where $\vec{u}$ is the velocity.

Unlike the previous methods that solve the time-dependent equations, the velocity-vorticity approach is specific to the particular representation of generalized stress tensor (turbulence model) that is used. Therefore, we shall carry out this discussion using the formulation of Chien and Schetz⁷⁶ as generalized by Markham and Lewis.⁷⁷ Using indicial notation, the equations of motion used in the Chien-Schetz formulation are

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (84)$$

$$\rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + X_i + \rho \frac{\partial}{\partial x_j} \left(\frac{\partial v u_i}{\partial x_j} + \frac{\partial v u_j}{\partial x_k} \right). \quad (85)$$

In this form, the stress tensor is completely symmetric and thus locally irrotational. Pressure can be eliminated from the problem by taking the curl of Eq. 85, leaving a three-component vector equation containing only velocity and vorticity. An additional three equations to close the set can be generated by taking the curl of Eq. 83 and applying continuity. Moreover, the six equations can all be put in the same general form

$$\frac{\partial}{\partial x_j \partial x_j} (a_\varphi \varphi_i) - b_\varphi u_j \frac{\partial \varphi_i}{\partial x_j} = S_{\varphi,i}, \quad (86)$$

where φ denotes velocity or vorticity, a_φ and b_φ are scalars depending on φ , and $S_{\varphi,i}$ is a source term. The specific form of these generic terms is given in Table III.E.10. To simplify the algebra, certain liberties have been taken in exchanging eddy viscosity and density with the partial derivatives.

In this formulation, all equations to be solved are made of the same type by eliminating pressure and its attendant difficulties. Furthermore, if the nonlinear terms are regarded as part of the source, then the six equations are all of Poisson type and can be solved by various techniques applicable to linear equations. This method has several disadvantages, however. First, the number of equations and variables has increased from four to six. Second, the boundary conditions on vorticity are just as difficult to apply as those on

TABLE III.E.10. Coefficients for the Velocity-Vorticity Formulation of Chien and Schetz

$a_u = v$	$a_w = 1$
$b_u = 1$	$b_w = 0$
$S_{w,i} = -\frac{1}{\rho} \frac{\partial X_j}{\partial x_k} \epsilon_{ijk} + \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} (v w_j) + \frac{\partial}{\partial x_j} \left(u_\ell \frac{\partial u_k}{\partial x_\ell} \right) \epsilon_{ijk}$	
$S_{u,i} = \frac{\partial w_j}{\partial x_k} \epsilon_{ijk}$	

pressure. Third, the formulation as given can solve steady problems only. Fourth, continuity is not rigidly enforced as it is in the VCP and MAC formulations, where specific steps are taken to maintain continuity. Finally, the formulation is restricted to an eddy-viscosity turbulence model.

The heat-energy equation can also be written in the generic form of Eq. 86, but again only if an eddy-

thermal-diffusivity transport model is used. In the Chien-Schetz formulation, the coupling between flow and heat transport is made by means of the body force term. Specifically, Chien and Schetz write

$$X_3 = \rho \beta (T - T_a) g, \quad (87)$$

where the index 3 denotes the vertical coordinate, β is the coefficient of volumetric expansion, g is the acceleration of gravity, and T_a is an ambient temperature. Using this assumption, the body-force terms of Table III.E.10 become

$$\frac{1}{\rho} \left(\frac{\partial X_3}{\partial x_2} - \frac{\partial X_2}{\partial x_3} \right) = g \beta \frac{\partial T}{\partial x_2}, \quad (88)$$

$$\frac{1}{\rho} \left(\frac{\partial X_1}{\partial x_3} - \frac{\partial X_3}{\partial x_1} \right) = -g \beta \frac{\partial T}{\partial x_1}, \quad (89)$$

and

$$\frac{1}{\rho} \left(\frac{\partial X_2}{\partial x_1} - \frac{\partial X_1}{\partial x_2} \right) = 0. \quad (90)$$

Boundary conditions are as follows. At the upstream boundary, the velocity and temperature are specified from which the vorticity components may be calculated from their definition. At solid walls, the temperature gradient and the normal component of vorticity are assumed to be zero and the no-slip condition is used. Boundary conditions for the other two components of vorticity are derived by integrating the defining equation from the wall node to a neighboring node assuming linear functions for the unknown distributions. At the downstream boundary, parallel flow is assumed. The free surface is assumed to be a rigid lid with no-slip conditions enforced. At the outfall, the velocity and temperature are prescribed from which vorticity is calculated.

A staggered grid system is not used, but rather all seven variables are defined at the same location. Central differences are used for

the diffusion and source terms, and first-order upwind differencing is used for the convection terms. The Gauss-Seidel method is used to solve the finite-difference equations.

f. Stream-function/Scalar-potential/Vorticity Methods
(Trent^{78,79})

This section considers an alternative formulation of the equations of motion that avoids the primitive variables. Our discussion begins by presenting the formulation in its most general terms. This is followed with an examination of two special cases of the general formulation and then with a brief discussion of the two Trent models that use this formulation.

Recall that any differentiable vector field $\vec{u}$ may be separated into irrotational and rotational components by defining a scalar ϕ and a vector field $\vec{\psi}$ such that

$$\vec{u} = \nabla\phi + \nabla \times \vec{\psi}. \quad (91)$$

The scalar ϕ is known as the scalar potential and corresponds to the irrotational component by the vector identity

$$\nabla \times \nabla\phi = 0. \quad (92)$$

Similarly, the vector $\vec{\psi}$ is known as the stream function and corresponds to the rotational part of the solution. Moreover, we have, by vector identity, that

$$\nabla \cdot \nabla \times \vec{\psi} = 0. \quad (93)$$

Note that an arbitrary constant can be added to the scalar potential and an arbitrary conservative vector field (a vector field that is the gradient of the scalar potential) can be added to the stream function. Therefore, to make the scalar potential and stream function unique, it is necessary to impose additional restrictions on ϕ and $\vec{\psi}$. This is usually done by specifying the value of ϕ at a single point and requiring that

$$\nabla \cdot \vec{\psi} = 0. \quad (94)$$

The two special cases of interest are (1) potential flow, which must be both inviscid and irrotational, and (2) nondivergent flow, in which $\nabla \cdot \vec{u} = 0$. In the former case, the governing equation can be shown, by taking the divergence of Eq. 91, to be

$$\nabla^2\phi = D, \quad (95)$$

where D is the divergence of the flow field. This is the familiar equation of potential flow. On the other hand, if the velocity field is nondivergent, then it follows that $\nabla\phi \equiv 0$ and

$$\vec{u} = \nabla \times \vec{\psi}. \quad (96)$$

Thus, the curl of the stream function can be used to replace velocity in the equation of motion. This accomplishes little in three dimensions, since the order of the differentiation is increased without significant simplification. However, the stream function is widely used in two dimensions, since it must always be normal to the velocity field and can therefore be represented as scalar ψ (e.g., $\vec{\psi} = (0, 0, \psi)$). Thus, we have the familiar stream-function representation in two dimensions in which

$$u = \frac{\partial \psi}{\partial y} \quad (97)$$

and

$$v = -\frac{\partial \psi}{\partial x} \quad (98)$$

In three dimensions, it is necessary to introduce the vorticity defined by

$$\vec{\omega} = \nabla \times \vec{u} \quad (99)$$

$$= \nabla \times \nabla \times \vec{\psi} \quad (100)$$

$$= \nabla(\nabla \cdot \vec{\psi}) - \nabla^2 \vec{\psi} \quad (101)$$

We can assume, without loss of generality, that $\nabla \cdot \vec{\psi} = 0$, so that

$$\nabla^2 \vec{\psi} = -\vec{\omega} \quad (102)$$

As in the previous section, the vorticity-transport equation is derived by taking the curl of the equation of motion. The result is

$$\frac{D\vec{\omega}}{Dt} = \vec{\omega} \cdot \nabla(\nabla \times \vec{\psi}) - D\vec{\omega} - (\nabla \rho) \times \frac{D}{Dt}(\nabla \times \vec{\psi}) + \nabla \times X + \nu \nabla^2 \vec{\omega} \quad (103)$$

Here, the viscosity has been assumed constant. Equations 102 and 103 represent a set of nonlinear, coupled partial differential equations for the two vectors $\vec{\omega}$ and $\vec{\psi}$. Due to the coupling and the nonlinearity, iterative methods are the only way to solve this set numerically. Once the stream function is calculated, the velocity can be found from Eq. 96, if the velocity field is non-divergent. Otherwise, Eq. 95 must also be solved and the velocity calculated from Eq. 96.

Hirasaki and Hellums⁸⁰ discuss the boundary conditions on φ and $\vec{\psi}$ when the normal velocity is specified. They recommend that the normal gradient of φ be set equal to the normal component of velocity; also, the normal component of the divergence of $\vec{\psi}$ as well as the tangential components of $\vec{\psi}$ should be set to zero at the boundary. They point out that these boundary conditions give that $\nabla \cdot \vec{\psi} = 0$.

Trent^{78,79} has developed a pair of two-dimensional thermal-plume models. One is a vertically averaged model using both the stream function and the scalar potential. The other is laterally averaged and uses only the stream function. The former allows for net nonzero flow across the boundary. Important applications include the study of flow with a time- and space-varying water level. The vertically averaged equations are derived allowing for a variable depth. The laterally averaged model can be used to study the selective withdrawal of stratified ambient water at the intake or density-induced currents in a cooling lake. The scalar potential is not included, since the lateral extent of the flow is taken as constant (unlike the varying water levels assumed above).

In his vertically averaged model, Trent allows either no-slip conditions, specified inflow/outflow conditions, or unconstrained outflow conditions on the boundaries. In the laterally averaged model, a free-slip, rigid-lid condition is used at the surface, a no-slip condition is applied along the bottom, and inflow/outflow conditions are used along the vertical edges.

In both models, Trent employs a staggered grid system for the finite-difference equations. The vorticity is evaluated at the center of the grid cell, the u velocity at the center of the left and right faces, the v velocity at the center of the top and bottom faces, and the stream function and scalar potential at the four corner nodes. First-order upwind differencing is used for all convective terms in the model, central differencing for all other terms. The finite-difference equations are solved by an iterative procedure for the steady-state solution.

g. Summary of Formulations and Comment

Among the alternative formulations used in the available differential-numerical thermal-plume models, the use of the primitive variables is the most popular. Only one three-dimensional model (that of Chien and Schetz) uses vorticity in combination with velocity, and only the two two-dimensional models developed by Trent use completely derived variables. Derived variables appear to be advantageous in two dimensions, but their utility in three dimensions is questionable. Moreover, the vorticity-velocity formulation has several disadvantages that were noted earlier. Among the most serious of these is its restriction to an eddy-viscosity turbulence model and the possibility that the velocity field may violate continuity.

The primitive-variable models are about equally split between two closely related techniques: The VCP and DNS formulations. Although strictly speaking the Brady-Geyer model is not of the VCP type, it can be argued that the balancing procedure used to correct velocity is equivalent to an iterative solution for the velocity-corrector potential. The main difference is that Brady and Geyer iterate directly on the velocity and thus save storage by avoiding a separate variable. Since the solution is intermediately updated, it is not clear how well the iteration converges. A time comparison with a VCP model would certainly be interesting.

The Pritchett-England-Taft model is the only truly three-dimensional model using the VCP method. The Macqueen model is vertically averaged, and the McGuirk-Spalding model uses a spatial-marching technique in the principal direction of flow (applicable for rivers). Similarly, the Spraggs-Street model is the only truly three-dimensional model using the DNS formulation. The Paul-Lick model is really quasi-three-dimensional, since it uses the hydrostatic form of the vertical transport equation. The Castellano-Tozzi model is two-dimensional with flow and energy completely uncoupled.

The Waldrop-Farmer model is unique into itself, since it solves the problem without auxiliary equations or corrective procedures and uses completely explicit forward differencing to obtain the new variables. As noted before, this procedure sacrifices some boundary conditions and thus cannot solve the fully elliptic problem. The approach is nevertheless quite interesting, and its limits of applicability are presently unknown.

The staggered grid system used by the MAC method is the most popular finite-difference system, since it offers many advantages. Other grid systems are used, however. Also, upwind differencing of the advection terms is generally preferred to completely FTCS differencing, although some models (that of Paul and Lick, for example) use the latter.

The purpose of this section was to describe in generic terms the various available formulations of differential-numerical plume models. Individual models were used to illustrate each formulation. The section has brought out the key concepts involved in these formulations and helped the reader to better understand the diverse spectrum of numerical models. The following section gives our opinion as to the important issues surrounding the use of differential-numerical models for thermal-plume prediction.

4. Important Issues Regarding the Numerical Modeling of Surface Thermal Plumes

There are a number of controversial issues regarding the application and further development of numerical models for surface thermal plumes. These issues arise in other applications of computational fluid dynamics as well and are consequently not uniquely the bane of plume models. Although much insight can be gained by studying the application of numerical methods to other fields such as boundary-layer theory, open channel or duct flow, and meteorology, one cannot necessarily extrapolate experience from one field to another. Our opinion is that thermal-plume investigators should test in the thermal-plume simulation the best ideas developed in related problem areas. Some of the borrowed ideas and methods will not work out well and should be discarded; others will succeed. This approach, along with the generation and testing of new methods, is probably the best approach to improving the state of the art of numerical modeling of surface thermal discharges.

Unfortunately, numerous areas require research. Some of these are discussed below with our suggestions on how to best resolve the controversial issues. These suggestions may often appear obvious to the reader, but many of the simplest suggestions have not yet been tried, primarily due to the relatively short length of time these models have been under development. The order of presentation does not indicate an order of importance.

a. Model Formulation in Terms of Primitive or Derived Variables

The relative merit of the various formulations of the governing equations is a major question of interest. Section III.E.3 reviewed the available formulations and contrasted the major differences. With the current state of development, one cannot specify a "best" formulation for surface-thermal-plume applications. In certain problems, one formulation may appear more theoretically satisfying than the others. For example, the McGuirk-Spalding⁵⁰ method of marching in one direction and using the VCP method in each of many normal planes appears to be a cheap and efficient way to solve problems of coflow discharges in unidirectional currents. Their method, however, is probably inadequate for a fully elliptic thermal-plume problem in three dimensions. Special models that take advantage of simplified conditions may well prevail until a general fully three-dimensional model can be developed and verified. Even then, such a model should be used only for those situations in which no simplified model will be adequate. Unfortunately, too little testing has been carried out to better assess virtues and faults of the available models. Only after more experience is accumulated can one better evaluate the models on an objective basis.

Our recommendation is that the different formulations described in Sec. III.E.3 should be compared using sets of good-quality field data taken under widely differing field situations. Only models at the same stage of development should be compared; i.e., a particular formulation should not be rejected as inferior because the coding of that formulation was not as efficient as the others tested. The testing of the full set of numerical models with the same data is the only way to identify a preferable formulation. Even then, a sensitivity study of each model to its differencing technique, turbulence parameters, boundary conditions, and solution technique is needed to determine why a model did well or poorly in the competition. Only after such additional work with the models is completed can we properly judge individual formulations.

b. Methods of Differencing the Governing Equations

Among the methods of discretizing the equations, central differences are commonly used to represent the diffusion and pressure terms. Upwind (one-sided) differencing is usually used for the advection terms, but two models (Brady and Geyer,³ Paul and Lick²) employ central differences for these terms. We suspect that the FTCS scheme is acceptable whenever turbulent diffusion and buoyancy are the dominant mechanisms of dispersion and

advection is not significant, but is probably not acceptable when strong inertial forces govern the streamwise flow. An upwind-differencing scheme of some kind is probably the best approach when convection is mainly responsible for establishing the streamwise distribution of the dependent variables. This supposition is based upon general experience in solution of the Navier-Stokes equations.^{81,82} For thermal-plume problems, the value of upwind differencing must be verified by trial.

Some doubt exists as to whether the usual form for first-order (and second-order) upwind differencing for the advection terms is adequate in all plume flows. Two papers by Raithby^{81,82} indicate that false or numerical diffusion can appear when upwind differencing is used. Raithby states^{81,82} that a large error may result in those regions of the flow where the grid lines and velocity directions are not closely aligned. Also, upwind differencing led to significant inaccuracies in those flow problems where large gradients of the dependent variable existed in the crossflow direction. Both of the above types of inaccuracy were found to be reduced when the grid lines and flow direction were closely aligned. In boundary-layer problems this requirement is not restrictive, but in recirculating flow problems (such as most thermal-plume problems) the violation of these conditions may lead to significant inaccuracies, a valid criticism of the upwind-difference approximation.

Raithby has developed two new schemes that, to a large extent, remove these errors in those problems where convection and diffusion are largely responsible for establishing the flow field. They are the skew upwind-difference scheme (SUDS) and the skew upwind-weighted-difference scheme (SUWDS). Each of these schemes uses the angle of the flow with respect to the grid to estimate fluxes by advection.

Raithby applied the upwind and SUDS schemes to a two-dimensional finite-difference solution of a pair of interacting jets. Raithby considered two parallel, two-dimensional, nonbuoyant (slot) jets discharging from a plane wall into a semi-infinite stagnant receiving body. The two merged into a single jet with a recirculation zone created between the jets and a sharp curvature produced in their centerlines as they competed for the same ambient fluid. In this problem, the flow moved across grid lines and large gradients in the dependent variables existed in the crossflow direction. Compared to experimentally determined velocity distributions, the upwind-differencing scheme seemed to generate numerical dispersion in some regions. The flow acted as if the viscosity were very high. Although the major features of the flow appeared to be correctly modeled, the magnitudes of the velocities were rather poorly predicted. Raithby states⁸² that applying the SUDS scheme greatly improved predictions at little, if any, extra cost. [Both solutions employed a (k, ϵ) turbulence model using the classical upwind differencing scheme.]

Our feeling is that each numerical plume model should be studied to ascertain the effects of changing the differencing scheme among the

popular and promising approaches. For example, to be tried are central differencing, first-order upwind differencing, second-order upwind differencing, skew upwind differencing, skew upwind-weighted differencing, and others in the literature. The relative merits of each for the various classes of plume problems (advection dominant, diffusion dominant, etc.) should be determined as well as relative computational costs. An investigation much like that done by Atias, Wolfshtein, and Israeli⁸³ in studying the efficiency and accuracy of various differencing schemes applied to the same fluid-flow problem would be very useful. In this study, the central differencing (with ADI or Gauss-Seidel iteration), first-order upwind differencing (with Gauss-Seidel iteration), second-order upwind differencing (with Gauss-Seidel iteration), and exponential differencing (with Gauss-Seidel iteration) schemes were all compared by solving the two-dimensional vorticity equation for two problems: (1) a moving plate causing a recirculating eddy inside a square cavity and (2) the impingement of a jet on flat plate normal to its direction of discharge.

c. Validity of Hydrostatic Approximation

The statement of the hydrostatic approximation was given in Sec. III.E.1.e along with a brief discussion of its validity. The weak form of the approximation is the one generally used. The hydrostatic approximation greatly simplifies the calculation of pressure. The three-dimensional pressure field is made essentially two-dimensional (the "surface" pressure), with the hydrostatic equation providing a simple equation for calculating its variation in the third (i.e., vertical) dimension.

The validity of this approximation in a numerical plume model is a controversial matter for high-velocity discharges in which significant momentum is transferred vertically. The inertial and diffusion terms neglected in the vertical momentum equation may then be of the same order of magnitude as the pressure and gravity terms retained in the hydrostatic form.

The best way to test this approximation for a given problem is to develop two otherwise identical models, one with the approximation and the other without. Ideally then, a comparison of predictions would indicate the importance of the neglected terms. This is an expensive proposition, however, in terms of development and programming labor. A less satisfactory, but also less expensive, approach is to run the model with the hydrostatic approximation and use the predicted velocity field to evaluate the order of magnitude of the inertial and diffusion terms. This approach is tricky, however, and does not always work.

The hydrostatic approximation is particularly powerful in combination with the rigid-lid condition whenever both are appropriate, since the combination significantly reduces computer time. The rigid-lid condition removes surface gravity waves and their corresponding small time scales,

allowing a much larger time step in the solution procedure. The rigid lid requires a two-dimensional pressure distribution to be calculated at the level of the undisturbed free surface from which the hydrostatic equation is used to calculate the pressure field in three dimensions. The ascending order of generality for surface thermal plume problems can be represented as follows:

- (1) Both the hydrostatic and the rigid-lid conditions are valid.
- (2) Only the rigid-lid condition is valid.
- (3) Neither the rigid-lid nor the hydrostatic approximations are valid.

The types of thermal-plume problems that fall into categories 1-3 are presently unknown.

d. The Simulation of Turbulence

As noted in Sec. III.E.1.g, four types of turbulence models may be used to represent the turbulent jet diffusion as well as mixing due to ambient turbulence. They are

- (1) Algebraic models based on the turbulent eddy-viscosity approach.
- (2) One equation models on turbulent energy; or two-equation models using the turbulent energy and a second dependent variable (energy-dissipation rate, length scale, etc.) based on the turbulent eddy-viscosity approach.
- (3) Multi-equation models of stress transport.
- (4) Stochastic or statistical models.

All numerical models to date for heated surface discharges have employed an algebraic model, except

- (1) The Pritchett-England-Taft model,⁵¹ which uses a one-equation model for turbulent kinetic energy based on the turbulent eddy-viscosity approach.
- (2) The McGuirk-Spalding model,⁵⁰ which uses a (k, ϵ) two-equation turbulence model also based on the eddy-viscosity approach.

For nonbuoyant jets, Raithby⁸² refers to the work of J. Militzer in employing a (k, ϵ) turbulence model for two interacting slot jets. To our knowledge the above three examples represent the total experience with models of the nonalgebraic type for thermal-plume (or very closely related) problems.

Unfortunately, no systematic study has been carried out to determine which type of turbulence models gives the best performance for the

thermal-plume simulation. Although the algebraic models are the simplest and most widely used, they are also the most subject to criticism theoretically in terms of their general applicability to plume problems (see Sec. III.E.1.g). We suspect that the simulation of turbulence is of major importance to the proper performance of any numerical thermal-plume model. The task of testing different theories should not be undervalued or underestimated. We can draw the general conclusion that no model of turbulence has yet been proven to be superior for thermal-plume applications, primarily because of the general lack of experience with these models as a whole. The inclusion of the more complicated turbulence models into thermal-plume codes adds significantly to the overall computer costs of the model. The proper simulation of turbulence will probably remain a major difficulty in the development of good numerical thermal-plume models and might even be a major obstacle to the increasing utility of these models.

Another complicating factor is that the effects of buoyancy on turbulence are only poorly understood. In the algebraic models, the general approach is to reduce vertical turbulent diffusion according to the local Richardson number. The way in which this should be done for thermal plumes is unknown. The only available information comes from oceanographic or similar studies of dubious application to thermal plumes. Even with the more complex models, the nature of turbulence in a stratified flow is difficult to determine. Consequently, the interaction of buoyancy and turbulence in thermal plumes is a key issue that requires much further study.

Although the simulation of turbulence is generally very important to the proper development of a plume model, there may well be situations in which plume turbulence and ambient turbulence are masked by large buoyancy or advection effects. For example, a high-velocity discharge into a rapid current might increase the magnitude of the advection terms in the equations of motion to the point where diffusion is sufficiently masked, and as a result, any turbulence model that gives ballpark answers will not adversely affect predictions. In the presence of significant buoyancy effects as for discharges with very low initial densimetric Froude numbers, the pressure and gravity terms may essentially govern the flow. These speculations, however, need to be tested with data. The reader should bear in mind that when testing turbulence theories, one should first check for numerical dispersion from the advection terms, since "numerical diffusion" from those terms can sometimes overwhelm, or at least mask, the real (physical) turbulent diffusion.

e. Role of Boundary Conditions and Size of Grid

The specification of boundary conditions that provide realistic, accurate, and stable solutions can be very difficult in any fluid-dynamics problem. This is in spite of the fact that defining some kind of plausible conditions at the boundaries is rather easy. The importance of boundary conditions for surface-thermal-plume problems still remains a mystery. This problem

is complicated by the fact that the proper boundary conditions can depend upon parameters internal to the Navier-Stokes equations (e.g., Reynolds number), the interior-point differencing method, the other boundary conditions, and sometimes the initial conditions. The best way to study the adequacy of a given set of boundary conditions is to carry out computational experiments using various plausible alternatives. Some questions that presently remain unanswered regarding the boundary conditions are:

(1) What are the most physically plausible boundary conditions for walls (shoreline, bottom), upstream boundaries, downstream open-water boundaries, offshore open-water boundaries, and the free surface?

(2) What is the best treatment for downstream open-water boundary conditions? Is it the zero-gradient condition, is it a linear extrapolation, is it an upwind difference method, etc.?

(3) Is coordinate stretching, to increase the distance to the open-water boundaries, a helpful approach, considering the truncation error associated with variable-spacing grid system?

(4) How small a computational grid can one choose (to save computer time) without the size of the grid becoming a factor in the problem by restricting the flow? Apparently a sensitivity analysis is warranted here for some sample plume problems.

(5) Are boundary conditions more accurately specified in one formulation of the governing equations than another (see Sec. III.E.3)?

(6) Under what conditions are free-surface and rigid-lid conditions appropriate?

It should be obvious that these kinds of questions are not specific to thermal-plume problems. However, due to the complexity of the Navier-Stokes solution, and especially the relationship of the boundary conditions to other items such as the interior-point differencing method, independent investigations of boundary conditions are best undertaken separately for each model formulation. The body of information thus generated will help in future development efforts in the thermal-plume area.

It is popular in some circles to severely criticize numerical modeling due to the usual difficulty in specifying proper boundary conditions. Our feeling is that if physically plausible boundary conditions are chosen that do not overly restrict the flow, then the adverse effects will be minimal. This speculation should be tested by numerical experiments. Additionally, the boundary-condition problem is not unique to the numerical model of a thermal plume. Alternative techniques of modeling a surface thermal plume also face similar boundary-condition problems. For example, a laboratory model of a surface plume must be carried out with the dubious influence of the nearby tank walls and the usually arbitrary boundary flow control at some of the tank sides.

f. Applicability of Parabolic Models

Although a surface thermal discharge is truly a boundary-value problem, some investigators have noted certain special situations in which they expect a parabolic approximation to be valid.

Patankar and Spalding¹⁷ have described three properties of a flow that allow the use of a parabolic approximation in one of the three directions of flow. They are:

- (1) There is a predominant direction of advection, i.e., there is no reverse flow in that direction.
- (2) The diffusion of momentum, heat, mass, etc., is negligible in that direction.
- (3) The downstream pressure has little influence on the upstream flow.

If these three conditions are met, the principal direction of flow becomes a one-way street in which upstream conditions can determine downstream flow properties, but not vice versa. Such behavior simplifies the numerical solution, since it permits an integration scheme that marches from an upstream station to a downstream one.

McGuirk and Spalding⁵⁰ have noted that the above circumstances are closely approximated for a thermal discharge in coflow with a river. On this basis, they ignore downstream boundary conditions and solve the equations as a series of two-dimensional elliptic problems. The longitudinal and cross-stream pressures are uncoupled allowing a quasi-two-dimensional solution for the three-dimensional pressure field. Computation times on the order of hours associated with solving the elliptic problem are thus reduced to minutes.

Although we find little fault with the parabolic approximation for such coflow applications, the number of situations in which the approximation is valid is very limited. For example, we would not expect the McGuirk-Spalding approach to be extendable to more general problems, such as a jet in a crossflow. Although the McGuirk-Spalding model does allow circulations in the plane normal to the flow, it allows none in the direction of flow. How far one can stretch the parabolic approximation and still achieve state-of-the-art predictions is not presently known.

An interesting application of the parabolic approximation is made by Waldrop and Farmer.^{66,67} Although their model is developed for general application, the dropping of the vertical-acceleration term from the vertical-momentum equation allows Waldrop and Farmer to write the momentum (and heat) equations as a sequence of parabolic equations. An explicit finite-difference scheme is then used to solve the equations (see Sec. II.L).

The parabolic nature of the equation set allows the boundary conditions that would ordinarily be enforced at the offshore and downstream vertical boundary planes to be relaxed in the numerical solution. Thus the model does not handle backwash effects. A major question is whether the solution algorithm preserves in any way the spatially elliptic nature of the governing equations. This in turn leads to the fundamental question: For what kind of application is the Waldrop-Farmer approach appropriate? Based on existing evidence (see Sec. II.L), we can only speculate that, for conditions of large outfall momentum or crossflow inertia, the parabolic approximation may give reasonable results. For example, reasonable predictions were obtained for the Holston River application (see Sec. II.L.9) in which a large river current swept the plume downstream. We have doubts that the model will perform well under conditions in which the offshore and downstream boundary conditions are important, e.g., a discharge into a sheltered cove. Although the model has predicted three-dimensional circulations (see Sec. II.L.6), a question arises concerning its general accuracy whenever the downstream and offshore boundary conditions are important enough to require a three-dimensional elliptic solution. Only further testing of the Waldrop-Farmer model will establish its range of applicability.

Our feeling is that parabolic methods are limited in practical application. How far one may extend such models is not known and should be a point of further study.

g. Achieving a Solution of the Steady-plume Problem by Solving the Unsteady Equations

As noted in Sec. III.E.3.a, the most common iterative scheme for calculating a steady thermal plume is the asymptotic-time technique. In this way, the steady solution is obtained by relaxing the time-dependent equations beginning with some arbitrary and presumably unimportant initial condition. Normally, the boundary conditions are held steady during the process, although it is actually only necessary that the boundary conditions eventually become steady. Surprisingly then, most of the successful numerical models for steady thermal plumes are built around the time-dependent governing equations.

Some of the advantages of using the time-dependent equations are:

- (1) The extension of the model to time-dependent problems such as plant startup is easily accomplished.
- (2) Existence and uniqueness of the solution are somewhat less troublesome on physical grounds if the unsteady equations are used.

The major disadvantage of using the asymptotic-time approach in achieving a steady-state solution is that solving the unsteady equations may require more computational time than a solution of the steady-state equations directly.

h. Considerations of Stability and Convergence

Questions of stability in the time-dependent numerical models and questions of convergence for all numerical plume models are seldom addressed in the published descriptions of these models and, when considered, are never seriously treated. This is unfortunate, since much can be learned from a study of these questions regarding the adequacy of the numerical scheme.

An accurate finite-difference simulation of the surface thermal plume must have the following characteristics:

(1) Stability: The decay in time of machine round-off errors creeps into the solution.

(2) Consistency: The finite-difference equations approach the corresponding governing partial differential equations as Δx , Δy , Δz , and Δt approach zero.

(3) Truncation-error convergence: The solution of the finite-difference equations approaches the solution of the corresponding governing partial differential equations as Δx , Δy , Δz , and Δt approach zero.

(4) Iteration convergence: An acceptable solution of the discretized equations in finite-difference form is attained (i.e., convergence) to within some tolerance.

For a solution of the governing flow equations to be acceptable, all these characteristics are necessary. As noted above, no existing model has treated these characteristics seriously in published work.

The three major methods¹ of studying the stability of a numerical scheme for a given set of partial differential equations are

- (1) Discrete perturbation stability analysis.
- (2) Von Neumann stability analysis.
- (3) Hirt's stability analysis.

The first method introduces a discrete perturbation into the finite-difference equations at an arbitrary point at some time level, and its effect is followed. Stability is indicated if the perturbation dies out. This procedure provides insight valuable in formulating boundary-condition treatments. In the second method (the von Neumann stability analysis), a finite Fourier-series expansion is made of the solution of the model equations, and the decay or amplification of each mode is considered separately to determine stability or instability. This method is the most dependable and provides information not only on damping but also on the phase relationships of the finite-difference equations and the resulting dispersion errors (phase errors). In the

third method (Hirt's stability analysis), the terms of the finite-difference equations are expanded in a Taylor series in order to develop continuum partial differential equations, the stability of which is determined from known stability properties of the continuum equations. This method also sheds light on dispersion errors and on the finite-difference behavior often referred to as the "artificial-viscosity" effect. Each of the analyses (others also exist¹) has its uses and can often complement one of the others.

The above analyses are helpful, but are not totally adequate for practical computations. In fact, most stability analyses are carried out on some linearized form of the finite-difference equations, and, as a result, much complexity contained in the nonlinearity is lost. Instabilities can occur due to nonlinearity and to nonconstant coefficients.

In actual fluid-dynamics problems, stability restrictions determined from linearized equations are locally applied. The mesh is scanned for the most restrictive stability limitations, and the resulting permissible time step is used throughout the mesh. The typical and prudent practice is to use some fraction, usually 0.8 or 0.9 of this analytically indicated maximum time step.¹ A major difficulty facing numerical modelers is that it may be impossible to distinguish between what might be called a true instability and just a very poor rate of convergence.

An interesting discussion of some of the practical aspects of stability in numerical-fluid-flow models is given by Leendertse and Liu.⁸⁴ They state that although an investigator developing multidimensional numerical-fluid-flow models nearly always has problems with stability of the computation, it is not necessarily a shortcoming of the model. Indeed, real flows are generally unstable and cannot always be resolved properly on the usual computational grid.

Leendertse and Liu have traced their difficulties with instabilities to the following causes:

- (1) Improper mathematical formulation of the physical process.
- (2) Improper description of the boundary conditions by which the problem became ill-posed.
- (3) The characteristics of the finite-difference solution method.
- (4) Use of incorrect methods for describing the effect of sub-gridsize fluid motions.
- (5) Use of incorrect coefficients for describing the effect of sub-gridsize fluid motions.
- (6) Violation of the stability criteria belonging to the chosen solution method.

Leendertse and Liu note that it is often difficult to locate the place and time of a growing instability. Even when they have been located, removing the instability is often a trial-and-error procedure in terms of changing the formulation, coefficients, or solution method.

The concept of stability is closely related to the concepts of consistency and (truncation-error) convergence. There are numerous equivalence theorems connecting the two concepts for linear systems. Although such theorems for linear systems provide guidelines for the nonlinear flow equations, they cannot be applied directly. The possibility of nonunique solutions to nonlinear equations is enough to prevent direct application of these theorems. Numerical experimentation is usually the only practical alternative in studying the stability and convergence of a numerical scheme.

Two obvious requirements for convergence of the nonlinear (or linear) equations are that the finite-difference equations are stable, in some sense, and consistent. Unfortunately, there are no definitive criteria for truncation-error convergence.

Examining the iterative-convergence behavior of a numerical scheme (perhaps when the iterations are to achieve a steady state) is a very subjective matter. Most investigators select a numerical value of the error parameter ϵ that coincides with their subjective judgment of convergence. Iterations are carried out until an error criterion based upon that error parameter is satisfied. A practical approach for estimating convergence to steady state is the criterion that

$$\max_{ijk} |\zeta_{ijk}^{n+1} - \zeta_{ijk}^n| < \epsilon,$$

where ζ represents a particular dependent variable, n is the time level, and the maximum is taken over all grid points. Roache¹ discusses some additional practical suggestions in determining convergence criteria.

A good choice of initial conditions can often help the convergence. This help could be insignificant, however, if very strict convergence criteria such as $\epsilon = 10^{-6}$ are being used and the initial guess is only within an order of magnitude of the correct solution. Sometimes poor initial conditions can lead to true nonlinear instabilities, as can occur¹ in an explicit asymptotic-time approach (such as DNS or VCP) if the associated Poisson-equation convergence (for pressure or for velocity-corrector potential) is inadequate.

We feel that the questions of stability and convergence have not been adequately addressed in present thermal-plume numerical models. Typically, a model is run for a particular application with the computations halted, not when convergence was necessarily achieved, but when a previously

determined computer run time was exceeded. It is not known if the model would have ever actually converged or if the convergence is slow or fast. Another point requiring study is the use of an accelerated convergence scheme. Waldrop and Farmer^{66,67} sometimes use such a scheme to speed up convergence of the heat-energy equation as it lags behind the flow-field convergence. Such acceleration schemes may violate the conservation laws at intermediate times. Another point that needs study is whether two significantly different initial conditions can lead to the same steady-state solution (assuming fixed and identical boundary conditions, of course). Comparing predictions with two different initial conditions provides information on the uniqueness and well-posedness questions of the model formulation and answers the question as to whether a steady-state solution can indeed be achieved from the unsteady equations in a unique manner.

i. Increased Number of Input Parameters

In comparison to the integral technique, the numerical model requires a larger number of input parameters to carry out a computer run. This is due to the fact that the numerical model treats more physical phenomena and is more intimately based upon fundamental principles. However, this leads to a problem. A number of the input quantities are not known with any significant degree of precision. The turbulence parameters have already been mentioned, and even if a complex turbulence model is used to generate them, there is still considerable doubt as to their correctness. Other quantities in doubt are the precise temperature and velocity profiles at the outfall. These are usually unknown due to the complex flow in the open channel upstream of the outlet cross section, and must usually be estimated. An excellent example of an unknown input quantity is the coefficient that relates wind stress to wind speed in the usual formulation. The question arises: Of what use is a precise numerical model of plume dispersion treating wind effects if we cannot properly estimate the wind stress from the wind speed based upon environmental data? The hope with these numerical models is, of course, that the uncertainty in the input parameters will not mask the benefits of the more detailed treatment the numerical model provides.

The above discussion raises a deeper question as to whether the numerical model, which purports to handle the microscopic physics, is a better approach. Certainly the large computation times required by the model work in its disfavor. However, in our opinion, the alternatives are worse in terms of our future predictive capability. The phenomenological models do not have a sufficient data base for accurate predictions; a sufficiently large data base will probably never be available. The integral models ignore the microscopic physics and try only to simulate a few of the most important processes that hopefully provide prediction of the gross plume characteristics. The problem with the integral approach is that you must assume a priori which processes are going to be important in a particular application. For example, the typical integral model assumes that inertial forces dominate buoyancy, with jet entrainment dominant and bottom effects of no importance.

These assumptions, implicit in the usual integral approach, have proven to be a complete failure for practical applications. Attempts to patch up the integral technique to include buoyancy and bottom effects have failed due to lack of data (for the buoyancy simulation) and the use of assumptions inconsistent with the integral formulation (for bottom effects). Numerical models, on the other hand, generally do not assume that certain processes are dominant before the simulation takes place. The philosophy there is that we do not know which phenomena are important and that the numerical simulation of the plume dispersion will provide that information as a prediction. The cost, of course, is in increased computer time and, as a sidelight, in increased detail of the predictions, some of which is not of usual interest. For example, the prediction of free-surface gravity waves in a free-surface numerical plume model is not of any practical use.

We feel that, in spite of the increased cost and doubt on input quantities, the numerical model provides the best future direction for modeling surface thermal plumes. The phenomena are too complex to be treated accurately by simpler methods.

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IV. VERIFICATION EFFORTS FOR SURFACE DISCHARGE MODELS

A. Introduction

From a totally pragmatic viewpoint, the ultimate value of any mathematical model is determined by its ability to predict prototype plumes. From a more theoretical viewpoint, however, the isolation of important modeling concepts and design principles must be a fundamental objective. In the latter vein, neither the phenomenological nor the calibrated integral approaches, which are, in effect, well-motivated curve fittings, are totally satisfying. In a practical sense, though, these models represent the most reliable alternative presently available for site-evaluation work. Since the yardstick by which we measure a model's worth is its ability to replicate the data, model-data comparisons are of fundamental importance within this report.

Model-data comparisons naturally begin with the acquisition and reduction of data. Several sets of field and laboratory data are available for model-data comparison, although not all have been yet used for this purpose. Section B of this chapter describes many of the available data sets that can be used for model-data comparisons. The section is included to acquaint the reader with the range of comparisons currently possible.

Not all model-data comparisons reflect on the practical value of the models. Some are carried out to calibrate the model and do not represent a fair evaluation of its predictive capabilities. Such comparisons only test the model's ability to be successfully fit to the data. Other calculations are conducted more as modeling exercises than as objective model-data comparisons. This is particularly true of the differential-numerical models in which the objective often is to merely demonstrate that a successful numerical algorithm has been obtained. In Sec. C, five models of the integral and phenomenological approach are compared to laboratory data in an attempt at verification under idealized, laboratory-controlled conditions. The complicating effects of wind, boundary interference, and a transient ambient waterbody are eliminated, or at least minimized. Each model evaluated is itself idealized in that the complicating features, which are indeed present for prototype plumes, are not simulated in the model. Thus the comparison of the models to laboratory data is meant to test the individual model's assumptions, because the conditions under which data were taken closely match the idealizations made in the models.

On the other hand, Secs. D-G of this chapter describe four major model-verification efforts with the emphasis on objective testing of each model's practicality in terms of predicting prototype data. Here the model idealizations are put to the test. To be discovered is how well the model replicates the real world, and, as a result, the utility of the model as a design and sitting tool is revealed. The four major efforts described in Secs. D-G are:

1. Work done at Pilgrim Nuclear Power Station with the Stolzenbach-Harleman and to a lesser extent the Pritchard models (Sec. D).
2. Work done at the Oskarshamnsverket site in Sweden with the Prych model (Sec. E).
3. Work done at Point Beach Nuclear Power Station with four integral models and one phenomenological model (comparisons made to jet and plume data). Additionally, one comparison is given of the same models with jet data taken at the Palisades Nuclear Power Station (Sec. F).
4. Work done on the Po River in Italy with certain rapid river models (Sec. G).

Limited comparisons of this kind have already been described in Chapter II, Part One, as they pertain to individual models.

Unfortunately, objective model-data comparisons are difficult to achieve, because the data and model calculations are not directly comparable.

First, a link must be formed between the abstract world of the models and the physical world, from which the data are obtained. The ways of forming this link are numerous, and the model user introduces a certain arbitrariness into the comparisons. Thus, model-data comparisons test not only the models themselves, but also the way in which the models are applied.

Normally, the gap between the model and the data is bridged partly by extension and interpretation of the data and partly by the extension and interpretation of the model, as shown schematically in Fig. IV.A.1. Both the data and the model are necessarily distorted in this process. Since the earliest models were of the jet-integral type, the data traditionally have become forced into the jet conceptualization by abstracting such components as center-line decay of temperature and velocity, jet width, and jet trajectory. Since the methods of model-data comparison used in this report and elsewhere in the literature are not necessarily the best, serious consideration should be given to alternative methods of presentation.

Fig. IV.A.1. Model-data Comparison Process

Model-data comparisons are seen in proper perspective only when considered within the context of the difficulties associated with such comparisons. For convenience in our discussion, we have divided the problems into those involved in data manipulation and those involved in preparing model predictions.

1. Collection and Reduction of Data

There are four principal methods of acquiring data on thermal plumes: surface-vessel plume surveys, tracer-dye studies, aerial surveys by remote thermal imaging, and drogue studies. Each has its advantages, limitations, and experimental uncertainties. Therefore, the fundamental ingredients of each method are described, followed by a discussion of the difficulties associated with that method.

The most common means of "measuring" a thermal plume is the surface-vessel plume survey, a method that provides the most comprehensive and reliable three-dimensional temperature data achievable. In this technique, a boat equipped with several temperature sensors (normally thermistors are used) located at various depths travels along a predetermined course, or with sophisticated navigational equipment over an arbitrary course, collecting and recording temperature and position data continuously. These data are later digitized and then transferred to two-dimensional "maps" of the discharge site.

Physical data on thermal plumes, like other related data, are subject to stochastic and systematic error. Although accuracy depends on the equipment used, normally one can associate an uncertainty of $\pm 0.5^{\circ}\text{C}$ with temperature measurements, and, unless a blunder is made in navigational surveying of the site, position data normally have imperceptible error on the scale of a typical thermal-plume map.

Surface-vessel surveys provide only limited, point-temperature data from which the behavior of the plume may be discerned. Having a large number of points improves the picture of the plume that can be reconstructed; unfortunately, however, they also increase the effort and expense of the survey. Furthermore, since boat-based plume maps require an hour or more to accumulate, they do not represent an instantaneous picture of the plume, but rather a time-averaged view that can be blurred by such phenomena as plume meandering. Fixed-point temperature measurements also vary with time. (See discussion of thermal fronts and other transient characteristics in Chapter III.) Boat-based plume maps are normally acquired by sampling each station at a single instant, and therefore can be much distorted by the large transients encountered in certain regions of the plume.

Assuming that meaningful point-temperature data can be obtained, the uninterpreted presentations of such data are rather unenlightening. The data can be made more graphic by qualitatively sketching in isotherms on two-dimensional horizontal and vertical plume maps. These contours illustrate the three-dimensional temperature distribution of the plume by means of cross-sectional viewing.

Tracer-dye studies are carried out in a manner similar to that just described for boat-survey maps. With suitable equipment, the two surveys can be made simultaneously. Dye is a passive pollutant, its presence having no effect on the overall flow field. Temperature, on the other hand, is an active pollutant. Transport of the two is normally assumed to be similar, however, and dye studies are often used to investigate heat-dispersion mechanisms. In a dye study, the dye is typically injected into the plant cooling-water system just before the cooling water reaches the condensers. This procedure (hopefully) fosters full mixing of dye with the cooling water as the flow passes through the condenser units. In other dye studies, a point or line source of fluorescent dye is located in the discharge channel or at some convenient point in the plume, although in these studies, the complete plume is not usually mapped. Plume dye concentrations are selectively mapped for later calculation of dispersion coefficients inside and (usually) outside the plume. Instead of, or in addition to, temperature measurements, these dye concentrations are determined by sampling the water at a given depth and measuring the fluorescence of the sample with a fluorometer. Unfortunately, the measured fluorescence can depend not only on the dye concentration but also on the temperature of the water, on the solar exposure that the dye has received, and on the amount of natural fluorescence in the sample. Correction for all of these effects is generally possible, but their existence makes measurement of all concentrations difficult.

In all dye tests, the concentrations measured are related to those discharged, from which local dilution (and temperature) can be deduced. A stated advantage in the use of dye, along with measurements of temperature for plumes, is that dye gives a clearer picture of the ambient temperature variation because it distinguishes heated water that has been discharged from water that has been heated by solar radiation. In addition to fluorometer measurements, aerial photographs are sometimes used to record the dye plume.

In measurements of diffusivity using a dye, concentration profiles at various cross sections of the dye plume are used to calculate turbulent dye diffusivities. In overall dilution studies using a dye, concentration maps similar to isotherm maps are constructed by drawing isoconcentration lines.

Remote thermal (infrared) scanning produces direct and essentially instantaneous pictures of surface isotherms. Unfortunately, remote methods sense only the topmost surface of the waterbody where conductive, convective, and radiative heat exchange is occurring. Thermal imaging is therefore unable to provide information on the three-dimensional distribution of temperature in the waterbody. Furthermore, under adverse conditions the surface temperatures may not be representative of even those measured just below the surface. Nor are the effects of films or slicks on the surface well known. A good example of the sensitivity of thermal images to surface disturbances is the fact that a boat wake can persist for upwards of an hour. Despite these difficulties, thermal imaging is a powerful tool to use in analyzing thermal plumes.

In a typical application of this technique, an airplane, outfitted with a sophisticated optical scanning system, rapidly scans the discharge site for electromagnetic radiation in a wavelength band of 8-13 microns. The intensity of radiation in this wavelength band varies with surface temperature and is commonly converted to an electrical signal by means of a semiconductor detector such as a cryogenically cooled mercury-cadmium-telluride chip. A black-body temperature reference can be intermittently placed in the field of the thermal imagery to provide a calibration of the system.

The electrical output of the radiation detector can be recorded in any of several ways. One method is to produce a continuous-tone, black-and-white photographic image of the site, with each shade of gray corresponding to a different temperature. Alternatively, a continuous record of the scan can be made on a magnetic tape or chart recorder. Artificial image enhancement is often used to abstract isotherm contours from the continuous-output record.

Thermal imagery is a sophisticated technique, and though many interesting questions concerning its use deserve discussion, we are examining only the most obvious issues. First, the intervening atmosphere has an effect on the recorded image. The magnitude of this effect depends on both the flight altitude (distance from the surface whose temperature is being measured) and the prevailing meteorological conditions, particularly the relative humidity. Water vapor in the air attenuates the intensity of radiation reaching the sensor, thereby lowering the apparent surface temperature. A counterposing effect takes place because the vapor in the air is emitting radiation at the ambient air temperature. When the air temperature is greater than the water surface temperature, the radiation emitted by the atmospheric vapor increases the apparent surface temperature. These problems can be minimized by flying at several altitudes or by ground-truth measurements, which are independent determinations of the surface temperature at one or more points.

Additional uncertainties arise in the processing and recording of the thermal imagery. The spectral response of the detector and the gain of the amplifying system are potential sources of error. However, because of the narrow band of wavelengths considered and the typically high quality of the electronics, these difficulties are usually considered minimal. When photographic records are made, the attendant problems become important. Good imagery relies on a uniform coating of photographic emulsion for each sheet of film and on the uniform processing of such film. Both the age of the film and the temperature at which it has been stored can affect the image density.

Neither the surface-vessel survey nor the remote thermal image can resolve all the intricacies of the isotherms. However, the contours obtained from these methods are believed to represent the general shape and size of the isotherms. Isotherm maps are particularly useful for visualizing the qualitative nature of the plume, but have been generally used for comparison

with mathematical models. Therefore, further reduction of the data is common, eventually to numerical form for comparison with models. One of the most widely used means of accomplishing reduction is to planimeter the area within surface isotherms. Since area is an integral measurement and since integral measurements are particularly relaxed toward experimental uncertainty, area within surface isotherm has become the standard method of measuring plume size. In comparing plumes taken under differing environmental conditions, the concept of temperature excess is used. The temperature excess is the arithmetic difference between the measured temperature and the temperature that would have been measured had the discharge not been present. Unfortunately, the latter value is unknown and must be guessed. The usual procedure is to measure the water temperature far from discharge and regard that temperature as the so-called "ambient" temperature. The difficulty here is that, even at points remote to the discharge, the water body is not isothermal and no single ambient exists. A further danger lies in the possibility that the point selected for ambient-temperature measurement may actually be on the fringe of the plume and thus not be representative of the ambient-temperature range. Although common, the use of the temperature-excess concept necessarily and arbitrarily distorts the data.

The temperature excess itself is not entirely satisfactory for comparing individual plume areas or for comparing model calculations with plume data. A further distortion of the data is commonly introduced when the local temperature excess is normalized by the discharge-temperature excess to produce the so-called "temperature-excess ratio." Again, the discharge temperature excess is not known and must be guessed. Just as in the case of the receiving waterbody, the effluent is assumed to be isothermal at the point of discharge, which is only approximately true. Furthermore, in surface-vessel surveys the outfall temperature can change during the period of measurement.

To illustrate how uncertainties in the ambient and discharge temperatures manifest themselves in the temperature-excess ratio, we are presenting a simple calculation. Suppose that the nominal ambient temperature is 20°C with an uncertainty of 1°C and that the nominal discharge temperature is 30°C with an uncertainty of 0.5°C . The discharge temperature excess is then 10°C with an uncertainty of about 1.5°C . For an isotherm of 23°C with an uncertainty of 0.5°C , the temperature excess is 3°C with an uncertainty of 1°C . The temperature excess ratio is 0.3 ± 0.12 , an uncertainty of about 40%. This calculation does not, of course, account for systematic errors. Data are almost always presented as points, even though the uncertainty in their exact value is rather large.

Although there is considerable variation, the most widely used method of presenting area data is to plot temperature-excess ratio as a function of cumulative area within the corresponding isotherm. Such plots are normally made on either semi- or full-logarithmic scales because of the need

to cover many decades of area measurement. Even though this method is widely used it has many disadvantages. One is the fact that a dimensionless temperature ratio is plotted on the ordinate and a dimensional quantity is plotted on the abscissa. Furthermore, many find the plots awkward since it is usually more acceptable to view area as a function of temperature and not temperature as a function of area as the plots suggest. Nevertheless, this method of presentation is common.

When based on the assumption that the plume is a spreading jet, locating features such as jet trajectory, centerline decay of temperature, and jet width becomes possible. The isotherm maps are often suggestive of motion, which is usually the basis of drawing centerlines or trajectories. Once the centerline has been established, the decay of temperature is abstracted by measuring the distance along the centerline to each isotherm of excess temperature. These data are then plotted as temperature-excess ratio versus centerline distance. Because the exact shape of the trajectories and the isotherms is not known, centerline temperature decay is uncertain. Jet widths are found by moving along the centerline until an isotherm of excess temperature is crossed and then measuring the perpendicular distance to the isotherm of half that excess temperature. Widths are by far the most uncertain, for much interpolation is needed to determine their values. Normally widths are plotted as a function of centerline distance to correspond to the predictions of the integral jet models. A serious deficiency of present model-data comparisons is that very little has been done to produce and compare model-predicted isotherm maps to those obtained from the data.

Drogue studies provide a vastly different perspective of the plume than do surface-vessel surveys, dye studies, or thermal images. Drogues are floats normally fitted with an easily observable surface marker that can be discharged into, and presumably will move with, the plume. Shore-based or aerial sightings can be used to record the drogue's position as a function of time and thus make possible the direct measurement of trajectory. Additionally, a drogue can also be equipped with a temperature-measuring and -recording device for measurement of temperature decay. Although they do not give much insight into the three-dimensional dispersion of heat, drogues do provide direct information on jet properties.

In summary, boat and aerial surveys of the plume provide important information on the dilution of heat in the waterbody through horizontal and vertical isotherm maps. Areas within isotherms of excess temperature can be obtained from these maps, as can trajectory, centerline temperature decay, and widths. The uncertainties in these data are often large but overlooked in making model-data comparisons. Drogue studies provide direct information on trajectory and, if temperature monitoring drogues are used, on temperature decay.

2. Problem Schematization and Preparation of Model Predictions

Models usually allow only very specific geometries that often do not match the problem at hand, making necessary the schematization of the

outfall configuration as the one postulated by the model. This process is usually not a straightforward matter. Whereas most models assume that the outfall is a rectangular channel discharging effluent at the surface of the waterbody, prototype discharge structures are often irregular and may discharge above or slightly below the surface. Nor is the exit velocity profile usually uniform, as many models assume. The specifications of equivalent outfall widths, depths, and densimetric Froude numbers are potential sources of much error in model-data comparisons.

The bottom topography of a prototype site is usually quite irregular and may strongly influence dilution. Unfortunately, the jet-integral models have been tested mostly at sites not representative of the idealizations made in their development. Three-dimensional models that assume a limitless ambient should not be applied to sites where significant bottom interaction occurs. Furthermore, jet-integral models are inappropriate for low-velocity discharges where momentum effects do not completely dominate.

Another difficulty with the use of jet-integral models is the selection of an ambient current. Many of these models assume that the jet experiences a uniform cross flow just as if the jet were a solid body. The variability and spatial dependence of the current makes its selection difficult.

The differential-numerical models conceptually require less schematization than do the integral models, but, in practice, some schematization is still necessary to accommodate the framework of the model. This need is particularly true in those models that are not written for general geometries. Consider the Pilgrim Nuclear Power Station, where a free overfall exists part of the time. This condition cannot be included in a numerical-model simulation without some arbitrary assumption, which is a potential source of error. Besides, certain models are unable to handle the irregular bottom topography and must assume either a flat or a smooth sloping bottom.

Along with the need for schematization, there is the problem of selecting model parameters appropriate to the given discharge. In many instances, the recommendations of the authors of the models are either too vague or unrelated to the problem to be helpful. Often an arbitrary choice must be made. A model may perform well or poorly just on the basis of a single parameter selection. And the selection in itself of model parameters is often hampered by insufficient information from field studies, for wind and/or current profiles are often not measured, making analysis of the data difficult.

In summary: misapplication of analytical models is a major concern in model-data comparisons; arbitrary decisions, usually involved in the preparation of model calculations, can significantly affect a model's performance; and uncertainties in measured values mean that predictions of jet characteristics within about 50% of the data should be considered acceptable.

B. Data Collection and Analysis Only

1. National Environmental Research Center (Corvallis, Oregon),
U. S. Environmental Protection Agency: Analysis of Hydraulic
Model and Field Data

Shirazi^{1,2} made a detailed analysis of laboratory data and some selected field data for the dispersion of surface thermal discharges. Table IV.B.1 provides a list of the sources and the ranges of basic parameters that were studied, and also identifies the type of discharge studied and the source of data. The basic parameters determined for each data set are listed below.

- a. Initial densimetric Froude number of the jet

$$F = \frac{U_o}{\sqrt{\frac{\Delta \rho}{\rho_o} g H_o}},$$

where

U_o = initial jet velocity,

g = acceleration of gravity,

$\Delta \rho$ = density difference between jet and ambient fluid,

ρ_o = density of ambient fluid,

and

H_o = depth of discharge if rectangular.

Substitute jet diameter D if jet is circular or semicircular; or jet hydraulic radius $R_H = B_o H_o / [2(B_o + H_o)]$ if the jet is of irregular shape. Here,

B_o = full channel width,

and

H_o = channel or jet depth.

- b. Reynolds number of the jet

$$R_N = \frac{U_o D_o}{\nu} \text{ or } \frac{U_o H_o}{\nu} \text{ or } \frac{U_o R_H}{\nu},$$

where

ν = kinematic viscosity.

TABLE IV.B.1. Summary of Experimental Parameters from the Shirazi Study of Laboratory and Field Data (from Shirazi¹)

Author	R_N	R	F	A	λ	α	Jet	Comments
Bergstedt and Stefan ³	10^4 - 10^5	0	1.9-19.7	2.0	∞	1.57		Lab
Dornhelm, Nouel, and Wiegel ⁴	7×10^3	0	2.3-2.4	1.0	∞	1.57		Lab
Frigo and Frye ⁵	10^6	0-0.37	1.0-2.6	2.6	-	1.03-2.41		Lake
Hayashi and Shuto ⁶	3×10^3 - 5.6×10^3	0	1.2-3.5	1.0	∞	1.57		Lab
Motz and Benedict ⁷	5.2×10^3 10^5 - 10^7	0.18-0.73 0.5-0.75	5.4-6.5 0.3-4.6	1.0 31, 83	∞ -	2.36-1.57 1.62-2.06		Lab River
Simmons et al. ⁸	10^5 - 10^7	0.39-1.24	1.3-3.8	4.8-7.2	-	1.62		River
Stefan, Hayakawa, and Schiebe ⁹	1.5×10^3 - 9.4×10^3	0	0.62-4.1	3.03-3.23	∞	1.57		Lab
Stolzenbach and Harleman ¹⁰	7×10^4 - 1.2×10^5	0-0.4	1.0-11.6	0.35-5.7	∞ and 50-400	1.57		Lab
Tamai and Wiegel ¹¹	5.5×10^3 - 2.05×10^4	0	2.4-11.3	1.0	∞	1.57		Lab
Weil ¹²	4.07×10^3 - 1.31×10^4	1.0	6.7-35.9	2.0	∞	3.14		Lab
Wiegel, Mobarek, and Jen ¹³	7×10^2 - 9.3×10^4	0	5.0-50.0	4.0, 10.0	∞ and 50-400	1.57		Lab

c. Aspect ratio

$$A = \frac{B_o}{H_o}$$

d. Bottom slope λ defined as the ratio of horizontal to vertical variation.

e. Angle α of the jet discharge with respect to the current. (This angle is expressed in radians and is zero for coflow.)

f. Ratio R of ambient velocity to initial jet velocity.

The above are the outfall and environmental parameters that were evaluated for each data set.

For each laboratory or field test, the data were reduced by Shirazi in terms of the following plume characteristics:

a. x/D and y/D , representing the nondimensional distance along and perpendicular to the ambient current.

b. S/D , the nondimensional centerline distance.

c. TEMP, the ratio of centerline temperature excess (above ambient) to the outfall temperature excess (above ambient).

d. W/D , the nondimensional plume half-width.

e. H/D , the nondimensional plume half-depth.

Table IV.B.2 lists data reduced by Shirazi and identified by the respective investigations. Table IV.B.2 is separated into three sections by grouping the data as follows:

Group 1 (G1): surface discharge into stagnant deep water without bottom slope.

Group 2 (G2): surface discharge into stagnant water with a sloping bottom.

Group 3 (G3): surface discharge into deep moving water.

The data presented by Shirazi are analyzed by Shirazi,^{1,2} but are actually used to calibrate a model described by Shirazi and Davis.¹⁴

The few comments following place the data in proper perspective. Considerable scatter sometimes appears among the data, even for relatively similar environmental and outfall conditions. This scatter is due in part to the many different scales under which the data were acquired: between different laboratory tests and between the laboratory and field tests. A second

TABLE IV.B.2. Surface Jet Data (G1) (from Shirazi¹)

AUTHOR(S)--- HENGSTFOT & STEFAN
DATE PUBLISHED--- UNPUBL.

X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D
(R= 0 F= 7.54 A= 2.00 ALPHA=1.57)						(R= 0 F=11.34 A= 2.00 ALPHA=1.57)						(R= 0 F= 2.99 A= 2.00 ALPHA=1.57)					
0	2.5	2.5	1.0000			0	2.5	2.5	1.0000			0	2.7	2.7	1.0000		
0	5.4	5.4	1.0000			0	5.4	5.4	1.0000			0	5.4	5.4	1.0000		
0	8.2	8.2	1.0000			0	8.0	8.0	.7500			0	8.0	8.0	.7500		
0	10.8	10.8	.9000			0	10.8	10.8	.6000			0	11.0	11.0	.7000		
0	13.6	13.6	.7200			0	13.6	13.6	.4800			0	13.4	13.4	.5700		
0	16.0	16.0	.6500			0	16.0	16.0	.4200			0	16.0	16.0	.6200		
0	22.0	22.0	.5000			0	22.0	22.0	.3400			0	21.6	21.6	.4500		
0	27.0	27.0	.4300			0	27.0	27.0	.3000			0	27.2	27.2	.3400		
0	30.0	30.0	.2900			0	30.0	30.0	.2600			0	32.0	32.0	.4000		
0	38.0	38.0	.2500			0	38.0	38.0	.2100			0	36.0	36.0	.3700		
0	43.0	43.0	.2100			0	43.0	43.0	.1800			0	44.0	44.0	.3300		
												0	48.0	48.0	.3000		
(R= 0 F= 8.35 A= 2.00 ALPHA=1.57)						(R= 0 F=16.44 A= 2.00 ALPHA=1.57)						(R= 0 F= 3.78 A= 2.00 ALPHA=1.57)					
0	2.5	2.5	1.0000			0	2.5	2.5	1.0000			0	4.0	4.0	1.0000		
0	5.4	5.4	1.0000			0	5.4	5.4	1.0000			0	6.6	6.6	.9000		
0	8.2	8.2	1.0000			0	8.0	8.0	.8000			0	8.0	8.0	.8000		
0	10.8	10.8	.8900			0	10.8	10.8	.7400			0	11.0	11.0	.6500		
0	16.0	16.0	.5600			0	16.0	16.0	.4000			0	13.4	13.4	.6500		
0	22.0	22.0	.4300			0	22.0	22.0	.3300								
0	27.0	27.0	.3300			0	27.0	27.0	.2600								
0	30.0	30.0	.2600			0	30.0	30.0	.2000								
0	38.0	38.0	.1900			0	38.0	38.0	.1500								
0	43.0	43.0	.1420			0	43.0	43.0	.1150								
0	48.0	48.0	.0950			0	48.0	48.0	.0800								
0	64.0	64.0	.0460			0	64.0	64.0	.0480								
(R= 0 F= 9.17 A= 2.00 ALPHA=1.57)						(R= 0 F=19.67 A= 2.00 ALPHA=1.57)						(R= 0 F= 2.16 A= 2.00 ALPHA=1.57)					
0	2.5	2.5	1.0000			0	2.5	2.5	1.0000			0	2.7	2.7	1.0000		
0	5.4	5.4	1.0000			0	5.4	5.4	1.0000			0	5.4	5.4	1.0000		
0	10.8	10.8	.9500			0	10.8	10.8	.8200			0	8.0	8.0	.8500		
0	13.6	13.6	.8200			0	13.6	13.6	.6700			0	11.0	11.0	.7500		
0	16.0	16.0	.7200			0	16.0	16.0	.5600			0	13.4	13.4	.5400		
0	22.0	22.0	.5100			0	22.0	22.0	.4000			0	16.0	16.0	.5400		
0	27.0	27.0	.4200			0	27.0	27.0	.3000			0	16.0	16.0	.5400		
0	30.0	30.0	.2900			0	30.0	30.0	.2400			0	21.6	21.6	.4300		
0	38.0	38.0	.2600			0	38.0	38.0	.1800			0	24.0	24.0	.3800		
0	43.0	43.0	.1800			0	43.0	43.0	.1420			0	27.2	27.2	.4300		
0	48.0	48.0	.1670			0	48.0	48.0	.1000			0	32.0	32.0	.3300		
0	64.0	64.0	.0950			0	64.0	64.0	.0480			0	36.0	36.0	.3300		
0	80.0	80.0	.0660			0	80.0	80.0	.0480			0	44.0	44.0	.2800		
(R= 0 F=11.34 A= 2.00 ALPHA=1.57)						(R= 0 F=19.67 A= 2.00 ALPHA=1.57)						(R= 0 F= 2.16 A= 2.00 ALPHA=1.57)					
0	2.5	2.5	1.0000			0	2.5	2.5	1.0000			0	2.7	2.7	1.0000		
0	5.4	5.4	1.0000			0	5.4	5.4	1.0000			0	5.4	5.4	1.0000		
0	10.8	10.8	.8900			0	10.8	10.8	.8200			0	8.0	8.0	.7500		
0	13.6	13.6	.7500			0	13.6	13.6	.6700			0	11.0	11.0	.7500		
0	16.0	16.0	.6200			0	16.0	16.0	.5600			0	13.4	13.4	.5400		
0	22.0	22.0	.4200			0	22.0	22.0	.4000			0	16.0	16.0	.5400		
0	27.0	27.0	.3300			0	27.0	27.0	.3000			0	21.6	21.6	.4300		
0	30.0	30.0	.2500			0	30.0	30.0	.2400			0	24.0	24.0	.3800		
0	38.0	38.0	.2100			0	38.0	38.0	.1800			0	27.2	27.2	.4300		
0	43.0	43.0	.1340			0	43.0	43.0	.1150			0	32.0	32.0	.3300		
0	48.0	48.0	.0950			0	48.0	48.0	.0800			0	36.0	36.0	.3300		
												0	44.0	44.0	.2800		
												0	48.0	48.0	.2200		

NUMBER OF LINES = 87

NUMBER OF LINES = 43

AUTHOR(S)--- DORNHELM, NOUËL & WIEGEL
DATE PUBLISHED--- JUNE 1972

AUTHOR(S)--- FRIGO & FRYE
DATE PUBLISHED--- OCT. 1972

X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D
(R= 0 F= 2.40 A= 1.00 ALPHA=1.57)						(R= 0 F= 1.2R A= 2.61 ALPHA=1.57)						(R= 0 F= 1.04 A= 2.61 ALPHA=1.57)					
0	4.3	4.3	.8500			0	0	0	1.0000			0	0	0	1.0000		
0	10.5	10.5	.6500			3.6	49.5	49.6	.4022	67.34		6.0	29.6	30.2	.6737	48.09	
0	20.0	20.0	.4900			6.0	81.4	81.6	.4886	173.16		11.9	45.9	47.5	.5684	73.27	
0	20.0	20.0	.5000			13.2	125.2	126.0	.3750	171.49		20.7	63.8	67.5	.4632	99.89	
0	30.0	30.0	.3500			14.8	142.9	143.8	.3750	176.12		44.3	88.1	101.4	.4105	106.55	
0	30.0	30.0	.3300			19.2	199.9	201.0	.3750	128.02		74.1	106.6	136.4	.3579	107.30	
0	40.0	40.0	.3000			23.6	296.3	297.5	.2841	202.02		125.2	132.0	193.5	.3053	114.71	
0	40.0	40.0	.3100									183.8	157.1	257.2	.2526	117.66	
0	62.0	62.0	.1900									238.5	176.7	314.7	.2000	177.59	
0	1.1	1.1	.8800									296.3	179.1	372.7	.1474	201.26	
0	2.1	2.1	.8500														
0	4.2	4.2	.8000														
0	11.0	11.0	.6200														
0	21.0	21.0	.5100														
0	21.0	21.0	.5000														
0	31.0	31.0	.3600														
0	31.0	31.0	.3500														
0	42.0	42.0	.3200														
0	48.0	48.0	.3000														
0	63.0	63.0	.1800														
0	1.1	1.1	.8000														
0	11.0	11.0	.5300														
0	21.0	21.0	.4450														
0	31.0	31.0	.3840														
0	42.0	42.0	.3200														
0	63.0	63.0	.2900														
0	85.0	85.0	.2050														

NUMBER OF LINES = 28

NUMBER OF LINES = 27

TABLE IV.B.2 (Contd.) (G1)

 AUTHOR(S)--- TANAI, WIEGEL & TORNBORG
 DATE PUBLISHED--- OCT. 1969

X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D
(R=	0	F=8.40	A=1.00	ALPHA=1.57)		(R=	0	F=3.30	A=1.00	ALPHA=1.57)		(R=	0	F=9.50	A=1.00	ALPHA=1.57)	
0	3.0	3.0	1.0000	.50		0	2.0	2.0	.9400	.60		0	38.0	38.0	.2000		
0	5.0	5.0	.8000	.50		0	4.0	4.0	.9800	.36		0	51.0	51.0	.1800		
0	6.3	6.3	.7000	.46		0	6.0	6.0	.8000	.35		(R=	0	F=8.40	A=1.00	ALPHA=1.57)	
0	8.0	8.0	.6000	.44		0	7.4	7.4	.7000	.34		0	6.5	6.5	.8000		
0	12.0	12.0	.5000	.44		0	9.0	9.0	.6000	.35		0	7.7	7.7	.7000		
0	15.0	15.0	.4000	.36		0	12.0	12.0	.5000	.35		0	9.4	9.4	.6000		
0	20.0	20.0	.3000	.36		0	16.0	16.0	.4000	.40		0	12.5	12.5	.5000		
0	24.0	24.0	.2500	.36		0	22.5	22.5	.3000	.50		0	17.0	17.0	.4000		
0	35.0	35.0	.2000	.36		0	30.0	30.0	.2500			0	23.3	23.3	.3000		
(R=	0	F=9.50	A=1.00	ALPHA=1.57)		(R=	0	F=11.30	A=1.00	ALPHA=1.57)		0	47.0	47.0	.2000		
0	6.2	6.2	.8000	.46		0	3.0	3.0	.7000	1.66		(R=	0	F=11.30	A=1.00	ALPHA=1.57)	
0	7.7	7.7	.7000	.45		0	5.0	5.0	.6900	1.55		0	2.9	2.9	.6000		
0	9.3	9.3	.6000	.50		0	7.0	7.0	.7000	1.45		0	5.0	5.0	.6200		
0	12.0	12.0	.5000	.55		0	10.0	10.0	.6000	1.18		0	8.0	8.0	.6400		
0	16.0	16.0	.4000	.57		0	14.0	14.0	.6000	1.00		0	13.0	13.0	.6000		
0	22.0	22.0	.3000	.57		0	17.0	17.0	.5000	.95		0	17.0	17.0	.5000		
0	25.0	25.0	.2500	.57		0	22.0	22.0	.4000	.89		0	23.0	23.0	.4000		
0	38.0	38.0	.2000	.63		0	29.0	29.0	.3000	.95		0	31.0	31.0	.3000		
0	51.0	51.0	.1700			0	41.5	41.5	.2000	1.15		0	39.0	39.0	.2000		
(R=	0	F=11.00	A=1.00	ALPHA=1.57)		0	60.0	60.0	.1500			0	48.0	48.0	.1500		
0	3.0	3.0	.9800	.98		0	78.0	78.0	.1200			0	73.0	73.0	.1000		
0	4.8	4.8	.9000	.85		(R=	0	F=2.40	A=1.00	ALPHA=1.57)		(R=	0	F=2.70	A=1.00	ALPHA=1.57)	
0	6.6	6.6	.8000	.83		0	3.5	3.5	1.0000			0	2.8	2.8	1.0000		
0	8.0	8.0	.7000	.80		0	5.0	5.0	.9000			0	4.4	4.4	.9000		
0	10.7	10.7	.6000	.85		0	8.0	8.0	.8000			0	5.8	5.8	.8000		
0	13.0	13.0	.5000	.85		0	14.0	14.0	.7000			0	7.5	7.5	.7000		
0	16.5	16.5	.4000	.75		0	20.0	20.0	.7000			0	9.0	9.0	.6200		
0	23.0	23.0	.3000	.75		0	30.0	30.0	.6000			0	12.0	12.0	.6300		
0	35.0	35.0	.2000	.60		0	40.0	40.0	.5300			0	17.0	17.0	.7000		
0	52.0	52.0	.1500			0	70.0	70.0	.6000			0	20.0	20.0	.7000		
(R=	0	F=2.50	A=1.00	ALPHA=1.57)		0	2.9	2.9	1.0000			0	30.0	30.0	.7200		
0	3.0	3.0	1.0000	.40		0	3.4	3.4	.9000			0	38.0	38.0	.7000		
0	4.0	4.0	.9000	.40		0	5.0	5.0	.8000			0	48.0	48.0	.6000		
0	6.0	6.0	.8000	.40		0	7.0	7.0	.7000			0	57.0	57.0	.5000		
0	12.0	12.0	.7300	.30		0	10.5	10.5	.6000			(R=	0	F=3.30	A=1.00	ALPHA=1.57)	
0	20.0	20.0	.7800	.30		0	15.0	15.0	.5400			0	1.5	1.5	.9000		
0	30.0	30.0	.7200	.25		0	20.0	20.0	.5400			0	2.0	2.0	.9400		
0	38.0	38.0	.7000	.20		0	23.0	23.0	.5000			0	3.5	3.5	.9900		
0	50.0	50.0	.6000	.28		0	30.0	30.0	.4000			0	4.7	4.7	.9000		
(R=	0	F=2.60	A=1.00	ALPHA=1.57)		0	40.0	40.0	.3450			0	6.2	6.2	.8000		
0	2.5	2.5	1.0000	.30		0	60.0	60.0	.3500			0	8.1	8.1	.7000		
0	3.4	3.4	.9000	.30		(R=	0	F=9.50	A=1.00	ALPHA=1.57)		0	10.0	10.0	.6000		
0	4.3	4.3	.8000	.30		0	3.0	3.0	.9000			0	13.5	13.5	.5000		
0	7.4	7.4	.7000	.30		0	3.5	3.5	.8000			0	17.0	17.0	.4000		
0	19.0	19.0	.6000	.19		0	4.2	4.2	.7000			0	22.0	22.0	.3000		
0	30.0	30.0	.4000	.16		0	8.0	8.0	.6000			0	30.0	30.0	.2000		
0	43.0	43.0	.5000	.17		0	11.5	11.5	.5000								
0	50.0	50.0	.4500			0	16.0	16.0	.4000								
						0	22.0	22.0	.3000								

NUMBER OF LINES = 133

 AUTHOR(S)--- WIEGEL & MOBARAK
 DATE PUBLISHED--- NOV. 1966

X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D	
(R=	0	F=50.00	A=4.00	ALPHA=1.57)		(R=	0	F=26.00	A=4.00	ALPHA=1.57)		(R=	0	F=26.00	A=10.00	ALPHA=1.57)		
0	41.0	41.0	.2700	5.96		0	57.8	57.8	.0910			0	82.0	82.0	.3550	4.10		
(R=	0	F=41.00	A=4.00	ALPHA=1.57)		(R=	0	F=23.00	A=4.00	ALPHA=1.57)		(R=	0	F=27.00	A=10.00	ALPHA=1.57)		
0	62.0	62.0	.2700	12.10		0	83.0	83.0	.2300	6.03		0	131.0	131.0	.1470	11.50		
(R=	0	F=28.00	A=4.00	ALPHA=1.57)		0	124.0	124.0	.0960	9.35		0	197.0	197.0	.1100	17.40		
0	41.0	41.0	.2430	7.05		(R=	0	F=18.00	A=4.00	ALPHA=1.57)		(R=	0	F=25.00	A=10.00	ALPHA=1.57)		
(R=	0	F=24.00	A=4.00	ALPHA=1.57)		0	166.0	166.0	.0860	12.50		0	328.0	328.0	.1020	19.70		
0	62.0	62.0	.1960	6.44		0	207.0	207.0	.0610	15.60		0	262.0	262.0	.0920	16.40		
(R=	0	F=18.00	A=4.00	ALPHA=1.57)		(R=	0	F=28.00	A=4.00	ALPHA=1.57)								
0	21.0	21.0	.4000	6.23		0	41.0	41.0	.3520	6.10								
(R=	0	F=23.00	A=4.00	ALPHA=1.57)		(R=	0	F=24.00	A=4.00	ALPHA=1.57)								
0	83.0	83.0	.1780	20.69		0	83.0	83.0	.1460	18.70								
(R=	0	F=18.00	A=4.00	ALPHA=1.57)		(R=	0	F=6.00	A=4.00	ALPHA=1.57)								
0	124.0	124.0	.1020	10.15		0	41.0	41.0	.2600	29.80								
(R=	0	F=16.00	A=4.00	ALPHA=1.57)		(R=	0	F=5.00	A=4.00	ALPHA=1.57)								
0	166.0	166.0	.0790	4.16		0	83.0	83.0	.2350	39.40								
(R=	0	F=10.00	A=4.00	ALPHA=1.57)		0	166.0	166.0	.0620	20.70								
0	166.0	166.0	.0710	17.18		0	41.0	41.0	.3360	24.00								
(R=	0	F=32.00	A=4.00	ALPHA=1.57)		(R=	0	F=16.00	A=4.00	ALPHA=1.57)								
0	166.0	166.0	.0760	8.39		0	41.0	41.0	.2860	9.30								
(R=	0	F=21.00	A=4.00	ALPHA=1.57)		(R=	0	F=18.00	A=4.00	ALPHA=1.57)								
0	145.0	145.0	.0880	4.68		0	83.0	83.0	.1430	20.70								
(R=	0	F=22.00	A=4.00	ALPHA=1.57)		(R=	0	F=14.00	A=4.00	ALPHA=1.57)								
0	104.0	104.0	.1280	6.75		0	83.0	83.0	.1500	21.80								
(R=	0	F=21.00	A=4.00	ALPHA=1.57)		(R=	0	F=27.00	A=10.00	ALPHA=1.57)								
0	21.0	21.0	.3900	2.70		0	66.0	66.0	.3070	6.59								
0	83.0	83.0	.0620	9.35		(R=	0	F=18.00	A=10.00	ALPHA=1.57)								
(R=	0	F=20.00	A=4.00	ALPHA=1.57)		0	131.0	131.0	.2170	26.20								
0	124.0	124.0	.0960	17.00		(R=	0	F=22.00	A=10.00	ALPHA=1.57)								
0	207.0	207.0	.0430	5.20		0	197.0	197.0	.1460	52.40								
(R=	0	F=24.00	A=4.00	ALPHA=1.57)		(R=	0	F=25.00	A=10.00	ALPHA=1.57)								
0	166.0	166.0	.1000	15.60		0	263.0	263.0	.1030	46.00								
0	228.0	228.0	.0380			0	328.0	328.0	.1180									
(R=	0	F=27.00	A=4.00	ALPHA=1.57)		0	328.0	328.0	.0790	65.50								
0	207.0	207.0	.0870	21.40		(R=	0	F=27.00	A=10.00	ALPHA=1.57)								
0	269.0	269.0	.0530			0	33.0	33.0	.6350	4.59								
(R=	0	F=26.00	A=4.00	ALPHA=1.57)		(R=	0	F=26.00	A=10.00	ALPHA=1.57)								
0	21.0	21.0	.2200	.52		0	33.0	33.0	.6000	4.82								

NUMBER OF LINES = 48

TABLE IV.B.2 (Contd.) (G2 and G3)

AUTHOR(S)--- WIEGEL & MORAREK
DATE PUBLISHED--- NOV. 1964

X/D	Y/D	S/D	TEMP	W/D	H/D
(R= 0 F=25.00 A=10.00 ALPHA=1.57)	0	0.6	0.6	.8700	11.48
(R= 0 F=22.00 A=10.00 ALPHA=1.57)	0	13.1	13.1	.5300	31.15
(R= 0 F=25.00 A=10.00 ALPHA=1.57)	0	19.7	19.7	.3800	34.43
(R= 0 F=27.00 A=10.00 ALPHA=1.57)	0	26.3	26.3	.3100	59.02
(R= 0 F=28.00 A=10.00 ALPHA=1.57)	0	32.8	32.8	.2800	42.62
(R= 0 F=29.00 A=10.00 ALPHA=1.57)	0	45.9	45.9	.1950	75.00
(R= 0 F=23.00 A=10.00 ALPHA=1.57)	0	6.6	6.6	.8460	14.40
(R= 0 F=24.00 A=10.00 ALPHA=1.57)	0	13.1	13.1	.5000	31.37
(R= 0 F=27.00 A=10.00 ALPHA=1.57)	0	19.7	19.7	.3650	48.50
(R= 0 F=25.00 A=10.00 ALPHA=1.57)	0	26.3	26.3	.2150	48.98
(R= 0 F=28.00 A=10.00 ALPHA=1.57)	0	32.8	32.8	.1900	40.44
(R= 0 F=27.00 A=10.00 ALPHA=1.57)	0	39.4	39.4	.1950	73.61
(R= 0 F=12.00 A=10.00 ALPHA=1.57)	0	13.1	13.1	.6900	37.38
(R= 0 F=22.00 A=10.00 ALPHA=1.57)	0	13.1	13.1	.4650	23.36
(R= 0 F=25.00 A=10.00 ALPHA=1.57)	0	26.3	26.3	.3900	61.90
(R= 0 F=26.00 A=10.00 ALPHA=1.57)	0	39.4	39.4	.4150	76.97
(R= 0 F= 6.00 A=10.00 ALPHA=1.57)	0	13.1	13.1	.6650	29.51
(R= 0 F= 8.00 A=10.00 ALPHA=1.57)	0	13.1	13.1	.5100	41.28
(R= 0 F=19.7 19.7 .4450 52.31	0	19.7	19.7	.4450	52.31
(R= 0 F= 5.00 A=10.00 ALPHA=1.57)	0	13.1	13.1	.5550	26.07
(R= 0 F= 8.00 A=10.00 ALPHA=1.57)	0	13.1	13.1	.5500	21.01

AUTHOR(S)--- FRIGO & FRYE
DATE PUBLISHED--- OCT. 1972

X/D	Y/D	S/D	TEMP	W/D	H/D
(R= .184 F= 1.24 A= 2.61 ALPHA=1.72)	0	0	0	1.0000	
	0	41.4	41.5	.6023	29.60
	8.8	62.1	64.1	.4886	62.16
	14.7	76.0	77.4	.5227	69.56
	21.4	88.8	93.7	.4886	77.70
	29.5	103.6	110.6	.4318	80.65
	37.0	116.2	125.4	.3750	72.52
	48.1	131.8	144.5	.3409	72.52
	62.9	151.7	166.4	.3182	69.56
	78.4	168.6	182.3	.2955	68.80
	95.5	182.0	214.2	.2614	106.55
	214.7	201.2	334.9	.1818	151.69
(R= .368 F= 1.16 A= 2.61 ALPHA=1.03)	0	0	0	1.0000	
	0	23.0	23.1	.6381	14.78
	8.6	35.4	36.4	.6381	22.19
	21.6	42.7	41.2	.5472	65.85
	39.1	47.3	71.2	.4476	50.32
	45.8	50.2	79.5	.4476	57.73
	65.0	62.1	101.1	.4000	62.91
	87.2	74.8	126.8	.3524	68.81
	114.7	84.6	157.7	.3048	91.40
	126.6	93.2	170.4	.2810	85.10
	208.1	93.2	361.4	.1143	
	139.8	97.6	500.3	.2571	98.42
	144.2	104.4	510.2	.2571	
	226.3	106.4	592.4	.1619	107.30
(R= .184 F= 2.31 A= 2.61 ALPHA=2.61)	0	0	0	1.0000	
	4.4	53.3	53.6	.6196	
	13.9	68.1	71.2	.5543	
	23.5	74.0	82.6	.5109	
	35.4	78.4	95.3	.4674	
	50.2	85.2	111.6	.4348	
	70.1	95.5	134.1	.4022	
	94.7	105.9	160.9	.3678	
	125.0	119.9	194.3	.2935	
	156.1	142.1	232.6	.2717	
	198.3	177.6	287.8	.2391	
	245.0	247.8	372.1	.1848	
	0	0	0	1.0000	
	7.6	29.5	30.5	.7283	53.27
	16.3	45.8	49.1	.6739	53.27
	29.5	65.7	73.0	.6196	55.50
	41.4	82.8	93.9	.5652	57.73
	57.7	102.0	119.1	.5109	56.98
	70.9	114.7	137.5	.4457	50.20
	80.0	123.5	150.2	.4022	50.93
	105.1	145.8	183.8	.3476	35.50
	135.4	177.6	227.8	.2935	37.01
	175.3	207.2	277.5	.1848	

NUMBER OF LINES = 49

X/D	Y/D	S/D	TEMP	W/D	H/D
(R= 0 F= 8.00 A=10.00 ALPHA=1.57)	0	19.7	19.7	.5250	35.80

NUMBER OF LINES = 22

AUTHOR(S)--- STOLZENBACH & HARLEMAN
DATE PUBLISHED--- 1971

X/D	Y/D	S/D	TEMP	W/D	H/D
(R= 0 F= 1.13 A= 2.70 ALPHA=1.57)	0	1.4	1.4	.9900	
	0	5.9	5.9	.9900	
	0	9.7	9.7	.8500	
	0	14.3	14.3	.7900	
	0	18.1	18.1	.7400	
	0	22.7	22.7	.7000	
	0	27.6	27.6	.6500	
	0	35.9	35.9	.5600	
(R= 0 F= 1.13 A= 2.70 ALPHA=1.57)	0	1.4	1.4	.9900	
	0	5.9	5.9	.9900	
	0	9.7	9.7	.8500	
	0	14.3	14.3	.7900	
	0	18.1	18.1	.7400	
	0	22.7	22.7	.7000	
	0	27.6	27.6	.6500	
	0	35.9	35.9	.5600	
(R= 0 F= 1.13 A= 2.70 ALPHA=1.57)	0	1.4	1.4	.9900	
	0	5.9	5.9	.9900	
	0	9.7	9.7	.8500	
	0	14.3	14.3	.7900	
	0	18.1	18.1	.7400	
	0	22.7	22.7	.7000	
	0	27.6	27.6	.6500	
	0	35.9	35.9	.6200	
(R= 0 F= 2.70 A= .35 ALPHA=1.57)	0	.6	.8	.9900	.42
	0	3.6	3.6	.9200	4.15
	0	6.2	6.2	.8500	7.06
	0	8.7	8.7	.8300	8.31
	0	12.5	12.5	.3100	10.38
	0	14.5	14.5	.3100	
	0	17.5	17.5	.3000	12.46
	0	22.9	22.9	.2800	17.86

AUTHOR(S)--- STOLZENBACH & HARLEMAN
DATE PUBLISHED--- 1971

X/D	Y/D	S/D	TEMP	W/D	H/D
(R= .258 F= 1.16 A= 2.30 ALPHA=1.57)	0	1.8	1.8	.9900	
	0	7.8	7.8	.8300	
	0	12.9	12.9	.6400	
	0	20.5	20.5	.8200	
	2.2	25.8	28.2	.5200	
	3.2	33.4	33.9	.5100	
	9.7	38.6	42.2	.5300	
	6.9	49.5	53.5	.4800	
	10.7	62.3	66.8	.4400	
	11.8	75.2	79.8	.4400	
(R= .386 F= 1.20 A= 2.30 ALPHA=1.57)	0	1.8	1.8	.9900	
	0	7.8	7.8	.8800	
	0	12.9	12.9	.8500	
	0	20.5	20.5	.7200	
	4.3	25.8	27.3	.7200	
	2.2	33.4	33.9	.5700	
	2.1	38.6	48.6	.7200	
	5.3	49.5	51.9	.6100	
	7.5	62.3	64.9	.5500	
	9.7	75.2	78.0	.5300	
(R= .405 F= 1.88 A= 2.30 ALPHA=1.57)	0	1.8	1.8	.9900	
	0	7.8	7.8	.8700	
	0	12.9	12.9	.7900	
	0	20.5	20.5	.6700	
	4.3	25.8	27.3	.6300	
	3.2	33.4	35.0	.6600	
	7.5	38.6	41.8	.6600	
	12.8	49.5	53.9	.5400	
	24.7	62.3	71.4	.4600	
	30.0	75.2	85.3	.4400	
(R= .115 F= 4.10 A= .51 ALPHA=1.57)	0	1.8	1.8	.9500	
	0	8.1	8.1	.5200	
	0	14.1	14.1	.4000	
	0	20.1	20.1	.3200	
	0	25.7	25.7	.2800	
	0	31.3	31.3	.2600	
	6.0	37.8	40.2	0	0
	6.1	47.9	50.3	.3000	
	11.1	50.4	55.8	0	0
	12.8	60.5	66.1	.2500	
	25.2	100.8	108.2	.2100	
(R= .087 F= 5.85 A= .51 ALPHA=1.57)	0	1.7	1.7	.9800	
	0	7.9	7.9	.5700	
	0	13.6	13.6	.4200	
	0	19.7	19.7	.2800	

X/D	Y/D	S/D	TEMP	W/D	H/D
(R= 0 F= 2.70 A= .35 ALPHA=1.57)	0	.8	.8	.9900	.42
	0	3.6	3.6	.9200	4.15
	0	6.2	6.2	.8500	7.06
	0	8.7	8.7	.8300	8.31
	0	12.5	12.5	.3600	10.38
	0	14.5	14.5	.3100	
	0	17.5	17.5	.3000	12.46
	0	22.9	22.9	.2800	17.86
(R= 0 F= 2.70 A= .35 ALPHA=1.57)	0	.8	.8	.9900	.42
	0	3.6	3.6	.9200	4.15
	0	6.2	6.2	.8500	7.06
	0	8.7	8.7	.8300	8.31
	0	12.5	12.5	.3600	10.38
	0	14.5	14.5	.3100	
	0	17.5	17.5	.3000	12.46
	0	22.9	22.9	.2800	17.86
(R= 0 F=11.60 A= 1.05 ALPHA=1.57)	0	3.9	3.9	.8300	3.56
	0	16.4	16.4	.5100	7.12
	0	29.9	29.9	.4300	9.96
	0	42.7	42.7	.3800	17.08
	0	54.1	54.1	.3500	34.16
	0	67.6	67.6	.3200	34.16
	0	79.7	79.7	.3000	37.00
	0	105.3	105.3	.2600	46.97
(R= 0 F=11.60 A= 1.05 ALPHA=1.57)	0	3.9	3.9	.7800	3.56
	0	16.4	16.4	.5400	12.81
	0	29.9	29.9	.4300	19.21
	0	42.7	42.7	.3500	29.18
	0	54.1	54.1	.3300	27.75
	0	67.6	67.6	.2500	
	0	79.7	79.7	.2500	63.34
	0	105.3	105.3	.2400	
(R= 0 F=11.60 A= 1.05 ALPHA=1.57)	0	3.9	3.9	.9000	3.56
	0	16.4	16.4	.4600	4.27
	0	29.9	29.9	.3400	13.52
	0	42.7	42.7	.2800	18.50
	0	54.1	54.1	.2400	14.22
	0	67.6	67.6	.1700	24.41
	0	79.7	79.7	.1700	45.54
	0	105.3	105.3	.1200	69.74

NUMBER OF LINES = 72

X/D	Y/D	S/D	TEMP	W/D	H/D
(R= .087 F= 5.85 A= .51 ALPHA=1.57)	0	25.1	25.1	.2400	
	0	30.6	30.6	.2400	
	2.5	36.9	37.4	0	0
	4.1	39.4	40.4	.2100	
	12.3	49.2	53.1	0	0
	19.9	59.1	65.6	.2000	
	23.6	61.6	70.1	0	0
	39.8	78.8	93.7	.1900	
	61.2	98.5	122.8	.1900	
(R= .115 F= 6.60 A= .51 ALPHA=1.57)	0	1.8	1.8	.9500	
	0	8.1	8.1	.9500	
	0	14.1	14.1	.4000	
	0	20.1	20.1	.2800	
	6.6	25.7	28.6	.2200	
	79.2	60.4	64.3	0	0
	38.8	62.5	78.1	0	0
	75.0	90.7	124.0	.2200	
	94.4	105.8	146.6	.1500	

NUMBER OF LINES = 63

TABLE IV.B.2 (Contd.) (G3)

 AUTHON(S)--- NOTZ & BENEDICT
 DATE PUBLISHED--- 1971

X/D	Y/D	S/D	TEMP	W/D	M/D	X/D	Y/D	S/D	TEMP	W/D	M/D	X/D	Y/D	S/D	TEMP	W/D	M/D
(R=.730 F=5.60 A=1.00 ALPHA=1.56)						(R=.230 F=5.60 A=1.00 ALPHA=1.56)						(R=.190 F=5.60 A=1.00 ALPHA=2.06)					
0	.2	.2	1.0000			0	0	0	1.0000			0	0	0	1.0000		
.3	.4	.5	1.0000			5.0	4.5	7.5	.2800			.9	1.4	1.7	1.0000		
.4	.7	.8	.9700									1.9	2.1	3.0	.6000		
.9	.9	1.4	.8500			(R=.200 F=5.60 A=1.00 ALPHA=1.56)						2.9	2.8	4.1	.4500		
1.9	1.4	2.5	.7500			0	0	0	1.0000			3.9	3.1	5.2	.4400		
2.9	1.6	3.5	.6300			0	.4	.4	1.0000			5.0	3.9	6.5	.3900		
3.9	1.6	4.5	.5400			0	.7	.7	.9800			8.9	5.3	10.7	.2900		
5.0	2.0	5.6	.4800			0	.7	.7	.9800			(R=.660 F=5.60 A=1.00 ALPHA=2.35)					
6.0	2.1	6.6	.4700			0	1.1	1.1	.9500			0	0	0	1.0000		
8.9	2.6	9.7	.3300			.4	1.4	1.7	.8500			0	.1	.1	.9800		
11.9	2.9	12.6	.2600			.9	2.4	2.8	.6000			.4	.2	.6	.8800		
(R=.440 F=5.60 A=1.00 ALPHA=1.56)						1.9	4.1	4.7	.5000			.9	.4	1.1	.7600		
0	0	0	.9700			2.9	4.2	5.7	.4000			1.9	.6	2.1	.6700		
0	.4	.4	.9700			3.9	4.8	6.9	.3600			2.9	.7	3.1	.6500		
0	.4	.5	1.0000			5.0	5.7	8.3	.3200			3.9	.9	4.1	.6100		
.2	.7	.8	.9800			8.9	7.8	12.8	.2300			5.0	.9	5.2	.5700		
.4	.9	1.2	.9700			(R=.670 F=5.60 A=1.00 ALPHA=2.06)						8.9	1.2	9.1	.5000		
.4	1.1	1.3	.9700			0	0	.1	1.0000			(R=.420 F=5.60 A=1.00 ALPHA=2.35)					
.9	1.4	1.9	.8300			.2	.2	.3	.9800			0	.2	.5	1.0000		
1.9	2.2	3.2	.6700			.4	.2	.5	.9600			.4	.2	.5	1.0000		
2.9	2.6	4.2	.5700			.9	.3	1.0	.8500			.9	.6	1.1	.9000		
3.9	2.7	5.2	.4700			1.9	.5	2.0	.6700			1.9	1.1	2.2	.8200		
5.0	3.3	6.4	.4100			2.9	.5	3.0	.5900			2.9	1.2	3.2	.7800		
6.0	3.1	7.5	.3700			3.9	.8	4.1	.5700			3.9	1.6	4.3	.7200		
8.9	3.9	10.5	.2700			5.0	.9	5.1	.5700			5.0	1.9	5.4	.6900		
(R=.300 F=5.60 A=1.00 ALPHA=1.56)						8.9	1.6	9.2	.5100			6.0	2.1	6.4	.6300		
0	0	0	.9900			(R=.440 F=5.60 A=1.00 ALPHA=2.06)						8.9	2.8	9.5	.5300		
.2	.5	.6	1.0000			.4	.4	.6	1.0000			(R=.300 F=5.60 A=1.00 ALPHA=2.35)					
.2	.7	.7	.9800			.8	.6	1.0	.9900			.4	.2	.5	1.0000		
.2	.7	.8	1.0000			.9	.7	1.2	.9000			.9	.6	1.1	.9200		
.4	.8	1.0	.9600			1.9	1.3	2.4	.8400			1.1	.4	1.3	.9800		
.3	.9	1.2	.9700			2.9	1.9	3.5	.8000			1.9	.9	2.3	.9000		
.9	1.6	2.1	.7900			3.9	1.9	4.5	.7300			2.9	1.4	3.4	.6600		
1.9	2.1	3.2	.6400			5.0	2.2	5.6	.6900			3.9	1.9	4.5	.5800		
2.9	2.8	4.5	.5700			6.0	2.4	6.6	.6500			5.0	2.1	5.6	.5100		
3.9	3.1	5.5	.5300			8.9	3.8	9.9	.5700			6.0	2.4	6.6	.6600		
5.0	3.4	6.6	.4700			(R=.230 F=5.60 A=1.00 ALPHA=2.06)						8.9	3.6	9.8	.4000		
6.0	3.3	7.6	.4400			0	.1	.2	1.0000			(R=.230 F=5.60 A=1.00 ALPHA=2.35)					
8.9	4.1	10.7	.2900			.4	.4	.7	1.0000			.4	.2	.5	1.0000		
(R=.230 F=5.60 A=1.00 ALPHA=1.56)						.9	.7	1.2	.9900			.9	.7	1.2	.9900		
0	.2	.2	1.0000			1.9	1.8	2.7	.7500			.9	.7	1.2	.9700		
0	.6	.6	1.0000			2.9	2.1	3.7	.5500			1.4	.9	1.7	.9700		
0	.7	.7	.9900			3.9	2.8	4.9	.5300			1.9	1.2	2.3	.9200		
.2	1.1	1.1	.9900			5.0	2.9	6.0	.4300			2.9	2.1	3.6	.7200		
.4	1.2	1.4	.9600			(R=.190 F=5.60 A=1.00 ALPHA=2.06)						3.9	2.4	4.7	.5700		
.9	1.4	1.7	.8700			0	0	.1	.9800			5.0	2.8	5.8	.5100		
.9	2.2	2.6	.7000			.4	.7	.8	1.0000			8.9	4.6	10.1	.3600		
1.9	3.3	4.1	.5200			.4	.8	1.0	.9900			(R=.180 F=5.60 A=1.00 ALPHA=2.35)					
2.9	3.3	5.1	.4300			(R=.270 F=5.60 A=1.00 ALPHA=1.56)						.1	0	.1	.9800		
3.9	3.8	6.2	.4000			0	0	0	1.0000			(R=.190 F=5.60 A=1.00 ALPHA=2.06)					
(R=.300 F=5.60 A=1.00 ALPHA=2.06)						11.9	7.0	14.3	.2100			59.8	14.1	64.3	.1500		
.2	0	.2	1.0000			23.9	11.9	27.2	.1700			(R=.660 F=5.60 A=1.00 ALPHA=2.35)					
.6	.2	.6	.8800			35.8	11.9	39.2	.1300			6.0	.9	6.0	.5600		
.3	.2	.9	.9900			59.8	20.7	64.7	.1600			6.0	.9	6.0	.5200		
.4	.5	1.2	.9400			(R=.200 F=5.60 A=1.00 ALPHA=1.56)						11.9	.9	12.1	.4900		
.9	.8	1.8	.9100			0	0	0	1.0000			11.9	.9	13.0	.3800		
1.9	1.4	3.0	.7700			11.9	10.0	15.7	.1800			23.9	3.1	25.2	.3800		
2.9	1.9	3.7	.7800			23.9	11.9	27.9	.1900			35.8	3.1	37.1	.2500		
3.9	2.4	4.8	.6300			35.8	16.1	40.4	.1500			47.9	5.3	40.4	.2300		
5.0	3.1	6.1	.5600			59.8	20.7	64.9	.1100			59.8	5.3	61.2	.1800		
8.9	4.6	10.4	.4200			(R=.670 F=5.60 A=1.00 ALPHA=2.06)						(R=.420 F=5.60 A=1.00 ALPHA=2.35)					
(R=.440 F=5.60 A=1.00 ALPHA=1.56)						6.0	1.1	6.1	.5500			11.9	3.3	12.4	.4600		
.2	0	.2	1.0000			6.0	1.2	6.2	.5500			11.9	3.3	12.4	.4900		
.6	.2	.6	.8800			11.9	1.7	12.2	.4100			11.9	3.1	12.6	.4300		
.3	.2	.9	.9900			11.9	1.7	12.2	.4100			23.9	5.3	24.8	.3400		
.4	.5	1.2	.9400			23.9	3.1	24.2	.2800			35.8	7.5	36.9	.3000		
.9	.8	1.8	.9100			35.8	3.1	36.2	.2300			47.9	9.7	49.2	.2200		
1.9	1.4	3.0	.7700			47.9	5.3	48.4	.2000			(R=.300 F=5.60 A=1.00 ALPHA=2.35)					
2.9	1.9	3.7	.6600			59.8	5.3	60.3	.1700			11.9	3.8	12.5	.3400		
3.9	2.4	5.2	.5300			(R=.440 F=5.60 A=1.00 ALPHA=2.06)						11.9	3.8	12.5	.3300		
5.0	2.7	6.3	.4900			11.9	4.5	12.7	.5000			11.9	3.1	13.2	.2700		
6.0	2.8	7.3	.3800			11.9	3.1	14.1	.3700			23.9	5.3	25.4	.2500		
8.9	3.8	10.5	.3400			23.9	5.3	26.3	.3100			35.8	7.5	37.6	.2000		
11.9	4.4	13.5	.2700			35.8	7.5	38.4	.2300			47.9	9.7	49.8	.1700		
11.9	4.4	13.5	.3100			47.9	9.7	50.7	.2100			59.8	11.9	61.9	.1400		
11.9	3.1	14.8	.2500			59.8	9.7	62.5	.1600			(R=.230 F=5.60 A=1.00 ALPHA=2.35)					
(R=.730 F=5.60 A=1.00 ALPHA=1.56)						(R=.300 F=5.60 A=1.00 ALPHA=2.06)						6.0	3.1	9.8	.5100		
11.9	2.9	12.3	.2600			23.9	5.3	24.5	.2200			6.0	3.1	6.9	.4400		
23.9	3.1	24.2	.1600			35.8	9.7	37.2	.2800			11.9	4.6	13.0	.3100		
35.8	3.0	36.2	.1200			47.9	9.7	49.3	.2100			11.9	4.6	13.0	.3300		
47.9	5.3	48.5	.1000			59.8	9.7	61.2	.1700			23.9	25.2	36.8	.2400		
59.8	7.5	60.6	.1000			(R=.230 F=5.60 A=1.00 ALPHA=2.06)						35.8	37.2	53.7	.1700		
(R=.440 F=5.60 A=1.00 ALPHA=1.56)						6.0	3.6	7.0	.4600			(R=.180 F=5.60 A=1.00 ALPHA=2.35)					
11.9	4.2	12.4	.2700			6.0	3.1	7.5	.4000			6.0	3.4	6.9	.4200		
11.9	4.2	12.6	.2600			11.9	5.3	13.6	.3000			6.0	3.1	7.2	.4500		
23.9	9.7	25.8	.1700			11.9	5.3	13.6	.3100			11.9	5.3	13.5	.3400		
35.8	9.7	37.8	.1200			11.9	5.3	13.9	.2800			23.9	7.5	25.7	.2100		
47.9	9.7	49.8	.1100			23.9	9.7	26.3	.1800			35.8	11.9	38.4	.1500		
59.8	7.5	61.9	.0900			35.8	9.7	38.3	.1800			59.8	11.9	62.4	.1100		
(R=.300 F=5.60 A=1.00 ALPHA=1.56)						59.8	14.1	62.6	.1400			(R=.190 F=5.60 A=1.00 ALPHA=2.06)					
11.9	4.5	12.7	.2500			6.0	4.1	7.3	.3700			(R=.180 F=5.60 A=1.00 ALPHA=2.35)					
11.9	4.4	12.8	.2400			6.0	5.3	8.5	.3500			NUMBER OF LINES = 249					
23.9	9.7	25.9	.1400			11.9	5.8	14.4	.2400								
35.8	9.7	27.9	.1300			11.9	5.3	14.9	.2200								
47.9	9.7	49.9	.1200			23.9	10.9	28.2	.1900								
59.8	9.7	61.8	.1200			35.8	10.9	40.1	.1500								
(R=.230 F=5.60 A=1.00 ALPHA=1.56)						(R=.190 F=5.60 A=1.00 ALPHA=2.06)						(R=.180 F=5.60 A=1.00 ALPHA=2.35)					
6.0	5.1	7.9	.3000			6.0	4.1	7.3	.3700								

TABLE IV.B.2 (Contd.) (G3)

AUTHOR(S)--- SIMMONS, KUESER, WRIGHT & JOURIS
DATE PUBLISHED--- DEC. 1972

X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D
(R=.388 F=1.32 A=7.18 ALPHA=1.62)						(R=.556 F=1.72 A=7.18 ALPHA=1.62)						(R=.746 F=1.72 A=7.18 ALPHA=1.62)					
22.3	25.9	34.2	.8100			30.0	15.0	36.2	.8871	51.00		19.0	9.0	21.1	.9785	50.00	
44.5	61.8	76.4	.5400			50.0	25.0	58.6	.8602	70.00		29.0	9.0	31.2	.9677	52.00	
198.9	147.9	253.3	.2700			76.0	40.0	88.7	.8333	80.00		40.0	9.0	42.2	.9570	45.00	
(R=.556 F=1.72 A=7.18 ALPHA=1.62)						98.0	45.0	111.3	.8065	82.00		50.0	7.0	52.5	.9301	34.00	
127.1	53.3	137.8	.8100			128.0	65.0	147.4	.7258	88.00		60.0	5.0	62.7	.9140	25.00	
198.9	108.4	228.4	.5400			155.0	80.0	178.3	.6452	98.00		71.0	4.0	73.8	.9032	20.00	
375.5	143.6	408.5	.2700			185.0	85.0	208.8	.5376	115.00		82.0	4.0	84.9	.8602	22.00	
(R=.746 F=1.72 A=7.18 ALPHA=1.62)						215.0	100.0	242.4	.4839	120.00		89.0	5.0	92.0	.8441	30.00	
129.3	17.9	129.9	.8100			255.0	105.0	282.8	.4301	120.00		98.0	6.0	101.1	.8226	45.00	
137.9	26.6	343.9	.5400			306.0	108.0	327.9	.3725			106.0	7.0	109.2	.8065	50.00	
182.4	26.6	548.8	.2700			340.0	110.0	368.0	.2688			115.0	8.0	118.3	.5376	51.00	
(R=1.235 F=3.04 A=4.75 ALPHA=1.62)						415.0	125.0	444.5	.2151			131.0	10.0	134.5	.4570	50.00	
43.7	2.9	43.8	.9400			480.0	140.0	511.3	.1505			148.0	11.0	151.6	.3871	51.00	
310.7	5.7	310.9	.4700			530.0	150.0	562.3	.1075			160.0	12.0	163.6	.3226	58.00	
(R=.917 F=3.79 A=4.75 ALPHA=1.62)						585.0	151.0	617.4	.0538			169.0	12.0	172.7	.2688	65.00	
5.7	4.3	7.1	.9600			(R=.388 F=1.32 A=7.18 ALPHA=1.62)						(R=.704 F=2.69 A=7.18 ALPHA=1.62)					
35.2	7.1	36.6	.4800			10.0	30.0	31.6	.8065	80.00		11.0	8.0	13.6	.9500	28.00	
(R=.855 F=2.34 A=5.38 ALPHA=1.62)						20.0	41.0	46.5	.6720	100.00		25.0	10.0	27.8	.8800	29.00	
6.5	6.5	9.2	.8300			30.0	51.0	60.7	.5376	120.00		39.0	11.0	41.9	.8000	38.00	
24.8	8.1	27.6	.5600			(R=.599 F=2.31 A=7.18 ALPHA=1.62)						51.0	11.0	53.9	.7500	42.00	
304.0	6.5	306.9	.2800			10.0	5.0	11.2	.9167	22.00		64.0	11.0	67.0	.6700	45.00	
(R=.758 F=2.28 A=5.91 ALPHA=1.62)						22.0	8.0	23.6	.8333	24.00		76.0	12.0	79.1	.6000	46.00	
8.9	8.9	12.6	.8100			40.0	12.0	42.1	.7500	30.00		90.0	13.0	93.2	.5500	47.00	
36.6	7.1	40.4	.5400			50.0	20.0	54.9	.6667	32.00		101.0	18.0	104.9	.5000	50.00	
336.9	7.1	340.7	.2700			62.0	27.0	68.9	.5833	40.00		NUMBER OF LINES = 98					
(R=.704 F=2.59 A=7.18 ALPHA=1.62)						80.0	37.0	89.5	.4167	58.00							
104.8	30.9	109.3	.5000			98.0	45.0	109.3	.3750	62.00							
165.1	46.7	171.6	.3000			120.0	50.0	131.9	.3167								
(R=.870 F=1.68 A=7.18 ALPHA=1.62)						136.0	58.0	149.8	.2917								
58.2	22.3	62.3	.7500			150.0	60.0	164.0	.2500								
173.8	50.6	183.7	.3800			(R=.870 F=1.68 A=7.18 ALPHA=1.62)											
278.6	81.9	290.4	.2300			9.0	12.0	15.0	.9248	31.00							
(R=.599 F=2.31 A=7.18 ALPHA=1.62)						20.0	18.0	27.6	.8647	35.00							
27.3	12.9	30.2	.8300			30.0	19.0	37.7	.8271	36.00							
88.3	37.3	95.9	.4200			40.0	22.0	48.2	.7519	39.00							
157.2	57.4	167.8	.2500			50.0	26.0	59.0	.7143	42.00							
(R=.599 F=2.30 A=7.18 ALPHA=1.62)						61.0	30.0	70.7	.6767	48.00							
22.3	8.6	23.9	.8700			72.0	34.0	82.5	.6391	52.00							
118.5	39.5	125.0	.4300			83.0	39.0	94.6	.6015	54.00							
150.1	48.8	158.0	.2600			93.0	40.0	104.7	.5639	58.00							
(R=.556 F=1.72 A=7.18 ALPHA=1.62)						105.0	41.0	116.8	.5263	60.00							
10.0	12.5	16.0	.9140	41.00		112.0	43.0	124.2	.5113	55.00							
						124.0	45.0	136.4	.4737	56.00							
						132.0	48.0	145.0	.4511	52.00							
						141.0	49.0	154.1	.4135	41.00							
						150.0	51.0	163.3	.3759	38.00							
						(R=.746 F=1.72 A=7.18 ALPHA=1.62)											
						9.0	5.0	10.3	.9944	40.00							

AUTHOR(S)--- WEIL
DATE PUBLISHED--- JUNE 1972

X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D
(R=1.000 F=26.43 A= 2.00 ALPHA=3.14)						(R=1.000 F=13.33 A= 2.00 ALPHA=3.14)						(R=1.000 F=16.40 A= 2.00 ALPHA=3.14)					
11.1	0	11.1	2.30			135.2	0	135.8	9.70			11.0	0	11.1	.7200		
20.4	0	20.4	3.33									22.0	0	22.1	.4500		
27.8	0	27.9	3.74			(R=1.000 F=13.70 A= 2.00 ALPHA=3.14)						45.0	0	45.2	.3300		
37.0	0	37.2	4.19			5.5	0	5.5	.9000			68.0	0	68.2	.2500		
44.8	0	45.0	4.52			11.0	0	11.1	.6800			90.0	0	90.3	.1750		
57.8	0	58.0	5.00			22.0	0	22.1	.5800			135.0	0	135.3	.1600		
74.1	0	74.4	5.48			45.0	0	45.2	.4300			200.0	0	200.4	.0980		
90.4	0	90.8	6.07			68.0	0	68.2	.2250			(R=1.000 F=19.00 A= 2.00 ALPHA=3.14)					
101.5	0	101.9	6.37			135.0	0	135.3	.2500			5.5	0	5.5	.9000		
111.1	0	111.6	6.63			(R=1.000 F= 9.70 A= 2.00 ALPHA=3.14)						11.0	0	11.1	.7200		
123.0	0	123.5	6.89			5.5	0	5.5	.7600			22.0	0	22.1	.5000		
135.2	0	135.8	7.11			11.0	0	11.1	.6500			45.0	0	45.2	.3500		
(R=1.000 F=18.51 A= 2.00 ALPHA=3.14)						22.0	0	22.1	.5700			68.0	0	68.2	.2150		
11.1	0	11.1	2.52			45.0	0	45.2	.3250			90.0	0	90.3	.1420		
20.4	0	20.4	3.70			68.0	0	68.2	.3600			135.0	0	135.3	.1250		
27.8	0	27.9	4.41			135.0	0	135.3	.2050			(R=1.000 F=14.00 A= 2.00 ALPHA=3.14)					
37.0	0	37.2	4.96			205.0	0	205.3	.1200			5.5	0	5.5	.7500		
44.8	0	45.0	5.33			270.0	0	270.4	.0900			11.0	0	11.1	.7300		
57.8	0	58.0	6.00			(R=1.000 F= 7.40 A= 2.00 ALPHA=3.14)						22.0	0	22.1	.5300		
74.1	0	74.4	6.52			5.5	0	5.5	.8500			45.0	0	45.2	.3300		
90.4	0	90.8	7.11			11.0	0	11.1	.7000			68.0	0	68.2	.2050		
101.5	0	101.9	7.41			22.0	0	22.1	.5500			90.0	0	90.3	.1700		
111.1	0	111.6	7.63			45.0	0	45.2	.4150			135.0	0	135.3	.1500		
123.0	0	123.5	7.85			68.0	0	68.2	.3250			(R=1.000 F=16.50 A= 2.00 ALPHA=3.14)					
135.2	0	135.8	8.07			135.0	0	135.3	.2550			4.1	0	4.1	.8500		
(R=1.000 F=15.22 A= 2.00 ALPHA=3.14)						(R=1.000 F= 6.80 A= 2.00 ALPHA=3.14)						8.1	0	8.1	.7000		
11.1	0	11.1	2.81			5.5	0	5.5	.8000			16.5	0	16.5	.6400		
20.4	0	20.4	4.00			11.0	0	11.1	.7500			32.5	0	32.7	.4500		
27.8	0	27.9	4.59			22.0	0	22.1	.6000			65.0	0	65.2	.1800		
37.0	0	37.2	5.19			45.0	0	45.2	.4400			98.0	0	98.3	.3000		
44.8	0	45.0	5.52			68.0	0	68.2	.2600			(R=1.000 F=11.50 A= 2.00 ALPHA=3.14)					
57.8	0	58.0	6.00			135.0	0	135.3	.1450			4.1	0	4.1	.9000		
74.1	0	74.4	6.59			(R=1.000 F=27.00 A= 2.00 ALPHA=3.14)						8.1	0	8.1	.7500		
90.4	0	90.8	7.33			5.5	0	5.5	.9200			16.5	0	16.6	.6300		
101.5	0	101.9	7.78			11.0	0	11.1	.6500			32.5	0	32.7	.5100		
111.1	0	111.6	8.07			22.0	0	22.1	.4800			65.0	0	65.2	.3250		
123.0	0	123.5	8.52			45.0	0	45.2	.2950			98.0	0	98.3	.3300		
135.0	0	135.6	8.89			68.0	0	68.2	.2160			(R=1.000 F=19.10 A= 2.00 ALPHA=3.14)					
(R=1.000 F=13.33 A= 2.00 ALPHA=3.14)						135.0	0	135.3	.2000			5.5	0	5.5	.6500		
11.1	0	11.1	3.04			(R=1.000 F=10.40 A= 2.00 ALPHA=3.14)						11.0	0	11.1	.5500		
20.4	0	20.4	4.19			5.5	0	5.5	.9400			22.0	0	22.1	.3000		
27.8	0	27.9	4.96			11.0	0	11.1	.7000			45.0	0	45.2	.1500		
37.0	0	37.2	5.63			22.0	0	22.1	.4700			90.0	0	90.2	.0630		
44.8	0	45.0	6.00			45.0	0	45.2	.3100			135.0	0	135.3	.0300		
57.8	0	58.0	6.81			68.0	0	68.2	.1740			(R=1.000 F=13.60 A= 2.00 ALPHA=3.14)					
74.1	0	74.4	7.48			(R=1.000 F=13.60 A= 2.00 ALPHA=3.14)						5.5	0	5.5	.5100		
90.4	0	90.8	8.15			5.5	0	5.5	.9400								
101.5	0	101.9	8.52			11.0	0	11.1	.7000								
111.1	0	111.6	8.89			22.0	0	22.1	.4700								
123.0	0	123.5	9.33			45.0	0	45.2	.3100								
						68.0	0	68.2	.1740								

TABLE IV.B.2 (Contd.) (G3)

(CONTINUED)
AUTHOR(S)--- WEIL

X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D
(R=1.000 F=10.40 A= 2.00 ALPHA=3.14)						(R=1.000 F=13.60 A= 2.00 ALPHA=3.14)						(R=1.000 F=16.80 A= 2.00 ALPHA=3.14)					
135.0 0 135.3 .1300						11.0 0 11.1 .6300						135.0 0 135.3 .0830					
(R=1.000 F=16.10 A= 2.00 ALPHA=3.14)						22.0 0 22.1 .5000						(R=1.000 F=26.70 A= 2.00 ALPHA=3.14)					
5.5 0 5.5 .8900						45.0 0 45.2 .2200						5.5 0 5.5 .8900					
11.0 0 11.1 .7400						90.0 0 90.2 .0630						11.0 0 11.1 .6800					
22.0 0 22.1 .5500						135.0 0 135.3 .0300						22.0 0 22.1 .5300					
45.0 0 45.2 .3600						(R=1.000 F=15.30 A= 2.00 ALPHA=3.14)						45.0 0 45.2 .2900					
68.0 0 68.2 .4100						4.1 0 4.1 .9600						90.0 0 90.2 .2000					
135.0 0 135.3 .2450						8.1 0 8.1 .7500						135.0 0 135.3 .0800					
(R=1.000 F=13.90 A= 2.00 ALPHA=3.14)						16.5 0 16.6 .6300						200.0 0 200.3 .0430					
5.5 0 5.5 .8900						32.5 0 32.7 .4750						(R=1.000 F=19.20 A= 2.00 ALPHA=3.14)					
11.0 0 11.1 .6800						65.0 0 65.2 .2400						5.5 0 5.5 .8900					
22.0 0 22.1 .5200						98.0 0 98.3 .2000						11.0 0 11.1 .7200					
45.0 0 45.2 .2900						(R=1.000 F= 9.50 A= 2.00 ALPHA=3.14)						22.0 0 22.1 .5600					
68.0 0 68.2 .2080						4.1 0 4.1 .9600						45.0 0 45.2 .3600					
135.0 0 135.3 .1200						8.1 0 8.1 .8100						90.0 0 90.2 .1800					
(R=1.000 F=35.90 A= 2.00 ALPHA=3.14)						16.5 0 16.6 .7000						135.0 0 135.3 .1150					
5.5 0 5.5 .9000						32.5 0 32.7 .5400						(R=1.000 F=15.50 A= 2.00 ALPHA=3.14)					
11.0 0 11.1 .6400						65.0 0 65.2 .3400						5.5 0 5.5 .8400					
22.0 0 22.1 .4800						98.0 0 98.3 .2400						11.0 0 11.1 .6600					
45.0 0 45.2 .3400						(R=1.000 F=32.30 A= 2.00 ALPHA=3.14)						22.0 0 22.1 .5100					
68.0 0 68.2 .2650						5.7 0 5.7 .9400						45.0 0 45.2 .3300					
90.0 0 90.3 .2150						11.0 0 11.1 .7800						90.0 0 90.2 .1700					
135.0 0 135.3 .1500						22.0 0 22.1 .5100						135.0 0 135.3 .1000					
200.0 0 200.4 .1350						45.0 0 45.2 .2200						(R=1.000 F=13.50 A= 2.00 ALPHA=3.14)					
270.0 0 270.4 .0425						90.0 0 90.2 .1080						5.5 0 5.5 .8000					
(R=1.000 F=23.90 A= 2.00 ALPHA=3.14)						135.0 0 135.3 .0430						11.0 0 11.1 .7100					
5.5 0 5.5 .9500						(R=1.000 F=23.30 A= 2.00 ALPHA=3.14)						22.0 0 22.1 .5800					
11.0 0 11.1 .7000						5.7 0 5.7 .9700						45.0 0 45.2 .3700					
22.0 0 22.1 .5300						11.0 0 11.1 .7400						90.0 0 90.2 .1900					
45.0 0 45.2 .3200						22.0 0 22.1 .4800						135.0 0 135.3 .1000					
68.0 0 68.2 .2400						45.0 0 45.2 .2400						(R=1.000 F=12.70 A= 2.00 ALPHA=3.14)					
90.0 0 90.3 .2350						90.0 0 90.2 .1050						5.5 0 5.5 .8600					
101.0 0 101.3 .2050						135.0 0 135.3 .0570						11.0 0 11.1 .7500					
135.0 0 135.4 .1730						(R=1.000 F=19.60 A= 2.00 ALPHA=3.14)						22.0 0 22.1 .5800					
200.0 0 200.4 .1030						5.7 0 5.7 .9400						45.0 0 45.2 .4200					
(R=1.000 F=10.60 A= 2.00 ALPHA=3.14)						11.0 0 11.1 .7500						90.0 0 90.2 .2040					
5.5 0 5.5 .8900						22.0 0 22.1 .4600						135.0 0 135.3 .1650					
11.0 0 11.1 .7300						45.0 0 45.2 .2900						(R=1.000 F= 9.50 A= 2.00 ALPHA=3.14)					
22.0 0 22.1 .5300						90.0 0 90.2 .0830						5.5 0 5.5 .8400					
45.0 0 45.2 .3300						135.0 0 135.3 .0620						11.0 0 11.1 .7200					
68.0 0 68.2 .2000						(R=1.000 F=16.80 A= 2.00 ALPHA=3.14)						22.0 0 22.1 .6500					
90.0 0 90.3 .1700						5.7 0 5.7 .9000						45.0 0 45.2 .3800					
135.0 0 135.3 .1300						11.0 0 11.1 .7200						90.0 0 90.2 .1900					
200.0 0 200.4 .0750						22.0 0 22.1 .4700						135.0 0 135.3 .1200					
(R=1.000 F=16.80 A= 2.00 ALPHA=3.14)						45.0 0 45.2 .2300						(R=1.000 F= 7.80 A= 2.00 ALPHA=3.14)					
5.5 0 5.5 .8900						90.0 0 90.2 .1350						5.5 0 5.5 .9000					

(CONTINUED)
AUTHOR(S)--- WEILAUTHOR(S)--- MOTZ
DATE PUBLISHED--- 1971

X/D	Y/D	S/D	TEMP	W/D	H/D	X/D	Y/D	S/D	TEMP	W/D	H/D
(R=1.000 F= 7.80 A= 2.00 ALPHA=3.14)						(R= .500 F= 4.55 A=31.00 ALPHA=2.06)					
11.0 0 11.1 .8000						75.0 75.0 106.1 .8200 25.00 1.03					
22.0 0 22.1 .6100						300.0 137.5 339.6 .4900 52.75 .62					
45.0 0 45.2 .4500						937.5 137.3 577.1 .3800 50.50 1.24					
90.0 0 90.2 .2150						762.5 187.5 807.6 .3300 70.00 .91					
135.0 0 135.3 .1020						1000.0 270.0 1059.0 .3100 79.00					
(R=1.000 F= 6.70 A= 2.00 ALPHA=3.14)						1275.0 0 1444.4 .0990					
5.5 0 5.5 .8600						1500.0 0 1669.4 .1300					
11.0 0 11.1 .7900						(R= .665 F= 1.56 A=31.00 ALPHA=1.62)					
22.0 0 22.1 .4400						75.0 62.5 97.6 .8100					
45.0 0 45.2 .3400						300.0 95.0 125.0 .5600 41.50					
90.0 0 90.2 .1950						937.5 125.0 564.8 .4000 50.75					
NUMBER OF LINES = 256						762.5 127.5 789.4 .4200 62.00					
						1000.0 138.5 1027.1 .2500 64.50					
						1275.0 0 1335.0 .1500					
						1500.0 0 1560.0 0 0 0					
						(R= .753 F= .83 A=31.00 ALPHA=1.62)					
						75.0 43.3 98.8 .7400					
						300.0 60.6 312.3 .5000 36.92					
						937.5 147.1 565.0 .4700 57.12					
						762.5 210.6 798.8 .4100 73.55					
						1000.0 233.7 1037.4 .2800 83.08					
						1275.0 0 1398.3 .0700 115.67					
						1500.0 0 1623.3 .0800 119.71					
						(R= .575 F= .30 A=83.00 ALPHA=2.06)					
						16.8 33.6 37.6 1.0000					
						302.5 210.1 373.4 .9000					
						773.1 252.1 845.9 .8000					
						974.8 252.1 1047.6 .7250					
						NUMBER OF LINES = 25					

significant scatter-related factor is that not all important parameters were measured for each survey--for example, the level of jet and ambient turbulence. A third factor, experimental error induced by instrumentation or technique, was illustrated when Shirazi found that stagnant tests in the laboratory (or in the field) provide questionable data, due to heat buildup or transient effects. Shirazi did not discuss each set of data critically in terms of its acquisition or its consistency with other similar data, so any potential user must refer to the original work for his own assessment or treat all the data as equally valid. The listing of experimental data in Table IV.B.2 should be useful to future investigators developing plume models or studying the physics of heated surface discharges.

2. Chesapeake Bay Institute, The Johns Hopkins University: Hydraulic-model Studies of a Buoyant Surface Jet in a Crossflow

To aid in the understanding of the near-field mechanical mixing that occurs when a buoyant surface jet discharges into a flowing ambient, Carter et al.¹⁵ have carried out some hydraulic-model studies. They used a 40 x 10 x 1-ft flume, with a rectangular discharge opening, oriented at right angles with respect to a uniform flow along the length of the flume (see Fig. IV.B.1). For runs in which data are reported, the outfall temperature excess θ_0 was 10 F° and the ambient current u_a was 0.18 ft/sec. Temperature measurements within the plume were made with a moving thermistor that traversed the plume approximately perpendicular to the plume centerline at various distances from the outfall. Vertical temperature measurements were also made in selected cases at specific points along the plume centerline.

Fig. IV.B.1. Schematic Drawing of a Buoyant Surface Three-dimensional Bent-over Jet.
(Adapted from Carter et al.¹⁵)

The program of hydraulic-model tests carried out by Carter et al.¹⁵ is given in Table IV.B.3, with runs divided into two types. The first type is designated as two-dimensional, with the depth of the discharge opening h_0 and the bottom depth D_w about equal (approximately 2 in.). Various discharge flow rates Q and discharge widths b_0 (0.2-2.0 in.) were used to produce aspect ratios $A_r = h_0/b_0$ in the range of about 1-11 and discharge velocity ratios R (ratio of discharge velocity q_0 to ambient velocity u_a) in the range of 1.75-8.81. For the nine runs presented, including centerline trajectory and centerline temperature decay data, velocity ratios R fall into three groups with

TABLE IV.B.3. The Carter-Schiemer-Regier Test Program^a (from Carter et al.¹⁵)

Test Desig.	Q, gpm	θ_o , °F	D_w , ft	h_o , ft	b_o , ft	A_r	α_o , deg	q_o , ft sec ⁻¹	u_a , ft sec ⁻¹	R	Re	F
A	4	10	0.167	0.174	0.163	1.07	90	0.31	0.18	1.75	5,020	3.50
B	4	10	0.167	0.174	0.065	2.68	90	0.79	0.18	4.38	12,799	8.92
C	4	10	0.167	0.174	0.0325	5.35	90	1.58	0.18	8.78	25,517	17.78
D	2	10	0.167	0.174	0.0812	2.14	90	0.32	0.18	1.80	5,233	3.65
E	2	10	0.167	0.174	0.0325	5.35	90	0.79	0.18	4.37	12,718	8.86
F	2	10	0.167	0.174	0.0163	10.71	90	1.58	0.18	8.81	25,629	17.86
G	4	10	0.833	0.175	0.065	2.69	90	0.78	0.18	4.36	12,765	8.82
I	4	10	0.667	0.175	0.065	2.69	90	0.78	0.18	4.36	12,765	8.82
K	4	10	0.500	0.174	0.065	2.68	90	0.79	0.18	4.39	12,750	8.89
M	4	10	0.333	0.172	0.065	2.64	90	0.80	0.18	4.45	12,767	9.08
O	4	10	0.333	0.082	0.136	0.60	90	0.80	0.18	4.53	6,101	13.22
Q	4	10	0.333	0.250	0.045	5.56	90	0.79	0.18	4.47	18,436	7.46
S	4	10	0.667	0.169	0.065	2.60	90	0.81	0.18	4.52	12,765	9.29
T	4	10	0.667	0.167	0.065	2.57	90	0.82	0.18	4.57	12,766	9.46
V	4	10	0.833	0.167	0.065	2.57	90	0.82	0.18	4.57	12,766	9.46
W	4	10	0.833	0.167	0.065	2.57	90	0.82	0.18	4.57	12,766	9.46
Y	4	10	0.500	0.167	0.065	2.57	90	0.82	0.18	4.57	12,766	9.46
A-II	0.48	20	0.333	0.072	0.084	0.852	0	0.18	0.18	1.00	1,207	2.17
B-II	0.336	30	0.333	0.050	0.084	0.59	0	0.18	0.18	1.00	838	2.07
C-II	0.336	30	0.333	0.050	0.084	0.59	0	0.18	0.18	1.00	838	2.07
D-II	0.336	30	0.333	0.048	0.084	0.59	0	0.18	0.18	1.00	804	2.12
E-II	0.336	30	0.333	0.048	0.084	0.59	0	0.18	0.18	1.00	804	2.12
F-II	1.06	10	0.167	0.167	0.083	2.00	0	0.18	0.18	1.00	2,799	2.07
G-II	4	10	0.168	0.179	0.065	2.75	90	0.77	0.18	4.26	12,786	8.38
H-II	4	10	0.167	0.185	0.043	4.35	90	1.14	0.18	6.32	19,551	12.43
I-II	2	10	0.168	0.175	0.146	1.20	90	0.17	0.18	0.97	2,770	1.91
J-II	4	10	0.168	0.178	0.146	1.22	90	0.34	0.18	1.91	5,635	3.80

^aNotation:

- Q = volumetric flow rate of water from the outlet discharge
 θ_o = excess temperature (above ambient) at the outfall
 D_w = depth of water in the tank (ambient receiving-body depth)
 h_o = outfall depth
 b_o = outfall width
 A_r = aspect ratio of the outlet; i.e., $A_r \equiv h_o/b_o$
 α_o = initial angle of discharge jet and ambient current
 q_o = initial jet velocity
 u_a = velocity of ambient current
 R = ratio of initial jet velocity, q_o , to ambient velocity, u_a
 Re = Reynolds number of discharged water
 $Re \equiv q_o h_o / \nu$, where ν is the kinematic viscosity of water
 F = Froude number of discharged water
 $F \equiv q_o / \sqrt{\Delta \rho / \rho g h_o}$

average values of 1.8, 4.3, and 8.0. In addition, lateral temperature distributions at various distances from the outfall are presented for three cases corresponding to each of the three groups.

The second type of test run is designated as three-dimensional with the bottom of the flume lowered below the bottom of the discharge. A constant flow rate Q of 4 gal/min was maintained for these runs. The size and shape of the discharge were kept approximately constant (except for two cases) to produce a nominal aspect ratio of 2.6, a nominal velocity ratio of 4.5, and a nominal densimetric Froude number F of 9.2. The 11 runs of this type are divided into four groups corresponding to different ratios, nominally 5, 4, 3, and 2, of flume depth to outfall depth. In addition to lateral and vertical temperature distributions for three cases selected from these four groups, centerline trajectory and centerline temperature-decay data are also presented.

Carter et al.¹⁵ note that, due to interaction with the near-shore tank wall on the leeward side of the jet, a large eddy with a counter current near the wall was produced. They observed that, due to the subsequent recirculation, the velocity and temperature profiles are neither symmetrical about the centerline, nor similar along the jet centerline. The offshore portions of the profiles did, however, appear to be similar both horizontally and vertically. Indeed, it is the offshore lateral temperature profiles that are presented with the data discussed above. Carter et al.¹ also attempt to examine, in light of their data, some of the assumptions used in integral model analysis of surface jets.

As a continuation of their study of heated surface jets discharging into a crossflow, Carter and Regier¹⁶ present 27 additional hydraulic-model runs carried out in the flume previously discussed. (See Table IV.B.4 for the program of tests in this study.) Only runs of the three-dimensional type (receiving water several times as deep as the outfall depth) were carried out, and more detailed temperature measurements were made at several depths, as opposed to the one preselected depth as in the earlier measurements. The runs were designed to study vertical growth, lateral spread, centerline trajectory, and centerline dilution (temperature decay) as a function of densimetric Froude number F , velocity ratio $R = q_0/u_0$, and aspect ratio $A_r = h_0/b_0$. Asymmetry of the lateral temperature profiles was not as extreme as in the previous runs; hence, both sides of the distribution could be used to determine the width. No attempt was made to study the influence of bottom slope, and all measurements are assumed to be in the near field, where surface heat loss should be negligible. All runs (except two) have a Reynolds number Re greater than 3000 ($q_0 h_0/\nu$; ν is the kinematic-viscosity coefficient), so that the jet can be considered to be fully turbulent and the results independent of Re .

The main objective of the Carter-Regier report was to approach the problem of jet-ambient interaction empirically and thus to obtain from the experimental data the dependence of the jet characteristics on F , R , and A_r .

TABLE IV.B.4. The Carter-Regier Test Program^a (from Carter et al.¹⁶)

Test Desig.	Q, gpm	θ_o , °F	D _w , ft	h _o , ft	b _o , ft	A _r	α_o , deg	q _o , ft sec ⁻¹	u _a , ft sec ⁻¹	R	F	R _e
<u>Series V</u>												
A-V	4	10	0.50	0.173	0.066	2.63	90	0.787	0.184	4.28	9.35	12,665
B-V	4	10	0.67	0.164	0.065	2.53	90	0.835	0.184	4.54	11.34	12,739
C-V	4	10	0.83	0.166	0.065	2.55	90	0.827	0.184	4.49	10.71	12,770
D-V	4	10	0.33	0.164	0.064	2.55	90	0.849	0.184	4.61	11.13	12,952
E-V	4	10	0.17	0.166	0.065	2.55	90	0.827	0.184	4.49	11.21	12,770
F-V	4	10	0.17	0.168	0.163	1.03	90	0.325	0.184	1.77	4.13	5,079
G-V	4	10	0.17	0.169	0.033	5.21	90	1.621	0.184	8.81	20.97	25,484
H-V	4	10	0.50	0.169	0.033	5.15	90	1.614	0.184	8.77	20.80	25,374
I-V	4	10	0.50	0.169	0.163	1.04	90	0.325	0.184	1.77	3.97	5,109
J-V	4	10	0.83	0.166	0.163	1.02	90	0.329	0.184	1.79	4.16	5,080
K-V	4	10	0.83	0.166	0.033	5.10	90	1.653	0.184	8.99	20.00	25,525
<u>Series VI</u>												
A-VI(1)	2	10	0.83	0.083	0.167	0.50	90	0.321	0.199	1.61	5.02	2,478
A-VI(2)	2	10	0.83	0.081	0.167	0.48	90	0.330	0.190	1.74	6.17	2,487
A-VI(3)	4	10	0.83	0.081	0.167	0.48	90	0.659	0.190	3.47	12.32	4,965
B-VI(1)	2	10	0.83	0.085	0.054	1.57	90	0.968	0.199	4.86	14.74	7,654
B-VI(2)	2	10	0.83	0.085	0.055	1.55	90	0.953	0.190	5.02	14.63	7,535
C-VI	2	10	0.83	0.087	0.033	2.62	90	1.554	0.199	7.81	25.24	12,577
<u>Series X</u>												
A-X	2	10	0.83	0.084	0.053	1.60	90	1.008	0.193	5.22	17.28	7,875
B-X	1	10	0.83	0.065	0.040	1.63	90	0.857	0.193	4.44	17.25	5,181
C-X	4	10	0.83	0.111	0.068	1.60	90	1.162	0.193	6.02	18.06	11,980
D-X	4	10	0.83	0.126	0.126	1.00	90	0.563	0.193	2.92	8.00	6,588
E-X	4	10	0.83	0.097	0.096	1.01	90	0.962	0.193	4.98	15.51	8,651
F-X	4	10	0.83	0.219	0.044	4.95	90	0.922	0.193	4.78	9.94	18,762
G-X	4	10	0.83	0.116	0.113	1.03	90	0.681	0.193	3.53	9.66	7,369
H-X	4	10	0.83	0.088	0.171	0.51	90	0.596	0.193	3.09	9.90	4,853
J-X	4	10	0.83	0.116	0.113	1.03	90	0.684	0.093	7.35	10.31	8,515
K-X	4	10	0.83	0.115	0.113	1.02	90	0.689	0.366	1.88	10.34	7,371

^aThe same notation is used here as in Table IV.B.3.

To this end, the experimental runs cover aspect ratios in the range of 0.48-5.21, densimetric Froude numbers in the range of 3.97-25.24, and velocity ratios in the range of 1.61-8.99. Only selected parts of the data are presented in their report, and in most cases these have been scaled by powers and products of F , R , and A_T to extract and verify the desired functional dependences. However, enough information is given to extract details for a few individual runs.

3. Argonne National Laboratory: Data Collection and Analysis of Three Years' Data at Point Beach and Palisades Power Plants

A group at Argonne National Laboratory has carried out an extensive program of thermal-plume mapping and analysis^{5,17-19} at several sites on Lake Michigan. Over the three-year period 1971-1973, 22 plumes at Point Beach Nuclear Power Station Unit 1, 12 plumes at the Palisades Nuclear Generating Plant, 2 plumes at the Waukegan Generating Station, 1 at the State Line Generating Station, and 6 double plumes at Point Beach when both Units 1 and 2 were operating, were measured and analyzed. The goal of the program was to increase our understanding of surface thermal-plume behavior by analyzing large amounts of data taken under a wide range of environmental conditions and to provide information on plume characteristics that might be used to verify mathematical models for thermal-plume prediction.

Three-dimensional temperature data were obtained from a moving boat, equipped with a 3-m long submerged boom with thermistors at every 0.5 m, traveling a serpentine path from the outfall to the far reaches of the plume. The position of the boat at any instant was determined with a Motorola Mini-Ranger positioning system consisting of two shore-based transponders and a receiver/transmitter unit and range console on board the boat. Although ambient lake current, wind speed and direction, air temperature, humidity, and lake surface conditions were usually measured in conjunction with the plume mapping, occasionally some of this information is missing.

Isotherm maps were drawn from plots of measurements made by the boat and these are presented in Refs. 5, 17, and 18. Sample isotherm plots from each site are shown in Figs. IV.B.2-4, and Table IV.B.5 lists the outfall information for each of the plants for which data were acquired and analyzed. All these maps were then analyzed to extract plume characteristics: centerline trajectory, centerline temperature decay, temperature half-widths, surface isotherm areas, and temperature half-depths along the centerline. The results are reported in graphical form in Ref. 19.

Tables IV.B.6-8 summarize the conditions for which data were acquired at the four sites: Point Beach, Palisades, Waukegan, and State Line. The large amounts of data taken at Point Beach and Palisades were each divided into five major categories, depending primarily upon the ambient lake current and secondarily upon wind and lake-surface conditions. The scatter

Fig. IV.B.2. Sample Isotherm Plot for Waukegan Plumes

TABLE IV.B.5. Outfall Information for Power Plants Studied

Plant	Point Beach ^{a,b} (Each unit identical unless otherwise noted)	Palisades ^a	Waukegan	State Line
Capacity (MWe)	497	715	1100	992
Maximum Discharge Flow Rate (m ³ /s)	25.1	25.5	55.2	52.5
Maximum Temperature Drop across Condensers (deg C)	10.7	13.9	7.1	5.8
Outfall Width (m)	10.7	28.3	30.5	92.8
Outfall Depth (m)	4.2	2.1	1.5	3.0 ^c
Average Outfall Velocity (m/s)	0.56	0.43	1.2	0.19
Initial Densimetric Froude Number (Nominal)	2.5	2.3	9.7	1.0
Aspect Ratio ^d	2.5	13.5	20.1	30.9
Angle to Shoreline	60° (Unit 1) 120° (Unit 2)	90°	90°	90°
Number of Plumes Measured				
Unit 1 only	23	15	2	1
Unit 2 only	2	-	-	-
Unit 1 and 2	7	-	-	-

^aNuclear Power Plant.^bThere are four cases listed under Unit 1 only in which Unit 2 is partially operational. For these cases typical parameters for Unit 2 are: power load = 69 MWe, discharged flow rate = 13.9 m³/s, maximum temperature drop across condensers = 4.2 deg C, average outfall velocity = 0.31 m/s, initial densimetric Froude Number = 1.98. Those values are also typical for the two plumes for which only Unit 2 was in operation.^cApproximate value due to irregular variation of discharge depth.^dAspect Ratio = $\frac{\text{outfall width}}{\text{outfall depth}}$

Fig. IV.B.3. Sample Isotherm Plot for Palisades Plumes (left) and State Line Plume (right)

Fig. IV.B.4. Sample Isotherm Plot for Point Beach Unit 1 Plumes (left) and Point Beach Units 1 and 2 Plumes (right)

TABLE IV.B.6. Thermal Plumes Measured at Palisades during 1972

Plume	Date	Time	Outfall Temp. (°C)	Ambient Temp. (°C)	Excess Temp. (degC)	Disch. Flow (m³/s)	Disch. Vel. (cm/s)	Magn. of Current (cm/s)	Dir. of Current (°cw from north)	Magn. of Wind (m/s)	Dir. of Wind (°cw from north)	Lake Conditions	Wave Height (m)	R	F _O	$\frac{K}{\rho c_p}$ (m/s x 10 ⁻⁶)
CATEGORY I (a): Moderate Shore-parallel Current; Light, Moderate Wind; Calm Lake Conditions																
A	13 Jun 72	1242-1443	21.6	14.5	7.1	25.6	43.1	13.3	012	2.7	265	Calm	0.0-0.3	0.31	2.62	4.1
B	13 Jun 72	1516-1615	21.8	15.8	6.0	25.6	43.1	22.7	000	2.6	265	Calm	0.0	0.53	2.79	4.0
D	20 Jun 72	1027-1109	20.6	14.5	6.1	25.6	43.1	15.0	025	3.6	220	Calm	0.0-0.3	0.35	2.87	5.0
E	20 Jun 72	1110-1200	21.2	14.9	6.3	25.6	43.1	27.7	030	3.2	220	Calm	0.0-0.3	0.64	2.78	4.6
F	20 Jun 72	1330-1425	21.7	15.8	5.9	25.6	43.1	27.1	025	3.0	220	Calm	0.0-0.3	0.65	2.81	4.4
G	20 Jun 72	1430-1515	21.4	16.7	4.7	25.6	43.1	27.1	025	2.7	220	Calm	0.0-0.3	0.65	3.12	4.4
I (b): Moderate Shore-parallel Current; Moderate Wind; Rough Lake Conditions																
H	17 Jul 72	1600-1655	29.3	22.2	7.1	25.6	43.1	19.4	030	4.5	220	Swells	0.3-1.0	0.45	2.19	7.6
CATEGORY II: Large Shore-parallel Current; Moderate Wind; Rough Lake Conditions																
X	7 Aug 72	1730-1900	19.4	11.3	8.1	25.6	43.1	90	205	4.4	340	Rough	1-2	1.9	2.69	5.2
Y	8 Aug 72	1415-1745	21.5	12.5	9.0	25.6	43.1	40-70	025	4.6	231	Rough	1	1.3	2.41	5.6
Z	11 Oct 72	1410-1630	22.0	13.5	8.5	25.6	43.1	60-80	025	6.0	213	Rough	1	1.6	2.42	7.2
CATEGORY III: Zero Current; Zero Wind; Calm Lake Conditions																
I	18 Jul 72	1952-2055	29.3	22.0	7.3	25.6	43.1	0.0	—	0.0	—	Calm	0.0	0.0	2.17	1.4
CATEGORY IV: Low-Moderate Onshore Current; Light Wind; Calm Lake Conditions																
J	19 Jul 72	1546-1710	28.9	23.5	5.4	25.6	43.1	10.0	170	1.8	010	Calm	0.0	0.23	2.49	4.0
CATEGORY V: Low, Moderate Offshore Current; Low, Moderate Wind; Calm, Rough Lake Conditions																
C	19 Jun 72	1630-1730	19.6	14.9	4.7	25.6	43.1	14.4	010	2.0	165	Calm	0.0	0.33	3.30	3.4
K	10 Oct 72	1625-1748	22.2	12.9	9.3	25.6	43.1	8.9	345	2.3	150	Calm	0.0-0.3	0.21	2.33	4.3
L	13 Oct 72	1015-1155	22.0	12.6	9.4	25.6	43.1	11.1	010	3.4	205	Rough	0.3-1.0	0.26	2.33	4.5

Notes:

R = Ratio of Ambient Velocity u_a to the Discharge Velocity u_o .F_O = Initial Densimetric Froude Number. $\frac{K}{\rho c_p}$ = Surface-heat-transfer Parameter.

TABLE IV.B.7. Thermal Plumes Measured at Point Beach Unit 1 during 1971 and 1972

Plume	Date	Time	Outfall Temp. (°C)	Ambient Temp. (°C)	Excess Temp. (degC)	Disch. Flow (m³/s)	Disch. Vel. (cm/s)	Magn. of Current (cm/s)	Dir. of Current (°cw from north)	Magn. of Wind (m/s)	Dir. of Wind (°cw from north)	Lake Conditions	Wave Height (m)	R	F _o	$\frac{K}{\rho c_p}$ (m/s x 10 ⁻⁶)
CATEGORY I: Strong North Current; Moderate Wind																
T	21 Jul 71	1508-1629	21.8	11.1	10.7	25.1	57.	18.	335	6.7	205	Moderately Calm	0.3-0.6	0.32	2.13	7.2
W	1 Sep 71	1445-1727	26.5	17.4	9.1	25.1	57.	—	355	4.9	135	Moderately Calm	0.3-0.6	—	1.98	7.0
A	17 May 72	1646-1806	18.9	7.9	11.0	25.1	56.	8.3	000	4.5	190	Calm	0.0-0.3	0.15	2.31	4.8
CATEGORY II: Moderate North Current; Moderate Wind																
R	20 Jul 71	1632-1805	21.5	12.6	8.9	25.1	57.	9.0	015	6.5	205	—	0.0-0.3	0.16	2.29	7.4
U	21 Aug 71	1530-1631	26.2	17.6	8.6	25.1	57.	—	350	2.7	045	—	0.6-0.9	—	2.04	4.4
V	1 Sep 71	0950-1130	27.1	17.4	9.7	25.1	57.	—	360	4.9	135	Moderately Calm	0.3-0.6	—	1.91	7.0
G	11 Jul 72	1245-1355	20.5	10.9	9.6	25.1	56.	15.0	300	4.3	175	Choppy	0.3-0.6	0.27	2.24	5.1
I	6 Sep 72	1115-1300	25.2	15.0	10.2	24.7	55.	18.8	020	5.2	205	Choppy	0.6-1.0	0.34	1.87	6.7
CATEGORY III (a): Zero Current; Light, Moderate Wind																
S	21 Jul 71	1020-1117	20.5	11.5	9.0	25.1	57.	—	—	3.4	205	Moderately Calm	0.3-0.6	—	2.36	4.3
C	24 May 72	0925-1040	21.3	10.7	10.6	25.1	56.	0.0	—	1.4	015	Calm	0.0	0.0	2.11	2.5
D	24 May 72	1045-1145	21.3	11.9	9.4	25.1	56.	0.0	—	2.2	015	Calm	0.0	0.0	2.19	3.3
E	24 May 72	1145-1225	21.3	11.0	10.3	25.1	56.	0.0	—	2.0	015	Calm	0.0	0.0	2.13	3.0
F	7 Jul 72	1830-2015	20.5	11.3	9.2	25.1	56.	—	—	—	—	—	—	—	2.27	1.2
J	9 Sep 72	1525-1630	24.5	14.5	10.0	24.7	55.	0.0	—	4.8	175	Choppy	0.0-0.3	0.0	1.92	6.2
III (b): Small Current; Light Wind																
H	13 Jul 72	1805-1903	20.3	13.3	7.0	25.1	56.	2.2	245	2.4	165	Calm	0.0	0.039	2.52	3.6
L	29 Sep 72	1545-1705	20.4	11.2	9.2	24.7	55.	2.2	300	2.6	300	Calm	0.3-0.6	0.040	2.24	3.6
III (c): Zero Current; Zero Wind																
B	18 May 72	1645-1751	18.0	9.5	8.5	24.7	55.	0.0	—	—	—	Calm	0.0	0.0	2.53	1.2
CATEGORY IV: Moderate South Current; Light, Moderate Wind																
O	20 May 71	1438-1550	17.5	8.1	9.4	25.1	57.	9.0	125	6.7	315	—	0.0-0.3	0.16	2.64	6.6
P	20 May 71	1740-1850	17.5	8.2	9.3	25.1	57.	9.0	125	5.6	270	—	0.0-0.3	0.16	2.65	5.8
X	28 Oct 71	1402-1539	21.2	12.8	8.4	25.1	57.	—	170	2.0	155	Slightly Choppy	0.0-0.3	—	2.36	3.2
CATEGORY V: Strong South Current; Moderate Wind																
Q	25 Jun 71	1000-1215	20.1	11.0	9.1	25.1	57.	—	—	6.5	045	Rough	0.6-0.9	—	2.38	7.0
Y	28 Oct 71	2000-2100	21.2	12.6	8.6	25.1	57.	15.0	180	3.1	050	Calm	0.0-0.3	0.26	2.34	3.2

Notes:

R = Ratio of Ambient Velocity u_a to the Discharge Velocity u_o .F_o = Initial Densimetric Froude Number. $\frac{K}{\rho c_p}$ = Surface-heat-transfer Parameter.

TABLE IV.B.8. Thermal Plumes Measured at Point Beach Units 1 and 2 during 1973 and Waukegan and State Line during 1971

Plume	Date	Time	Outfall Temp. (°C)	Ambient Temp. (°C)	Excess Temp. (degC)	Disch. Flow (m ³ /s)	Disch. Vel. (cm/s)	Magr. of Current (cr./s)	Dir. of Current (°cw from north)	Magn. of Wind (m/s)	Dir. of Wind (°cw from north)	Lake Conditions	Wave Height (m)	R	F _o	$\frac{K}{\rho C_p}$ (m/s x 10 ⁻⁶)
POINT BEACH --- DOUBLE PLUMES																
	5 Jun 73	1150-1400		7.4				5.6	353	5.6	232	Calm	0.3			5.7
A			17.3		9.9	24.3	54.							0.10	2.47	
B			17.3		9.9	24.3	60.							0.09	2.89	
	6 Jun 73	1120-1255		9.6				0.0	—	3.8	202	Calm	0.0-0.3			4.4
C			17.7		8.1	24.3	54.							0.0	2.56	
D			17.5		7.9	24.3	60.							0.0	3.04	
	7 Jun 73	1102-1230		7.7				6.1	115	7.0	265	Slightly Choppy	0.3			6.8
E			17.4		9.7	24.3	54.							0.11	2.47	
F			17.1		9.4	24.3	60.							0.10	2.95	
	10 Aug 73	1416-1500		7.5				—	—	3.8	273	Slightly Choppy	0.3			4.2
I			17.4		9.9	24.3	54.							—	2.46	
J			17.8		10.3	24.3	60.							—	2.79	
	23 Aug 73	1156-1304		17.6				—	—	2.1	250	Calm	0.0-0.3			3.8
K			28.5		10.9	24.3	54.							—	1.66	
L			28.7		11.1	24.3	60.							—	1.91	
	12 Jun 73	1515-1647		10.1				5.4	150	2.4	028	Calm	0.0-0.3			3.3
G			18.5		8.4	24.3	54.							0.17	2.44	
H			18.2		8.1	24.3	60.							0.16	2.92	
WAUKEGAN																
A	2 Jun 71	1140-1320	18.8	10.2	8.6	45.9	99.	—	—	4.7	270	—	0.0-0.3	—	7.29	5.3
B	2 Jun 71	1443-1723	18.3	10.2	8.1	44.4	96.	—	—	3.8	270	—	0.0-0.3	—	7.36	4.5
STATE LINE																
C	4 Aug 71	1227-1550	25.3	17.5	7.8	52.4	19.	—	—	4.2	045	—	0.0-1.0	—	~ 1.	6.2

Notes:

R = Ratio of Ambient Velocity u_a to Discharge Velocity u_o .F_o = Initial Densimetric Froude Number. $\frac{K}{\rho C_p}$ = Surface-heat-transfer Parameter.

of plume characteristics within many of the categories apparently provides evidence that the details of plume dispersion depend upon slight variations in environmental factors not easily identified, measured, or least of all, modeled. Then, the data within each category were averaged, and the results were compared among different categories and sites. Under similar environmental conditions, the Waukegan site demonstrated the most rapid plume dilution in terms of centerline temperature decay and isotherm areas due to its large outfall densimetric Froude number (≈ 9.7). Conversely, the State Line site with its low initial densimetric Froude number (≈ 1) showed the least rapid plume dispersion. The Point Beach Unit 1 and Palisades discharges, on the other hand, although having nearly equal heat-rejection rates and Froude numbers (≈ 2.4), had plume characteristics that varied significantly, due mainly to the large difference in outfall aspect ratio and secondarily to the magnitude of site-dependent ambient currents. This much larger aspect ratio accounted for the Palisades plumes having a generally slower centerline-temperature decay and larger areas than Point Beach.

The data at Point Beach are compared to integral and phenomenological model predictions in Sec. F of this chapter.

4. Virginia Institute of Marine Sciences: Data Collection and Analysis of Thermal Plumes at the Surry Plant on the James River

A group at the Virginia Institute of Marine Science headed by C. S. Fang has designed and carried out an extensive thermal-monitoring program for a tidal estuarine region of the James River in Virginia.²⁰ The area studied is in the vicinity of Hog Point near the Surry Nuclear Power Plant. This plant is made up of two 822-MWe units that employ once-through cooling, taking water from the downstream side of Hog Point and discharging it through a surface outfall on the upstream side of the point. Clearly, salinity structure and tidal oscillations, as well as ambient turbulence, buoyant forces, and surface heat loss, will affect the dispersion of the heated effluent. The first unit went into operation late in December 1972 and the second unit in March 1973. Two years (1971 and 1972) of preoperational data were obtained to establish an existing environmental baseline with which to compare data obtained during plant operation.^{21,22} To date, one year (1973) of operational data has been published.²³ A report presenting the 1974 plume measurements is in preparation.

Most of the data consists of measurements taken from a moving boat. The boat traversed preset transects such as shown in Fig. IV.B.5. During the preoperational runs, only the path on the upstream side of Hog Point was traveled and the small zigzags in front of the outfall were not included. When the plant was in operation, the small zigzags were added to obtain more detail, and on three occasions, the path on the downstream side of the point was also traversed to determine the extent of plant influence. Runs were taken approximately twice a week through the two preoperational years. During 1973, when the plant was in operation, 49 runs were taken between March 27 and October 18. It took about 1 hr 40 min for the boat to travel the course, and

Fig. IV.B.5. Boat Transects and Area of James River under Study. (Adapted from C. S. Fang et al.²⁴)

typical summer plumes; these measurements were made at low-water slack at the 0.5-ft level on August 7, 1973. Water temperature decreased very rapidly within 1000-1500 yards of the outfall. In many of these cases, determination of complete, closed isotherms was not possible because of the coarseness of the grid laid out by the fixed preset transects. Monthly averages of the water temperatures for each transect are also presented for each of the three years. The 1973 data indicate higher average water temperatures and salinities than either 1971 or 1972. The slight thermal stratification of about 1 F° in the top 6 ft found in 1971 and 1972 was absent in 1973. The higher salinity resulting from the transfer of the more saline downstream water to the upstream discharge of the plant tended, in some cases, to cause the warm effluent to sink and to remain at the bottom due to its higher density.

Ambient water temperature for 1973 ranged from 1.2 to 5.1 F° higher than in 1971 and 1972, which could have been due to natural heating since air temperatures were also higher. Or, the locations at which ambient measurements were made may not have been out of the range of influence of the plant, which clearly would affect the ambient temperatures reported.

In addition to the data taken by the moving boat, meteorological and water data were also collected during the monitoring program by an instrumented tower (Tower 6, Fig. IV.B.5), located in the river about 1000 yards north of the discharge. The quantities measured include: water temperature at three depths (0.5, 3, and 6 ft), air temperature at two heights (6 and 26 ft), relative humidity at two heights (6 and 26 ft), net radiation through the water surface, wind speed at three heights (6, 12, and 24 ft), wind direction, and water pressure at 8 ft. The resulting tower data were not presented in the reports reviewed here.

runs were made at different tidal stages: low slack, flood, high slack, and ebb. Measurements from the boat included water temperature at three depths (0.5, 3, and 6 ft), air temperature at two heights (3 and 6 ft), the dew-point temperature, and, during the preoperational runs, the salinity. During 1973, salinity data were taken manually only at selected stations due to equipment malfunction. The data from these boat runs are tabulated in Refs. 21-23. For the 1973 runs made during plant operation, isotherm plots of the 0.5-ft water-temperature data are also presented.²³

Figure IV.B.6 is an example of an isotherm plot for one of the

Fig. IV.B.6. Temperature Distribution on August 7, 1973; Low Slack Water. (Adapted from Parker et al.²³)

To our knowledge the plume data taken at Surry exist in tabulated form and horizontal isotherms. No further reduction has been done into jet characteristics: centerline, widths, or areas. Areas have been tabulated for the 1974 data, but these results have not yet been published.

Before the construction of the Surry Plant, Carpenter and Pritchard²⁵ made predictions of the expected excess-temperature distributions using a hydraulic model of the James River estuary. One purpose of the monitoring program of Fang et al.²⁴ was to compare these predictions to actual measurements in order to check the hydraulic-modeling results. Comparisons made are presented in Refs. 23 and 27. The actual plumes were found to be 41-93% smaller than the model predicted and the predictions for the lower excess temperatures, more accurate than for the higher temperatures. The data indicated that the heat dissipates more rapidly in the near-field region than the hydraulic model predicts. Parker²⁶ points out that this may be due to the distorted nature of the model, which cannot properly simulate the entrainment of ambient water in the near-field region.

5. Kuhlman-Prahl Laboratory Study of Surface Thermal Plumes at Low Densimetric Froude Numbers²⁷

Kuhlman and Prahl²⁷ used a 6 by 20-ft tank to carry out 23 laboratory tests as a parametric study of low-densimetric-Froude number, heated surface discharges. Four additional laboratory tests were made in an attempt to simulate the entrance of the Cuyahoga River into Lake Erie, a very low initial-densimetric-Froude-number case.

The runs made and parameters varied are listed in Table IV.B.9. The laboratory flume used for the parametric studies is adequately represented by the schematic of Carter's flume given in Fig. IV.B.6.

Figure IV.B.7 is a schematic of the Kuhlman-Prahl model of the Cuyahoga River entrance into the Cleveland Harbor and Lake Erie. The vertical dimensions of the model were stretched by a factor of four to prevent tank-water depths from becoming too small (0.1-0.2 in.) for measurement purposes, and the depth of the river outlet was taken as equal to that of the receiving basin. An additional parameter varied in this study was the width of the receiving body. In all test cases, the heated discharge was directed at 90° to the shoreline, into a shore-parallel current. In fact, all tests were made with a flat bottom in the plume equal to the discharge depth.

Kuhlman and Prahl presented measurements of temperature, velocity, dye, and turbulence. Isotherm maps were constructed at various vertical levels from both thermistor and scanning-infrared-camera data; the two methods gave consistent measurements. Velocities were measured using a dual-sensor, compensated, hot-film anemometer; one sensor is here used to measure temperature fluctuations in the flow. These fluctuations are then

TABLE IV.B.9. Listing of Tank Tests Carried Out by Kuhlman and Prahl
(from Kuhlman and Prahl²⁷)

$\bar{Q}_j$, gpm	$\bar{Q}_c$, gpm	ΔT , °F	$\bar{h}$, in.	$\bar{b}_0$, in.	$\bar{W}$, in.	Fr_0	R	Re	α	Fr	W
1.380	30.4	21.5	1.96	1.96	71.5	0.825	0.60	2500	1.0	0.05	36.5
1.40	20.6	22.0	1.96	1.96	71.5	0.825	0.40	2500	1.0	0.05	36.5
1.40	10.3	22.0	1.96	1.96	71.5	0.834	0.20	2500	1.0	0.05	36.5
1.54	34.0	11.1	1.96	1.96	71.5	1.31	0.60	2500	1.0	0.05	36.5
1.80	12.3	5.0	2.10	2.10	71.5	1.97	0.20	2500	1.0	0.055	34.0
1.80	37.0	5.0	2.10	2.10	71.5	1.97	0.60	2500	1.0	0.055	34.0
1.80	24.7	5.0	2.10	2.10	71.5	1.97	0.40	2500	1.0	0.055	34.0
1.38	5.81	21.6	1.96	1.96	13.75	0.825	0.60	2500	1.0	0.05	7.0
1.38	12.52	21.7	1.96	1.96	29.4	0.825	0.60	2500	1.0	0.05	15.0
1.38	20.9	21.5	1.96	1.96	49.0	0.825	0.60	2500	1.0	0.05	25.0
0.28	5.01	21.7	1.0	1.0	18.2	0.91	1.0	1000	1.0	0.055	18.2
0.28	10.03	22.2	1.0	1.0	35.75	0.89	1.0	1000	1.0	0.055	36.0
0.28	15.04	22.5	1.0	1.0	53.6	0.89	1.0	1000	1.0	0.055	54.0
0.28	17.7	22.5	1.0	1.0	63.0	0.89	1.0	1000	1.0	0.055	63.0
0.126	2.26	21.0	1.0	0.445	8.0	0.93	1.0	1000	0.45	0.055	18.1
0.561	10.03	22.2	1.0	1.96	35.75	0.89	1.0	1000	1.96	0.055	18.2
0.561	6.02	22.2	1.0	1.96	35.75	0.89	0.6	1000	1.96	0.055	18.2
0.842	15.04	23.0	1.0	3.0	53.6	0.88	1.0	1000	3.0	0.055	18.0
0.842	9.03	23.0	1.0	3.0	53.6	0.88	0.6	1000	3.0	0.055	18.0
1.07	20.06	22.3	1.0	3.75	71.5	0.90	1.0	1000	3.75	0.055	18.7
1.07	12.03	22.3	1.0	3.75	71.5	0.90	0.6	1000	3.75	0.055	18.7
0.276	20.15	20.7	1.96	1.96	71.5	0.17	2.0	500	1.0	0.01	36.5
0.276	10.07	20.7	1.96	1.96	71.5	0.17	1.0	500	1.0	0.01	36.5
<u>Cuyahoga River Model</u>											
0.4	17.7	12.8	0.61	1.36	54.4	2.71	1.07	917	2.23	0.126	41.5
0.2	8.85	26.9	0.61	1.36	54.4	0.94	1.07	460	2.23	0.063	41.5
0.2	5.01	26.9	0.61	1.36	54.4	0.94	0.6	460	2.23	0.063	41.5
0.2	8.85	26.0	0.61	1.36	60.0	0.94	0.97	460	2.23	0.063	45.8

no harbor breakwall

List of Symbols

- $\bar{b}_0$ = plume width at point of discharge
 Fr = Froude number = $\bar{U}_j / \sqrt{g\bar{h}}$
 Fr_0 = initial densimetric Froude number = $\bar{U}_j / \sqrt{(\Delta\rho/\rho)g\bar{h}}$
 $\bar{g}$ = acceleration of gravity
 $\bar{h}$ = channel depth
 $\bar{Q}_c$ = crossflow volume flow rate
 $\bar{Q}_j$ = initial discharge volumetric flow rate
 R = crossflow-to-plume velocity ratio
 Re = plume Reynolds number
 W = nondimensional crossflow channel width ($=\bar{W}/\bar{b}_0$)
 $\bar{W}$ = crossflow channel width
 α = plume aspect ratio
 ΔT = temperature difference (above ambient)
 $\Delta\rho$ = local density difference between ambient and plume
 $\bar{\rho}$ = ambient density

used to electronically compensate anemometer readings for the nonisothermal nature of the flow. Additional velocity measurements were made, some at the surface, as near to the air-water interface as was possible with the hot-film probe, and some at various depths below the surface. Also measured were turbulence-intensity profiles and turbulent-energy spectra for various points in the plume. To help in understanding plume dispersion, dye was injected into the plume and the resulting pattern was photographed.

Fig. IV.B.7. Schematic of Kuhlman-Prahl Tank Model of Cuyahoga River Entering the Cleveland Harbor Area and Lake Erie. (Adapted from Kuhlman and Prahl.²⁷)

Kuhlman and Prahl observed a Kelvin-Helmholtz instability at the plume-ambient interface for initial densimetric Froude numbers less than 0.52. At these densimetric Froude numbers, a cold-water wedge was seen entering the discharge channel. The tank studies showed that a series of transverse line vortices grew rapidly to extend over the entire plume depth, and definite bands of alternately warm and cold fluid were noticeable at the surface. Apparently, cooler crossflow fluid was brought to the surface by the rotation and mixing of the vortices. The Kuhlman-Prahl tests show that, for initial densimetric Froude numbers larger than 0.52, a regular array of vortices exists near the outfall, but breaks down into a turbulence farther out from the discharge. As the initial densimetric Froude number is increased, the point of breakdown into a randomized flow moves nearer the discharge mouth. Kuhlman and Prahl suggest that the presence of these vortices significantly affects the mixing process and dispersion of the plume.

The Kelvin-Helmholtz phenomenon was not seen in the laboratory tests made for the Cuyahoga River case ($Fr_0 > 0.52$ in the model), although aerial infrared measurements of the prototype showed the vortex phenomenon ($Fr_0 \approx 0.17$ in the prototype). However, the other characteristics of the plume measured by aerial infrared photography generally matched those of the laboratory model.

Kuhlman and Prahl have modified the Motz-Benedict integral model⁷ to treat low-densimetric-Froude-number discharges. This quasi-three-dimensional model assumes lateral similarity profiles for temperature

and velocity, and profiles are assumed to be constant with depth. Plume-depth variation is included semi-empirically, based upon the tank results. The Kuhlman-Prahl tank results and field data taken at the Point Beach Power Plant⁵ were compared with the model.

6. Koester Experimental Study of Surface and Submerged Single-port Thermal Discharges²⁸

To determine the effect of the free surface and a shallow bottom on jet characteristics, Koester²⁸ carried out a parametric study of a round jet discharge located at the surface and at various submergences. This study was done to assist in the design of the once-through cooling system for the Atlantic Off-shore Nuclear Generating Station planned for the coast of New Jersey.

Fig. IV.B.8. Schematic of Heated Discharge Modeled in Tank Tests by Koester. (Adapted from Koester,²⁸)

A schematic of the experimental setup is shown in Fig. IV.B.8, and tests performed are listed in Table IV.B.10. As indicated in the table, the initial densimetric Froude number F_0 was varied from 4.9 to 15.7, the relative depth of water H/D_0 was varied from 1.9 to 6.3, the nondimensional crossflow velocity v/u_0 ranged from 0.020 to 0.074, and the relative submergence z/H was varied from 0 (outfall at the surface) to 1 (outfall at bottom).

The experiments were performed in a 60 x 40 x 1½-ft basin containing a fixed array of 98 thermistors. An additional thermistor was used to monitor the hot water in the head tank; another was placed in the discharge. To obtain three-dimensional temperature data, the vertical position of the fixed array was varied.

The 16 surface-discharge tests of Table IV.B.9 are the ones of interest here. For the stagnant-ambient cases, Koester found that dilution increased with densimetric Froude number and decreased with basin depth. Two additional effects were noted. First, the effects of bottom friction on the vertical velocity profile were evident from dye injections. Second, the restriction placed on the amount of available entrainment water by the shallow bottom caused the reentrainment of some of the partly diluted plume water and higher plume temperatures.

Koester compared his experimental results with predictions of the Stolzenbach-Harleman model. The comparison of predicted and observed centerline-temperature decays were generally good for the "deep-water,"

TABLE IV.B.10. Experimental Data of Parametric Study by Koester (from Koester²⁸)

Run No.	Physical Parameters							Nondimensional Parameters							Experimental Results $\frac{\Delta T_{\max}}{\Delta T_0}$
	H, ft	v, fps	u_0 , fps	$\sqrt{h_0 b_0}$, ft	D_0 , ft	ΔT_0 , °F	F_0	F_0	z/H	v/ u_0	Re	$\frac{h_{\max}}{H}$	H/ D_0	H/ $\sqrt{h_0 b_0}$	
1	0.67	0	1.13	0.07	0.106	16.7	17.9	14.2	0	0	12017	0.75	6.3	10	0.46
2	0.67	0	1.13	0.07	0.106	16.8	17.5	13.9	0.33	0	12017	-	6.3	10	0.37
3	0.67	0	1.13	0.07	0.106	18.4	16.7	13.2	0.67	0	12017	-	6.3	10	0.26
4	0.67	0	1.13	0.07	0.106	17.0	17.8	14.1	1.0	0	12017	-	6.3	10	0.18
5	0.48	0	1.13	0.07	0.106	17.6	17.3	13.7	0	0	12017	1.0	5.0	7.2	0.50
6	0.48	0	1.13	0.07	0.106	15.6	19.8	15.7	0.33	0	12017	-	5.0	7.2	0.37
7	0.48	0	1.13	0.07	0.106	16.5	17.5	13.8	0.67	0	12017	-	5.0	7.2	0.26
8	0.48	0	1.13	0.07	0.106	16.9	17.0	13.5	1.0	0	12017	-	5.0	7.2	0.19
9	0.40	0	1.13	0.07	0.106	16.6	19.1	15.1	0	0	12017	1.34	3.8	6	0.37
10	0.40	0	1.13	0.07	0.106	-	-	-	0.33	0	12017	-	3.8	6	-
11	0.40	0	1.13	0.07	0.106	17.9	18.4	14.5	0.67	0	12017	-	3.8	6	0.33
12	0.40	0	1.13	0.07	0.106	16.7	19.0	15.0	1.0	0	12017	-	3.8	6	0.20
13	0.20	0	1.13	0.07	0.106	16.4	18.2	14.4	0	0	12017	2.55	1.9	3	0.41
14	0.20	0	1.13	0.07	0.106	17.3	17.2	13.6	0.5	0	12017	-	1.9	3	0.44
15	0.20	0	1.13	0.07	0.106	16.4	18.3	14.5	1.0	0	12017	-	1.9	3	0.35
16	0.79	0	0.82	0.08	0.125	16.1	11.5	9.1	0	0	10191	0.42	6.3	10	0.37
17	0.79	0	0.82	0.08	0.125	16.1	11.7	9.3	0.33	0	10191	-	6.3	10	0.50
18	0.79	0	0.82	0.08	0.125	16.3	11.1	8.8	0.67	0	10191	-	6.3	10	0.20
19	0.79	0	0.82	0.08	0.125	15.7	11.4	9.0	1.0	0	10191	-	6.3	10	0.16
20	0.47	0	0.82	0.08	0.125	17.4	11.0	8.7	0	0	10191	0.78	3.8	6	0.46
21	0.47	0	0.82	0.08	0.125	17.7	10.9	8.7	0.33	0	10191	-	3.8	6	0.43
22	0.47	0	0.82	0.08	0.125	16.3	11.5	9.1	0.67	0	10191	-	3.8	6	0.25
23	0.47	0	0.82	0.08	0.125	17.1	11.0	8.7	1.0	0	10191	-	3.8	6	0.25
24	0.24	0	0.82	0.08	0.125	16.6	11.0	8.7	0	0	10191	1.54	1.9	3	0.44
25	0.24	0	0.82	0.08	0.125	16.6	11.0	8.7	0.5	0	10191	-	1.9	3	0.41
26	0.24	0	0.82	0.08	0.125	16.4	11.1	8.8	1.0	0	10191	-	1.9	3	0.31
27	0.82	0	0.49	0.10	0.161	16.1	6.2	4.9	0	0	7912	0.33	4.5	8	0.53
28	0.82	0	0.49	0.10	0.161	16.5	5.9	4.6	0.33	0	7912	-	4.5	8	0.32
29	0.82	0	0.49	0.10	0.161	16.1	6.3	5.0	0.67	0	7912	-	4.5	8	0.32
30	0.82	0	0.49	0.10	0.161	16.6	6.2	4.9	1.0	0	7912	-	4.5	8	0.21
31	0.61	0	0.49	0.10	0.161	15.7	6.4	5.1	0	0	7912	0.45	3.8	6	0.59
32	0.61	0	0.49	0.10	0.161	15.2	6.4	5.1	0.33	0	7912	-	3.8	6	0.41
33	0.61	0	0.49	0.10	0.161	14.9	6.5	5.2	0.67	0	7912	-	3.8	6	0.41
34	0.61	0	0.49	0.10	0.161	15.7	6.4	5.1	1.0	0	7912	-	3.8	6	0.32
36	0.31	0	0.49	0.10	0.161	16.8	6.3	5.1	0	0	7912	0.88	1.9	3	0.57
37	0.31	0	0.49	0.10	0.161	17.8	6.0	4.8	0.5	0	7912	-	1.9	3	0.57
38	0.31	0	0.49	0.10	0.161	15.2	6.7	5.3	1.0	0	7912	-	1.9	3	0.42
50	0.40	0.060	1.13	0.07	0.106	17.8	16.3	12.9	0	0.049	12017	1.25	3.8	6	0.22
51	0.40	0.060	1.13	0.07	0.106	16.9	17.1	13.6	1.0	0.049	12017	-	3.8	6	0.14
54	0.47	0.047	0.82	0.08	0.125	-	-	-	0	0.058	10191	0.78	3.8	6	0.25
55	0.47	0.047	0.82	0.08	0.125	16.7	11.2	8.9	1.0	0.058	10191	-	3.8	6	0.19
56	0.61	0.036	0.49	0.10	0.161	17.0	6.0	4.8	0	0.074	7912	0.45	3.8	6	0.48
57	0.61	0.036	0.49	0.10	0.161	16.8	6.0	4.8	1.0	0.074	7912	-	3.8	6	-
60	0.40	0.060	1.13	0.07	0.106	17.4	16.5	13.0	0	0.020	12017	1.25	3.8	6	0.34
61	0.40	0.060	1.13	0.07	0.106	16.3	17.4	13.7	1.0	0.020	12017	-	3.8	6	0.22
64	0.47	0.047	0.82	0.08	0.125	16.7	11.2	8.9	0	0.023	10191	0.78	3.8	6	0.28
65	0.47	0.047	0.82	0.08	0.125	16.4	11.3	9.0	1.0	0.023	10191	-	3.8	6	0.20
66	0.61	0.036	0.49	0.10	0.161	16.4	6.1	4.8	0	0.030	7912	0.45	3.8	6	0.48
67	0.61	0.036	0.49	0.10	0.161	16.2	6.1	4.8	1.0	0.030	7912	-	3.8	6	0.32

List of Symbols

- A = discharge-channel aspect ratio, h_0/b_0
 b_0 = one-half the width of the rectangular outfall channel equivalently, for a round discharge, $b_0 = \pi D^2/(2D) = \pi D/2$
 D_0 = diameter of round discharge
 F_0 = densimetric Froude number of the discharge $F_0 = u_0/\sqrt{g(\Delta\rho/\rho_a)D_0}$
 g = acceleration of gravity
 H = maximum allowable penetration of the jet (depth of receiving body of water)
 h_0 = depth of discharge channel (equivalently, for a round discharge, $h_0 = D_0$)
 Re = discharge Reynolds number $Re = u_0 D_0/\nu$
 u_0 = discharge velocity
 v = ambient crossflow velocity
 z = vertical coordinate direction measured from the free surface
 $\Delta T_{\max}$ = maximum temperature difference at the surface between the discharge and ambient water
 ΔT_0 = temperature difference between the discharge and the ambient water
 $\Delta\rho$ = difference in densities between outfall and ambient water
 ρ_a = ambient density
 ν = kinematic viscosity of water

Also,

$$F_0' = F_0 A^{1/4} = u_0/\sqrt{g(\Delta\rho/\rho_a)\sqrt{h_0 b_0}}$$

 $h_{\max}$ = maximum plume depth as predicted by the Stolzenbach-Harleman model

$$\frac{h_{\max}}{H} = 0.42 F_0' \sqrt{h_0 b_0}/H$$

stagnant-ambient cases. Based on his data, Koester found that the maximum plume depth and centerline-temperature decay predictions were adequate (for all these high F_o runs) if

$$\frac{H}{D_o} \geq 3.3 \ln \frac{h_{\max}}{0.45 D_o},$$

where H is the depth of the basin, D_o is the diameter of the discharge, and $h_{\max}$ is the maximum plume depth predicted by the model. Areas and widths were not well predicted by the model, however.

The laboratory tests with the circular part at the surface and in the presence of an ambient current were carried out by varying initial densimetric Froude numbers but maintaining the relative depth as fixed. Koester found that increasing the Froude number had a positive effect on dilution until $h_{\max}/H \approx 1$. For $h_{\max}/H > 1$, the jet becomes attached to the bottom and a further increase in Froude number further blocks the ambient flow, increasing the area of jet attachment to the bottom. As a result, all the crossflow cannot pass to the lee side of the jet. A reentrainment eddy was found to occur with higher plume temperatures resulting from this reentrainment. Koester also found that plume deflection and temperature dilution increase with increasing crossflow velocity v/u_o once $h_{\max}/H$ remains constant. The Stolzenbach-Harleman model compared poorly with the data, partially because of the relatively shallow depths for which the model was tested.

The original report contains more details and describes results for intermediate and near-bottom discharges.

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C. Comparison of Integral/Phenomenological Models to Laboratory Data

1. Introduction

It is most appropriate that we begin our verification efforts of the integral and phenomenological models by comparing model predictions with experimental data obtained in the laboratory. These comparisons test the assumptions and validity of the models under conditions closely matching those idealized in model development. In contrast, comparisons with field data are summarized in Secs. IV.D-G and test not only the validity of model assumptions once again, but also the degree to which the model idealizations match the prototype situation under consideration. Our previous study of these models (see Chapter II) has revealed that they are idealized in many ways. Key prototype effects such as bottom interference, wind interaction, and transient conditions are often neglected. Comparisons with laboratory data allow the model assumptions to be tested free of the complicating factors present at all prototype sites.

Comparisons with laboratory data, however, do not comprise a total test. The danger in relying solely on such comparisons stems from the fact that all the models considered contain at least one parameter whose value must be obtained from experimental data. Into these so-called empirical coefficients is lumped much of the poorly understood physics. As a result, a model is tuned to the type of data used in its calibration. A model with fitting coefficients more appropriate to field data may perform better in prototype comparisons than with laboratory data. Because of this bias, one must look first for trends and second for precise quantitative values. A model that gives an absurd trend in a key jet parameter in comparison with laboratory data will most likely have the same problem in comparison with prototype data. This point is made more lucid if we regard the sources of model-data discrepancies as consisting of three types: (1) those due to model-development flaws such as when the model cannot predict the trend in data taken under conditions idealized by the model, (2) those due to idealizations made in the mathematical model that were not accurate for the prototype application at hand, and (3) those due to discrepancies related to inadequate tuning of the model to the laboratory or field data at hand. This section addresses the first and third causes of errors as related to laboratory comparisons. Sections IV.D-G present large field-data verification efforts that throw more light on the problems of applying these models to real-world situations, i.e., the second and third causes of errors.

Two sets of laboratory data were chosen as a basis for model comparisons in this section. Table IV.C.1 presents the specific parameters varied in the tank tests chosen for model-data comparisons. The first set of data, taken at MIT by Stolzenbach and Harleman when their surface-discharge model was developed, included surface discharges into deep water into both a stagnant and a moving ambient. Tests were also made over a sloping bottom,

but, since the models to be evaluated do not treat bottom effects, those tests will not be considered. The Stolzenbach-Harleman data along with other laboratory data and some field data, have been reduced to a common format by Shirazi and are presented here, along with numerous other sets of experimental data in the Shirazi data analysis discussed in Sec. IV.B.

TABLE IV.C.1. Laboratory Data Used in Integral and Phenomenological Model Comparisons^a

	F_o	A	R	D_w/h_o	$K/(\rho c_p u_o)$
MIT Data--Run 3	2.2	2.3	0.0	∞	6.2×10^{-5}
MIT Data--Run 6	4.4	5.8	0.0	∞	4.2×10^{-5}
MIT Data--Run 10	9.6	1.0	0.0	∞	1.9×10^{-5}
MIT Data--Run 22	1.9	2.3	0.18	∞	9.5×10^{-5}
MIT Data--Run 25	6.6	0.51	0.055	∞	2.8×10^{-5}
CBI Data--Run G	8.8	0.37	0.23	4.8	0
CBI Data--Run M	9.1	0.38	0.22	1.9	0
CBI Data--Run B	8.9	0.37	0.23	1.0	0
CBI Data--Run H-X	9.9	1.96	0.32	∞	0
CBI Data--Run A-VI(1)	5.0	2.0	0.62	∞	0

^a F_o = initial densimetric Froude number (usual definition);

A = ratio of full width to depth;

R = ratio of ambient to discharge velocity;

D_w/h_o = ratio of receiving-water depth to outfall depth;

$K/(\rho c_p u_o)$ = nondimensional surface heat-transfer coefficient, where u_o is the initial jet velocity.

The second set of laboratory experiments to be used for model-evaluation purposes consists of the Carter-Schiemer-Regier² and Carter-Regier³ tests (see Table IV.C.1). These surface-discharge tests were generally conducted with moderate-to-large initial densimetric Froude numbers and ambient currents. Water-depth changes in the Carter-Schiemer-Regier² tests indicate how jet characteristics change with bottom depth. The details of the Carter-Schiemer-Regier² and Carter-Regier³ experimental studies are also discussed in Sec. IV.B. The MIT (Stolzenbach-Harleman) and Chesapeake Bay Institute (Carter et al.) data were chosen for comparison, as they represent data taken over a wide range of parameters. More importantly, the data were taken by reliable methods, are consistent with other laboratory data, and are, in general, internally self-consistent.

The integral models chosen for study here are those of Stolzenbach and Harleman,^{1,4} Prych,⁵ and Shirazi and Davis.⁶ The two phenomenological models of Pritchard^{7,8} were also considered. The models chosen for verification are discussed in Chapter II and probably represent the most popular and promising integral and phenomenological models available in the literature.

2. Comparison of Model Predictions to Stolzenbach-Harleman Laboratory Data

The Stolzenbach-Harleman data were taken in a receiving basin 46 x 28 x 1.5 ft. The temperature-measurement system consisted of a wooden probe frame containing 59 thermistor probes in fixed locations. Eight other thermistors were placed in various inflow and outflow pipes to monitor outfall and ambient temperature. The frame was moved up and down to provide horizontal profiles at various depths below the surface. Stolzenbach and Harleman established a control of the flow at the boundaries: the two sides and rear of the basin. If a stagnant ambient discharge were being set up, there was concern about keeping the basin level and preventing large eddies from forming in the ambient fluid. Stolzenbach and Harleman thus withdrew 100 gpm from the rear of the receiving basin through a base manifold and returned an equal flow of 50 gpm through the side walls. This was done for all "stagnant" case runs. Thus a very slow ambient current was set up from the side walls to the back of the basin, which hopefully prevented a buildup of heat in the tank, reduced the eddying, and did not distort the jet structure. To create an ambient crossflow, on the other hand, water was pumped out of one side channel and into the other. An entire experimental run was completed in about 1 hr.

The MIT data chosen for study in this section are listed in Table IV.C.1. There are stagnant runs of low, intermediate, and high initial densimetric Froude numbers. Test runs with a moderate ambient current and low and high initial densimetric Froude numbers are also studied and compared with model predictions.

The laboratory situations had to be schematized in order to prepare model predictions. First, the Shirazi-Davis and Prych models were run with values of zero for the ambient-turbulence coefficients. This is Prych's recommendation, which is reasonable as ambient-turbulence effects should be negligible in these laboratory tests. The Shirazi-Davis model was run twice, first with the generic recommendations for these coefficients as given by Shirazi and Davis, and second with zero values. The choice of zero for the coefficients was more appropriate and gave consistently better predictions for the laboratory data we tested; so the results presented are for zero ambient-turbulence coefficients.

A second difficulty handled was the choice of a single uniform ambient-current velocity to represent the nonuniform current simulated in the laboratory tests. Stolzenbach and Harleman simulated ambient currents in their flume in the form presented in Fig. IV.C.1. This distribution of crossflow velocity was measured in the tank by timing submerged floats. The solid line in the figure represents the function used to represent the data in the original model-data comparisons of the Stolzenbach-Harleman model given in Ref. 1. For the present comparisons, a uniform ambient current

Fig. IV.C.1. Crossflow Velocity in MIT Experimental Basin. (Adapted from Stolzenbach and Harleman.¹)

was assumed in all model calculations. That value of ambient current was determined by first examining the data for the farthest centerline distance for which the temperature data were measured and then averaging the ambient current (using the functional form) from the outfall to that distance. The errors incurred in such an approximation are hoped and expected to be small. The Stolzenbach-Harleman model simulates a variable crossflow, but the Prych and Shirazi-Davis models require alteration to do so. In any case, the two phenomenological models of Pritchard assume a uniform crossflow and cannot be altered. A uniform crossflow velocity for all models was thus thought to be the best common choice and the most practical.

In the following comparison of model predictions to data, the initial densimetric Froude number F_0 (usual definition), the aspect ratio A (ratio of

full outfall width to depth), and the crossflow velocity R (ratio of ambient to discharge velocity) are noted in each of the figures.

In the forthcoming figures Pritchard's Model No. 1 is plotted for cases in which an ambient current was present. Since this model does not treat ambient-current effects, we ran the code assuming no current was present. Although Pritchard does suggest the application of his model for $R < 0.1$, we present the computation for a larger R for academic comparison only. This fact should be kept in mind in reading the later discussion of results. Also, we applied Pritchard's recommendations for the computation of widths in his first model (see Sec. II.F) directly to the calculation of widths for his second model. His second model only predicts a "total width," with no instructions on the calculation of half-widths as needed in these model-data comparisons. Pritchard has subsequently stated to us⁹ that the calculational procedure does not extend to his second model. Here again, caution should be used in interpreting the results. Surprisingly, the widths predicted by this method for his second model were adequate, at least in the comparisons with the MIT data.

Plotted for the stagnant ambient runs are surface centerline-temperature decay, surface-temperature half-widths, and half-depths. The surface centerline-temperature decay is plotted in terms of θ_c/θ_0 , where θ_c

is the centerline-temperature rise (above ambient) and θ_0 is the outfall-temperature rise (above ambient). The half-width of the jet, W_T , at any centerline distance was determined from the data as the distance between the centerline and the lateral position where the surface-temperature rise (above ambient) is half the centerline-temperature rise. The data plotted are actually the average of the left and right half-widths as determined from the data analysis. Half-depths in the data were determined by plotting, for each x along the centerline, the vertical profile of temperature (excess). The half depth, D_T , then, is the depth at which the centerline-temperature rise is reduced to half the surface centerline-temperature rise. For the comparisons with ambient currents, only surface centerline trajectories and surface centerline temperatures were determined and presented in the data; consequently, in these cases, the model-data comparisons are presented only for those two jet characteristics.

The first comparison, given in Figs. IV.C.2-4, is for an initial densimetric Froude number of 2.2 and a stagnant ambient. For centerline-temperature decay, the experimental data are not sufficiently refined to tell if any model has the right trend. The Shirazi-Davis and Pritchard No. 1 models are clearly too conservative, nearly by a factor of 2. The Stolzenbach-Harleman model does well until it reaches a plateau at $\theta_c/\theta_0 \approx 0.45$. The model then predicts essentially no dilution over a distance of $30 h_0$, which is highly unrealistic. Section II.C discusses in some detail the plateau often seen in centerline-temperature decay predictions of the Stolzenbach-Harleman model.

Prych and Stolzenbach-Harleman width predictions, as given in Fig. IV.C.2, are poor. These models underpredict widths for S/h_0 less than about 30, and then grossly overpredict widths for large values of S/h_0 . Incorrect

Fig. IV.C.2. Comparison of Surface Centerline-temperature-decay Predictions of Models to MIT Laboratory Data: Low Initial Densimetric Froude Number, No Ambient Current

Fig. IV.C.3. Comparison of Surface-temperature Half-width Predictions of Models to MIT Laboratory Data: Low Initial Densimetric Froude Number, No Ambient Current

Fig. IV.C.4. Comparison of Centerline-temperature Half-depth Predictions of Models to MIT Laboratory Data: Low Initial Densimetric Froude Number, No Ambient Current

lateral-spreading assumptions are the probable cause. The other three models also underpredict widths, the best agreement occurring for the Pritchard Model No. 2.

Concerning depth predictions, the Prych and Stolzenbach-Harleman models predict too small an asymptotic depth. Reduced depths result from the large widths, since mass, momentum, and energy must be conserved. The

Shirazi-Davis model makes a fairly good prediction of plume depth; however, the model predicts too hot and narrow a plume. Pritchard's Model No. 1 cannot predict the stratification observed in the plume data, although his model No. 2 gives a reasonable depth prediction here, certainly an improvement over that of Model No. 1.

Figures IV.C.5-7 present comparisons for an intermediate Froude number, $F_0 = 4.4$. Here Pritchard's Model No. 1 is much too conservative in

Fig. IV.C.5. Comparison of Surface Centerline-temperature-decay Predictions of Models to MIT Laboratory Data: Intermediate Densimetric Froude Number, No Ambient Current

Fig. IV.C.6. Comparison of Surface-temperature Half-width Predictions of Models to MIT Laboratory Data: Intermediate Densimetric Froude Number, No Ambient Current

Fig. IV.C.7. Comparison of Centerline-temperature Half-depth Predictions of Models to MIT Laboratory Data: Intermediate Densimetric Froude Number, No Ambient Current

its prediction of centerline-temperature decay. The model of Shirazi and Davis also underpredicts dilution rather significantly. The Stolzenbach-Harleman model, again, predicts temperatures well until a temperature plateau is reached. (At $\theta/\theta_0 \approx 0.3$, essentially no dilution occurs over a distance of $70h_0$.) The Prych model performed well in both trend and magnitude. In this case, Pritchard's Model No. 2 is an improvement over his first model, but the results are not as good as those of Prych.

Concerning widths (Fig. IV.C.6), the Prych and Stolzenbach-Harleman models again give the wrong trends: first, underprediction, then a tendency for gross overprediction. Pritchard's Model No. 2 again provides the best width predictions. Pritchard's Model No. 1 and the Shirazi-Davis model grossly underpredict widths.

Prych gives the best predictions of depth, although the predicted plume is too shallow far from the discharge. Again, this is probably due to the large lateral spreading. The Stolzenbach-Harleman model also seems to incorrectly predict asymptotic depth with too great a slope in the far field and consequently too much vertical shear. A large slope is not indicated by the data. Both Pritchard models and the Shirazi-Davis simulation indicate a correct trend, but the predicted plumes are too deep asymptotically. In Pritchard's second model, the asymptotic depth is a function of F_0 . From the data, D_T/h_0 does not show large variations as the Froude number changes. Based upon these data, the form for the asymptotic depth seems to be incorrect.

Figures IV.C.8-10 compare the high-initial-densimetric-Froude-number case, $F_0 = 9.6$, with model predictions. Clearly, Pritchard's Model No. 1 significantly underpredicts centerline-temperature decay. The other

Fig. IV.C.8. Comparison of Surface Centerline-temperature-decay Predictions of Models to MIT Laboratory Data: High Densimetric Froude Number, No Ambient Current

Fig. IV.C.9. Comparison of Surface-temperature Half-width Predictions of Models to MIT Laboratory Data: High Densimetric Froude Number, No Ambient Current

Fig. IV.C.10. Comparison of Centerline-temperature Half-depth Predictions of Models to MIT Laboratory Data: High Densimetric Froude Number, No Ambient Current

models do fairly well, although the data are not refined enough to determine which model is best. Again, the Prych and Stolzenbach-Harleman models underpredict, then overpredict, widths.

Regarding depth predictions, the Prych model has the best trend (plume dips down, then rises up). The Stolzenbach-Harleman model has numerical difficulties; the other models give an incorrect trend.

Figures IV.C.11 and 12 compare the low-densimetric-Froude number, ambient-current case ($F_0 = 1.9$, $R = 0.18$) with model predictions.

Fig. IV.C.11. Comparison of Surface Centerline-temperature-decay Predictions of Models to MIT Laboratory Data: Low Initial Densimetric Froude Number, Moderate Ambient Current

Fig. IV.C.12. Comparison of Surface Centerline-trajectory Predictions of Models to MIT Laboratory Data: Low Initial Densimetric Froude Number, Moderate Ambient Current

Isotherms abstracted from the experimental data (not shown here) indicate no cutoff of inshore entrainment in this case. If the ambient current were large enough, a cutoff of inshore entrainment would occur, since the outlet in the tank tests is flush with the simulated shoreline. The Shirazi-Davis model accurately predicts centerline temperatures; the other models overpredict dilution. Pritchard's Model No. 1 is the second most accurate, although it does not consider crosscurrent effects. It is not clear why all models (except the Shirazi-Davis one) overpredict dilution in this case. The Shirazi-Davis model predicts best for the trajectory. The Stolzenbach-Harleman and Prych models appear to be continually diverging from the data in their trajectory predictions. An overprediction of dilution implies the entrainment of greater ambient momentum than the data show, indicating a greater bending. This is the most logical explanation for the model-data discrepancies.

Figures IV.C.13 and 14 are plots of model predictions and data for a larger initial densimetric Froude number ($F_0 = 6.6$, $R = 0.055$). The Shirazi-Davis model predicts the best centerline temperatures. The other models make such predictions satisfactorily, with the possible exception of Pritchard's Model No. 2. The Stolzenbach-Harleman simulation failed beyond a centerline distance of $20h_0$. Trajectory predictions for the models were quite good; the Stolzenbach-Harleman model predictions are plotted, of course, only to a distance of $20h_0$ from the discharge. (See Sec. II.C for a discussion of the computational difficulties with that model.)

3. Comparisons with Hydraulic Data Obtained by Carter et al.^{2,3}

Carter et al.^{2,3} of the Chesapeake Bay Institute (CBI) at The Johns Hopkins University have carried out hydraulic-model studies of heated surface

Fig. IV.C.13. Comparison of Surface Centerline-temperature-decay Predictions of Models to MIT Laboratory Data: Moderate Initial Densimetric Froude Number, Low Ambient Current

Fig. IV.C.14. Comparison of Surface Centerline-trajectory Predictions of Models to MIT Laboratory Data: Moderate Initial Densimetric Froude Number, Low Ambient Current

jets discharging into a flowing ambient. (See Sec. IV.B for further details concerning these studies.) Five typical cases from the many runs made at CBI have been selected. These five cases include variations in receiving-water depth, initial densimetric Froude number, and outfall aspect ratio as studied by Carter et al.^{2,3} The results of these runs have been compared with four models for surface thermal discharges: the models of Stolzenbach and Harleman,^{1,4} Prych,⁵ Shirazi and Davis,⁶ and Pritchard's Model No. 2.⁸ Details of these models are discussed in Chapter II.

First, three cases designated by Carter et al.² as Runs G, M, and B will be considered. These three cases are essentially the same, except for the ratio of the depth of the receiving water D_W to the depth of the outfall h_O . This ratio, D_W/h_O , was 4.8 for Run G, 1.9 for Run M, and 1.0 for Run B. In all three cases, the initial densimetric Froude number F_O was about 8.9, the outfall aspect ratio A (b_O/h_O , where b_O is the full width of the outfall) was about 0.37, and the ratio R of ambient current to outfall velocity was about 0.23. The Shirazi-Davis, Prych, and Stolzenbach-Harleman models do not take into account bottom interaction and therefore yield essentially the same predictions for all three cases. Pritchard's Model No. 2 considers Run B to be bottom-limited and thus yields predictions that are different when compared to Runs G and M; this model does not attempt to predict the centerline trajectory of the plume. Figures IV.C.15-24 present the data and the results of the model calculations for the three cases. Note that, in these and all succeeding figures, quantities with dimensions of length are divided by h_O , the depth of the outfall, which for these first three tests was about 0.17 ft.

As seen in Figs. IV.C.15, 18, and 22, the Shirazi-Davis model predicts the trajectory much better than the Prych model, but this may not be a fair comparison for the models in that the data may be affected by the nearness of the far shore, an effect that is not included in any of the models. Apparently due to the relatively strong crosscurrent, the Stolzenbach-Harleman computer code fails at a short distance from the outfall. Even the initial trend

Fig. IV.C.15. Comparison of Surface Centerline-trajectory Predictions of Models to CBI Laboratory Data: $D_W/h_O = 4.8$

Fig. IV.C.16. Comparison of Surface Centerline-temperature-decay Predictions of Models to CBI Laboratory Data: $D_W/h_0 = 4.8$

Fig. IV.C.17. Comparison of Surface-temperature Half-width Predictions of Models to CBI Laboratory Data: $D_W/h_0 = 4.8$

Fig. IV.C.18. Comparison of Surface Centerline-trajectory Predictions of Models to CBI Laboratory Data: $D_W/h_0 = 1.9$

Fig. IV.C.19. Comparison of Surface Centerline-temperature-decay Predictions of Models to CBI Laboratory Data: $D_W/h_0 = 1.9$

Fig. IV.C.20. Comparison of Surface-temperature Half-width Predictions of Models to CBI Laboratory Data: $D_W/h_0 = 1.9$

Fig. IV.C.21. Comparison of Centerline-temperature Half-depth Predictions of Models to CBI Laboratory Data: $D_W/h_0 = 1.9$

Fig. IV.C.22

Comparison of Surface Centerline-trajectory Predictions of Models to CBI Laboratory Data: $D_W/h_0 = 1.0$

Fig. IV.C.23

Comparison of Surface Centerline-temperature-decay Predictions of Models to CBI Laboratory Data: $D_W/h_0 = 1.0$

Fig. IV.C.24. Comparison of Surface-temperature Half-width Predictions of Models to CBI Laboratory Data: $D_W/h_0 = 1.0$

does not agree with the data. Bear in mind that, when large currents are present, some of the assumptions/idealizations made in the Stolzenbach-Harleman, Prych, and Shirazi-Davis models are no longer applicable, such as flow and temperature similarity, equal entrainment on windward and leeward sides of the jet, and the cutoff of inshore entrainment as the plume interacts with the nearshore. These models are thus suspect at the larger ranges in ambient-current velocities.

Figures IV.C.16, 19, and 23 show the centerline-temperature decay as a function of distance along the centerlines for these three cases. The data indicate that interaction with the shallow bottom produces a far less rapid temperature decay, reaching, for example, one-quarter the outfall temperature excess 2.8 times farther away in Run B than in Runs G and M. The Pritchard model is conservative in all three cases, but clearly does exhibit the proper trend when bottom interaction occurs. In the deep-water cases (G and M), the Prych model predicts the centerline-temperature decay very well, but it far overestimates this decay in the shallow water (B). Conversely, the Shirazi-Davis model greatly overpredicts the centerline temperature decay in all three cases, especially the shallow-water case. The Stolzenbach-Harleman initial trend is far too conservative, except in Run B, where the shallow bottom apparently brings the data up near the model prediction. The model does not consider bottom effects.

Figures IV.C.17, 20, and 24 present the half-widths of the temperature distribution W_T . This is the lateral distance perpendicular to the jet centerline where the temperature excess drops to one-half its centerline value. For these three cases, Carter et al.² found that the lateral temperature profiles were not symmetric, due to entrainment and possible recirculation of heated water in the region between the jet and the near shore. The data therefore reflect only the half-width as determined from measurements on the offshore side of the plume. Although the data do not seem to show any clear trends, shallow water (Run B) appears to produce a slightly wider distribution. The Pritchard model overpredicts the widths by a factor of about two beyond the initial regional value ($S/h_0 \gtrsim 10$). Note that the Pritchard model does not directly predict widths as defined here. It does, however, predict the length and maximum width of each isotherm. The half-widths presented here are calculated assuming the existence of bullet-shaped isotherms, as Pritchard assumed in his first model. For these cases, the jet is significantly bent by the crosscurrent and the isotherms are clearly distorted. These widths, therefore, may describe the plume predictions poorly, but some representation was needed in order to make a width comparison for the model with the data. The Prych- and Shirazi-Davis-model predictions are similar; both do fairly well. The Stolzenbach-Harleman predictions are nearly the same as those of Prych over the short distance for which the model runs.

For Run M only, Carter et al.² present data on the half-depth D_T of the temperature distribution. The data indicate that the plume is buoyant

and floats close to the surface after initially entraining and spreading vertically. The Prych model gives the most reasonable predictions of depth; the Pritchard model yields a plume three times too deep asymptotically. The Pritchard model predicts the initial downward growth, but does not account for the subsequent floating trend. And, probably unrealistically, the other models predict a drastic initial decrease in depth. In particular, the Shirazi-Davis model predicts that the plume will grow rapidly in the vertical direction, a fact not borne out by the data. In this case, the water, although deeper than the outfall depth, was fairly shallow, a circumstance that could influence the vertical growth of the plume. This conclusion could not, however, account for such a large discrepancy in depth.

To examine the influence of initial densimetric Froude number on the jet, Runs H-X and A-VI(1) of Carter and Regier³ can be compared. Each is a deep-water case with the receiving water 10 times as deep as the outfall depth. Also, each had an aspect ratio of about 2.0 and a discharge depth h_0 of about 0.085 ft. Run H-X had an initial densimetric Froude number of 5.0 and a velocity ratio of 0.62. Carter and Regier point out that the initial Reynolds number for Run A-VI(1) is less than 3000, so that there is some concern as to whether this was a fully turbulent jet. Since data concerning the centerline trajectory are not included in the report of Carter and Regier for Run A-VI(1), no comparison can be made. Figure IV.C.25 presents the data for the centerline trajectory for Run H-X. Multiple values of the Y coordinate at a given X coordinate indicate the variability in the position of the centerline according to Carter and Regier.³ We note that the data span the predictions of the Shirazi and Davis and Prych models, and that the bending predicted by the Stolzenbach-Harleman model falls far short of the data.

Fig. IV.C.25. Comparison of Surface Centerline-trajectory Predictions of Models to CBI Laboratory Data: High Initial Densimetric Froude Number, Moderate Ambient Current, Deep Water

Figures IV.C.26-31 show the rest of the data for these two cases along with the four model predictions. In particular, Figs. IV.C.26 and 29 show

the centerline-temperature decays. The data look similar for the two cases, although the high-initial-densimetric-Froude-number case (Run H-X) shows a slightly more rapid decay. Pritchard's Model No. 2 is far too conservative in both cases, although it does reflect the trend that the high-Froude-number case decays more rapidly. The Shirazi-Davis and Prych predictions are similar, but the Shirazi-Davis model is best in the near-field region, where the drop-off is rapid, and the Prych model is best farther from the outfall, where the decay is more gradual. The Stolzenbach-Harleman model seems to give erroneous results, especially in the higher-current case [Run A-VI(1)].

Figures IV.C.27 and 30 show the temperature half-width data for these two cases. Again, the Pritchard model overpredicts the widths by a factor of two beyond the initial region. The Shirazi-Davis and Prych predictions are again fairly similar, but significantly underpredict the widths. They do, however, account for the trend toward a wider distribution for Run H-X.

The half-depths of the jets are shown in Figs. IV.C.28 and 31. The data for the higher-Froude-number case (Run H-X, $F_0 = 9.9$) show a jet that is two or three times as deep as the lower-Froude-number case [Run A-VI(1), $F_0 = 5.0$]. All the models predict this trend, but none of them adequately describe the data. In both cases, the data indicate that, although the jet initially grows in the vertical directions, it eventually becomes significantly shallower. The Pritchard and Prych models do show this trend, but the Shirazi-Davis model predicts a jet that continues to grow vertically.

Finally, the effect of the outfall aspect ratio can be considered by comparing Runs G and H-X. Run G had an aspect ratio of 0.37; Run H-X had an aspect ratio of 1.96. Both may be considered to be deep-water cases with Froude numbers of about 9. The centerline trajectories have already been examined in the previous comparisons. Note here that the initial depth of the discharge h_0 for Run G was twice as large as for Run H-X. Thus the scaling

Fig. IV.C.26. Comparison of Surface Centerline-temperature-decay Predictions of Models to CBI Laboratory Data: High Initial Densimetric Froude Number, Moderate Ambient Current, Deep Water

Fig. IV.C.27. Comparison of Surface-temperature Half-width Predictions of Models to CBI Laboratory Data: High Initial Densimetric Froude Number, Moderate Ambient Current, Deep Water

Fig. IV.C.28. Comparison of Centerline-temperature Half-depth Predictions of Models to CBI Laboratory Data: High Initial Densimetric Froude Number, Moderate Ambient Current, Deep Water

Fig. IV.C.29. Comparison of Surface Centerline-temperature-decay Predictions of Models to CBI Laboratory Data: Moderate Initial Densimetric Froude Number, Large Ambient Current

Fig. IV.C.30. Comparison of Surface-temperature Half-width Predictions of Models to CBI Laboratory Data: Moderate Initial Densimetric Froude Number, Large Ambient Current

Fig. IV.C.31. Comparison of Centerline-temperature Half-depth Predictions of Models to CBI Laboratory Data: Moderate Initial Densimetric Froude Number, Large Ambient Current

factors are different in the two sets of figures. The data for the centerline temperature decay (Figs. IV.C.16 and 28), adjusted for the factor of two in the scaling factor h_0 , show almost no dependence on this difference in aspect ratios. The Pritchard model predicts a significant dependence on aspect ratio, being only slightly conservative in the smaller-aspect-ratio case (Run G), but far too conservative in the large-aspect-ratio case (Run H-X). The Prych and Shirazi-Davis models also predict a slightly less rapid temperature decay for the larger-aspect-ratio case (Run H-X). Although this is not borne out by the data, the shift is small and both models give fairly good centerline-temperature decay predictions. The Prych model results, however, appear to be slightly better.

The half-widths of the temperature distributions appear in Figs. IV.C.17 and 27. The data for the large aspect ratio (Run H-X, $A = 1.9$) show a temperature distribution that is almost 50% wider than the lower aspect ratio case (Run G, $A = 0.37$). As noted before, the Pritchard-model predictions are far too wide in both cases. The Shirazi-Davis model exhibits the wrong trend, predicting a slightly narrower width for the larger aspect ratio. The Prych model shows the same trend as the data and, of the four models, best predicts the widths. However, it slightly underpredicts the widths for the larger aspect ratio. Depth data are not presented by Carter et al.² for Run G, so no comparison can be made.

4. Conclusions

The results of the model-data comparisons carried out for the MIT data are basically inconclusive in the sense that no model proved itself clearly

superior to the others. The Prych model performed slightly better than the other models, in spite of the incorrect trends it predicted for plume widths.

For the comparisons to data taken under stagnant ambient conditions, the two Pritchard models and the Shirazi-Davis model predicted plumes that were generally too narrow and too hot. The predicted depths for these models had incorrect trends; the nearly constant depths predicted over the major portion of the plume were in contrast to the variable-depth plume observed in the data.

The Prych and Stolzenbach-Harleman models predicted centerline temperature reasonably well; the Stolzenbach-Harleman model predictions were not reliable beyond the initiation of the characteristic plateau usually observed in its centerline temperature predictions. The trend in plume widths was incorrect for both these models, as they underpredict plume spreading for a short distance from the outfall and then grossly overpredict it. Predicted depths of these models were roughly of the right trend, but became asymptotically too small as the plume-width predictions grew too large. Since these models are based upon conservation laws of mass and momentum, a rapid decrease in depth must accompany a large increase in width.

The comparisons of the model predictions to the MIT data with ambient currents revealed the clear superiority of the Shirazi-Davis model. Remember, however, that these comparisons were only in terms of centerline temperature and trajectory data, since only those parameters were measured. Pritchard's Model No. 1 was second best in terms of accuracy. The other models predicted too great a dilution in the two cases studied.

Results of the model comparisons to the selected CBI data revealed that the Shirazi-Davis and Prych models do poorly for situations in which the discharge is made into a shallow receiving basin. Neither model takes bottom effects into account. The CBI data indicate that bottom interaction can have significant effects on the centerline-temperature decay. Pritchard's Model No. 2 allows for bottom interaction and fairly well describes the centerline-temperature decay. In contrast with the MIT data comparisons, Pritchard's Model No. 2 far overpredicts the widths, and this may, as stated earlier, be an artifact of the way width was defined. The data of Carter et al.² were used in the calibration of the Pritchard model, so this test is not truly objective. It is also apparent that the Stolzenbach-Harleman model does not perform satisfactorily in the larger-Froude-number, larger-ambient-current situations considered in the CBI data. Since the model gave the most erroneous results in the highest-crosscurrent case [Run A-VI(1), $R = 0.62$], the large crosscurrents probably limit the direct usability of the model to this set of data. (See the discussion of the Stolzenbach-Harleman model in Chapter II.) Of the four models considered, the Prych model best predicts the subset of the CBI data considered in this report.

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D. Pilgrim Nuclear Generation Station: Verification of Stolzenbach-Harleman and Pritchard Models

The Pilgrim Nuclear Power Station affords an interesting case study of thermal-plume investigations, because a broad spectrum of techniques has been used to analyze its thermal discharge. Beginning with preoperational studies, the theoretical and experimental investigations have included:

1. Hydraulic-model studies carried out by the Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics of the Massachusetts Institute of Technology (MIT).^{1,2}
2. Preoperational analytical model calculations using the Stolzenbach-Harleman model made by MIT.³
3. Preoperational dye studies of plume dilution carried out at the Pilgrim site by Zone Research, Inc.⁴
4. Baseline studies of the site, including boat surveys of the depth, temperature, and salinity fields along with drogue studies of jet trajectory (pump operation only) performed by MIT.^{6,7}
5. Postoperational thermal- and dye-plume mappings done by Vast, Inc.⁶ by means of surface vessel.
6. Aerial overflights with remote thermal maps of the plume conducted by Coastal Research Corporation, coordinated with the surface-vessel measurements made by Vast, Inc.⁶
7. Surface-vessel surveys of the thermal plume by MIT.⁷⁻¹⁰
8. Studies of the tidal and wind-induced current at the Pilgrim site performed by International Marine Science and Engineering.¹¹
9. Comparison of Stolzenbach-Harleman model calculations to boat-based plume data conducted by MIT.⁷⁻¹⁰
10. Comparison of MIT and Coastal Research Corporation data to predictions of the Pritchard phenomenological model carried out by Boston Edison.¹³
11. Comparison of MIT boat-based and Aero-Marine remotely sensed plume data by Stone & Webster.¹⁰
12. Comparison of the STUFF numerical model to one Coastal Research Corporation infrared thermal map carried out by Systems Science and Software.
13. Presently ongoing studies by EG&G (not discussed).

Since the thrust of this section is model-data comparison, our discussion centers on items 9 and 10 above. Results of the other studies are only highlighted. The STUFF model predictions are compared with data in the

review of that model in Chapter II and will not be repeated. The work summarized here is discussed in greater detail in the reports comprising the Pilgrim investigations as listed at the end of this section.

1. The Site

The Pilgrim Nuclear Power Station is located on the western shore of Cape Cod Bay near Plymouth, Massachusetts.² The Bay, formed by the protrusion of Cape Cod into the Atlantic Ocean, is, very roughly, 20 miles square and opens to the sea on the north. The seasonal weather patterns at the site are typical of New England climates, although the Atlantic Ocean provides some temperance. Stagnant atmospheric conditions are rare and wind speed and direction are variable, the most frequent wind direction being off-shore with onshore winds occurring about one-third of the time.

Several features of the Pilgrim site complicate the analysis of the thermal plume. Although the details of these factors are, for the most part, unique to Pilgrim, they illustrate the type of modeling problem typically encountered.

Figure IV.D.1 is a schematic view of the site. In that figure, note that the intake is located on the shore about one-quarter mile from the discharge. Water flows from the intake into a lagoon adjacent to the discharge and separated from it by a stone breakwater. The positioning of the intake and discharge suggests that recirculation can be a serious problem at the site.

Fig. IV.D.1. Schematic View of the Pilgrim Nuclear Power Station Site. (Adapted from Doret et al.⁷)

Vast, Inc., measured the amount of recirculation over a two-week period in December 1972, using fluorescent dye released in the discharge canal. As shown in the results of this study presented in Fig. IV.D.2, recirculation varies between 0 and roughly 28%, reaching 20% on three occasions.

Although no absolute correlation is justified by the Vast measurements, accompanying meteorological data indicate that recirculation is enhanced by a north wind. In the Environmental Report for Unit 2, Boston Edison states¹³ that, based on the frequency of contributing meteorological conditions, noticeable recirculation of partly diluted plume water into the intake lagoon is expected to occur about 10% of the time. In addition, direct recirculation, either as seepage through or as waves overtopping the breakwater, is also possible, but is generally thought to be a minor effect.

Fig. IV.D.2. Recirculation at the Pilgrim Site as Measured by Vast, Inc., during December 1972. (Adapted from Marine Ecology Studies.⁶)

Again, referring to Fig. IV.D.1, note that two shoals extend out from the shoreline, one on each side of the discharge. These shoals can be expected to influence the behavior of the thermal plume as well as the current structure in the immediate vicinity of the plant. In addition, offshore bottom contours permit significant bottom interaction during low tide. The vertical cross section of Fig. IV.D.3 shows that the bottom slope varies from about 3:100 at the outfall to 7:1000 at 3000 ft. At low tide, the water is shallow and

Fig. IV.D.3

Depth of Water as a Function of Distance Offshore. (Adapted from Doret et al.⁷)

the initial densimetric Froude number is large (>10). Field data discussed later indicate that the plume mixes vertically until it contacts the bottom, where it remains attached for several hundred feet. Additionally, the data indicate that no bottom contact is made at high tide, due to the deeper water and the greater effects of buoyancy on vertical mixing.

As a result of wind influence superimposed on tide influence, the structure of the current near the plant is complex. O'Hagan,¹¹ among others, has investigated the current structure. In his study, a single current meter was moored 1 ft below the surface about 1000 ft offshore at a point within the region sheltered by two shoals. Subtracting the calculated tidal component from the total observed current, O'Hagan attempted to correlate this reduced current speed and direction with the instantaneous wind speed and direction recorded by a 300-ft meteorological tower near the plant. Though wind history was not included in the correlation, the residual current was found to correlate well with wind speed and direction, varying in magnitude from 0.2 to 2.0% of the steady wind speed, the greatest number of measured values being near

1.0%. In most instances, the wind-induced current was much larger than the tidal current. Droque and dye studies of the ambient current conducted in anticipation of plant construction support these observations.

Fig. IV.D.4. Schematic View of the Pilgrim Discharge Structure. (Adapted from Doret et al.⁷)

Therefore, during part of the tidal cycle, the level of water in the discharge is higher than in the receiving bay, and free open-channel flow results. For tides below mean low water, a free overfall exists.

No satisfactory way exists in which to schematize this outfall as a rectangular canal discharging at the water surface, as supposed by the popular integral models. Consequently, the definitions of model parameters such as outfall width, outfall depth, discharge velocity, and initial densimetric Froude number are potential sources of considerable error.

Tidal changes produce a time-varying densimetric Froude number and aspect ratio. These transient discharge conditions, coupled with changing environmental parameters, may prevent a steady plume from ever becoming

established. Thus, use of the common steady-state integral models is only an approximate treatment. It is interesting to compare model predictions with data taken at various stages of the tidal cycle.

2. Summary of Laboratory and Field Studies of the Pilgrim Discharge

In 1969, Harleman and Stolzenbach conducted hydraulic-model studies of the then-proposed Pilgrim Station. A distorted hydraulic model was used to produce turbulent flow and allow representation of a fairly large horizontal extent on a laboratory scale. Accordingly, the model was scaled 40 to 1 in the vertical dimension and 250 to 1 in the horizontal dimension. On a prototype scale, the MIT model covered about 2 miles in the shore-parallel direction, about 1 mile in the offshore direction, and about 60 ft in water depth. Further details of model construction are given by Harleman and Stolzenbach¹ and by the Boston Edison Company.²

Using the hydraulic model, Harleman and Stolzenbach investigated the effects of tidal variation, ambient current, and discharge rate on plume behavior. Based on 9 model plumes, they concluded:

- a. Tidal variation and discharge rate have no significant effect on the plume.
- b. An ambient current (varied between 0 and 0.6 knot in the laboratory tests) bends the jet trajectory, but has little effect on temperature decay.
- c. Recirculation directly into the intake is not a serious problem at the site.

A criticism of the hydraulic model tests carried out by Harleman and Stolzenbach stems from the use of distorted scaling for near-field simulations. Scaling principles were used to maintain representative turbulence and simultaneously provide significant surface area for heat loss. Unfortunately, the scaling used in the hydraulic model for Pilgrim produces a bottom slope that is too steep by a factor of $25/4$, for, as noted earlier, a significant amount of bottom interaction occurs during low tide. Since greater vertical mixing is allowed in the hydraulic tank, the laboratory plume should be smaller than its prototype counterpart. When the hydraulic-model study was conducted by Harleman and Stolzenbach, very few data were available by which to assess the role of bottom interference. Hence, based on the data of Wiegel et al.,¹⁵ the investigators concluded that a distortion of $25/4$ would not change the validity of the model. Recently, however, Stolzenbach¹⁶ stated that laboratory studies of near-field dilution should more properly be carried out in large, undistorted basins that preserve the local topography of the site. This episode illustrates one of the principal difficulties with hydraulic models: The investigator must select beforehand which similarity laws will be retained in scaling and which will be violated.

Zone Research, Inc.,⁴ conducted dye studies at the site in order to provide field data on available dilution prior to the plant startup. Fluorescent dye was introduced into the discharge with the pumps operating, and the dye plume was surveyed with a boat-based fluorometer. These early measurements provided some field data for impact-evaluation work.

Vast, Inc.,⁶ conducted both surface-vessel plume surveys and dye-tracer studies at the Pilgrim Station. Rhodamine dye was introduced into the plant intake. Then, two boats equipped with fluorometers, temperature sensors, and Motorola Mini-Ranger navigation systems were used to survey the thermal and dye plumes. Seven plume maps were made on three dates in December 1972. The investigators found, during their measurements, that wind-induced current effects were dominant over tidal effects. Unfortunately, due to such unfavorable weather conditions as strong wind, high waves, and icing of equipment, the study was somewhat limited.

Temperature and dye measurements were made along relatively fixed transects. Data reported cover an average of three trips along the transect, with successive passes matched by aligning concentration peaks when such peaks could be discerned. The validity of this kind of averaging technique is somewhat questionable, since it is impossible to know whether changes with time observed in station readings (assuming the station can be exactly relocated) are stochastic or real.

Table IV.D.1 and Fig. IV.D.5 summarize the Vast, Inc., data. Here θ refers to the temperature excess; i.e., a given temperature above ambient and θ_0 is the temperature excess at the outfall. Table IV.D.1 gives the reported ambient and outfall conditions for each of the eight studies, and

TABLE IV.D.1. Outfall and Ambient Conditions for Plume Surveys Made by Vast, Inc.

Date of Survey, 1972	Time of Survey	Tidal Stage	Wind	Temperature, °F	
				Intake	Outfall
December 20	0719-1002	Flood	Calm	40.2	66.1
December 20	1301-1512	Ebb	NE, 6-7 mph 20-26 mph after 1500	40.6	64.0
December 28	0825-0947	Ebb	W, 2-5 mph	40.5	67.6
December 28	1002-1336	Ebb-Flood	N-NW, 3-15 mph	40.1	68.5
December 29	0804-0903	Ebb	NW, 9-11 mph	41.0	68.9
December 29	0924-1208	Ebb	NW, 9-18 mph	40.8	69.2
December 29	1340-1513 ^a	Flood	N, 7-15 mph	40.8	70.1
December 29	1516-1644	Flood	N, 7-15 mph	41.2	69.9

^aSurface isotherms could not be closed for this survey; consequently no isotherm areas were presented.

Fig. IV.D.5. Summary of Surface Isotherm-area Data Taken by Vast, Inc., during December 1972

Fig. IV.D.5 graphically portrays the measured plume areas as a function of excess-temperature ratio. Considerable variation among the data is evident.

On December 29, 1972, Coastal Research Corporation⁶ made aerial surveys of the Pilgrim thermal plume; simultaneous surface-vessel temperature measurements were made by Vast, Inc. During about 9 hr that encompassed all tidal stages, 41 overflights were made. Early winter weather conditions, however, adversely affected both the aerial and surface measurements, including the 3-ft surface waves that were present during most of the study. In all instances, the wind was observed to be the controlling factor in determining plume behavior.

From the 41 passes over the plant, 11 plume maps were constructed. These data are presented in temperature-decay and isotherm-area format in Figs. IV.D.6 and 7. Here, θ_c stands for the temperature excess measured at distance S along the jet centerline. These plots reveal the large scatter among the data, especially noticeable within the first 2000 ft from the discharge. For convenience in comparing different tidal stages, the data have been replotted in Figs. IV.D.8 and 9 using H for high tide, L for low tide, E for ebb tide, and F for flood tide. The data show no statistically significant difference with tidal stage, although the single ebb-tide measurement appears to be below the rest of the data. This may well be due to the fact that the ebb-tide measurement was made near the end of a rather long period of a strong NNW wind. The remaining data were all taken during a north wind of somewhat lower speed.

Fig. IV.D.6. Summary of Centerline Temperature-decay Data Obtained from Aerial Infrared Plume Maps Made by Coastal Research Corp., December 29, 1972

Fig. IV.D.7. Summary of Surface Isotherm-area Data Obtained from Aerial Infrared Plume Maps Made by Coastal Research Corp., December 29, 1972

Fig. IV.D.8. Replot of Centerline Temperature-decay Data by Tidal Stage.
(Coastal Research Corp. data taken December 29, 1972.)

Fig. IV.D.9. Replot of Surface Isotherm-area Data by Tidal Stage.
(Coastal Research Corp. data taken December 29, 1972.)

In addition to these quantitative results, two common plume characteristics were evident. First, the thermal images revealed one or more small pockets of warm water exterior to the main isotherm. Second, a sharp, upwind edge of the plume was apparent. Disjointed plumes with distinct upwind edges have been observed at a number of sites.

The thermal images appear to be inconsistent among themselves. Two incidents of inconsistent behavior are illustrated in Figs. IV.D.10-14. Three low-tide plume measurements all taken within less than half an hour are compared in Figs. IV.D.10 and 11. The difference between the 1302- and 1319-hr surveys is striking. For example, the plume areas are roughly three times as large in the 1319 survey as in the 1302 survey. A growth of a plume to three times its original size in only 17 min seems inconceivable to us. Figure IV.D.12 compares the 1302 and 1319 thermal-plume maps. This figure reveals that measured isotherms are elongated parallel to the shore in the 1319 survey; the earlier measurements show roughly circular isotherms clustered near the outfall. As an example, consider the 42°F isotherm. The edge of this isotherm moves about 5000 ft down the shore between the two surveys, indicating an edge velocity in excess of 4 ft/sec. Two high-tide comparisons made at 1724 and 1744 hr show a similar difference in plume temperature decay and areas as indicated in Figs. IV.D.13 and 14. Such results raise questions as to the validity of these thermal-plume maps.

Because of the above-stated uncertainty surrounding that data set, it is unfortunate that the 1302-hr Coastal Research Corp. plume data were selected for comparison with the STUFF numerical model (see Sec. II.K). Although that case appears representative of the total CRC data as shown in Figs. IV.D.7 and 8, any conclusions that could be drawn from the model-data comparisons are necessarily weakened by the questions surrounding the data.

MIT has conducted both pre- and postoperational surveys of the Pilgrim site,⁶⁻¹⁰ the former acquiring baseline data to assist in evaluating the environmental impact of the then-proposed discharge and the latter attempting to validate the Stolzenbach-Harleman model. Measurements have included both boat surveys and drogue studies.

During the preoperational studies conducted in April, June, and August 1972, simultaneous measurements of salinity and temperature as a function of depth were made at various stations near the outfall.⁶⁻⁷ An MIT-developed instrument that simultaneously measures water conductivity (a measure of salinity), temperature, and pressure (a measure of depth) was used to generate an analog tape and chart recording at the time of the data taking. The tapes were later electronically demodulated and digitized for computer analysis. The position of each station in the bay was ascertained by sextant sighting to three clearly visible objects whose positions could be established from U. S. Geological Survey Maps. From the sightings, the position of each station was geometrically determined relative to the known locations.

Fig. IV.D.10. Comparison of Centerline-temperature Decay for Three Successive Low-tide Plumes. (Coastal Research Corp. data taken December 29, 1972.)

Fig. IV.D.11. Comparison of Isotherm-area Data for Three Successive Low-tide Plumes. (Coastal Research Corp. data taken December 29, 1972.)

Fig. IV.D.13. Comparison of Centerline-temperature Decay for Two Successive High-tide Plumes. (Coastal Research Corp. data taken December 29, 1972.)

Fig. IV.D.14. Comparison of Surface Isotherm-area Data for Two Successive High-tide Plumes. (Coastal Research Corp. data taken December 29, 1972.)

During the April surveys, a uniform vertical temperature profile was observed; the June measurements revealed a distinct thermocline with a temperature difference across the thermocline of about 2 F°. The depth of the thermocline depended on the tidal stage and varied between 5 and 10 m. Salinity appeared to be uniform with depth in both cases. The August data, taken at ebb tide, showed both temperature and salinity stratification with a higher salt concentration measured near the surface. In all cases, the investigators noted that surface currents were controlled by the wind, concurring with previously described results reported by Vast, Inc. Subsurface currents were found to depend on tidal stage.

Drogue studies were made on four days during June and August 1972 while tests of the plant cooling system were being conducted. The drogues were deployed in the discharge canal and their pathlines charted to determine the trajectory of the nonbuoyant prototype plume. (Actually, the plume was negatively buoyant since cooler bottom water was being discharged.)

MIT made postoperational thermal-plume maps at the Pilgrim site on seven days during 1973 as part of their effort to validate the Stolzenbach-Harleman model.⁷⁻¹⁰ Surveys were made on January 26, March 25, April 19, July 2, July 3, August 30, and November 13, 1973. The plume maps were made using the "transect" method, which is based on setting out easily visible buoys at fixed and known locations in the bay. Boats then travel at known speeds along approximately straight paths (transects) between the buoys, collecting data continuously in time. This information is sufficient to specify the temperature at various positions along the transects.

Temperature measurements were made by towing an assembly that MIT refers to as the small-boat bathythermograph. The bathythermograph consists of two thermistors, one of which is towed one-half foot below the surface while the other is towed at a depth of 4 ft. MIT claims an accuracy of 0.1 C° and a response time of 0.1 sec for the thermistors.

In addition to and coordinated with the MIT surface-vessel plume surveys conducted August 30, 1973, a series of hourly infrared overflights was made by Aero-Marine Surveys. Simultaneously, ground-truth data and hourly vertical temperature profiles at seven MIT buoy locations were acquired by Marine Research, Inc. Stone & Webster Engineering Corp. studied the results of this combined field program in an attempt to assess the validity of surface-vessel and remotely sensed thermal-plume measurements. Based on the measurements reported, they conclude that:

a. A large discrepancy in both quantitative and qualitative aspects of a thermal plume can exist between boat surveys and aerial infrared plume maps.

b. Although nonsynoptic, boat measurements provide much greater information on the vertical extent of the plume.

c. Aerial infrared measurements sense only the top few millimeters of the surface and are thus strongly dependent on surface heat transfer.

d. Although neither measurement technique is totally satisfactory, surface-vessel measurements generally give a better estimate of the extent of the thermal plume.

These conclusions are in general agreement with numerous results reported in the literature. However, these results have special meaning for the August 30, 1973, thermal-plume surveys. The discrepancy between the two measurement techniques was large. During part of the day, for example, the two plumes appeared to be moving in different directions. The difference between measured temperatures 200 ft from the outfall was as much as 8 F°, and at 500 ft the discrepancy was as much as 6 F°. Even at 1500 ft, the difference was as much as 4 F°.

Aero-Marine Surveys state,¹⁰ however, that particularly adverse conditions prevailed on the date of the survey--high humidity, high air temperatures, and stagnant meteorological conditions were all present. These factors strongly influence surface heat transfer and thus the temperatures measured remotely. Indeed, very sharp vertical temperature gradients were observed near the surface.

3. Reduction of the MIT Data for Model-data Comparison

To facilitate model-data comparison, the MIT investigators have reduced their limited point data to more interpretable form by:

- a. Sketching in surface isotherms.
- b. Using these isotherms to generate temperature-decay curves.
- c. Drawing qualitative views of the vertical cross section along the plume centerline.

Their approach follows the standard avenues of data reduction. Although the same general criticisms to be presented here apply in some degree to most other field studies, the weaknesses are stated in the context of this specific program. First, some degree of interpolation and extension of the data is necessary to close isotherm contours, since temperature measurements are restricted to fixed transects. Errors thus produced in reported areas are generally argued to be small due to the integral nature of the area measure, although errors of 20-50% are conceivable.

A second criticism of the data is the lack of ambient current measurements. The investigators seek to minimize the importance of this omission through a sensitivity study of the Stolzenbach-Harleman model to low ambient currents. Stolzenbach-Harleman model calculations for various values of ambient current are shown in Fig. IV.D.15 as presented by Pagenkopf et al.⁹

A family of predicted temperature decay curves is shown for an initial densimetric Froude number of 1.7. Here, V/U_0 is the ratio of the ambient current speed V to the discharge velocity U_0 , F_0 is the initial densimetric Froude number, k is the heat-loss coefficient normalized by the product of the density of water and its specific heat, and A is the aspect ratio as defined by Stölzenbach and Harleman, i.e., h_0/b_0 , where h_0 is the depth of the discharge channel and b_0 is its half-width. There are two somewhat disquieting aspects of Fig. IV.D.15. First, only the stagnant case and the case of $V/U_0 = 0.105$ ran successfully for more than 200 ft. Why this is true is difficult to understand when three intermediate ($0 < V/U_0 < 0.105$) cases did not run successfully. Second, an obvious lack of consistency exists in the model calculations, since no apparent trend with ambient current speed is evident. All data that we have examined support the belief that plume characteristics change smoothly with current speed.

Fig. IV.D.15. Centerline Temperature-decay Predictions of the Stölzenbach-Harleman Model, Showing the Effects of Ambient Current.
(Adapted from Pagenkopf et al.⁹)

At this time, this behavior cannot be traced to its cause in the model formulation. One explanation may lie in the general difficulties encountered with the computer code, as discussed in Sec. II.C. Also given in that section is our opinion that the model should not be extended to large ambient currents such as $V/U_0 = 0.25$ and $V/U_0 = 0.50$ in Fig. IV.D.15, because several assumptions made in model development break down for these large currents.

A comparison of results given later in this section with results from Point Beach, as described in Sec. F of this chapter, shows the Pilgrim plume to be smaller than the one at Point Beach, even though the two plants are at about the same power level and initial densimetric Froude number (at Pilgrim high tide). This difference may well be due to the wind-tide-produced

shear current at Pilgrim. In short, we feel that the application of steady-plume, stagnant-ambient model predictions to a case where a transient, shear current probably exists is an unwise and basically unfair test of the model. Boston Edison¹⁷ states that a current meter was moored 1 ft below the surface in the vicinity of the plant during most of 1973 and that some data on current may be available from that record. These data may be useful in explaining some of the phenomena observed at Pilgrim. Moreover, we do not feel that model calculations, even if consistent, can be used to remove the need for current measurements, since wind-induced and tidal currents are known to be present.

Temperature-decay curves were drawn from the plume maps by measuring the approximate centerline distance to an isotherm. As with the isotherms themselves, considerable interpolation of the data was needed, since neither the exact shape of the isotherm nor the centerline trajectory were accurately known. Different interpolations of the data can lead to different decay curves. However, in lieu of a more exact method of making model comparisons, this technique serves the immediate need and is believed to be an acceptable way of portraying data trends.

The above two abstractions of the data are needed for quantitative comparison with the analytical model. Pagenkopf et al.⁹ also give a sequence of vertical temperature cross sections that provide valuable insight into the basic nature of the plume. The cuts that are made along the approximate centerline of the plume illustrate the roles that buoyancy and bottom interaction play. These cross sections aid in assessing the amount of bottom interaction that occurs and in locating the point of jet separation from the bottom.

Figures IV.D.16-26 are the plots presented by Pagenkopf et al.⁹ The symbol T_a is used here to denote the ambient temperature. These plots, which are useful in picturing the Pilgrim plume, make apparent that the high-tide plume (low initial densimetric Froude number) is highly buoyant, while the low-tide plume (high initial densimetric Froude number) exhibits prolonged bottom contact.

The MIT investigators did not compare measured plume widths with model predictions. Our experience has been that plume widths are particularly important for detecting anomalies in Stolzenbach-Harleman model predictions. In most cases, predicted widths are too large at lower isotherms, and this behavior of the model, we believe, is due to the buoyant lateral-spreading velocity that takes effect at lower isotherms. The role of this velocity is more easily seen from a plot of plume widths than from any other method of data presentation.

4. Schematizing the Outfall

As previously noted, it is not easy to schematize the Pilgrim outfall as the rectangular channel required by the Stolzenbach-Harleman model.

Fig. IV.D.16. Vertical Cross Section of the Pilgrim Plume for an Initial Densimetric Froude Number of 11.6. (Adapted from Pagenkopf et al.⁹) ANL Neg. No. 190-1528.

Fig. IV.D.17. Vertical Cross Section of the Pilgrim Plume for an Initial Densimetric Froude Number of 8.0. (Adapted from Pagenkopf et al.⁹) ANL Neg. No. 190-1529.

Fig. IV.D.18. Vertical Cross Section of the Pilgrim Plume for an Initial Densimetric Froude Number of 2.2. (Adapted from Pagenkopf et al.⁹) ANL Neg. No. 190-1530.

Fig. IV.D.19. Vertical Cross Section of the Pilgrim Plume for an Initial Densimetric Froude Number of 3.9. (Adapted from Pagenkopf et al.⁹) ANL Neg. No. 190-1531.

Fig. IV.D.20. Vertical Cross Section of the Pilgrim Plume for an Initial Densimetric Froude Number of 3.2. (Adapted from Pagenkopf et al.⁹) ANL Neg. No. 190-1532.

Fig. IV.D.21. Vertical Cross Section of the Pilgrim Plume for an Initial Densimetric Froude Number of 1.9. (Adapted from Pagenkopf et al.⁹) ANL Neg. No. 190-1533.

Fig. IV.D.22. Vertical Cross Section of the Pilgrim Plume for an Initial Densimetric Froude Number of 6.8. (Adapted from Pagenkopf et al.⁹) ANL Neg. No. 190-1534.

Fig. IV.D.23. Vertical Cross Section of the Pilgrim Plume for an Initial Densimetric Froude Number of 2.6. (Adapted from Pagenkopf et al.⁹) ANL Neg. No. 190-1535.

Fig. IV.D.24. Vertical Cross Section of the Pilgrim Plume for an Initial Densimetric Froude Number of 2.3. (Adapted from Pagenkopf et al.⁹) ANL Neg. No. 190-1536.

Fig. IV.D.25. Vertical Cross Section of the Pilgrim Plume for an Initial Densimetric Froude Number of 2.9. (Adapted from Pagenkopf et al.⁹) ANL Neg. No. 190-1537.

Fig. IV.D.26. Vertical Cross Section of the Pilgrim Plume for an Initial Densimetric Froude Number of 4.6. (Adapted from Pagenkopf et al.⁹) ANL Neg. No. 190-1538.

The situation was complicated by the presence of several large boulders that fell into the discharge canal. MIT adopted the following procedure to determine discharge width, depth, and densimetric Froude number. First, the depth of water in the channel is carefully measured using a tide gauge. From this height and the known positions of the fallen boulders, the effective discharge area is calculated. The width of the discharge is then taken to be the ratio of the effective discharge area to the measured depth. Plant-reported data are used to obtain the discharge temperature and to obtain the discharge flow rate from which the discharge velocity is calculated. The danger in using such data is that there may be a lack of calibration agreement between the plant and field systems.

This information is sufficient to calculate model parameters such as initial densimetric Froude number and aspect ratio. Table IV.D.2 summarizes the parameter values used in model calculations. The first four columns are defined in accordance with the model idealization as described in Sec. II.C. The last column is an estimate of maximum plume depth, assuming that free vertical entrainment could occur. This depth is calculated with a formula derived from the Stolzenbach-Harleman³ model:

$$h_{\max} = \begin{cases} 0.42\sqrt{h_0 b_0} F' + 1, & F' \leq 3.0, \\ 0.42\sqrt{h_0 b_0} F', & F' > 3.0, \end{cases}$$

where

$$F' = F_0 A^{1/4}.$$

TABLE IV.D.2. Summary of Discharge Parameters for Pilgrim Data

Date	Aspect Ratio	Tidal Stage	Initial Densimetric Froude Number	Discharge Velocity, ft/sec	Discharge Depth, ft	Maximum Plume Depth as Predicted by Model
4-19-73	0.47	High	1.7	2.3	8.2	8.0
1-26-73	0.33	High	2.3	1.9	7.5	9.0
4-19-73	0.46	High	2.7	3.2	6.8	7.5
3-25-73	0.48	Mid	4.5	6.0	5.0	11.5
1-26-73	0.25	Mid	4.8	3.2	5.0	14
1-26-73	0.23	Mid	5.8	3.7	4.5	16
3-25-73	0.39	Mid	6.7	7.3	4.1	15
1-26-73	0.19	Mid	8.9	5.0	3.5	20
4-19-73	0.39	Low	10.9	7.5	4.1	24
4-19-73	0.39	Low	11	7.5	4.1	24
3-25-73	0.39	Low	11	7.3	4.1	24
3-25-73	0.39	Low	11.3	7.3	4.1	25
1-26-73	0.14	Low	15.9	7.4	2.5	27
7-2-73	0.39	Low	11.6	7.6	4.1	25
7-3-73	0.37	Low	11.5	8.0	3.9	24
8-30-73	0.45	Mid	9.3	6.0	4.7	19
8-30-73	0.44	Mid	8.7	6.1	4.6	18
8-30-73	0.44	Mid	8.5	6.1	4.6	18
7-3-73	0.47	Mid	7.7	6.0	5.2	20
8-30-73	0.46	Mid	6.7	5.8	4.9	17
8-30-73	0.47	Mid	7.2	4.9	5.3	16
11-13-73	0.47	Mid	4.6	2.7	7.9	18.4
11-13-73	0.47	Mid	4.5	2.6	8.1	18.5
8-30-73	0.46	Mid	3.8	3.1	6.6	7.8
8-30-73	0.46	Mid	3.7	3.5	6.2	12
11-13-73	0.48	Mid	3.4	2.1	9.1	10.8
7-3-73	0.46	Mid	3.2	2.9	7.3	8.6
7-3-73	0.46	Mid	3.2	2.9	7.6	9
11-13-73	0.49	High	2.9	1.8	9.8	10.9
11-13-73	0.5	High	2.8	1.8	10.0	11
11-13-73	0.5	High	2.6	1.7	10.2	11
11-13-73	0.5	High	2.5	1.6	10.5	11
11-13-73	0.5	High	2.3	1.6	10.8	11
7-3-73	0.48	High	2.3	2.2	8.7	9
8-30-73	0.47	High	2.2	2.2	8.0	7.7
7-3-73	0.48	High	2.1	2.1	8.9	9
8-30-73	0.48	High	1.9	1.8	8.9	8.2
8-30-73	0.49	High	1.9	1.5	9.8	8.5
8-30-73	0.5	High	1.3	1.4	10.1	8.5

5. Presentation and Analysis of Results

In addition to the Stolzenbach-Harleman model⁵ described in Chapter II (referred to by MIT as the "theory without bottom slope"), an earlier version of the model³ ("theory with bottom slope") was compared with the data. This earlier version contains a treatment of bottom interference that was found lacking in comparisons with hydraulic-model data. Figure IV.D.27 gives the results of these comparisons. Plotted in that figure are measured and predicted jet widths for several cases showing the effects of bottom slope in the hydraulic tank. Here, S_x is the bottom slope in vertical feet per horizontal foot. For both the high and low initial densimetric Froude numbers, the data show an approximately linear growth with the high initial densimetric Froude number indicating the greatest scatter. The three theory-with-bottom-slope predictions give widths much smaller than those represented by the data, but the qualitative

nature of these predictions is correct. The theory without bottom slope ($S_x = \infty$) gives widths that grow rapidly with centerline distance. Furthermore, the asymptotic behavior of the predicted curves is incorrect, that is, progressively larger values of bottom slope trend away from the limiting case of infinite bottom slope. Also note (see more detailed discussion in Chapter II) that we were never able to get the code for the theory with bottom slope to run successfully on our computer.

Fig. IV.D.27. Comparison of Stolzenbach-Harleman-predicted Plume Widths and Hydraulic-model Data, Including Both the Theory with and the Theory without Bottom Slope. (Adapted from Stolzenbach and Harleman.³)

In the first report on the Pilgrim field studies,⁷ MIT tried a rather unusual method of calculating isotherm areas from analytical model results. Centerline-temperature-decay curves from the theory without bottom slope and width predictions from the theory with bottom slope were "combined" to determine isotherm areas. Pasting together two separate model predictions has no theoretical justification. The comparisons were not significantly improved by this procedure, and the "combined theory" was later abandoned.

Fewer than a third of the 39 plume surveys have been reduced to temperature-decay form, compared to the analytical model, and then published. The missing data could be helpful in further testing the Stolzenbach-Harleman model. Published temperature-decay curves are given in Figs. IV.D.28-37 as

Fig. IV.D.28. Comparison of Stolzenbach-Harleman Predicted Centerline-temperature Decay with Field Data from the Pilgrim Site; High Tide, April 19, 1973. (Adapted from Doret et al.⁷)

Fig. IV.D.29. Comparison of Stolzenbach-Harleman Predicted Centerline-temperature Decay with Field Data from the Pilgrim Site; High Tide, August 30, 1973. (Adapted from Pagenkopf et al.⁹)

Fig. IV.D.30. Comparison of Stolzenbach-Harleman Predicted Centerline-temperature Decay with Field Data from the Pilgrim Site; High Tide, November 13, 1973. (Adapted from Pagenkopf et al.⁹)

Fig. IV.D.31. Comparison of Stolzenbach-Harleman Predicted Centerline-temperature Decay with Field Data from the Pilgrim Site; High Tide, January 26, 1973. (Adapted from Doret et al.⁷)

Fig. IV.D.32. Comparison of Stolzenbach-Harleman Predicted Centerline-temperature Decay with Field Data from the Pilgrim Site; High Tide, April 19, 1973. (Adapted from Doret et al.⁷)

Fig. IV.D.33. Comparison of Stolzenbach-Harleman Predicted Centerline-temperature Decay with Field Data from the Pilgrim Site; High Tide, July 3, 1973. (Adapted from Pagenkopf et al.)

Fig. IV.D.34. Comparison of Stolzenbach-Harleman Predicted Centerline-temperature Decay with Field Data from the Pilgrim Site; Mid-tide, August 30, 1973. (Adapted from Pagenkopf et al.⁹)

Fig. IV.D.35. Comparison of Stolzenbach-Harleman Predicted Centerline-temperature Decay with Field Data from the Pilgrim Site; Low Tide, July 3, 1973. (Adapted from Pagenkopf et al.⁹)

Fig. IV.D.36. Comparison of Stolzenbach-Harleman Predicted Centerline-temperature Decay with Field Data from the Pilgrim Site; Low Tide, April 19, 1973. (Adapted from Pagenkopf et al.⁹)

Fig. IV.D.37. Comparison of Stolzenbach-Harleman Predicted Centerline-temperature Decay with Field Data from the Pilgrim Site; Low Tide, March 25, 1973. (Adapted from Pagenkopf et al.⁹)

adapted from Doret et al.⁷ and Pagenkopf et al.⁹ The figures are arranged by increasing discharge densimetric Froude number.

Figures IV.D.28-33 give the temperature-decay curves for the high-tide cases. The success of the model ranges from poor to fair to excellent. In each case, one curve fits the data rather well. Unfortunately, the model calculation is not always the same. The best overall agreement belongs to the theory with bottom slope. This result is curious, since the cross sections of Figs IV.D.16-26 show that buoyancy restricts bottom contact at low densimetric Froude numbers.

The case of lowest initial densimetric Froude number shown in Fig. IV.D.28 has several illustrative features. First, the data indicate a very sharp decay beginning at a centerline distance of about 150 ft. Over the next 100 ft, the temperature excess drops more than 50%. A highly buoyant jet does not usually exhibit such good dilution. Second, the run for the theory without bottom slope for zero ambient current has a temperature plateau at θ_c/θ_o approximately equal to 0.6. Most field data with which we are familiar show no such plateau, but, rather, exhibit a smooth decay of temperature excess from its initial value at the end of the flow establishment to its asymptotic value far from the outfall. We return to this point later.

Figure IV.D.34 shows the temperature decay of what could be called a midtide plume ($F_o = 0.3$). Due to the rapidly changing physical parameters at midtide, the adequacy of a steady-state model is the most doubtful for these cases. As expected, the model-data comparisons are poor.

Also note that here there is no temperature plateau such as that seen in the high-tide predictions. Our experience has been that predictions will show no plateau if the initial densimetric Froude number is above about 6. Stolzenbach⁹ suggests that his model is more reliable for larger densimetric Froude numbers.

At low tide (Figs. IV.D.35-37), the initial densimetric Froude number is large (greater than 10) while the aspect ratio (outfall depth over half-width) is small. Physical parameters are more nearly constant at low tide, because, whenever the tide is below about 4 ft, the depth of flow in the discharge canal is the critical depth of a free-sloping open channel. The initial densimetric Froude number and aspect ratio remain approximately constant for an hour or more. Thus, the use of a steady-state model is perhaps acceptable. Yet, because of the high initial densimetric Froude number, significant bottom interaction is possible. For example, with an initial densimetric Froude number of 11.3, the value of h_{max} is 35 ft, which is greater than the depth of water anywhere in the region of the survey. Low-tide comparisons are thus far too optimistic. Even the theory with bottom slope fails to come within the scatter of the data.

Figures IV.D.38-43 are plots of measured isotherm areas organized by date. The data show much scatter among themselves with no definitely

Fig. IV.D.38

Measured Surface Isotherm-area Data
for Surveys Taken at the Pilgrim Sta-
tion on January 20, 1973

Fig. IV.D.39

Measured Surface Isotherm-area Data
for Surveys Taken at the Pilgrim Sta-
tion on March 25, 1973

Fig. IV.D.40

Measured Surface Isotherm-area Data
for Surveys Taken at the Pilgrim Sta-
tion on April 19, 1973

Fig. IV.D.41. Measured Surface Isotherm-area Data for Surveys Taken at the Pilgrim Station on July 2 and 3, 1973

Fig. IV.D.42. Measured Surface Isotherm-area Data for Surveys Taken at the Pilgrim Station on August 30, 1973

Fig. IV.D.43. Measured Surface Isotherm-area Data for Surveys
Taken at the Pilgrim Station on November 13, 1973

consistent trend with tidal stage (initial densimetric Froude number) established. In preparing the Final Environmental Statement for Pilgrim Units 2 and 3,^{12,14} the regulatory staff conducted a statistical analysis of the plume areas and concluded that there is no statistically significant difference between high- and low-tide plumes.

Measured plume areas and model predictions are compared in Figs. IV.D.44-48. The most striking feature of these graphs is the fact that, with the possible exception of the August case, model predictions are too small.

Fig. IV.D.44. Comparison of Stolzenbach-Harleman-predicted Plume Areas with Field Data Taken at the Site January 26, 1973
(Adapted from Pagenkopf et al.⁹)

Fig. IV.D.45. Comparison of Stolzenbach-Harleman-predicted Plume Areas with Field Data Taken at the Site April 19, 1973. (Adapted from Pagenkopf et al.⁹)

Fig. IV.D.46. Comparison of Stolzenbach-Harleman-predicted Plume Areas with Field Data Taken at the Site July 3, 1973. (Adapted from Pagenkopf et al.⁹)

Fig. IV.D.47. Comparison of Stolzenbach-Harleman-predicted Plume Areas with Field Data Taken at the Site August 30, 1973. (Adapted from Pagenkopf et al.⁹)

Fig. IV.D.48. Comparison of Stolzenbach-Harleman-predicted Plume Areas with Field Data Taken at the Site November 13, 1973. (Adapted from Pagenkopf et al.⁹)

at higher isotherms and too large at lower isotherms: The S-like character of the model predictions is not generally observed in the data. The plateau in each isotherm area occurs at about the same temperature excess ratio as the one in centerline distance noted previously.

Pagenkopf et al. have noted the general lack of correlation between plume areas and tidal stage and suggest that wind is an important controlling factor at the site. To substantiate this hypothesis, they present a sequence of three graphs that show wind speed and measured area within the 5 C° isotherm as functions of time. Two of these plots are shown in Figs. IV.D.49 and 50.

Fig. IV.D.49. Comparison of Area within the 5 and 8 C° Excess Isotherms with Wind Speed and Air Temperature for July 3, 1973. (Adapted from Pagenkopf et al.⁹)

Fig. IV.D.50

Comparison of Area within the 5 and 9 C° Excess Isotherms with Wind Speed for January 26, 1973. (Adapted from Pagenkopf et al.⁹)

The July 3, 1973 case (Fig. IV.D.49) shows a strong correlation between area and wind speed. A change in wind speed from 0 to 15 mph apparently produces an order of magnitude decrease in plume area. The January 26, 1973, case (Fig. IV.D.50) shows an almost negligible correlation of wind speed with isotherm area. The isolated episodes portrayed in these plots are not totally convincing. The arbitrary choice of the 5 C° isotherm and the failure to consider the stochastic nature of the data make the results rather inconclusive. More supportive of the belief that wind is a significant factor at the site are the visual observations reported by a number of different investigators under differing circumstances.

The similarity of high- and low-tide plumes and the observation that wind influences plume size reflect negatively on integral models that consider only such discharge parameters as densimetric Froude number and aspect ratio and ignore key environmental parameters.

Boston Edison¹³ compared Pritchard's phenomenological model to selected conglomerate data at high-, low-, and midtidal stages. In general, the model underpredicted dilution at high tide and overpredicted dilution at low tide. At low tide, both the Stolzenbach-Harleman and Pritchard models predict a strong dilution that is not evidenced by the data. As many investigators have pointed out, this state may well be due to the bottom interference that occurs at low tide.

6. Summary of Model-data Comparisons and Comment

To summarize the Pilgrim model-data comparisons, we have plotted on a single graph all available isotherm area data. The justification for presenting the aggregate data without consideration of tidal stage is the previously noted fact that measurements show no statistically significant difference between data from various tidal stages. Figure IV.D.51 compares the data with three MIT predictions:

a. Hydraulic-model studies of the discharge conducted in August 1968 by Harleman and Stolzenbach and presented in the Applicant's Environmental Report,² September 1970.

b. Stolzenbach-Harleman analytical-model predictions for a high-tide case, namely, January 26, 1973, $IF_0 = 2.3$.

c. Stolzenbach-Harleman analytical-model predictions for a low-tide case, namely, March 25, 1973, $IF_0 = 11.0$.

To fully appreciate this plot, several features of the data and model calculations must be recalled. First, the Vast and Coastal Research Corp. data were all taken during December 1972 and represent, therefore, winter-plume measurements. In contrast, the MIT data were taken during January, March, April, July, August, and November 1973 and represent nearly all seasons of the year. Second, the hydraulic-model predictions were made prior to

plant operation, using a distorted model, before the undesirability of distortion in near-field hydraulic models was fully recognized. Third, the analytical-model predictions do not include the treatment of bottom interaction appearing in the original work of Stolzenbach and Harleman.

Fig. IV.D.51. Comparison of All Presently Available Data from the Pilgrim Plant with Selected Stolzenbach-Harleman-model Calculations

The hydraulic-model prediction lies within the range of the data and accurately portrays their trend. Neither of the analytical-model predictions do as well: The low-tide (high initial densimetric Froude number) prediction significantly overpredicts dilution, failing to come within even the two-order-of-magnitude scatter among the data; the high-tide prediction falls closer to the data, but misrepresents their trend. The previously noted temperature plateau appears at a relative excess temperature of about 0.6, but the data show no such plateau. In our opinion, the fact that the predictions are conservative at lower isotherms does not outweigh the model's failure to qualitatively replicate the trend in the data.

Figures IV.D.52-54 compare predicted and measured surface isotherms for typical high-, mid-, and low-tide cases. Each offers unique insights into the Stolzenbach-Harleman model. Consider first the high-tide comparison of Fig. IV.D.52. The first four predicted isotherms are smaller than their measured counterparts, but generally have the right shape. Isotherms for which $\theta/\theta_0 = 0.50$ or less are quantitatively and qualitatively in disagreement with measured isotherms. Moreover, the dramatic change in isotherm shape occurs at $\theta/\theta_0 = 0.6$ or roughly the same place as the temperature plateau pointed out in Figs. IV.D.31 and 51. Our explanation of these occurrences follows. At the higher isotherms, dilution is approximately correctly predicted by classical jet theory, i.e., nonbuoyant jet theory, modified according to the results of Ellison and Turner. As the local densimetric Froude number drops

with dilution, a point is eventually reached where the fictitious lateral-spreading velocity postulated in model development dominates, causing the plume to spread rapidly and unnaturally. Resulting isotherms take the shape of coathangers. This artificial spreading greatly distorts the aspect ratio of the jet, cutting off dilution. A thin, flat, nondiluting plume results. The above sequence of events is at odds with observed plume behavior.

Fig. IV.D.52. Comparison of Measured and Stolzenbach-Harleman Surface Isotherms at High Tide

Fig. IV.D.53. Comparison of Measured and Stolzenbach-Harleman-predicted Surface Isotherms at Midtide

Fig. IV.D.54. Comparison of Measured and Stolzenbach-Harleman-predicted Surface Isotherms at Low Tide

For the midtide case shown in Fig. IV.D.53, the predicted plume is much too small. Also interesting in this comparison is the shape of the isotherm at $\theta/\theta_0 = 0.20$. We speculate that the bulge in the middle of this isotherm is also due to the artificial spreading velocity assumed in model development. Surface heat loss is the most likely reason for the isotherm not assuming the coathanger shape as in the previous case. The low-tide comparison (Fig. IV.D.54) is dramatic in showing the vast overprediction of dilution made by the model.

The question to be asked is, "Does the Stolzenbach-Harleman model really apply to Pilgrim and, if not, are the model-data comparisons fair?" The answer, as we see it, is a qualified no. The Stolzenbach-Harleman model does not apply to Pilgrim, since it does not consider what now appears to be the predominant phenomena at the site. The qualification arises from the anomalies that the predictions show. We feel that the appearance of temperature plateaus in both temperature decay and area predictions and the erratic behavior of the model as crosscurrent is varied are strong indictments of the model that cannot be attributed to the site.

Having concluded that the model does not apply at Pilgrim, one is tempted to ask, "Where then does the model apply?" The model-data comparisons presented here provide little assistance in answering that question. We can, however, restate our current opinion, which is based on our experience with the model. First, we have serious doubts about the adequacy of the model for nonstagnant cases. Second, we feel the model should be restricted to initial densimetric Froude numbers greater than about 6. Third, we have found that excessive lateral widths can be expected whenever the local densimetric Froude number becomes about 2 or less. Fourth, we recommend that

the model not be used to analyze shallow-water discharges where restriction of vertical entrainment can occur. These limitations rule out most existing surface discharge sites.

Figure IV.D.55 compares the data with four Pritchard predictions:

- a. Model calculations for January 26, 1973. Here, again $F_0 = 2.3$. We have used the same outfall width and depth as in the Stolzenbach-Harleman calculation and have assumed no reentrainment.
- b. Same as a, except that we have assumed a background temperature excess that is 10% of the discharge temperature excess.
- c. Model calculations for March 25, 1973. Here, again $F_0 = 11.0$. We have used the same outfall width and depth as in the Stolzenbach-Harleman calculation and have assumed no reentrainment.
- d. Same as c, except that we have assumed a background temperature excess that is 10% of discharge temperature excess.

Fig. IV.D.55. Comparison of All Available Data from the Pilgrim Plant with Selected Pritchard-model Predictions

Several features of this plot warrant comment. First, all predictions were prepared allowing full vertical entrainment of the plume to Pritchard's critical mixing depth of 10 ft. The model relies somewhat on the insight of the user and on his ability to select the proper options within the framework set forth by Pritchard. Calculations were also prepared assuming various possible limitations on vertical entrainment. The principal effect of these variations was the slight displacement of the shoulder in each curve corresponding to the point at which vertical entrainment ceases. On lower isotherms the effect was small, and these results are not shown.

Second, although Pritchard's model is not based on a densimetric Froude number, it still predicts at low tide improved dilution that is not evidenced by the data. This condition suggests that although the densimetric Froude number is not explicitly included in the model, the application of Pritchard's methodology appears to have the same effect.

Third, the Pritchard calculations tend to envelop the data, although in some cases the data do show a more pronounced shoulder than do the model calculations. As stated above, the model predictions would have been improved in this respect had some limitations been imposed on vertical entrainment.

Fourth, with the possible exception of the low-tide recirculation case, the overall agreement with the data is good. The predictions are qualitatively valid and of the right order of magnitude.

Figures IV.D.56-58 compare measured isotherms with those predicted by the Pritchard (Model No. 1) phenomenological model. The high-tide comparison is presented in Fig. IV.D.56. Here, the measured isotherms show a distinct east edge; i.e., isotherms are compacted on the east side of the plume. Sharp-edged plumes have often been observed in the field and are generally thought to be the result of wind and/or ambient-current interaction with the plume. Clearly, the model cannot predict this behavior, since neither wind nor ambient current were included in its development. Despite this weakness, the model gives a conservative estimate of plume size that should, in light of the present state of the art, be considered a successful prediction.

Figures IV.D.57-58 compare measured and predicted isotherm maps at mid and low tide. In these cases, the Pritchard model, like the one

Fig. IV.D.56. Comparison of Measured and Pritchard-predicted Surface Isotherms at High Tide

Fig. IV.D.57. Comparison of Measured and Pritchard-predicted Surface Isotherms at Midtide

Fig. IV.D.58. Comparison of Measured and Pritchard-predicted Surface Isotherms at Low Tide

of Stolzenbach and Harleman, greatly overpredicts dilution. The predictions would have been somewhat improved had the cutoff of vertical entrainment by bottom interaction been included in the schematization. In a sense, then, these predictions represent the model's worst performance. The isotherms measured at midtide again show a distinct edge, except that this time it is to the west of the outfall. In the high-tide case, the plume appears to be bending in a westward direction. Both of these observations suggest that wind and/or ambient currents are at work.

Another aspect of the comparisons deserves comment. Why do the data not exhibit greatly increased dilution at low tide? Based on the data acquired so far at the site, bottom interference is the most plausible explanation. However, the dissipation of momentum by the free overfall is a point that has been overlooked so far. Temperatures and velocities may be more validly measured at a point near the discharge, but within the plume, and used as the initial values for model calculations. The parameters cited for low-tide plumes may not be at all indicative of the characteristics of that plume if significant momentum dissipation occurs at the discharge point.

One final point to be emphasized is that the lack of current measurements makes it nearly impossible to identify important trends in the data. The data may have been more interpretable had they been grouped by tidal stage and current (direction and magnitude), a grouping that may have explained much of the scatter. Some effort could be made in that direction by using the meteorological data and the presumed correlation with current, although the indirect nature of such analyses would weaken any conclusions that could be drawn.

7. Summary and Conclusions

The discharge of the Pilgrim Nuclear Power Station is one of the most extensively studied in the world, and there is much to be learned from the associated laboratory, analytical, and field studies that have been carried out. The circumstances provide the rare opportunity to witness the success or failure of both hydraulic- and analytical-model studies.

The site is beset by strong wind effects, tidal currents, bottom interaction, and recirculation possibilities. The shape of the discharge canal and the shoreline is irregular, and a free overfall exists when the tide falls below mean low water. Many problems make a straightforward schematization of the outfall impossible and thus introduce many arbitrary considerations into the model-data comparisons.

The preoperational distorted hydraulic model contained an incorrect modeling of bottom slope. Bottom effects were then thought to be insignificant; recent prototype measurements have challenged that belief.

Prototype data have been collected by remote thermal imaging, surface-vessel measurements, and drogue studies. Of all the data obtained, only a few thermal plumes have been processed to temperature-decay form for comparison with the Stolzenbach-Harleman model. No width predictions and only a few area predictions have been compared with the data. Furthermore, the absence of ambient current measurements limits the utility of the data and the ability to discern important physical effects.

The data show that good dilution is possible at both high- and low-tidal stages. In fact, there is essentially no significant difference between

high- and low-tide plumes. Further, though confirmation is lacking, midtide plumes appear to be somewhat more expansive than either high- or low-tide plumes. A qualitative study of the available data shows that low-tide plumes exhibit much vertical mixing and prolonged bottom contact, while high-tide plumes are buoyant and do not have perceptible bottom interaction.

Wind is a major factor at Pilgrim. Wind-induced currents are usually much stronger than tidal currents and, based upon limited data, vary between 0.2 and 2.0% of the wind velocity as measured by the onsite meteorological tower. Data have revealed that, under certain conditions, the plume is sharply bent moving down current along the shore.

The comparison of the Stolzenbach-Harleman model to data showed it to be inadequate for this site, and the conclusion reached is that that model does not really apply to shallow-water discharges such as Pilgrim. The low-tide predictions were in considerable error, vastly overpredicting dilution. The high-tide predictions agreed more closely with the data, though they did show an anomalous temperature plateau. The model was found to be erratic when nonzero values of crossflow velocity were input. Finally, the treatment of bottom effects was shown to be inadequate, since the theory with bottom slope agreed better with the high-tide data where no interaction occurred than it did with low-tide data where significant bottom effects were present.

The Pritchard model generally represented the data well, although it, too, overpredicted dilution at low tide, but to a lesser extent than did the Stolzenbach-Harleman simulation. Moreover, it was not compared to individual data sets as was done with the Stolzenbach-Harleman model.

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E. Oskarshamnsverket: Verification of the Prych Model

The integral model of Prych¹ was developed at the Swedish Meteorological and Hydrological Institute (SMHI) in 1972. An extensive effort to compare the model predictions with field and tank data was carried out by Weil² at SMHI, and the results were published in 1974. The model-data comparisons encompassed the following:

(1) Prych-model calculations were compared with eight sets of field data taken at Oskarshamnsverket in Sweden, along with one set of data each from the Point Beach and Waukegan power plants on Lake Michigan. Centerline-temperature decays, isotherm areas, and isotherm configurations were compared with corresponding surface predictions.

(2) Isotherm-area predictions of the Prych model were compared to the phenomenological relationship of Asbury and Frigo³ for the Oskarshamnsverket site. The Prych model was also compared with two other simple phenomenological models: the Jen-Wiegel-Mobarek correlation⁴ for heated surface jets in deep water and the Carter-Pritchard analytical/phenomenological model.⁵ Centerline-temperature decay and surface-isotherm areas observed at Oskarshamnsverket were used for this comparison.

(3) Prych-model predictions were compared with power-law representations of laboratory and field data due to Shirazi.⁶ Trajectories, centerline-temperature decays, widths, and areas were compared for an imaginary power-plant discharge into a current.

(4) The Prych model was compared with the results of two hydraulic-model tests for a coflowing discharge in which ambient turbulence was the main factor in dispersion. Only centerline-temperature decays were compared.

(5) The Prych model was coupled with the near-field results of an undistorted hydraulic model for the Forsmark Power Plant in Sweden to yield intermediate- and far-field predictions where the hydraulic-model results were inapplicable. Centerline-temperature decay, areas, and trajectories were compared with those abstracted from field data taken at the plant.

Each of these verification efforts is critically discussed.

1. Model-data Comparisons at the Oskarshamnsverket Power Plant

a. The Oskarshamnsverket Site

The major effort of Weil² was the analysis of the Oskarshamnsverket data from which eight model-data comparisons were made. A study of the site indicates that Oskarshamnsverket is by no means ideal for model evaluations. The difficulties associated with the site are perhaps a major reason why the comparisons were inconsistent and generally inconclusive. Our interest in the comparisons lies in discovering the reasons for the inconsistent results. We soon see that the Oskarshamnsverket site, like all

other sites; is not at all ideal for the application of integral models. The typical situation here is that bottom effects are not present, yet difficulties in schematizing the effluent discharge to the Baltic Sea are very severe.

Oskarshamnsverket is a 460-MWe power plant operating on the southeast coast of Sweden. At that maximum power level, the flow rate of the discharge is $22 \text{ m}^3/\text{sec}$, with an initial temperature excess of about 10°C . The advantage of model-data comparisons at this site centers around the fact

that the receiving water in the Baltic is relatively deep (6-18 m deep, once 200 m offshore). Also, shoreline obstructions are few, and a large amount of temperature data has been acquired during normal operation of the plant. A major disadvantage of this site is that the discharge is not made directly into the Baltic Sea, but, rather, is made through the Hamnefjärden inlet, which is approximately 1200 m long and 100 m wide with a 3-m-deep sill located 200 m from the sea (see Fig. IV.E.1). The heated effluent is partially diluted in its traverse through the inlet, and hence the major portion of its momentum is dissipated. Also, the profiles of velocity and temperature at the outfall are probably not uniform, as assumed in the Prych model, but must be schematized as being so. To apply the Prych model, Weil had to simulate a rectangular discharge by recalculating the excess temperature (above ambient) and the flow rate.

Fig. IV.E.1. The Oskarshamnsverket Site.
(Adapted from Weil.²)

Figure IV.E.1 shows the location of Oskarshamnsverket, along with its two intakes and the plant discharge into the Hamnefjärden inlet.

In the figure, a small arrow locates the point of discharge of the plant. This arrow is about 500 m upstream of the sill in the southwest corner of the Hamnefjärden inlet. About 800 m due south of that arrow is a small peninsula. The two arrows there represent the two inlets to the plant, each drawing water from a side of the promontory. The power plant itself is located between those arrows. At times, one or both of these inlets were used to supply cooling water to the plant. An open canal, $\approx 800 \text{ m}$ long, directs the heated effluent from the plant to the Hamnefjärden inlet for disposal. The inlet has almost zero freshwater inflow from rain and runoff.

b. Outfall Schematization

To aid in the calculation of outlet dimensions, flow rate, and temperature, measurements were undertaken at the Hamnefjärden inlet.

Vertical velocity and temperature data were acquired at one location along Transect 3 on May 3, 1972; these data and the resulting calculations done by Weil represent Step 1 in Table IV.E.1. Later, on September 27-28, 1972, vertical velocity and temperature profiles were made at three to five locations along each of Transects 1, 2, and 3. The results are discussed as Step 2 of Table IV.E.1.

We feel that the measurement procedure and the calculations based upon it are clearly inadequate to define parameters for the Hamnefjärden outlet flow. First, Step 1 is doubtful because it does not account for horizontal variations in velocity and temperature across Transect 3. Also not clear is whether the vertical profiles were taken at the center of the horizontal temperature and velocity distributions to represent measured maximum values. In actuality, the measurements were taken at the physical center of the channel inlet. Even so, a single vertical profile cannot adequately determine the flow through that large an outlet.

Second, the measurements in Step 2 yielded inconsistent results and were insufficiently refined horizontally and vertically. By continuity, the integrated flow rates through each cross section of Transects 1, 2, and 3 should all be $22 \text{ m}^3/\text{sec}$, the same as the power-plant flow rate. They are not equal to the plant flow rate, and the discrepancies very likely involve the lack of horizontal and vertical resolution at each of the three transects, as the sparse measurements made prevented continuity from being satisfied. The surface temperatures measured in Transects 1-3 indicated that dilution had occurred, yet the profiles for Transects 1 and 2 did not show any return flow. This lack of consistency indicated that the measurement network used vertically and horizontally was too loose. More definition was needed, especially near the bottom and certainly laterally near the center of the horizontal distribution.

The above two problems make schematizing the effluent discharge difficult and the calculational procedure somewhat arbitrary.

A third difficulty involves the treatment of the horizontal and vertical variation of velocity and temperature, assuming they are measured with a sufficiently refined mesh at Transect 3. How to handle these variations is a typical problem in outfall schematization. Integrating the flow and heat across the outlet to obtain a uniform "averaged" profile as input to an integral model is the usual approach and is probably adequate. Spatially variable velocity and temperature at an outlet are probably significant only within the zone of flow establishment. The lack of measurement resolution compounds the difficulties of determining single values for outlet temperature and velocity.

Weil noted that his schematization of the Hamnefjärden outlet was artificial in a number of ways. One of these ways was that fixing Q_0 , H_0 , and $2B_0$ as constants for each field survey somewhat arbitrarily determined

TABLE IV.E.1. Measurements and Calculations
Made at the Hamnefjärden Inlet to Determine
Input Parameters for the Prych Model

Step 1

Measurements were made on May 3, 1972, of the vertical profile of temperature and velocity at the center of the channel where Hamnefjärden opens to the Baltic Sea. The results of the measurements appear in Fig. IV.E.2; the measurements are located at the center of Transect 3 in Fig. IV.E.3. Transect 3 also represents the location of the origin of the model calculations.

Volume and heat flux were calculated from the measured velocity and temperature profiles to determine the overall mixing at Transect 3. The velocity profile indicated a substantial return flow of Baltic Sea water below the 3-m depth mark. The return flow enters the inlet at near-ambient temperature (6.5°C) and with a mean velocity of about 5 cm/sec. The effective width of the temperature and velocity profiles at Transect 3 was carried out as follows:

a. The velocity profile was integrated over the total 7-m depth to obtain an average velocity of 0.05 m/sec. The effective outlet width (no dilution assumed from plant to Transect 3) was then given by

$$2B_0 = \frac{\text{plant flow}}{u_0 H_0} = \frac{22.0}{(0.05)(7.0)} = 69 \text{ m.}$$

b. A similar calculation for heat flux gave $2B_0 = 57 \text{ m}$ as the effective width of the temperature profile.

c. The two width values were then averaged to give $2B_0 = 63 \text{ m}$. With this value, the velocity profile of the upper layer was integrated to the zero velocity depth ($H_0 = 3 \text{ m}$) to get an estimate of the discharge flow in just that upper layer. The calculation yielded $Q_0 = 33.4 \text{ m}^3/\text{sec}$. This value was used in Prych-model calculations. The effective dilution from the plant to Transect 3 was then $22/33.4 = 0.66$. So Weil determined that the effective $\Delta\rho$ at Transect 3 was 0.66 times the outlet density difference between the plant discharge and ambient water.

Step 2

A more thorough, though still inadequate, set of measurements was made on September 27-28, 1972, at Transects 1, 2, and 3 by the Chalmers Tekniska Högskola. These measurements yielded conflicting data as to the true mixing characteristics of the inlet. Measurement locations for that survey are shown by X's in Fig. IV.E.3. Integration of the three velocity profiles at Transect 1 yielded a net mass flow rate of $30.2 \text{ m}^3/\text{sec}$ with no return flow indicated; for Transect 2, an integrated flow rate of $23.8 \text{ m}^3/\text{sec}$ with no return flow was calculated. Weil suspects that the discrepancy may be due to (1) the transient nature of the flow over the two-day survey period or (2) the measured velocities not being normal to the section because of geographic constraints, or (3) the uncertainty in the flow conditions near the shoreline. Heat-flux calculations showed the same type of anomalies. Integrated velocities in the upper layer of Transect 3 yielded $39.4 \text{ m}^3/\text{sec}$. Some return flow did exist, but it was of insufficient magnitude to allow flow integration over the entire depth.

Although no return flow was observed from the measurements taken at Transect 1, the integrated flow yielded 30.2, not 22, m^3/sec . If the Transect 1 flow value represents the undiluted plant flow, then the dilution at Transect 3 is calculated as $30.2/39.4 = 0.77$. If the integrated flow at Transect 3 is correct, then the dilution there is $22/39.4 = 0.56$. Unable to resolve the inconsistency, Weil settles for the calculated dilution of 0.66 to represent the overall mixing inside Hamnefjärden.

The true width of Transect 3 is 67 m as compared to the calculated value 63 m. This width yields a flow rate of $33.1 \text{ m}^3/\text{sec}$ instead of 33.4, the difference being quite insignificant.

Fig. IV.E.2

Temperature and Velocity Profiles at Mouth of Hamnefjärden, Case A. (Adapted from Weil.²)

Fig. IV.E.3. Buoy Stations in Outer Part of Hamnefjärden, Oskarshamnsverket, September 1972. (Adapted from Weil.²)

the initial densimetric Froude number; the densimetric Froude number will change only through variation of $\Delta\rho$. Weil states that it is more likely that the inlet geometry determines the initial densimetric Froude number and that the overall mixing rate adjusts itself to maintain the Froude number. With this argument, the geometry of the inlet would more or less be destroying the effluent's jet momentum to the point where the densimetric Froude number at Transect 3 is almost fixed. The densimetric Froude number at the entrance to the Baltic would thus be very nearly the same for a range of power-plant-operation conditions. Reduction of the densimetric Froude number to nearly 1 at Transect 3 indicates that the effluent plume to the Baltic is a more nearly buoyant wedge, rather than a momentum-dominated jet.

In the Prych-model calculations, the overall mixing taking place in the Hamnefjärden inlet was accounted for by defining the $\Delta\rho$ of the outlet effluent to be 0.66 times that of the plant density difference (above ambient). Weil assumes that the mixing mechanics of plume dispersion beyond the inlet depends on a densimetric Froude number at the inlet; however, the plume-temperature decay for a particular plume would be more accurately represented if one began with the measured temperature (difference above ambient) at the outfall to the Baltic.

Examination of Fig. IV.E.2 reveals a sharp variation in temperature in the upper layer, making it difficult to determine a single value for the outlet-temperature excess to be used in the preparation of model calculations. The jog in the vertical temperature profile evident in this figure could easily give a 20% error in the calculated value of initial temperature excess, whereas that jog probably does not affect the overall mixing characteristics as the water flows out of the inlet. Because of these alternative effects, Weil chose $\Delta\rho$ from the overall mixing in the Hamnefjärden and the temperature difference from plume measurements. These independent choices of density and temperature differences are obviously inconsistent. However, Weil felt that if he assumed that the initial temperature differences were reflected in the bulk mixing across the little inlet section, it would not be worth reporting any of the predictive comparisons because the uncertainty in Hamnefjärden mixing might obscure the entire accuracy of the plume hydrodynamics in the Baltic. Thus, he permitted the inconsistency in $\Delta\rho$ and ΔT_O to help alleviate the problem of nonuniform temperature difference.

The angle of discharge (71°) was determined by drawing a line through the maxima of the three lateral velocity profiles measured at Transects 1-3 in Step 2 (see Table IV.E.1). This line was approximate, as only three to five velocity measurements were made at each station. The angle between that line and a straight line representing the shoreline was defined to be the angle of discharge. The line through the inlet turned out to be nearly perpendicular to the sill.

Weil's difficulty in determining Q_O , $\Delta\rho$, ΔT_O , H_O , and $2B_O$ exemplifies the usual difficulty in applying mathematical models of thermal plumes (especially of the integral type) to prototype situations. We feel that the schematization for the Oskarshamnsverket discharge is ill-determined due to data limitations; consequently, the calculations for Q_O , $\Delta\rho$, ΔT_O , H_O , and $2B_O$ are arbitrary. Numerous velocity and temperature profiles should have been taken at the Hamnefjärden outlet during each plume survey of the conditions at Transect 3. Only a fine grid of measurements could properly define the outflow and temperatures for each plume survey. Significant error is bound to result when only four vertical profiles are measured, no matter where they are selected. This poor schematization, we feel, is a major cause of the lack of consistency in the model-data comparisons, as we indicate later. We also feel that no prototype site is ideal for applying integral models. All sites have either the presence of a shallow bottom in the receiving water, an interfering

shoreline, or significant wind or transient effects. Even if the Hamnefjärden outlet conditions were known exactly, there would remain a schematization difficulty (although a lesser one) of defining uniform velocity and temperature profiles and a rectangular outlet geometry for input into the Prych integral model. That problem always exists for prototype sites.

c. The Model-data Comparisons and Discussion of the Field Data

Table IV.E.2 summarizes the basic information given by Weil² on the prototype data for the 10 Prych-model comparisons, and Table IV.E.3

TABLE IV.E.2. Prototype Data Situations for the Oskarshamnsverket, Point Beach, and Waukegan Power Plants. (Adapted from Weil.²)

Case	Location	Date	Time	Current, m/sec	Ambient Temperature, °C
A	Oskarshamn.	3 May 1972	0900-1300	0.091 SW	6.5
B	Oskarshamn.	3 October 1972	0830-1300	0.033 NE	11.5
C	Oskarshamn.	3 October 1972	1530-1640	0.105 NE	11.5
D	Oskarshamn.	4 October 1972	0820-1130	0.07 NE	11.5
E	Oskarshamn.	4 October 1972	1225-1710	0.07 NE	11.5
F	Oskarshamn.	5 October 1972	0725-1100	0.03 NE	11.5
G	Oskarshamn.	14 November 1972	1000-1430	0.075 SE	7.0
H	Oskarshamn.	20 December 1972	0845-1300	0.04 SSE	4.5
I	Point Beach	20 July 1971	1632-1805	0.0915 N	12.7
J	Waukegan	12 August 1970	1200-1357	0.18 S	23.5

TABLE IV.E.3. Correlation of Predicted Results with Oceanographic Parameters. (Adapted from Weil.²)

Input Parameter	Case									
	A	B	C	D	E	F	G	H	I	J
Month	May	Oct	Oct	Oct	Oct	Oct	Nov	Dec	July	Aug
V direction	SW	NE	NE	NE	NE	NE	SE	SSE	N	S
V, m/sec	0.091	0.033	0.105	0.07	0.07	0.03	0.075	0.04	0.092	0.18
V/u ₀	0.52	0.19	0.59	0.4	0.4	0.17	0.42	0.24	0.16	0.12
W direction	NE	SSE	S	SSW	WSW	SW	NW	WSW	S	E
W, m/sec	4	2	4-5	8	5-6	4-6	5-7	5-6	6.5	3
V/W, %	2.3	1.7	2.3	0.9	1.3	0.6	1.3	0.7	1.4	5.8
Power, MWe	415	450	450	450	450	450	460	460	480	970
Q ₀ , m ³ /sec	33.4	33.4	33.4	33.4	33.4	33.4	33.4	33.4	25.0	53.2
u ₀ , m/sec	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.56	1.47
2B ₀ , m	63.0	63.0	63.0	63.0	63.0	63.0	63.0	63.0	10.7	25.0
H ₀ , m	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	4.2	1.45
θ, °	109 ^a	71	71	71	71	71	39	56	79	90
ΔT, C°	11.5	5.5	4.5	6.0	5.5	4.0	4.0	5.0	8.8	7.0
Δρ, kg/m ³	0.76	1.0	1.0	0.96	0.97	1.02	0.61	0.47	1.53	1.92
F ₀	1.18	1.03	1.03	1.05	1.05	1.02	1.32	1.51	2.22	8.9
Correlation					Goodness of Fit ^b					
T vs distance	-,0	0	-	+	0,+	-	-	0,+	-	-
T vs area	-,0	0	-	0,+	0,+	-	-	0,+	-	-
Trajectory	0	-	+	-,+	+	+	0	-	+	+
Overall	Good	Good	Poor	Fair	Fair	Fair	Fair	Good	Fair	Poor

^aAdjusted to 90° for numerical model.

^bPlus (+) values indicate Prych-model temperatures exceeding field values; minus (-) values, the opposite. When two values are given, the plotted results cross. Plus trajectory values indicate model plumes becoming parallel to ambient current more rapidly than in the field.

lists the pertinent input parameters for the Prych model for those cases. As previously stated, mixing in the Hamnefjärden inlet was considered constant for all dates. Similarity of the velocity profile at Transect 3 from the two field surveys indicated to Weil that similarity was a good assumption. However, as noted, the outfall-temperature excess (temperature at Transect 3 minus ambient temperature) was measured at each survey and did not follow the dilution factor 0.66 in reduction. Surface temperatures apparently did not follow as closely the consistent reduction expected for the overall dilution at Transect 3.

The Prych model was modified slightly by Weil before calculations were made. First, the most important change was made in the differential equation for nonbuoyant horizontal spreading. This equation originally contained¹ the density difference $\Delta\rho$. Weil reasoned that the nonbuoyant spreading should be independent of $\Delta\rho$ and, therefore, that these terms were inappropriate. Also, minor errors in the code were corrected; some changes were made in small details and in the way in which some empirical constants were selected. To determine the effects of these changes in the model, Weil applied the original and the revised codes to a hypothetical situation at Oskarshamnsvverket of the plant discharging directly into the Baltic Sea. The results are shown in Figs. IV.E.4 and 5. Weil concluded that the revisions reduced plume widths slightly, but had little effect on centerline temperatures. The revised version of the model was used in the Weil study.

Figures IV.E.6-25 are plots of surface-isotherm area, surface centerline-temperature decay, and surface-isotherm contours for the data and Prych-model predictions. Table IV.E.3 summarizes the results of these comparisons. Here the symbols +, -, and 0, respectively, are used to indicate whether the Prych model overpredicts, underpredicts, or satisfactorily predicts the field values.

Fig. IV.E.4. Effects of Prych Program Revisions on Model Predictions for Surface Isotherm Areas and Centerline-temperature Decay. Data for Oskarshamnsvverket, Restricted Outlet $Q_0 = 22 \text{ m}^3/\text{sec}$. (Adapted from Weil.²)

Fig. IV.E.5. Effects of Prych Program Revisions on Model Predictions for Plume Width and Depth Data for Oskarshamnsverket, Restricted Outlet, $Q_o = 22 \text{ m}^3/\text{sec}$. (Adapted from Weil.²⁾)

Fig. IV.E.7. Relative Temperature vs Enclosed Area and Distance, Case A. (Adapted from Weil.²)

Fig. IV.E.8. Direct Comparison of Isotherms, Case B. (Adapted from Weil.²)

Fig. IV.E.9. Relative Temperature vs Enclosed Area and Distance, Case B. (Adapted from Weil.²)

FIELD
DATA

0 500 1000 m

PRYCH
MODEL

OSKARSHAMNSVERKET

3 OCT. 1972 P.M.

Fig. IV.E.10. Direct Comparison of Isotherms, Case C. (Adapted from Weil.2)

Fig. IV.E.11. Relative Temperature vs Enclosed Area and Distance, Case C. (Adapted from Weil,²)

OSKARSHAMNSVERKET
4 OCT 1972 AM.

Fig. IV.E.12. Direct Comparison of Isotherms, Case D. (Adapted from Weil.²)

Fig. IV.E.13. Relative Temperature vs Enclosed Area and Distance, Case D. (Adapted from Weil.²)

Fig. IV.E.14. Direct Comparison of Isotherms, Case E. (Adapted from Weil.²)

Fig. IV.E.15. Relative Temperature vs Enclosed Area and Distance, Case E. (Adapted from Weil,²)

Fig. IV.E.16. Direct Comparison of Isotherms, Case F. (Adapted from Weil.²)

Fig. IV.E.17. Relative Temperature vs Enclosed Area and Distance, Case F. (Adapted from Weil.²)

Fig. IV.E.18. Direct Comparison of Isotherms, Case G. (Adapted from Weil.²)

Fig. IV.E.19. Relative Temperature vs Enclosed Area and Distance, Case G. (Adapted from Weil.²)

OSKARSHAMNSVERKET
20 DEC 1972

Fig. IV.E.20. Direct Comparison of Isotherms, Case H. (Adapted from Weil.²)

Fig. IV.E.21. Relative Temperature vs Enclosed Area and Distance, Case H. (Adapted from Weil.²)

Fig. IV.E.22. Direct Comparison of Isotherms, Case I. (Adapted from Weil.²)

Fig. IV.E.23. Relative Temperature vs Enclosed Area and Distance, Case I. (Adapted from Weil.²)

Fig. IV.E.24. Direct Comparison of Isotherms, Case J. (Adapted from Weil.²)

Fig. IV.E.25. Relative Temperature vs Enclosed Area and Distance, Case J. (Adapted from Weil.²)

The Baltic bottom topography does not influence the plume because the water beyond the Hamnefjärden inlet is quite deep. This is the reason why the site was chosen to verify the Prych model. The plume measurements indicated no bottom interaction, and the Prych model correctly predicted no interaction. Only when the flow- and heat-rejection rates were substantially increased to simulate several additional units at the plant site did the Prych model indicate bottom interaction. Predicted asymptotic plume depths for a single plant varied over the range of 2-4 m. Measured asymptotic plume depths were 1.5-2 m.

Little thermal stratification exists at the Oskarshamnsverket site during most months of the year. The maximum summer stratification is on the order of $1\frac{1}{2}^{\circ}\text{C}$, with a fairly distinct thermocline. The top layer consumes most of the water column.

The SMHI plume measurements were taken during May, October, November, and December. In Swedish coastal waters, stratification, if any, is extremely weak and of no consequence during these months. Consequently, the plume is discharged into water of nearly uniform density.

In contrast to the schematization problems at Oskarshamnsverket, it was much simpler to compare the Prych model to the field data taken at Point Beach and Waukegan.

Outlet parameters for the Point Beach comparison were well defined. However, at Waukegan, the irregular discharge canal had to be approximated as a rectangular slot 25×1.45 m; a width of 25 m was obtained through approximate measurement, and the depth was obtained through conservation of initial momentum. The initial flow was adjusted from $Q_0 = 49.0$ to $53.2 \text{ m}^3/\text{sec}$ to account for the fact that the outlet temperature was lower than the reported plant-discharge-channel temperature.

From analyzing these comparisons, Weil concludes that the accuracy of the Prych model varies widely among the cases presented. Plumes I and J at Point Beach and Waukegan do have bottom effects that indeed distort the jet characteristics from the bottomless-lake condition the model is simulating. The Prych model overpredicts dilution, since the centerline temperatures and isotherm areas are much less than indicated from the data. However, it is doubtful that the large differences between model predictions and the data shown in these comparisons are caused entirely by bottom restrictions, as suggested by Weil.

A description of the method of data acquisition would be useful here in determining a possible source of error in the model-data discrepancies at Oskarshamnsverket. SMHI used a small boat with two thermistors mounted on the bow, one at a depth of 0.5 m and the other at 1.5 m. The boat traveled a zigzag pattern, crossing the plume centerline on every linear segment of the path. The transects were nearly normal to shore when the plume was bent by

the current and nearly parallel to shore at very low or zero currents when the plume was oriented directly offshore. Each plume study began with a first traverse near the outlet to the Baltic to a point about 200 m offshore; there the repetitive zigzag traverses of the plume axis began. The investigators determined the times when they were well outside the plume by observing the temperature data they were collecting. Just before the boat course was changed, the field crew would take a position fix. Position readings were then repeated at the end of each transect; the location of individual measurements between position fixes was determined assuming a constant boat velocity. The precise boat location was obtained by trilateration. The field crew was confident of its positioning because only small currents were present at Oskarshamnsverket, implying that such positioning methods are reasonably accurate.

Ambient-current measurements were made by tracking a series of drogues during the same time period that the plume was measured. Typically, 12-15 drogues were released just upstream of the plume. For example, when the plume was headed south, the drogues were dropped upcurrent about 1 km north of the discharge. The drogues were sited from shore and their positions fixed every 10 min, on average. Some drogues entered the plume; these drogues were ignored in the calculation of ambient current. Weil had no difficulty in choosing the unperturbed drogues and calculating a current from them. The results were reasonably consistent; accuracy probably ranged from 20 to 50%, a typical measurement having an accuracy of about 20%. The total time of travel for the drogues used in ambient-current calculations was between 1/2 and 1 hr.

Due to the numerous transects and boat fixes made (for positioning), a period of a few hours was needed to accumulate the field data. Currents in the area are tidal, but are very small. The maximum tidal-height variation is only a few centimeters.

d. Analysis and Critical Discussion of the Model-data Comparisons

As can be seen from a study of the comparisons for the Oskarshamnsverket case, the model is not in error in any consistent fashion for the eight cases compared. Weil concludes that the use of the model at low initial densimetric Froude numbers, as at Oskarshamnsverket, reveals no correlation between the model-data discrepancy and the environmental conditions present.

In our study of the comparisons presented, we have come to the conclusion that the model-data discrepancies are due to several causes.

(1) Inadequate Schematization of the Effluent Flow into the Baltic Sea. The Prych model requires a number of parameters to be specified before the computer code may be run. The outfall velocity, temperature,

and flow rate at the outlet must be known and specified. As noted in Subsection b above, the lack of sufficient horizontal and vertical resolution of velocity and temperature measurements at Transect 3 of the Hamnefjärden outlet make considerably doubtful the integrated value of flow- and heat-rejection rates to the Baltic Sea. Further complicating matters, the spatially variable outlet velocity and temperature had to be simulated as uniform profiles for input to the Prych model. The inconsistencies in the SMHI and Chalmers Tekniska Högskola data underscore the importance of obtaining precise velocity and temperature measurements at the outlet for each plume survey.

Weil tried to assess the effects of the imprecise description of input variables on the predictions of the Prych model. To do this, he varied the input parameters and made additional runs for Case A. He set H_0 , the depth of the upper layer at the inlet mouth, at 2.5, 3.0, and 4.0 m; u_0 , F_0 , and the heat-rejection rate were held constant. Thus, Q_0 and $\Delta\rho$ were adjusted for different amounts of mixing in Hamnefjärden. Figure IV.E.26 illustrates the results. Here, field data and predictive model results are plotted together.

Fig. IV.E.26. Sensitivity of Numerical Model to Input Variables, Oskarshamnsverket. (Adapted from Weil.²)

The conclusion of these tests is that the Prych model is sensitive, but not significantly so, to a variation in these input parameters. At a relative temperature of 0.4, the area changes by $\approx 25\%$ and the centerline distance by $\approx 45\%$, for H_0 varying from 2.5 to 4.0 m. Not plotted are comparisons made for changes in H_0 and F_0 , assuming a constant dilution in the inlet, and comparisons involving variations caused by changing only $\Delta\rho$. Model sensitivity to temperature for these changes were even less than shown in Figs. IV.E.24 and 25.

The discrepancy between the data and the model runs is much more significant than between runs of this sensitivity test. Though schematization is indeed a factor in the discrepancies, it apparently is not the only contributing factor. Although the sensitivity of the Prych model to different input parameters is only moderately significant, this does not mean that the real world is not sensitive to these kinds of changes in outfall conditions.

(2) Failure of Site to Match Idealizations Made in Model Development. Several idealizations made in the Prych model are not met in the Oskarshamnsverket case. Mentioned above are the nonuniform outlet velocity and temperature profiles. Also, there is shoreline interaction that occurs in the presence of most currents; the model does not treat this boundary interference. Transient effects due to changing currents and winds at the site violate the steady-state assumption made.

(3) Flaws in the Model Development. Several weaknesses in the Prych model development contribute to model-data discrepancies. First, the region of flow establishment is highly oversimplified in the model. At the time of model development, few data were available for an adequate simulation in that region. It is not even clear that a flow-establishment region should be used in simulating Oskarshamnsverket, since the profiles are already partly or fully developed at the mouth of the Hamnefjärden inlet. Assuming a Gaussian profile there, perhaps, would have provided better agreement with data. Unfortunately, the Prych model is oversimplified in precisely the region where the outlet profiles are not well known.

The second major model weakness involves the lateral-spreading assumption. As noted in Sec. II.E, the Prych model overpredicts plume lateral spreading at low local densimetric Froude numbers. This large lateral spreading is observed at Oskarshamnsverket by comparing predicted and measured results for plume isotherms; the predictions yield significantly wider isotherms than are indicated from the data. In the same vein, the total lateral spreading is probably incorrectly assumed to be the simple sum of buoyant and nonbuoyant parts.

A lesser model assumption violated is that the velocity and temperature profiles are similar. At low densimetric Froude numbers, as exist at Oskarshamnsverket, this kind of similarity is destroyed by buoyancy effects. Other model flaws are discussed in Sec. II.E.

(4) Inconsistencies in the Field Data. Examination of the conglomerate of field data from Oskarshamnsverket sometimes raise questions of physical validity and internal consistency. Some examples are as follows:

(a) For plume B, the current and wind were measured to be in the N-NE direction. From Fig. IV.E.8, the plume isotherms indicate that the plume is heading in a southwesterly direction. Weil explains the apparent discrepancy as due to a current reversal. The plume likely had been moving in a southwesterly direction, with the warm water located south of the plant. However, the current likely shifted from the SW to NE direction just before the start of the plume measurements. The plume water along the south coast was just beginning to return north as the ambient current was being measured. This explanation is plausible, but indicates a clear nonsteady situation of plume and ambient conditions.

(b) The change in orientation seems to be large for plumes B and C, although the times of measurement were not far apart. For $\theta/\theta_0 = 0.5$, the centerline distance to that isotherm changed from 900 to 2000 m and the isotherm areas changed from 2.0 to 0.5 km². Part of the explanation might be the fact that the current in Case B was already directed north after a current reversal, with the warm water still south of the plant. The reversing current is one reason why two sets of plume measurements were made that day.

(c) Plumes D and E were taken on the same day with the same ambient current, yet the plume orientations appear significantly different. To Weil, the cause did not seem to be instrument variability or data-taking methodology. The drogue measurements were not expected to be too far off as they are not instantaneous measurements but are integrated measurements over a period of 1/2 to 1 hr. This is in contrast to the unaveraged instantaneous temperature measurements made by a boat passing through the plume. The conglomerate of plume temperatures are neither synoptic nor averaged to remove temperature fluctuations due to turbulence. Consequently, some error is introduced. In spite of these temperature-measurement problems, we believe that the general shape and trend of the dispersing plume is represented by the boat measurements. Consequently, the orientation difference between plumes D and E still remains unexplained.

To better observe the consistency of the data, we drew Figs. IV.E.27 and 28, plots of the conglomerate of the Oskarshamnsverket data for centerline-temperature decay and surface-isotherm areas. Recall that plumes B-F have currents directed north, plume A is traveling south, and plumes G and H are directed offshore. The offshore-directed plumes have negligible boundary interference and should, in theory, be more quickly diluted than those directed longshore. This behavior is not observed in Figs. IV.E.27 and 28. Note that all the currents measured at Oskarshamnsverket are small. This certainly muddles the distinction between the measured plumes, as the differences between offshore, onshore, and longshore currents are slight. For low currents, reentrainment of heated water and the geometry of the area can

lead to heat buildup in the vicinity of the discharge. Probably, the plume trajectories, centerline temperatures, and isotherm areas are sensitive to the magnitude and direction of low ambient currents. Also, steady-state conditions are rare at these low currents, as reentrainment of heated-plume water back into the plume tends to cause a time-dependent behavior and difficulty in determining the proper temperature of water entering the plume.

Fig. IV.E.27. Conglomerate Plot of Axial-temperature Decay for the Eight Sets of Oskarshamnsverket Data.

Fig. IV.E.28. Conglomerate Plot of Surface-isotherm Areas for the Eight Sets of Oskarshamnsverket Data.

Figure IV.E.29 is a plot of surface-isotherm area from the Oskarshamnsverket field data with the Asbury-Frigo correlation derived from 23 plume surveys taken at six power plants on the Great Lakes. The Asbury-Frigo correlation was developed with data from several low-densimetric-Froude-number discharges with volumetric flow rates ranging

from 3.2 to 53.1 m³/sec. Clearly, Oskarshamnsverket fits into this category. The solid line represents the best fit, drawn by eye through the data; the dashed lines represent the envelope of the data for the 23 plumes. From this figure, the Oskarshamnsverket data appear to be consistent with field data taken from other plants having low-initial-densimetric-Froude-number discharges.

LETTERS ARE OSKARSHAMNSVERKET FIELD DATA, CASES A-H
 SOLID LINE IS ASBURY-FRIGO CORRELATION
 DOTTED LINES ARE DATA ENVELOPE OF ASBURY-FRIGO CURVE

Fig. IV.E.29. Comparison of Oskarshamnsverket Data to Asbury-Frigo Correlation. (Adapted from Weil.²)

The apparent internal inconsistencies in some of the field data described above do not, of course, disqualify their validity, since apparent inconsistencies may be found among virtually all field data taken at a specific site. The Oskarshamnsverket plume measurements do show trends in the data, and these trends alone are the ones of interest in model-data comparisons. Weil acknowledges that the data are indeed vulnerable, yet he has confidence in the way the data were actually measured. Weil feels confident that the overall picture of the plume collected on a particular day was a good description of the plume in the field. He also feels that the measurements, often made over a 1-3-hr period, were fairly steady-state; i.e., the field team were well concerned with the steady-state nature of the data collected. His assumption was verified by occasional repeat measurements of portions of a plume already mapped.

We feel that the discrepancies between the Prych-model predictions and the field data result from the fact that the model is really inapplicable to the Oskarshamnsverket site. The inherent assumptions made in the model development imply that the model is most accurately applied to large-densimetric-Froude-number discharges and deep-water receiving bodies. Thus, the poor correlation of model predictions to data is not the result of poor quality data or poor model development, but just the inapplicability of the idealizations to this site. This finding supports our basic conclusion: Integral

models such as the Prych model are too idealized for general application to prototype sites. Only general predictive estimates of plume parameters may be obtained. If only order-of-magnitude estimates are needed to satisfy the user, then the model may be considered satisfactory for his purpose.

e. Hydraulic-model Predictions Compared to Field Data and Prych-model Predictions

In addition to the field data and Prych-model predictions that appear in Figs. IV.E.7, 11, 13, and 15, hydraulic-model data are also plotted. A hydraulic model of the plant discharge and surrounding topography was built and run at Chalmers Tekniska Högskola, Göteborg, Sweden, at a horizontal scale 1:400 and a vertical scale 1:50. The hydraulic model was made for a plant discharge flow of 22 m³/sec and shore-parallel currents of 5-10 cm/sec. Different temperature differences ($\Delta\rho$ values) were tested. The entire Hamnefjärden inlet and discharge to the Baltic were modeled in this distorted model. Data from this model (surface measurements) were compared with field data (0.5-m measurements) and Prych-model predictions (surface predictions) for plumes A, C, D, and E. In the comparisons, plume temperatures from the laboratory model were normalized to achieve 1.0 for the measured values at the Hamnefjärden outlet mouth. For Case A, the hydraulic-model test used for comparison was one with a south-flowing current and a plant $\Delta\rho = 1.0 \text{ kg/m}^3$. From this, using Weil's 0.66 reduction factor, the $\Delta\rho$ at the inlet mouth is expected to be 0.66 kg/m^3 . For Cases C, D, E, the results of two laboratory tests were averaged to yield temperatures to be used for comparative purposes. Runs for plant density differences of 1.0 and 2.0 kg/m^3 were averaged to yield, at the inlet mouth, $\Delta\rho \approx [(1.0 + 2.0)/2] \times 0.66 = 0.99 \text{ kg/m}^3$. This averaging in temperatures was assumed to best match the $\Delta\rho$ of the outlet mouth of 1.0, 0.96, and 0.97 kg/m^3 of the field plumes (see Table IV.E.3).

We feel that the predictions of the hydraulic model compared poorly to the field data for the following reasons: First, the distorted model did not adequately simulate the jet mixing that took place in the Hamnefjärden inlet or the mixing that initially took place as the effluent entered the Baltic. Second, the field conditions do not precisely match the conditions modeled (or the average of those conditions) in the laboratory.

f. Effect of Selection of Ambient Temperature on Prych-model Predictions

Weil investigated the effect of the ambient temperature on model predictions at Oskarshamnsverket in an effort to assess the effect that specifying an erroneous ambient temperature might have on the model-data comparisons. In Fig. IV.E.30, the data for Case B are plotted with the nominal value of the ambient temperature, T_a , 11.5°C . The data are also plotted assuming T_a was ± 0.5 , ± 1 , $\pm 2^\circ\text{C}$ from that nominal T_a . Note that when the value of ambient temperature is increased, the centerline-temperature decay is more rapid. Also, the amount of change increases with distance from the outfall

where plume temperatures approach ambient, since the relative temperature $(T_{\text{isotherm}} - T_{\text{ambient}})/(T_{\text{outfall}} - T_{\text{ambient}})$ is plotted. In actuality, the specification of an accurate ambient temperature is a model problem not a field-data problem in that plume isotherms may be drawn from field data without knowing the value of the ambient temperature, while model predictions require that an ambient temperature be specified before such predictions can be calculated.

Fig. IV.E.30. Effect of Choice of Ambient Temperature on Field Data, Case B. (Adapted from Weil.²)

Weil states that, in most cases, T_a can be selected to a $\pm 0.5^\circ\text{C}$ accuracy. For Oskarshamnsverket Case B, changes in the ambient temperature of 0.5°C lead to errors of 10% in centerline distances at $T = 0.5$ and errors of 45% at $T = 0.2$. In Case B, the initial temperature excess is 5.5°C , while in most cases it is about 10°C ; thus, the changes in the latter case are only half those given for Case B. Therefore, a 0.5°C uncertainty in the choice of ambient temperature would lead to an error of about 5% of the local centerline distance to $T = 0.5$ and up to an error of 20% in the distance to $T = 0.2$. Weil recommends that, for conservative temperature predictions, the ambient temperature should be chosen on the low side when there is doubt as to the proper

ambient temperature. The conclusion from this sensitivity study is that error in the ambient temperature are not sufficient in themselves to explain the model-data discrepancies observed.

Weil discussed the difficulties in comparing subsurface-temperature measurements with surface predictions of mathematical models. This kind of comparison is often made, but the errors incurred are seldom if ever evaluated. At Oskarshamnsverket, temperature measurements made at a depth of 0.5 m were compared with surface predictions of the Prych model (Figs. IV.E.4-21). The question arises whether it is better to extrapolate the vertical temperature profiles of the data upward to estimate the surface temperature as predicted by the model or whether it is better to prepare model predictions for the 0.5-m depth and make the comparisons at that level. Weil concludes that the first option is the better of the two alternatives (although he agrees that the presentations in Figs. IV.E.6-25 are probably the best approach). His reasoning is given below. First, Weil notes that at Oskarshamnsverket, far from the outlet, temperature gradients near the surface are typically negligible in the top meter or two. Thus, errors in extrapolated surface temperatures would be nearly negligible in the far field, but would be significant in the near field, where larger vertical gradients exist. Recall that the Prych model assumes a half-Gaussian distribution of velocity and temperature. The sharp dropoff from the surface value (using the Gaussian distribution) may be a reasonable approximation near the discharge but would be inappropriate for the near-uniform, vertical profile of temperature far from the outfall. The model would be expected to give too sharp a vertical dropoff of temperature in the far field.

We disagree with Weil's answer to the question posed above. We believe that if the Prych model cannot predict the 0.5-m field data directly, a flaw in the model simulation is indicated. Extrapolating the subsurface data to surface values appears to be just a way of making the model performance appear better. Apparently the half-Gaussian profile for the vertical variation of velocity and temperature provides a much sharper dropoff than is warranted by the data in the far field and should be recognized as such, without being minimized by data-handling techniques. We propose that a direct comparison of surface predictions of the Prych model to the 0.5-m field data is most informative, with the less satisfactory alternative of comparing 0.5-m predictions with the data measured at the 0.5-m depth.

Figure IV.E.31 shows the effects of subsurface-data collection for the field measurements of Case B at Oskarshamnsverket. Here, temperatures measured at the 0.5-m depth were extrapolated by either linear or hand-drawn curves to determine the surface temperature. The top diagram of Fig. IV.E.22 shows the differences between 0.5-m measured values and the extrapolated surface values. Weil defines this as a "temperature error."

Nine surveys taken by SMHI in 1972 and five surveys taken by Chalmers Tekniska Högskola in October 1972 were used to provide the data for Fig. IV.E.32. Weil notes that the size of the differences depends very much on plume-layer thickness, which in turn depends on outlet densimetric Froude

number. Errors would be reduced by increasing the densimetric Froude number, since greater mixing will take place and smaller vertical gradients will exist in the plume layer. Absolute and fractional excess-temperature differences are plotted against radial distance from the source. Radial distance was chosen because most data points were not on plume centerlines. In each case, surface temperatures were found to be higher than or equal to 0.5-m-depth temperatures. Absolute values of the differences were generally less than 1.5°C and, for distances greater than 400 m from the outfall, were less than 0.6°C . The median difference was only 0.3°C , but this is biased by many data of zero error in the far field. Fractional error limits were within 45% out to 500 m and within 16% beyond that. The dashed lines in Fig. IV.E.32 represent the difference (absolute and fractional) of centerline-temperature predictions between 0.5-m and surface depths. Note that in the far field the Prych model predicts greater differences between surface and 0.5-m predictions, because of the smaller plume thickness and temperature excess. This overprediction reflects again on the half-Gaussian assumption for vertical variation of velocity and temperature. The prototype data show a more uniform thickness far from the outfall. Most important, the differences shown in Fig. IV.E.32 are only for problems in evaluating surface temperatures and not for possible errors in determining the outlet temperature, since no true outlet-temperature value can be known due to the stratified profile of temperature in the discharge point.

Fig. IV.E.31. Effects of Subsurface Data Collection, Case B. (Adapted from Weil.²)

Fig. IV.E.32. Temperature Errors due to Subsurface Measurement, Oskarshamnsverket. (Adapted from Weil.²)

A related point made by Weil is that one must be cautious with the application of the relative temperature variable in plotting model-data comparisons. There will be combined errors in the specification of ambient temperature and the specification of a single "average" temperature representing outfall temperature in any calculation of relative temperature. In the calculation of relative temperature $T = (T_s - T_a)/(T_o - T_a)$, errors in specifying T_s and T_a may either reinforce themselves to produce a larger error or nearly cancel in their combined effect. The total error depends principally on whether one is near or far from the outlet and on the magnitude and sign of the specific errors in T_o and T_a . The investigator should be cognizant of where, in his plume representation, the plotting of temperature in terms of the fractional expression, T , will cause problems. Weil concludes his discussion of the possible differences that may be caused by comparing Prych-model surface predictions for temperature to 0.5-m Oskarshamnsverket measurements by noting that the magnitude of the discrepancies involved is minor. Other causes, major in nature, must necessarily exist for the model-data differences. To reiterate we feel it best to directly compare model predictions at the surface to data acquired near or at the surface. Extrapolation should be studied only as an indication of the kinds of errors that might be inherent in such a direct comparison.

g. Evaluation of Verification Effort of Prych Model at Oskarshamnsverket

A few general comments from us about the Oskarshamnsverket comparisons are appropriate here. First, we feel that the main difficulty in applying the Prych model to Oskarshamnsverket is that the heated discharge is not really a jet. Rather, the initial jet momentum is largely dissipated in the Hamnefjärden inlet and, consequently, the outlet velocity to the Baltic is small. (The plant-discharge conditions are not given in Ref. 1 or 2, making it impossible to ascertain the precise reduction of initial momentum occurring within the inlet.) Models developed to treat the spread of density wedges⁷⁻⁹ are probably more appropriate for the Oskarshamnsverket discharge into the Baltic, since the plume entering the Baltic appears to be floating on the receiving water with an initial densimetric Froude number very close to 1. The Prych model is basically an extension of nonbuoyant-jet theory to those situations for which buoyancy is only a second-order effect. However, the Oskarshamnsverket situation is essentially a buoyant discharge with very little jet momentum. Many of the basic assumptions of the model are invalid in such an application. In essence, because of the low densimetric Froude number there, the Oskarshamnsverket site is a poor one to test the adequacy of the model.

A related difficulty stems from the fact that the Prych model runs into computational difficulties when the local densimetric Froude number approaches one. In the comparisons made by Weil, the initial densimetric Froude number ranged from 1.03 to 1.51.

Second, it may be that there were phenomena affecting plume dispersion at Oskarshamnsverket that the model did not simulate correctly. An important consideration, in this vein, is the transient environmental conditions evident from some sets of plume measurements. The time over which the field studies were taken at Oskarshamnsverket varied from about 1 to 4-3/4 hr with an average survey lasting about 3-3/4 hr. In such a tidally affected area, it is doubtful that steady-state conditions may be assumed for a survey of that duration.

The crude analysis of the zone of flow establishment also sheds doubt on the initial phase of mixing, which is additionally affected by the crude method of schematizing the outfall. Another difficulty was that insufficient measurements were made for properly defining the exit velocity and temperature profiles.

The Point Beach and Waukegan comparisons ranged from fair to poor. Although the outfall temperature and velocity were fairly well defined, interaction with the bottom occurred in both cases, violating the model idealization of no boundary interference. The cutoff of entrainment water and the restricted depth of the plume near the outfall for both plumes are certainly plausible explanations of the optimistic plume predictions.

2. Comparison of Prych Model with Simple Phenomenological Models Using the Oskarshamnsverket Conditions

Part of Weil's work in verifying the Prych model was to compare modeling approaches. The idea here was to compare integral and phenomenological approaches with the Oskarshamnsverket field data in the hope that one may start to narrow the choice of modeling approaches for the design of power plants. As we indicate later, Weil concludes that the phenomenological approach is still viable with the state of the art of integral modeling. Although Weil's study was concluded in 1974, we feel that his basic conclusion still remains true.

In the first comparison, Weil contrasts the Oskarshamnsverket data with the Prych predictions and the Asbury-Frigo correlation.⁸ The Asbury-Frigo correlation is an averaged plot of surface-area data collected from 23 surveys at six power plants on the Great Lakes. All the data considered by Asbury and Frigo were plotted as relative temperature ratio θ/θ_0 versus A/Q , where θ = isotherm-temperature excess above ambient, θ_0 = outfall-temperature excess, A = area within θ isotherm, and Q = discharge flow rate.

Asbury and Frigo plotted numerous groupings of the data of the form θ/θ_0 versus A/Q^a , where a is some power of the plant flow rate. The choice $a = 1$ yielded by far the best correlation. From the resulting plot of θ/θ_0 versus A/Q , they drew a curve by eyeball fit through the data. This Asbury-Frigo curve is often used as a phenomenological relationship for isotherm-area predictions. Figure IV.E.33 is a plot of the Prych predictions with the Asbury-Frigo curve and its data envelope for the same eight sets of

Fig. IV.E.33. Prych Model Predictions for Oskarshamnsverket Compared to Asbury-Frigo Correlation. (Adapted from Weil.²)

Oskarshamnsverket data that were plotted in Fig. IV.E.29. Here, Q has been taken as $33.4 \text{ m}^3/\text{sec}$ as previously described. Weil then compared Figs. IV.E.29 and 33. The conclusions are the following:

- a. In seven of eight surveys, the Prych model better fits the near-field data ($\theta/\theta_0 > 0.6$) than the Asbury-Frigo curve.
- b. In five of seven surveys, the Asbury-Frigo curve better fits the far-field data than the Prych model. (In one case, the Prych model had numerical difficulties and did not give far-field predictions.)
- c. Viewed on a log-log plot, the Asbury-Frigo curve better represented the data of five of seven cases than did the Prych model.
- d. The phenomenological model errors in the near field were found to be always smaller than the analytical model errors in the far field.
- e. The Prych model showed a tendency toward too low a mixing rate in the far field.

Weil concludes that, for Oskarshamnsverket, the Asbury-Frigo correlation, in general, gives better surface-area predictions than does the Prych model, but the difference in performance is small. At least within the range of flows and initial densimetric Froude numbers used to construct the Asbury-Frigo correlation, the correlation seems to allow reasonable predictions. For the design of the newer power plants with typically high densimetric Froude numbers, the dilution capability of the surface discharge should be significantly better than predicted by the Asbury-Frigo correlation, since the correlation is based upon data taken at older power plants with typically low initial densimetric Froude numbers and consequently lower dilution capability.

Weil also compared the laboratory results of Jen, Wiegel, and Mobarek⁴ and the Carter-Pritchard Model⁵ with the Oskarshamnsverket data and Prych-model predictions. Jen et al. measured the centerline-temperature decay of a round, buoyant surface-jet discharge into stagnant receiving water. Their tests were carried out for initial densimetric Froude numbers ranging from 18 to 180. Jen et al. found that for these high-densimetric-Froude-number discharges the relative temperature excess was inversely proportional to axial distance, as with three-dimensional, nonbuoyant jets. The empirical result of Jen et al. is plotted against the Oskarshamnsverket data of Cases D, E, and G in Fig. IV.E.34. Note that the empirical formula overestimates mixing, although the trend of temperature decay with distance is correct. Weil attributes the overestimation of dilution to the high aspect ratio of the discharge compared with that used in the Jen et al. study ($A = 21$ rather than $A = 1$ for a circular jet). The effect is partly counteracted by increased mixing due to ambient currents.

Weil notes that the specification of flow-establishment length is important and yet is unknown for a high-aspect-ratio discharge such as Oskarshamnsverket. This deficiency is only part of the problem. We suspect that the

high initial densimetric Froude numbers used in the study of Jen et al. are more important in explaining the difference. Large vertical mixing and vertical spreading were observed in the laboratory tests. Significant vertical mixing was not observed in the Oskarshamnsverket data. Instead, the plume stratified from its initial 3-m depth to an asymptotic depth of 1.5-2.0 m. The reduced vertical mixing in the Oskarshamnsverket data is the most likely reason for the higher temperatures observed.

Fig. IV.E.34. Comparison of Predictions of Two Simple Phenomenological Models with Oskarshamnsverket Data (top diagram) and with the Prych Model Run for the Same Data (bottom diagram). (Adapted from Weil.²)

The Carter-Pritchard phenomenological model is also plotted in Fig. IV.E.34. This model considers variations in the horizontal plane only, assuming no vertical mixing. In addition, there are four regions assumed to govern the dispersion. The first is a potential core with no mixing assumed. The second region initiates the mixing with the centerline temperature decaying to the minus one-fourth power of the centerline distance. Plume bending in the presence of an ambient current occurs in these first two regions. As bending is completed, the third region starts. There the temperature drops

as the centerline distance to the minus one-half power. The fourth and final region covers the far field, where the relative temperatures are less than 0.2. There the temperature drops in proportion to the distance to the minus one power. The comparison in Fig. IV.E.34 revealed to Weil that the core zone of the Carter-Pritchard model is too large. A probable reason for the large length predicted is that the length of the core region depends only on the outfall width ($6B_0$) and not on aspect ratio. Dilution compares well to the Prych predictions but is underestimated when compared to field data. The underestimation is probably due to the assumption of a two-dimensional jet (constant plume depth of 3 m) with no vertical entrainment.

Weil concludes that, at least for Oskarshamnsverket, the Prych model is an improvement over the correlation of Jen et al. and the simple analytical/phenomenological Carter-Pritchard model. He states that the Prych predictions are better, due mainly to the model's inclusion of the effects of aspect ratio, ambient current, and horizontal and vertical mixing. We feel that part of the reason that the Prych predictions are better here is that the Jen et al. correlation and the Carter-Pritchard model actually apply for larger densimetric-Froude-number discharges than Oskarshamnsverket at ≈ 1 . Comparisons for higher densimetric-Froude-number, deep-water discharges would be interesting, as all three models (Prych, Jen et al., and Carter and Pritchard) are more applicable in such cases.

3. Comparison of Prych Model to Trends in Laboratory and Field Data for Heated Surface Discharges

Another set of comparisons made by Weil involve the Shirazi data correlations described in Sec. IV.B. Recall that Shirazi presented power-law correlations for flows into stagnant receiving water, flows into a moving current, and flows over a sloping bottom.

Weil uses Shirazi's results for the third case to determine if the Prych model predicts the same trends as were determined from Shirazi's analysis of field and laboratory data. To accomplish this, Weil defines an imaginary power plant called SMHlverket. This plant is assumed to have outfall conditions $Q_0 = 113 \text{ m}^3/\text{sec}$, $\Delta\rho = 1.5 \text{ kg/m}^3$, $2B_0 = 21.0 \text{ m}$, $H_0 = 7.0 \text{ m}$, and $\theta_0 = 90^\circ$, with environmental parameters $V = 0.077 \text{ m/sec}$, $k = 0.00001 \text{ m/sec}$, $\epsilon_H = 1.0 \text{ m}^2/\text{sec}$, and $\epsilon_V = 0.001\epsilon_H$.

Weil uses the following parameters internal to the code in preparing Prych model predictions:

$$E_0 = 0.1, C_D = 0.2, C_F = 0.5, \Delta S = 50.0 \text{ m}, \text{ and } S_0 = 77.4 \text{ m}.$$

In this way, the basic nondimensional parameters in the code are $R = 0.1$, $A = 3$, $F_0 = 2.4$, and $E = 0.186$. Only a few results will be indicated. To satisfy additional interest, refer to Ref. 2.

For the conditions given above, Figs. IV.E.35 and 36 show the behavior of the plume parameters T , B/H_0 , and Y/H_0 versus S/H_0 . The emphasis made here in the comparisons by Weil is on the resulting trends or slope of the model and data functions plotted, not on specific numerical values. If Prych predicts the proper trends in the data, it is hoped that these results indicate that plume physics is being modeled properly for the case tested, and that numerical differences can be reduced by proper tuning of the model by adjusting model coefficients. Figure IV.E.35 shows that the trend of temperature versus distance compares well with the phenomenological relationship.

Fig. IV.E.35. Temperature and Plume Width vs Distance, SMHIVERKET and Laboratory and Field Data. (Adapted from Weil.²)

Fig. IV.E.36. Trajectory vs Distance, SMHIVERKET and Laboratory and Field Data. (Adapted from Weil.²)

The flow-establishment region predicted by the model is quite unrealistic in this case, which is actually quite typical. Also typical were the plume-width comparisons, as they showed different trends. Figures IV.E.35 and 36 support previous evidence that plume widths grow too rapidly in the model (see Sec. IV.C). Farther out from the discharge, the spreading rate compares better with the data; yet, as Weil notes, the abstracting of widths from the data is complicated by plume asymmetry.

Figure IV.E.36 shows that the Prych-model predictions of trajectory compare well with laboratory data and not so well with field data, except downstream after most of the bending has taken place. Weil suspects that the latter

negative result is due to boundary effects in the field. Weil does not show the variation in or predictions of layer depth H/H_0 , but states that it varies little, decreasing steadily from 0.65 to 0.28 as S/H_0 increases from 16 to 700. Shirazi does not present data for comparison. Weil concludes that the trend in model behavior with respect to axial distance is good, but plume widths tend to increase too quickly, with axis temperatures not strongly affected by plume width. Since conservation laws are satisfied by the model, overpredicted plume widths must necessarily imply underpredicted plume depths; this relationship must apply because, as Weil states, the axial temperatures are reasonably correct. This trend in Prych predictions is better observed in the comparisons of the model to laboratory data presented in Sec. IV.C.

Other figures, not presented here, show the sensitivity of the model to input variables R , θ_0 , A , F_0 , and E . Centerline temperatures, widths, trajectories, and plume depths were used in the study. Wherever possible, predictions from the Shirazi data correlation were also plotted and the trends examined. The results again revealed difficulties in the treatment of the flow-establishment region and plume-width predictions. Recall that Shirazi and Davis later modified the Prych model, improving the treatment of flow establishment and modifying the method of determining plume widths.

4. Verification of the Simulation of Turbulent Diffusion in the Prych Model with Open-channel Data.

A unique feature of the Prych model is the simulation of the effect of ambient turbulence on the plume predictions. The theoretical validity of that simulation is addressed in Sec. II.D. Weil attempted to test the model's ability to simulate mixing due to turbulence in the ambient fluid by comparing the model to mixing data that he himself took in previous open-channel tests made in a laboratory tank.¹⁰ For these tests, a heated effluent was discharged at the surface and coflow to a current flowing in an open channel. The discharge was from a semicircular nozzle and was at the same velocity as the current. In these tests, jet entrainment due to shear forces is absent and spreading results from buoyancy and ambient-turbulence effects only. Figure IV.E.37 compares some of these data with the Prych-model predictions.

Laboratory conditions for Runs 23 and 48 of Weil¹⁰ appear in Table IV.E.4. Note that ϵ_H is an empirical expression for the horizontal diffusion coefficient and u^* is the channel shear velocity. The vertical diffusion coefficient is taken as $(1/4)\epsilon_H$. The Prych model was run with the conditions given in Table IV.E.4, assuming (a) the laboratory model scale was 1:100, (b) the semicircular nozzle was scaled as a rectangle with the same width and area, and (c) the discharge was in coflow, initial angle of zero. Weil concludes from Fig. IV.E.37 that the overall comparison of the Prych-model predictions with the data for centerline-temperature decay is good, even though the model slightly overestimates the relative importance of the friction factor. The second graph in Fig. IV.E.37 shows that the Prych model yields identical temperature predictions, since only $\Delta\rho$ is changed. This independence of the

Fig. IV.E.37. Comparison of Numerical Model and Open-channel-mixing Data. (Adapted from Weil.²)

TABLE IV.E.4. Parameters for Two Open-channel Laboratory Tests of Coflowing Discharge

	$\Delta\rho$	F_o	$U_o = V$	d	D	C_f	ϵ_H	ϵ_V
Run 23	1.9	19.4	30.5	12.7	1.35	0.022	$0.027u^*d$	$(1/4)\epsilon_H$
Run 48	1.9	19.2	30.5	12.7	1.35	0.070	$0.027u^*d$	$(1/4)\epsilon_H$

$\Delta\rho$ = initial density difference between jet and ambient receiving water (kg/m^3)

F_o = initial densimetric Froude number

U_o = initial jet velocity (cm/sec)

V = ambient-current velocity (cm/sec)

D = equivalent jet diameter (cm)

d = depth of flow (cm)

C_f = Darcy-Weisbach friction factor

ϵ_H = horizontal diffusion coefficient (cm^2/sec)

ϵ_V = vertical diffusion coefficient (cm/sec)

u^* = channel shear velocity (cm/sec).

relative temperature to initial density differences was confirmed in the laboratory, as several runs were made there with otherwise identical conditions but only $\Delta\rho$ changed between laboratory tests.

5. Using the Prych Model to Extend Near-field Predictions of an Undistorted Hydraulic Model

Previous verification work indicates that one of the problems with the Prych model lies in its treatment of the plume physics in the vicinity of the outfall. Boundary effects (shoreline and bottom) and the inadequate treatment of the zone of flow establishment make initial predictions of the model questionable for prototype applications. Weil proposes that an undistorted hydraulic model of the site (if existing or planned) be used to provide near-field predictions. The advantage in using an undistorted model for near-field predictions is that it correctly scales the effects of jet momentum and buoyancy, including their interaction with boundary features. However, undistorted models do not properly scale turbulent mixing, interfacial shear, and surface heat-loss mechanisms, which are important in the later stages of plume dispersion.

Weil suggests that undistorted-model results be used out to a vertical cross section where the plume is entirely separated from the shoreline and bottom, but where plume-temperature and velocity profiles are still measurable in the hydraulic model. The profile of temperature and velocity at the cross section can then be used to determine initial conditions for the analytical model. In principle, the analytical model will then properly simulate ambient turbulence, interfacial friction, and surface heat loss, which are important to further plume dispersion, and thus extend near-field predictions into the beginning of the far field. Weil notes that it is difficult to verify this approach, since few undistorted models are large enough to extend into the far field. We add, moreover, that measurements of velocity in a laboratory model are difficult, especially when temperature gradients and temperature fluctuations are present, as in these plume models. Despite the difficulties, the idea is intriguing.

Weil gives a simple application of this matching idea to the Forsmark power plant on the northeast coast of Sweden, where a 1:75 undistorted scale model was set up by Statens Vattenfallsverk.¹¹ There the outfall flow rate was 200 m³/sec, the outfall-temperature excess was 10 C°, and the discharge was through a channel 43.5-m wide and 2.3-m deep. The initial jet velocity is then 2.0 m/sec. An ambient current of 10 cm/sec, directed nearly perpendicular to the outfall was simulated in the laboratory model.

Analyzing the 0.5-m temperature isotherms (see Fig. IV.E.38), Weil chose the 6 C° isotherm on the plume axis as the point at which to match the laboratory model and analytical models. At this location, 240 m from the outlet, the width of the plume was 188 m and the depth was 5.15 m. The local angle of the plume centerline with the current was 120°. Since velocities were not measured, the local plume flow rate was determined from

Fig. IV.E.38. Comparison of Temperature and Distance with Forsmark Undistorted Model. (Adapted from Weil.²)

$$Q = \frac{2Q_0}{T} = \frac{2(200)}{0.6} = 667 \text{ m}^3/\text{sec}.$$

This equation expresses a simple heat balance between the outfall and the point at which the relative temperature excess reaches 0.6. The Prych-model calculation is begun at this point without a zone of flow establishment. The ambient diffusion coefficients in the Prych model are set to zero, since the predictions are to be compared with hydraulic-model predictions at the lower isotherms. These comparisons appear in Figs. IV.E.38 and 39. The Prych predictions for centerline temperature and isotherm area are clearly improved by this technique, indicating that the major source of error in the model probably occurs in its treatment of the early stages of mixing. Also, the two cases of (1) the Prych predictions starting from the outlet and (2) the Prych predictions starting from the downstream cross section (Figs. IV.E.38 and 39) have the same trend, indicating that mixing is nearly independent of the dilution at that point. Unfortunately, the hydraulic-model results are not available beyond the 3 C° isotherm.

The improvement in trajectory predictions by beginning the model at the downstream cross-section interface is less marked. Weil suggests from the comparison that the Prych model underestimates far-field mixing nearer the 2 and 1 C° isotherms in this case. The fact that the predicted trajectory does not bend as much as is indicated by the data implies that the model is not entraining ambient momentum as it should. Consequently, the dilution predicted is not large enough. (The difference he states might also be due to significant ambient turbulent mixing in the hydraulic model, or else imprecision of current direction or persistent coastal effects of the ambient current in the hydraulic model.) Consequently, some jet characteristics appear to be improved by this extension of hydraulic-model predictions but others are not. A little disturbing in the comparison of Fig. IV.E.38 is the concave downtrend in the hydraulic-model data for centerline-temperature decay and the concave uptrend in the predictions. Although the magnitude of the difference between predicted and measured relative temperatures is not large, the slopes of the predictions are quite different. Only more data comparisons could reveal the reason, whether it be in data-taking methodology or in the modeling of plume physics. Thus it appears that errors in model simulation giving incorrect predictive trends, although not removed, may be reduced by this extension procedure.

Weil's idea of using the Prych model to extend the results of hydraulic models is interesting and, although the results presented are no proof of its success, the suggestion is certainly worthy of study. The benefit lies in getting a short distance away from the coastline to where we have more faith in the integral-model predictions, i.e., where the model idealizations are more likely to be satisfied. Off the coast, the local topography will give no trouble, and the velocity and temperature distribution should be more reasonably defined with an established profile. We feel that the advantage, however, lies only in applications where residual initial momentum is fairly large at the

Fig. IV.E.39. Comparison of Enclosed Area and Trajectory with Forsmark Undistorted Model. (Adapted from Weil.²)

downstream cross section of the plume and, moreover, where the plume is not fully bent toward the current. For plumes of nearly dissipated momentum, the local densimetric Froude number is nearly unity, portending future computational difficulties with the model as that number decreases to one. If the cross section is at a location where the plume is in its far-field dispersion, we believe that numerous other far-field models may be more appropriate to use. These all solve the single convective-diffusion equation by either finite differences,¹² Monte-Carlo techniques,¹³ or point-superposition methods.¹⁴ In such cases, the application of far-field models would be most appropriate.

6. Conclusions

Weil compared the Prych-integral-model predictions to eight sets of field data taken at the Oskarshamnsverket plant. The site did match the model's idealizations because the plume had no interaction with the bottom of the Baltic, and the ambient conditions often appeared steady, so that steady-state measurements were likely to be approximated satisfactorily. On the other hand, the site did not match the model idealizations because the plant discharged into an inlet where the effluent lost most of its momentum before entering the Baltic. At the entrance to the Baltic, the velocity and temperature distributions were also significantly nonuniform. The initial densimetric Froude number was very close to 1.0. Unfortunately, for this application, the model idealizes a rectangular discharge with uniform profiles and a jetlike discharge. (The model is developed as an extension of nonbuoyant jet theory.) The discharge to the Baltic is more of a buoyant density wedge than a jet discharge. In summary, we feel that the proper schematization of the discharge and the very low densimetric Froude number of the discharge became the major points of difficulty in applying the model to Oskarshamnsverket.

In the Oskarshamnsverket comparisons, Weil found that the overall accuracy of the Prych model was only fair. The comparison with the 10 sets of field data ranges from very good to very poor. Predicted temperatures are both too high and too low for the data considered; however, in general, centerline-temperature predictions are better than width predictions. Predictions of plume width were generally too large and sometimes greater than the local centerline length, which is the theoretical limit for low-densimetric-Froude-number discharges. On the other hand, Weil found that layer depth and trajectory predictions were reasonable.

No predominant source of the errors in the predictions could be identified by Weil. In the studies made, the model-data discrepancies were larger than one would expect from an incorrect choice of ambient temperature, from subsurface acquisition of data, or from incorrect specification of the input parameters. More important, there seemed to be no correlation between the sign or magnitude of the error and any discharge or hydrological parameter. Therefore, although the errors in prediction were often significant, no systematic discrepancy seemed to be present in the application of the model

to Oskarshamnsverket. Although shoreline effects sometimes contribute to model inaccuracies, Weil concludes that they probably do not account for the errors seen at Oskarshamnsverket. Adding to the discrepancy, Weil concludes, is the possibility of time-varying ambient conditions.

Adding to the puzzlement, we feel, is the method of outfall schematization. The Prych model is not very sensitive to different methods of calculating outfall parameters; this is in contrast to our feeling that some degree of sensitivity does exist in the real world. It is unfortunate that profiles of velocity and temperature at the outfall were not carefully measured in the field.

Our feeling on the model-data comparisons is a little stronger than is suggested by Weil: The model does not really apply to Oskarshamnsverket. The Prych model was developed for heated surface jets. The Oskarshamnsverket discharge is not a jet--it is really a buoyant density wedge as it enters the Baltic. Hence, density wedge models might be more suitable.

Although the Prych model can be run to low-initial-densimetric-Froude-number discharges like Oskarshamnsverket, we find that there are weaknesses present in the model formulation that are undoubtedly important in explaining the model-data discrepancies. These weaknesses include:

- a. The model predicts plumes that are generally too wide. The lateral spreading assumption on buoyancy is incorrect (see Sec. II.D). Also, the total lateral spreading is probably incorrectly assumed to be a simple sum of buoyant and nonbuoyant parts.
- b. The model assumes similar profiles laterally and vertically for velocity and temperature. At low densimetric Froude numbers, this similarity does not exist.
- c. The model has a poor simulation of the zone of flow establishment (initial mixing).
- d. The model assumes uniform outlet conditions that are not present at Oskarshamnsverket.

In spite of the shortcomings noted above, reinforced by the Weil study, the Prych model did appear capable of providing general estimates of the plume parameters. The field data acquired had some inconsistencies and presented some problems of interpretation; yet, on the whole, the field data adequately represented the trends in the data. The Shirazi-Davis calibration of the Prych model not only remedies some of its problems but also definitely improves it, as indicated by its predictions (not shown here).

The single Point Beach and Waukegan data comparisons to the Prych model once again revealed to us the idealized nature of the Prych model.

Boundary interference violated an important model idealization in these applications. This plume-boundary interaction was an important contribution to the poor comparison of the Prych model to field data taken at these two sites.

A significant part of Weil's work was to judge the relative merit of the integral and phenomenological approaches in predicting power-plant discharges. Again using the Oskarshamnsverket data, Weil found that the Asbury-Frigo correlation predicted surface areas better than the Prych model; the Prych model gave better predictions of centerline-temperature decay and surface-isotherm areas than the Jen-Wiegel-Mobarek phenomenological correlation and the Carter-Pritchard analytical/phenomenological model. Thus, based upon the limited testing done, Weil concludes that phenomenological models cannot be dismissed for power-plant design. We add that these correlations, based upon experimental data, are useful within the data range of their development. The success of the Asbury-Frigo correlation is most likely due to the fact that the correlation was developed from power-plant discharges of low densimetric Froude number, much like those at Oskarshamnsverket.

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F. Point Beach Nuclear Power Plant: Verification of Selected Surface-discharge Models

This section summarizes part of the work performed by the Energy and Environmental Systems Division as part of its effort to test available thermal-plume models with high-quality field data acquired at sites on the Great Lakes. Specifically, this section considers two segments of this work. First, one phenomenological and four popular integral models and six sets of jet data are compared. Second, three of the four integral models and one additional phenomenological model are compared with several sets of plume data (three-dimensional temperature measurements). Other parts of our verification work are summarized in other sections of this report, namely, Chapter II, Sec. VI.C, and Appendixes B and C.

1. Comparison with Jet Regime Data

Five popular thermal-plume models, reviewed in Chapter II, were chosen for comparison with prototype velocity and temperature data acquired at two sites on Lake Michigan. They are the Motz-Benedict model,¹ the Stolzenbach-Harleman model,² the Prych model,³ Pritchard Model No. 1,⁴ and the Shirazi-Davis model.⁵ This section summarizes the results of the comparison. Additional references on the subject include Paddock et al.,^{6,7} Frigo et al.,⁸ and Frye et al.⁹

The problem of model-data comparisons divides itself naturally into three parts. The first involves the acquisition of a comprehensive set of field measurements. The second pertains to the data reduction that must be done to allow meaningful comparison with the models. The third part is the actual preparation of model predictions and their comparison to the reduced data. Each part is considered in order.

a. Experimental Data; Velocity and Temperature Measurements

During the 1972 field year, Argonne National Laboratory conducted surveys of the jet regime of the thermal discharges at the Point Beach Nuclear Power Plant and at the Palisades Nuclear Generating Station. The particulars of these two outfall configurations appear in Figs. IV.F.1 and 2.

The Point Beach Plant has two units with a gross generating capacity of 505 MWe per unit. The once-through cooling system draws water from Lake Michigan at a normal maximum flow rate of 25.1 m³/sec per unit. At full power the heat-rejection rate is 2.68×10^5 kcal/sec for each unit. This corresponds to a maximum temperature rise across the condenser of 10.7 C°. The discharge flumes extend in different directions about 46 m into the lake at 60° to the shoreline. The width of each flume is 10.7 m, and the depth at the point of discharge is 4.2 m. The corresponding nominal outfall velocity is 0.56 m/sec.

Fig. IV.F.1. Point Beach Nuclear Power Plant, Two Creeks, Wisconsin. ANL Neg. No. 190-1395.

The Palisades Station consists of a single unit with a maximum gross generating capacity of 714 MWe. The once-through cooling system pumps water from Lake Michigan at $25.6 \text{ m}^3/\text{sec}$. The heat-rejection rate of the plant is $3.56 \times 10^5 \text{ kcal/sec}$, resulting in a temperature rise across the condensers of 13.9°C . The discharge canal is a diverging pile structure on the lake shore about 33 m long. It is initially 11.3-m wide and diverges to a width of 28.3 m at the point of discharge. The average depth at the outfall is 2.1 m, and the average discharge velocity is 0.43 m/sec.

Fig. IV.F.2. Palisades Nuclear Generating Station, South Haven, Michigan. ANL Neg. No. 190-1394.

Bottom contours in the vicinity of the plants are given in Figs. IV.F.3 and 4. On May 18, May 23, July 13, and September 9, 1972, surveys were taken at Point Beach and on June 14, July 19, and October 10, 1972, at the Palisades Plant.

A 5.5-m boat was outfitted to allow acquisition of three-dimensional temperature and velocity measurements. Briefly, the system consisted of a ducted current meter with a thermistor attached, a depth sounder, and a range positioning system, together with the electronics appropriate for reading and recording data on board the boat.

Anchors were located on opposite sides of the jet, and transects were made across the jet by pulling the boat from one anchor to the other. Very near the outfall, the tendency for the boat to drift with the jet was overcome by attaching a third line to the outfall structure itself. Ideally, the transects should be straight lines perpendicular to the flow; however, physical limitations have yielded curved and somewhat irregular paths. At several locations on the transects, the current-meter assembly was lowered over the side of the boat and suspended at 0.5-m intervals to a depth of 3.0 m or to the bottom. The first time the experiment was conducted, time variations were discovered in velocity and temperature; therefore, in order to obtain average values of velocity and temperature, along with any variations, strip-chart recorders were used to record current-meter and thermistor outputs.

The location of the boat was determined by a Motorola Mini-Ranger system, consisting of two shore-based transponders and a receiver-transmitter aboard the boat. Time-of-flight measurements, made automatically by the Mini-Ranger system, determine the distance to each transponder to an accuracy of about 3 m. Since land-based surveying provides the locations of the transponders, the position of the boat is easily determined by trilateration. The accuracy of a given position is generally taken as $\pm 3 \text{ m}$, although it depends on station location with respect to the transponders. The transponders, however, were placed so that most stations fall within the region of greatest accuracy.

Fig. IV.F.3. Approximate Depth Contours near Point Beach Outfall. ANL Neg. No. 190-878.

Fig. IV.F.4. Approximate Depth Contours near Palisades Outfall. ANL Neg. No. 190-880.

A Bendix Q-15 geomagnetic-ducted current meter with a YSI thermistor attached was used to measure the velocity and temperature of the water. The Q-15 has a five-bladed impeller that rotates freely in both directions and is enclosed in a duct. The ducted assembly aligns itself with the current due to an attached vane of adjustable length. Electronic averaging (over about 25 sec) of the number of impeller turns and the presence of the duct removes wave- and boat-motion effects. Current speed and direction were displayed on board by means of a readout unit, Model No. S-232.

Figures IV.F.5 and 6 give the results of measurements made for the stagnant lake at Point Beach on May 23 and for the typically small ambient current present on July 13. Figure IV.F.7 presents the jet for October 10 at Palisades, where larger currents are often seen.

Figures IV.F.5-7 show the results of measurements made at the 0.5-m depth. Each figure provides a table of velocity and temperature measurements and indicates the location of each station, where the velocity is represented vectorially. The centerline and width contours shown on each figure are the results of a fitting procedure described in the next subsection.

An important observation to make here is that a variety of factors, inherent in making measurements in the jet regime, may cause uncertainties in the data. These problems must be understood if proper use is to be made of the data.

An important aspect of this experimental uncertainty is the short-term variations in the velocity and temperature of the discharge jet. Oscillations due to local turbulence and the thermal front phenomenon are discussed in some detail in Chapter III. These oscillations at Point Beach and Palisades appear to have periods ranging from a few seconds to a few minutes. Since measurements at a particular point in the jet, by a traveling boat, are instantaneous, an unrepresentative value apparently might be obtained. At these two sites, the scale of the short-term temperature fluctuations is several Celsius degrees or less; the velocity variations are about 50% of the mean value or less. Fluctuations of this size were not present at all locations. A more typical value for the uncertainty in the temperature measurement is $\pm 0.5^\circ\text{C}$; a typical value for the uncertainty in the current speed is $\pm 20\%$.

Another source of uncertainty, in terms of data analysis, is the ambiguity resulting from longer-term and spatial variations of the ambient current and temperature. These numbers, necessarily assumed to be constants throughout the measurement (which lasted from 3 to 7 hr), vary in both time and space. Ambient-current measurements were typically made at a single location (at several depths) before and after the jet-regime measurements. Here again, current variations in time and position may lead, for a variety of reasons, to an unrepresentative value for ambient-current speed. (The direction of the current is generally thought to be more definite.) As noted above,

Fig. IV.F.5. Jet-regime Study for 0.5-m Depth at Point Beach Power Plant (Unit 1):
 May 23, 1972; 0945-1700 Hours. ANL Neg. No. 190-760 Rev. 1.

Fig. IV.F.6. Jet-regime Study for 0.5-m Depth at Point Beach Power Plant (Unit 1):
July 13, 1972, 1308-1706 Hours. ANL Neg. No. 190-762 Rev. 2.

Fig. IV.F.7. Jet-regime Study for 0.5-m Depth at Palisades Generating Station:
 October 10, 1972, 1025-1550 Hours. ANL Neg. No. 190-759 Rev. 1.

ambient-current speeds as reported may have an uncertainty of 20-50%; lower current speeds are the most uncertain. On some occasions, ambient-temperature measurements are as difficult to pin down as ambient-current speed. The ambient temperature was chosen to be the water temperature (at the appropriate depth) that appeared not to be influenced by the discharge water, yet was in the vicinity of the discharge. Unfortunately, on days when upwelling, downwelling, shoreline heating, or other disrupting phenomena occurred, the reported values of ambient water temperature may have an uncertainty of as much as 1-2 C°.

In the face of the previously discussed uncertainties in the data, instrumentation accuracy may not be very important, but for completeness, Table IV.F.1 lists the instrument specifications. Of special interest in terms of velocity measurements is the fact that, although a ducted current meter is used to null out disturbing vertical motion, shielding of the impeller occurs if the duct is not aligned with the flow. The importance of this finding remains unclear, but because the meter continuously averages the speed over a 25-sec period, any shielding would result in lower values of current speed.

TABLE IV.F.1. Instrument Accuracy

Instrument	Sensor	Accuracy	Threshold	Range	Time Constant	Resolution	Remarks
Bendix Q-15 Current Meter	Speed; impeller	±4% of full scale	3 cm/sec	0-1.0 knot; low scale 0-5.0 knots; high scale	25 sec; low scale 2.5 sec; high scale	1 cm/sec; low scale 5 cm/sec; high scale	
	Direction; compass with vane	±12°	-	0-360°	-	5°	Vane has adjustable length; 0.3-3.0 m
Temperature Recorder ^a	Thermistor	±0.5C°	-	0-30°C	~2.5 sec	0.2C°	Consisting of a Rustrak Model No. 2133 temperature recorder and YSI No. 409 thermistor probe
Temperature Recorder ^b	Thermistor	±0.2C°	-	0-50°C	~2.5 sec	0.1C°	Consisting of YSI No. 709 probe and digital readout built at Argonne
Motorola Mini-Ranger	-	±3 m	-	0.1-35 km	-	1 m	Accuracy applies to each range mea- surement. System accuracy varies with position of transponders relative to boat. For most of these measurements, ±3 m would apply.

^aUsed in temperature measurements up to and including July 19, 1972.

^bUsed in temperature measurements after July 19, 1972.

b. Data Analysis

To examine the features of the temperature and velocity distributions in the near-field region of the discharges, and to facilitate comparisons with analytical-model predictions, appropriate functional forms for the distributions were selected and fitted to the field measurements. These activities were performed as an attempt to smooth out fluctuations among the data points. Fitting was accomplished in two horizontal dimensions and at each vertical level. For the lateral excess temperature and excess velocity profiles, Gaussian distributions were chosen. The excess temperature θ is the difference between the measured temperature and the ambient temperature at that depth. The excess velocity u_E is defined as the component of the measured velocity parallel to the centerline of the jet, minus the component of the ambient current parallel to the centerline. For these purposes, the ambient current was taken to be shore parallel. The widths of these profiles were

assumed to grow linearly with distance from the outfall. After a constant initial region, the centerline-temperature excess θ_c and the centerline-velocity excess u_{cE} were assumed to decay as $1/\sqrt{s}$, where s is the distance from the outfall as measured along the centerline. For the centerline trajectories, a quadratic form was chosen.

A preliminary examination of the Point Beach data indicated that such forms might represent the available data fairly well. Also, for this initial attempt at data smoothing, simple forms with limited numbers of free parameters were thought to be appropriate. Clearly, unless the actual functional forms are known, any smoothing procedure of this type will have limitations and biases.

According to these functional forms, the temperature excess at a lateral distance η from the temperature centerline at a distance s from the outfall is given by

$$\theta(\eta, s) = \theta_c(s) \exp \left[-(\ln 2) \left(\frac{\eta}{W_T(s)} \right)^2 \right] = \theta_c(s) \left(\frac{1}{2} \right) (\eta / W_T)^2,$$

where

$$\theta_c(s) = \begin{cases} \theta_o A & \text{for } s \leq \alpha b_o / A^2 \\ \theta_o \sqrt{\frac{\alpha b_o}{s}} & \text{for } s > \alpha b_o / A^2 \end{cases}$$

and

$$W_T(s) = C b_o + \gamma s.$$

In the above expression, θ_o is the initial temperature excess at the outfall, b_o is the width of the outfall, and W_T is the half-width of the temperature distribution. The half-width is defined as the lateral distance at which θ drops to half of θ_c . The dimensionless parameters A , α , C , and γ are free parameters to be determined by fitting to the field measurements. The temperature centerline is specified by a parabola that passes through the center of the outfall. This introduces two more free parameters for a total of six. The excess velocity distribution is specified by an analogous set of expressions and introduces another set of six free parameters. The θ_o is replaced by u_o , which is the outfall velocity averaged over the entire outfall.

The temperature and velocity functions described above, each with only six free parameters, are fairly restrictive. As a consequence, only the general trends of the resulting fitted functions can be considered to be significant, since the details are artifacts of the functions selected.

The fitting procedure was carried out on the computer by a minimization technique. Those values that minimized the root-mean-square deviation of the functions from the data were chosen as the final values of the parameters. The sets of measurements at each depth taken during the four jet studies at the Point Beach outfall and one of the studies at the Palisades outfall (October 10, 1972) were each fitted independently. Measurements for the two other Palisades surveys did not lend themselves to this fitting procedure because the limited number of data points did not appear to characterize a major portion of the jet region. That is, the jet seemed to be significantly wider than the region surveyed.

Overall average deviations correspond to about 7% of the average initial temperature excess and average outfall velocity, respectively. The average deviation of the fitted function from the temperature data varied from about 0.2 to 1.1 C°, with an overall average deviation of 0.7 C° for all five studies. The velocity deviation varied from 1 to 8 cm/sec, with an overall average deviation of 4 cm/sec.

Some of the fitting results for the Point Beach studies are shown together in Figs. IV.F.8-11 so that they may be compared with each other. The circles on the curves of θ_c/θ_o (Fig. IV.F.8) indicate the approximate locations of the field-measurement transects.

Figure IV.F.8 shows the centerline-temperature and -velocity decays resulting from the fits to the measurements taken at the 0.5-m depth. The dropoff rates are approximately the same for each survey, and they are approximately the same for both temperature and velocity, except for the velocity results of July 13, 1972. This difference is presently unexplained. Indeed, with this one exception, the differences are probably within the errors associated with the experimental measurements and fitting procedure. On all four dates, the Point Beach Plant was operating at essentially the same power level and discharge flow rate.

The temperature distribution across the outfall was measured and found to be quite uniform. Therefore the excess-temperature ratio θ_c/θ_o plotted in the upper half of Fig. IV.F.8 must be 1.0 at $s = 0$. Since this restriction was not placed on the fitting function, not much significance should be attached to the details contained in the first 100 m of the fitting results. Velocity distributions at the outfall were also measured, but were not found to be uniform. In fact, velocities as high as 60% above the average outfall velocity were observed and show up on the lower half of Fig. IV.F.8, where the velocity ratio exceeds 1.0 for small values of s . Again, not much significance should be attached to the details of the results in this initial region.

Figure IV.F.9 represents the corresponding temperature and velocity half-widths. The temperature distribution, on the average, is twice as wide as the velocity distribution.

Fig. IV.F.8. Centerline-temperature Excess and Velocity Decays Resulting from Fits to Point Beach Jet Data. ANL Neg. No. 190-950.

Fig. IV.F.9. Half-widths of Temperature and Velocity Distributions Resulting from Fits to Point Beach Jet Data. ANL Neg. No. 190-961.

Fig. IV.F.10. Centerline-temperature Excess and Velocity Excess as a Function of Depth Resulting from Fits to Point Beach Jet Data (Normalized to 0.5-m results). ANL Neg. No. 190-953.

Fig. IV.F.11. Half-widths of Temperature and Velocity Distributions as a Function of Depth $s = 150$ m Resulting from Fits to Point Beach Jet Data. ANL Neg. No. 190-945.

Figure IV.F.10 shows the centerline-temperature and -velocity excesses as functions of depth relative to the 0.5-m results. Due to the nature of the functional forms chosen, these quantities are independent of distance from the outfall after the initial region, which was always less than 100 m. Within the accuracy of these results, the excesses are fairly constant with depth. The temperature results of September 9, 1972, do show an exception. Because there is no clear dropoff with depth, a characteristic depth of the jet cannot be extracted from this aspect of the data.

Figure IV.F.11 shows the widths as a function of depth 150 m from the outfall; this distance was chosen because it lies approximately in the middle of the range over which measurements were taken. These widths have a decreasing trend tending toward zero at a depth of 2.5-3.0 m. The temperature widths are approximately constant for the first 1.0-1.5 m and then decrease; the velocity widths decrease uniformly.

c. Comparisons with Model Predictions

The basic characteristics of the five integral models under study are summarized in Table IV.F.2. Calculations based on these models have been carried out for the outfall and ambient conditions that prevailed during the five jet studies for which fitting results were obtained. Table IV.F.3 summarizes the discharge, environmental, and model parameters used in the application of the models to all five prototype cases. Model parameters such as entrainment and drag coefficients were chosen in a systematic manner based upon the recommendations of the authors of the model.

TABLE IV.F.2. Basic Characteristics of the Five Integral Models Compared to Argonne Data

Model ^a	Dimensionality	Ambient Current	Buoyancy
Pritchard	2	No	No
Motz-Benedict	2	Yes	No
Stolzenbach-Harleman	3	Yes	Yes
Prych	3	Yes	Yes
Shirazi-Davis	3	Yes	Yes

^aThe Pritchard model is a synthesis of previous theoretical and physical modeling results for buoyant and nonbuoyant jets in an integral format, complemented by results of field data from some existing plants (see Sec. II.F).

The Motz-Benedict model is a simple two-dimensional integral analysis based upon the entrainment and drag-force concepts. Integral equations of mass, momentum, and energy are used (see Sec. II.B).

The Stolzenbach-Harleman and Prych models couple the flow and energy integral equations with simulations for entrainment, drag force (Prych only), and buoyancy (see Secs. II.C and II.D).

The Shirazi-Davis model is a revised version of the Prych model with calibration of unknown coefficients to hydraulic model and prototype field data (see Sec. II.E).

None of these models treat bottom or wind effects.

TABLE IV.F.3. Model Input Parameters for Jet Measurements Obtained on May 18, May 23, July 13, September 9, and October 10, 1972, for Point Beach and Palisades Power Plants

General Data and Parameters	Point Beach Unit No. 1				Palisades
	May 18	May 23	July 13	September 9	October 10
Ambient Water Temperature (0.5-meter depth)	9.2°C	14.3°C	13.0°C	16.3°C	13.0°C
Ambient Current Speed	0	0	5.7 cm/sec	2.2 cm/sec	0
Average Wind Speed	1.9 m/sec	1.0 m/sec	1.5 m/sec	3.7 m/sec	2.0 m/sec
Width of Outfall	10.7 m	10.7 m	10.7 m	10.7 m	28.3 m
Depth of Outfall (average)	4.2 m	4.2 m	4.2 m	4.2 m	2.1 m
Angle of Outfall with Shore	60°	60°	60°	60°	90°
Discharge Flow Rate	24.7 m ³ /sec	25.1 m ³ /sec	24.7 m ³ /sec	24.7 m ³ /sec	25.6 m ³ /sec
Outfall Temperature	17.7°C	21.6°C	20.3°C	24.5°C	22.2°C
Average Outfall Velocity	54.7 cm/sec	55.5 cm/sec	54.7 cm/sec	54.7 cm/sec	42.3 cm/sec
Initial Froude Number	2.57	2.37	2.42	2.06	2.27
Surface-heat-transfer Coefficient	242 kcal/m ² -day-C°	197 kcal/m ² -day-C°	166 kcal/m ² -day-C°	459 kcal/m ² -day-C°	281 kcal/m ² -day-C°
<u>Additional Parameters for Pritchard Model</u>					
Inverse Spreading Rate	6.0	6.0	6.0	6.0	6.0
Critical Mixing Depth	4.2 m	4.2 m	4.2 m	4.2 m	3.0 m
Temperature Excess of Recirculated Water	0	0	0	0	0
<u>Additional Parameters for Motz-Benedict Model</u>					
Length of Flow-establishment Region	55.5 m	55.5 m	30.4 m	45.3 m	147.2 m
Angle at End of Flow-establishment Region	60°	60°	49.9°	52.3°	90°
Width at End of Flow-establishment Region	8.5 m	8.5 m	8.5 m	8.5 m	22.6 m
Entrainment Coefficient	0.05	0.05	0.307	0.126	0.04
Drag Coefficient	0.5	0.5	0.5	0.5	0.5
<u>Additional Parameters for Stolzenbach-Harleman Model</u>					
Aspect Ratio	0.79	0.79	0.79	0.79	0.151
<u>Additional Parameters for Prych Model</u>					
Entrainment Coefficient	0.1	0.1	0.1	0.1	0.1
Drag Coefficient	0.2	0.2	0.2	0.2	0.2
Interfacial-shear-stress Coefficient	0.5	0.5	0.5	0.5	0.5
<u>Additional Parameters for Shirazi-Davis Model</u>					
Entrainment Coefficient	0.5	0.5	0.5	0.5	0.5
Drag Coefficient	1.0	1.0	1.0	1.0	1.0
Shear Coefficient	0.0	0.0	0.0	0.0	0.0
Horizontal Diffusion Coefficient	0.02	0.02	0.02	0.02	0.02
Vertical Diffusion Coefficient	0.004	0.004	0.004	0.004	0.004
Spreading Coefficient	1.4	1.4	1.4	1.4	1.4

Where the authors' recommendations were unclear, predictions were prepared based on what their intentions were judged to be. Emphasis was placed on testing the models fairly in terms of their current predictive capabilities, and no attempt was made to fit the models to the data. Thus the failure of a given model to correctly predict jet characteristics is to be viewed as the combination of poor modeling principles and a general lack of knowledge of the important empirical numbers needed by the model.

Note that we are applying these idealized models to a real-world situation. Idealizations such as no wind, no bottom interaction, and no transient effects are almost never the case in real life. Thus, at Point Beach and Palisades, we are also testing the validity of these idealizations. Significant bottom interaction does occur in the data and is bound to prejudice results of the model-data comparisons. However, the practical usefulness of these models depends solely on their ability to predict prototype situations.

The model calculations and the data-fitting results have been compared relative to jet centerline trajectories, centerline-temperature decay, temperature half-widths, centerline-velocity decay, velocity half-widths, areas within isotherms, centerline temperature and velocity variation with depth, and temperature- and velocity-width variation with depth. A full presentation of these comparisons for all five jet studies would go far beyond the limitations of this report. The complete comparisons with detailed comments and evaluations are contained in Paddock et al.⁴ Some of the 0.5-m depth fitting results are presented here (see Figs. IV.F.12-23), along with the surface predictions of the models. Again, the circles on Figs. IV.F.13, 17, and 21 indicate the approximate location of the field-measurement transects.

Fig. IV.F.12

Centerline Trajectories Resulting from the Fitting Procedure and Model Calculations for Point Beach: May 23, 1972. ANL Neg. No. 190-936.

Fig. IV.F.13. Centerline-temperature Excess and Velocity Decays Resulting from the Fitting Procedure and Model Calculations for Point Beach: May 23, 1972. ANL Neg. No. 190-949 Rev. 1.

Fig. IV.F.14. Half-widths of Temperature and Velocity Distributions Resulting from the Fitting Procedure and Model Calculations for Point Beach: May 23, 1972. ANL Neg. No. 190-946 Rev. 1.

Fig. IV.F.15. Isotherm Areas Resulting from the Fitting Procedure and Model Calculations for Point Beach: May 23, 1972. ANL Neg. No. 190-938 Rev. 1.

Fig. IV.F.16. Centerline Trajectories Resulting from the Fitting Procedure and Model Calculations for Point Beach: July 13, 1972. ANL Neg. No. 190-940 Rev. 1.

Fig. IV.F.17. Centerline-temperature Excess and Velocity Decays Resulting from the Fitting Procedure and Model Calculations for Point Beach: July 13, 1972. ANL Neg. No. 190-958 Rev. 1.

Fig. IV.F.18. Half-widths of Temperature and Velocity Distributions Resulting from the Fitting Procedure and Model Calculations for Point Beach: July 13, 1972. ANL Neg. No. 190-942 Rev. 1.

Fig. IV.F.19. Isotherm Areas Resulting from the Fitting Procedure and Model Calculations for Point Beach: July 13, 1972. ANL Neg. No. 190-943 Rev. 1.

Fig. IV.F.20. Centerline Trajectories Resulting from the Fitting Procedure and Model Calculations for Palisades: October 10, 1972. ANL Neg. No. 190-957.

Fig. IV.F.21. Centerline-temperature Excess and Velocity Decays Resulting from the Fitting Procedure and Model Calculations for Palisades: October 10, 1972. ANL Neg. No. 190-960 Rev. 1.

Fig. IV.F.22. Half-widths of Temperature and Velocity Distributions Resulting from the Fitting Procedure and Model Calculations for Palisades: October 10, 1972. ANL Neg. No. 190-959 Rev. 1.

Fig. IV.F.23

Isotherm Areas Resulting from the Fitting Procedure and Model Calculations for Palisades: October 10, 1972. ANL Neg. No. 190-937 Rev. 1.

Of the five models considered in this study, the Pritchard No. 1 simulation compared best with the present jet data. Model-data discrepancies usually indicated conservative predictions on the part of the Pritchard analysis. The success of this model is due in part to the persistent efforts of Pritchard to calibrate his model with field and hydraulic data.

The greatest difficulty in the application of the Motz-Benedict model lies in the selection of model parameters--in particular, the entrainment coefficient. The authors of the model presented data that were meant to be used as a guide in selecting this parameter. The data, however, are not consistent and show much scatter. Preliminary results indicate that, if model parameters are suitably chosen, the Motz-Benedict model can give conservative predictions of jet characteristics. No evidence is available to indicate that parameters that fit the model to data taken at Point Beach and Palisades will work equally well at other sites. Until the model has been calibrated with large amounts of data and a consistent algorithm has been specified for selecting model parameters, the Motz-Benedict model cannot be used reliably.

The Stolzenbach-Harleman model predicts a lateral spreading rate that is far too large. If the Stolzenbach-Harleman predictions for width were accurate, the presence of bottom effects should provide data with even larger spreading rates due to the reduced vertical dilution capability. The difficulty in the width predictions may be traced, in part, to the model assumptions for the lateral spreading velocity, which is based more on intuition than

on any data. The too-rapid decay rate predicted by the model can be partly attributed to the significant bottom effects at the Point Beach and Palisades sites, but the excessive spreading rate is suspect. When ambient currents are present, there are also difficulties that arise in the accuracy of the differential equations developed by the authors of the model (see Sec. II.C).

The Prych model also appears to have difficulty with its treatment of lateral spreading. Large widths and optimistic centerline-temperature predictions appear in the comparisons. These large widths may also be traced to an incorrect assumption on lateral spreading velocity. The numerical difficulties of the Stolzenbach-Harleman code are not present in this model, due to the simpler form of the model equations.

The Shirazi-Davis simulation has improved the lateral spreading prediction of the Prych model, yet still gives a faster decay of temperature and velocity than is warranted by the data. From the very limited comparisons carried out to date, this model appears to be the most appropriate one of those tested to use for the larger currents (as the Pritchard model No. 1 will not apply).

Shirazi and Davis⁵ indicate that their model should compare best to the mean of data taken under similar outfall and environmental conditions since much scatter does exist for data taken under apparently identical conditions. As this is true for all the models reviewed, we turn to comparisons of the same models (with Pritchard model No. 2) to sets of plume data, each set taken under similar conditions. We find that the basic conclusions of the comparisons made here remain true.

2. Comparisons with Plume Data

Several integral models for the prediction of the characteristics and behavior of thermal plumes from shoreline surface discharges into large bodies of receiving water have been compared to field data taken near the Point Beach Unit 1 outfall. The acquisition and analysis of the data are described in detail in Frigo et al.^{8,10} and Kyser et al.¹¹ and therefore will be discussed only briefly below. The comparisons with the models do not appear elsewhere and hence are discussed in more detail.

During the 1971 and 1972 field years, a group from the Energy and Environmental Systems Division at Argonne National Laboratory mapped 22 plumes. (See Sec. IV.B for a more detailed discussion.) Three-dimensional temperature data were obtained from a moving boat equipped with a 3-m submerged boom with thermistors every 0.5 m. The boat traveled a serpentine path from the outfall to the far reaches of the plume. The position of the boat at any instant was determined with a Motorola Mini-Ranger positioning system, consisting of two shore-based transponders and a receiver/transmitter unit and range console on board the boat. Ambient lake current, wind speed and direction, air temperature, humidity, and lake-surface conditions were usually measured in conjunction with the plume mapping.

Isotherm maps were drawn from plots of the boat measurements. These maps were analyzed to extract plume characteristics: centerline trajectory, centerline-temperature decay, temperature half-width (defined as the lateral distance from the centerline at which the temperature excess drops to half its centerline value), and surface-isotherm areas. Due to plume bending and shoreline interaction, the widths on either side of the plume centerline were found to be different. These widths have been treated separately and are designated "left side" and "right side" as one faces lakeward from the discharge. The 22 plumes were divided into five major categories, depending primarily on ambient lake current and the associated plume bending. Table IV.F.4 lists the plumes by category, along with the outfall and ambient conditions. Each plume is designated by a letter for identification purposes.

Calculations based on five integral models were carried out for each of the five categories using average parameters for each category. Categories I and II were assigned nominal north currents of 15 and 10 cm/sec, respectively. Category III was classified as stagnant with no ambient current. Category IV and V were assigned south currents of 10 and 15 cm/sec, respectively. The five models considered were those of Shirazi and Davis,⁵ Pritchard (Model No. 1),⁴ Pritchard (Model No. 2),¹² Prych,³ and Stolzenbach and Harleman.² Model parameters such as entrainment and drag coefficients were chosen to be those recommended by the authors of the individual models. The comparisons are considered category by category.

Figures IV.F.24-27 show the model predictions and the data for the three plumes of Category I. The data within this group form a very tight band with little scatter. In these cases, the discharge is opposing the ambient current and the Prych and Stolzenbach-Harleman models do not perform well. Not seen in the data is the usual plateau in the centerline-temperature predictions of the Stolzenbach-Harleman model. Reasons for the plateau are discussed in more detail in Secs. II.C and IV.D. The Shirazi-Davis model had to be extrapolated from computer runs with larger currents, because the model ran into numerical difficulty for this small current case as the local densimetric Froude number approached unity. (The extrapolation procedure is described in Shirazi and Davis⁵ and in Sec. II.E.) The point at which the local densimetric Froude number reached unity is indicated by a cross bar on the plots of the model predictions. This point occurred 47 m from the outfall for Category I, as noted, for example, in Fig. IV.F.24. Although the details of the centerline-temperature decay are not described, the overall predictions of this model are quite good, as seen in the isotherm area comparison of Fig. IV.F.27. The Pritchard No. 1 model was not designed to be used in the presence of a significant current and probably should not have been used. The centerline-temperature decay (Fig. IV.F.24), however, is very well described.

Also in reasonable agreement with the data are the width and isotherm-area predictions. Note that the Pritchard models do not directly predict widths as defined here. They do, however, predict the lengths and

TABLE IV.F.4. Thermal Plumes Measured at Point Beach Unit 1 during 1971 and 1972

Plume	Date	Time	Outfall Temp. (°C)	Ambient Temp. (°C)	Excess Temp. (degC)	Disch. Flow (m³/s)	Disch. Vel. (cm/s)	Magn. of Current (cm/s)	Dir. of Current (°CW from north)	Magn. of Wind (m/s)	Dir. of Wind (°CW from north)	Lake Conditions	Wave Height (m)	R	F _O	$\frac{K}{\rho C_p}$ (m/s x 10 ⁻⁶)
CATEGORY I: Strong North Current; Moderate Wind																
T	21 Jul 71	1508-1639	21.8	11.1	10.7	25.1	57.	18.	335	6.7	205	Moderately Calm	0.3-0.6	0.32	2.13	7.2
W	1 Sep 71	1445-1727	26.5	17.4	9.1	25.1	57.	—	355	4.9	135	Moderately Calm	0.3-0.6	—	1.98	7.0
A	17 May 72	1646-1806	18.9	7.9	11.0	25.1	56.	8.3	000	4.5	190	Calm	0.0-0.3	0.15	2.31	4.8
CATEGORY II: Moderate North Current; Moderate Wind																
R	20 Jul 71	1632-1805	21.5	12.6	8.9	25.1	57.	9.0	015	6.5	205	—	0.0-0.3	0.16	2.29	7.4
U	31 Aug 71	1530-1631	26.2	17.6	8.6	25.1	57.	—	350	2.7	045	—	0.6-0.9	—	2.04	4.4
V	1 Sep 71	0950-1130	27.1	17.4	9.7	25.1	57.	—	360	4.9	135	Moderately Calm	0.3-0.6	—	1.91	7.0
G	11 Jul 72	1245-1355	20.5	10.9	9.6	25.1	56.	15.0	300	4.3	175	Choppy	0.3-0.6	0.27	2.24	5.1
I	6 Sep 72	1115-1300	25.2	15.0	10.2	24.7	55.	18.8	020	5.2	205	Choppy	0.6-1.0	0.34	1.87	6.7
CATEGORY III (a): Zero Current; Light, Moderate Wind																
S	21 Jul 71	1020-1117	20.5	11.5	9.0	25.1	57.	—	—	3.4	205	Moderately Calm	0.3-0.6	—	2.36	4.3
C	24 May 72	0925-1040	21.3	10.7	10.6	25.1	56.	0.0	—	1.4	015	Calm	0.0	0.0	2.11	2.5
D	24 May 72	1045-1145	21.3	11.9	9.4	25.1	56.	0.0	—	2.2	015	Calm	0.0	0.0	2.19	3.3
E	24 May 72	1145-1225	21.3	11.0	10.3	25.1	56.	0.0	—	2.0	015	Calm	0.0	0.0	2.13	3.0
F	7 Jul 72	1830-2015	20.5	11.3	9.2	25.1	56.	—	—	—	—	—	—	—	2.27	1.2
J	9 Sep 72	1525-1630	24.5	14.5	10.0	24.7	55.	0.0	—	4.8	175	Choppy	0.0-0.3	0.0	1.92	6.2
III (b): Small Current; Light Wind																
H	13 Jul 72	1805-1903	20.3	13.3	7.0	25.1	56.	2.2	245	2.4	165	Calm	0.0	0.039	2.52	3.6
L	29 Sep 72	1545-1705	20.4	11.2	9.2	24.7	55.	2.2	300	2.6	300	Calm	0.3-0.6	0.040	2.24	3.6
III (c): Zero Current; Zero Wind																
B	18 May 72	1645-1751	18.0	9.5	8.5	24.7	55.	0.0	—	—	—	Calm	0.0	0.0	2.53	1.2
CATEGORY IV: Moderate South Current; Light, Moderate Wind																
O	20 May 71	1438-1550	17.5	8.1	9.4	25.1	57.	9.0	125	6.7	315	—	0.0-0.3	0.16	2.64	6.6
P	20 May 71	1740-1850	17.5	8.2	9.3	25.1	57.	9.0	125	5.6	270	—	0.0-0.3	0.16	2.65	5.8
X	28 Oct 71	1402-1539	21.2	12.8	8.4	25.1	57.	—	170	2.0	155	Slightly Choppy	0.0-0.3	—	2.36	3.2
CATEGORY V: Strong South Current; Moderate Wind																
Q	25 Jun 71	1600-1215	20.1	11.0	9.1	25.1	57.	—	—	6.5	045	Rough	0.6-0.9	—	2.38	7.0
Y	28 Oct 71	2000-2100	21.2	12.6	8.6	25.1	57.	15.0	180	3.1	050	Calm	0.0-0.3	0.26	2.34	3.2

Notes:

R = Ratio of Ambient Velocity u_a to the Discharge Velocity u_o .F_O = Initial Densimetric Froude Number. $\frac{K}{\rho C_p}$ = Surface-heat-transfer Parameter.

Fig. IV.F.24. Centerline-temperature Decay for Category I

Fig. IV.F.25. Left-side Temperature Half-widths for Category I

Fig. IV.F.26. Right-side Temperature Half-widths for Category I

Fig. IV.F.27. Isotherm Areas for Category I

maximum widths of each isotherm. Pritchard suggests that these isotherms will be "bullet-shaped," and, at least for his first model, he gives a prescription (see Sec. II.F) for calculating these widths. The widths presented here for both models are based on the assumption of the existence of the bullet-shaped isotherms; hence, since the widths plotted for Model No. 2 are based on our assumption that the prescription given for Model No. 1 also applies to Model No. 2, these resulting width plots are to be used with caution.

For some of these cases, the plume is significantly bent by the cross current, resulting in isotherms that clearly will be distorted. This determination of plume width may not be a good one when an ambient current is present, but a representation such as this is useful in comparing the trend in model predictions with the data and other models. The Pritchard No. 2 model, although designed to take ambient current into account to some extent, does not compare quite as well.

The results for Category II are presented in Figs. IV.F.28-31. Again, the ambient current opposes the discharge and the Prych and Stolzenbach-Harleman models fail to give reasonable results. The data exhibit more scatter

Fig. IV.F.28. Centerline-temperature Decay for Category II

Fig. IV.F.29. Left-side Temperature Half-widths for Category II

Fig. IV.F.30. Right-side Temperature Half-widths for Category II

Fig. IV.F.31. Isotherm Areas for Category II

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than in the previous category, but certain trends can be observed. The centerline-temperature decay is not quite as rapid, probably because the smaller current brings less cold ambient water into the plume. Also, lower ambient turbulence is likely to be present at times of lower currents. Again the Shirazi-Davis model had to be extrapolated from calculations using higher currents. The trend of less rapid centerline temperature decay is somewhat accounted for by these predictions. The width predictions are too small, but the overall area predictions are quite reasonable. The Pritchard No. 1 model results are essentially the same as for Category I, since ambient current is not taken into account. Although no differences are predicted, the results are still reasonable. The Pritchard No. 2 model, which considers ambient current, exhibits an incorrect shift in the centerline-temperature decay and plume areas, giving results that are far too optimistic.

Figures IV.F.32-35 show the data alone for Category III. So much scatter was shown that this category was tentatively divided into three subcategories, as indicated by the centerline-temperature decays in Fig. IV.F.32. The solid curves represent averages of the members of the subcategories, the dashed curve is a subcategory with only one member, and the diagonal cross-hatching indicates the range of one standard deviation for these averages. Apparently under stagnant or near-stagnant conditions, currents and winds, even though small, can have a significant effect on plume dispersion. Even small current or small wind can aid significantly in preventing a building of heat in the vicinity of the plant. Category III(b) with a small current (~ 2 cm/sec) exhibits a much more rapid centerline-temperature decay and isotherm areas that are two to three times smaller than Category III(a) with apparently no ambient current at all.

Fig. IV.F.32. Centerline-temperature-decay Data for Category III

Fig. IV.F.33. Left-side Temperature Half-width Data for Category III

Fig. IV.F.34. Right-side Temperature Half-width Data for Category III

Fig. IV.F.35. Isotherm-area Data for Category III

Figures IV.F.36-39 show the model predictions along with the data. The Prych and Stolzenbach-Harleman models yield questionable results, especially with regard to plume widths (see Figs. IV.F.37 and 38). Here again the Shirazi-Davis model was extrapolated from computer runs with nonzero currents. The large scatter in the data makes the comparisons difficult to evaluate. The isotherm areas predicted by this model appear to be somewhat small, especially at the higher temperatures. The predictions of the Pritchard No. 1 model are not too different from the Shirazi-Davis results, although the model does predict larger areas for the higher temperatures. For no ambient current, the Pritchard No. 2 model attempts to assess the effect of bottom interaction by taking into account the slope of the bottom. Calculations were prepared assuming both a flat bottom and a sloped bottom, typical of that

Fig. IV.F.36. Centerline-temperature-decay Data for Category III Along with Model Predictions

Fig. IV.F.37. Left-side Temperature Half-width Data for Category III Along with Model Predictions

Fig. IV.F.38. Right-side Temperature Half-width Data for Category III Along with Model Predictions

Fig. IV.F.39. Isotherm-area Data for Category III Along with Model Predictions

found at Point Beach. The predictions that include bottom slope yield areas that are much too small. This may not be a fair comparison, because the bottom at Point Beach actually decreases in depth at first before it starts to drop off, undoubtedly causing significant bottom interaction that the model cannot be expected to take into account.

Category IV results are shown in Figs. IV.F.40-42. In this case, the ambient current and discharge are in the same general direction, and the plume is quickly pushed against the south shore. The subsequent interaction with the shore makes it impossible to extract a temperature half-width on the right side of the centerline. Except for the two Pritchard models, all the models seriously overpredict the rate of centerline temperature decay, especially in the first 1000 m. Isotherm areas are also too small by about an order

Fig. IV.F.40. Centerline-temperature-decay Data for Category IV

Fig. IV.F.41. Temperature Half-widths (Left Side) for Category IV

Fig. IV.F.42. Isotherm Areas for Category IV

of magnitude. The shoreline interaction mentioned above is no doubt responsible for cutting off inshore entrainment and causing the reentrainment of already mixed plume water, which would decrease the centerline decay and increase areas. The Pritchard No. 1 model, although again not designed for cases with significant ambient currents, produced the best overall characterization of the data.

An interesting observation about the Pritchard No. 2 model is made at this point. The model does not take into account the direction of the ambient current, only its magnitude. One would therefore expect that the predictions for Categories II and IV should be similar, yet comparison of Figs. IV.F.28 and 40 shows that they are indeed different. This must be due to the different average initial temperature excesses used in these two cases and the resulting difference in initial densimetric Froude numbers. Category II had a Froude number of 2.07, and Category IV had a value of 2.55. So large a difference in predictions for so small a difference in initial conditions is difficult to accept.

The last category to be considered is Category V with strong south currents. An extremely large but unmeasured current existed while plume Q was being mapped. The centerline bent back to the shore within about 500 m of the outfall and hugged the shoreline for many kilometers downstream. The other plume in this category, plume Y, was mapped in the presence of a measured current of 15 cm/sec. This plume also followed the shoreline fairly closely. The results are shown in Figs. IV.F.43-45. The centerline decays are even more gradual than for Category IV. All the models severely underpredict the areas, and, for the most part, the centerline temperature decays. Except for the two Pritchard models, all the models expect the stronger current to produce larger dilutions, narrower widths, and smaller areas due to the large amounts of cold ambient water brought in by the current. The interference of the south shore apparently negates this effect.

Fig. IV.F.43. Centerline-temperature-decay Data for Category V

Fig. IV.F.44. Temperature Half-widths (Left Side) for Category V

Fig. IV.F.45. Isotherm Areas for Category V

An overall examination of the performance of the five integral models shows that none of them adequately describe all the categories encountered in this study. The Pritchard No. 1 model, although not designed to be used in most of these cases and giving essentially the same results for all five categories, gives the best overall description of the 22 plumes observed. In the two categories (IV and V) for which the Prych and Stolzenbach-Harleman models were able to make meaningful predictions, they failed to agree with the data. This disagreement may have been due in part to the interaction of the physical plume with the shore, which none of the models takes into account. The Pritchard No. 2 model, which is a generalization and extension of the original model No. 1 to cover interactions with bottom and ambient current, did not succeed in improving upon model No. 1. The Shirazi-Davis model had to be extrapolated in three out of the five cases, because of which the results, according to authors of the model, are to be viewed with caution. The model predictions did, nonetheless, agree reasonably with the data and did predict some of the trends observed. The question remains as to whether these favorable aspects will be repeated at new sites with different outfall and environmental parameters.

3. Conclusions from Jet and Plume Data Comparisons

The model-data comparisons indicate that, as environmental conditions change at the Point Beach Power Plant, the models (integral and phenomenological) do not correctly represent the trends in the field data. The models do not treat some of the significant factors affecting plume dispersion at both Point Beach and Palisades. Of these, bottom and shoreline interference, wind effects, transient effects, and ambient stratification are the most significant omissions. In essence, then, the models are too idealized, as they assume a limitless uniform-temperature receiving body and a steady-state plume. Despite the nonideal nature of the sites, they are typical of the numerous existing shoreline discharges.

Errors due to the idealized nature of the models are compounded by inaccuracies in the model development. These types of errors are seen more clearly in Sec. IV.C as the models are compared to laboratory data that closely match the idealizations made in the model. Discrepancies occurring between model and data are also due to errors in data acquisition: instrument inaccuracy and oscillations in measured plume velocities and temperatures. However, we feel that the data errors are not severe enough to destroy the proper trends in the data as environmental conditions change. These trends have been generally consistent at the Point Beach site. The discrepancies, we feel, are due mostly to model errors and not data errors. The models may, at best, be used only as general estimates of plume characteristics.

More specifically, the models generally overpredict dilution, as the presence of the bottom and shoreline cut off entrainment water that is assumed to be present in the model. Larger currents mean more dilution to

these idealized models, yet this does not occur (south-directed currents at Point Beach), since the shoreline cuts off entrainment water on the lee side of the plume. Surface isotherm areas are generally underpredicted when boundary interferences occur.

Summarizing the performance of the individual models:

(1) The Pritchard No. 1 model and the Shirazi-Davis models are, overall, the most accurate, despite being sometimes far off when compared to field data.

(2) The Pritchard No. 2 model typically predicts too great a dilution; as a whole, the model is not an improvement over the Pritchard No. 1 model.

(3) The Stolzenbach-Harleman and Prych models generally overpredict dilution and greatly overpredict plume widths. The Stolzenbach-Harleman model has a plateau in its centerline-temperature decay, not observed in the field data.

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G. Piacenza and Ostiglia Power Stations: Verification of Selected Rapid River Models

1. Introduction

The Center for Computational and Experimental Studies (CISE) in Milan, Italy, has been working under contract since 1971 for the National Organization for Electrical Energy (ENEL) to map the thermal plumes from the Piacenza and Ostiglia power stations on the Po River, especially during the critical situation of low river flow. One further aim of the work is to research the mixing phenomena in the near field and to follow with verification and improvement of near-field models. The conditions under study involve the Po River, which, throughout all periods of the year, has an ambient current speed greater than the initial discharge velocity for the two power plants under examination. This situation is referred to as the rapid-river case. In a rapid-river situation, the thermal plume originating from a shoreline discharge is quickly bent over and pushed up against the shore. Inshore entrainment is consequently shut off, producing narrow elongated isotherms that hug the shoreline. A rapid-river discharge probably occurs often enough in U. S. rivers to be considered here. The CISE research team has mounted an effort¹⁻⁶ to map the velocity and temperature fields of rapid-river plumes from these two sites and to translate that knowledge into a viable mathematical model.

Figure IV.G.1a qualitatively illustrates the rapid-river jet. In that figure, u_H represents the ambient current speed and m_0 stands for the ratio of ambient to discharge velocities. As shown in Fig. IV.G.1a, the jet velocity has, at least qualitatively, an inverted Gaussian profile. This profile is inconsistent with the traditional assumptions of the popular integral models discussed previously, such as Motz-Benedict,⁷ Stolzenbach-Harleman,⁸ Prych,⁹ or Shirazi-Davis,¹⁰ making them unsuitable for such river situations. Figure IV.G.1b illustrates a case in which the discharge velocity is larger than the ambient current speed, indicating the usual Gaussian velocity distributions assumed for such a jet. The two idealized distributions are compared in Fig. IV.G.1c.

2. Field Work

The CISE plume mapping began¹ in the spring of 1971 with a scoping study of the Piacenza plume. Figure IV.G.2 illustrates the discharge and bottom topography in the vicinity of the Piacenza discharge. The dashed contours indicate an island that separates two branches of the river; the boundary varies with river stage. Also illustrated are the Emilia and Levante intakes, whose water passes through the plants and is discharged at the same surface outfall canal. The Emilia and Levante substations each have two units; the total complex of four units makes up the Piacenza steam power station. The points A and B in Fig. IV.G.2 represent the "cold" and "hot" stations, respectively, from which the ambient and outfall temperatures are monitored continuously.

The CISE uses two mobile platforms and two fixed stations for measurements of velocity and temperature. The main mobile platform is a flat-bottom boat for transverse plume measurements for distances greater than 10 m from the nearshore river bank. The second mobile platform is an automatic crane that moves on land and is used to obtain vertical temperature profiles within 10 m of the shore and for longitudinal distances less than 100 m downstream. The CISE instrumentation is summarized in Tables IV.G.1-4.

Table IV.G.5 summarizes the 10 field surveys carried out at the Piacenza and Ostiglia Power Plants. Noteworthy is the slow variation of ambient conditions from one day to the next, allowing a more extended survey to take place over a period of 2-3 days. The March-April 1971 scoping surveys of the Piacenza Plant indicated that upcurrent of the discharge, the water temperature did not vary appreciably with depth. Also, a temperature survey on August 23, 1971, indicated that at an upstream location, $X = -20$, $Y = 35$ (see Table IV.G.2 for nomenclature), the temperature varied only 0.1°C vertically as the measurement depths varied from 0.25 to 2.80 m in steps of 0.4 m.

Fig. IV.G.1a. Qualitative Configuration of a Jet When $m_0 > 1$. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1317.

Fig. IV.G.1b. Qualitative Configuration of a Jet When $m_0 < 1$. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1317.

Fig. IV.G.1c

Qualitative Variation of Transverse Velocities for $m_0 > 1$ and $m_0 < 1$ (as simulated by the Vanderbilt models^{7,11}). (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1317.

TABLE IV.G.2. Position Location on the Po River

The generic point P in the river is marked out by three coordinates X, Y, and Z defined in the following way:

-- X is the longitudinal distance to a river cross section as measured along the right river bank, from the midsection of the discharge channel. Each section has been chosen (see Fig. IV.G.2), in the first approximation, perpendicular to the river current and, further, is identified by its distance X and by the angle ϕ which it forms with North.

-- Y is the lateral distance to the point P measured from the line of separation between the water and the right river bank.

-- Z is the depth of the point of measurement from the surface of the water.

According to the adopted definitions, the coordinates Y-Z of the point P vary with the level of the river and the adopted system of reference is not Cartesian.

The pole on the boat that carries the transducers is aligned with stakes previously fixed on both river shores for identifying the various sections of measurement.

The field studies of 1971 included river cross sections at X = 16, 28, 38, 50, 64, 94, 112, 212, and 350 m downcurrent; in 1972, sections 147, 162, 178, 244, 265, 295, 500, 670, 985, 1690, and 2150 m were added. The choice of some of these latter sections was, in part, motivated by the possibility of effecting a good anchoring. Along each vertical section, the Y measurements are spaced typically at 2.0 m; the Z measurements are typically 0.4 m from one vertical location to another.

TABLE IV.G.3. Equipment for Temperature and Velocity Measurements

-- Platinum-resistance thermometers: Type BSE 712J4 5020-2 of the Rosemount Engineering Company; nominal resistance at 0°C, $100.0 \pm 0.1 \Omega$; precision Type II, response time, <2 sec.

-- Thermocouples: Chromel-Alumel type; thermocoax 2ABc10 of Philips; response time, 0.05-0.1 sec.

The platinum-resistance thermometers are used for the base temperature measurements, and the thermocouples are utilized for the measurement of temperature oscillations about the base values determined. The measurement precision of the resistance thermometers is considered to be $\pm 0.01^\circ\text{C}$.

The temperature sensors are protected in a tube of stainless steel; the rapidity of response is claimed to be practically unaltered. The maximum error of the temperature measurements is $\pm 0.1^\circ\text{C}$.

-- OTT current meter: (a) Type 10.002 with impeller 125 mm in diameter and 0.25 m long; range of measurements, 0.25-2.50 m/sec; (b) Type 10.002, with screw 125 mm in diameter and 0.50 long; range of measurements, 0.5-5.00 m/sec.

-- SIAP current meter: Type 4001 with impeller 125 mm in diameter and 0.25 m long; range of measurements, 0.35-3.5 m/sec.

TABLE IV.G.4. Data-acquisition System

All measurement signals coming from the fixed "hot" and "cold" stations and from the transducers mounted on the automatic crane are sent to a single substation mounted on a van that is equipped with the following instruments:

-- HP 3450 digital voltmeter: $5\frac{1}{2}$ digits with the possibility of reading the resistance directly, having a precision $\pm(0.01\%$ of the actual reading + 0.01% of full scale) and connected to a printer with maximum printing speed of 10 measurements per second.

-- Dynograph: Type R. S. Beckman galvanometer with two channels having a maximum frequency of response of 80 Hz and connected to the thermocouples for recording the rapid oscillations of temperature, which can occur in the initial phase of mixing of the plume and ambient currents.

-- Leeds and Northrup Speedomax XL: Type W/L AZAR chart recorder with two channels, connected to thermocouples installed upstream and in the discharge canal in order to obtain a continuous recording of the "hot" and "cold" reference temperatures.

-- SIAP CE6010 rotation counter, connected to the current meters with the possibility of resolution of up to one-half turn due to an incorporated electronic divisor; the measurement time is determined by means of a chronometer with a Type CE6012 synchronized interrupter.

The boat is equipped with the following instruments:

-- TE 350 digital voltmeter: $4\frac{1}{2}$ digits with a precision of $\pm(0.01\%$ of the actual reading + 0.01% of full scale)

-- A counter of identical characteristics to that used in the velocity measurements carried out on the riverbank by the crane.

TABLE IV.G.5. Summary of Data Acquisition from Ten Field Surveys Carried Out by CISE/ENEL

Survey Number	Plant	Date	Power Level	Location of Transects (x)	Transverse River Distances (y)	Vertical Distances Measured	Report Number that Data Is Contained
<u>Temperature Measurements Only</u>							
1	Piacenza	3/30-31/71	252-350 MWe	only 11 points in the river were temperatures measured with depth			1
2	Piacenza	4/5-6/71	338-390 MWe	only 7 points in the river were temperatures measured with depth			1
3	Ostiglia	5/24-25/71	194-302 MWe	temperature was measured with depth at 44 points in the river			3
4a	Piacenza	8/30-31/71	350-380 MWe	17, 20, 26, 38, 63, 112, 237 m	2-6 lateral points ranging from 1-23 m	1-6 vertical points ranging from 0.2-3.05 m	2A/B
5	Piacenza	9/1-2/71	380-390 MWe	17, 23, 28, 38, 63, 112, 185, 212, 237, 270 m	1-7 lateral points ranging from 2-23 m	1-7 vertical points ranging from 0.25-2.65 m	2A/B
6a	Piacenza	9/9-10/71	390-460 MWe	17, 38, 63, 94, 112 m	3-7 lateral points ranging from 2-27 m	1-6 vertical points ranging from 0.25-3.65 m	2A/B
7	Piacenza	9/21-22/71	460 MWe	17, 38, 112, 212, 350, 420, 800, 1100, 1600 m	3-6 lateral points ranging from 5-150 m	1-5 vertical points ranging from 0.25-3.25 m	2A/B
8	Piacenza	10/11-12/71	400-415 MWe	17, 28, 38, 50, 63, 94, 112, 212, 350, 420, 800 m	5-11 lateral points ranging from 1-100 m	1-10 vertical points ranging from 0.25-3.60 m	2A/B
<u>Velocity Measurements Only</u>							
4b	Piacenza	8/20/71	—	-20 m	6, 11, 16, 21, 27, 32, 37, 42, 48.5, 53.5, 58.5, 63.5, 68.5, 73.5 m	0.6 y_{max}	2A/B
4c	Piacenza	8/23/71	—	-20 m	15, 25, 35, 50, 63, 74, 81, 100.5 m	0.6 y_{max}	2A/B
6b	Piacenza	9/7/71	—	-20 m	7, 11, 16, 21, 26, 31, 36, 41, 46, 49, 54, 59, 64, 69, 74, 79 m	0.6 y_{max}	2A/B
4d	Piacenza	8/27/71	—	14 m	5 lateral points ranging from 2-14 m	4-7 vertical points ranging from 0.35-3.20 m	2A/B
4e	Piacenza	8/30/71	—	38 m	4 lateral points ranging from 2-12 m	2-6 vertical points ranging from 0.4-5.0 m	2A/B
4f	Piacenza	8/31/71	—	26 m	3 lateral points ranging from 6-12 m	6 vertical points ranging from 0.4-2.95 m	2A/B
6c	Piacenza	9/10/71	—	17, 38, 94 m	2 lateral points ranging from 5-12 m	4-7 vertical points ranging from 0.25-3.35 m	2A/B
<u>Temperature and Velocity Measurements</u>							
9	Piacenza	8/25-25/72 (temperature measurements)	680-690 MWe	10, 28, 38, 50, 64, 94, 112, 147, 162, 178, 244, 265, 295, 300, 500, 670, 985, 1690, 2150 m	1-12 lateral points ranging from 1-90 m	1-10 vertical points ranging from 0.25-3.90 m	5A/B
		8/28-29/72 (velocity measurements)	—	16, 28, 38, 50, 64, 92, 112 m	1-11 lateral points ranging from 2-22 m	2-10 vertical points ranging from 0.25-4.0 m	5A/B
		8/11/72 (velocity measurements)	—	-20 m 64 m	6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, 63, 68, 73, 78, 83, 96 m 17 lateral points ranging from 13-100 m	0.6 y_{max} 0.6 y_{max}	
10	Ostiglia	9/27-28/72 (temperature measurements)	320 MWe	0, 3, 6, 9, 12, 81, 137, 190, 237, 291, 352, 444, 514, 576, 636	1-4 lateral points ranging from 9-218 m	4-5 vertical points ranging from 0.25-5.0 m	6
		10/4/72 (velocity measurements)	—	at discharge location	9, 20, 28, 45 m	5-9 vertical points ranging from 0.70-4.1 m	6

Moreover, the upcurrent temperature varied little with upstream position. Temperature measurements were taken at points C, D, E, and F (see Fig. IV.G.2) at depths of 0.25 and 1.0 m during the August 1972 tests. The temperatures at these points matched that of position A to within the sensitivity of the resistance thermometers a few hundredths of a degree Celsius. This established the validity of position A chosen as the "cold" station. Typically, the temperature at A (at 0.25-m depth) varied less than 1 C° during the 7-hr period typical of the field survey. Since each plume temperature was taken at a different time, each comparison of temperatures is made to the ambient temperature determined at A, denoted T_A , at the same instant. This was effected by an interpolation of various values measured at A with time. The CISE authors justify their interpolation procedure by citing the small amplitude and regularity of the variations of T_A in the period of one day.

To further prove that conditions may be assumed constant for the majority of multiday surveys, the authors of the project provide statistics of some major parameter variations for the typical survey of August 23-25, 1972. During these three days, the river level H varied 0.16 m, giving a variation of about 7% in river discharge flow. The power output of the Piacenza plant varied 10 MWe maximum, equivalent to a 1.5% variation. The excess temperature ΔT between the "hot" and "cold" stations varied about 1%. The plant flow rate was estimated to vary 4%. The authors assumed that the above variations were of modest importance, so that the entire survey can be considered to have been carried out with constant reference conditions.

The outfall temperature is itself an important parameter in characterizing the reference conditions and reflects the variation in power level of the units. Since the measured canal temperatures vary less than about ± 0.2 C° from each other or about 2.5% of the excess temperature (outfall minus intake temperature), the CISE authors thus expected that a single location in the discharge channel ("hot" station B) would suitably represent the outfall temperature.

The temperature difference $\Delta T = T_B - T_A$ and the discharge flow rate are representative of the power level of the plant. The flow rate is not measured directly but deduced, with an inevitable margin of uncertainty from nominal values of the circulating flow; this discharge flow is changed only in the case of large power variations by the number of pumps in operation (one or two per unit). An analysis of ΔT for numerous surveys disclosed that the value of ΔT in a single day was about constant, which is in accord with a constant power load. The small variations that did occur could be accounted for by variations in discharge flow rate, power level, and/or pressure across the condensers. The maximum difference in ΔT is related to variations of the power-plant power level between different survey dates. The CISE authors prove this through an example: September 1, 1971, had a $\Delta T \approx 8.0$ C° and a power level of 380 MWe, and October 11, 1971, had a $\Delta T \approx 8.8$ C° and a power level of 415 MWe; a simple calculation reveals that these values of power level are consistent.

Variation in ΔT as well as power level for a particular survey are generally very significant if the data are taken over a long period of time (more than several hours and up to a few days). The actual assessment of variations in ambient and outfall conditions as done here is very necessary when the data are to be used for model verification and improvement.

Temperatures in the discharge canal were studied in some detail on August 24, 1972. Figure IV.G.3 gives a plan and section view of the Piacenza discharge channel with the locations G, H, I, L, M, N, O, and B at which temperatures were measured. Table IV.G.6 summarizes the recorded temperatures. Note that "hot" station B is always higher by 0.2-0.4 C° than the temperatures at any other point. The CISE authors speculate as to possible reasons for this variation.

(1) The two units of the Levante Plant discharge separately into the canal, and anytime the units operate with different power levels, the discharge flows will be at different temperatures. A variation of 3 MWe is translatable in the Levante case to ~ 0.16 C° temperature difference in the cooling water. During the August 1972 field survey, these two units discharged their condenser water with a temperature difference of 0.6-0.7 C°.

(2) Also discharged from the Levante Station into the channel is the service water, through a system of pipes that separates the collecting tanks of the condenser water of the two Levante units. The temperature of this water is unknown, yet can be considerably different than the water coming from the condensers.

(3) The Emilia Central Station also discharges into the canal at a temperature that can be quite different than the preceding discharges.

TABLE IV.G.6. Measured Temperatures in Piacenza Discharge Canal on August 8, 1972 (from Rota and Zavatarelli⁵)

Hour	H River Level, m	Position	Z, m	T, °C	T _B , °C
1445	42.58 ↓	G	0.50	26.9	27.1
1450		G	1.00	26.8	27.1
1500		H	0.50	26.9	27.1
1505		H	1.00	26.8	27.1
1510		I	0.50	26.9	27.2
1515		I	1.00	26.8	27.2
1520		L	0.50	26.9	27.2
1525		L	1.00	26.9	27.2
1530		M	0.50	26.9	27.2
1535		M	1.00	26.9	27.2
1540		N	0.50	27.0	27.2
1545		N	1.00	27.0	27.2
1550		O	0.50	27.1	27.3
1555		O	1.00	27.1	27.3

Fig. IV.G.3. Plan and Section View of the Condenser-water Discharge Channel at Piacenza.
(Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1357.

The discharge canal therefore is the confluence of four discharges of different temperatures. A certain mixing distance is required for complete uniformity in temperature. Even for equilibrium power levels of the units involved, that mixing distance can be larger than the length of the discharge canal itself.

3. Analysis of River Plume Data

Table IV.G.5 clearly indicates that considerable data have been collected in the thermal-plume studies at Piacenza and Ostiglia.* The Piacenza data show many similar characteristics from one date to the next, because ambient and outfall conditions do not vary significantly. Most of the data were taken during low flow conditions of the Po River to evaluate the most critical conditions for the discharge into the river. The 1971 Piacenza plumes were measured when the plant was operating at about the 300-400-MWe range; the 1972 data were taken when the plant was at a 680-690-MWe level.

A brief summary of the results of the August 23-25, 1972, survey at Piacenza is instructive at this point to uncover the physics of rapid-river plume dispersion and to show motives for the assumptions made in the mathematical models for rapid-river plumes. This discussion is followed by some general conclusions about the plumes measured on the Po River.

With the large ambient current present, the Piacenza plumes will, after a few meters, be directed downstream and confined to the shoreline.

Fig. IV.G.4

Schematic Representation of a Jet against a Wall (Shoreline). (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1306.

The CISE authors approximate this situation for data-discussion purposes using an identical jet directed downcurrent and positioned at the nearshore line (see Fig. IV.G.4). (This is also the Abramovich simulation¹² for confined jets discussed later.) From this approximation the jet can be considered divided into the following zones in the longitudinal direction:

-- Initial zone: the part of the jet in which there exists at least one point characterized by a temperature or velocity equal to that at the mouth of the discharge.

-- Main zone: the part of the jet in which the profiles of velocity and temperature are similar and easily expressible in terms of analytical functions.

*The Ostiglia data are not as suitable for model comparisons due to the presence of interfering piers of an overhead railway within about 80 m of the outfall. The railroad pillars interfere with the plume flow field to some extent.

Between the above two zones is distinguished a third one, called the transition zone, in which the physical characteristics are not easily determinable.

Laterally, the initial zone is distinguished by:

-- Potential core: that part of the initial zone in which the same temperature and velocities are found as in the discharge canal. The potential core is limited to extend from the discharge to the end of the initial region. By definition, it is characterized by zero temperature and velocity gradients.

-- Jet boundary layer: that portion of the initial zone between the potential core and the ambient river water. This jet boundary layer is characterized by strong gradients of temperature and velocity.

From the Piacenza measurements, it was easy to verify that the initial and main regions did exist and that the Abramovich simulation (referred to above) of a rapid-river discharge does have merit. The latter is discussed in more detail when the Abramovich model for rapid-river discharges is reviewed.

Figure IV.G.5 sketches the river-level variation above sea level over July and August 1972. Only toward the middle and end of August was the river level sufficiently near low water ($H < 43$ m) that near-critical conditions were considered to be present; those conditions were significant enough for the CISE field research. Figure IV.G.5 also represents the variation in river discharge as a function of level. On the graph are reported experimental points obtained during the 1971 and 1972 field surveys. Figure IV.G.6 reports the variations in ambient temperature measured at A for the survey dates August 23-25. During the first day, this temperature remained practically constant at about 19.2°C ; during the second day, an increase of about 0.8°C was observed from 18.4 to 19.2°C ; and for the third day, there was an increase of about 1.1°C from 18.8 to 19.9°C . As stated above, each plume temperature is taken with reference to the ambient temperature at that instant, for the purpose of calculating the excess temperatures.

Figure IV.G.7 reports measurements of ambient velocity and river flow taken on August 8, 1972. Figure IV.G.7 also illustrates the bottom profile, the values of ambient velocity measured to a depth equal to 0.6 of the maximum depth for 16 vertical measurements, and curves of integration constructed to determine the river discharge. These measurements were made on August 8, when the plant was on reduced power.

Figures IV.G.8 and 9 sketch vertical isotherms in cross sections at 16 and 64 m downcurrent respectively. Some lateral diffusion has clearly taken place, as the width is greater at 64 m. However, since the plume is nearly fully mixed from surface to bottom due to turbulence in the jet and entrained ambient water and as generated by the bottom, lateral density gradients are apparently unimportant and do not add additional lateral spreading. Figure IV.G.8 represents a cross section in the initial region ($X \leq 40$ m) of this survey.

Fig. IV.G.5a. Level of the Po River for July-August 1972. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1349 Rev. 1.

Fig. IV.G.5b. Variation of Flow in the Po River as a Function of Its Level. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1349 Rev. 1.

Fig. IV.G.6. Variation of the Temperature at Position A ("Cold" Station) in the Course of a Day. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1356.

Fig. IV.G.7. Bottom Profile of the Po River and Values of Velocity Measured at 0.6 of the Depth at 16 Vertical Locations with a Curve of Integration to Determine the River Flow. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1354.

Fig. IV.G.8. Measured Excess Temperature ΔT and Variation of Some Isotherms Obtained by Interpolation for the Section X = 16 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1348.

Fig. IV.G.9. Measured Excess Temperature ΔT and Variation of Some Isotherms Obtained by Interpolation for the Section X = 64 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1350.

Figures IV.G.10-15 sketch lateral temperature decays at depths of 0.25, 1.0, 1.8, and 2.2 m for numerous traverses across the plume. Cross sections in the initial region show a region of constant temperature followed by a rapid dropoff to ambient temperature. These decay curves tend to flatten out further down current, with no apparent region of constant temperature adjacent to the shore. Temperature curves with maximum points indicate that the jet centerline is slightly displaced from the river bank.

Figures IV.G.16-18 plot horizontal isotherms at depths 0.25, 1.80, and 2.60 m; Fig. IV.G.19 shows the lateral and downstream extent of the 1 C° isotherm. Note that the isotherms vary little with depth and that the regions of high temperatures, laterally and longitudinally, are close to the outfall.

Figures IV.G.20-23 plot longitudinal temperature decays at different depths for fixed lateral distances of 4, 6, 8, 10, 14, 16, and 18 m. For values of Y of 4-8 m, the variations of the curves $\Delta\bar{T}$ as a function of X are similar from one depth to another. The variation of these curves is in good agreement with the turbulent-jet theory of Abramovich discussed below. According to this theory, the discharge temperature is maintained in the potential core portion of the initial region, and beyond that core into the boundary-layer portion, a rapid decay of temperature is predicted. For Y = 8 m, the discharge $\Delta\bar{T}$ is conserved, for each depth analyzed, up to X ≈ 40 m. For Y > 8 m, the hypothesis of parallel jets visualized in Fig. IV.G.4 begins to falter. For Y = 10 m, in fact, the curves begin to exhibit a maximum indicating an off-riverbank centerline. That the axis of the potential core is about 8 m from the right shoreline of the river is approximately affirmed.

Vertical velocity profiles are given in Figs. IV.G.24-26. Some of these figures show somewhat irregular variations due to the considerable non-uniformities of the river bottom. The presence of prominences that penetrate from the shore into the river cause eddies and rotational vortices, which produce local counter currents with respect to the discharge and the river. (See open circles at Y = 4 m in Fig. IV.G.25.)

Figures IV.G.27-29 report velocity and temperature variations as a function of Y on the same graph, relative to sections X = 16, 28, 38, 50, 64, 94, and 112 m and for two depths: Z = 0.7 and 2.0 m. For Z = 2.0 m, curves corresponding to sections X = 28 and 50 m are missing. These curves could be used for a theoretical analysis in the light of calculational models presently in existence.

A few of the general conclusions made by CISE upon studying the totality of the Piacenza plumes can now be summarized.

- (1) The configuration of the plume for all the surveys carried out appears fairly similar.
- (2) The plume tends to maintain itself on the surface in the sense that the plume gets narrower and contracts considerably with distance from the surface.

Fig. IV.G.10. Variation of Lateral Excess Temperature $\overline{\Delta T}$ for Fixed Values of X at $Z = 0.25$ m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1362.

Fig. IV.G.11. Variation of Lateral Excess Temperature $\overline{\Delta T}$ for Longitudinal Distances $X = 28, 50,$ and 94 m at Depth of 1.00 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1341.

Fig. IV.G.12. Variation of Lateral Excess Temperature $\overline{\Delta T}$ for Longitudinal Distances $X = 28, 50,$ and 94 m at Depth of 2.20 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1337.

Fig. IV.G.13. Variation of Lateral Excess Temperature $\overline{\Delta T}$ for Longitudinal Distances $X = 147, 162, 178, 212, 244, 265, 295,$ and 350 m at Depth of 0.25 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1345.

Fig. IV.G.14. Variation of Lateral Excess Temperature $\overline{\Delta T}$ for Longitudinal Distances $X = 147, 162, 178, 212, 244, 265, 295$, and 350 m at Depth of 1.00 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1338.

Fig. IV.G.15. Variation of Lateral Excess Temperature $\overline{\Delta T}$ for Longitudinal Distances $X = 147, 162, 178, 212, 244, 265, 295$, and 350 m at Depth of 1.80 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1340.

Fig. IV.G.16. Variation of Horizontal Isotherms for $X < 350$ m at Depth of 0.25 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1365.

Fig. IV.G.17. Variation of Horizontal Isotherms for $X < 350$ m at Depth of 1.80 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1363.

Fig. IV.G.18. Variation of Horizontal Isotherms for $X < 350$ m at Depth of 2.60 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1366.

Fig. IV.G.20. Variation of Excess Temperature $\overline{\Delta T}$ as a Function of X for Lateral Distances Y = 4, 8, 10, and 14 m and Depths of 0.25, 1.00, and 1.80 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1361.

Fig. IV.G.21. Variation of Excess Temperature $\overline{\Delta T}$ as a Function of X for Lateral Distances $Y = 16$ and 18 m at Depths of 0.60 , 1.40 , and 2.20 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1344.

Fig. IV.G.22. Variation of Excess Temperature $\overline{\Delta T}$ as a Function of X for Lateral Distances $Y = 6, 8, 10$, and 14 m and Depths of $0.60, 1.40$, and 2.20 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1343.

Fig. IV.G.23. Variation of Excess Temperature $\overline{\Delta T}$ as a Function of X for Lateral Distances $Y = 16$ and 18 m and Depths of 0.25 , 1.00 , and 1.80 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1339.

Fig. IV.G.24. Vertical Velocity Profiles in the Vicinity of the Discharge for Lateral Distances $Y = 4, 6, 8, 10,$ and 12 m at $X = 28$ m Down Current. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1355.

Fig. IV.G.25. Vertical Velocity Profiles in the Vicinity of the Discharge for Lateral Distances $Y = 4, 6, 8, 10, 12, 13, 14, 16, 18,$ and 20 m at $X = 64$ m Down Current. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1360.

Fig. IV.G.26. Vertical Velocity Profiles in the Vicinity of the Discharge for Lateral Distances $Y = 7, 9, 11, 13, 15, 17,$ and 19 m at $X = 112$ m Down Current.
 (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1351.

Fig. IV.G.27. Comparison between Variations of Excess Temperature and Velocity as a Function of Lateral Distance Y for Longitudinal Distances $X = 16, 38, 64$, and 94 m at Depth of 0.70 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1358.

Fig. IV.G.28. Comparison between Variations of Excess Temperature and Velocity as a Function of Lateral Distance Y for Longitudinal Distances $X = 28, 50$, and 112 m at Depth of 0.70 m and for $X = 112$ m at Depth of 2.00 m. (Adapted from Rota and Zavatarelli.⁵) ANL Neg. No. 190-1359.

Fig. IV.G.29. Comparison between Variations of Excess Temperature and Velocity as a Function of Lateral Distance Y for Longitudinal Distances $X = 16, 38, 64,$ and 94 m at Depth of 2.00 m. (Adapted from Rota and Zavatarelli,⁵) ANL Neg. No. 190-1353.

(3) The zone of interest for thermal perturbations, especially for the highest excess temperatures, is rather restricted and concentrated in the vicinity of the right-hand shore.

(4) The maximum thermal gradients on axes X and Y are in the vicinity of the discharge.

(5) In the initial zone ($X < 40$ m for the August 1972 survey), the variations in the velocity and temperature curves can be subdivided along the Y axis in three parts, of which the central one has strong slope and the other two have modest gradients (see Figs. IV.G.27-29).

(6) Different from the predictions of jet theory (seen later), the three parts of the two curves do not always coincide.

(7) Generally, the velocity of the undisturbed river is reached for values of Y at which the thermal disturbance is still appreciable. This implies that heat is transported faster than momentum.

(8) The plume studies of 1971 and 1972 were done at differing power levels; the 1971 surveys had Piacenza at ~ 400 MW, and during 1972, the plant was operating at nearly 700 MW. Comparing horizontal isotherms from the two years revealed that, once the power was increased, a notable enlargement appeared in the longitudinal extent of the plume defined by the $\Delta T = 7, 5,$ and 3°C isotherms; on the other hand, the $\Delta T = 1^\circ \text{C}$ isotherm presented only modest variations. The enlargement of the plume, in the transverse direction, is much more modest, being a few meters and becoming noticeable only in the initial region of the jet in relation, presumably, to the greater velocity of the discharge.

(9) The boundary-layer zone of the initial region of the jet had an immediate separation from the bank for the 1972 survey, compared with its protrusion in the 1971 survey. The boundary-layer zone of the jet can be determined approximately by examining the recordings of the thermocouples, because that region is characterized by strong turbulent mixing. This presents ample oscillations in temperature, as happened for points in the August 1972 survey for $Y = 8-10$ m.

4. Model-verification Work

The model-verification work done by CISE involves three integral-type models for thermal discharges into rapid-river currents. The first is the Vanderbilt University model developed by Benedict, Polk, Yandell, and Parker.¹¹ It is a modification of the Motz-Benedict model discussed in Chapter II. The second model is meant to be a CISE improvement⁴ on the Vanderbilt model and circumvents the need for entrainment and drag coefficients that are generally unknown. However, these models introduce a parameter K that itself requires data for its determination. The third, and most promising model evaluated was developed and presented by Abramovich in his classic treatise, The Theory of Turbulent Jets.¹² As additionally indicated, all three models can be applied to surface thermal discharges into large ambient currents. However, the Abramovich work has the advantage of allowing a complete predictive calculation without additional empirical or experimental data as well as providing the best comparisons with Po River thermal-plume data. Each model formulation is discussed briefly with its associated data comparisons.

a. The Vanderbilt Model

The Vanderbilt simulation¹¹ assumes a plume configuration and velocity field as sketched in Fig. IV.G.1 for $m_0 > 1$. The model is two-dimensional, assuming no variation with depth and entrainment occurring only at the sides of the jet. In this rapid-river case, the initial assumptions and equations are modified, starting from the Motz-Benedict simulation, since the jet velocity is now less than the ambient velocity. This "velocity deficiency" below ambient is the quantity assumed to have a Gaussian shape and to be similar section-to-section and is expressed as

$$u_H \cos \beta - u = (u_H \cos \beta - U) \exp(-\eta^2/b^2), \quad (1)$$

where

u = total jet velocity at any point,

u_H = ambient current (assumed uniform),

β = angle of jet axis to shoreline,

U = total centerline velocity,

η = lateral distance from centerline,

and

b = width of jet.

The continuity equation is simulated as

$$\frac{d}{ds} \left(\int_A u \, dA \right) = CE(u_H \cos \beta - U), \quad (2)$$

where

A = cross-sectional area of jet normal to centerline,

s = longitudinal distance from outfall,

C = circumference of jet in cross section exposed to ambient water,

and

E = entrainment coefficient.

Substituting Eq. 1 into Eq. 2 gives, with $C = 2Z_0$ (Z_0 being the depth of the jet),

$$\frac{d}{ds} [b(2u_H \cos \beta - \sqrt{\pi} U')] = 2EU', \quad (3)$$

where

$$U' = u_H \cos \beta - U.$$

If x is along the direction of the ambient current, then the x -momentum equation becomes

$$\frac{d}{ds} \left(\int_A u^2 \, dA \cos \beta \right) = CE(u_H \cos \beta - U)u_H + F_D \sin \beta. \quad (4)$$

with u from Eq. 1, $C = 2Z_0$, and

$$F_D = \frac{C_d u_H^2 Z_0 \sin^2 \beta}{2}, \quad (5)$$

Eq. 4 becomes

$$\frac{d}{ds} \left[2b \cos \beta \left(u_H^2 \cos^2 \beta - U' u_H \sqrt{\pi} \cos \beta + \frac{U'^2}{2} \sqrt{\frac{\pi}{2}} \right) \right] = 2EU' u_H + \frac{C_d u_H^2 \sin^2 \beta}{2}.$$

The y-momentum equation is

$$\frac{d}{ds} \left(\int_A u^2 \sin \beta \, dA \right) = -F_D \cos \beta, \quad (6)$$

which, upon substitution of Eqs. 1 and 5 into Eq. 6, becomes

$$\begin{aligned} \frac{d}{ds} \left[b \sin \beta \left(u_H^2 \cos^2 \beta - U' u_H \sqrt{\pi} \cos \beta + \frac{U'^2}{2} \sqrt{\frac{\pi}{2}} \right) \right] \\ = \frac{-C_d u_H^2 \sin \beta \cos \beta}{4}. \end{aligned} \quad (7)$$

The heat-energy equation is

$$\frac{d}{ds} \left(\int_A u T' \, dA \right) = 0. \quad (8)$$

Here surface heat loss is neglected due to the rapid dilution expected and the small surface isotherm areas produced. The authors assume a Gaussian distribution for T' given by

$$T' = T \exp \left(-\frac{\eta^2}{b^2} \right). \quad (9)$$

Upon substitution for u and T' , this gives*

$$\frac{d}{ds} \left[\left(u_H \cos \beta - \frac{U'}{\sqrt{2}} \right) T b \right] = 0. \quad (10)$$

All area integrals are converted to integrals from $-b$ to $+b$ at a cross section. Whenever exponentials were being integrated over that range, however, the authors replaced $(-b, b)$ by $(-\infty, \infty)$, since only minor differences occur and the integrations become simpler.

The equations of jet geometry are unchanged from the $m_0 < 1$ simulation, being

$$\frac{dx}{ds} = \cos \beta, \quad (11a)$$

*There is a slight nomenclature change here. The x , y , and T in this section were written in the previous discussion of the data as X , Y , and ΔT . This reflects the nomenclature of the original reports MISTER 1-6.

and

$$\frac{dy}{ds} = \sin \beta. \quad (11b)$$

Introducing

$$S = Es, \quad X = Ex, \quad Y = Ey,$$

and

$$M = \left(u_H^2 \cos^2 \beta - U' u_H \sqrt{\pi} \cos \beta + \sqrt{\pi/8} U'^2 \right) b, \quad (12)$$

the equations become

equation of continuity:

$$\frac{d}{dS} \left[(2u_H \cos \beta - \sqrt{\pi} U') b \right] = 2U'; \quad (13)$$

conservation of momentum along x:

$$\frac{d}{dS} (M \cos \beta) = U' u_H + \frac{1}{4E} C_d u_H^2 \sin^2 \beta; \quad (14)$$

conservation of momentum along y:

$$\frac{d}{dS} (M \sin \beta) = -\frac{1}{4E} C_d u_H^2 \sin \beta \cos \beta; \quad (15)$$

equation of conservation of energy:

$$\frac{d}{dS} \left[\left(u_H \cos \beta - \frac{U'}{\sqrt{2}} \right) \Delta T b \right] = 0; \quad (16)$$

and equations of jet geometry:

$$\left. \begin{aligned} \frac{dX}{dS} &= \cos \beta \\ \frac{dY}{dS} &= \sin \beta \end{aligned} \right\} \quad (17)$$

The unknowns in the equations are U' , ΔT , β , b , X , and Y . Thus, there are six unknowns and six equations, all a function of jet distance S . Additionally, there are two experimentally defined coefficients, E and C_d , required by the model.

Note the similarity between the above equations derived for $m_0 > 1$ and the equations for $m_0 < 1$ (derived by Motz and Benedict):

equation of continuity:

$$\frac{d}{ds}(Ub) = \frac{2E}{\sqrt{\pi}}(U - u_H \cos \beta); \quad (18)$$

conservation of momentum along x :

$$\frac{d}{ds}(U^2b \cos \beta) = \frac{2\sqrt{2}E}{\sqrt{\pi}}(U - u_H \cos \beta)u_H + \sqrt{\frac{2}{\pi}} \frac{C_d u_H^2 \sin^2 \beta}{2}, \quad (19)$$

conservation of momentum along y :

$$\frac{d}{ds}(U^2b \sin \beta) = -\sqrt{\frac{2}{\pi}} \frac{C_d u_H^2 \sin \beta \cos \beta}{2}; \quad (20)$$

equation of energy conservation:

$$\frac{d}{ds}(U\Delta Tb) = 0; \quad (21)$$

and equations of geometry:

$$\left. \begin{aligned} \frac{dx}{ds} &= \cos \beta \\ \frac{dy}{ds} &= \sin \beta \end{aligned} \right\} \quad (22)$$

The Gaussian assumptions for velocity have, at least qualitatively, the variation presented in Fig. IV.G.1. Note that, for $m_0 = 1$, the two variations in the model simulations are not symmetric in fact, while for $m_0 < 1$, the jet velocity tends to zero when the transverse coordinate tends to infinity. (In reality, the width of the jet is less than the thickness that would render u and u_H equal.) For $m_0 > 1$, the same velocity tends to u_H . This fact explains the nonsymmetry of the equations in the two cases.

Both systems have been solved by computer using Runge-Kutta techniques. The codes are presently available from Benedict, who is now at Tulane University.

The major disadvantages of the Benedict et al. model lie in the areas:

(1) Application to practical cases is severely hampered since the experimental values of E and C_d are not at our disposal as they were derived from laboratory and field tests for $m_0 < 1$.

(2) No information is available on the zone of flow establishment. For $m_0 < 1$, the Motz-Benedict graphs, developed from physical-model tests, are available.

(3) The Gaussian-velocity-distribution assumption of the model for $m_0 > 1$ is conceptually incorrect. The jet for the rapid-river case is thrust against the shore of the river after a short distance. Consequently, the velocity difference is maximum against the wall (because here the velocity of the jet is zero) and not on the geometrical axis of the jet as the Gaussian-velocity assumption indicates.

b. Modification of the Vanderbilt Model

The next step taken by CISE was to modify the model of Benedict et al. to the point that it no longer required E and C_d as input. The authors⁴ have introduced an experimental parameter K taken from the geometrical structure of the jet, supposing that

$$\frac{db}{ds} = K, \quad b = K(s - s_N) + b_N, \quad (23)$$

is valid in the region of established flow. Here s_N and b_N are the abscissa and half-width, respectively, at the end of the region of flow establishment (initial and transition regions). Introduction of the spreading parameter K sidesteps the need for an entrainment coefficient. However, experimental data are still needed to determine that parameter K . (As additionally indicated, the improvement of this model over the model of Benedict et al. is really only marginal, since the unknown entrainment coefficient E is essentially "renewed" as the spreading rate K , which is also unknown.)

For $m_0 > 1$, the jet is immediately flattened against the shore and therefore the coefficient C_d has a rather small influence on the whole phenomenon. Even the angle β , which in this case coincides with $K/2$, has a very small effect and can be considered constant. A schematic representation of the jet appears in Fig. IV.G.30.

Fig. IV.G.30

Geometrical Configuration of the Jet, Hypothesized through the Application of the Vanderbilt University Model. (Adapted from Cavanna and Folli,⁴) ANL Neg. No. 190-1309.

Lack of knowledge of jet behavior in the initial and transition regions of the jet does not permit a theoretical derivation of relationships for determination of s_N and b_N ; such conditions must be introduced empirically, as was done here.

The CISE version of the Benedict model uses only three differential equations. The assumptions on temperature are also different, being

$$\left. \begin{aligned} \Delta T &= \Delta T_m, & -b < n < 0; \\ \Delta T &= \Delta T_m e^{-(n/2b)^2} & b > n > 0. \end{aligned} \right\} \quad (24)$$

Here n refers to lateral distance measured from shore. The assumption of a constant-temperature-region transverse to shore followed by a rather rapid dropoff was deduced from the Po River thermal-plume data. Heat is dissipated only at the region of the plume in which the jet and ambient currents come in contact, the shore acting as a screen. Therefore the supposition is that, in the first (transverse) half of the jet, the temperature is maintained constant and only in the second half does it begin to decay according to a Gaussian variation twice that of velocity, in agreement with the theory of Taylor. The transverse velocity field is still expressed as

$$\Delta u = \Delta u_m e^{-(n/b)^2}, \quad -b < n < b. \quad (25)$$

The three equations used in the model are:

x-momentum equation:

$$\frac{d}{ds} \left[\int_{-b}^b u^2 \cos \beta \, dA \right] = \underbrace{\frac{d}{ds} \left[u_H \int_{-b}^b u \, dA \right]}_{\text{rate of change of x-momentum of entrained fluid}} + F_D \sin \beta; \quad (26)$$

the angle β is small, so it is assumed constant and $\cos \beta \approx 1$, $\sin \beta \approx \beta$;

y-momentum equation:

$$\frac{d}{ds} \left[\int_{-b}^b u^2 \sin \beta \, dA \right] = -F_D \cos \beta; \quad (27)$$

here,

$$F_D = \frac{C_d u_H^2 z_o \sin \beta}{2} \approx \frac{C_d}{2} u_H^2 z_o \beta; \quad (28)$$

conservation of energy:

$$\frac{d}{ds} \left[\int_{-b}^0 u' T' dn \right] = 0. \quad (29)$$

The resulting equations, after substitution for Δu and ΔT are made, become:

conservation of momentum along x:

$$2u_H^2 \frac{db}{ds} + I_2 \frac{d}{ds} (b \Delta u_m^2) - 4I_1 u_H \frac{d}{ds} (b \Delta u_m) = 2u_H \frac{d}{ds} [b(u_H - I_1 \Delta u_m)], \quad (30)$$

conservation of momentum along y:

$$2u_H^2 \frac{d}{ds} (b\beta) + I_2 \frac{d}{ds} [b(\Delta u_m)^2 \beta] - 4I_1 u_H \frac{d}{ds} (b\beta \Delta u_m) = -\frac{C_d}{2} u_H^2 \beta, \quad (31)$$

and conservation of energy:

$$\frac{d}{ds} (\Delta T_m) = -\frac{\Delta T_m}{b(u_H - I_1 \Delta u_m)} \frac{d}{ds} [b(u_H - I_1 \Delta u_m)]. \quad (32)$$

The result of drag force on the jet is negligible because the jet is against the wall and traveling with the current. Consequently, the C_d term in the y-momentum equation can be dropped. In the above equations, I_1 and I_2 are integrals of the transverse Gaussian velocity distribution:

$$I_1 = \frac{1}{2} \int_{-1}^1 f(n/b) d(n/b)$$

and

$$I_2 = \frac{1}{\sqrt{2}} \int_{-\sqrt{2}}^{\sqrt{2}} f(n/b) d(n/b),$$

where

$$\Delta u = \Delta u_m f(n/b).$$

The function f was taken as $e^{-(n/b)^2}$. The two momentum equations can be solved to yield

$$\Delta u_m = A - A^2 - \sqrt{\Delta u_0 (2A - \Delta u_0) h_0 / b}, \quad \Delta u_0 < A, \quad (33)$$

and

$$\Delta u_m = A + \sqrt{A^2 - \Delta u_o(2A - \Delta u_o)b_o/b}, \quad \Delta u_o > A, \quad (34)$$

where

$$A = u_H I_1 / I_2. \quad (35)$$

The heat equation can be solved to give the longitudinal temperature variation in the established flow region

$$\frac{\Delta T_m}{\Delta T_o} = \frac{b_o}{b} \frac{u_H - I_1 \Delta u_o}{u_H - I_1 \Delta u_m}.$$

This modified Vanderbilt model was compared to the October 1971 data from the Piacenza plant. The results appear in Figs. IV.G.31-33. From Fig. IV.G.31a, which represents the variation of temperature in the longitudinal direction at a fixed 6-m distance from the river bank, a value of s_N equal to about 45-50 m was determined. From Eqs. 33-34, with $b_o \approx 5$ m, a value of $b_N = 4.7$ m can be deduced. From Fig. IV.G.31b, in which isotherms are given as functions of transverse and longitudinal coordinates at the same depth, a value of K is deduced whose value is about 0.037.

With these data, the results of the modified Vanderbilt model are reported in Figs. IV.G.32 and 33, together with experimental points. Concerning Fig. IV.G.31a, the analysis of the experimental data brought the authors to use Eq. 34 for $0 < s < s_N$ and Eq. 33 for $s > s_N$.

Although the results obtained with the above model were fairly good, the CISE authors felt that this model was not yet satisfactory for the following reasons:

- (a) The necessity of determining experimentally the abscissa and the relative width of the jet at the transition section.
- (b) The assumption of a velocity profile that, for a jet against the wall, is conceptually erroneous.

c. The Abramovich Model

The two models previously treated have, as already described, several gaps in their conceptual developments. For that reason, the CISE authors turned to the work of G. N. Abramovich,¹² in which, among other things, the Russian author treats in detail the case of the so-called "parallel jets." Parallel jets, in this context, refers to jets in which the initial velocity is parallel to the velocity of the ambient current. Abramovich discusses the theory of free parallel jets, that is, those not bound by a wall. This theory can,

with excellent approximation, be reduced to the case of rapid-river discharges along a river bank. The connection lies in the fact that, after a very brief transition region (of the order of 2-3 m for Piacenza), the discharged jet bends to become completely pushed up against the wall, in keeping with the condition of "parallel jets." Abramovich also discusses this application of the free-jet theory to a discharge directed parallel and contiguous to a straight wall (shore-line, in this case). The main results of the Abramovich "free-jet" theory will be summarized, followed by the modifications of the theory required by the presence of a solid wall. Abramovich discusses the two-dimensional theory of free and confined jets for $u_H < u_0$. Only the latter case is reviewed here.

Fig. IV.G.31a

Longitudinal Variation of Maximum Temperature of the Protruding Jet during the Experimental Field Study of October 11-12, 1971, Used for the Determination of s_N . (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1308.

Fig. IV.G.31b. Plane Isotherms (of excess temperature) Measured October 11-12, 1971, Used for the Determination of K . (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1316.

Fig. IV.G.32. Longitudinal Variation of Maximum Temperature According to the Modified Vanderbilt Model in Relation to the Field Data of Piacenza of October 1971. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1304.

Fig. IV.G.33. Transverse Variation of Temperature According to the Modified Vanderbilt Model in Relation to the Field Data at Piacenza of October 1971. (The experimental points are normalized by the theoretical $\Delta T_{\max}$.) (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1307.

Fig. IV.G.34

Qualitative Representation of the Longitudinal Profile of Maximum Velocity in a Jet. (Adapted from Cavanna and Folli.⁴)
ANL Neg. No. 190-1311 Rev. 1.

As noted before, Abramovich divides a free jet into three regions: initial, transition, and main (see Fig. IV.G.34). A sketch of the initial region appears in Fig. IV.G.35. Numerous experiments have shown that the velocity, temperature, and concentration profiles in the boundary layer between two currents is similar in each transverse section; on the basis of such observations, one can determine the proper law of variation. A fundamental point in the Abramovich theory consists of the determination of the law according to which the width b of the boundary layer increases as a function of longitudinal coordinate x of the jet between two (parallel) currents of infinite extent. Abramovich argues, from similarity considerations, that

$$\frac{db}{dx} = C \frac{|1 - m|}{1 + m},$$

where $m = u_2/u_1$, and u_2 and u_1 are the velocities of the jet and ambient currents, respectively. This formula is valid even if the two jets are not of infinite width, but then u_1 and u_2 are not constant. In particular, it is valid even for a jet of finite dimension in an infinite current; in such a case, in the initial region, b represents the width of the two boundary layers while $m = m_0 = u_H/u_0$; in the main region, b is the half-width of the jet and $m = u_H/u_m$, where u_m is the velocity of the jet (centerline) axis. Since u_m is variable along the axis, m varies with longitudinal coordinate x . Air-jet experiments (see Fig. IV.G.36) have shown there is an interval of values of m within which

Fig. IV.G.35a

Schematic Representation of the Boundary Layer between Two Parallel Currents. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1342.

Fig. IV.G.35b

Schematic Representation of the Initial Region of the Jet. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1342.

Fig. IV.G.36

Schematic of the Boundary Layer
between Two Plane Parallel Jets.
(Adapted from Cavanna and Folli.⁴)
ANL Neg. No. 190-1352.

db/dx depends scarcely at all on m , particularly for $0.3 \leq m \leq 3$. The result here is

$$\frac{db}{dx} = C \frac{|1 - m_0|}{1 + m_0}$$

along the whole jet, and means that it is an effectively linear growth of the jet width with plume centerline distance for these values of m . This is similar to the classical theory of jets discharging into stagnant environments.

Figure IV.G.36 shows six sketches from photographs of the boundary layer between two plane air jets at $T_a = 318^\circ\text{K}$ and $T_o = 462^\circ\text{K}$, obtained by Zestkov et al.¹² The velocity of the heated jet was 100 m/sec, and that of the cold jet varied from 0 to 60 m/sec; the corresponding values of m are indicated in the figure. When m was increased from 0 to 0.34, the thickness of the jet boundary layer diminished considerably, though the thickness remained nearly unchanged when m was further increased to 3.0.

The formulas Abramovich derives for the geometry, velocity, and temperature distribution for the free jet are summarized in Table IV.G.7. Uniform profiles of velocity and temperature were assumed at the orifice for this theory. A more detailed discussion of the similarity considerations and the momentum and continuity integral equations used appears in Refs. 4 and 12.

The calculational formulas derived by Abramovich referred only to uniform-initial-velocity profiles; in actual cases, however, the effects of initial nonuniformities ought to be treated as resulting from a nonuniform discharge flux. In particular, the discharge channel velocity is zero at the walls, which can notably influence the structure of the jet, as illustrated in Fig. IV.G.41.

From the Abramovich theory, it follows that the major changes with respect to the uniform-velocity case occur in the values of x_H and x_N . In regard to the initial region, everything proceeds as if the discharge point had been moved backwards a quantity x_{OH} ; this distance is difficult to determine. For the extreme case of a turbulent flux, completely developed, x_{OH} has the variation reported in Fig. IV.G.42. The calculational procedure for nonuniform discharge flux is summarized in Table IV.G.8.

TABLE IV.G.7. Free-jet Theory of Abramovich

Initial Region (see Figs. IV.G.34, 35, 37, and 38)

Geometry:

$$1. \quad \frac{y_1}{b} = 0.416 + 0.134m_o.$$

$$2. \quad \frac{y_2}{b} = -0.584 + 0.134m_o.$$

$$3. \quad b(x) = 0.27 \frac{|1 - m_o|}{1 + m_o} x,$$

$$b = y_1 + y_2.$$

$$4. \quad \frac{b_H}{b_o} = \frac{1}{0.416 + 0.134m_o}.$$

5. Figure IV.G.37 plots the graph of $\bar{x}_H = x_H/b_o$. It also may be calculated from

$$\bar{x}_H = \frac{1 + m_o}{0.27(1 - m_o)(0.416 + 0.134m_o)}.$$

Velocity profile:

$$u = u_o, \quad -\frac{b_o}{2} < y < y_2;$$

$$\frac{u_o - u}{u_H - u_o} = f(\eta) = (1 - \eta^{1.5})^2, \quad y_2 < y < y_1;$$

where

$$\eta = \frac{y - y_2}{b};$$

$$u = u_H, \quad y > y_1.$$

Temperature profile:

$$\Delta T = \Delta T_o, \quad -\frac{b_o}{2} < y < y_2;$$

TABLE IV.G.7 (Contd.)

$$\frac{\Delta T}{\Delta T_0} = \eta, \quad y_2 < y < y_1;$$

$$\Delta T = 0, \quad y > y_1.$$

See Fig. IV.G.38 for verification of temperature profile in the initial region of the jet.

Transition Region (see Figs. IV.G.34 and 39)

Geometry:

$$1. \quad \frac{b_N}{b_0} = \frac{1}{A_2 + m_0(A_1 - A_2)},$$

$$A_1 = 0.45, \quad A_2 = 0.316.$$

2. Figure IV.G.39 plots the graph of $\bar{x}_N = x_N/b_0$ for values of m_0 . (Use two top curves only for uniform temperature and velocity profile at the outfall.)

Velocity and temperature profiles:

Although Abramovich analyzes this region, the Italian authors prefer to avoid the complexity and, as a first approximation, correct the first and third zones as shown in Fig. IV.G.31a; this has been supported from the application of the model.

Main Region (see Figs. IV.G.34 and 40)

The geometry, velocity, and temperature distributions are calculated by choosing a value of centerline velocity u_m , and then deriving a location x , width b , and centerline temperature ΔT_m associated with that u_m . Then lateral profiles for that centerline location are determined with the following steps:

$$1. \quad A_1 = 0.45, \quad A_2 = 0.316, \quad \alpha = A_2/A_1 = 0.7.$$

$$2. \quad m = u_H/u_m.$$

$$3. \quad P^2 = \frac{4A_2(1-m)}{m^2 A_1^2}.$$

TABLE IV.G.7 (Contd.)

4. Solve for b in

$$\frac{\Delta u_m}{\Delta u_o} = \frac{u_m - u_H}{u_o - u_H} = \frac{mA_1}{2A_2(1 - m)} \left(\sqrt{1 + \frac{P^2 b_o}{b}} - 1 \right).$$

5. For $m < 1$,

$$\begin{aligned} \pm c \left(\frac{x}{b_o} - \frac{x_o}{b_o} \right) &= \frac{b}{b_o} + \frac{A_2 P^2}{A_1} \left[\left(\frac{2}{P^2} \frac{b}{b_o} + 1 \right) \sqrt{\frac{1}{P^2} \frac{b}{b_o} \left(\frac{1}{P^2} \frac{b}{b_o} + 1 \right)} \right. \\ &\quad \left. - \ln \left(\sqrt{\frac{1}{P^2} \frac{b}{b_o} + 1} + \sqrt{\frac{1}{P^2} \frac{b}{b_o}} \right) + \frac{2}{P^4} \left(\frac{b}{b_o} \right)^2 \right]. \end{aligned}$$

Here $x_o = 0$, and $c = 0.22$. For $m > 1$,

$$\begin{aligned} \pm c \left(\frac{x}{b_o} - \frac{x_o}{b_o} \right) &= b - b_N + \frac{\alpha}{P^2} \left\{ 2 \left(\frac{b}{b_o} + P^2 \right) \sqrt{\frac{b}{b_o} \left(\frac{b}{b_o} + P^2 \right)} \right. \\ &\quad \left. - \left(2 \frac{b_N}{b_o} + P^2 \right) \sqrt{\frac{b_N}{b_o} \left(\frac{b_N}{b_o} + P^2 \right)} \right. \\ &\quad \left. - P^4 \ln \left[\frac{\sqrt{\frac{b}{b_o} + P^2} + \sqrt{\frac{b}{b_o}}}{\sqrt{\frac{b_N}{b_o} + P^2} + \sqrt{\frac{b_N}{b_o}}} \right] + 2 \left[\left(\frac{b}{b_o} \right)^2 - \left(\frac{b_N}{b_o} \right)^2 \right] \right\}. \end{aligned}$$

Here $x_o = x_N$, and $c = 0.22$.

6. Lateral velocity field:

$$\frac{\Delta u}{\Delta u_m} = \frac{u - u_H}{u_m - u_H} = \left[1 - \left(\frac{y}{b} \right)^{1.5} \right]^2 = \left(1 - \xi^{1.5} \right)^2$$

for $y > 0$.

TABLE IV.G.7 (Contd.)

7. Lateral temperature field:

$$\frac{\Delta T}{\Delta T_m} = \sqrt{\frac{\Delta u}{\Delta u_m}}, \quad \frac{\Delta T}{\Delta T_m} = 1 - \left(\frac{y}{b}\right)^{1.5}.$$

See Fig. IV.G.40 for verification of above temperature profile in the main region of the jet. The length y_c in the figure is the distance from the axis to the point at which the velocity equals half the sum of the extreme values ($\Delta u_c = 0.5\Delta u_m$).

Alternatively, for centerline temperature decay:

$$\frac{\Delta T_m}{\Delta T_o} = \frac{\Delta u_m}{\Delta u_o} \frac{(\Delta u_m/\Delta u_o)A_2(1 - m_o) + A_1m_o}{(\Delta u_m/\Delta u_o)B_2(1 - m_o) + B_1m_o} = K_T \frac{\Delta u_m}{\Delta u_o},$$

where

$$K_T = \frac{(\Delta u_m/\Delta u_o)A_2(1 - m_o) + A_1m_o}{(\Delta u_m/\Delta u_o)B_2(1 - m_o) + B_1m_o},$$

$$B_1 = 0.6,$$

and

$$B_2 = 0.368.$$

Note: If (and whenever) $0.3 \leq m \leq 3.0$,

1. The initial and transition region formulas are valid.
2. For the main region

$$(a) \quad b - b_N = 0.22 \frac{|m_o - 1|}{m_o + 1} (x - x_N),$$

$$m_o = \frac{u_H}{u_o},$$

and b_N and x_N are as before.

$$(b) \quad \frac{\Delta u_m}{\Delta u_o} = \frac{u_m - u_H}{u_o - u_H} = \frac{m_o A_1}{2A_2(1 - m_o)} \left(\sqrt{1 + P^2 \frac{b_o}{b}} - 1 \right),$$

TABLE IV.G.7 (Contd.)

where

$$P^2 = \frac{4A_2(1 - m_0)}{m_0^2 A_1^2}$$

(c) Formulas for the transverse velocity and temperature profile remain unchanged.

Apply these formulas from the critical point x_K when the value 0.3 or 3.0 is attained.

Fig. IV.G.37

Variation of the Length of the Initial Region of a Jet (uniform flux) as a Function of the Ratio of Velocities $m_0 = u_H/u_0$. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1303 Rev. 1.

Fig. IV.G.38

Transverse Profile of Temperature in the Initial Region of a Jet. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1315.

Fig. IV.G.39

Variation in the Position of the Transition Section due to an Initial Nonuniformity of Flux. The curve on top refers to an initial uniform flux, the one below to an initial flux that is turbulent and completely developed. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1312 Rev. 1.

Fig. IV.G.40

Transverse Profile of Temperature in the Boundary Layer of the Main Region of the Jet. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1301.

Fig. IV.G.41

Effect on the Boundary Layer of a Jet
of an Initial Nonuniformity of Flux

Fig. IV.G.42

Variation in the Length of the Initial Region
due to an Initial Nonuniformity of the Flux.
(Adapted from Cavanna and Folli.⁴) ANL
Neg. No. 190-1310 Rev. 1.

TABLE IV.G.8. Modification of Abramovich Free-jet Theory for
Initially Nonuniform Velocity and Temperature Profiles

The following coefficients are consequently introduced by Abramovich concerning the initial nonuniformity of the discharge:

$$n_{1u} = \int_0^1 \frac{u_o}{u_{om}} \frac{dy}{b_o},$$

$$n_{2u} = \int_0^1 \left(\frac{u_o}{u_{om}} \right)^2 \frac{dy}{b_o},$$

and

$$n_T = \int_0^1 \frac{u_o}{u_{om}} \frac{\Delta T}{\Delta T_{om}} \frac{dy}{b_o}.$$

The quantities with the index "o" are the average values in the initial section and the index "om" refer to the maximum values (on the axis in the case of a true jet). Redefined are

$$P^2 = \frac{4A_2(n_{2u} - n_{1u}m_o)}{m_o^2 A_1^2},$$

$$K_T = \frac{n_T(1 - m_o)}{n_{2u} - n_{1u}m_o} \frac{(\Delta u_m / \Delta u_o) A_2(1 - m_o) + A_1 m_o}{(\Delta u_m / \Delta u_o) B_2(1 - m_o) + B_1 m_o},$$

and

$$b_N = \frac{n_{2u} - m_o n_{1u}}{(1 - m_o)[A_2 + m_o(A_1 - A_2)]}.$$

The new length of the initial zone is obtained from

$$x'_H = x_H - x_{OH},$$

where x'_H is the new value and x_H is that calculated for uniform discharge. For turbulent discharge flux, completely developed, $\bar{x}_{OH} = x_{OH}/b_o$ is obtained from Fig. IV.G.42 and x_H is determined from Fig. IV.G.37. The distance to the end of the transition region is determined directly from Fig. IV.G.39.

The calculational procedure remains the same as for the uniform-flux case with the formulas of Table IV.G.7.

The above theoretical treatment has an extremely simple application in the case of a jet against the wall. In this specific case, one has the formation of two boundary layers: the first one at the wall, and the second at the interface of the discharge and ambient current. The jet can, again, be divided into three regions (see Fig. IV.G.4). The first region of flow, the length of which is designated x_H , ends at the section where the boundary layers of the jet and the wall join. In this section, with coordinate x_N , the region of flow begins in which the axial velocity u_m is changed from the value u_0 to u_H . This region of flow, by analogy with the accompanying jet, is called the main region. The region of flow contained between the sections, having the coordinates x_H and x_N , is the transition region. Along the length of this region the thicknesses of the jet increase approximately according to the same law as in the first region. Also, the velocity profiles maintain themselves as similar in the different sections of the main region of the jet.

It is immediately observed that with a jet not bound to the wall, one can distinguish an axis of symmetry (the centerline of the jet) about which the velocity profile is centered. This is really not possible for the non-idealized rapid-river discharge. According to Abramovich, however, it is the same wall that functions as an axis of symmetry: One can visualize a treatment of a half-jet, centered in its complexity at the wall.

Table IV.G.9 summarizes the formulas used for predicting jet dispersion parallel to a wall. Some minor simplifications of the Abramovich formulas have been made by the Italian authors in their application to Piacenza.

In Table IV.G.9, the formula for the lateral velocity profile was derived by assuming that the velocity profile in each cross section consists of segments of the velocity profiles in the jet boundary layer and the wall's boundary layer, with those profiles merging at $y = \theta$. The conditions for the determination of θ are that the values of the velocity and the values of the derivatives of the velocity with respect to the y coordinate, calculated for both profiles, are equal. The "one-seventh law" assumes that the wall boundary layer is turbulent.

The velocity profile may be approximated in the manner described in Table IV.G.9 in the interval $1.22 < m < \infty$. At $m = 1.22$, the ratio of the velocities at $y = \theta$ becomes

$$\frac{u_\theta}{u_H} = \frac{u_\theta}{u_m} \frac{u_m}{u_H} = \frac{1}{m} \frac{u_\theta}{u_m} = 0.92,$$

so that the velocity at the contact point of the two profiles differs from the value of the undisturbed flow by only 8%, and thus the flow is almost completely uniform. Thus the calculations are ended there.

TABLE IV.G.9. Confined-jet Theory of Abramovich

Initial RegionGeometry:

$$1. \quad y_1'/b = 0.416 + 0.134m_o.$$

$$2. \quad y_2'/b = -0.584 + 0.134m_o.$$

$$3. \quad b(x) = 0.27 \frac{|1 - m_o|}{1 + m_o} x;$$

$$b = y_1' + y_2'.$$

$$4. \quad \frac{x_H}{b_o} = \frac{1}{\frac{\delta_H}{x_H} + 0.27(0.416 + 0.134m_o) \frac{|1 - m_o|}{1 + m_o}};$$

$$\frac{\delta_H}{x_H} = \frac{0.37}{Re_{x_H}^{0.2}} \quad (\text{relative thickness of turbulent boundary layer at the shoreline});$$

$$Re_{x_H} = \frac{\rho_o u_o x_H}{\mu}.$$

The quantities x_H and δ_H are calculated by successive approximations. First, x_H/b_o is found for $\delta_H = 0$, then δ_H/x_H is determined and the value x_H/b_o is more accurately defined. Usually two successive approximations are sufficient to determine x_H . The Italian authors have preferred to use the free-jet method for calculating x_H for the Piacenza case.

Velocity profile:

$$0 < y < \delta, \quad \text{one assumes } u/u_o = (y/b)^{1/7}.$$

$$\delta < y < y_1, \quad \text{one assumes a constant velocity } u_o.$$

$$y_1 < y < y_2, \quad \text{one assumes } \frac{u}{u_o} = 1 - (1 - m_o) \left[1 - \left(\frac{y - y_2}{b} \right)^{1.5} \right]^2.$$

Temperature profile:

This is not addressed by Abramovich; the Italian investigators use

$$\frac{\Delta T_m}{\Delta T_o} = \frac{\Delta u_m}{\Delta u_o}$$

TABLE IV.G.9 (Contd.)

in all regions. The transverse temperature excess is not discussed by either Abramovich or the Italian authors. The free-jet relationship of Table IV.G.7 is expected to apply where the free jet simulated is of initial width $2b_0$.

Transition Region

Abramovich derives the formulas

$$\frac{b_N}{b_0} = \left[\frac{1}{m_0}(1 - m_0) + 0.0975 \left(\frac{1}{m_0} \right)^2 \frac{\delta_H}{b_0} \right] \frac{1}{0.316 \left(1 - \frac{1}{m_0} \right) \left(\frac{1}{m_0} + 0.425 \right)}$$

and

$$\frac{x_N}{b_0} = \frac{b_N/b_0 - 1}{0.22 \frac{m_0 - 1}{1 + 1/m_0} \left(\frac{0.584}{m_0} - 0.134 \right)}$$

for the abscissa and width of the transition regions, respectively. The Italian investigators use instead the free-jet formula and method described in Table IV.G.7. The same procedure is used to smooth the approximate temperature and velocity field in that region as discussed in Table IV.G.7.

Main Region

Geometry:

$$\text{For } \kappa = \frac{1}{m}, \quad \kappa_0 = \frac{1}{m_0},$$

$$\begin{aligned} \frac{x}{b_0} - \frac{x_N}{b_0} &= 10[\kappa_0(1 - \kappa_0) + C_4\kappa_0^2][F_7(\kappa) - F_7(\kappa_0)] \\ &\quad + 1.13\kappa_0(1 - \kappa_0)[F_8(\kappa) - F_8(\kappa_0)], \end{aligned}$$

$$\frac{b}{b_0} = [\kappa_0(1 - \kappa_0)C_4\kappa_0^2] \frac{1}{0.316(1 - \kappa)(\kappa + 0.425)} + \frac{0.25\kappa_0(1 - \kappa_0)}{1 - \kappa},$$

where

$$C_4 = 0.097 \frac{\delta_H}{b_0} - 0.0435 \frac{1 - \kappa_0}{\kappa_0} F_6(\kappa_0);$$

TABLE IV.G.9 (Contd.)

$$F_6(\kappa_0) = -1.4 \ln(1 - \kappa_0) - 0.6(\kappa_0 + 0.425) + \frac{0.58}{\kappa_0 + 0.425};$$

$$F_7(\kappa_0) = \ln \frac{1 - \kappa}{\kappa + 0.425} + \frac{1}{(1 - \kappa)^2} - \frac{1}{1 - \kappa} + \frac{0.425}{\kappa + 0.425};$$

$$F_8(\kappa) = \frac{\kappa}{(1 - \kappa)^2}.$$

The Italian investigators use for b the free-jet formula, where the free-jet initial width is $2b_0$. Figure IV.G.43 shows no significant difference between the two methods.

Velocity profile:

The velocity in this region ($x > x_N$) is expressed as a superposition of profiles that one could have in a single boundary layer and in a free jet having the wall as an axis of symmetry. The quantity θ is calculated from

$$\theta^{1.5} = 0.475 - \sqrt{(0.475)^2 - \frac{0.05}{m - 1}}, \quad m > 1.22.$$

Once m is known from $m = u_H/u_m$, the calculation of the profile then follows from θ :

$$\frac{u}{u_1} = (y/\delta)^{1/7}, \quad 0 < y < \theta,$$

and

$$\frac{u}{u_m} = m + (1 - m)(1 - \xi^{1.5})^2, \quad \theta < y < b,$$

where u_1 is the velocity at the edge of the wall boundary layer, the thickness of which is equal to δ .

Temperature profile:

Abramovich does not discuss this; the Italian authors use the longitudinal decay

$$\frac{\Delta T_m}{\Delta T_0} = \frac{\Delta u_m}{\Delta u_0},$$

TABLE IV.G.9 (Contd.)

and for the lateral decay,

$$\frac{\Delta T}{\Delta T_m} = 1 - (y/b)^{1.5}.$$

Note: For $0.3 \leq m \leq 3$, the Italian authors calculate the width b from

$$b = b_N + 0.22 \frac{|m_0 - 1|}{m_0 + 1} (x - x_N), \quad x \geq x_N,$$

from analogy with the free-jet case.

Fig. IV.G.43

The Variation of the Dimensionless Thickness of a Jet b/b_0 with x/b_0 . (The dashed line corresponds to the problem of the expansion of a free jet in an accompanying flow.) (Adapted from Abramovich.¹²) ANL Neg. No. 190-1347 Rev. 1.

Application of the above calculational method of Abramovich for confined jets was made to the field data from Piacenza for October 1971 and October 1972. The October 1972 data are yet to be published by CISE. The data used for the application of the model are those reported in Tables IV.G.10 and 11.

For the 1971 data at Piacenza, the calculational method was described in some detail. The velocity ratio m_0 is

$$m_0 = \frac{u_H}{u_0} \approx \frac{1.7}{0.54} = 3.13.$$

With this value of m_0 and expecting that, in the discharge canal, turbulence is completely developed (i.e., the effects of the entrance have been exhausted),

TABLE IV.G.10. Data Used in the Application of the Model to the Field Data of Piacenza 1971

Minimum width of discharge canal	7 m
Maximum width of discharge canal	9 m
Initial width of assumed jet	$b_0 = 8$ m
Average velocity of discharge	$u_0 = 0.54$ m/sec
Average velocity of the river	$u_H = 1.7$ m/sec
Initial ratio of velocities	$m_0 = 3.13$
Excess temperature at discharge	$T_0 = 8.8$ C°

TABLE IV.G.11. Data Used in the Application of the
Model to the Field Data of Piacenza 1972

Average discharge velocity	$u_o = 0.88 \text{ m/sec}$
Average river velocity	$u_H = 1.87 \text{ m/sec}$
Initial ratio of velocities	$m_o = 2.12$
Excess temperature at discharge	$T_o = 8.1^\circ \text{C}$
Other data as in Table IV.G.10	

the values of x_H (end of the initial region) and x_N (end of region of transition) can be found, from the graphs of Figs. IV.G.37, 39, and 42, to be

$$x_H = (x_H)_{\text{uniform}} - x_{OH} = 42 \text{ m}; \quad x_N = 54 \text{ m}.$$

The jet, as is experimentally observed, moves against the wall within a very short distance (2-3 m); it can be treated as a half-jet concentrated in its complexity at the wall. The width of such a half-jet at its section of transition can, with good approximation, be calculated by

$$b_N = \frac{b_o}{A_2 + m_o(A_1 - A_2)} = \frac{8}{0.316 + 3.13(0.450 - 0.316)} \approx 10.9 \text{ m}.$$

As for the value of b in the main region ($x \geq x_N$), observe that with $m_o = 3.13$ one is already in the range of values of m_o ($0.3 \leq m \leq 3$), for which the considerations of the note on Table IV.G.7 are valid. Then,

$$b = b_N + 0.22 \frac{|m_o - 1|}{m_o + 1} (x - x_N), \quad x \geq x_N;$$

that is,

$$b = 10.9 + 0.113(x - 54), \quad x \geq x_N.$$

The transverse profile of temperature $\Delta T/\Delta T_m$ as a function of y/b can be determined now as well as the longitudinal T_m as a function of x from the formulas of Table IV.G.9. The temperature variation as a function of x is reported in Fig. IV.G.44; the agreement with experimental values is good. The transverse profile $\Delta T/\Delta T_m$ in the main region is given by

$$\frac{\Delta T}{\Delta T_m} = 1 - \left(\frac{y}{b}\right)^{1.5}.$$

In Fig. IV.G.45, the quantity $(1 - \Delta T/\Delta T_m)$ is given (on a semilog plot, it is a straight line) as a function of y/b_o . Even on this graph, one can see the good agreement of the model with experimental values; practically all results are contained in a band of 20% around the theoretical value of $\Delta T/\Delta T_m$.

Fig. IV.G.44. Longitudinal Variation of Maximum Temperature According to the Abramovich Model in Relation to the Field Data at Piacenza in October 1971. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1313.

Fig. IV.G.45. Transverse Variation of Maximum Temperature According to the Abramovich Model in Relation to the Field Data at Piacenza of October 1971. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1300.

Along the shore for about 30-40 m from the discharge, a small area is noticeable in which the water is practically stagnated. We can correct for such a beach zone by considering the corresponding points (those in Fig. IV.G.45 provided with arrows) to be much nearer the bank than they are. Doing this, the said points will reenter the 20% band.

Figure IV.G.46 reports the variation of some isotherms. The zone of transition has not been treated analytically, but the initial and main regions have been linked together; the margin of error is sufficiently small to be acceptable.

Fig. IV.G.46. Variation of Some Isotherms According to the Abramovich Model in Relation to the Field Data at Piacenza of October 1971. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1302.

Figures IV.G.47 and 48 show the results obtained in connection with the October 1972 field survey. In this second case, the value of m_0 is assumed to be 2.12 (see Table IV.G.10). As seen from Figs. IV.G.47 and 48, the rendering of the experimental data is decisively worse than in the preceding case.

The reason for this difficulty lies in the fact that the assumed value of m_0 , substantially important in the determination of the initial length of transition, is very sensitive to the measurement of the velocity of the ambient and discharge currents comprising m . To illustrate the validity of the

Fig. IV.G.47. Longitudinal Variation in Maximum Temperature According to the Abramovich Model in Relation to the Field Data for Piacenza of October 1971. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1305.

Fig. IV.G.48. Transverse Variation of Temperature According to the Abramovich Model in Relation to the Field Data of Piacenza of October 1972. (Adapted from Cavanna and Folli.⁴) ANL Neg. No. 190-1314.

above statement, the theoretical variation that would result by modifying the assumed value of m_0 by 10% is also plotted. Observe, in particular, that an error of at least 10% in the measurement of river velocity, and of 10% additionally in the jet velocity, carries with it errors of about 25% in m_0 . Variations of this order probably cannot be such as to substantially modify the variation of the physical phenomena. The CISE authors therefore conclude that, for $m_0 = 2.12$, one is already in the range in which a jet model is no longer in a position to describe, in a complete way, the dynamics of transport.

d. Conclusions

From the verification work done by CISE, some well-defined conclusions and directions were drawn. Among them are:⁴

- (1) The exact theoretical treatment of mixing in terms of the nonbuoyant interaction of two fluids in coflow does not give a formulation of the problem of significant practical use; yet it is valuable in shedding light on the phenomena that govern the mixing as well as sensitizing the research to the difficulties of the problem.
- (2) Present analytical models* are based on hypotheses either of diffusion alone (current and jet velocity equal vectorially) or of interaction of jets (current and jet velocity very much different) alone.
- (3) In general, diffusion models** are marginally important for the study of the heated plume in the vicinity of the discharge. The difficulty

*The publication date of MISTER No. 4 is February 1973.

**Far-field (diffusion) models are discussed in MISTER No. 4, but were not reviewed here.

in the use of such models results from the scanty knowledge of the eddy diffusivities. (The CISE evaluation of diffusion models was not extensive and restricted only to the two- and three-dimensional Edinger-Polk models.¹³⁾

(4) The simple jet models for the coflowing discharge that were studied by CISE are generally applicable shortly after the discharge and are easy to use to calculate the velocity and temperature fields. However, as the parameters that come into play increase, the correlations for them seem to be less known. Among the jet models for coflowing jets, the Abramovich model is preferable because it allows a complete predictive calculation without additional empirical or experimental data. The data comparisons for such a model have demonstrated that a good agreement exists for ratios of velocity (river to discharge velocity) greater than 3 and only fair agreement for ratios of velocity near 2. Even if this confirms the progressive lack of validity of the jet model as the ratio of velocity decreases, it is still useful to be able to compare with experimental data in a systematic way.

The CISE work has not made a systematic examination of existing models in use by various groups working in this area of thermal discharges. This will be done in their future research, even though they foster some doubts on the utility of using such models for Italian conditions characterized by velocity ratios greater than 1.

5. Critical Discussion

The major question that arises from the data-acquisition procedure is whether the data have been taken in a synoptic fashion. Table IV.G.5 indicates that the temperature surveys were made for the Piacenza plant during a 2-day period in 1971 and a 3-day period in 1972. The reports on the 1971 surveys (MISTER 2A/B) indicate that a 0.5-1.0 C° difference in ambient temperature was typical during the period of the survey. In one case, the September 9-10, 1971 survey, a 2.5 C° difference in ambient temperature existed between the two days. For the 1972 survey at Piacenza during August 23-25, the ambient temperature differed by as much as 1.5 C° at different times during the measurements (see Fig. IV.G.6). In spite of these differences, data from a survey encompassing 2 or 3 days was considered to be collected synoptically.

To ease the difficulty caused by a varying ambient temperature, the Italian investigators maintained a resistance thermometer and thermocouple 10 m upstream of the discharge to continuously monitor ambient temperature. Each plume temperature was then taken with reference to the ambient temperature at that instant for the purpose of calculating the excess temperatures. This technique aids in mitigating the effect of a nonconstant ambient temperature. This is not completely satisfactory, as comparisons of excess temperatures measured at the same location on consecutive days of a survey reveal up to 0.5 C° differences. Table IV.G.12 lists excess temperatures measured at the same location on consecutive days for the same survey. Although the data available are limited to only two surveys, they indicate that differences can often be nonnegligible.

TABLE IV.G.12. Excess Temperatures Measured at Same Location on Consecutive Days for the Same Survey

Survey Date	X, m	Y, m	Z, m	$\bar{\Delta T}$, C°	T _A ^a	T _B ^b
August 30, 1971	237	7	0.20	1.4	22.1	30.1
			0.60	1.5		
			1.00	1.3		
			1.50	1.3		
August 31, 1971	237	7	0.25	1.6	22.2	30.2
			0.75	1.6		
			1.25	1.6		
			1.75	1.6		
September 1, 1971	63	13	0.25	4.9	22.0	29.8
			0.65	5.0		
		16	0.25	3.5		
			1.05	2.0		
			1.85	1.3		
			2.25	1.3		
		19	0.25	2.5		
			1.05	1.0		
September 2, 1971	63	13	0.25	4.7	21.9	30.0
			0.65	5.0		
		16	0.25	3.8		
			1.05	2.5		
			1.85	1.1		
			2.25	0.8		
		19	0.25	1.6		
			1.05	0.6		

^aT_A = ambient temperature.

^bT_B = outfall temperature.

Plumes are obviously not fixed entities remaining in the same location for days. Conceivably, ambient flow and discharge conditions are continually changing, though probably not at a very considerable rate, causing the plume to change position from one day to the next. The same longitudinal, lateral, and vertical location in the plume is not at the same point on different days. Ambient temperature changes affect the initial densimetric Froude number of the discharge; changing river-flow conditions affect plume entrainment and bending. Changing plant power level and flow affect the excess temperatures as well as the densimetric Froude number. The precise effect of the densimetric Froude number is probably minimal due to the large currents and the fully mixed conditions that result.

To be considered also in any evaluation of the synoptic character of a plume survey is the difficulty in determining an average temperature at a single location due to oscillations measured within the plume. This was discussed in Chapter III for the CISE surveys. Suffice it to say that temperature oscillations of up to 3-4 C° were common in the boundary-layer region of the plume. Core and near-ambient regions of the plume showed little temperature oscillations. These oscillations seem to plague all boat measurements of temperature and have to be considered as part of the measurement error. Temperature measurements at a single location for one minute aided in averaging out these oscillations. However, a more synoptic study could

have been accomplished with reference to changing ambient conditions by a moving-boat procedure, or by aerial infrared study, or by a large number of anchored in situ thermographs and current meters, etc. Use of a radar-range system would locate the boat at any instant in its motion. Velocity measurements cannot be acquired in this manner, however. A combination of the moving boat and profiling techniques would have provided the synoptic surveys best used in model/data comparisons and the detailed plume studies required for the basic physical understanding needed.

The temperature oscillations in the plume were treated in a more thorough fashion in the last two surveys at Piacenza. First, in surveys 5-7, oscillations were noted only by recording maximum and minimum temperatures observed at a particular location. The temperature at that point was determined by arithmetically averaging these numbers. In survey 8 and the August 23-25, 1972, comprehensive survey, 30 temperature measurements were recorded in 30 sec for the first survey and 30 measurements were made

TABLE IV.G.13. Typical Peak-to-Peak Amplitude of Temperature Oscillations Seen at Piacenza for Different Mean Excess Temperatures (from Ref. 2)

$\bar{\Delta T}$, C°	Peak-to-Peak Amplitude	$\bar{\Delta T}$, C°	Peak-to-Peak Amplitude
>8	1.4	5-6	6.5
7-8	5.0	3-5	7.0
6-7	5.3	1-3	5.7

in 60 sec for the latter survey. These recordings were manipulated to obtain a mean and standard deviation that represented the measured temperatures. Concerning the type of temperature variations observed, Table IV.G.13 reports the peak-to-peak amplitude of oscillations observed for different values of average excess temperature $\bar{\Delta T}$. In the longitudinal direction, the temperature-oscillation phenomenon strongly attenuated for $X >$

100 m. In the transverse direction, there is a first zone of 6 m in width near the river bank in which the temperatures are nearly stationary, an intermediate zone of 6-12 m in which there are noticeable oscillations, and an outer zone in which the oscillation phenomenon is attenuated. Velocity measurements in that intermediate zone indicated that that zone corresponded to the maximum intensity of mixing between the plume and ambient current. It is stated in MISTER 5A/B⁵ that an additional 60 measurements will stabilize the mean and standard deviation to $\pm 1\%$ of the values for the first 30 measurements. Consequently, the profiling technique used will induce a greater error from ambient temperature variation than by the averaging technique of the temperature oscillations, at least for the last two surveys done at Piacenza.

Concerning the analytical modeling work done by CISE, we agree that, due to lack of knowledge of E and C_d , or S_N and K , the Vanderbilt University model and its modified version developed by CISE are inadequate for future prototype predictions. Their determination as a function of input parameters such as m_0 requires considerable data that do not exist, either as prototype or hydraulic model data; correlation formulas for these parameters will otherwise be inadequate. In any case, a comparison of these two models with other Piacenza data would be useful in determining the variation expected in the parameters E and C_d , or S_N and K , at a single site as environmental conditions change.

It is interesting to compare the spreading rate K determined from the modified Vanderbilt University model for a rapid river with the spreading rate as predicted from the Motz-Benedict model⁷ (Sec. II. B) for a discharge into a stagnant lake. From the CISE work, $K = 0.037$, while an equivalent K using the Motz-Benedict model turns out to be 0.090 (using the analytical solution to the Motz-Benedict model for zero current, and $E = 0.04$). The 0.037 value is in the right ballpark and should be lower than the stagnant lake value of the plume spreading rate, as the reduced shear in the rapid river plumes causes less spreading due to entrainment. Spreading due to diffusion is acknowledged to be quite slow.

From the two Abramovich model comparisons, the model appears to be viable only for very large currents, where the assumption of two-dimensionality is most valid. For smaller currents, with m_0 still larger than 1, the plume is expected to be more three-dimensional near the outfall with a greater influence of buoyancy on lateral spreading. For $m_0 = 3.13$, temperature could apparently be treated as a tracer in two-dimensional confined flow with negligible effects of density differences. The Abramovich model does not treat the effect of ambient turbulence, which should be important in determining plume dispersion for the region beyond the direct vicinity of the outfall. There, the jet velocity has increased to near-ambient values and jet entrainment due to shear has nearly ceased. This is another problem area in the application of the Abramovich model to prototype rapid-river sites.

Since most rapid-river cases in the U. S. do not usually yield a value of m_0 as large as 3, a different approach would more likely be required to produce a useful and more general model for rapid rivers. Any of the three-dimensional numerical models could be applied to this rapid-river case with the hope that it would be general enough to treat any three-dimensionality of the plume should it exist for lower values of m_0 . [The Till¹⁴ model, however, needs some additional improvement before it can be applied with any confidence even for rapid-river plumes (see Chapter II).] These models would also handle the initial bending of the jet with its initial dilution before the plume turns parallel to the shore and interacts with it. There is little experience in this area of three-dimensional, numerical-model application, and many runs of the computer code are likely to be needed before eddy diffusivities and boundary conditions can best be determined.

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*MISTER is an acronym for the Italian "misure termiche" or thermal measurements.

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V. OBSERVATIONS ON THE STATE OF THE ART

This report summarizes the Argonne experience with mathematical models for buoyant plumes resulting from heated surface discharges. The goal was to determine the ability of idealized mathematical models to reliably assess the size and character of prototype plumes. Since our direct experience is limited, our work was heavily supplemented with a critical review of published and unpublished material.

It is concluded that, of the models so far compared with prototype data, none has been shown adequate over a wide range of conditions. Moreover, available models were found useful for only generalized estimates of plume characteristics; precise predictions are not currently possible.

This conclusion was reached through comparison of model predictions with laboratory and field data. Comparisons with laboratory data enable one to test the validity of the model within the geometry and conditions assumed in model development. Comparisons with field data indicate the extent to which prototype effects, not accounted for in model formulation, limit the utility of the model.

A pertinent factor to be considered in making model-data comparisons is the generally poor overall quality of the data. Existing field data come from a few similar sites, with only a small range in outfall parameters covered. Both hydraulic-model and prototype-field data often show order-of-magnitude scatter, even for apparently similar conditions. Part of this scatter is certainly attributable to stochastic errors of measurement, but a significant part is also due to a failure to recognize all the factors that affect the data. A far more consistent explanation of the data would be possible if all the contributing factors could be accounted for. Data are often taken under different physical scales without a knowledge of ambient turbulence levels or the effect of transient environmental conditions. The dearth of good quality data limits both the ability to test models thoroughly and the basis from which to make good assumptions in model development.

Despite the acknowledged limitations, the data, in aggregate, present a good qualitative picture of prototype plumes. Therefore, in carrying out model-data comparisons, one should place greatest emphasis on the trends of the data and less emphasis on their numerical accuracy. Models that fail to describe the qualitative trends in the data have been heavily criticized in this report on that account. Another consideration in assessing the viability of the models should be the degree of sophistication in their respective mathematical techniques. However, subjective likes and dislikes for a model's formulation should not weigh equally with its ability to predict the right trends in the data.

The major reason for the generally poor performance of existing models seems to be a limited understanding of plume physics. An important

related reason is the inadequate assumption that plume characteristics are controlled by discharge parameters alone. Experience has shown that prototype plumes are controlled partly by discharge parameters and partly by the nature and history of the receiving waterbody. The effects of ambient turbulence, boundary (bottom and shoreline) interference, and wind-induced shear currents are often omitted in the formulation of mathematical models. These effects are clearly not second order, for models that do not identify and correctly treat the key physical mechanisms cannot be expected to predict reliably. Additional basic studies are needed to understand these phenomena and their interrelationship with buoyancy and entrainment, and thereby provide a more sound basis for model development.

Most of the verification work now completed concerns what might be called first-generation, mathematical-plume models. These first-generation models fall into two broad classes: integral and phenomenological, although the dividing line between these two groups is rather diffuse. Phenomenological models rely heavily on the direct use of data and on the formulator's ability to glean important trends from the data. Integral models also use data through empirically determined coefficients, but generally put greater reliance on a conceptualization of the physics implemented through the use of mathematical formulas.

The performance of the integral models has, in general, been disappointing. These models neglect important phenomena in their development. Even the phenomena that are included--entrainment, buoyancy, and ambient turbulence (in some models)--are treated by using functional forms that often are arbitrary or are based upon intuitive guesswork. These forms are an inherent part of the integral model simulation, yet cannot be adequately selected, due to the aforesaid poor understanding of plume physics. One way to test the adequacy of such intuitive treatments is through the comparison of model predictions with data taken under conditions similar to those assumed in model development. Comparisons with hydraulic-model data have shown that integral models often fail to correctly predict important trends in the data, even under idealized conditions. When comparisons are poor, one is often at a loss to identify which of the numerous assumptions made in developing the model is at fault.

A marked deficiency of the integral models is that many of the phenomena that are important in prototype plume dispersion destroy the flow similarity upon which the integral models are founded. The models cannot be extended to account for these effects, which are alien to the integral formulation. More basic studies would be useful in understanding the effects and interaction of entrainment, buoyancy, and ambient turbulence, but we feel that key prototype effects such as wind effects, shear currents, and boundary interference are beyond the capability of the integral approach.

Although the overall performance of the integral models has been disappointing, their accuracy in a particular case depends upon how well the

problem at hand matches the idealizations of the model. For example, the integral models perform better for high-initial-densimetric-Froude-number discharges in deep water where buoyancy is a second-order effect and boundary interference is nonexistent, reducing the problem to nearly nonbuoyant jet theory.

The phenomenological models implicitly handle the complex mechanisms of plume dilution and their interaction, provided that a sufficiently broad set of experimental data can be first obtained and then synthesized into a useful form. Well-constructed phenomenological models can generally be expected to give reasonable predictions of dilution and other gross plume properties when applied to situations similar to those included in the data base. Their applicability outside the range of parameters covered in their development can be assessed only through comparisons with data. At present, well-developed phenomenological models are available for only a limited number of applications. More good quality data, covering a broader range of conditions and a greater understanding of existing data, are necessary to produce better phenomenological models.

Among first-generation models, the Shirazi-Davis and Pritchard No. 1 simulations appear to be superior to the others reviewed in this report in the limited prototype testing that has taken place. From comparisons with available field data, these models have been shown to be capable of predicting correct trends with roughly factor-of-two accuracy in the centerline distance to a given isotherm, factor-of-two accuracy in widths, and factor-of-five accuracy in isotherm areas. Given the limitations of the data and the general lack of understanding of plume physics, these types of predictions represent the state of the art. We emphasize again that these statements are based on limited testing; there is no guarantee that the two models will do as well in other applications. Notably, these two models are the only ones that have been extensively calibrated with data. An added convenience is that both models are easy to use. The Shirazi-Davis model has a well-documented computer code; a set of nomograms available in workbook form can be used for determining predictions. The Pritchard No. 1 model can also be applied without resorting to a computer calculation, although its algorithmic nature makes it easy to program. We do not feel that ease of application is an important criterion for evaluating models, but such an attribute is always welcome. Although these models are perhaps the best available, they are nevertheless limited in their ability to handle key prototype effects. The Shirazi-Davis model, for example, is unable to handle boundary interference, and the Pritchard No. 1 model considers none of the effects of ambient crossflow. Neither model can treat transient effects.

The Stolzenbach-Harleman and Prych models gave reasonable predictions for centerline-temperature decay for deep-water situations, but gave inadequate predictions for jet width and isotherm areas in all cases. In fact, the Prych model provided the best predictions for laboratory data (see Sec. IV.C), and the Shirazi-Davis model performed second best. We may

explain the Prych-model superiority in the laboratory comparisons as due perhaps to the choice by Prych of experimental coefficients that were determined predominantly from previous experiments in the laboratory. The Shirazi-Davis and Pritchard No. 1 models were calibrated by their authors to both laboratory and field data on thermal plumes. Hence, the experimental coefficients chosen by Shirazi-Davis and Pritchard undoubtedly reflect some of the complicating features affecting prototype plumes that are nonexistent or different for laboratory plumes (e.g., different scales of turbulence, influence of boundary interaction, wind effects, etc., that appear in the prototype data but are not easily quantified). The differences that occur in experimental data taken under apparently identical conditions in the field and laboratory underscore the importance and usefulness of carrying out both types of data comparisons of analytical-model predictions.

The second-generation, thermal-plume models attempt the numerical solution of the generalized equations of flow. These models, which make fewer assumptions on the nature of plume dispersion and therefore offer the potential of covering many more phenomena, are only now becoming available for testing with prototype data. Unfortunately, the few existing applications of these models have been generally inconclusive for various reasons, such as a poor schematization, a poor choice of diffusivities, too small a computational grid, or an incomplete or inadequate set of data for comparison. The poor results thus far obtained are partly attributable to these facts. More information on the choice of diffusivities and better schematizations are needed.

Despite the early developmental stage of available numerical models, many prototype sites of current and future interest are too complicated to be accommodated by the simplistic integral and phenomenological models. In these complex situations, a numerical model may be the only alternative.

Several steps should be taken to clarify the issues surrounding the numerical models. First, the simulation of turbulent mixing should be carefully investigated. This investigation should properly begin with a sensitivity study of existing models to eddy diffusion coefficients and with a determination of the amount of numerical dispersion inherent in each solution scheme. Turbulent energy transport or other models of turbulent mixing should be tried. Some efforts in these directions have already been made. Tank measurements of thermal plumes including measurements of plume Reynolds stresses would be helpful; in this way predictions can be tested with eddy viscosities and diffusivities known. Various turbulence theories can thus be tested also. Second, the model algorithms should be tested in simple problems where established solutions (perhaps analytical) exist. A set of benchmark problems of increasing difficulty would foster the comparison of various techniques and the development of better models. Third, the role of the boundary conditions should be established. These conditions are often arbitrarily prescribed, and the effects of such specifications are not presently known. Fourth, efforts should be made to reduce the computer requirements or,

equivalently, to increase the scope of problems that can be handled with current computer time and memory requirements. Increased efficiency would make the numerical models more generally available and would speed model development. Whether any significant reduction in computer requirements is achievable is presently uncertain.

Numerical modeling of thermal plumes holds perhaps the greatest promise of the future and should therefore be pursued. If work on these models continues at its present or, better yet, an accelerated rate and if the physical and analytical investigations set forth above are carried out, then numerical plume models should become a valuable licensing instrument in the next few years.

To determine which model best suits his intended use, a potential user should review all available material on the models under investigation. The considerations involved in selecting a model for a particular application are too numerous and subjective to be presented in short form. Therefore, we make no recommendations or unqualified endorsements. To do this would misrepresent our view of the state of the art.

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We wish to thank especially our colleagues Dr. John Tokar and Sam Zivi for their technical suggestions and moral support in helping us to overcome the difficulties in completing the report.

APPENDIX A

Sample Computer-code Output for Selected Models

Sample calculations have been prepared using the computer codes for several of the integral and phenomenological surface-discharge plume models. The models considered were those of Motz and Benedict, Prych, Shirazi and Davis, and Pritchard (Nos. 1 and 2). Calculations have not been prepared for the Stolzenbach-Harleman Model due to the ambiguity as to which version is correct (see discussion in Sec. II,C). All calculations have been based on the discharge and ambient conditions listed in Table A.1. These conditions are the ones that prevailed at Point Beach Unit 1 on May 18, 1972 (1645-1751 hrs), when Argonne National Laboratory conducted a plume-mapping survey.¹ Point Beach Unit 1 is a 505-MWe unit located near Two Creeks, Wisconsin, on the western shore of Lake Michigan. The unit employs once-through cooling and discharges the heated water by means of a flume that extends about 46 m into the lake at an angle of 60° with respect to the shoreline.

TABLE A.1. Discharge Parameters and Ambient Conditions Used in Preparing Model Predictions

Discharge Parameters:	
Discharge flow rate	24.7 m ³ /sec
Width of discharge	10.7 m
Depth of discharge	4.2 m
Discharge velocity (mean)	54.7 cm/sec
Discharge temperature excess	8.5 C°
Ambient Conditions:	
Ambient current speed	0
Ambient temperature	9.2°C
Average relative humidity	75%
Average wind speed	5 mph
Average air temperature	11.5°C
Sky conditions	Clear
Additional Parameters:	
Initial densimetric Froude number	2.57
Discharge aspect ratio (depth/width)	0.39
Current ratio (ambient current/ discharge velocity)	0
Surface heat-loss coefficient	242 kcal/(m ² -day-C°)

1. Motz-Benedict Model

The model parameters used in preparing the Motz-Benedict model predictions are given in Table A.2. The actual computer output is reproduced in Fig. A.1, most of the initial output being self-explanatory, and integration results along the jet centerline are tabulated. Some of the columns of interest are labeled as follows:

S Nondimensional distance along jet centerline
 ARC(M) Distance along centerline (m)

TEMP(C)	Centerline temperature (°C)
EX TEMP(C)	Centerline excess temperature (C°)
U(CM/S)	Centerline velocity (cm/sec)
B(M)	Width parameter (m)
X(M)	x coordinate of centerline (in direction of ambient current or parallel to shoreline) (m)
Y(M)	y coordinate of centerline (perpendicular to direction of ambient current or in offshore direction) (m)
BETA	Angle centerline makes with x axis (degrees)
TIME(S)	Time of travel along centerline from point of discharge (sec).

Areas within isotherms of specific excess temperatures are then tabulated along with coordinates of these isotherm lines, which are crudely plotted on the line printer.

Computer-calculation results are plotted in Fig. A.2 in three graphs as follows: The top graph is a plot of centerline excess-temperature ratio θ/θ_0 (local excess temperature above ambient, divided by the outfall excess temperature) as a function of s , the distance along the centerline measured from the point of discharge. The centerline velocity ratio, u_c/u_0 (local velocity in excess of component of ambient velocity parallel to jet centerline divided by the mean outfall velocity) is also plotted and here coincides with the excess-temperature-ratio curve. The center graph shows the temperature and velocity half-widths as a function of centerline distance. The temperature half-width W_T is defined as the lateral distance from the centerline to the point where the excess temperature has dropped to one-half its centerline value. The velocity half-width is defined in an analogous way. The bottom graph is a plot of the half-depth of the temperature distribution as a function of distance along the centerline. The half-depth D is defined as the depth at which the local excess temperature reaches one-half its surface centerline value. The contour of the actual lake bottom along the centerline of the discharge is also shown on this plot. (The Motz-Benedict model is two-dimensional in character and assumes that the depth remains constant at its initial value.

TABLE A.2. Model Parameters Used in Preparing Motz-Benedict Model Predictions

Entrainment coefficient, E	0.05
Drag coefficient, C_D	0.5
Length of flow establishment, L	5.2
Jet-width ratio, r	1.6
Jet-angle ratio, r'	1.0

MOTZ-BENEDICT MODEL

POINT BEACH, MAY 18, 1972. STANDARD PARAMETERS.

OUTFALL PARAMETERS-

HALF-WIDTH OF OUTFALL, BOP= 5.4 METERS
 DEPTH OF OUTFALL, ZD= 4.2 METERS
 NOMINAL OUTFALL VELOCITY, UO= 54.7 CM/SEC
 TEMPERATURE EXCESS AT THE OUTFALL, TO= 8.5 DEG.C
 ANGLE OF OUTFALL WITH CURRENT, BETAP= 60.0 DEG.
 VOLUMETRIC DISCHARGE RATE, Q= 24.7 CU.M/SEC

AMBIENT CONDITIONS AND MODEL PARAMETERS-

AMBIENT VELOCITY, UA= 0.0 CM/SEC
 AMBIENT TEMPERATURE, TAM= 9.2 DEG.C
 LENGTH OF ESTABLISHMENT ZONE, SE= 55.5 METERS
 ENTRAINMENT COEFFICIENT, E= 0.050
 DRAG COEFFICIENT, CD= 0.500
 HEAT LOSS COEFFICIENT, FK= 242.0 KCAL/DAY-SQ.M-DEG.C
 RATIO OF JET ANGLE AT EOFE TO OUTFALL ANGLE, BENDPEP= 1.000
 RATIO OF JET WIDTH AT EOFE TO OUTFALL WIDTH, BODPEP= 1.6
 RATIO OF TEMPERATURE WIDTH TO VELOCITY WIDTH= 1.0

TYPE OF RUN= 3.0

S	ARC(M)	TEMP(C)	EX TEMP(C)	U(CM/S)	B(M)	B1/2T(M)	B1/2U(M)	X(M)	Y(M)	BETA	TIME(S)
0.366	55.46	17.70	8.50	54.70	8.56	7.13	7.13	27.73	48.03	60.00	101.39
0.376	56.98	17.62	8.42	54.16	8.73	7.27	7.27	28.49	49.34	60.00	104.18
0.386	58.49	17.53	8.33	53.64	8.90	7.41	7.41	29.25	50.66	60.00	106.99
0.396	60.07	17.45	8.25	53.11	9.08	7.56	7.56	30.04	52.02	60.00	109.95
0.406	61.65	17.37	8.17	52.60	9.26	7.71	7.71	30.83	53.39	60.00	112.93
0.417	63.29	17.29	8.09	52.08	9.44	7.86	7.86	31.65	54.81	60.00	116.07
0.428	64.93	17.21	8.01	51.57	9.63	8.02	8.02	32.47	56.23	60.00	119.24
0.439	66.64	17.14	7.94	51.07	9.82	8.18	8.18	33.32	57.71	60.00	122.56
0.450	68.35	17.06	7.86	50.57	10.01	8.34	8.34	34.17	59.19	60.00	125.92
0.462	70.12	16.98	7.78	50.07	10.21	8.50	8.50	35.06	60.73	60.00	129.45
0.474	71.90	16.91	7.71	49.59	10.41	8.67	8.67	35.95	62.26	60.00	133.01
0.486	73.74	16.83	7.63	49.10	10.62	8.84	8.84	36.87	63.86	60.00	136.75
0.498	75.59	16.76	7.56	48.63	10.83	9.02	9.02	37.79	65.46	60.00	140.53
0.511	77.51	16.68	7.48	48.15	11.05	9.20	9.20	38.75	67.12	60.00	144.50
0.524	79.43	16.61	7.41	47.68	11.26	9.38	9.38	39.71	68.79	60.00	148.50
0.537	81.42	16.54	7.34	47.21	11.49	9.57	9.57	40.71	70.52	60.00	152.71
0.550	83.42	16.47	7.27	46.76	11.72	9.75	9.75	41.71	72.25	60.00	156.96
0.564	85.50	16.39	7.19	46.30	11.95	9.95	9.95	42.75	74.04	60.00	161.42
0.577	87.57	16.32	7.12	45.85	12.18	10.14	10.14	43.79	75.84	60.00	165.93
0.591	89.73	16.25	7.05	45.40	12.43	10.35	10.35	44.87	77.71	60.00	170.66
0.606	91.89	16.19	6.99	44.96	12.67	10.55	10.55	45.95	79.58	60.00	175.45
0.620	94.14	16.12	6.92	44.52	12.92	10.76	10.76	47.07	81.53	60.00	180.47
0.635	96.39	16.05	6.85	44.09	13.18	10.97	10.97	48.19	83.47	60.00	185.54
0.651	98.72	15.98	6.78	43.65	13.44	11.19	11.19	49.36	85.50	60.00	190.86
0.666	101.06	15.92	6.72	43.23	13.71	11.41	11.41	50.53	87.52	60.00	196.24
0.682	103.49	15.85	6.65	42.80	13.98	11.64	11.64	51.74	89.62	60.00	201.89
0.698	105.92	15.79	6.59	42.39	14.25	11.87	11.87	52.96	91.73	60.00	207.59

Fig. A.1. Computer Output of Motz-Benedict Code

0.715	108.45	15.72	6.52	41.97	14.54	12.10	12.10	54.22	93.92	60.00	213.58
0.731	110.97	15.66	6.46	41.57	14.82	12.34	12.34	55.49	96.10	60.00	219.63
0.749	113.60	15.59	6.39	41.16	15.12	12.59	12.59	56.80	98.38	60.00	225.99
0.766	116.23	15.53	6.33	40.76	15.42	12.84	12.84	58.11	100.66	60.00	232.40
0.784	118.96	15.47	6.27	40.36	15.73	13.09	13.09	59.48	103.02	60.00	239.14
0.802	121.69	15.41	6.21	39.97	16.03	13.35	13.35	60.85	105.39	60.00	245.95
0.821	124.54	15.35	6.15	39.57	16.35	13.62	13.62	62.27	107.85	60.00	253.09
0.840	127.38	15.29	6.09	39.19	16.68	13.88	13.88	63.69	110.31	60.00	260.31
0.859	130.34	15.23	6.03	38.80	17.01	14.16	14.16	65.17	112.87	60.00	267.89
0.879	133.29	15.17	5.97	38.43	17.34	14.44	14.44	66.65	115.44	60.00	275.55
0.899	136.37	15.11	5.91	38.05	17.69	14.73	14.73	68.18	118.10	60.00	283.59
0.919	139.44	15.05	5.85	37.68	18.04	15.02	15.02	69.72	120.76	60.00	291.71
0.940	142.64	15.00	5.80	37.31	18.40	15.32	15.32	71.32	123.53	60.00	300.24
0.961	145.84	14.94	5.74	36.95	18.76	15.62	15.62	72.92	126.30	60.00	308.85
0.983	149.16	14.88	5.68	36.59	19.13	15.93	15.93	74.58	129.18	60.00	317.90
1.005	152.49	14.83	5.63	36.23	19.51	16.24	16.24	76.25	132.06	60.00	327.03
1.028	155.95	14.77	5.57	35.88	19.90	16.57	16.57	77.98	135.06	60.00	336.63
1.051	159.41	14.72	5.52	35.53	20.29	16.89	16.89	79.70	138.05	60.00	346.32
1.074	163.01	14.67	5.47	35.18	20.70	17.23	17.23	81.50	141.17	60.00	356.49
1.098	166.61	14.61	5.41	34.84	21.10	17.57	17.57	83.30	144.28	60.00	366.77
1.123	170.35	14.56	5.36	34.50	21.52	17.92	17.92	85.17	147.53	60.00	377.56
1.147	174.09	14.51	5.31	34.16	21.95	18.27	18.27	87.04	150.77	60.00	388.46
1.173	177.98	14.45	5.25	33.83	22.39	18.64	18.64	88.99	154.14	60.00	399.91
1.199	181.87	14.40	5.20	33.50	22.82	19.00	19.00	90.94	157.51	60.00	411.48
1.225	185.92	14.35	5.15	33.17	23.28	19.38	19.38	92.96	161.01	60.00	423.62
1.252	189.97	14.30	5.10	32.85	23.74	19.76	19.76	94.98	164.52	60.00	435.88
1.280	194.18	14.25	5.05	32.52	24.21	20.16	20.16	97.09	168.16	60.00	448.76
1.308	198.39	14.20	5.00	32.21	24.69	20.55	20.55	99.19	171.81	60.00	461.77
1.336	202.77	14.15	4.95	31.89	25.18	20.97	20.97	101.38	175.60	60.00	475.43
1.365	207.15	14.11	4.91	31.58	25.68	21.38	21.38	103.57	179.39	60.00	489.23
1.395	211.70	14.06	4.86	31.27	26.19	21.80	21.80	105.85	183.34	60.00	503.72
1.425	216.26	14.01	4.81	30.97	26.70	22.23	22.23	108.13	187.28	60.00	518.35
1.457	220.99	13.96	4.76	30.66	27.24	22.68	22.68	110.50	191.38	60.00	533.72
1.488	225.73	13.92	4.72	30.37	27.77	23.12	23.12	112.86	195.49	60.00	549.25
1.520	230.66	13.87	4.67	30.07	28.33	23.59	23.59	115.33	199.75	60.00	565.55
1.553	235.58	13.83	4.63	29.78	28.88	24.05	24.05	117.79	204.02	60.00	582.02
1.586	240.71	13.78	4.58	29.48	29.46	24.53	24.53	120.35	208.46	60.00	599.31
1.620	245.83	13.74	4.54	29.20	30.04	25.01	25.01	122.92	212.90	60.00	616.77
1.655	251.16	13.69	4.49	28.91	30.64	25.51	25.51	125.58	217.51	60.00	635.12
1.691	256.49	13.65	4.45	28.63	31.24	26.01	26.01	128.25	222.13	60.00	653.64
1.727	262.03	13.60	4.40	28.35	31.87	26.53	26.53	131.02	226.93	60.00	673.10
1.764	267.58	13.56	4.36	28.07	32.49	27.05	27.05	133.79	231.73	60.00	692.75
1.802	273.34	13.52	4.32	27.80	33.15	27.60	27.60	136.67	236.72	60.00	713.39
1.840	279.11	13.48	4.28	27.53	33.80	28.14	28.14	139.55	241.72	60.00	734.23
1.879	285.11	13.43	4.23	27.26	34.47	28.70	28.70	142.55	246.91	60.00	756.13
1.919	291.10	13.39	4.19	26.99	35.15	29.26	29.26	145.55	252.10	60.00	778.24
1.960	297.34	13.35	4.15	26.73	35.85	29.85	29.85	148.67	257.51	60.00	801.46
2.001	303.58	13.31	4.11	26.47	36.56	30.44	30.44	151.79	262.91	60.00	824.91
2.044	310.07	13.27	4.07	26.21	37.29	31.05	31.05	155.03	268.53	60.00	849.55
2.086	316.56	13.23	4.03	25.95	38.02	31.66	31.66	158.28	274.15	60.00	874.43
2.131	323.31	13.19	3.99	25.70	38.78	32.29	32.29	161.65	279.99	60.00	900.56
2.175	330.06	13.15	3.95	25.45	39.54	32.92	32.92	165.03	285.84	60.00	926.95
2.222	337.08	13.11	3.91	25.20	40.34	33.58	33.58	168.54	291.92	60.00	954.67
2.268	344.10	13.07	3.87	24.95	41.13	34.24	34.24	172.05	298.00	60.00	982.67
2.316	351.40	13.04	3.84	24.71	41.95	34.93	34.93	175.70	304.32	60.00	1012.07
2.364	358.70	13.00	3.80	24.47	42.78	35.61	35.61	179.35	310.64	60.00	1041.77
2.414	366.30	12.96	3.76	24.23	43.63	36.33	36.33	183.15	317.22	60.00	1072.96
2.464	373.89	12.92	3.72	23.99	44.49	37.04	37.04	186.95	323.80	60.00	1104.47
2.516	381.79	12.89	3.69	23.76	45.38	37.78	37.78	190.90	330.64	60.00	1137.56
2.568	389.69	12.85	3.65	23.53	46.27	38.53	38.53	194.85	337.48	60.00	1170.98
2.623	397.91	12.82	3.62	23.29	47.20	39.30	39.30	198.96	344.60	60.00	1206.08
2.677	406.13	12.78	3.58	23.07	48.13	40.07	40.07	203.06	351.72	60.00	1241.54

Fig. A.1 (Contd.)

2.733	414.68	12.75	3.55	22.84	49.09	40.87	40.87	207.34	359.12	60.00	1278.78
2.789	423.23	12.71	3.51	22.62	50.06	41.68	41.68	211.61	366.53	60.00	1316.39
2.848	432.12	12.68	3.48	22.40	51.06	42.51	42.51	216.06	374.23	60.00	1355.90
2.907	441.01	12.64	3.44	22.18	52.07	43.35	43.35	220.51	381.93	60.00	1395.80
2.968	450.26	12.61	3.41	21.96	53.11	44.22	44.22	225.13	389.94	60.00	1437.72
3.029	459.51	12.58	3.38	21.75	54.15	45.09	45.09	229.76	397.95	60.00	1480.05
3.092	469.14	12.54	3.34	21.53	55.24	45.99	45.99	234.57	406.29	60.00	1524.52
3.156	478.76	12.51	3.31	21.32	56.32	46.89	46.89	239.38	414.62	60.00	1569.43
3.221	488.77	12.48	3.28	21.11	57.45	47.83	47.83	244.39	423.29	60.00	1616.60
3.287	498.78	12.44	3.24	20.91	58.58	48.77	48.77	249.39	431.96	60.00	1664.25
3.356	509.20	12.41	3.21	20.70	59.76	49.75	49.75	254.60	440.98	60.00	1714.30
3.425	519.61	12.38	3.18	20.50	60.93	50.73	50.73	259.81	450.00	60.00	1764.85
3.496	530.45	12.35	3.15	20.30	62.16	51.75	51.75	265.22	459.38	60.00	1817.96
3.568	541.28	12.32	3.12	20.10	63.38	52.77	52.77	270.64	468.76	60.00	1871.59
3.642	552.55	12.29	3.09	19.90	64.65	53.83	53.83	276.28	478.52	60.00	1927.94
3.716	563.82	12.26	3.06	19.71	65.92	54.88	54.88	281.91	488.28	60.00	1984.84
3.793	575.55	12.23	3.03	19.52	67.25	55.99	55.99	287.77	498.44	60.00	2044.63
3.871	587.27	12.20	3.00	19.33	68.57	57.09	57.09	293.64	508.59	60.00	2105.01
3.951	599.47	12.17	2.97	19.14	69.95	58.23	58.23	299.74	519.16	60.00	2168.44
4.032	611.67	12.14	2.94	18.95	71.32	59.38	59.38	305.84	529.73	60.00	2232.51
4.115	624.37	12.11	2.91	18.76	72.75	60.57	60.57	312.18	540.72	60.00	2299.82
4.199	637.06	12.08	2.88	18.58	74.19	61.76	61.76	318.53	551.71	60.00	2367.79
4.286	650.26	12.05	2.85	18.40	75.68	63.00	63.00	325.13	563.15	60.00	2439.22
4.373	663.47	12.02	2.82	18.22	77.17	64.25	64.25	331.73	574.58	60.00	2511.35
4.463	677.21	12.00	2.80	18.04	78.72	65.54	65.54	338.60	586.48	60.00	2587.14
4.554	690.95	11.97	2.77	17.86	80.27	66.83	66.83	345.47	598.38	60.00	2663.68
4.648	705.25	11.94	2.74	17.69	81.88	68.17	68.17	352.62	610.76	60.00	2744.11
4.742	719.54	11.91	2.71	17.51	83.49	69.51	69.51	359.77	623.14	60.00	2825.34
4.841	734.42	11.89	2.69	17.34	85.17	70.91	70.91	367.21	636.02	60.00	2910.69
4.939	749.29	11.86	2.66	17.17	86.85	72.31	72.31	374.65	648.91	60.00	2996.89
5.041	764.77	11.83	2.63	17.00	88.60	73.76	73.76	382.38	662.31	60.00	3087.47
5.143	780.25	11.81	2.61	16.84	90.34	75.22	75.22	390.12	675.71	60.00	3178.95
5.249	796.35	11.78	2.58	16.67	92.16	76.73	76.73	398.18	689.66	60.00	3275.08
5.355	812.46	11.76	2.56	16.51	93.98	78.24	78.24	406.23	703.61	60.00	3372.17
5.465	829.22	11.73	2.53	16.34	95.87	79.82	79.82	414.61	718.13	60.00	3474.20
5.576	845.98	11.71	2.51	16.19	97.76	81.39	81.39	422.99	732.64	60.00	3577.24
5.691	863.42	11.68	2.48	16.03	99.73	83.03	83.03	431.71	747.75	60.00	3685.53
5.806	880.86	11.66	2.46	15.87	101.70	84.67	84.67	440.43	762.85	60.00	3794.90
5.925	899.02	11.63	2.43	15.71	103.75	86.37	86.37	449.51	778.57	60.00	3909.85
6.045	917.17	11.61	2.41	15.56	105.79	88.08	88.08	458.58	794.29	60.00	4025.94
6.170	936.06	11.58	2.38	15.41	107.92	89.85	89.85	468.03	810.65	60.00	4147.95
6.294	954.95	11.56	2.36	15.26	110.06	91.63	91.63	477.47	827.01	60.00	4271.18
6.424	974.51	11.54	2.34	15.10	112.27	93.47	93.47	487.30	844.04	60.00	4400.70
6.553	994.27	11.51	2.31	14.96	114.49	95.32	95.32	497.14	861.06	60.00	4531.51
6.688	1014.73	11.49	2.29	14.81	116.80	97.24	97.24	507.37	878.79	60.00	4669.02
6.823	1035.20	11.47	2.27	14.66	119.11	99.17	99.17	517.60	896.51	60.00	4807.88
6.963	1056.50	11.45	2.25	14.52	121.52	101.17	101.17	528.25	914.95	60.00	4953.87
7.104	1077.80	11.42	2.22	14.38	123.92	103.17	103.17	538.90	933.40	60.00	5101.30
7.250	1099.97	11.40	2.20	14.23	126.42	105.25	105.25	549.99	952.60	60.00	5256.29
7.396	1122.14	11.38	2.18	14.09	128.92	107.33	107.33	561.07	971.80	60.00	5412.83
7.548	1145.22	11.36	2.16	13.95	131.53	109.50	109.50	572.61	991.79	60.00	5577.40
7.700	1168.30	11.34	2.14	13.82	134.13	111.67	111.67	584.15	1011.78	60.00	5743.61
7.859	1192.33	11.31	2.11	13.68	136.84	113.93	113.93	596.17	1032.59	60.00	5918.37
8.017	1216.36	11.29	2.09	13.55	139.55	116.19	116.19	608.18	1053.40	60.00	6094.87
8.182	1241.38	11.27	2.07	13.41	142.38	118.54	118.54	620.69	1075.06	60.00	6280.45
8.347	1266.39	11.25	2.05	13.28	145.20	120.89	120.89	633.20	1096.73	60.00	6467.89
8.518	1292.44	11.23	2.03	13.15	148.14	123.33	123.33	646.22	1119.29	60.00	6664.98
8.690	1318.49	11.21	2.01	13.02	151.08	125.78	125.78	659.24	1141.84	60.00	6864.05
8.869	1345.61	11.19	1.99	12.89	154.14	128.33	128.33	672.80	1165.33	60.00	7073.39
9.048	1372.73	11.17	1.97	12.76	157.20	130.88	130.88	686.37	1188.82	60.00	7284.83
9.234	1400.97	11.15	1.95	12.64	160.38	133.53	133.53	700.49	1213.28	60.00	7507.20
9.420	1429.22	11.13	1.93	12.51	163.57	136.18	136.18	714.61	1237.74	60.00	7731.79

Fig. A.1 (Contd.)

9.614	1458.63	11.11	1.91	12.39	166.89	138.95	138.95	729.31	1263.21	60.00	7968.02
9.808	1488.04	11.09	1.89	12.27	170.21	141.71	141.71	744.02	1288.68	60.00	8206.62
10.010	1518.68	11.07	1.87	12.14	173.67	144.59	144.59	759.34	1315.21	60.00	8457.60
10.211	1549.31	11.05	1.85	12.03	177.12	147.46	147.46	774.65	1341.74	60.00	8711.09
10.422	1581.22	11.03	1.83	11.90	180.72	150.46	150.46	790.61	1369.37	60.00	8977.78
10.632	1613.13	11.02	1.82	11.79	184.32	153.46	153.46	806.56	1397.01	60.00	9247.13
10.851	1646.36	11.00	1.80	11.67	188.07	156.58	156.58	823.18	1425.79	60.00	9530.53
11.070	1679.60	10.98	1.78	11.56	191.82	159.70	159.70	839.80	1454.58	60.00	9816.77
11.299	1714.23	10.96	1.76	11.44	195.73	162.96	162.96	857.12	1484.57	60.00	10117.96
11.527	1748.86	10.94	1.74	11.33	199.64	166.21	166.21	874.43	1514.56	60.00	10422.18
11.765	1784.94	10.93	1.73	11.21	203.71	169.60	169.60	892.47	1545.81	60.00	10742.34
12.002	1821.02	10.91	1.71	11.10	207.78	172.99	172.99	910.51	1577.05	60.00	11065.71
12.250	1858.62	10.89	1.69	10.99	212.02	176.52	176.52	929.31	1609.61	60.00	11406.07
12.498	1896.22	10.87	1.67	10.88	216.27	180.05	180.05	948.11	1642.17	60.00	11749.85
12.756	1935.40	10.86	1.66	10.77	220.69	183.74	183.74	967.70	1676.11	60.00	12111.74

T(DEG.C)	T/T0	AREA(SQ.M)	AREA(ACRES)	AREA(SQ.KM)	TIME(SEC)
8.0000	0.9412	6.19770 02	1.53150-01	6.19770-04	119.83
7.0000	0.8235	8.87560 02	2.19320-01	8.87560-04	174.45
6.0000	0.7059	1.41720 03	3.50190-01	1.41720-03	271.71
5.0000	0.5882	2.65140 03	6.55170-01	2.65140-03	462.72
4.0000	0.4706	6.07070 03	1.50010 00	6.07070-03	894.21
3.0000	0.3529	1.85660 04	4.58780 00	1.85660-02	2100.07
2.0000	0.2353	9.20230 04	2.27390 01	9.20230-02	6970.68
1.6667	0.1961	1.88100 05	4.64800 01	1.88100-01	11872.97

COORDINATES OF 1.6667 DEG.C ISOTHERM IN METERS

X	Y	X	Y	X	Y	X	Y
37.19	42.57	18.27	53.49	38.11	43.79	18.87	54.90
39.03	45.01	19.47	56.30	39.98	46.28	20.09	57.77
40.94	47.55	20.71	59.23	41.93	48.88	21.37	60.75
42.92	50.20	22.02	62.27	43.94	51.58	22.69	63.85
44.97	52.95	23.37	65.42	46.04	54.39	24.08	67.07
47.11	55.82	24.79	68.71	48.22	57.31	25.52	70.41
49.33	58.80	26.26	72.12	50.48	60.35	27.03	73.89
51.63	61.91	27.80	75.67	52.83	63.52	28.60	77.51
54.02	65.14	29.40	79.35	55.26	66.82	30.23	81.27
56.50	68.50	31.07	83.18	57.79	70.25	31.94	85.18
59.08	72.00	32.81	87.17	60.42	73.82	33.72	89.24
61.76	75.64	34.63	91.31	63.15	77.53	35.57	93.46
64.54	79.43	36.52	95.61	71.98	89.61	41.62	107.15
69.11	113.26	53.53	133.80	109.76	142.14	68.22	166.13
134.67	177.43	86.32	205.34	164.68	220.55	108.67	252.89
200.81	273.29	136.27	310.55	244.28	337.80	170.40	380.45
296.52	416.78	212.68	465.18	359.21	513.56	265.15	567.87
434.31	532.33	330.46	692.29	523.97	778.35	412.09	842.95
630.47	958.39	514.75	1025.20	755.53	1181.50	645.45	1245.06
896.86	1461.62	817.37	1507.51				

COORDINATES OF 2.0000 DEG.C ISOTHERM IN METERS

X	Y	X	Y	X	Y	X	Y
36.65	42.88	18.81	53.18	37.55	44.11	19.42	54.58
38.46	45.34	20.04	55.98	39.40	46.62	20.67	57.43
40.34	47.90	21.31	58.88	41.31	49.23	21.98	60.39

Fig. A.1 (Contd.)

42.29	50.56	22.64	61.91	43.30	51.95	23.33	63.48
44.32	53.33	24.03	65.05	45.37	54.77	24.75	66.68
46.42	56.22	25.47	68.31	47.52	57.72	26.23	70.01
48.61	59.22	26.98	71.70	49.74	60.78	27.76	73.47
50.88	62.34	28.55	75.23	52.06	63.97	29.37	77.07
53.23	65.59	30.19	78.90	54.46	67.28	31.04	80.80
55.68	68.98	31.89	82.71	56.95	70.74	32.78	84.69
58.22	72.50	33.67	86.67	59.54	74.33	34.60	88.73
60.86	76.16	35.53	90.78	62.23	78.07	36.50	92.92
63.59	79.98	37.47	95.06	70.92	90.23	42.68	106.53
87.76	114.04	54.88	133.02	108.04	143.14	69.94	165.14
132.47	178.70	88.52	204.07	161.85	222.19	111.49	251.26
197.15	275.39	139.92	308.44	239.51	340.55	175.17	377.69
290.22	420.41	218.98	461.54	350.76	518.45	273.61	562.99
422.67	639.05	342.10	685.57	507.23	788.02	428.82	833.28
603.95	973.70	541.27	1009.89				

COORDINATES OF 3.0000 DEG.C ISOTHERM IN METERS

X	Y	X	Y	X	Y	X	Y
35.30	43.66	20.16	52.40	36.17	44.91	20.81	53.78
37.04	46.16	21.45	55.16	37.95	47.46	22.13	56.59
38.95	48.76	22.80	58.03	39.79	50.11	23.50	59.52
40.73	51.46	24.20	61.01	41.71	52.87	24.93	62.55
42.68	54.28	25.66	64.10	43.70	55.74	26.42	65.71
44.71	57.21	27.19	67.32	45.76	58.73	27.98	68.99
46.81	60.26	28.78	70.67	47.90	61.84	29.61	72.40
48.99	63.43	30.44	74.14	50.12	65.08	31.30	75.95
51.25	66.74	32.17	77.75	52.43	68.46	33.07	79.63
53.60	70.18	33.97	81.51	54.82	71.97	34.92	83.46
56.04	73.76	35.86	85.41	57.30	75.62	36.84	87.43
58.56	77.49	37.82	89.46	59.88	79.43	38.85	91.57
61.19	81.37	39.87	93.67	68.19	91.80	45.41	104.96
84.25	116.06	58.39	131.00	103.50	145.76	74.48	162.52
126.53	182.12	94.46	200.64	153.99	226.72	119.35	246.72
186.54	281.52	150.54	302.31	224.72	349.09	189.96	369.15
268.13	433.17	241.07	448.79				

COORDINATES OF 4.0000 DEG.C ISOTHERM IN METERS

X	Y	X	Y	X	Y	X	Y
34.17	44.31	21.29	51.75	35.01	45.58	21.97	53.11
35.85	46.84	22.64	54.47	36.73	48.16	23.34	55.89
37.60	49.48	24.05	57.30	38.51	50.85	24.78	58.78
39.42	52.22	25.51	60.25	40.36	53.65	26.28	61.78
41.30	55.07	27.05	63.30	42.28	56.56	27.84	64.89
43.25	58.05	28.64	66.48	44.26	59.59	29.48	68.13
45.28	61.14	30.31	69.78	46.32	62.75	31.18	71.49
47.37	64.36	32.05	73.21	48.46	66.04	32.96	74.99
49.55	67.72	33.87	76.77	50.68	69.47	34.82	78.62
51.80	71.21	35.77	80.47	52.97	73.03	36.76	82.39
54.14	74.85	37.75	84.31	55.35	76.75	38.79	86.31
56.56	78.64	39.82	88.31	57.82	80.61	40.90	90.38
59.07	82.59	41.98	92.45	65.77	93.20	47.83	103.56
81.03	117.93	61.62	129.13	99.12	148.29	78.86	159.98
120.35	185.69	100.64	197.08	144.60	232.15	128.74	241.30

Fig. A.1 (Contd.)

COORDINATES OF 5.0000 DEG.C ISOTHERM IN METERS

X	Y	X	Y	X	Y	X	Y
33.13	44.91	22.33	51.15	33.95	46.19	23.03	52.49
34.76	47.48	23.74	53.84	35.60	48.81	24.47	55.24
36.45	50.15	25.20	56.64	37.32	51.54	25.97	58.09
38.19	52.93	26.74	59.54	39.10	54.37	27.54	61.05
40.00	55.82	28.34	62.56	40.94	57.33	29.18	64.12
41.88	58.84	30.02	65.69	42.85	60.41	30.89	67.32
43.82	61.98	31.77	68.94	44.83	63.62	32.68	70.63
45.83	65.25	33.60	72.32	46.87	66.96	34.55	74.07
47.91	68.66	35.51	75.83	48.99	70.44	36.51	77.65
50.07	72.22	37.51	79.47	51.18	74.07	38.55	81.36
52.29	75.92	39.60	83.25	53.45	77.85	40.69	85.21
54.60	79.78	41.79	87.17	55.79	81.79	42.93	89.21
56.98	83.80	44.08	91.24	63.30	94.63	50.30	102.13
77.45	119.99	65.19	127.07	93.31	151.64	84.67	156.63

COORDINATES OF 6.0000 DEG.C ISOTHERM IN METERS

X	Y	X	Y	X	Y	X	Y
32.11	45.50	23.35	50.56	32.89	46.80	24.09	51.88
33.67	48.11	24.83	53.21	34.48	49.46	25.60	54.59
35.28	50.82	26.37	55.96	36.12	52.23	27.17	57.39
36.95	53.64	27.98	58.82	37.82	55.11	28.82	60.31
38.68	56.59	29.67	61.79	39.57	58.12	30.55	63.33
40.46	59.66	31.44	64.87	41.38	61.26	32.36	66.47
42.30	62.86	33.29	68.06	43.25	64.53	34.26	69.72
44.19	66.20	35.23	71.37	45.17	67.94	36.25	73.09
46.15	69.68	37.27	74.81	47.16	71.50	38.34	76.59
48.16	73.32	39.41	78.37	49.20	75.21	40.54	80.21
50.23	77.11	41.67	82.05	51.29	79.09	42.85	83.97
52.35	81.07	44.04	85.87	53.44	83.14	45.29	87.85
54.52	85.22	46.54	89.82	60.11	96.47	53.49	100.29

COORDINATES OF 7.0000 DEG.C ISOTHERM IN METERS

X	Y	X	Y	X	Y	X	Y
31.00	46.14	24.46	49.92	31.73	47.47	25.24	51.22
32.47	48.80	26.03	52.52	33.23	50.18	26.85	53.87
33.98	51.57	27.67	55.21	34.76	53.01	28.53	56.61
35.53	54.46	29.40	58.00	36.33	55.97	30.31	59.45
37.12	57.49	31.22	60.89	37.94	59.07	32.18	62.39
38.74	60.65	33.15	63.88	39.57	62.30	34.17	65.42
40.39	63.96	35.20	66.96	41.22	65.70	36.29	68.55
42.04	67.44	37.39	70.13	42.87	69.27	38.56	71.76
43.67	71.12	39.75	73.37	44.46	73.06	41.04	75.03
45.19	75.03	42.39	76.65	45.81	77.17	43.92	78.26

COORDINATES OF 8.0000 DEG.C ISOTHERM IN METERS

X	Y	X	Y	X	Y	X	Y
29.56	46.98	25.90	49.08	30.19	48.36	26.79	50.33
30.81	49.76	27.69	51.56	31.42	51.22	28.65	52.82
32.00	52.71	29.65	54.07	32.52	54.31	30.77	55.32
32.82	55.03	32.11	56.44				

Fig. A.1 (Contd.)

Fig. A.2. Predictions of Motz-Benedict Model. ANL Neg. No. 190-1098.

The areas within isotherms of specific excess temperature are tabulated in Table A.3.

TABLE A.3. Isotherm Areas Predicted
by Motz-Benedict Model

Excess Temperature, C°	Area, m ²
8	620
7	888
6	1417
5	2651
4	6071
3	18566
2	92023

2. Prych Model

The model parameters used in preparing the Prych-model predictions are given in Table A.4. In Fig. A.3 the actual computer output is reproduced with the input parameters listed at the top of each page of output and labeled as follows:

U	outlet velocity (m/sec)
V	ambient current velocity (m/sec)
2B	width of outlet (m)
H	depth of outlet (m)
Q	discharge flow rate (m ³ /sec)
F	initial densimetric Froude number
DR	difference in density between ambient and discharge water (kg/m ³)
TH	angle of discharge with respect to ambient current direction (degrees)
1000*K	1000 times the atmospheric heat-transfer coefficient (m/sec)
E	entrainment coefficient

TABLE A.4. Model Parameters Used in Preparing
Prych-model Prediction

Entrainment coefficient, E_o	0.1
Drag coefficient, C_D	0.2
Interfacial shear coefficient, C_F	0.5

FLOATING WARM WATER JETS -- PRYCH CALCULATIONS FOR POINT BEACH UNIT NO. 1 MAY 18, 1972

U= 0.5470 2R= 10.67 Q= 24.7 DR= 1.12 1000*K= 0.00280 CD= 0.20 EH= 1.00 HS= 1.0/1.0
 V= 0.0 H= 4.24 F= 2.54 TH= 60.0 E= 0.1000 CF= 0.50 EV= 1.00 VS= 1.0/1.0

S	X	Y	TH	T	U	U+VCT	Q	HT	H	B	F	RI	IHLF
0.0	21.5	37.2	60.0	8.500	0.537	0.537	2.000	1.000	2.08	28.2	2.374	0.111	0
5.0	24.0	41.5	60.0	7.088	0.430	0.430	2.398	1.000	2.40	36.6	1.934	0.168	2
10.0	26.5	45.8	60.0	6.250	0.365	0.365	2.720	1.000	2.58	45.5	1.690	0.219	2
15.0	29.0	50.2	60.0	5.692	0.323	0.323	2.986	1.000	2.65	54.9	1.541	0.264	2
20.0	31.5	54.5	60.0	5.291	0.293	0.293	3.212	1.000	2.67	64.6	1.444	0.300	2
25.0	34.0	58.8	60.0	4.985	0.271	0.271	3.410	1.000	2.66	74.7	1.380	0.329	2
30.0	36.5	63.1	60.0	4.740	0.253	0.253	3.585	1.000	2.62	85.0	1.334	0.352	2
35.0	39.0	67.5	60.0	4.538	0.240	0.240	3.745	1.000	2.57	95.5	1.301	0.370	2
40.0	41.5	71.8	60.0	4.366	0.228	0.228	3.893	1.000	2.53	106.3	1.275	0.385	2
45.0	44.0	76.1	60.0	4.216	0.218	0.218	4.031	1.000	2.48	117.2	1.254	0.398	2
50.0	46.5	80.5	60.0	4.084	0.210	0.210	4.161	1.000	2.43	128.3	1.237	0.410	2
55.0	49.0	84.8	60.0	3.966	0.202	0.202	4.285	0.999	2.39	139.7	1.221	0.420	2
60.0	51.5	89.1	60.0	3.859	0.196	0.196	4.403	0.999	2.34	151.2	1.208	0.430	2
65.0	54.0	93.5	60.0	3.762	0.190	0.190	4.516	0.999	2.30	163.0	1.196	0.438	2
70.0	56.5	97.8	60.0	3.672	0.184	0.184	4.626	0.999	2.26	174.9	1.184	0.447	2
75.0	59.0	102.1	60.0	3.590	0.179	0.179	4.732	0.999	2.23	187.2	1.174	0.455	2
80.0	61.5	106.4	60.0	3.513	0.174	0.174	4.835	0.999	2.19	199.7	1.164	0.462	2
82.5	62.7	108.6	60.0	3.477	0.172	0.172	4.885	0.999	2.17	206.0	1.160	0.466	3
85.0	64.0	110.8	60.0	3.442	0.170	0.170	4.935	0.999	2.16	212.4	1.155	0.470	3
87.5	65.2	112.9	60.0	3.408	0.167	0.167	4.984	0.999	2.14	218.9	1.151	0.473	3
90.0	66.5	115.1	60.0	3.375	0.165	0.165	5.032	0.999	2.12	225.5	1.147	0.477	3
92.5	67.7	117.3	60.0	3.343	0.163	0.163	5.080	0.999	2.11	232.1	1.143	0.480	3
95.0	69.0	119.4	60.0	3.312	0.162	0.162	5.127	0.999	2.09	238.8	1.139	0.483	3
97.5	70.2	121.6	60.0	3.282	0.160	0.160	5.174	0.999	2.08	245.6	1.135	0.486	3
100.0	71.5	123.8	60.0	3.252	0.158	0.158	5.220	0.999	2.06	252.5	1.131	0.489	3
102.5	72.7	125.9	60.0	3.224	0.156	0.156	5.266	0.999	2.05	259.4	1.128	0.493	3
105.0	74.0	128.1	60.0	3.196	0.154	0.154	5.312	0.999	2.03	266.4	1.125	0.496	3
107.5	75.2	130.3	60.0	3.169	0.153	0.153	5.357	0.999	2.02	273.6	1.121	0.498	3
110.0	76.5	132.4	60.0	3.143	0.151	0.151	5.401	0.999	2.00	280.8	1.118	0.501	3
112.5	77.7	134.6	60.0	3.117	0.150	0.150	5.445	0.998	1.99	288.0	1.115	0.504	3
115.0	79.0	136.8	60.0	3.092	0.148	0.148	5.489	0.998	1.98	295.4	1.112	0.507	3
117.5	80.2	138.9	60.0	3.068	0.147	0.147	5.533	0.998	1.96	302.9	1.109	0.510	3
120.0	81.5	141.1	60.0	3.044	0.145	0.145	5.576	0.998	1.95	310.5	1.106	0.512	3
122.5	82.7	143.2	60.0	3.020	0.144	0.144	5.619	0.998	1.93	318.1	1.103	0.515	3
125.0	84.0	145.4	60.0	2.997	0.142	0.142	5.662	0.998	1.92	325.9	1.100	0.517	3
127.5	85.2	147.6	60.0	2.975	0.141	0.141	5.704	0.998	1.91	333.7	1.098	0.520	3
130.0	86.5	149.7	60.0	2.953	0.140	0.140	5.746	0.998	1.90	341.7	1.095	0.522	3
132.5	87.7	151.9	60.0	2.931	0.138	0.138	5.788	0.998	1.88	349.8	1.093	0.525	3
135.0	89.0	154.1	60.0	2.910	0.137	0.137	5.830	0.990	1.87	358.0	1.090	0.527	3
137.5	90.2	156.2	60.0	2.889	0.136	0.136	5.871	0.998	1.86	366.2	1.088	0.530	3
140.0	91.5	158.4	60.0	2.869	0.135	0.135	5.912	0.998	1.85	374.6	1.085	0.532	3
142.5	92.7	160.6	60.0	2.849	0.133	0.133	5.953	0.998	1.83	383.2	1.083	0.534	3
145.0	94.0	162.7	60.0	2.830	0.132	0.132	5.994	0.998	1.82	391.8	1.081	0.536	3
147.5	95.2	164.9	60.0	2.810	0.131	0.131	6.035	0.998	1.81	400.6	1.079	0.539	3
150.0	96.5	167.1	60.0	2.791	0.130	0.130	6.076	0.998	1.80	409.5	1.077	0.541	3
152.5	97.7	169.2	60.0	2.773	0.129	0.129	6.116	0.998	1.79	418.5	1.074	0.543	3
155.0	99.0	171.4	60.0	2.754	0.128	0.128	6.156	0.997	1.77	427.6	1.072	0.545	3
157.5	100.2	173.6	60.0	2.736	0.127	0.127	6.197	0.997	1.76	436.9	1.070	0.547	3
160.0	101.5	175.7	60.0	2.718	0.126	0.126	6.237	0.997	1.75	446.4	1.068	0.549	3
162.5	102.7	177.9	60.0	2.701	0.125	0.125	6.277	0.997	1.74	456.0	1.066	0.551	3

Fig. A.3. Computer Output of Prych Code

FLOATING WARM WATER JETS -- PRYCH CALCULATIONS FOR POINT BEACH UNIT NO. 1 MAY 18, 1972

J= 0.5470 29= 10.67 Q= 24.7 DR= 1.12 1000*K= 0.00280 CD= 0.20 EH= 1.00 HS= 1.0/1.0
 V= 0.0 4= 4.24 F= 2.54 TH= 60.0 E= 0.1000 CF= 0.50 EV= 1.00 VS= 1.0/1.0

S	X	Y	TH	T	U	U+VCT	Q	HT	H	B	F	RI	IMLF
165.0	104.0	180.1	60.0	2.684	0.124	0.124	6.317	0.997	1.73	465.7	1.064	0.553	3
167.5	105.2	182.2	60.0	2.666	0.123	0.123	6.357	0.997	1.72	475.6	1.063	0.555	3
170.0	106.5	184.4	60.0	2.650	0.122	0.122	6.397	0.997	1.71	485.6	1.061	0.557	3
172.5	107.7	186.6	60.0	2.633	0.121	0.121	6.437	0.997	1.69	495.9	1.059	0.559	3
175.0	109.0	188.7	60.0	2.617	0.120	0.120	6.477	0.997	1.68	506.2	1.057	0.561	3
176.3	109.6	189.8	60.0	2.608	0.119	0.119	6.497	0.997	1.68	511.5	1.056	0.562	4
177.5	110.2	190.9	60.0	2.600	0.119	0.119	6.517	0.997	1.67	516.8	1.056	0.562	4
178.8	110.8	192.0	60.0	2.592	0.118	0.118	6.536	0.997	1.67	522.1	1.055	0.563	4
180.0	111.5	193.0	60.0	2.584	0.118	0.118	6.556	0.997	1.66	527.5	1.054	0.564	4
181.3	112.1	194.1	60.0	2.576	0.117	0.117	6.576	0.997	1.66	533.0	1.053	0.565	4
182.5	112.7	195.2	60.0	2.569	0.117	0.117	6.596	0.997	1.65	538.5	1.052	0.566	4
183.8	113.3	196.3	60.0	2.561	0.116	0.116	6.616	0.997	1.65	544.0	1.052	0.567	4
185.0	114.0	197.4	60.0	2.553	0.116	0.116	6.636	0.997	1.64	549.6	1.051	0.568	4
186.3	114.6	198.5	60.0	2.545	0.115	0.115	6.656	0.997	1.63	555.2	1.050	0.568	4
187.5	115.2	199.5	60.0	2.537	0.115	0.115	6.676	0.996	1.63	560.9	1.049	0.569	4
188.8	115.8	200.6	60.0	2.530	0.115	0.115	6.696	0.996	1.62	566.6	1.048	0.570	4
190.0	116.5	201.7	60.0	2.522	0.114	0.114	6.716	0.996	1.62	572.4	1.048	0.571	4
191.3	117.1	202.8	60.0	2.514	0.114	0.114	6.736	0.996	1.61	578.2	1.047	0.572	4
192.5	117.7	203.9	60.0	2.507	0.113	0.113	6.756	0.996	1.61	584.1	1.046	0.573	4
193.8	118.3	205.0	60.0	2.499	0.113	0.113	6.776	0.996	1.60	590.0	1.045	0.573	4
195.0	119.0	206.0	60.0	2.492	0.112	0.112	6.796	0.996	1.60	596.0	1.045	0.574	4
196.3	119.6	207.1	60.0	2.484	0.112	0.112	6.816	0.996	1.59	602.1	1.044	0.575	4
197.5	120.2	208.2	60.0	2.477	0.111	0.111	6.836	0.996	1.59	608.2	1.043	0.576	4
198.8	120.8	209.3	60.0	2.470	0.111	0.111	6.856	0.996	1.58	614.3	1.043	0.577	4
200.0	121.5	210.4	60.0	2.462	0.111	0.111	6.877	0.996	1.58	620.5	1.042	0.577	4
201.3	122.1	211.4	60.0	2.455	0.110	0.110	6.897	0.996	1.57	626.8	1.041	0.578	4
202.5	122.7	212.5	60.0	2.448	0.110	0.110	6.917	0.996	1.57	633.1	1.041	0.579	4
203.8	123.3	213.6	60.0	2.441	0.109	0.109	6.937	0.996	1.56	639.5	1.040	0.580	4
205.0	124.0	214.7	60.0	2.433	0.109	0.109	6.957	0.996	1.56	646.0	1.039	0.580	4
206.3	124.6	215.8	60.0	2.426	0.109	0.109	6.978	0.996	1.55	652.5	1.039	0.581	4
207.5	125.2	216.9	60.0	2.419	0.108	0.108	6.998	0.996	1.55	659.1	1.038	0.582	4
208.8	125.8	217.9	60.0	2.412	0.108	0.108	7.018	0.996	1.54	665.8	1.037	0.582	4
210.0	126.5	219.0	60.0	2.405	0.107	0.107	7.039	0.996	1.53	672.5	1.037	0.583	4
211.3	127.1	220.1	60.0	2.398	0.107	0.107	7.059	0.996	1.53	679.3	1.036	0.584	4
212.5	127.7	221.2	60.0	2.391	0.107	0.107	7.080	0.996	1.52	686.1	1.035	0.585	4
213.8	128.3	222.3	60.0	2.384	0.106	0.106	7.100	0.996	1.52	693.0	1.035	0.585	4
215.0	129.0	223.4	60.0	2.377	0.106	0.106	7.121	0.995	1.51	700.0	1.034	0.586	4
216.3	129.6	224.4	60.0	2.370	0.105	0.105	7.141	0.995	1.51	707.1	1.034	0.587	4
217.5	130.2	225.5	60.0	2.363	0.105	0.105	7.162	0.995	1.50	714.3	1.033	0.587	4
218.8	130.8	226.6	60.0	2.356	0.105	0.105	7.183	0.995	1.50	721.5	1.032	0.588	4
220.0	131.5	227.7	60.0	2.349	0.104	0.104	7.204	0.995	1.49	728.8	1.032	0.589	4
220.6	131.8	228.2	60.0	2.345	0.104	0.104	7.214	0.995	1.49	732.4	1.032	0.589	5
221.3	132.1	228.8	60.0	2.342	0.104	0.104	7.224	0.995	1.49	736.1	1.031	0.589	5
221.9	132.4	229.3	60.0	2.338	0.104	0.104	7.235	0.995	1.49	739.9	1.031	0.590	5
222.5	132.7	229.9	60.0	2.335	0.103	0.103	7.245	0.995	1.48	743.6	1.031	0.590	5
223.8	133.3	230.9	60.0	2.328	0.103	0.103	7.266	0.995	1.48	751.1	1.030	0.591	4
225.0	134.0	232.0	60.0	2.321	0.103	0.103	7.287	0.995	1.47	758.7	1.030	0.591	4
225.6	134.3	232.6	60.0	2.318	0.102	0.102	7.298	0.995	1.47	762.6	1.029	0.591	5
226.3	134.6	233.1	60.0	2.315	0.102	0.102	7.308	0.995	1.47	766.4	1.029	0.592	5
226.9	134.9	233.6	60.0	2.311	0.102	0.102	7.319	0.995	1.47	770.3	1.029	0.592	5

Fig. A.3 (Contd.)

FLOATING WARM WATER JETS -- PSYCH CALCULATIONS FOR POINT BEACH UNIT NO. 1 MAY 18, 1972

U= 0.5470 2R= 10.67 Q= 24.7 DR= 1.12 1000*K= 0.00280 CO= 0.20 FH= 1.00 HS= 1.0/1.0
V= 0.0 H= 4.24 F= 2.54 TH= 60.0 E= 0.1000 CF= 0.50 EV= 1.00 VS= 1.0/1.0

S	X	Y	TH	T	U	U+VCT	Q	HT	H	B	F	RI	IHLF
227.5	135.2	234.2	60.0	2.308	0.102	0.102	7.329	0.995	1.46	774.2	1.029	0.592	5
228.8	135.8	235.3	60.0	2.301	0.101	0.101	7.351	0.995	1.46	782.1	1.028	0.593	4
230.0	136.5	236.3	60.0	2.294	0.101	0.101	7.372	0.995	1.45	790.0	1.027	0.594	4
230.5	136.8	236.9	60.0	2.291	0.101	0.101	7.382	0.995	1.45	794.1	1.027	0.594	5
231.3	137.1	237.4	60.0	2.287	0.101	0.101	7.393	0.995	1.45	798.1	1.027	0.594	5
231.9	137.4	238.0	60.0	2.284	0.101	0.101	7.404	0.995	1.45	802.2	1.027	0.594	5
232.5	137.7	238.5	60.0	2.281	0.100	0.100	7.414	0.995	1.44	806.2	1.026	0.595	5
233.1	138.0	239.1	60.0	2.277	0.100	0.100	7.425	0.995	1.44	810.4	1.026	0.595	5
233.9	138.3	239.6	60.0	2.274	0.100	0.100	7.436	0.995	1.44	814.5	1.026	0.595	5
234.4	138.6	240.1	60.0	2.271	0.100	0.100	7.447	0.995	1.43	818.6	1.026	0.596	5
235.0	139.0	240.7	60.0	2.267	0.100	0.100	7.458	0.995	1.43	822.8	1.025	0.596	5
235.5	139.3	241.2	60.0	2.264	0.099	0.099	7.468	0.995	1.43	827.0	1.025	0.596	5
236.3	139.6	241.8	60.0	2.261	0.099	0.099	7.479	0.995	1.43	831.3	1.025	0.596	5
236.9	139.9	242.3	60.0	2.257	0.099	0.099	7.490	0.995	1.42	835.5	1.025	0.597	5
237.5	140.2	242.8	60.0	2.254	0.099	0.099	7.501	0.995	1.42	839.8	1.024	0.597	5
238.1	140.5	243.4	60.0	2.251	0.099	0.099	7.512	0.994	1.42	844.1	1.024	0.597	5
238.8	140.8	243.9	60.0	2.247	0.098	0.098	7.523	0.994	1.42	848.4	1.024	0.598	5
239.4	141.1	244.5	60.0	2.244	0.098	0.098	7.534	0.994	1.41	852.8	1.024	0.598	5
240.0	141.5	245.0	60.0	2.241	0.098	0.098	7.545	0.994	1.41	857.2	1.024	0.598	5
240.5	141.8	245.5	60.0	2.237	0.098	0.098	7.556	0.994	1.41	861.6	1.023	0.598	5
241.3	142.1	246.1	60.0	2.234	0.098	0.098	7.567	0.994	1.41	866.0	1.023	0.599	5
241.9	142.4	246.6	60.0	2.231	0.098	0.098	7.578	0.994	1.40	870.5	1.023	0.599	5
242.5	142.7	247.2	60.0	2.227	0.097	0.097	7.589	0.994	1.40	875.0	1.023	0.599	5
243.1	143.0	247.7	60.0	2.224	0.097	0.097	7.600	0.994	1.40	879.5	1.022	0.600	5
243.9	143.3	248.3	60.0	2.221	0.097	0.097	7.611	0.994	1.40	884.0	1.022	0.600	5
244.4	143.6	248.8	60.0	2.217	0.097	0.097	7.622	0.994	1.39	888.6	1.022	0.600	5
245.0	144.0	249.3	60.0	2.214	0.097	0.097	7.633	0.994	1.39	893.2	1.022	0.600	5
245.6	144.3	249.9	60.0	2.211	0.096	0.096	7.644	0.994	1.39	897.9	1.021	0.601	5
246.3	144.6	250.4	60.0	2.208	0.096	0.096	7.655	0.994	1.39	902.5	1.021	0.601	5
246.9	144.9	251.0	60.0	2.204	0.096	0.096	7.667	0.994	1.38	907.2	1.021	0.601	5
247.5	145.2	251.5	60.0	2.201	0.096	0.096	7.678	0.994	1.38	911.9	1.021	0.601	5
248.1	145.5	252.0	60.0	2.198	0.096	0.096	7.689	0.994	1.38	916.7	1.021	0.602	5
248.8	145.8	252.6	60.0	2.194	0.096	0.096	7.700	0.994	1.38	921.5	1.020	0.602	5
249.1	146.0	252.9	60.0	2.193	0.095	0.095	7.706	0.994	1.38	923.9	1.020	0.602	6
249.4	146.1	253.1	60.0	2.191	0.095	0.095	7.712	0.994	1.37	926.3	1.020	0.602	6
249.7	146.3	253.4	60.0	2.189	0.095	0.095	7.717	0.994	1.37	928.7	1.020	0.602	6
250.0	146.5	253.7	60.0	2.188	0.095	0.095	7.723	0.994	1.37	931.1	1.020	0.602	6
250.6	146.8	254.2	60.0	2.185	0.095	0.095	7.734	0.994	1.37	936.0	1.020	0.603	5
251.3	147.1	254.7	60.0	2.181	0.095	0.095	7.746	0.994	1.37	940.9	1.020	0.603	5
251.9	147.4	255.3	60.0	2.178	0.095	0.095	7.757	0.994	1.36	945.9	1.019	0.603	5
252.5	147.7	255.8	60.0	2.175	0.094	0.094	7.769	0.994	1.36	950.8	1.019	0.604	5
252.8	147.9	256.1	60.0	2.173	0.094	0.094	7.775	0.994	1.36	953.3	1.019	0.604	6
253.1	148.0	256.4	60.0	2.171	0.094	0.094	7.780	0.994	1.36	955.9	1.019	0.604	6
253.4	148.2	256.6	60.0	2.170	0.094	0.094	7.786	0.994	1.36	958.4	1.019	0.604	6
253.8	148.3	256.9	60.0	2.168	0.094	0.094	7.792	0.994	1.36	960.9	1.019	0.604	6
254.4	148.6	257.5	60.0	2.165	0.094	0.094	7.803	0.994	1.35	966.0	1.018	0.604	5
255.0	149.0	258.0	60.0	2.162	0.094	0.094	7.815	0.994	1.35	971.1	1.018	0.604	5
255.3	149.1	258.3	60.0	2.160	0.094	0.094	7.821	0.994	1.35	973.7	1.018	0.604	6
255.6	149.3	258.5	60.0	2.158	0.094	0.094	7.827	0.994	1.35	976.2	1.018	0.605	6
255.9	149.4	258.8	60.0	2.157	0.093	0.093	7.832	0.994	1.35	978.8	1.018	0.605	6

Fig. A.3. (Contd.)

F. JATING WARM WATER JETS -- PRYCH CALCULATIONS FOR POINT BEACH UNIT NO. 1 MAY 18, 1972

UJ= 0.5470 2B= 10.67 Q= 24.7 DR= 1.12 1000*K= 0.00280 CD= 0.20 EH= 1.00 HS= 1.0/1.0
 V= 0.0 H= 4.24 F= 2.54 TH= 60.0 E= 0.1000 CF= 0.50 EV= 1.00 VS= 1.0/1.0

S	X	Y	TH	T	U	U+VCT	Q	HT	H	B	F	RI	IHLF
256.3	149.6	259.1	60.0	2.155	0.093	0.093	7.838	0.994	1.35	981.4	1.018	0.605	6
256.9	149.9	259.6	60.0	2.152	0.093	0.093	7.850	0.994	1.34	986.6	1.018	0.605	5
257.5	150.2	260.2	60.0	2.148	0.093	0.093	7.862	0.994	1.34	991.9	1.017	0.605	5
257.8	150.4	260.4	60.0	2.147	0.093	0.093	7.868	0.994	1.34	994.5	1.017	0.605	6
258.1	150.5	260.7	60.0	2.145	0.093	0.093	7.873	0.994	1.34	997.2	1.017	0.606	6
258.4	150.7	261.0	60.0	2.144	0.093	0.093	7.879	0.994	1.34	999.8	1.017	0.606	6
258.8	150.8	261.2	60.0	2.142	0.093	0.093	7.885	0.994	1.34	1002.5	1.017	0.606	6
259.4	151.1	261.8	60.0	2.139	0.093	0.093	7.897	0.993	1.33	1007.9	1.017	0.606	5
260.0	151.5	262.3	60.0	2.135	0.092	0.092	7.909	0.993	1.33	1013.2	1.016	0.606	5
260.3	151.6	262.6	60.0	2.134	0.092	0.092	7.915	0.993	1.33	1016.0	1.017	0.606	6
260.6	151.8	262.9	60.0	2.132	0.092	0.092	7.921	0.993	1.33	1018.7	1.017	0.606	6
260.9	151.9	263.1	60.0	2.130	0.092	0.092	7.927	0.993	1.33	1021.4	1.016	0.607	6
261.3	152.1	263.4	60.0	2.129	0.092	0.092	7.933	0.993	1.33	1024.2	1.016	0.607	6
261.9	152.4	264.0	60.0	2.126	0.092	0.092	7.945	0.993	1.32	1029.7	1.016	0.607	5
262.5	152.7	264.5	60.0	2.122	0.092	0.092	7.957	0.993	1.32	1035.2	1.015	0.608	5
262.7	152.8	264.6	60.0	2.121	0.092	0.092	7.960	0.993	1.32	1036.6	1.016	0.607	7
262.8	152.9	264.8	60.0	2.121	0.092	0.092	7.963	0.993	1.32	1038.0	1.016	0.607	7
263.0	152.9	264.9	60.0	2.120	0.091	0.091	7.966	0.993	1.32	1039.4	1.016	0.607	7
263.1	153.0	265.0	60.0	2.119	0.091	0.091	7.969	0.993	1.32	1040.8	1.016	0.607	7
263.4	153.2	265.3	60.0	2.117	0.091	0.091	7.975	0.993	1.32	1043.6	1.016	0.607	6
263.8	153.3	265.6	60.0	2.116	0.091	0.091	7.981	0.993	1.31	1046.5	1.016	0.608	6
264.4	153.6	266.1	60.0	2.112	0.091	0.091	7.993	0.993	1.31	1052.1	1.015	0.608	5
265.0	154.0	266.7	60.0	2.109	0.091	0.091	8.005	0.993	1.31	1057.8	1.014	0.609	5
265.2	154.0	266.8	60.0	2.108	0.091	0.091	8.008	0.993	1.31	1059.3	1.015	0.608	7
265.3	154.1	266.9	60.0	2.108	0.091	0.091	8.011	0.993	1.31	1060.7	1.015	0.608	7
265.5	154.2	267.1	60.0	2.107	0.091	0.091	8.014	0.993	1.31	1062.2	1.015	0.608	7
265.6	154.3	267.2	60.0	2.106	0.091	0.091	8.017	0.993	1.31	1063.6	1.015	0.608	7
265.9	154.4	267.5	60.0	2.104	0.091	0.091	8.023	0.993	1.31	1066.5	1.015	0.608	6
266.3	154.6	267.7	60.0	2.103	0.091	0.091	8.029	0.993	1.30	1069.4	1.015	0.608	6
266.9	154.9	268.3	60.0	2.099	0.090	0.090	8.042	0.993	1.30	1075.2	1.015	0.609	5
267.5	155.2	268.8	60.0	2.096	0.090	0.090	8.054	0.993	1.30	1081.0	1.013	0.611	5
267.7	155.3	269.0	60.0	2.095	0.090	0.090	8.057	0.993	1.30	1082.6	1.014	0.609	7
267.8	155.4	269.1	60.0	2.094	0.090	0.090	8.060	0.993	1.30	1084.0	1.014	0.609	7
268.0	155.4	269.2	60.0	2.094	0.090	0.090	8.063	0.993	1.30	1085.5	1.014	0.609	7
268.1	155.5	269.4	60.0	2.093	0.090	0.090	8.066	0.993	1.30	1087.0	1.014	0.609	7
268.4	155.7	269.6	60.0	2.091	0.090	0.090	8.073	0.993	1.30	1090.0	1.014	0.609	6
268.8	155.8	269.9	60.0	2.089	0.090	0.090	8.079	0.993	1.29	1093.0	1.014	0.609	6
269.1	156.0	270.2	60.0	2.088	0.090	0.090	8.085	0.993	1.29	1096.0	1.014	0.609	6
269.4	156.1	270.4	60.0	2.086	0.090	0.090	8.091	0.993	1.29	1099.0	1.014	0.609	6
269.7	156.3	270.7	60.0	2.085	0.090	0.090	8.097	0.993	1.29	1102.0	1.014	0.610	6
270.0	156.5	271.0	60.0	2.083	0.090	0.090	8.104	0.993	1.29	1105.1	1.014	0.610	6
270.2	156.5	271.1	60.0	2.082	0.089	0.089	8.107	0.993	1.29	1106.6	1.014	0.610	7
270.3	156.6	271.3	60.0	2.081	0.089	0.089	8.110	0.993	1.29	1108.1	1.014	0.610	7
270.5	156.7	271.4	60.0	2.080	0.089	0.089	8.113	0.993	1.29	1109.7	1.014	0.610	7
270.6	156.8	271.5	60.0	2.080	0.089	0.089	8.116	0.993	1.29	1111.2	1.014	0.610	7
270.9	156.9	271.8	60.0	2.078	0.089	0.089	8.123	0.993	1.29	1114.3	1.014	0.610	6
271.3	157.1	272.1	60.0	2.076	0.089	0.089	8.129	0.993	1.28	1117.4	1.014	0.610	6
271.9	157.4	272.6	60.0	2.073	0.089	0.089	8.141	0.993	1.28	1123.5	1.013	0.611	5
272.5	157.7	273.2	60.0	2.070	0.089	0.089	8.155	0.993	1.28	1129.5	1.014	0.609	5
272.6	157.7	273.2	60.0	2.069	0.089	0.089	8.156	0.993	1.28	1130.3	1.014	0.610	8

Fig. A.3. (Contd.)

FLOATING WARM WATER JETS -- PRYCH CALCULATIONS FOR POINT BEACH UNIT NO. 1 MAY 18, 1972

U= 0.5470 2R= 10.57 O= 24.7 DR= 1.12 1000*K= 0.00280 CD= 0.20 FH= 1.00 HS= 1.0/1.0
V= 0.0 H= 4.24 F= 2.54 TH= 60.0 E= 0.1000 CF= 0.50 EV= 1.00 VS= 1.0/1.0

S	X	Y	TH	T	U	U+VCT	Q	HT	H	B	F	RI	IHLF
272.7	157.8	273.3	60.0	2.069	0.089	0.089	8.158	0.993	1.28	1131.1	1.013	0.610	8
272.7	157.8	273.4	60.0	2.068	0.089	0.089	8.159	0.993	1.28	1131.9	1.013	0.610	8
272.8	157.9	273.4	60.0	2.068	0.089	0.089	8.161	0.993	1.28	1132.6	1.013	0.610	8
273.0	157.9	273.6	60.0	2.067	0.089	0.089	8.164	0.993	1.28	1134.2	1.013	0.611	7
273.1	158.0	273.7	60.0	2.066	0.089	0.089	8.167	0.993	1.28	1135.8	1.013	0.611	7
273.4	158.2	274.0	60.0	2.065	0.089	0.089	8.174	0.993	1.28	1139.0	1.013	0.611	6
273.8	158.3	274.2	60.0	2.063	0.088	0.088	8.180	0.993	1.27	1142.2	1.013	0.611	6
274.1	158.5	274.5	60.0	2.061	0.088	0.088	8.186	0.993	1.27	1145.3	1.013	0.611	6
274.4	158.6	274.8	60.0	2.060	0.088	0.088	8.193	0.993	1.27	1148.6	1.013	0.611	6
274.5	158.7	274.9	60.0	2.059	0.088	0.088	8.196	0.993	1.27	1150.2	1.013	0.611	7
274.7	158.8	275.0	60.0	2.058	0.088	0.088	8.199	0.993	1.27	1151.8	1.013	0.611	7
274.8	158.9	275.2	60.0	2.057	0.088	0.088	8.202	0.993	1.27	1153.4	1.013	0.611	7
275.0	159.0	275.3	60.0	2.056	0.088	0.088	8.206	0.993	1.27	1155.0	1.013	0.611	7
275.3	159.1	275.6	60.0	2.055	0.088	0.088	8.212	0.993	1.27	1158.2	1.013	0.611	6
275.6	159.3	275.9	60.0	2.053	0.088	0.088	8.219	0.993	1.27	1161.5	1.012	0.611	6
275.9	159.4	276.1	60.0	2.052	0.088	0.088	8.225	0.993	1.26	1164.8	1.012	0.612	6
276.3	159.6	276.4	60.0	2.050	0.088	0.088	8.232	0.993	1.26	1168.0	1.012	0.612	6
276.4	159.7	276.5	60.0	2.049	0.088	0.088	8.235	0.993	1.26	1169.7	1.012	0.612	7
276.6	159.7	276.7	60.0	2.048	0.088	0.088	8.238	0.993	1.26	1171.3	1.012	0.612	7
276.7	159.8	276.8	60.0	2.047	0.088	0.088	8.241	0.993	1.26	1173.0	1.012	0.612	7
276.9	159.9	276.9	60.0	2.047	0.088	0.088	8.244	0.993	1.26	1174.7	1.012	0.612	7
277.2	160.0	277.2	60.0	2.045	0.088	0.088	8.251	0.993	1.26	1178.0	1.012	0.612	6
277.5	160.2	277.5	60.0	2.043	0.087	0.087	8.257	0.992	1.26	1181.3	1.012	0.612	6
277.7	160.3	277.6	60.0	2.042	0.087	0.087	8.261	0.992	1.26	1183.0	1.012	0.612	7
277.8	160.4	277.8	60.0	2.042	0.087	0.087	8.264	0.992	1.26	1184.7	1.012	0.612	7
278.0	160.4	277.9	60.0	2.041	0.087	0.087	8.267	0.992	1.26	1186.3	1.012	0.612	7
278.1	160.5	278.0	60.0	2.040	0.087	0.087	8.271	0.992	1.26	1188.0	1.012	0.612	7
278.4	160.7	278.3	60.0	2.038	0.087	0.087	8.277	0.992	1.25	1191.4	1.012	0.612	6
278.8	160.8	278.6	60.0	2.037	0.087	0.087	8.284	0.992	1.25	1194.8	1.012	0.612	6
278.9	160.9	278.7	60.0	2.036	0.087	0.087	8.287	0.992	1.25	1196.5	1.012	0.612	7
279.1	161.0	278.8	60.0	2.035	0.087	0.087	8.290	0.992	1.25	1198.2	1.012	0.612	7
279.2	161.1	279.0	60.0	2.034	0.087	0.087	8.294	0.992	1.25	1199.9	1.012	0.612	7
279.4	161.1	279.1	60.0	2.033	0.087	0.087	8.297	0.992	1.25	1201.6	1.012	0.612	7
279.7	161.3	279.4	60.0	2.032	0.087	0.087	8.303	0.992	1.25	1205.0	1.011	0.613	6
280.0	161.5	279.6	60.0	2.030	0.087	0.087	8.310	0.992	1.25	1208.5	1.011	0.613	6
280.2	161.5	279.8	60.0	2.029	0.087	0.087	8.313	0.992	1.25	1210.2	1.011	0.613	7
280.3	161.6	279.9	60.0	2.028	0.087	0.087	8.317	0.992	1.25	1211.9	1.011	0.613	7
280.5	161.7	280.1	60.0	2.028	0.087	0.087	8.320	0.992	1.25	1213.7	1.011	0.613	7
280.6	161.8	280.2	60.0	2.027	0.087	0.087	8.323	0.992	1.25	1215.4	1.011	0.613	7
280.8	161.8	280.3	60.0	2.026	0.087	0.087	8.327	0.992	1.24	1217.2	1.011	0.613	7
280.9	161.9	280.5	60.0	2.025	0.086	0.086	8.330	0.992	1.24	1218.9	1.011	0.613	7
281.1	162.0	280.6	60.0	2.024	0.086	0.086	8.333	0.992	1.24	1220.7	1.011	0.613	7
281.3	162.1	280.7	60.0	2.023	0.086	0.086	8.337	0.992	1.24	1222.4	1.011	0.613	7
281.6	162.2	281.0	60.0	2.022	0.086	0.086	8.343	0.992	1.24	1225.9	1.011	0.613	6
281.9	162.4	281.3	60.0	2.020	0.086	0.086	8.350	0.992	1.24	1229.5	1.011	0.614	6
282.0	162.4	281.3	60.0	2.020	0.086	0.086	8.352	0.992	1.24	1230.3	1.011	0.613	8
282.0	162.5	281.4	60.0	2.019	0.086	0.086	8.353	0.992	1.24	1231.2	1.011	0.613	8
282.1	162.5	281.5	60.0	2.019	0.086	0.086	8.355	0.992	1.24	1232.1	1.011	0.613	8
282.2	162.5	281.5	60.0	2.018	0.086	0.086	8.357	0.992	1.24	1233.0	1.011	0.613	8
282.3	162.6	281.7	60.0	2.018	0.086	0.086	8.360	0.992	1.24	1234.8	1.011	0.613	7

Fig. A.3 (Contd.)

FLOATING WARM WATER JETS -- PRYCH CALCULATIONS FOR POINT BEACH UNIT NO. 1 MAY 18, 1972

U= 0.5470 2R= 10.67 Q= 24.7 DR= 1.12 1000*K= 0.00280 CD= 0.20 FH= 1.00 HS= 1.0/1.0
 V= 0.0 H= 4.24 F= 2.54 TH= 60.0 E= 0.1000 CF= 0.50 EV= 1.00 VS= 1.0/1.0

S	X	Y	TH	T	U	U+VCT	Q	HT	H	B	F	RI	IMLF
282.5	162.7	281.8	60.0	2.017	0.086	0.086	8.364	0.992	1.24	1236.6	1.011	0.613	7

AREAS OF EXCESS TEMPERATURE FOR
 PRYCH CALCULATIONS FOR POINT BEACH UNIT NO. 1 MAY 18, 1972

REL. TEMP.	AREA
0.43	0.3440 06
0.85	0.2680 06
1.28	0.2110 06
1.70	0.1580 06
2.13	0.8840 05
2.55	0.3780 05
2.98	0.1760 05
3.40	0.9220 04
3.83	0.5420 04
4.25	0.3490 04
4.68	0.2510 04
5.10	0.1920 04
5.53	0.1550 04
5.95	0.1290 04
6.38	0.1070 04
6.80	0.9420 03
7.23	0.7740 03
7.65	0.6680 03
8.08	0.5350 03
8.50	0.2290 03

Fig. A.3 (Contd.)

CD	form drag coefficient
CF	interfacial shear-stress coefficient
EH	multiplicative adjustment factor for ambient horizontal-diffusion coefficient
EV	multiplicative adjustment factor for ambient vertical-diffusion coefficient
HS	horizontal model scale
VS	vertical model scale

Integration results along the jet centerline and areas within isotherms of specific excess temperatures are then tabulated. Some of the columns of interest are labeled as follows:

S	coordinate along jet centerline ($S = 0$ at end of region of flow establishment) (m)
X	x coordinate of centerline (parallel to ambient current) (m)
Y	y coordinate of centerline (perpendicular to ambient current, offshore direction) (m)
TH	angle between centerline and x direction (degrees)
T	centerline excess temperature ($^{\circ}\text{C}$)
U	centerline excess velocity (m/sec)
U+VCT	centerline velocity in direction of centerline (m/sec)
H	characteristic thickness of jet (m)
B	characteristic width of jet (m)
F	gross densimetric Froude number
RI	Richardson number on jet centerline.

The results of the computer calculation, along with the results of the Shirazi-Davis model for ease in comparing these two related models, are plotted in the next section.

3. Shirazi-Davis Model

The model parameters used in preparing the Shirazi-Davis-model predictions are given in Table A.5. In Fig. A.4 the actual computer output is reproduced, with the input parameters listed at the top of each page of output and labeled as follows:

FRO	initial densimetric Froude number
2BO/HO	aspect ratio of outfall

V/UO	ratio of ambient to outfall velocity
ANGLE	angle of discharge with respect to shoreline or direction of ambient current (degrees)
SURFACE H	dimensionless surface-heat-loss parameter
E	entrainment coefficient
CD	drag coefficient
CF	interfacial shear coefficient
RE	Reynolds number of discharge
EV	ratio of vertical to horizontal turbulent-exchange coefficient
EH	horizontal turbulent-exchange coefficient

Integration results along the jet centerline and areas within isotherms of specific temperature ratios are then tabulated. Some of the columns of interest are labeled as follows:

S	distance along centerline, in units of the initial depth at the point of discharge (h_0)
X	x coordinate of centerline (parallel to shoreline or ambient current), in units of h_0
TH	angle between jet centerline and x direction (degrees)
T	centerline excess-temperature ratio
U	centerline velocity ratio
H	characteristic depth of jet, in units of h_0
B	characteristic width of jet, in units of h_0
F	gross densimetric Froude number
RI	Richardson number on jet centerline

The results of the computer calculation, along with the results from the similar calculation based on the Prych model, are plotted in Fig. A.5.

TABLE A.5. Model Parameters Used in Preparing Shirazi-Davis-model Predictions

Entrainment coefficient, E_0	0.05
Drag coefficient, C_D	1.0
Interfacial shear coefficient, C_F	0.0

FRQ = 2.6 ZBC/HO = 2.5 V/UD = 0.0 ANGLE = 60.0 SURFACE H = 0.512E-05
 E = 0.0500 CD = 1.0000 CF = C.C RE = C.128E 07 EV = 0.200E 00 EH = 0.200E-01

S	X	Y	TH	T	U	TIME	Q	HT	H	B	F	RI	IHLF
7.2	3.6	6.2	60.0	1.000	0.970	0.731E 01	2.000	1.000	0.60	5.6	2.145	0.136	0
7.4	3.7	6.4	60.0	0.975	0.943	0.757E 01	2.042	1.000	0.62	5.7	2.075	0.145	5
7.7	3.8	6.7	60.0	0.960	0.919	0.783E 01	2.083	1.000	0.64	5.8	2.013	0.155	5
7.9	4.0	6.9	60.0	0.943	0.897	0.811E 01	2.121	1.000	0.65	5.9	1.957	0.164	5
8.2	4.1	7.1	60.0	0.927	0.876	0.839E 01	2.158	1.000	0.67	6.0	1.906	0.172	5
8.4	4.2	7.3	60.0	0.912	0.857	0.868E 01	2.193	1.000	0.68	6.1	1.860	0.181	5
8.7	4.3	7.5	60.0	0.898	0.839	0.897E 01	2.226	1.000	0.69	6.2	1.817	0.190	5
9.2	4.6	8.0	60.0	0.873	0.806	0.958E 01	2.290	0.999	0.72	6.4	1.741	0.207	4
9.7	4.8	8.4	60.0	0.851	0.777	0.102E 02	2.349	0.999	0.74	6.6	1.678	0.223	4
10.2	5.1	8.8	60.0	0.831	0.752	0.109E 02	2.404	0.999	0.76	6.9	1.623	0.238	4
10.7	5.3	9.3	60.0	0.814	0.729	0.115E 02	2.455	0.999	0.77	7.1	1.575	0.253	4
11.2	5.6	9.7	60.0	0.798	0.708	0.122E 02	2.504	0.999	0.78	7.3	1.533	0.267	4
11.7	5.8	10.1	60.0	0.784	0.689	0.129E 02	2.550	0.999	0.79	7.6	1.495	0.280	4
12.7	6.3	11.0	60.0	0.756	0.653	0.144E 02	2.641	0.998	0.82	8.0	1.422	0.310	3
13.7	6.8	11.8	60.0	0.734	0.625	0.160E 02	2.718	0.998	0.83	8.5	1.372	0.333	3
14.7	7.3	12.7	60.0	0.715	0.601	0.176E 02	2.790	0.998	0.84	9.0	1.330	0.354	3
15.7	7.8	13.6	60.0	0.699	0.580	0.193E 02	2.857	0.998	0.84	9.5	1.294	0.374	3
16.7	8.3	14.4	60.0	0.684	0.561	0.211E 02	2.919	0.998	0.84	10.0	1.264	0.392	3
17.7	8.8	15.3	60.0	0.670	0.544	0.229E 02	2.979	0.998	0.85	10.5	1.235	0.411	3
18.7	9.3	16.2	60.0	0.658	0.528	0.247E 02	3.035	0.998	0.85	11.0	1.210	0.428	3
19.7	9.8	17.0	60.0	0.646	0.514	0.266E 02	3.088	0.998	0.85	11.5	1.188	0.444	3
20.7	10.3	17.9	60.0	0.636	0.501	0.286E 02	3.138	0.998	0.85	12.0	1.168	0.460	3
21.7	10.8	18.8	60.0	0.626	0.490	0.306E 02	3.186	0.998	0.85	12.4	1.150	0.474	3
22.7	11.3	19.6	60.0	0.617	0.479	0.327E 02	3.232	0.998	0.85	12.9	1.134	0.488	3
23.7	11.8	20.5	60.0	0.609	0.465	0.348E 02	3.276	0.998	0.84	13.4	1.119	0.501	3
24.7	12.3	21.4	60.0	0.601	0.460	0.369E 02	3.318	0.998	0.84	13.9	1.106	0.513	3
25.7	12.8	22.2	60.0	0.594	0.452	0.391E 02	3.359	0.998	0.84	14.4	1.093	0.524	3
27.6	13.8	23.9	60.0	0.581	0.436	0.436E 02	3.435	0.997	0.84	15.3	1.069	0.548	2
29.6	14.8	25.7	60.0	0.569	0.422	0.483E 02	3.507	0.997	0.83	16.2	1.049	0.565	2
31.6	15.8	27.4	60.0	0.558	0.410	0.531E 02	3.573	0.997	0.82	17.1	1.035	0.585	2
33.6	16.8	29.1	60.0	0.549	0.402	0.580E 02	3.636	0.997	0.81	18.0	1.029	0.592	2
35.6	17.8	30.9	60.0	0.540	0.378	0.631E 02	3.692	0.997	0.84	18.9	0.964	0.675	2
35.7	17.8	30.9	60.0	0.540	0.378	0.632E 02	3.694	0.997	0.84	18.9	0.962	0.677	8
35.7	17.8	30.9	60.0	0.540	0.377	0.633E 02	3.695	0.997	0.84	19.0	0.959	0.681	8
35.7	17.9	30.9	60.0	0.540	0.377	0.633E 02	3.696	0.997	0.84	19.0	0.960	0.680	8
35.8	17.9	31.0	60.0	0.540	0.377	0.634E 02	3.696	0.997	0.84	19.0	0.958	0.682	8
35.8	17.9	31.0	60.0	0.539	0.376	0.635E 02	3.697	0.997	0.84	19.0	0.957	0.685	8
35.8	17.9	31.0	60.0	0.539	0.376	0.636E 02	3.698	0.997	0.84	19.0	0.955	0.687	8
35.8	17.9	31.0	60.0	0.539	0.375	0.637E 02	3.698	0.997	0.84	19.0	0.953	0.688	8
35.9	17.9	31.1	60.0	0.539	0.375	0.638E 02	3.699	0.997	0.84	19.0	0.952	0.692	8
35.9	18.0	31.1	60.0	0.539	0.374	0.639E 02	3.700	0.997	0.84	19.1	0.945	0.696	7
36.0	18.0	31.2	60.0	0.539	0.373	0.641E 02	3.702	0.997	0.84	19.1	0.946	0.700	7
36.1	18.0	31.2	60.0	0.539	0.372	0.643E 02	3.703	0.997	0.84	19.1	0.943	0.705	7
36.1	18.1	31.3	60.0	0.538	0.371	0.644E 02	3.704	0.997	0.85	19.2	0.940	0.709	7
36.3	18.1	31.4	60.0	0.538	0.369	0.648E 02	3.707	0.997	0.85	19.2	0.935	0.717	6

Fig. A.4. Computer Output of Shirazi-Davis Code

FLOATING WARM WATER JETS -- POINT BEACH UNIT NO. 1 18 MAY 72

PAGE 2

FRD = 2.6 2BC/HO = 2.5 V/UC = 0.0 ANGLE = 60.0 SURFACE H = 0.512E-05

E = 0.0500 CD = 1.0000 CF = 0.0 RE = 0.12EE 07 EV = 0.200E 00 EH = 0.200E-01

S	X	Y	TH	T	U	TIME	Q	HT	H	B	F	RI	IRLF
36.4	18.2	31.5	60.0	0.536	0.367	0.651E 02	3.709	0.997	0.85	19.3	0.930	0.725	6
36.5	18.3	31.6	60.0	0.537	0.366	0.654E 02	3.711	0.997	0.85	19.3	0.925	0.733	6
36.6	18.3	31.7	60.0	0.537	0.364	0.658E 02	3.713	0.997	0.85	19.4	0.920	0.740	6
36.9	18.4	31.9	60.0	0.536	0.361	0.665E 02	3.718	0.997	0.86	19.5	0.911	0.755	5
37.1	18.6	32.2	60.0	0.536	0.358	0.672E 02	3.722	0.997	0.86	19.6	0.903	0.765	5
37.4	18.7	32.4	60.0	0.535	0.355	0.679E 02	3.726	0.997	0.86	19.7	0.894	0.783	5
37.6	18.8	32.6	60.0	0.535	0.353	0.686E 02	3.729	0.997	0.86	19.8	0.887	0.797	5
38.1	19.1	33.0	60.0	0.534	0.347	0.700E 02	3.736	0.997	0.87	20.1	0.873	0.823	4
38.6	19.3	33.4	60.0	0.533	0.343	0.714E 02	3.743	0.997	0.87	20.3	0.861	0.846	4
39.1	19.6	33.9	60.0	0.532	0.339	0.729E 02	3.750	0.997	0.88	20.5	0.850	0.867	4
39.6	19.8	34.3	60.0	0.531	0.335	0.744E 02	3.757	0.997	0.88	20.7	0.841	0.887	4
40.1	20.1	34.7	60.0	0.530	0.332	0.759E 02	3.764	0.997	0.88	21.0	0.832	0.905	4
40.6	20.3	35.2	60.0	0.529	0.328	0.774E 02	3.771	0.997	0.88	21.2	0.824	0.923	4
41.6	20.8	36.0	60.0	0.527	0.322	0.805E 02	3.785	0.997	0.88	21.6	0.809	0.958	3
42.6	21.3	36.9	60.0	0.525	0.317	0.836E 02	3.798	0.997	0.88	22.1	0.796	0.988	3
43.6	21.8	37.8	60.0	0.523	0.312	0.868E 02	3.811	0.997	0.88	22.5	0.785	1.016	3
44.6	22.3	38.6	60.0	0.521	0.307	0.900E 02	3.824	0.997	0.88	22.9	0.775	1.042	3
45.6	22.8	39.5	60.0	0.520	0.303	0.932E 02	3.836	0.997	0.88	23.4	0.766	1.067	3
46.6	23.3	40.4	60.0	0.518	0.299	0.966E 02	3.849	0.997	0.88	23.8	0.758	1.091	3
48.6	24.3	42.1	60.0	0.515	0.291	0.103E 03	3.873	0.997	0.88	24.6	0.743	1.136	2
50.6	25.3	43.8	60.0	0.512	0.285	0.110E 03	3.896	0.996	0.87	25.5	0.730	1.176	2
52.6	26.3	45.5	60.0	0.509	0.279	0.117E 03	3.919	0.996	0.87	26.3	0.719	1.213	2
54.6	27.3	47.3	60.0	0.506	0.274	0.125E 03	3.941	0.996	0.86	27.1	0.709	1.248	2
56.6	28.3	49.0	60.0	0.503	0.269	0.132E 03	3.963	0.996	0.86	27.9	0.700	1.280	2
58.6	29.3	50.7	60.0	0.500	0.264	0.139E 03	3.984	0.996	0.85	28.7	0.692	1.310	2
62.6	31.3	54.2	60.0	0.495	0.256	0.155E 03	4.024	0.996	0.84	30.2	0.677	1.367	1
66.6	33.3	57.6	60.0	0.490	0.249	0.171E 03	4.063	0.996	0.83	31.8	0.665	1.416	1
70.5	35.3	61.1	60.0	0.486	0.242	0.187E 03	4.100	0.995	0.83	33.3	0.655	1.462	1
74.5	37.3	64.5	60.0	0.481	0.237	0.203E 03	4.136	0.995	0.82	34.7	0.646	1.503	1
78.5	39.3	68.0	60.0	0.477	0.231	0.221E 03	4.170	0.995	0.81	36.2	0.638	1.541	1
82.5	41.3	71.5	60.0	0.473	0.227	0.238E 03	4.203	0.995	0.80	37.6	0.631	1.576	1
90.5	45.3	78.4	60.0	0.466	0.219	0.274E 03	4.266	0.994	0.78	40.4	0.618	1.638	0
98.5	49.2	85.3	60.0	0.459	0.212	0.311E 03	4.325	0.994	0.77	43.0	0.608	1.694	0
106.5	53.2	92.2	60.0	0.453	0.205	0.349E 03	4.381	0.993	0.76	45.7	0.600	1.743	0
114.4	57.2	99.1	60.0	0.448	0.200	0.389E 03	4.435	0.992	0.75	48.2	0.592	1.787	0
122.4	61.2	106.0	60.0	0.442	0.195	0.429E 03	4.486	0.992	0.73	50.7	0.586	1.827	0
130.4	65.2	112.9	60.0	0.437	0.191	0.470E 03	4.534	0.991	0.72	53.2	0.580	1.863	0
138.4	69.2	119.8	60.0	0.432	0.187	0.513E 03	4.581	0.990	0.72	55.6	0.575	1.897	0
146.4	73.2	126.8	60.0	0.428	0.183	0.556E 03	4.626	0.990	0.71	58.0	0.570	1.927	0
154.3	77.2	133.7	60.0	0.424	0.180	0.600E 03	4.670	0.989	0.70	60.3	0.566	1.956	0
162.3	81.2	140.6	60.0	0.419	0.177	0.644E 03	4.712	0.988	0.69	62.6	0.562	1.982	0
170.3	85.2	147.5	60.0	0.416	0.174	0.690E 03	4.753	0.988	0.68	64.9	0.559	2.007	0
178.3	89.1	154.4	60.0	0.412	0.171	0.736E 03	4.793	0.987	0.68	67.1	0.556	2.030	0
186.3	93.1	161.3	60.0	0.408	0.169	0.783E 03	4.831	0.986	0.67	69.3	0.553	2.052	0
194.3	97.1	168.2	60.0	0.405	0.167	0.831E 03	4.869	0.985	0.66	71.5	0.550	2.072	0

Fig. A.4 (Contd.)

FRO = 2.6 280/HO = 2.5 V/UO = C.O ANGLE = 60.0 SURFACE T = 0.512E-05

E = 0.0500 CD = 1.0000 CF = 0.0 RE = 0.128E 07 EV = 0.200E 00 EH = 0.200E-01

S	X	Y	TH	T	U	TIME	Q	HT	H	B	F	RI	IHLF
202.2	101.1	175.1	60.0	0.401	0.164	0.879E 03	4.905	0.984	0.66	73.7	0.547	2.092	0
210.2	105.1	182.0	60.0	0.398	0.162	0.928E 03	4.941	0.983	0.65	75.8	0.545	2.110	0
218.2	109.1	189.0	60.0	0.395	0.160	0.977E 03	4.976	0.983	0.65	77.9	0.543	2.127	0
226.2	113.1	195.9	60.0	0.392	0.158	0.103E 04	5.010	0.982	0.64	80.0	0.541	2.144	0
234.2	117.1	202.8	60.0	0.389	0.157	0.108E 04	5.043	0.981	0.64	82.1	0.539	2.160	0
242.1	121.1	209.7	60.0	0.386	0.155	0.113E 04	5.076	0.980	0.63	84.1	0.537	2.175	0
250.1	125.1	216.6	60.0	0.383	0.153	0.118E 04	5.108	0.979	0.63	86.1	0.535	2.189	0
258.1	129.1	223.5	60.0	0.381	0.152	0.123E 04	5.139	0.978	0.62	88.2	0.533	2.203	0
266.1	133.1	230.4	60.0	0.378	0.150	0.129E 04	5.170	0.977	0.62	90.2	0.532	2.216	0
274.1	137.0	237.3	60.0	0.375	0.149	0.134E 04	5.201	0.976	0.61	92.2	0.530	2.229	0
282.0	141.0	244.3	60.0	0.373	0.148	0.139E 04	5.230	0.975	0.61	94.1	0.529	2.241	0
290.0	145.0	251.2	60.0	0.370	0.146	0.145E 04	5.260	0.974	0.61	96.1	0.527	2.253	0
298.0	149.0	258.1	60.0	0.368	0.145	0.150E 04	5.288	0.973	0.60	98.0	0.526	2.264	0
306.0	153.0	265.0	60.0	0.366	0.144	0.156E 04	5.317	0.972	0.60	100.0	0.525	2.276	0
314.0	157.0	271.9	60.0	0.363	0.143	0.161E 04	5.345	0.971	0.60	101.9	0.524	2.286	0
322.0	161.0	278.8	60.0	0.361	0.141	0.167E 04	5.372	0.970	0.59	103.8	0.522	2.297	0
329.9	165.0	285.7	60.0	0.359	0.140	0.173E 04	5.399	0.969	0.59	105.7	0.521	2.307	0
337.9	169.0	292.6	60.0	0.357	0.139	0.178E 04	5.426	0.968	0.59	107.6	0.520	2.316	0
345.9	173.0	299.5	60.0	0.355	0.138	0.184E 04	5.452	0.967	0.58	109.4	0.519	2.326	0
353.9	177.0	306.5	60.0	0.353	0.137	0.190E 04	5.478	0.966	0.58	111.3	0.518	2.335	0
361.9	180.9	313.4	60.0	0.350	0.136	0.196E 04	5.504	0.964	0.58	113.1	0.517	2.344	0
369.8	184.9	320.3	60.0	0.348	0.135	0.202E 04	5.529	0.963	0.58	115.0	0.516	2.353	0
377.8	188.9	327.2	60.0	0.346	0.134	0.208E 04	5.554	0.962	0.57	116.8	0.515	2.362	0
385.8	192.9	334.1	60.0	0.345	0.133	0.214E 04	5.578	0.961	0.57	118.6	0.514	2.370	0
393.8	196.9	341.0	60.0	0.343	0.133	0.220E 04	5.603	0.960	0.57	120.5	0.513	2.378	0
401.8	200.9	347.9	60.0	0.341	0.132	0.226E 04	5.627	0.959	0.57	122.3	0.512	2.386	0
409.7	204.9	354.8	60.0	0.339	0.131	0.232E 04	5.651	0.957	0.56	124.1	0.512	2.394	0
417.7	208.9	361.7	60.0	0.337	0.130	0.238E 04	5.674	0.956	0.56	125.8	0.511	2.402	0
425.7	212.9	368.7	60.0	0.335	0.129	0.244E 04	5.697	0.955	0.56	127.6	0.510	2.409	0
433.7	216.9	375.6	60.0	0.333	0.128	0.250E 04	5.720	0.954	0.56	129.4	0.509	2.417	0
441.7	220.9	382.5	60.0	0.332	0.128	0.256E 04	5.743	0.953	0.56	131.2	0.508	2.424	0
449.6	224.8	389.4	60.0	0.330	0.127	0.263E 04	5.766	0.951	0.55	132.9	0.508	2.431	0
457.6	228.8	396.3	60.0	0.328	0.126	0.269E 04	5.788	0.950	0.55	134.7	0.507	2.438	0
465.6	232.8	403.2	60.0	0.327	0.125	0.275E 04	5.810	0.949	0.55	136.4	0.506	2.445	0
473.6	236.8	410.1	60.0	0.325	0.125	0.282E 04	5.832	0.948	0.55	138.2	0.506	2.452	0
481.6	240.8	417.0	60.0	0.323	0.124	0.288E 04	5.853	0.946	0.55	139.9	0.505	2.459	0
489.6	244.8	423.9	60.0	0.322	0.123	0.294E 04	5.875	0.945	0.55	141.6	0.504	2.465	0
497.5	248.8	430.9	60.0	0.320	0.123	0.301E 04	5.896	0.944	0.54	143.3	0.503	2.472	0
505.5	252.8	437.8	60.0	0.319	0.122	0.307E 04	5.917	0.942	0.54	145.1	0.503	2.478	0
513.5	256.8	444.7	60.0	0.317	0.121	0.314E 04	5.938	0.941	0.54	146.8	0.502	2.485	0
521.5	260.8	451.6	60.0	0.315	0.121	0.321E 04	5.959	0.940	0.54	148.5	0.502	2.491	0
529.5	264.8	458.5	60.0	0.314	0.120	0.327E 04	5.979	0.938	0.54	150.2	0.501	2.497	0
537.4	268.7	465.4	60.0	0.312	0.120	0.334E 04	5.999	0.937	0.54	151.9	0.500	2.503	0
545.4	272.7	472.3	60.0	0.311	0.119	0.341E 04	6.020	0.936	0.53	153.5	0.500	2.509	0
553.4	276.7	479.2	60.0	0.309	0.118	0.347E 04	6.040	0.934	0.53	155.2	0.499	2.516	0

Fig. A.4 (Contd.)

FLOATING WARM WATER JETS -- POINT BEACH UNIT NO. 1 18 MAY 72

PAGE 4

FRQ = 2.6 2BC/HQ = 2.5 V/UO = 0.0 ANGLE = 60.0 SURFACE H = 0.512E-05

E = 0.0500 CD = 1.0000 CF = C.C RE = 0.128E 07 EV = 0.200E 00 EH = 0.200E-01

S	X	Y	TH	T	U	TIME	Q	HT	H	B	F	RI	IHLF
561.4	280.7	486.1	60.0	0.308	0.118	0.354E 04	6.059	0.933	0.53	156.9	0.499	2.521	C
569.4	284.7	493.1	60.0	0.306	0.117	0.361E 04	6.079	0.932	0.53	158.6	0.498	2.527	0
577.3	288.7	500.0	60.0	0.305	0.117	0.368E 04	6.098	0.930	0.53	160.2	0.497	2.533	0
585.3	292.7	506.9	60.0	0.304	0.116	0.375E 04	6.118	0.929	0.53	161.9	0.497	2.539	0
593.3	296.7	513.8	60.0	0.302	0.116	0.381E 04	6.137	0.927	0.53	163.5	0.496	2.545	0
601.3	300.7	520.7	60.0	0.301	0.115	0.388E 04	6.156	0.926	0.52	165.2	0.496	2.551	0
609.3	304.7	527.6	60.0	0.299	0.115	0.395E 04	6.175	0.925	0.52	166.8	0.495	2.556	0
617.2	308.7	534.5	60.0	0.298	0.114	0.402E 04	6.194	0.923	0.52	168.5	0.495	2.562	0
625.2	312.6	541.4	60.0	0.297	0.114	0.409E 04	6.212	0.922	0.52	170.1	0.494	2.567	0
632.2	316.6	548.3	60.0	0.295	0.113	0.416E 04	6.231	0.920	0.52	171.8	0.494	2.573	0
641.2	320.6	555.3	60.0	0.294	0.113	0.423E 04	6.249	0.919	0.52	173.4	0.493	2.579	C
649.2	324.6	562.2	60.0	0.292	0.112	0.430E 04	6.267	0.917	0.52	175.0	0.492	2.584	0
657.2	328.6	569.1	60.0	0.291	0.112	0.438E 04	6.285	0.916	0.52	176.6	0.492	2.590	0
665.1	332.6	576.0	60.0	0.290	0.111	0.445E 04	6.303	0.914	0.52	178.2	0.491	2.595	0
673.1	336.6	582.9	60.0	0.289	0.111	0.452E 04	6.321	0.913	0.52	179.9	0.491	2.600	0
681.1	340.6	589.8	60.0	0.288	0.110	0.459E 04	6.339	0.911	0.51	181.5	0.490	2.606	C
689.1	344.6	596.7	60.0	0.286	0.110	0.466E 04	6.357	0.910	0.51	183.1	0.490	2.611	0
697.1	348.6	603.6	60.0	0.285	0.109	0.474E 04	6.374	0.908	0.51	184.7	0.489	2.617	0
705.0	352.6	610.5	60.0	0.284	0.109	0.481E 04	6.392	0.907	0.51	186.3	0.489	2.622	0
713.0	356.6	617.5	60.0	0.283	0.108	0.488E 04	6.409	0.905	0.51	187.9	0.488	2.627	0
721.0	360.5	624.4	60.0	0.281	0.108	0.496E 04	6.426	0.904	0.51	189.5	0.488	2.632	0
729.0	364.5	631.3	60.0	0.280	0.108	0.503E 04	6.443	0.902	0.51	191.0	0.487	2.638	0
737.0	368.5	638.2	60.0	0.279	0.107	0.511E 04	6.460	0.901	0.51	192.6	0.487	2.643	C
744.9	372.5	645.1	60.0	0.278	0.107	0.518E 04	6.477	0.899	0.51	194.2	0.486	2.648	0
752.9	376.5	652.0	60.0	0.277	0.106	0.526E 04	6.494	0.898	0.51	195.8	0.486	2.653	0
760.9	380.5	658.9	60.0	0.275	0.106	0.533E 04	6.510	0.896	0.51	197.4	0.485	2.659	C
768.9	384.5	665.8	60.0	0.274	0.105	0.541E 04	6.527	0.895	0.50	198.9	0.485	2.664	0
776.9	388.5	672.7	60.0	0.273	0.105	0.548E 04	6.543	0.893	0.50	200.5	0.485	2.669	0
784.8	392.5	679.6	60.0	0.272	0.105	0.556E 04	6.560	0.892	0.50	202.1	0.484	2.674	C
792.8	396.5	686.6	60.0	0.271	0.104	0.563E 04	6.576	0.890	0.50	203.6	0.484	2.679	0
800.8	400.5	693.5	60.0	0.270	0.104	0.571E 04	6.592	0.889	0.50	205.2	0.483	2.685	C

Fig. A.4 (Contd.)

AREAS OF EXCESS TEMPERATURE FOR
POINT BEACH UNIT NO. 1 18 MAY 72

REL. TEMP.	AREA	
0.05	0.256E 06	PARTIAL AREA - CALCULATION VALID TO T =0.270
0.10	0.203E 06	PARTIAL AREA - CALCULATION VALID TO T =0.270
0.15	0.164E 06	PARTIAL AREA - CALCULATION VALID TO T =0.270
0.20	0.129E 06	PARTIAL AREA - CALCULATION VALID TO T =0.270
0.25	0.915E 05	PARTIAL AREA - CALCULATION VALID TO T =0.270
0.30	0.413E 05	
0.35	0.151E 05	
0.40	0.509E 04	
0.45	0.163E 04	
0.50	0.543E 03	
0.55	0.248E 03	
0.60	0.154E 03	
0.65	0.103E 03	
0.70	0.731E 02	
0.75	0.552E 02	
0.80	0.422E 02	
0.85	0.330E 02	
0.90	0.257E 02	
0.95	0.194E 02	
1.00	0.917E 01	

Fig. A.4 (Contd.)

Fig. A.5. Predictions of Prych and Shirazi-Davis Models

The same quantities that are defined and discussed in connection with the Motz-Benedict model (Sec. 1 of this appendix) are plotted.

The areas within isotherms of specific excess temperatures are tabulated in Table A.6 for both the Prych and Shirazi-Davis models.

TABLE A.6. Isotherm Areas Predicted by
Prych and Shirazi-Davis Models

Excess Temperature, C°	Prych Area, m ²	Shirazi-Davis Area, m ²
8	569	360
7	858	665
6	1,270	1,246
5	2,070	3,098
4	4,740	19,970
3	1.7×10^4	2.5×10^5
2	1.1×10^5	-

4. Pritchard Model No. 1

The model parameters used in preparing the Pritchard (No. 1) model predictions are given in Table A.7. The actual computer output is reproduced in Fig. A.6, most of the initial output being self-explanatory, followed by three pages of intermediate output. Final temperature distribution characteristics are then tabulated. The output is self-explanatory: excess temperatures are in C°, areas in square meters, and distances in meters. Centerline distances to specific excess temperatures and half-widths of the temperature distribution are tabulated next, and, finally, the coordinates of specific excess temperature isotherms are listed. The y coordinate is in the offshore direction along the centerline of the plume, and the x coordinate is in a lateral direction, perpendicular to the plume centerline. These isotherms, and the half-widths discussed above, are based on Pritchard's schematization of the isotherms as "bullet-shaped."

TABLE A.7. Model Parameters Used in Preparing
Predictions Used in Pritchard Model No. 1

Inverse spreading rate, n	6.0
Critical mixing depth, D	4.2 m
Recirculation temperature excess, ϕ	0

```

AMBIENT TEMP = 9.200 DEGREES C
INITIAL EXCESS TEMP = 8.500 DEGREES C
DISCHARGE WIDTH = 10.670 M.
DISCHARGE DEPTH = 4.200 M.
DISCHARGE VELOC = 54.700 CM./SEC
RATE OF HEAT REJECTION = 0.180000D 06 KCAL/SEC
WIND SPEED = 1.900 MPS.
ASSUMED TEMPERATURE OF RECIRCULATING WATER = 0.0 DEGREES C
AV= 6.000 AM= 0.200 AL= 0.0
D= 4.200 AC= 0.215 ALPHA= 1.000
COMPUTING OPTION = 0 MAREA= 0

```

Fig. A.6

Computer Output of Code
for Pritchard Model No. 1

EXCESS TEMP DEG. C	INIT DIST M.	CORR TEMP DEG. C	CORR DIST M.	AREA SQ. M.	DELT AREA SQ. M.
8.50	64.02	8.50	64.02	881.19	881.19
8.40	65.55	8.40	65.55	923.91	42.72
8.20	68.79	8.20	68.79	1017.40	93.49
8.00	72.27	8.00	72.27	1123.01	105.62
7.80	76.03	7.80	76.03	1242.70	119.69
7.60	80.08	7.60	80.08	1378.77	136.07
7.40	84.47	7.40	84.47	1533.98	155.21
7.20	89.23	7.20	89.23	1711.65	177.68
7.00	94.40	7.00	94.40	1915.81	204.16
6.80	100.03	6.80	100.03	2151.34	235.53
6.60	106.19	6.60	106.19	2424.21	272.86
6.40	112.93	6.40	112.93	2741.74	317.53
6.20	120.33	6.20	120.33	3113.00	371.26
6.00	128.48	6.00	128.48	3549.28	436.28
5.80	137.50	5.80	137.50	4064.75	515.47
5.60	147.50	5.60	147.50	4677.28	612.53
5.40	158.62	5.40	158.62	5409.67	732.39
5.20	171.06	5.20	171.06	6291.18	881.51
5.00	185.02	5.00	185.02	7359.79	1068.61
4.80	200.76	4.80	200.76	8665.24	1305.45
4.60	218.59	4.60	218.59	10273.40	1608.16
4.40	238.92	4.40	238.92	12272.55	1999.15
4.20	262.21	4.20	262.21	14782.52	2509.97
4.00	289.09	4.00	289.09	17968.24	3185.72
3.80	320.32	3.80	320.32	22060.28	4092.04
3.60	356.90	3.60	356.90	27386.43	5326.15
3.40	400.12	3.40	400.12	34421.50	7035.07
3.20	451.70	3.20	451.70	43867.77	9446.27
3.00	513.94	3.00	513.94	56788.51	12920.74
2.80	589.98	2.80	589.98	74836.48	18047.97
2.60	684.24	2.60	684.24	100658.88	25822.40
2.40	803.03	2.40	803.03	138643.83	37984.94
2.20	955.67	2.20	955.67	196360.79	57716.97
2.00	1156.36	2.00	1156.36	287491.84	91131.04
1.80	1427.61	1.80	1427.61	438182.96	150691.12
1.60	1700.53	1.60	1700.53	621738.40	183555.45
1.40	1943.46	1.40	1943.46	812066.49	190328.08
1.20	2267.38	1.20	2267.38	1105312.72	293246.23
1.00	2720.85	1.00	2720.85	1591650.32	486337.60

Fig. A.6 (Contd.)

MEAN TEMPS DEG. C	INTERPOLATED VAPOR PRESSURES MM. HG	HEAT TRANSFER COEFF. BTU/SQ. FT./DAY	HEAT TRANSFER COEFFICIENT KGM CAL/SQ. M./DAY
17.70	15.1929	-844.5540	-2290.6755
17.65	15.1456	-839.0452	-2275.7340
17.50	15.0035	-822.5251	-2230.9268
17.30	14.8141	-800.5134	-2171.2246
17.10	14.6247	-778.5188	-2111.5689
16.90	14.4404	-756.7884	-2052.6297
16.70	14.2612	-735.3222	-1994.4072
16.50	14.0820	-713.8731	-1936.2309
16.30	13.9028	-692.4410	-1878.1008
16.10	13.7236	-671.0259	-1820.0168
15.90	13.5494	-649.8700	-1762.6360
15.70	13.3802	-628.9734	-1705.9583
15.50	13.2110	-608.0937	-1649.3266
15.30	13.0418	-587.2309	-1592.7407
15.10	12.8726	-566.3851	-1536.2006
14.90	12.7079	-545.7741	-1480.2977
14.70	12.5477	-525.3980	-1425.0319
14.50	12.3875	-505.0388	-1369.8117
14.30	12.2273	-484.6963	-1314.6370
14.10	12.0671	-464.3706	-1259.5079
13.90	11.9114	-444.2797	-1205.0156
13.70	11.7602	-424.4236	-1151.1600
13.50	11.6090	-404.5842	-1097.3497
13.30	11.4578	-384.7615	-1043.5847
13.10	11.3066	-364.9554	-989.8649
12.90	11.1597	-345.3743	-936.7553
12.70	11.0171	-326.0182	-884.2559
12.50	10.8745	-306.6786	-831.8014
12.30	10.7319	-287.3557	-779.3919
12.10	10.5893	-268.0492	-727.0273
11.90	10.4506	-248.9483	-675.2200
11.70	10.3158	-230.0528	-623.9700
11.50	10.1810	-211.1738	-572.7646
11.30	10.0462	-192.3113	-521.6039
11.10	9.9114	-173.4651	-470.4878
10.90	9.7805	-154.8244	-419.9287
10.70	9.6535	-136.3890	-369.9265
10.50	9.5265	-117.9699	-319.9687
10.30	9.3995	-99.5672	-270.0552

Fig. A.6 (Contd.)

ISOTHERM NUMBER	THETA DEG C	DELT GAMMAS, KGM CAL/DAY	DELT AREAS-CORR SQ. M.	DELT GAMMAS-CORR KGM CAL/DAY	TOTAL AREAS SQ. M.
42	8.50	2018521.5	881.08	2018259.54	881.08
41	8.40	97211.9	42.71	97198.69	923.79
40	8.20	208567.0	93.47	208535.82	1017.26
39	8.00	229322.2	105.60	229284.57	1122.86
38	7.80	252727.9	119.67	252682.33	1242.53
37	7.60	279292.2	136.04	279236.75	1378.57
36	7.40	309548.8	155.17	309481.19	1533.74
35	7.20	344021.7	177.63	343939.00	1711.38
34	7.00	383436.5	204.11	383334.85	1915.48
33	6.80	428668.5	235.46	428543.06	2150.94
32	6.60	480958.8	272.78	480803.21	2423.72
31	6.40	541690.4	317.41	541496.26	2741.13
30	6.20	612331.4	371.11	612087.90	3112.25
29	6.00	694888.3	436.09	694580.93	3548.34
28	5.80	791857.9	515.21	791467.28	4063.55
27	5.60	906731.5	612.20	906231.40	4675.75
26	5.40	1043673.2	731.93	1043027.61	5407.68
25	5.20	1207508.3	880.90	1206667.66	6288.58
24	5.00	1404835.3	1067.77	1403730.53	7356.35
23	4.80	1644222.1	1304.28	1642755.40	8660.63
22	4.60	1937858.2	1606.53	1935888.30	10267.16
21	4.40	2301341.6	1996.82	2298662.08	12263.98
20	4.20	2754310.3	2506.60	2750616.18	14770.58
19	4.00	3324573.4	3180.77	3319404.83	17951.35
18	3.80	4050571.7	4084.62	4043221.43	22035.97
17	3.60	4989299.3	5314.78	4978648.33	27350.75
16	3.40	6220800.6	7017.24	6205038.72	34368.00
15	3.20	7857420.7	9417.58	7833554.21	43785.57
14	3.00	10070318.9	12873.16	10033234.08	56658.73
13	2.80	13121369.1	17966.35	13062027.90	74625.08
12	2.60	17435800.7	25676.83	17337509.98	100301.91
11	2.40	23701463.8	37713.33	23531988.66	138015.29
10	2.20	33058238.5	57182.72	32752238.54	195197.96
9	2.00	47534311.5	90012.38	46950811.36	285210.34
8	1.80	70898331.3	148165.88	69710235.28	433376.22
7	1.60	77080193.4	179589.38	75414726.16	612965.59
6	1.40	70407400.5	185376.43	68575654.57	798342.03
5	1.20	93829615.8	283904.15	90840442.40	1082246.18
4	1.00	131338006.2	466901.09	126089077.36	1549147.27

Fig. A.6 (Contd.)

FINAL TEMPERATURE DISTRIBUTION:

THETA = EXCESS TEMPERATURE
 A-THETA = AREA WITHIN THE THETA ISOTHERM
 L-THETA = LENGTH OF THE THETA ISOTHERM
 B-THETA = MEAN WIDTH OF THE THETA ISOTHERM
 T-THETA = TIME(SEC) TO ISOTHERM

THETA	A-THETA	L-THETA	B-THETA	T-THETA
8.50	881.08	64.02	16.00	77.97
8.40	923.79	65.55	16.39	80.79
8.20	1017.26	68.79	17.20	86.85
8.00	1122.86	72.27	18.07	93.52
7.80	1242.53	76.02	19.01	100.90
7.60	1378.57	80.07	20.02	109.08
7.40	1533.74	84.46	21.12	118.16
7.20	1711.38	89.22	22.30	128.29
7.00	1915.48	94.39	23.60	139.60
6.80	2150.94	100.02	25.01	152.28
6.60	2423.72	106.17	26.54	166.55
6.40	2741.13	112.91	28.23	182.65
6.20	3112.25	120.31	30.08	200.90
6.00	3548.34	128.47	32.12	221.66
5.80	4063.55	137.48	34.37	245.39
5.60	4675.75	147.47	36.87	272.62
5.40	5407.68	158.59	39.65	304.04
5.20	6288.58	171.02	42.76	340.47
5.00	7356.35	184.97	46.24	382.97
4.80	8660.63	200.70	50.18	432.85
4.60	10267.16	218.53	54.63	491.77
4.40	12263.98	238.83	59.71	561.88
4.20	14770.58	262.11	65.53	645.98
4.00	17951.35	288.95	72.24	747.73
3.80	22035.97	320.15	80.04	872.01
3.60	27350.75	356.67	89.17	1025.41
3.40	34368.00	399.81	99.95	1216.99
3.20	43785.57	451.28	112.82	1459.38
3.00	56658.73	513.35	128.34	1770.60
2.80	74625.08	589.15	147.29	2176.88
2.60	100301.91	683.02	170.76	2717.39
2.40	138015.25	801.21	200.30	3452.36
2.20	195197.96	952.84	238.21	4477.41
2.00	285210.34	1151.76	287.94	5950.38
1.80	433376.22	1419.75	354.94	8143.65
1.60	612965.59	1688.49	422.12	10572.00
1.40	798342.03	1926.97	481.74	13034.14
1.20	1082245.18	2243.59	560.90	16804.91
1.00	1549147.27	2684.28	671.07	23006.22

K(0.5) = -67.72 KGM CAL/SQ. M./DAY.
 GAMMA(1) = 621536324.35 KGM CAL/DAY
 DELTA-ARFA(0.5) = 220468420.73 SQ. M.

Fig. A.6 (Contd.)

ISOTHERMS FROM PRITCHARD MODEL

Y (M.)	TC (DEG.)	T/T0	WIDTH(1/2) (M.)
0.0	8.50	1.0000	5.34
72.27	8.00	0.9412	34.31
82.22	7.50	0.8824	39.00
94.39	7.00	0.8235	44.72
109.47	6.50	0.7647	51.80
128.47	6.00	0.7059	60.73
152.88	5.50	0.6471	72.20
184.97	5.00	0.5882	87.27
228.34	4.50	0.5294	107.61
288.95	4.00	0.4706	136.00
377.32	3.50	0.4118	175.70
513.35	3.00	0.3529	220.93
738.58	2.50	0.2941	269.38
1151.76	2.00	0.2353	335.53
1799.82	1.50	0.1765	444.46
2684.28	1.00	0.1176	657.27

ISOTHERM TEMPERATURE EXCESS = 8.0 C-DEG.
 ISOTHERM AREA = 1122.86 SQ. M. (0.277 ACRES)

Y(M.)	X(M.)	Y(M.)	X(M.)
0.0	5.335	54.201	9.033
2.981	6.202	56.459	8.963
5.962	6.953	58.718	8.747
8.943	7.589	60.976	8.374
11.924	8.109	63.234	7.823
14.905	8.513	65.493	7.052
17.886	8.802	67.751	5.975
20.867	8.976	70.009	4.373
23.848	9.033	72.268	0.0

ISOTHERM TEMPERATURE EXCESS = 7.0 C-DEG.
 ISOTHERM AREA = 1915.48 SQ. M. (0.473 ACRES)

Y(M.)	X(M.)	Y(M.)	X(M.)
0.0	5.335	70.792	11.799
3.894	6.850	73.741	11.706
7.787	8.163	76.691	11.424
11.681	9.274	79.640	10.938
15.574	10.183	82.590	10.218
19.468	10.890	85.540	9.210
23.361	11.395	88.489	7.804
27.255	11.698	91.439	5.712
31.148	11.799	94.389	0.000

Fig. A.6 (Contd.)

ISOTHERM TEMPERATURE EXCESS = 6.0 C-DEG.
 ISOTHERM AREA = 3548.34 SQ. M. (0.877 ACRES)

Y(M.)	X(M.)	Y(M.)	X(M.)
0.0	5.335	96.351	16.058
5.299	7.848	100.365	15.932
10.599	10.027	104.380	15.549
15.898	11.870	108.394	14.887
21.197	13.378	112.409	13.907
26.496	14.550	116.424	12.536
31.796	15.388	120.438	10.622
37.095	15.891	124.453	7.774
42.394	16.058	128.468	0.0

ISOTHERM TEMPERATURE EXCESS = 5.0 C-DEG.
 ISOTHERM AREA = 7356.35 SQ. M. (1.818 ACRES)

Y(M.)	X(M.)	Y(M.)	X(M.)
0.0	5.335	138.731	23.122
7.630	9.504	144.511	22.940
15.260	13.117	150.292	22.388
22.891	16.174	156.072	21.434
30.521	18.675	161.853	20.024
38.151	20.621	167.633	18.049
45.781	22.010	173.414	15.294
53.411	22.844	179.194	11.194
61.042	23.122	184.975	0.0

ISOTHERM TEMPERATURE EXCESS = 4.0 C-DEG.
 ISOTHERM AREA = 17951.35 SQ. M. (4.436 ACRES)

Y(M.)	X(M.)	Y(M.)	X(M.)
0.0	5.335	216.716	36.119
11.919	12.550	225.746	35.836
23.839	18.803	234.775	34.972
35.758	24.094	243.805	33.483
47.677	28.423	252.835	31.280
59.597	31.790	261.865	28.196
71.516	34.195	270.895	23.891
83.436	35.638	279.925	17.486
95.355	36.119	288.954	0.0

ISOTHERM TEMPERATURE EXCESS = 3.0 C-DEG.
 ISOTHERM AREA = 56658.73 SQ. M. (14.001 ACRES)

Y(M.)	X(M.)	Y(M.)	X(M.)
0.0	5.335	385.013	64.169
21.176	19.124	401.055	63.666
42.351	31.075	417.097	62.131

Fig. A.6 (Contd.)

63.527	41.187	433.140	59.486
84.703	49.460	449.182	55.572
105.879	55.895	465.224	50.092
127.054	60.492	481.266	42.444
148.230	63.250	497.309	31.066
169.406	64.169	513.351	0.000

ISOTHERM TEMPERATURE EXCESS = 2.0 C-DEG.
 ISOTHERM AREA = 285210.34 SQ. M. (70.477 ACRES)

Y(M.)	X(M.)	Y(M.)	X(M.)
0.0	5.335	863.823	143.970
47.510	37.828	899.815	142.841
95.021	65.988	935.808	139.399
142.531	89.816	971.801	133.464
190.041	109.312	1007.793	124.682
237.551	124.475	1043.786	112.387
285.062	135.306	1079.779	95.228
332.572	141.804	1115.771	69.699
380.082	143.970	1151.764	0.000

ISOTHERM TEMPERATURE EXCESS = 1.0 C-DEG.
 ISOTHERM AREA = 1549147.27 SQ. M. (382.803 ACRES)

Y(M.)	X(M.)	Y(M.)	X(M.)
0.0	5.335	2013.207	335.534
110.726	82.725	2097.090	332.903
221.453	149.797	2180.974	324.880
332.179	206.550	2264.858	311.049
442.905	252.985	2348.741	290.581
553.632	289.100	2432.625	261.927
664.358	314.897	2516.508	221.935
775.085	330.375	2600.392	162.440
885.811	335.534	2684.276	0.0

Fig. A.6 (Contd.)

Results of the computer calculation are plotted in Fig. A.7, and the same quantities that were defined and discussed in connection with the Motz-Benedict model (Sec. 1 of this appendix) are plotted. Because the velocity distribution is not dealt with in the Pritchard model, velocity half-widths are not calculated. The areas within isotherms of specific excess temperatures are tabulated in Table A.8.

5. Pritchard Model No. 2

The model parameters used in preparing the Pritchard (No. 2) model predictions are given in Table A.9. The actual computer output is reproduced in Fig. A.8, most of the output being self-explanatory. Final temperature

TABLE A.8. Isotherm Areas Predicted by Pritchard Model No. 1

Excess Temperature, C°	Area, m ²
8	1123
7	1915
6	3548
5	7356
4	17951
3	56659
2	285210

TABLE A.9. Model Parameters Used in Preparing Predictions Used in Pritchard Model No. 2

Depth of water, D_{wo}	4.2 m
Bottom slope, S_x	0.01
Critical mixing depth, $h_{1/2}$	3.2 m
Velocity ratio, R	∞

Fig. A.7. Predictions of Pritchard Model No. 1

PRITCHARD MODEL NO. 2 -- POINT BEACH UNIT NO. 1 18 MAY 72

INPUT DATA-

DISCHARGE RATE 24.7 CMS
 DEPTH OF DISCHARGE 4.2 METERS
 WIDTH OF DISCHARGE 10.7 METERS
 HEAT REJECTION RATE 7.558E 08 KCAL/HR
 INITIAL TEMPERATURE RISE 8.5 DEG.C
 DEPTH OF WATER 4.2 METERS
 BOTTOM SLOPE 1.000E-02
 AMBIENT CURRENT SPEED 0.0 MPS
 AMBIENT TEMPERATURE 9.2 DEG.C
 INTAKE TEMPERATURE 9.2 DEG.C
 WIND SPEED 4.3 MPH
 SALT CONCENTRATION 0.0 PPM
 RUN PARAMETER 2.0

CALCULATED PARAMETERS-

DISCHARGE VELOCITY 0.55 MPS
 INITIAL DENSIMETRIC FROUDE NUMBER 2.56
 ASPECT RATIO 0.39
 RATE OF VERTICAL GROWTH 4.3918E-03
 CRITICAL DEPTH 3.2 METERS
 BASIC LENGTH SCALE 6.7 METERS
 DELRHO/RHO 1.1174E-03
 VALUE OF N 3.9620

TEMP DEG.C	RATIO	DIST METERS	WIDTH METERS	AREA SQ.M	K KCAL/SQ.M C.DEG-DAY	COR.DIST METERS	COR.WIDTH METERS	COR.AREA SQ.M	DEPTH METERS	M	BETA
8.0	0.9412	31.86	15.69	4.298E 02	268.0	31.86	15.69	4.298E 02	3.221	0.156	3.001
7.0	0.8235	47.57	18.14	7.422E 02	264.5	47.56	18.14	7.421E 02	3.221	0.156	3.001
6.0	0.7059	67.51	23.86	1.385E 03	261.1	67.50	23.86	1.385E 03	3.221	0.323	2.102
5.0	0.5882	99.04	34.05	2.900E 03	257.7	99.03	34.05	2.900E 03	3.221	0.323	2.102
4.0	0.4706	158.32	53.20	7.243E 03	254.5	158.29	53.19	7.240E 03	3.221	0.323	2.102
3.0	0.3529	289.86	95.68	2.385E 04	251.4	289.74	95.64	2.383E 04	3.221	0.323	2.102
2.0	0.2353	679.78	221.62	1.296E 05	248.3	678.74	221.28	1.292E 05	3.221	0.323	2.102
1.0	0.1176	1526.98	495.26	6.504E 05	245.4	1519.67	492.89	6.442E 05	3.221	0.323	1.000

UNBENT PLUME, DEPTH LIMITED FROM 0.0 TO 6.313E 01 OR FROM RATIO 1.00000 TO 0.79217

Fig. A.8. Computer Output of Code for Pritchard Model No. 2. ANL Neg. No. 190-1326.

distribution is specified by the columns labeled corrected distance (COR.DIST), corrected width (COR.WIDTH), and corrected areas (COR.AREA). The distance is the distance to the end of a specific isotherm, the width is the maximum total width of the isotherm, and the area is the area within the isotherm.

The results of the computer calculation are plotted in Fig. A.9, in three graphs as follows: The top graph is a plot of centerline excess temperature ratio as a function of distance along the centerline of the plume; the center graph shows the maximum total width of the isotherms of specific excess temperatures plotted at the centerline distance at which that specific excess temperature is reached; and the bottom graph is a plot of the depth of the plume, which, in this case, is simply the critical depth defined by Pritchard. Areas within isotherms of specific excess temperatures are tabulated in Table A.10.

TABLE A.10. Isotherm Areas Predicted
by Pritchard Model No. 2

Excess Temperature, C°	Area, m ²
8	430
7	742
6	1385
5	2900
4	7240
3	23830
2	129200

Fig. A.9. Predictions of Pritchard Model No. 2. ANL Neg. No. 190-1335.

Reference

1. A. A. Frigo, D. E. Frye, and J. V. Tokar, Field Investigations of Heated Discharges from Nuclear Power Plants on Lake Michigan: 1972, ANL/ES-32 (Mar 1974).

APPENDIX B

Summary of Model Predictions Prepared under
Subcontract to Argonne National Laboratory1. Introduction

Given the failure of the popular integral models to predict thermal plumes accurately, we sought to test the capabilities of other techniques such as differential-numerical modeling and subchannel-balancing methods. The most expedient way to complete the testing of these complex models, with the personnel available to us, was to sponsor the individual authors of the models in the preparation of predictions for several cases where complete sets of field data existed.

In early 1974, when this matter was being considered, there were two published models directly applicable to thermal plumes in Lakes. One was the model of Paul and Lick developed at Case Western Reserve under the sponsorship of the Environmental Protection Agency. The other was the model of Waldrop and Farmer developed under Navy sponsorship at Louisiana State University. Additionally, Till at the University of Missouri at Columbia was completing work on his model for river plumes using the subchannel-balancing technique and Science Applications, Inc., was working on a general three-dimensional hydrodynamics code that could be adapted to thermal-plume applications. We therefore contacted the individuals involved in each of these efforts. Waldrop and Farmer declined our invitation because their model was not sufficiently developed for such competition. The cost of modifying and running the Science Applications, Inc., model was prohibitive. However, negotiations were successful with Paul and Lick and with Till, and contracts were let for these authors to prepare model predictions for the Point Beach Nuclear Power Station. We provided each group with all the information necessary for them to prepare model predictions, but did not provide the measured thermal-plume maps themselves.

This appendix contains the material supplied to us to fulfill these contracts, edited to conform to our publication requirements. Some repetition of earlier sections is unavoidable, since we wanted the appendix to be complete in itself to avoid burdening the reader with having to continually refer to earlier sections.

2. Application of Till Model to Point Beacha. Introduction

The immediate objective of this investigation is to develop a numerical method for the accurate prediction of isotherms resulting from the discharge of the Point Beach Power Plant condenser cooling water into Lake Michigan. The ultimate objective is the development of a mathematical model

for predicting thermal discharges into a lake. Most of the significant physical features associated with the heated discharge and the dispersing-dissipating medium are considered. The proposed model reflects five primary processes that lead to the dispersion and dissipation of the heated effluent. These processes are: the initial spreading caused by the momentum difference of the heated discharge, buoyancy, convection in the direction of the discharge, dispersion due to natural eddy turbulence, and heat loss to the atmosphere.

Only a single-port discharge is considered. However, a multiple-port discharge, if parallel, and surface as well as subsurface discharges also can be considered. Other pertinent factors are that (1) the natural body of water may be stratified or unstratified; and (2) the temperature distribution is determined as a function of the initial temperature excess, mass flow rate, method of release, turbulence, and atmospheric conditions.

Included in this report are single-port discharge studies for stagnant lake (Case 1) and single-port discharge studies for conditions of lake current and wind shear (Case 2), the results being produced as numerical and plotted data. Also included are discussions of the changes made to the original river model to adapt it for the lake situation, advantages and disadvantages of the model as it now exists, and possible improvements for both a lake and a river situation. The Case 1 and Case 2 work was performed at the University of Missouri-Rolla.

Henceforth and throughout this report, the code used to make the determinations of the heated effluent dispersion will be called TEMPL (Thermal Enrichment Modeling Program for Lake) for the lake application model and TEMPR (Thermal Enrichment Modeling Program for River) for the river-application model.

b. List of Symbols

A_x Longitudinal cross-sectional area (L^2)

A_y Lateral cross-sectional area (L^2)

A_z Vertical cross-sectional area (L^2)

F Force

g Gravitational acceleration

g_c Gravitational constant (ML/FT^2)

H Energy

h Enthalpy (H/M)

k von Karman constant

L Length

M	Mass
m	Mass flow rate (M/t)
p	Pressure (F/L^2)
Q	Heat-rate addition or loss per unit time
R_i	Richardson number (dimensionless)
T	Temperature
t	Time
u_x	Longitudinal velocity (L/t)
u_y	Lateral velocity (L/t)
u_z	Vertical velocity (L/t)
w	Convective crossflow between adjacent subchannels (M/tL)
w'	Turbulent crossflow between adjacent subchannels (M/tL)
x	Longitudinal space variable (L)
y	Lateral space variable (L)
z	Vertical space variable (L)
ϵ_m	Eddy diffusivity of momentum (L^2/t)
ϕ	Slope of river (deg)
ρ	Density
τ_w	Wind shear (F/L^2)
τ_x	Friction shear (F/L^2)

c. Theoretical Considerations

The model in this investigation is developed to simulate the dispersion and dissipation of heated effluent from the Point Beach Power Plant into Lake Michigan. A nodal representation is created by developing longitudinal sections in the direction of the outfall and subdividing each section laterally and vertically into subchannels.

The conservation equations are applied at each subchannel with turbulent and convective crossflow connections between subchannels. These connections are the paths by which mass, momentum, and energy are exchanged between adjacent subchannels and provide necessary coupling for the equations. The solution procedure consists of achieving a mass, momentum, and energy balance between the subchannels at each section in sequence until the desired number of sections has been investigated.

(1) Conservation Equations. The equations are developed in a manner similar to those developed in H. Till's dissertation¹ for use in the river case, except that wind shear and tidal-induced crosscurrent have been added. The equations are derived by performing a balance on a subchannel.

A subdivision of a section, known as a subchannel, has adjacent to it one to four similar subchannels. These adjacent subchannels are situated laterally and vertically and are coupled by the mass, momentum, and energy mixing between them. There are two forms of mixing, one due to pressure differences and the other due to natural turbulence. Balancing the pressure between adjacent subchannels enables differences to be diminished by a subchannel donating or accepting mass, momentum, and energy through convective and turbulent mixing. Turbulent mixing exchanges only momentum and energy between subchannels, subchannels containing the greater amount donating to those with the lesser amount. Turbulent mixing is not considered for the mass balance, because the time average of mass transfer due to turbulent conditions is zero. Figure B.1 illustrates interconnected subchannels with convective and turbulent crossflows, which are superimposed to produce net crossflows. By use of these basic assumptions, the steady-state conservation equations take the following form:

Continuity Equation

$$\frac{d\dot{m}_i}{dx} = - \sum_{J=1}^M w_{iJ}, \quad i = 1, 2, 3, \dots, N,$$

where N is the maximum number of subchannels, and M is the maximum number of subchannels adjacent to the subchannel of interest. These are

Fig. B.1. Subchannels with Convective and Turbulent Crossflows

usually the four subchannels that surround the subchannel being considered. This will be the situation unless otherwise stated. The subchannel denoted i is the one under consideration, and subchannels j , k , l , and m , are the subchannels interconnected with i by convective crossflows. The continuity condition states that the rate of flow change in a subchannel is equal to the net transport of mass into or from the subchannel.

Energy Equation

$$\dot{m}_i \frac{dh_i}{dx} = \dot{Q}_i + \sum_{J=1}^M w'_{iJ} (h_J - h_i) + \sum_{J=1}^M \begin{cases} 0, & \text{if } w_{iJ} \geq 0; \\ w_{iJ} (h_i - h_J), & \text{if } w_{iJ} < 0; \end{cases} \quad i = 1, 2, 3, \dots, N.$$

The energy equation is composed of the energy-transport mechanisms to and from the subchannel. The first term on the right side of the equation is the heat energy gained or lost by the subchannel from exterior causes, such as the addition of heated effluent at a power plant discharge and the surface heat loss to the atmosphere. The second term is the turbulent transport of energy between subchannel i and interconnected adjacent subchannels j , k , l , and m . Energy is always transported by turbulence from the subchannel with the greater amount to the one with the lesser amount. The third term is the convective crossflow of the energy by pressure differences between subchannel i and interconnected subchannels j , k , l , and m . When energy is leaving the subchannel under consideration, it produces no change in that subchannel's energy; therefore, only the transport into the subchannel is considered.

Equations of Motion

The equations of motion are solved for the pressure terms so that these equations can be used conveniently to determine the convective crossflows. The component pressures referred to may be only a small fraction of the actual hydrostatic pressures. The reference hydrostatic pressures are removed from the calculated values. Part of the exchange of mass, energy, and momentum between a subchannel and those subchannels immediately adjacent and interconnected is based on the pressure differences between the subchannels. Since the magnitude of the pressure terms at a nodal point should be the same for the three equations of motion, equating respective longitudinal and transverse equations through their own pressure terms generates the required new set of equations needed to solve for the convective crossflows. This set contains as many simultaneous equations as there are subchannel interconnections and as many unknown convective crossflows.

Calculations for convective crossflows, vertical and lateral, are made as follows: To determine the vertical convective crossflows, the difference between the pressures for two vertically adjacent subchannels, based on values obtained from their x equations of motion, is equated to the derivative

of the z equation of motion. This allows vertical motion to be included in the determination of the vertical convective crossflows. To determine the lateral convective crossflows, the difference between the pressures for two laterally adjacent subchannels, based on values obtained from their x equations of motion, is equated to the derivative of the y equation of motion. This allows lateral motion to be included in the determination of the lateral convective crossflows.

x Equation of Motion

$$\frac{dp_i}{dx} = \frac{(\tau_x + \tau_{wx})\Delta x}{A_x} - \frac{2\dot{m}_i}{A_x g_c} \frac{du_{xi}}{dx} - \frac{u_{xi}^2}{g_c} \frac{dp_i}{dx} + \sum_{J=1}^M \frac{w'_{iJ}}{A_x g_c} (u_{xJ} - u_{xi})$$

$$- \sum_{J=1}^M \begin{cases} \frac{w_{iJ}}{A_x g_c} u_{xi}, & \text{if } w_{iJ} \geq 0; \\ \frac{w_{iJ}}{A_x g_c} u_{xJ}, & \text{if } w_{iJ} < 0. \end{cases}$$

The first and second terms on the right side of the equation are shearing-stress terms due to the viscosity and the wind at the surface. The third and fourth terms are developed by the differentiation of the momentum acceleration term. The fifth term is the turbulent crossflow between subchannel i and interconnected adjacent subchannels j, k, l , and m . Momentum is always transported by turbulence from the subchannel with the greater amount to the one with the lesser amount. The sixth term is the convective crossflow between subchannel i and interconnected adjacent subchannels j, k, l , and m . The sign of a convective crossflow depends on whether the crossflow is entering or leaving a particular subchannel.

y Equation of Motion

The y equation of motion is significant if large flows in the lateral direction occur, wind shear develops, or a crosscurrent is present. The y equation of motion is expressed as

$$\frac{dp_i}{dy} = \frac{\tau_{wy}\Delta y}{A_y} - \frac{2\dot{m}_{yi}}{A_y g_c} \frac{du_{yi}}{dy} - \frac{u_{yi}^2}{g_c} \frac{dp_i}{dy} - \frac{\rho u_x}{dx} \frac{du_y}{dx} + \sum_{J=1,m}^M \frac{w'_{iJ}}{A_y g_c} (u_{yJ} - u_{yi})$$

$$- \sum_{J=1,m}^M \begin{cases} \frac{w_{iJ}}{A_y g_c} u_{yi}, & \text{if } w_{iJ} \geq 0; \\ \frac{w_{iJ}}{A_y g_c} u_{yJ}, & \text{if } w_{iJ} < 0. \end{cases}$$

The first term on the right side of the equation is the wind-shear term, the second and third are acceleration terms, the fourth term is a convection in the longitudinal direction, and the fifth and sixth terms are turbulent and convective transport terms. Convection by the usual process is considered only in the vertical direction.

z Equation of Motion

The z equation of motion is significant for the buoyancy of the heated water and is expressed as

$$\frac{dp_i}{dy} = \rho_i \cos \varphi - \frac{2\dot{m}_{zi}}{A_z g_c} \frac{du_{zi}}{dz} - \frac{u_{zi}^2}{g_c} \frac{d\rho_i}{dz} - \frac{\rho u_x}{g_c} \frac{du_z}{dx} + \sum_{J=j,k}^M \frac{w'_{iJ}}{A_z g_c} (u_{zJ} - u_{zi})$$

$$- \sum_{J=j,k}^M \begin{cases} \frac{w_{iJ}}{A_z g_c} u_{zi}, & \text{if } w_{iJ} \geq 0; \\ \frac{w_{iJ}}{A_z g_c} u_{zJ}, & \text{if } w_{iJ} < 0. \end{cases}$$

The first term on the right side of the equation is the gravitational term, the second and third are acceleration terms, the fourth term is a convection component in the longitudinal direction, and the fifth and sixth terms are for turbulent and convective transport. Convection by the usual procedure is considered only in the lateral direction.

(2) Turbulent Model. The turbulent model follows that used by A. Wada,² which was based on O. I. Mamayev's work for diffusion in the sea. The vertical eddy diffusivity of momentum is assumed to be of the form

$$\epsilon_m = A_1 + A_0 e^{-C_m R_i},$$

where A_1 , A_0 , and C_m are empirical coefficients for momentum in the transverse directions and R_i is the local Richardson number. This approach is assumed to be a rough approximation because better information was not available. The coefficients are considered to be spatially constant. Typical values of the coefficients are $C_m = 0.4$, $A_0 = 38.736 \text{ lb}_m/\text{hr-ft}$, and $A_1 = 387.36 \text{ lb}_m/\text{hr-ft}$. These are for the z direction. The Richardson number is taken to be

$$R_i = \frac{g(\rho_o - \rho)}{\rho u_z^2}.$$

The lateral eddy diffusivity in Lake Michigan near Point Beach has been measured to be about $500 \text{ cm}^2/\text{sec}$, and this is the constant value used.³

(3) Heat Loss to Atmosphere. A heated body of water cools primarily by losing heat to the atmosphere. All heated water bodies lose heat to the atmosphere by long-wave back radiation and evaporation, and receive heat primarily through short-wave solar radiation and long-wave atmospheric radiation. Depending on the relative temperatures, heat conduction can go either way. The heat-exchange mechanisms, along with their range in magnitude of monthly average values for northern latitudes, are illustrated in Fig. B.2.⁴

Fig. B.2. Mechanisms of Heat Transfer across a Water Surface. (Adapted from Edinger and Geyer.⁴)

The net rate at which heat crosses the water surface is

$$\Delta H = (H_s + H_a - H_{sr} - H_{ar}) - (H_{br} + H_e \pm H_c).$$

When there is no transfer of energy across the water surface, an equilibrium condition is said to be established. The heated discharge, superimposed upon the natural conditions, unbalances this equilibrium and dissipates heat to the atmosphere. The equation used to express the net rate of heat exchange is

$$\Delta H = -K(T_s - E),$$

where K is the heat-exchange coefficient, T_s is the water-surface temperature, and E is the equilibrium temperature. The heat-exchange coefficient is determined using the equation

$$K = 15.7 + (0.26 + \beta)(a + bW).$$

Values from the Lake Hefner Studies were used for the empirical coefficients for evaporation, which are $a = 0.0$ and $b = 11.4$.^{5,6} The proportionality coefficient, β , for the temperature vapor-pressure approximation is known once the temperature range is established. The equilibrium temperature is calculated by the method established by Edinger and Geyer.⁴

(4) Surface Wind Shear. For the wind-shear model, the classical relationship between wind speed and shear stress on a body was applied:

$$\tau_w = \rho_a C_D W^2,$$

where

τ_w = surface wind stress,

ρ_a = density of overlaying air,

C_D = drag coefficient (initial guess, 0.5),

and

W = wind speed (measured 10 ft above surface).

For application to the x equation of motion,

$$\tau_{wx} = \tau_w \sin \varphi,$$

and application to the y equation of motion,

$$\tau_{wy} = \tau_w \cos \varphi,$$

where φ is the angle of wind direction with the shore. For inclusion in the dp/dx and the dp/dy equations the terms are

$$+ \frac{\tau_{wx}\Delta x}{A_x} \text{ and } + \frac{\tau_{wy}\Delta y}{A_y}.$$

(5) Mass, Energy, and Pressure Integration Method. The longitudinal integration of motion is performed as an initial-value problem using finite differences. The necessary initial conditions of lake and outfall mass flow rate, temperature, and pressure for the first spatial step are known, as are the characteristics of the discharge. In the first spatial section, perpendicular or oblique transport is handled by controlling the convective crossflows.

Boundary conditions, primarily the meteorological ones, are applied at each spatial section. Integration is accomplished by the implicit Euler predictor-corrector method. The conservation equations in their iterative form are:

Continuity

$$\dot{m}_{i,n+1}^{(k+1)} = \dot{m}_{i,n} + \frac{\Delta x}{2} \left\{ f_m(x_n, \dot{m}_n) + f_m[x_{n+1}, \dot{m}_{n+1}^{(k)}] \right\}.$$

Energy

$$h_{i,n+1}^{(k+1)} = h_{i,n} + \frac{\Delta x}{2} \left\{ f_h(x_n, h_n) + f_h[x_{n+1}, h_{n+1}^{(k)}] \right\}.$$

Pressure

$$p_{i,n+1}^{(k+1)} = p_{i,n} + \frac{\Delta x}{2} \left\{ f_p(x_n, p_n) + f_p[x_{n+1}, p_{n+1}^{(k)}] \right\}.$$

Derivative values for the mass, energy, and pressure in these equations are denoted by f_m , f_h , and f_p ; k and $k + 1$ denote the iteration counter and n and $n + 1$, which is the same as x and $x + \Delta x$, the longitudinal spatial step. The above three equations require predictor forms to start at each new spatial step, for which the Euler explicit form is used. These equations are:

Continuity

$$\dot{m}_{i,n+1}^{(k)} = \dot{m}_{i,n} + \Delta x f_m(x_n, \dot{m}_n).$$

Energy

$$h_{i,n+1}^{(k)} = h_{i,n} + \Delta x f_h(x_n, h_n).$$

Pressure

$$p_{i,n+1}^{(k)} = p_{i,n} + \Delta x f_p(x_n, p_n).$$

Convergence to within an established tolerance for values produced by the corrector equations is followed by a step forward to the next spatial step. A final solution for the corrector form of the conservation equations at each spatial step is achieved by iteration, and the derivatives developed for these equations are used as the initial conditions for the succeeding predictor step.

For lake sections other than those at which a thermal discharge occurs, convective crossflows are determined by pressure differences created between adjacent subchannels. The pressure differences from the x equations of motion for two adjacent subchannels are set equal to the pressure differentials derived from the y and z equations of motion. Pressures subtracted from two laterally adjacent subchannels are set equal to the pressure derivative obtained from the lateral equation of motion and from two vertically adjacent subchannels to the vertical equation of motion. The equations resulting for the lateral and vertical directions are similar to the Euler equations and provide a means for integration in the transverse directions as well as the additionally necessary equations for solving for the diversion crossflows. For the lateral case

$$(p_i - p_j)_{x+\Delta x} = \left. \frac{dp}{dy} \right|_{x+\Delta x} \Delta y,$$

where p_j represents the pressure at the previous lateral subchannel as progression proceeds from left to right for successive layers. For the vertical case,

$$(p_i - p_m)_{x+\Delta x} = \left. \frac{dp}{dz} \right|_{x+\Delta x} \Delta z,$$

where p_m represents the pressure at the previous vertical subchannel as progression proceeds from top to bottom. This procedure produces an equation for each existing path of crossflow within a system of equations, which is then solved iteratively for the diversion crossflows.

The total iteration-loop calculation starts by determining values for mass, energy, and pressure at spatial step $x + \Delta x$. With these results, the pressure equations are applied as described above, and convective crossflows are obtained. The next procedural steps concern testing for convergence to acceptable tolerances, which include the values for the conservation equations. (For ease of expressing which iteration loop is being discussed, the iteration for the conservation equations is called the outer iteration loop, and the iteration for the diversion crossflows is called the inner iteration loop.) If such convergence is achieved for the outer loop, similar testing is applied to the convective crossflows, resulting in the respective convergence of the y - and z -pressure equations. Both the inner and outer iteration loops must converge to within an acceptable tolerance before the calculation can advance to the next section.

Further explanation of the numerical solution appears in Appendix B of H. Till's thesis.¹

d. Problems Encountered and Improvements in Computing

(1) Computing Problems. The original approach to the solution of a lake discharge was to view it as a discharge into a river with no current. Two difficulties were encountered. The first was the inability of the river model to handle low or zero flows. Such situations produced large differential changes in the energy equation and, hence, problems in determining water-property conditions. (Subroutine PROP could not establish water temperature, density, viscosity, and conditions for saturated water vapor.) These changes arise because, in the turbulent and convective transport terms of the energy equation, the flow appears in the denominator, and both the turbulent-mixing, or eddy-diffusivity, term and the convective-crossflow term appear in the numerator. Therefore, if the lateral movement of heated water is large compared to the longitudinal flow, large energy differentials ensue and subroutine PROP fails. It was assumed that lateral movement would not be the primary mode of flow and that, for small longitudinal flow, there would be an accompanying small movement laterally and vertically. In a lake discharge, such need not be the case. This problem was originally circumvented by not applying the terms for turbulent and convective mixing unless the flow met some minimum criteria; for example, $\dot{m} \geq 10^4 \text{ lb}_m/\text{hr}$. This condition proved to be not totally acceptable because it would not allow energy to be transported transversely unless a minimum flow was achieved. For a river this would not be the case, because the transverse movement of energy is directly related to the forward flow of water in both the turbulent and the convective crossflows.

The second and most notable difficulty was the inability of the model to maintain for an extended longitudinal distance the transverse movement of the water. Stagnant cases were set up, and the transverse movement of the water was insufficient. Terms that were assumed insignificant when the model was applied to the river became very significant when applied to the lake. The discharge, starting out in the original 60° angle, would soon tend to move longitudinally; i.e., the correct direction would not be maintained, and the water would begin moving out into the lake. The equations were made more complete by the addition of more terms, and the actual flows in the lateral and vertical directions were considered by including $\dot{m}_y$, $\dot{m}_z$, and u_y , u_z terms.

The transverse-momentum equations were developed in a more complete way and applied in subroutine CRECTR instead of DXFLO. In this fashion the transverse pressures and flows were put into the iteration scheme as they should be. Because of these difficulties, the straight-out-into-the-lake marching technique was abandoned and a marching technique in the direction of flow was applied.

To adapt the original river model to a lake situation, zero diagonal elements had to be removed from the solution matrix. Such a situation prevailed when the flow in two adjacent subchannels was zero, resulting in a zero mass and momentum crossflow. To leave these terms in the matrix solution would result in a singular matrix and therefore no solution. Hence, they

were removed and the solution matrix was assembled to include only subchannel connections with crossflows. After the solution was completed for each iteration, it was reassembled so that the solution crossflows applied to the actual locations. This total mechanism was accomplished by applying a sub-routine called MATSET.

(2) Solution Technique. The original technique for a solution for the crossflows was not effective in the lake situation because diagonal dominance, a necessary condition for solution by the successive-displacement technique, was not consistently achieved. Another solution approach was applied that was successful. This technique is described as follows:

$$Aw = B,$$

where

A is the coefficient matrix,

w is the crossflow solutions,

and

B is the pressure-difference terms;

$$A = (A^* + D),$$

where

D is the diagonal of the coefficient matrix

and

A* is all of the off diagonal terms;

$$(A^* + D)w = B,$$

$$Dw = B - A^*w,$$

$$w = D^{-1}(B - A^*w^i),$$

$$w^{i+1} = D^{-1}(B - A^*w^i),$$

where

i is the iteration number.

(3) List-pointer Processing Technique. Application of the model to a lake required many subchannels. To make the model more accommodating to this situation, variable subchannel dimensions were applied, instead of the constant subchannel dimensions as in the river case. Because the use of a

great number of subchannels requires large amounts of computer core space, a solution technique had to be applied that would reduce this storage requirement. A list-pointer processing technique was applied that reduced both core requirements and run time. In this technique, all zero elements are excluded and two elements are required in order to establish a coefficient-matrix element.

The diagonal coefficient matrix in subroutine DXFLO was dimensioned number of channels by number of connections. Since each channel has at most four connections, the matrix can be dimensioned number of channels by eight: four locations for the values and four locations for pointers to the connection associated with the value. For a set of 100 channels and 200 connections, the storage requirements conventionally would be 80,000 bytes. Using list processing, the matrix requires 3200 bytes, a saving of over 75,000 bytes.

By similar reasoning, the off-diagonal matrix can be reduced in dimension from number of connections by number of connections, to number of connections by 15 for a set of 200 connections. The matrix is reduced from 160,000 to 12,000 bytes. The subchannel connection matrix was reduced from 160,000 to 800 bytes. For a system of 100 channels and 200 connections, memory requirements were reduced by 320,000 bytes.

Before these changes were made, the program could model the outfall to a distance of about 200 ft at a cost of \$56. After these changes were made, the code could model 1000 ft for about \$10. Both run time and memory requirements have been reduced significantly.

(4) Data-input Improvements. The program input has been simplified by making the code locate and number subchannels and subchannel connections. Also, the system can be changed during the forward-marching process. This alteration was used in the determination of isotherms in the lake situation because of the slope of the lake bed. The subchannel system was consistently made larger. Since the input-card quantity was unnecessarily large in the original river case, the simplified program setup and reduced input-card volume has been an improvement.

e. Subchannel Arrangement and Input Data

The longitudinal direction was made the direction of the discharge. A top view of the approximate arrangement of subchannels is illustrated in Fig. B.3. The subchannels increase 1.52 m per width step as they approach the sides. The three center subchannels are 10.7 m in width, and the outer subchannels approach 25 m in width. The length of the subchannels is increased from 6.1 to 15.2 m per step. Figure B.4, an elevated view of the subchannel arrangement, shows the variation in subchannel depths. Also indicated are the subchannels receiving the discharge. The temperatures of the discharges are 16.7°C with a lake ambient temperature of 9.6°C for Case 1, and

20.2°C with a lake ambient temperature of 13.5°C for Case 2. The discharge flow rates were 24.7 m³/sec or 0.53 m/sec for Case 1 and 25.1 m³/sec or 0.56 m/sec for Case 2.

Fig. B.3. Top View of Subchannel Arrangement

Fig. B.4. Elevated View of First Region Subchannel Arrangement

f. Discussion of Results

The results indicate that the heated effluent moves primarily out into the lake at a 60° angle. There is relatively little lateral dispersion. No effects of the wind and current in Case 2 were perceived. The wind speed was 0-2 m/sec at 135° from north, and the lake current was 2.2 cm/sec at 245° from north. Case 1 was a stagnant case, and no wind or lake current was considered. Isotherms created in Case 1, at the surface and at depths of 0.5, 1, and 3 m are illustrated in Figs. B.5-8. Isotherms for Case 2 and the same positions are illustrated in Figs. B.9-12.

Fig. B.5
Surface Isotherms for Case 1

Fig. B.6
Case 1 with Isotherms at 0.5-m Depth

Fig. B.7
Case 1 with Isotherms at 1.0-m Depth

Fig. B.8

Case 1 with Isotherms at 3.0-m Depth

Fig. B.9
Surface Isotherms for Case 2

Fig. B.10

Case 2 with Isotherms at 0.5-m Depth

Fig. B.11. Case 2 with Isotherms at 1.0-m Depth

Fig. B.12. Case 2 with Isotherms at 3.0-m Depth

g. Suggestions for Future Work

The results in general indicate that the model has the potential for lake use, but more work is required to bring it up to an acceptable degree of accuracy.

The relative magnitudes of the equations of motion must be analyzed thoroughly. The normal interaction between the terms does not appear to be correct. Pressure grows in subchannels that are transporting fluid out and tends to decrease when fluid is transported in. The opposite result is sought and does occur, but in a lethargic manner. This situation allows the code to work, but not in a totally correct fashion. To be operationally correct, the crossflows for each step should be sufficient to bring the pressures toward zero. The terms in the equations of motion seem to be analytically correct. In the formula translation to the computer, however, a thorough understanding of the computer's handling of them was not investigated, and it is here that some problems may lie. One suggestion for relieving the small terms dealing with pressures is to multiply the respective equations through by A_{xgc} , A_{ygc} , and A_{zhc} . This produces numbers much larger in magnitude, and results have indicated that these numbers are handled more accurately by the computer. However, because of the cross-sectional areas involved, difficulties may arise in equating the longitudinal-momentum equation to the transverse-momentum equation.

Because the code is directional, the straight-out-into-the-lake marching technique with the oblique discharge was not completely successful. Since the code is not totally three-dimensional, work must be done to bring it closer to that state. As the code now stands, it cannot handle the especially important lake-situation case in which the transverse flow is greater than the longitudinal flow. True three-dimensionality most likely can be achieved within the framework of the existing technique. With the improvements that have been made, the river case will be improved. A more appropriate turbulent model must be applied to both the river case and the lake case.

The correction to the model for low flow conditions would include molecular diffusion that would allow for energy exchange between two adjacent subchannels when either one or both contained water in a stagnant or a very-slow-moving condition.

Investigations should be performed treating the model variously as either a near-field, an intermediate-field, or a far-field model. In the near field, the turbulent mixing could be ignored due to the dominance of the convective transport; in the intermediate region, they could both be applied; and in the far field, the turbulent mixing will dominate and the convective transport will have been essentially completed and can be ignored. These applications should help achieve solutions more easily and possibly more accurately.

3. Application of Paul-Lick Model to Point Beach

a. Introduction

The work of Paul and Lick^{1,2} is extended to provide a more realistic model for thermal outfalls and river discharges. In particular, previous restrictions to constant-depth basins, symmetric outfalls and rivers, stagnant outer flows, and constant vertical eddy coefficients are relaxed. The model is applied to the Point Beach Power Plant outfall on Lake Michigan where the actual lake bottom topography is used, wind stress is permitted to interact with the water, the flow in the lake and the Coriolis force are permitted to act on the plume, and the vertical eddy coefficient is assumed dependent on the local vertical temperature gradient.

b. List of Symbols

- A_H Horizontal eddy-diffusion coefficient of momentum
- A_V Vertical eddy-diffusion coefficient of momentum
- b_o Full width of outfall
- B_H Horizontal eddy diffusion coefficient of heat
- B_V Vertical eddy diffusion coefficient of heat
- Fr Froude number, $U_o / \sqrt{gh_o}$
- $F(z)$ Similarity profile
- $f(\Delta T)$ Equation of state
- g Acceleration of gravity
- g_1, g_2, g_3 Specific temperature and velocity profiles
- h Variable depth of receiving basin
- h_o Initial depth of outfall
- k Coriolis parameter
- K Heat-exchange coefficient
- P Nondimensional pressure, $P = \frac{\bar{P}Re}{\rho_o g h_o Fr^2}$
- P' Depth-averaged pressure
- Pr Turbulent Prandtl number, A_H/B_H
- P_s Nondimensional surface pressure
- Re Turbulent Reynolds number, $u_o b_o / A_H$
- R_o Nondimensional Coriolis parameter, $R_o = \frac{k b_o^2}{A_H}$

- t Nondimensional time, $t = \frac{\bar{t}A_H}{b_o^2}$
- T Nondimensional temperature from $\Delta T = \frac{\bar{T} - T_E}{T_E}$
- T_E Equilibrium temperature
- U Vertically averaged u velocity
- u Nondimensional velocity in x direction, $u = \frac{\bar{u}}{u_o}$
- U_o Outfall velocity (at $\bar{x} = 0, \bar{y} = 0, \bar{z} = 0$)
- $\bar{u}$ $u/F(z)$
- V Vertically averaged v velocity
- v Nondimensional velocity in y direction, $v = \frac{\bar{v}}{u_o}$
- $\bar{v}$ $v/F(z)$
- w Nondimensional velocity in z direction, $w = \frac{\bar{w}b_o}{u_o h_o}$
- x, y, z Nondimensional Cartesian coordinates, $x = \frac{\bar{x}}{b_o}, y = \frac{\bar{y}}{b_o},$
 $z = \frac{\bar{z}}{h_o}$
- α Inclination
- β B_V/B_H , also similarity constant
- ΔT Nondimensional temperature difference,
 $\Delta T = (T - T_E)/T_E$
- $\Delta \rho$ Nondimensional density difference, $\Delta \rho = \frac{\bar{\rho} - \bar{\rho}(T_E)}{\bar{\rho}(T_E)}$
- ζ Surface elevation
- γ A_V/A_H
- ρ Density
- $\rho(T_E)$ Density at equilibrium temperature
- σ Reduced vertical coordinate, z/h
- τ_x Wind stress at surface in x direction
- τ_y Wind stress at surface in y direction
- τ_x^B Bottom stress in x direction

- τ_y^B Bottom stress in y direction
 (-) Refers to dimensional quantity

c. Numerical Model

The basic equations for the numerical model are derived from the time-dependent, three-dimensional equations of motion for a viscous, heat-conducting fluid. The geometry of the problem is as shown in Fig. B.13. The following assumptions are made:

- (1) The pressure is assumed to vary hydrostatically; i.e.,

$$\frac{\partial P}{\partial z} = \rho g.$$

- (2) The rigid-lid approximation is made; i.e., $w(z = 0) = 0$.

- (3) The Boussinesq approximation is made; the density variations are small and can be neglected, except in the gravity term.

- (4) Heat sources and sinks in the fluid are neglected.

- (5) Eddy coefficients account for the turbulent and molecular diffusion effects in both the momentum and energy equations. The horizontal coefficient is constant, but the vertical coefficient depends on the local vertical temperature gradient.

- (6) The variations in the bottom topography are gradual.

Fig. B.13
 Geometry for Three-dimensional Jet

The following discussion gives reasons for the use of various approximations. The hydrostatic approximation is made because, for the cases in which the model will be used, variations in the vertical velocity are small enough so that the neglected terms in the vertical-momentum equation are second order, compared to the gravity terms. This approximation has been used extensively in the modeling of oceanic basins and lakes,⁹⁻¹⁴ and it has been shown to be valid in the modeling of river discharges.⁸ An important consequence of its use is that the order of the system of equations, and thus the computational effort required for solution, is reduced.

The rigid-lid approximation is used to damp out surface gravity waves that would otherwise be present. Time scales associated with the gravity waves are small compared with other relevant time scales in the problem. Including these smaller time scales would greatly increase the computational time required for a solution. For example, for the discharge of the Cuyahoga River into Lake Erie, inclusion of surface gravity waves in the numerical model has been estimated⁸ to increase the computational time by at least two orders of magnitude. Only the surface variations associated with gravity waves are neglected in this approximation.

The Boussinesq approximation allows the fluid to be treated as incompressible. The density differences encountered in the application of the model being small, the approximation is valid; and the coupling between the momentum and energy equations is retained. A consequence of the Boussinesq approximation is that the energy equation reduces to a balance between convection and diffusion.

All heat inputs and outputs to the model are assumed to occur at the boundaries of the solution domain, and heat transfer by radiation is consequently treated as a surface heat flux.

Turbulence is important in most large-scale geophysical, fluid-mechanical problems. Due mainly to a lack of understanding of the fundamental mechanisms involved, the means of incorporating turbulence into numerical models has so far remained relatively simple, particularly through the use of constant eddy coefficients.⁷⁻¹⁴ Because vertical temperature variation has been found to affect the effective vertical-eddy-coefficient value,¹⁵⁻¹⁷ the coefficient is not taken to be constant but to be a function of the local vertical temperature gradient.

The present numerical model allows for variations in the depth of the basin into which the discharges are made. A standard numerical procedure to fit the variable depth into a model is to vary the number of vertical points in the computational mesh according to the local depth.¹⁸ A seemingly more complicated procedure, although simpler in many respects, is to stretch the vertical coordinate to the local depth. The equations are transformed according to

$$x \leftrightarrow \bar{x},$$

$$y \leftrightarrow \bar{y},$$

and

$$z \leftrightarrow z/h(x, y).$$

Although the equations to be solved appear more complicated, they are solved over a constant-depth basin in the transformed system. This greatly reduces programming complexities and in the long run makes the inclusion of depth variation simpler.

The assumption of gradual variations in the depth allows the use of a reduced form of transformed diffusion terms. Because the transformation used is nonconformal, these terms involve cross-derivatives of the spatial coordinates. The terms containing derivatives of the depth are neglected with respect to those terms containing only the depth. This approximation is used in meteorological problems when topographic variations are included.^{19,20}

The resulting system of transformed equations, in nondimensional form, is

$$\begin{aligned} \frac{1}{h} \frac{\partial(hu)}{\partial x} + \frac{1}{h} \frac{\partial(hv)}{\partial y} + \frac{\partial \Omega}{\partial \sigma} &= 0, \\ \frac{\partial u}{\partial t} + \text{Re} \left[\frac{1}{h} \frac{\partial(hu^2)}{\partial x} + \frac{1}{h} \frac{\partial(huv)}{\partial y} + \frac{\partial \Omega u}{\partial \sigma} \right] + R_o v &= - \frac{\partial P}{\partial x} \\ &- \frac{\text{Re}}{\text{Fr}^2} \left[h \int_0^\sigma \frac{\partial \Delta \rho}{\partial x} d\sigma + \frac{\partial h}{\partial x} \left(\int_0^\sigma \Delta \rho d\sigma - \sigma \Delta \rho \right) \right] \\ &+ \frac{1}{h} \left[\frac{\partial}{\partial x} \left(h \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial u}{\partial y} \right) \right] + \left(\frac{b_o}{h_o} \right)^2 \frac{1}{h^2} \frac{\partial}{\partial \sigma} \left(\gamma \frac{\partial u}{\partial \sigma} \right), \\ \frac{\partial v}{\partial t} + \text{Re} \left[\frac{1}{h} \frac{\partial(huv)}{\partial x} + \frac{1}{h} \frac{\partial(hv^2)}{\partial \sigma} + \frac{\partial \Omega v}{\partial \sigma} \right] - R_o u &= - \frac{\partial P}{\partial y} \\ &- \frac{\text{Re}}{\text{Fr}^2} \left[h \int_0^\sigma \frac{\partial \Delta \rho}{\partial y} d\sigma - \frac{\partial h}{\partial y} \left(\int_0^\sigma \Delta \rho d\sigma - \sigma \Delta \rho \right) \right] \\ &+ \frac{1}{h} \left[\frac{\partial}{\partial x} \left(h \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial v}{\partial y} \right) \right] + \left(\frac{b_o}{h_o} \right)^2 \frac{1}{h^2} \frac{\partial}{\partial \sigma} \left(\gamma \frac{\partial v}{\partial \sigma} \right), \end{aligned}$$

and

$$\begin{aligned} \text{Pr} \left\{ \frac{\partial \Delta T}{\partial t} + \text{Re} \left[\frac{1}{h} \frac{\partial(hu \Delta T)}{\partial x} + \frac{1}{h} \frac{\partial(hv \Delta T)}{\partial y} + \frac{\partial \Omega \Delta T}{\partial \sigma} \right] \right\} &= \frac{1}{h} \left[\frac{\partial}{\partial x} \left(h \frac{\partial \Delta T}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial \Delta T}{\partial y} \right) \right] \\ &+ \left(\frac{b_o}{h_o} \right)^2 \frac{1}{h^2} \frac{\partial}{\partial \sigma} \left(\beta \frac{\partial \Delta T}{\partial \sigma} \right), \end{aligned}$$

where

$$\Delta \rho = f(\Delta T),$$

$$\frac{1}{h} \frac{\partial P}{\partial \sigma} = \frac{\text{Re}}{\text{Fr}^2} (1 + \Delta \rho),$$

$$\sigma = z/h(x, y),$$

and

$$\Omega = \frac{1}{h} \left[w - \sigma \left(u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} \right) \right] = \frac{d\sigma}{dt}.$$

The conservative form of the convective terms is used, as Arakawa²¹ has found this to be advantageous for numerical computations. Density is taken as a function of temperature only, and for applications to thermal outfalls,⁸ pressure effects on density are not important. The energy equation is written in terms of temperature differences. (The effect of round-off error will be less in the evaluation of the derivatives if the differences are used.)

The rigid-lid condition is difficult to apply in a numerical solution of the above system of equations. To alleviate this difficulty, we can derive an additional equation (a Poisson equation for the surface pressure), which contains the rigid-lid condition. This derivation is accomplished by taking the divergence of the vertically integrated, horizontal-momentum equations and using the vertically integrated continuity and hydrostatic-pressure equations. The Poisson equation is

$$\begin{aligned} \frac{\partial^2 P_s}{\partial x^2} + \frac{\partial^2 P_s}{\partial y^2} = & - \frac{\partial}{\partial x} \int_0^1 A \, d\sigma - \frac{\partial}{\partial y} \int_0^1 B \, d\sigma - \frac{\partial}{\partial t} [w(\sigma = 0)] \\ & + \frac{1}{h} \frac{\partial h}{\partial x} \int_0^1 A' \, d\sigma + \frac{1}{h} \frac{\partial h}{\partial y} \int_0^1 B' \, d\sigma, \end{aligned}$$

where

$$\begin{aligned} A = & \text{Re} \left(\frac{1}{h} \frac{\partial hu^2}{\partial x} + \frac{1}{h} \frac{\partial huv}{\partial y} + \frac{\partial \Omega u}{\partial \sigma} \right) + R_{0v} + \frac{\text{Re}}{Fr^2} \left[h \int_0^\sigma \frac{\partial \Delta \rho}{\partial x} \, d\sigma \right. \\ & \left. + \frac{\partial h}{\partial x} \left(\int_0^\sigma \Delta \rho \, d\sigma - \sigma \Delta \rho \right) \right] - \frac{1}{h} \frac{\partial}{\partial x} \left(h \frac{\partial u}{\partial x} \right) - \frac{1}{h} \frac{\partial}{\partial y} \left(h \frac{\partial u}{\partial y} \right) - \left(\frac{b_0}{h_0} \right)^2 \frac{1}{h^2} \frac{\partial}{\partial \sigma} \left(\gamma \frac{\partial u}{\partial \sigma} \right), \\ B = & \text{Re} \left(\frac{1}{h} \frac{\partial huv}{\partial x} + \frac{1}{h} \frac{\partial hv^2}{\partial y} + \frac{\partial v \Omega}{\partial \sigma} \right) - R_{0u} + \frac{\text{Re}}{Fr^2} \left[h \int_0^\sigma \frac{\partial \Delta \rho}{\partial y} \, d\sigma \right. \\ & \left. + \frac{\partial h}{\partial y} \left(\int_0^\sigma \Delta \rho \, d\sigma - \sigma \Delta \rho \right) \right] - \frac{1}{h} \frac{\partial}{\partial x} \left(h \frac{\partial v}{\partial x} \right) - \frac{1}{h} \frac{\partial}{\partial y} \left(h \frac{\partial v}{\partial y} \right) - \left(\frac{b_0}{h_0} \right)^2 \frac{1}{h^2} \frac{\partial}{\partial \sigma} \left(\gamma \frac{\partial v}{\partial \sigma} \right), \end{aligned}$$

$$A' = -A - \frac{\partial P_s}{\partial x},$$

and

$$B' = -B - \frac{\partial P_s}{\partial y}.$$

The term P_s is the integration constant resulting from the vertical integration of the hydrostatic-pressure equation and is the surface pressure, i.e., the pressure at the surface $z = 0$. It is a function of both x and y . This surface pressure can be interpreted in terms of the equivalent hydrostatic height of water above or below the surface $z = 0$.

In this way, surface displacements neglecting gravity waves can be compared between this rigid-lid model and the equivalent free-surface model.

To eliminate accumulative error in the satisfaction of the continuity equation, a corrective procedure⁸ is used in the numerical solution. Therefore the vertical velocity at the surface $z = 0$ has not been set to zero in the right-hand side of the Poisson equation for the surface pressure.

The following boundary conditions are used:

$$\text{Inlet: } \begin{cases} u = g_1(y, z); \\ v = g_2(y, z); \\ \Delta T = g_3(y, z). \end{cases}$$

$$\text{Shore: } \begin{cases} u = 0; \\ v = 0; \\ \frac{\partial \Delta T}{\partial n} = 0. \end{cases}$$

$$\text{Bottom: } \begin{cases} u = 0; \\ v = 0; \\ w = 0; \\ \frac{\partial \Delta T}{\partial z} = 0. \end{cases}$$

$$\text{Surface:} \quad \left\{ \begin{array}{l} \frac{\partial u}{\partial z} = \tau_x; \\ \frac{\partial v}{\partial z} = \tau_y; \\ \frac{\partial \Delta T}{\partial z} = K \Delta T; \\ w = 0. \end{array} \right.$$

$$\text{Outer boundary, no outer flow:} \quad \left\{ \begin{array}{l} \frac{\partial u}{\partial n} = 0; \\ \frac{\partial v}{\partial n} = 0; \\ \frac{\partial \Delta T}{\partial n} = 0. \end{array} \right.$$

Outer boundary, outer flow: same as above, except flow is specified along one boundary.

Pressure Conditions: $\frac{\partial P}{\partial n} = \text{integrated x- or y-momentum equation,}$

specify pressure level at one point.

The functional forms g_1 , g_2 , and g_3 are the specified velocity and temperature profiles across the outfall inlet. The bottom and shore are taken as no-slip, impermeable, insulated surfaces. A heat-transfer condition proportional to the temperature difference⁸ and a wind-dependent stress are imposed at the surface. Normal-derivative pressure-boundary conditions are derived from the appropriate vertically integrated momentum equation. For the numerical problem, open-water boundaries must be taken at some finite distance. How the boundary conditions are specified at these boundaries depends somewhat on the whim of the investigator. Roache²² discusses the outer-boundary conditions for incompressible flow. His main conclusion is that the actual boundary conditions used are important only in that they do not severely restrict the flow. The boundary conditions used are that the normal derivatives of the velocities and the temperature are zero. The only variation to this occurs when there is an assumed flow in the lake. For these situations, the actual flow is specified along one of the boundaries.

d. The Two-dimensional, Depth-averaged Model

Once the three-dimensional version is understood, the development of the two-dimensional, time-dependent model is conceptually simple. In this

model, the three-dimensional equations are reduced to two-dimensional equations by an integration or averaging over depth. The vertical structure and the effects of buoyancy are lost, yet the gross features of the flow may be generally indicated. Vertical averaging in the two-dimensional model greatly simplifies the equations and reduces computer time considerably.

The dispersion of heat in the model is calculated, approximately, from the integrated energy equation. In this two-dimensional model, the momentum and continuity equations are uncoupled from the energy equation and must be solved first at each time step; the resulting velocities are used as input to the energy equation.

Paul and Lick assume a similarity profile, F , for velocity in the vertical direction. In this way,

$$u(x, y, z) = \bar{u}(x, y)F(z),$$

$$v(x, y, z) = \bar{v}(x, y)F(z),$$

and

$$w(x, y, z) \equiv 0.$$

Vertically averaged velocities are defined by

$$U = \int_{-h}^{\zeta} u \, dz \approx \int_{-h}^0 u \, dz$$

and

$$V = \int_{-h}^{\zeta} v \, dz \approx \int_{-h}^0 v \, dz.$$

The nonlinear terms may be integrated from the bottom $z = -h$ to the surface $z = \zeta$. For example,

$$\begin{aligned} \int_{-h}^{\zeta} u \frac{\partial u}{\partial x} dz &= \bar{u} \frac{\partial \bar{u}}{\partial x} \int_{-h}^{\zeta} F^2(z) dz \\ &= \frac{\bar{u} \frac{\partial \bar{u}}{\partial x} \int_{-h}^{\zeta} F^2(z) dz}{\left(\int_{-h}^{\zeta} F(z) dz \right)^2}. \end{aligned}$$

When integrated from $z = -h$ to $z = \zeta$, the basic three-dimensional equations of the Paul-Lick model become:

$$\text{Continuity: } \frac{\partial \zeta}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0;$$

$$\begin{aligned} \text{x momentum: } \frac{\partial U}{\partial t} + \beta \left(\frac{\partial U^2}{\partial x} + \frac{\partial UV}{\partial y} \right) - R_o V &= -\frac{1}{\rho} \frac{\partial P}{\partial x} \\ &+ A_H \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) + \rho \tau_x - \rho \tau_x^B; \end{aligned}$$

$$\begin{aligned} \text{y momentum: } \frac{\partial V}{\partial t} + \beta \left(\frac{\partial UV}{\partial x} + \frac{\partial V^2}{\partial y} \right) + R_o U &= -\frac{1}{\rho} \frac{\partial P}{\partial y} \\ &+ A_H \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) + \rho \tau_y - \rho \tau_y^B; \end{aligned}$$

where

$$P(x, y, t) = \int_{-h}^{\zeta} P(x, y, z, t) dz$$

and

$$\beta = \frac{\int_{-h}^{\zeta} F^2(z) dz}{\left[\int_{-h}^{\zeta} F(z) dz \right]^2}.$$

An estimate or reasonable guess is made for the function $F(z)$ to evaluate the β integral.

To reduce computer time, Paul and Lick again make the rigid-lid approximation. In this two-dimensional model, the rigid-lid approximation is included by setting $\partial \zeta / \partial t = 0$ in the continuity equation. The continuity, x-momentum, and y-momentum equations are solved explicitly as was done in the three-dimensional version of the model. Paul and Lick have also generalized this model to treat variable-bottom topography in which the two-dimensional nature of the model is kept, each resulting velocity component representing an average over the local depth. The same coordinate stretching procedure that was used in the three-dimensional model (see Sec. II.K) was also used in the two-dimensional case for treating variable-bottom topography.

e. Numerical Procedure

The general arrangement of variables in the grid system is almost identical to that used by Paul and Lick.⁸ Horizontal velocities are defined at integral nodal points, temperature is defined at half-integral nodal points in the horizontal and integral nodal points in the vertical, and surface pressure

is defined at half-integral nodal points in the horizontal. Figure B.14 shows typical horizontal and vertical grid sections and the relative positions of the variables.

Fig. B.14. Arrangement of Variables in Grid Sections

For the derivation of the finite-difference equations, variables are sometimes required at points where they are not defined. In these circumstances, the undefined quantity is taken as the simple average of the neighboring values.

The finite-difference approximations to the equations are derived by integrating the equations over nodal cells. Either the midpoint or the trapezoidal-integration rule is used to evaluate these integrals. Typical nodal cells are indicated in Fig. B.15. Part of the rationale behind the arrangement of the variables is to provide cells that lend themselves to easy use of the integration rules.

Fig. B.15. Typical Nodal Cells for Grid System

The Euler explicit-time scheme is used exclusively in the present model. In this scheme, the time derivative is written as a simple forward time difference and the rest of the equation is evaluated with the previously calculated values. The three-time-level explicit-time scheme used by Paul and Lick⁸ has been found not as stable as the Euler explicit scheme. Even the filtering procedure suggested by Bryan¹⁰ for the three-time-level scheme did not provide enough stability when inertially dominated flows are calculated.

The following scheme is used for solving the difference equations:

- (1) Values from the previous time step are assumed to be available.
- (2) Surface pressure is calculated with the right-hand side of the equation evaluated from the previous time-step values.
- (3) Temperature is calculated by the explicit time scheme.
- (4) Density is calculated from the equation of state.
- (5) Horizontal velocities are calculated by the explicit time scheme.
- (6) Vertical velocity is calculated by vertically integrating the continuity equation from the bottom.
- (7) The present time step is now complete.

At each time step the Poisson equation for the surface pressure is solved by the alternating-direction-implicit (ADI) method,^{23,24} a method that has been particularly efficient in previous calculations.⁸

The forcing term in the Poisson equation for the surface pressure involves a time derivative of the vertical velocity at the surface. Under the rigid-lid condition, this velocity and its vertical derivatives are zero; however, the numerical vertical integration of the continuity equation can yield some nonzero value for the vertical velocity at the surface. Deviation from zero indicates that the continuity equation is not satisfied exactly by the difference solution. If not accounted for, this error in the satisfaction of the continuity equation can grow in time.

To correct for this error, we used a procedure devised by Hirt and Harlow,²⁵ in which the term $(\partial/\partial t)[\Omega(z = 0)]$ is first written as a simple backward time difference. The value of Ω at the present time step is then set to zero, and the one from the previous time step is set to the value calculated by integration of the continuity equation. This corrective procedure was effective in river-discharge problems.^{7,8}

After the temperatures are calculated by the explicit time scheme, they are checked to see if static stability is satisfied, i.e., if the temperatures

decrease monotonically downward (assuming that density increases with decreasing temperature). When a static instability is encountered, an infinite mixing procedure is used;⁸ i.e., the temperature is arithmetically averaged over any unstable region.

f. Applications of the Numerical Model to Point Beach

The grid layout used for the Point Beach Power Plant thermal outfall is shown in Fig. B.16. Near the outfall the grid spacing is constant, but it increases away from the outfall. Finally, the grid spacing becomes constant in each direction. Such a grid system is used to pick up details of the flow near the inlet where quantities change rapidly, and to avoid an excessive number of points away from the inlet where quantities are changing a great deal more slowly. Constant spacing is used in the vertical direction.

Fig. B.16. Horizontal Grid Layout for Point Beach Outfall

The shoreline, where the water depth becomes about 1 m, is the one used, not the one where the water depth goes to zero. Extremely small depths are avoided because the coordinate stretching transformation is singular for zero depths. The bottom depths used in the model were determined by overlaying the grid system on a map of the lake-bottom topography and reading the appropriate values.

Actually, the Point Beach discharge canal extends out into the water at 60° . This geometry is approximated in the model by a rectangular channel, perpendicular to the shore, with outflow velocity at 60° to the shore. Figure B.17 compares the discharge configuration of the actual outfall and that used in the model. A check of this discharge used in the model was run on a two-dimensional, constant-density, constant-depth version of the model to see

if the outfall would continue to go at 60° to the shore; results indicated that this schematization did not change the direction of the flow.

Fig. B.17. Comparison of Discharge Configurations

The extremely large difference between the minimum and maximum grid spacing for this application can create problems in the solution of the equations because the allowable step size in the time integration is limited by the smallest grid spacing. This difficulty is overcome by dividing the grid into two regions. One region is near the inlet, where the grid spacing is small; the other region includes the rest of the grid, where the spacings are much larger. Thus, different time steps can be used in different regions. For example, where the wind is zero, the ratio between the largest and smallest grid spacing is about 26, so that, for every 26 time steps in the inner region, only one time step is required in the outer region.

The vertical eddy coefficient is taken as dependent on the local vertical temperature. The actual form used is similar to that used in an application of the numerical model to the wind-driven and thermally driven circulation in a small pond.²⁶ It is

$$A_v = a + b \frac{\partial T}{\partial Z},$$

where a and b are constants.

The constant a is chosen so that in the absence of temperature gradients, the eddy coefficient is equal to that which would be used in a constant-eddy-coefficient model.

The stress acting on the water surface due to the wind is calculated by the formulas developed by Wilson.²⁷ These formulas have been used successfully in numerical calculations of wind-driven circulations in lakes and small ponds.^{11,26,28}

The values used for the horizontal and the vertical eddy-coefficient expressions were partially determined by a trial-and-error procedure. A general behavior for the outfall is determined, i.e., how fast the velocity decays. Based upon previous experience, values for the coefficients are selected. These values are then run in a two-dimensional, constant-density (but variable-depth) version of the model.

(1) Application No. 1. The model is applied to Point Beach Power Plant outfall on Lake Michigan. The outfall extends out into the lake approximately 60° to the shore. For this calculation, the actual lake-bottom topography (see Fig. B.18) is used and the vertical eddy coefficient is taken as a function of the local vertical temperature gradient. Plant and environmental conditions submitted by Argonne²⁹ for this run are described in Table B.1. These physical parameters were acquired during a field survey on May 18, 1972. Table B.2 summarizes the relevant parameters for input into the Paul-Lick model.

Parameters used in these calculations were considered and adapted as discussed here. The outfall-velocity profile was a smoothed average of the measured, with temperature taken as constant. The equilibrium temperature of the water body was taken as the measured temperature of the lake surface. The turbulent Prandtl number was taken as unity. The values for the eddy coefficients were chosen as follows: From outfall measurements, the distance required for the velocity to decay to one-half of the outfall value was determined. A reasonable value for the horizontal coefficient was picked based on previous experience. The ratio of the horizontal to vertical coefficient was then chosen so that it gave the same velocity decay as a two-dimensional boundary-layer jet with bottom friction. The functional form for the vertical eddy coefficient (see Table B.2) is the same as discussed by Sundaram et al.^{30,31} The surface heat-transfer coefficient is calculated from the work of Edinger and Geyer.³² The equation of state is obtained by a least-squares fit of density-versus-temperature data.

The results are shown in Figs. B.19-32. Horizontal velocities at the surface and at the 2.1-m depth are shown in Figs. B.19 and 22.

Fig. B.18. Bottom Topography at Point Beach Unit 1 as Simulated in the Model for May 18, 1972.

TABLE B.1. Measured Parameters Acquired during Field Survey of Point Beach Unit 1 Discharge of May 18, 1972

Outfall Dimensions: Depth = 4.2 meters
Width = 10.78 meters

$T_A = 9.5^\circ\text{C}$

$T_O = 18.0^\circ\text{C}$

$T_I = 7.2^\circ\text{C}$

$T_{DB} = 10.0^\circ\text{C}$

$Q = 24.7 \text{ m}^3/\text{sec}$

Plant Load = 495 MW_e

Ambient Current = 0 - 0

Wind = 0 - 0

Lake Surface Conditions = Calm

Cloud Cover = Clear

Relative Humidity = 75%

Depth (M)	T_A ($^\circ\text{C}$)
0	9.5
.5	9.2
1.0	8.5
1.5	8.3
2.0	7.7
2.5	7.6
3.0	7.5

Definition of Terms:

T_A : Ambient Temperature ($^\circ$ Centigrade)

T_O : Outfall Temperature ($^\circ$ Centigrade)

T_I : Intake Temperature ($^\circ$ Centigrade)

T_{DB} : Dry Bulb Temperature ($^\circ$ Centigrade)

Q : Discharge Flow Rate (meters³/sec)

U_A : Ambient Velocity (cm/sec) measured at 1 m depth

N.A.: Not Available - Not Measured

Current Dimensions: Clockwise angle from north towards the direction the current is flowing

Wind Direction: Clockwise angle from north from the direction the wind is coming from

Distribution of Outfall Velocities within the Channel:

Outfall Velocities (m/sec)

	← 2.67 →	← 1.78 →	← 1.78 →	← 1.78 →	← 2.67 →	
.75	.633	.572	.903	.572	.633	.75
1.25	.511	.663	.723	.663	.511	1.25
1.75	.602	.753	.663	.753	.602	1.75
2.25	.482	.602	.723	.602	.482	2.25
2.75	.482	.602	.693	.602	.482	2.75
3.25	.511	.542	.633	.542	.511	3.25
3.75	.333	.511	.602	.511	.333	3.75
4.2	.347	.421	.482	.421	.347	4.2

$Q = 25.09 \text{ m}^3/\text{sec}$

← 10.78 →

TABLE B.2. Physical Parameters for Input to Paul-Lick Model for Point Beach Outfall Application No. 1:
Zero Current and Wind

Flow rate	24.7 m ³ /sec
Outfall width	10.8 m
Outfall depth	4.2 m
Equilibrium temperature	9.5°C
Turbulent Prandtl number	1.0
Vertical eddy coefficient	$\left(50 + 200 \frac{\partial \Delta T}{\partial z}\right) \text{ cm}^2/\text{sec}$
Horizontal eddy coefficient	1000 cm ² /sec
Surface heat-transfer coefficient	30 W/m ² -°C
Wind stress	0
Temperature variation	8.5°C
Maximum velocity	0.9 m/sec
Turbulent Reynolds number	110
Froude number	0.14
Densimetric Froude number	4.2
Equation of state	$\Delta \rho = -1.25 \times 10^{-3} \Delta T$

Fig. B.19

Horizontal Velocities at Surface as
Predicted by Paul-Lick Model for
Point Beach Unit 1, May 18, 1972

Fig. B.20

Horizontal Velocities at 0.54-m Depth
as Predicted by Paul-Lick Model for
Point Beach Unit 1, May 18, 1972

Fig. B.21

Horizontal Velocities at 1.0-m Depth
as Predicted by Paul-Lick Model for
Point Beach Unit 1, May 18, 1972

Fig. B.22
Horizontal Velocities at 2.1-m Depth
as Predicted by Paul-Lick Model for
Point Beach Unit 1, May 18, 1972

Fig. B.23

Horizontal Velocities at 3.0-m Depth
as Predicted by Paul-Lick Model for
Point Beach Unit 1, May 18, 1972

Fig. B.24
Horizontal Isotherms at the Surface
as Predicted by Paul-Lick Model for
Point Beach Unit 1, May 18, 1972

Fig. B.25

Horizontal Velocities at Surface as Predicted for Point Beach Unit 1, May 18, 1972: Constant-temperature, Constant-depth, Two-dimensional Version of Paul-Lick Model, $Re = 11$, $\tau_x^B = \tau_y^B = 0.3$, $\alpha = 35^\circ$

Fig. B.26

Horizontal Velocities at Surface as Predicted for Point Beach Unit 1, May 18, 1972: Constant-temperature, Constant-depth, Two-dimensional Version of Paul-Lick Model, $Re = 11$, $\tau_x^B = \tau_y^B = 0.3$, $\alpha = 30^\circ$

Fig. B.27

Horizontal Velocities at Surface as Predicted for Point Beach Unit 1, May 18, 1972: Constant-temperature, Constant-depth, Two-dimensional Version of Paul-Lick Model, $Re = 11$, $\tau_x^B = \tau_y^B = 0.3$, $\alpha = 20^\circ$

Fig. B.28

Horizontal Velocities at Surface as Predicted for Point Beach Unit 1, May 18, 1972: Constant-temperature, Constant-depth, Two-dimensional Version of Paul-Lick Model, $Re = 11$, $\tau_x^B = \tau_y^B = 0.3$, $\alpha = 25^\circ$

Fig. B.29

Horizontal Velocities at the Surface as Predicted for Point Beach Unit 1, May 18, 1972: Constant-temperature, Variable-depth, Two-dimensional Version of Paul-Lick Model, $Re = 11$, $\tau_x^B = \tau_y^B = 0.3$, $\alpha = 20^\circ$

Fig. B.30

Horizontal Velocities at the Surface as Predicted for Point Beach Unit 1, May 18, 1972: Constant-temperature, Variable-depth, Two-dimensional Version of Paul-Lick Model, $Re = 11$, $\tau_x^R = \tau_y^R = 0.3$, $\alpha = 25^\circ$

Fig. B.31

Horizontal Velocities at the Surface as Predicted for Point Beach Unit 1, May 18, 1972: Constant-temperature, Variable-depth, Two-dimensional Version of Paul-Lick Model, $Re = 11$, $\tau_x^B = \tau_y^B = 0.3$, $\alpha = 30^\circ$

Fig. B.32

Horizontal Velocities at the Surface as Predicted for Point Beach Unit 1, May 18, 1972: Constant-temperature, Variable-depth, Two-dimensional Version of Paul-Lick Model, $Re = 11$, $\tau_x^B = \tau_y^B = 0.3$, $\alpha = 35^\circ$

The increased surface velocities to the left of the outfall centerline are probably the results of a changing bottom topography. Figure B.18 indicates that the water shallows more quickly on the left side of the outfall, forcing fluid to rise and resulting in larger velocities. The surface isotherms, shown in Fig. B.24, indicate a more jet-like behavior than did the earlier Cuyahoga River applications.⁷ Because of the somewhat larger initial densimetric Froude number, buoyancy is less important. The plume is fairly well mixed vertically near the outfall, resulting in a vertical eddy coefficient that is almost constant. The general pattern of the results is similar to that observed in the field.

(2) Discussion of Results of Application No. 1. The above simulation is made assuming zero wind and no ambient stratification. Ambient stratification was originally simulated in the run, but the time required to reach a solution would have been too long to be practicable.

Only the surface isotherms are presented, since the lower isotherms were similar.

Internal oscillations in the velocities were noted as time progressed for the region near the outfall. The period of these oscillations was close to that of an internal gravity wave traveling across the outfall.

The velocity vectors are not plotted at grid points, but are interpolated from grid points.

To indicate that the curving of the jet to the left (looking offshore) is due to the bottom topography, the constant-temperature, two-dimensional version of the model was run. (See Figs. B.25-28 compared to B.29-32.) The angle α refers to the inclination of the plotted profiles. The constant-depth case gives a symmetric jet; the variable-depth jet does curve to the left. The angle at which the results are plotted has much to do with how good or bad the results appear.

A nonsymmetric inlet profile was run to check the sensitivity of the trajectory of the jet to the distribution of outfall velocities. The constant-temperature 2-D model predicted profiles that were fuller on one side of the jet, yet the trajectory of this case remained nearly the same as the trajectory of the jet with symmetric outfall velocities. The two-dimensional results included were for $Re = 11$, but cases were run up to $Re = 110$ with the same relative bottom friction; the results indicated no change in trajectory.

(3) Application No. 2. Table B.3 describes plant and environmental conditions submitted by Argonne²⁹ for this run. A moderate current and wind were present. The physical parameters were acquired during a field survey on that date. Table B.4 summarizes the relevant parameters for input into the Paul-Lick model. Unless otherwise stated, the input conditions such as diffusivities and surface heat-transfer formulas are identical to the first case run. The results are shown in Figs. B.33-42.

(4) Discussion of Results of Application No. 2. Internal gravity waves generated in this case when the program was started up took an extremely long time to die out. The movement of isotherms back and forth as time progresses is observable. Near the outfall, the oscillations were restricted, but far from the outfall, these oscillations were larger.

Some averaged (eyeball) surface-isotherm patterns are plotted. The actual isotherm pattern is typical; however, the sharpening of the 6 and 7 C° isotherms back to the right (looking offshore) is due to a recirculation cell (see Figs. B.38 and 40). Due to the averaging required, meaningful horizontal isotherm plots were difficult to present at other levels. Instantaneous plots did indicate decreasing isotherm areas with depth.

Velocity plots at three different inclinations were included so that a more general picture could be presented.

(5) Rerun of Application No. 1.* In the hope that a better comparison would result, Paul and Lick decided to rerun²⁷ their model for Case 1, since the first run did not reach steady state. This second run made in March 1975 used 3 hr CPU time. Although Paul and Lick call this run a steady-state run,

*The text below was written by the present authors, not by Paul and Lick.

TABLE B.3. Measured Parameters Acquired during Field Survey of Point Beach Unit 1 Discharge of May 20, 1971

Outfall Dimensions: Depth = 4.2 meters
Width = 10.78 meters

$T_A = 8.3^\circ\text{C}$

$T_O = 17.5^\circ\text{C}$

$T_I = 8.9^\circ\text{C}$

$T_{DB} = 12.2^\circ\text{C}$

$Q = 25.1 \text{ m}^3/\text{sec}$

Plant Load = 478 MW_e

Ambient Current = 9.1 cm/sec -125°

Wind = 4.5 - 6.7 m/sec -270°

Lake Surface Conditions: 0- .3 m waves

Cloud Cover: N.A.

Relative Humidity: N.A.

Depth (M)	T_A ($^\circ\text{C}$)
0	8.3
.3	8.3
.9	8.3
1.8	8.2
2.7	8.2

Definition of Terms:

T_A : Ambient Temperature ($^\circ$ Centigrade)

T_O : Outfall Temperature ($^\circ$ Centigrade)

T_I : Intake Temperature ($^\circ$ Centigrade)

T_{DB} : Dry Bulb Temperature ($^\circ$ Centigrade)

Q : Discharge Flow Rate (meters³/sec)

U_A : Ambient Velocity (cm/sec) measured at 1 m depth

N.A.: Not Available - Not Measured

Current Dimensions: Clockwise angle from north towards the direction the current is flowing

Wind Direction: Clockwise angle from north from the direction the wind is coming from

Distribution of Outfall Velocities within the Channel:

Outfall Velocities (m/sec)

	← 2.67 →	← 1.78 →	← 1.78 →	← 1.78 →	← 2.67 →	
.75	.633	.572	.903	.572	.633	.75
1.25	.511	.663	.723	.663	.511	1.25
1.75	.602	.753	.663	.753	.602	1.75
2.25	.482	.602	.723	.602	.482	2.25
2.75	.482	.602	.693	.602	.482	2.75
3.25	.511	.542	.633	.542	.511	3.25
3.75	.333	.511	.602	.511	.333	3.75
4.2	.347	.421	.482	.421	.347	4.2

$Q = 25.09 \text{ m}^3/\text{sec}$

← 10.78 →

TABLE B.4. Physical Parameters for Input to Paul-Lick Model for Point Beach Application No. 2: Moderate Current and Wind

Flow rate	24.7 m ³ /sec
Maximum outfall velocity	0.9 m/sec
Outfall width	10.8 m
Outfall depth	4.2 m
Outfall temperature	17.5°C
Wind	4.5-6.7 m/sec; 270°
Ambient lake temperature (lake unstratified)	8.3°C
Ambient lake current	9.1 cm/sec, 125°
Vertical eddy coefficient	50 cm ² /sec
Horizontal eddy coefficient	1000 cm ² /sec
Surface heat-transfer coefficient	30 W/m ² -°C
Wind stress	1.0 dyne/cm ² , 270°
Equation of state	$\Delta\rho = -1.25 \times 10^3 \Delta T$
Turbulent Prandtl number	1.0
Turbulent Reynolds number	110
Froude number	0.14
Densimetric Froude number	3.7

Fig. B.33

Horizontal Velocities at Surface as Predicted by Paul-Lick Model for Point Beach Unit 1, May 20, 1971, $\alpha = 30^\circ$

Fig. B.34

Horizontal Velocities at 0.5-m Depth as Predicted by Paul-Lick Model for Point Beach Unit 1, May 20, 1971, $\alpha = 30^\circ$

Fig. B.35

Horizontal Velocities at 1.0-m Depth as Predicted by Paul-Lick Model for Point Beach Unit 1, May 20, 1971, $\alpha = 30^\circ$

Fig. B.36

Horizontal Velocities at 2.0-m Depth as Predicted by Paul-Lick Model for Point Beach Unit 1, May 20, 1971, $\alpha = 30^\circ$

Fig. B.37

Horizontal Velocities at 3.0-m Depth as Predicted by Paul-Lick Model for Point Beach Unit 1, May 20, 1971, $\alpha = 30^\circ$

Fig. B.38

Horizontal Velocities at Surface as Predicted by Paul-Lick Model for Point Beach Unit 1, May 20, 1971, $\alpha = 0^\circ$

Fig. B.39

Horizontal Velocities at 2.0-m Depth as Predicted by Paul-Lick Model for Point Beach Unit 1, May 20, 1971, $\alpha = 0^\circ$

Fig. B.40

Horizontal Velocities at Surface as Predicted by Paul-Lick Model for Point Beach Unit 1, May 20, 1971, $\alpha = 90^\circ$

Fig. B.41

Horizontal Velocities at 2.0-m Depth as Predicted by Paul-Lick Model for Point Beach Unit 1, May 20, 1971, $\alpha = 90^\circ$

Fig. B.42

Horizontal Isotherms at the Surface as Predicted by Paul-Lick Model for Point Beach Unit 1, May 20, 1971

the region beyond 200 m from the outfall was still contaminated with internal gravity waves. The problem still remains as to how to suppress those waves and/or how much additional computer time would be necessary for the plume to reach steady state. Apparently, the longer the run time, the larger the region that reaches steady state.

Figures B.43-46 show the results of this run which were discussed in Sec. II.K. Figure B.43 is a plot of the surface isotherms; Figs. B.44-46 are plots of the horizontal velocity distribution at 0.5-, 1.0-, and 2.0-m depths, respectively.

(6) Application of the Two-Dimensional Model to Point Beach.

Figures B.47 and 48 illustrate the time development of the integrated two-dimensional velocity field at Point Beach, assuming a constant bottom depth for the receiving lake. The conditions for the run are the same as those of the zero-wind and -current simulation of May 18, 1972 (Case 1). The flow was initiated by assuming an impulsive start for the discharge and steady-discharge conditions thereafter. The vortices generated on each side of the discharge as

the simulation begins are eventually swept out to infinity as the simulation continues in time. The vortices on the right side of the discharge (looking off-shore) are affected by the presence of the (right-hand) outfall corner; the left vortices are not. This is due solely to the Paul-Lick method of simulating the 60° angle of discharge by the presence of a 90° outfall discharging its effluent at a 60° angle. Paul and Lick note the same kind of phenomenon occurring in their three-dimensional variable-density simulation (Chapter II). This small distortion of the flow field near the outfall does not occur in the field, of course, since the flume as well as the discharge are directed at 60° to shore.

Fig. B.43

Surface Isotherm Predictions of the Paul-Lick Model for Point Beach Unit 1, May 18, 1972. (Second computer run.)

Fig. B.44

Horizontal Velocity Predictions at 0.5-m Depth for the Paul-Lick Model for Point Beach Unit 1, May 18, 1972. (Second computer run.)

0.5 METER DEPTH

Fig. B.45

Horizontal Velocity Predictions at 1.0-m Depth for the Paul-Lick Model for Point Beach Unit 1, May 18, 1972. (Second computer run.)

1.0 METER DEPTH

Fig. B.46

Horizontal Velocity Predictions at 2.0-m Depth
for the Paul-Lick Model for Point Beach Unit 1,
May 18, 1972. (Second computer run.)

Fig. B.47. Paul-Lick Two-dimensional Constant-depth Model Application to Point Beach Unit 1, May 18, 1972, Showing Surface Velocities at $t = 87.5$ sec (Top diagram) and 262.5 sec (Bottom diagram)

Fig. B.48. Paul-Lick Two-dimensional Constant-depth Model Application to Point Beach Unit 1, May 18, 1972, Showing Surface Velocities at $t = 937.5$ sec (Top diagram) and 1575.0 sec (Bottom diagram)

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APPENDIX C

Reprint of Published Paper on
Differential-Numerical Models for
Surface Thermal Plumes

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APPLICATION, VERIFICATION, AND IMPROVEMENT OF NUMERICAL MODELS FOR SURFACE THERMAL DISCHARGES FROM POWER-PLANT OUTFALLS

ANTHONY POLICASTRO, WILLIAM DUNN, and ROBERT PADDOCK

Argonne National Laboratory, Argonne, Illinois, U.S.A.

ABSTRACT

This paper evaluates seven numerical models for the prediction of surface thermal plumes from power-plant outfalls in terms of their present and potential capability for environmental-impact evaluation. Two of the models discussed are not presently competitive, yet represent interesting approaches. Two other models are from related problem areas and have formulations sufficiently general for future application to surface-discharge problems. The final three models are promising and have already been tested to a limited extent with prototype data.

Unfortunately, the comparisons of models to data attempted so far are few in number and have, in general, been poorly executed. The difficulties have typically involved the use of too small a computational grid, the poor choice of turbulence parameters, and the comparison to either incomplete or poor-quality data. The models discussed are currently in what might be called the germination stage. Therefore, the above-noted comparisons of model predictions with prototype data, although not sufficient for final judgments on the models, have aided in identifying major areas of needed work. Finally, recommendations are made for the general improvement of numerical models for surface thermal discharges.

LIST OF SYMBOLS

A_{H1}, A_{H2}	Longitudinal and lateral coefficients of eddy viscosity, respectively
A_V	Vertical coefficient of eddy viscosity
B_{H1}, B_{H2}	Longitudinal and lateral coefficients of eddy thermal diffusivity, respectively
B_V	Vertical coefficient of eddy thermal diffusivity
D	Molecular thermal-diffusion coefficient
E	Local turbulent kinetic energy
f	Function relating temperature to density in the equation of state
g	Acceleration due to gravity
J	Function of the spatial variables defined by integrating Ω , its spatial derivative Ω' , and a weighting function
P	Local pressure
Q	Boussinesq parameter = $\Delta\rho/\rho_E$
Ri	Local (gradient) Richardson number
T	Local temperature

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u, v, w	Components of velocity in x, y, and z directions, respectively
$\tilde{v}$	Tentative vector-velocity field
x, y	Horizontal coordinates
z	Vertical coordinate
α, β, γ	Slowly varying functions determined from data
ΔT	Excess temperature; reference temperature usually taken as the ambient temperature
ΔT_c	Local excess temperature of plume at its centerline
ΔT_o	Excess temperature of plume at the outfall
$\Delta \rho$	Density deficiency = $\rho - \rho_E$
λ	Macroscale of turbulent fluctuations
ν	Molecular kinematic viscosity
ρ	Local density
ρ_E	Reference density usually taken as ambient density
Ψ	Velocity-corrector potential
Ω	Generalized strain rate

INTRODUCTION

Waste-heat disposal from steam-electric generating facilities is a critical aspect of power-plant siting in the United States. According to U.S. law, waste heat discharged into navigable waters should be considered a pollutant, and as such must be controlled. Specifically, the Federal Water Pollution Control Act Amendments of 1972 forbid most thermal discharges after 1983. Variances will be granted, however, if it is determined that the mandates of the Act are more stringent than necessary to ensure the propagation of a balanced, indigenous population of shellfish, fish, and wildlife. In implementing the provisions of the Act, the U.S. Environmental Protection Agency will issue variances for thermal discharges on a case-by-case basis.

In a variance request, the onus of proof lies with the utility that is either using or proposing to use once-through cooling. A detailed analysis of thermal-plume characteristics, both temporally and spatially, is required. For operating plants, field measurements can be made, but for proposed plants or those under construction, the only recourse is theoretical and/or hydraulic-model studies to seek out the necessary information. This paper summarizes part of an effort carried out under the sponsorship of the U.S. Energy Research and Development Administration (formally the USAEC) to determine the reliability with which mathematical models can predict prototype plumes. A more detailed discussion of these efforts is given by Dunn, Policastro, and Paddock.¹

Available mathematical models for heated surface-discharge plumes fall into three large classes: integral, phenomenological, and differential-numerical.

The integral approach assumes a free jet and flow similarity and is therefore very limited in its applicability. Integral models can provide only general estimates of plume characteristics because they are too idealized for most prototype applications. Shoreline and bottom restrictions, wind effects, and transient environmental conditions complicate the problem beyond the capability of integral models.

Phenomenological models rely more heavily on the direct use of data and have been no more successful than integral models since the quantity of good-quality laboratory and field data amenable to the development of phenomenological

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models is small. As a result, well-developed phenomenological models exist for only a narrow range of situations.

The past few years have seen an upsurge of interest in modeling thermal plumes using advanced numerical techniques such as finite-difference methods of solving the generalized equations of flow and heat transport. Numerical modeling of thermal plumes holds great promise, since such models require fewer assumptions on the nature of plume dispersion and simultaneously enable additional physical mechanisms to be included, at least in concept. Treatment of shoreline and bottom interference, wind, ambient stratification, spatially variable outfall conditions, and ambient currents can all be incorporated into the model formulation. This paper discusses seven numerical models, existing applications to prototype surface thermal-plume situations for three of the most promising models, and the areas of needed improvement.

NUMERICAL MODEL FORMULATIONS

The discussion begins with two models that are, at present, not suitable for prototype applications, but have interesting approaches.

The model developed by Brady and Geyer² represents an early attempt at numerical modeling of thermal plumes. These authors sought to numerically solve the coupled, time-dependent equations of flow, heat conservation, and salt conservation. The hydrostatic approximation (pressure at a given point is due only to the weight of water above that point) was made, but the equation of vertical momentum conservation was retained. Since model development was stopped in 1972 before any concrete results were obtained, the model should be considered of historical interest only. Furthermore, the model does not use current state-of-the-art differencing schemes. The developers speculate that at least one additional year of effort is needed to make the model competitive.

Till³ attempted to numerically model thermal plumes using a concept that avoids the difficult Navier-Stokes equations in their usual form. The technique, which we will call subchannel balancing, consists of laying out rectangular parallelepiped subchannels in the direction of flow. At each location within the subchannel, the net mass, momentum, and heat-energy inflows are balanced against the time rate of change of the corresponding physical variable within the subchannel. This procedure leads to a set of several hundred coupled, non-linear ordinary differential equations, which are solved as an initial-value problem in a single independent spatial variable. The technique has not yet been shown adequate for problems where the flow is other than principally one-dimensional. In fact, the current formulation of the model has been disappointing, even for rivers, a situation apparently well suited to the subchannel balancing method. The technique is promising, since it is fairly easy to implement numerically and since it is considerably faster computationally than the other methods to be discussed. Further use of the model, however, would require a much improved formulation.

We will now discuss two other models, which, because of the generality of their approach, appear promising for application to surface thermal-plume problems, but which have not yet been so applied.

The Chien-Schetz model⁴ was developed to treat shallow and deep-water submerged discharges, typically in rivers. This model solves the three-dimensional, steady-state governing equations written in terms of vorticity, velocity, and temperature, with the Boussinesq approximation (density variations considered only insofar as they produce buoyancy) made. The introduction of vorticity removes pressure from the governing equations and further allows all the equations to be written in Poisson form. To do this, various terms are collected on the right-hand side of the equation and regarded as a known source. The hydrostatic approximation is not made, but the rigid-lid condition (zero surface-height elevations) is imposed. Because the governing equations are all written in Poisson

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form, standard central differencing and the SOR (successive-overrelaxation) method can be used. In evaluating the forcing function of the Poisson-type equations, the advection terms are upwind differenced. The computer code permits both uniform and variable grid spacing. Computer simulations have been published for a submerged turbulent jet in a crossflow (deep water) and for the laminar flow in the entrance region of a square channel. In both cases, agreement with experimental data was obtained. Continuing work involving model improvements as well as applications to submerged shallow-water heated discharges is about to be published. A user's manual is being prepared. Application to heated surface discharges involves only a change in boundary conditions and turbulence coefficients. Such a change in the model (computer code) is not considered to be a major one.

Wnek *et al.*⁵ has described a compressible three-dimensional thermal-hydraulics code (SCORE-EVET) for use in the design and analysis of nuclear reactors. The original development of the model was done at the Los Alamos Scientific Laboratory with further work done by Science Applications, Inc., of LaJolla, California, and the Aerojet Nuclear Corp., of Idaho Falls, Idaho. The model exists in several versions, but the one most suitable for application to thermal-plume problems is a code for incompressible flow known as VARMINT. The time-dependent, three-dimensional Navier-Stokes and heat-transport equations are solved using the MAC (marker and cell) method.⁶ Briefly, the MAC method is a finite-difference scheme in which pressure is defined at the nodes of the grid system and the velocities are defined at the face centers of the grid cells. Also, in the MAC method a set of massless marker particles is followed through the grid system by time-integrating the velocity field through which the particles move. The code is written to accept either no-slip, free-slip, inflow, outflow, or specified pressure boundary conditions. No attempts have been made to modify the VARMINT code for thermal-plume problems, although the code and its compressible version (SCORE-EVET) have been used for reactor analysis. Such modifications are not considered to be major.

We turn now to the formulation of three numerical models that have been tested with prototype field data from existing surface-discharge sites. The three models to be discussed below solve the traditional form of the three-dimensional, time-dependent equations of turbulent, incompressible (in the Boussinesq sense) flow and heat transport. The generic form of these equations is given in Table 1.

FORMULATION OF THE PAUL-LICK MODEL⁷⁻⁸

The Paul-Lick model uses the hydrostatic form of the vertical-momentum equation; i.e., only the gravity and vertical pressure-gradient terms are retained. Variable bottom is handled through a local stretching transformation that maps an irregular bottom onto one of constant depth. The more complicated equations thus obtained are not shown.

Figure 1 illustrates boundary conditions typically imposed on the solution domain when executing finite-difference solutions. In particular, Paul and Lick used no-slip and adiabatic conditions at the shore and bottom grid boundaries, wind-stress and surface-heat-loss conditions on the free surface, and zero second-order normal derivatives at the open-water boundaries. In addition, they used the rigid-lid condition (zero surface-height elevations). Such an assumption is desirable since it artificially damps out unwanted oscillations of the free surface and allows larger time steps to be used. To compensate for the restrictions of the rigid-lid condition, the authors introduced the surface pressure, which ostensibly corresponds to the pressure distribution that would have existed at the rigid-lid plane had surface elevations been allowed. An auxiliary two-dimensional Poisson-type equation is developed for the surface pressure distribution by taking the divergence of the Navier-Stokes equations and then integrating vertically over the domain of flow.

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Table 1. Generalized Governing Equations for Surface Thermal-Plume Problems

Conservation of Mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Conservation of Longitudinal Momentum

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left(A_{H1} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(A_{H2} \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left(A_V \frac{\partial u}{\partial z} \right)$$

Conservation of Lateral Momentum

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \frac{\partial}{\partial x} \left(A_{H1} \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left(A_{H2} \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left(A_V \frac{\partial v}{\partial z} \right)$$

Conservation of Vertical Momentum

$$\left\{ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right\} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + g + \left\{ \frac{\partial}{\partial x} \left(A_{H1} \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left(A_{H2} \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial z} \left(A_V \frac{\partial w}{\partial z} \right) \right\}$$

Conservation of Heat Energy

$$\frac{\partial \Delta T}{\partial t} + u \frac{\partial \Delta T}{\partial x} + v \frac{\partial \Delta T}{\partial y} + w \frac{\partial \Delta T}{\partial z} = \frac{\partial}{\partial x} \left(B_{H1} \frac{\partial \Delta T}{\partial x} \right) + \frac{\partial}{\partial y} \left(B_{H2} \frac{\partial \Delta T}{\partial y} \right) + \frac{\partial}{\partial z} \left(B_V \frac{\partial \Delta T}{\partial z} \right)$$

Equation of State

$$\rho = f(T)$$

- Notes: (1) The Paul-Lick and Waldrop-Farmer models make the hydrostatic approximation; consequently, terms in braces in the vertical momentum equation are eliminated.
- (2) In the present form of the Paul-Lick and Waldrop-Farmer models, $A_{H1} = A_{H2} = \text{const.}$, $B_{H1} = B_{H2} = \text{const.}$ The vertical viscosity and eddy diffusivity are variable.
- (3) In the Pritchett-England-Taft model, the turbulent energy equation is solved for the local turbulent kinetic energy at each time step using

$$\begin{aligned} \frac{\partial}{\partial t} (E) + \frac{\partial}{\partial x} (uE) + \frac{\partial}{\partial y} (vE) + \frac{\partial}{\partial z} (wE) &= \alpha \lambda \sqrt{2E} \Omega^2 - \beta (2E)^{7/6} J^{1/3} \\ &+ \frac{\partial}{\partial x} \left(\alpha \gamma \lambda \sqrt{2E} \frac{\partial E}{\partial x} \right) + \frac{\partial}{\partial y} \left(\alpha \gamma \lambda \sqrt{2E} \frac{\partial E}{\partial y} \right) + \frac{\partial}{\partial z} \left(\alpha \gamma \lambda \sqrt{2E} \frac{\partial E}{\partial z} \right) - \alpha \gamma \lambda \sqrt{2E} g \frac{\partial Q}{\partial z} \end{aligned}$$

From this solution at each time step, the diffusion terms in the momentum and heat energy equations are written as

$$\frac{\partial}{\partial x_j} \left[\left(\nu + \alpha \lambda \sqrt{2E} \right) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]$$

and

$$\frac{\partial}{\partial x_j} \left[(D + \alpha \gamma \lambda \sqrt{2E}) \frac{\partial Q}{\partial x_j} \right],$$

respectively.

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Fig. 1. Typical Boundary Conditions Specified in the Formulation of Numerical Models of Surface Thermal Plumes.

The six equations of Table 1, specialized to the Paul-Lick formulation, and the Poisson-type equation for surface pressure are solved numerically using finite-difference methods. Simple forward differencing in time and central differencing in space are used. The finite-difference form of the advection terms is conservative, but the generally preferred upwind differencing is not used. Moreover, the time-dependent equations are written in a completely explicit form, which allows them to be solved as an initial-value problem. The procedure of solving the time-dependent equations for the steady-state solution is sometimes referred to as the asymptotic-time technique. The numerical solution is executed through the following five-step algorithm:

- 1) The two-dimensional Poisson-type equation is used to calculate the surface pressure from existing values of the nodal velocities. The alternating-direction-implicit method is used for this purpose.
- 2) Using existing nodal velocities, the nodal temperatures are computed using an explicit finite-difference form of the time-dependent energy equation.
- 3) Densities are calculated from the equation of state, and then, starting with the surface pressures from Step 1 above, the pressure is calculated by integrating the density field vertically downward.
- 4) With updated values of the pressure and present nodal velocities, the time-dependent x - and y -momentum equations are solved for the u and v velocities, respectively, by an explicit finite-difference expression.
- 5) The vertical velocities are computed by vertical integration of the continuity equation.

A typical run of the model to steady-state solution takes about 3 hr of CPU time on an IBM 360 computer.

FORMULATION OF THE WALDROP-FARMER MODEL⁹⁻¹²

The Waldrop-Farmer model is similar to the Paul-Lick model in that it too solves the generalized equations of Table 1 using an explicit finite-difference analog of the continuous system. Both models use the hydrostatic form of the vertical momentum equation for thermal-plume problems, although the Waldrop-Farmer model has also been formulated⁹ to solve the full three-dimensional

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equations of flow. Even for thermal-plume problems, there are several important differences between the Paul-Lick and Waldrop-Farmer models in the boundary conditions and in the finite-difference solution.

The boundary conditions imposed in the Waldrop-Farmer model are the same as those of the Paul-Lick model on the bottom and at the shoreline. At the free surface, approximate free-surface boundary conditions are used instead of the rigid-lid conditions. During execution, a mass balance is performed on the topmost level of grid cells to obtain a distribution of equivalent free-surface elevations.

An interesting facet of the open-water boundary conditions used by Waldrop and Farmer is that only the upcurrent boundary conditions are ever enforced. Because the equations are cast in the form of a sequence of explicit integration steps, the offshore and downcurrent boundary conditions are relaxed. Thus, Waldrop and Farmer are not solving the boundary-value problem at each time step and no backwater effects from the region exterior to the grid can be predicted. Moreover, the physical meaning of the transient solution may be affected by the fact that boundary conditions are imposed on only two of the four vertical sides of the grid system.

In setting up the finite-difference grid, Waldrop and Farmer use a stretching transformation, which maps a constant cell dimension in the transformed domain into a variable cell dimension in the natural domain. The computational grid is small near the outfall and becomes larger and larger as one moves away from the discharge. The model developers argue that the use of a stretching transformation lessens the importance of the far boundary conditions and, in this sense, mitigates the omission of open-water boundary conditions.

By dropping the acceleration term from the vertical-momentum equation, Waldrop and Farmer cast the problem into a sequence of first-order integrations, each of which can be written in explicit finite-difference form using simple forward differencing of the first-order terms. Waldrop and Farmer used conservative upwind differencing of the advection terms instead of the central differencing used by Paul and Lick. When only steady-state values are sought, an overrelaxed iteration scheme is used.

The finite-difference expressions for the continuity, momentum, and heat-energy equations are solved in the following order:

- 1) The x-momentum equation is solved for the u-velocity component (x direction offshore, u velocity in direction of x axis).
- 2) The y-momentum equation is solved for the v-velocity component (y direction shore-parallel, v velocity in direction of y axis).
- 3) The conservation-of-energy equation is solved for the temperature at each node.
- 4) The continuity equation (time-dependent) is integrated from the bottom to the surface for the w-velocity component (w velocity in vertical or z direction).
- 5) The free-surface elevation is computed by performing a mass balance on the topmost layer of grid cells.
- 6) The z-momentum equation is solved in space (time-dependent) for the pressure field. The integration is carried out vertically downwards from the free surface. This integration and determination of pressure at each grid row becomes simpler when the hydrostatic approximation is made.

Only Steps 1-3 involve time integrations. The remaining steps are spatial integrations. A run to steady state for a typical thermal-plume problem takes about 3 hr of CPU time on an IBM 360 computer.

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FORMULATION OF THE PRITCHETT-ENGLAND-TAFT MODEL¹³⁻¹⁵

The Pritchett-England-Taft model (entitled STUFF) solves the Navier-Stokes and heat-energy equations shown in Table 1 without making the hydrostatic approximation. A major component of the model in its present form is the Gawain-Pritchett heuristic turbulence model¹⁵ from which eddy viscosities and eddy thermal diffusivities are calculated based on the local velocity field. As part of this turbulent-transport model, an equation of turbulent-energy conservation is developed. This equation, given in the notes of Table 1, is solved by Pritchett, England, and Taft along with the other governing equations.

The Navier-Stokes equations are solved in two steps using a combination of the Chorin¹⁶ and SMAC¹⁷ (simplified marker and cell) methods. In the first step, tentative values of the velocity components are computed using a finite-difference form of the equations with only the diffusion and advection terms included. The procedure here follows the treatment of Crowley¹⁸ in which the advection terms are forward-differenced for each cell; then a diffusion-like term is added to correct for the truncation error, which, if left uncorrected, would produce artificial diffusion of momentum. Once the tentative velocities are determined, the following Poisson-type equation is solved for the scalar velocity-corrector potential Ψ :

$$\nabla^2 \Psi = -\nabla \cdot \tilde{v},$$

where $\tilde{v}$ represents the tentative velocity field determined in the first step. This solution is accomplished by means of an overrelaxed Gauss-Seidel iteration procedure. The final values of the velocity are then found by adding the gradient of Ψ to the tentative velocity field. The advantage of using the velocity-corrector potential is that direct calculation of pressure is avoided.

Once the hydrodynamic equations are solved, the equations for the scalar variables -- fluid density, heat, and turbulent energy -- are tackled. The same procedure is used for each equation since all three have the same general form. The STUFF model uses a combination Eulerian/Lagrangian scheme to solve these scalar transport equations following the SMAC method.¹⁷ In this approach, a set of Lagrangian particles is followed through the Eulerian grid system thereby avoiding numerical diffusion. Derivatives are evaluated in the Eulerian grid based upon the average of the scalar variables held by all the Lagrangian particles in a given finite-difference cell at a given time step. Marker particles are not used for the solution for the velocity components from the hydrodynamic equations, since spatial gradients and hence numerical diffusion were thought to be less important in these equations.

Steady-state run times for surface thermal-plume problems are about 4 hr of CPU time on a Univac 1108.

COMPARISONS OF MODEL PREDICTIONS WITH PROTOTYPE FIELD DATA

Considered below are several comparisons of numerical model predictions with field data taken at existing surface-discharge sites. These few comparisons comprise essentially all the testing completed so far with prototype data. The models were also run by their developers for other prototype situations, but only as modeling exercises. For example, the Paul-Lick model was run for the Cuyahoga River entering Lake Erie, and the Waldrop-Farmer model was run for the Mississippi River entering the Gulf of Mexico. No data existed for either of these cases, so only the comparison of multiple runs as a sensitivity study was possible. The STUFF model was also run for other, not thermal plume, problems. Reference 1 discusses these modeling exercises and gives a more detailed description of the model-data comparisons cited below.

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PAUL-LICK MODEL

Argonne National Laboratory sponsored the preparation of Paul-Lick model calculations by the model developers for comparison with field data taken on two dates at the Point Beach Nuclear Power Station, located on the Western shore of Lake Michigan near Two Creeks, Wisconsin. Stagnant lake conditions prevailed on May 18, 1972;¹⁹ a moderate wind and current were present on May 20, 1971.²⁰ Predicted and measured isotherms for these dates are compared in Figs. 2 and 3. In the model calculations, the solution domain was divided into two regions. In the region near the outfall, a fine-grid mesh was used; in the rest of the solution domain, a somewhat coarser mesh was used. The ratio of the largest to the smallest grid-cell dimensions was 26. Since the largest stable time increment depends on the grid spacing, different time steps were used in the two regions of the grid system to improve the efficiency with which the time integrations could be carried out. For every 26 time steps taken in the inner region, one time step was taken in the outer region.

The comparison of model predictions with the field data as carried out by Argonne National Laboratory¹ was based on trajectory, centerline temperature decay, plume widths, and isotherm areas. Our comments on these comparisons are given below.

Fig. 2. Comparison of Surface Isotherms from Measurements and Predictions of Paul-Lick Model: Point Beach Power Plant, May 18, 1972.

Fig. 3. Comparison of Surface Isotherms from Measurements and Predictions of Paul-Lick Model: Point Beach Power Plant, May 20, 1971.

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1. The comparison of predicted and measured centerline temperature decay, width, and isotherm area was good for the first 200 m from the outfall. The major disappointment with these comparisons is that, after 3 hr of calculation on an IBM 360 computer, only this very small area near the outfall reached steady state. The solution at points farther out in the grid was contaminated by internal gravity waves, which are apparently slowly damped out as computation time is increased. The Paul-Lick model is not presently well suited to environmental impact work or even mixing-zone analyses, since excessively long computer runs are needed to bring the entire solution to steady state. More testing and analysis of the model are necessary to change this situation.

2. The turbulence parameters used in model calculations were poorly chosen. A value of the horizontal eddy viscosity was first selected from open-channel flow data. The ratio of horizontal to vertical eddy viscosities was then chosen so that centerline velocity predictions made with a two-dimensional, constant-temperature, depth-averaged version of the model matched the velocity decay of a previously measured Point Beach plume. The thermal eddy diffusivities were obtained by assuming a turbulent Prandtl number of one. The partial fitting of model predictions to earlier data from the same site lessens the objectivity of the comparisons. Despite this fitting, the turbulence parameters used in model calculations may be too large (in fact, by about an order of magnitude for the vertical ones), based on values generally cited in the literature.

3. The computational grid in both comparisons contains only a portion of the total plume. An 800-m distance to the far boundary is too small. The definable region of the plume is about 2500 m along the plume trajectory, and a grid of approximately this size would have been more realistic. The small grid dimensions no doubt increase the effects of the arbitrarily specified open-water boundary conditions and, therefore, cause us to question whether the conditions used are adequate.

4. The calculations for May 20, 1971 (nonzero-current case), do not show the compacting of isotherms and the resulting sharp edge on the upcurrent side of the plume seen in the data.

The comparison of Paul-Lick model predictions with prototype data is generally encouraging, but reveals that more work is needed to improve model formulation and application.

WALDROP-FARMER MODEL

The Waldrop-Farmer model was run by the model developers for comparison with two Point Beach plumes taken September 1, 1971.²⁰ An ambient current was present on that date, but was not measured. Waldrop and Farmer inadvertently misread the tabulation of plant and environmental data and consequently made model calculations using a zero ambient current and a condenser discharge rate of 771 cfs, although the flow rate was actually 885 cfs. Predicted and measured surface isotherms are compared in Fig. 4. However, because of the errors made in schematization, the comparisons should be considered solely on a trend or order-of-magnitude basis. The run in this case required about 3 hr of CPU time on an IBM 360 computer using a grid with 29 cells in the shore-parallel direction, 17 cells in the offshore direction, and 15 cells in the vertical direction.

The comparison is obviously poor in this case, with the predicted plume considerably smaller than the actual plume. For example, at $\Delta T/\Delta T_0 = 0.4$, the predicted isotherm area is too small by about a factor of seven. Also, given the assumption of stagnant ambient conditions, the predicted trajectory should not turn offshore as it does, but rather should remain straight at a 60° angle with the shore. (The Paul-Lick model also predicts a slight bending for the stagnant case.) This bending is perhaps due to the solid-wall boundary conditions imposed at the shoreline. The simulation of the nearshore as a solid wall,

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Fig. 4. Comparison of Surface Isotherms from Measurements and Predictions of Waldrop-Farmer Model, Point Beach Power Plant: Field Data Taken September 1, 1971, at 0950-1130 and 1445-1727 Hrs.

coupled with the oblique angle of discharge, might cause water to artificially pile up on the lee side of the plume thereby pushing the trajectory offshore.

Other factors possibly contributing to the poor comparison are:

1. The vertical diffusivities were arbitrarily chosen to be 20% of the horizontal diffusivities. This assumption overemphasized vertical mixing and may be partly responsible for the extremely rapid decay of temperature and velocity that is predicted.
2. As with the Paul-Lick comparison, too small a grid was used. The model developers now realize this and recommend that further runs be made with a grid large enough so that the physical variables are at least slowly changing at the grid boundaries.
3. The compactness of the lower isotherms in the model predictions is not physically realistic and is, in fact, directly opposite to the trend seen in the data. This qualitatively incorrect behavior may be due to the small grid as noted above or is perhaps an indication that the model was not run to steady state.

A rerun of model calculations with more realistic parameters and conditions would aid greatly in identifying the sources of model-data discrepancies.

A more recent application¹² of the model was more encouraging. This time the model was applied to the John Sevier Steam Plant located on the Holston River in Eastern Tennessee. Predicted and measured surface and vertical isotherms for a date in July of 1973 are compared in Fig. 5. The code was run for four different values of horizontal diffusivity -- 20, 40, 60, and 100 cm²/sec -- with vertical diffusivity assumed to be given by²¹

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$$A_V = A_{H1} (1 + 10 Ri)^{-1/2}; A_{H2} = A_{H1},$$

$$B_V = B_{H1} (1 + \frac{10}{3} Ri)^{-3/2}; B_{H2} = B_{H1},$$

where Ri is the local gradient Richardson number. These values are more realistic than the ones used for the Point Beach case since vertical stratification is, at least, considered. The best comparison was obtained for $60 \text{ cm}^2/\text{sec}$, and this is the one presented by Waldrop and Farmer¹² and shown in Fig. 5.

A comparison under winter conditions, again run with different diffusivities, yielded, at best, only reasonable plume predictions. The measured plume, for example, penetrated deeper into the river; i.e., it moved offshore some distance before bending downcurrent. The predicted plume bent quickly downcurrent and appeared to hug the shore much more than the actual plume did.

In both the summer and winter comparisons, model predictions within 800 m of the discharge were relatively insensitive to the choice of turbulence parameters used. At 800-1600 m, however, the model predictions did exhibit noticeable sensitivity to eddy viscosity. Near the outfall, turbulent diffusion may be dominated by inertial and buoyancy forces.

The Holston River comparisons are reasonably good and encourage the testing of the model with smaller ambient currents to determine if equally satisfactory results can be obtained. The success of the model in these applications is somewhat clouded by the "fitting" of the model to the data by varying the horizontal diffusion coefficients, but the results imply that good comparisons may be possible if diffusivities are properly chosen. How diffusivities should be selected *a priori* will probably always remain a major issue.

PRITCHETT-ENGLAND-TAFT MODEL

The Pritchett-England-Taft or STUFF model has been twice compared with prototype field data for surface thermal plumes. In one case, the authors ran their model for the May 20, 1971, data taken at the Point Beach Nuclear Power

Fig. 5. Comparison of Field Data and Predictions of Waldrop-Farmer Model for the John Sevier Steam Plant on the Holston River, July 1973. Isotherms at the Surface and Through a Vertical Cross Section.

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Station.²⁰ Our subsequent comparison of model predictions with the data showed that the predicted plume was much smaller than the observed plume. However, that verification effort is invalid since it was later discovered that the model was inadvertently run with the thermal-expansion coefficient of air instead of water.

The other presently available model-data comparison was sponsored by the Boston Edison Company for the Pilgrim Nuclear Power Station located on the tidally influenced Cape Cod Bay. The data selected for comparison were a set of aerial infrared measurements made at 1302 hr on December 29, 1972, by Coastal Research Corporation.²² Model predictions are compared with these data in Fig. 6. The computer simulation was carried out using a grid of 50 mesh intervals along the shore and 25 mesh intervals in the offshore direction, each 20 m long, and two mesh intervals in the vertical direction, 3 m long. Because of computer limitations, only two vertical cells were used. The salient features of this comparison are discussed here. More details are presented in Ref. 1. For reasons cited below, we do not feel that the comparison of STUFF model predictions with the Pilgrim data is a fair test of the model.

1. The aerial infrared data are of poor quality. The data selected for comparison are from one of 11 overflights reported for December 29, 1972, by the Coastal Research Corporation. Data taken only 17 min later than those chosen for comparison showed a twofold to fivefold increase in isotherm areas. This variation is virtually impossible, given the environmental conditions on that date.

2. Ambient currents were not measured on the date of the infrared overflights. Model predictions were made using an estimate of wind-induced and tidally induced currents. Moreover, the computation of wind-induced current is based on a probably invalid correlation of wind and current speeds. The best available information suggests that the wind-induced current was probably four times the value assumed for model calculations.

Fig. 6. Comparison of Surface Isotherms from Measurements and Predictions of Pritchett-England-Taft Model: Pilgrim Nuclear Power Station, December 29, 1972, at 1302 Hrs.

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3. The model simulated the outfall as 3 m in depth (the depth of one grid cell). At low tide, when the data used for comparison were taken, the depth is less than 1 m. A calculation of initial densimetric Froude number shows that the simulation was really carried out for high-tide conditions.

4. As with the Paul-Lick and Waldrop-Farmer comparisons previously presented, the computational grid was too small. Again, the boundary conditions used would be somewhat in error, although their effect cannot be assessed until a sensitivity analysis of the model is performed.

5. The infrared technique used for making temperature measurements senses only the top millimeter of the surface. In contrast, the STUFF model calculations gave temperatures averaged over the top 3 m (one grid cell). It is difficult to interpret a comparison of these physically different quantities.

In spite of all these problems with the simulation, the STUFF model performed surprisingly well in terms of isotherm area, centerline temperature decay, and width. The model deserves further testing.

SUMMARY AND CONCLUSIONS

Numerical models for surface thermal plumes are essentially in a germination stage of development. The Brady-Geyer and Till models have interesting ideas in their development, but require more work before the models become competitive. The Chien-Schetz model and the VARMINT code have formulations that appear promising; yet these models need some modification before they may be applied directly to surface thermal-plume problems.

The Paul-Lick, Waldrop-Farmer, and Pritchett-England-Taft models already have experience in predicting prototype surface plumes. Unfortunately, the comparisons of models to data attempted so far are few in number and have, in general, been poorly executed. The difficulties have typically involved the use of too small a computational grid, a poor choice of turbulence parameters, and the use of either incomplete or poor-quality data. The comparisons described in this paper are not sufficient to make final judgments on the models, yet have aided in identifying major areas of needed work in model development and model application.

Our recommendations for future improvement of the available models are:

1. Compare the models to prototype data that are complete with outfall and ambient parameters carefully measured. Each comparison to prototype data should involve several runs of the model for sensitivity studies to ascertain the cause of discrepancies between the models and the data.
2. Evaluate different theories of turbulence in these models to determine which one(s) appear to represent plume dispersion best.
3. Determine the role of the boundary conditions in affecting plume predictions. Boundary conditions are often arbitrarily prescribed, with the effects of such specifications not presently known. Testing the sensitivity of the models to their boundary conditions, especially at the free-surface and open-water boundaries would be useful. A related matter is the sensitivity of the models to the size of the computational grid; an important issue is to determine a size that minimizes unnatural effects from the grid boundaries, yet avoids wasting computer time with unnecessary computational effort.
4. Study the finite-difference scheme itself for its conservative and transportive properties and for numerical dispersion.
5. Study the effect of boundary-condition relaxation, typically at the open-water ends of the computational region. A major question to be answered is whether the relaxation of one or more boundary conditions implies that the thermal-plume problem is no longer well posed. If the problem is indeed not

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well posed, the transient calculations with the model would have no physical meaning.

6. Test the predictions of the models with laboratory data under controlled conditions. Laboratory conditions would limit the number of phenomena affecting the plume and allow specific phenomena to be studied. Laboratory studies including the measurement of Reynolds stresses would be helpful; in this way, predictions can be tested with viscosities and diffusivities known.

7. Test the formulations of the models on simple problems for which established (perhaps analytical) solutions are available. Establishing a set of benchmark problems of increasing difficulty and encompassing a wide range of conditions for which good-quality laboratory or field data exist would foster the comparison of various numerical techniques and the development of better plume models. These benchmark problems would be helpful, although not completely satisfactory, since there is no guarantee that a model solving those test problems well would necessarily be as successful in a new case, principally because of new complexities that may arise.

8. Develop methods to increase the efficiency of the numerical schemes. Efforts should be made to reduce the computer requirements or equivalently to increase the scope of problems that can be handled with current computer time and memory resources. This would make the numerical models more generally available and would speed model development. At present, it is uncertain whether any significant reduction in computer requirements is achievable.

Mathematical modeling of thermal plumes by numerical means holds the greatest promise for the future and should therefore be pursued. If work on these models continues at its present or, better yet, an accelerated rate, and if the physical and analytical investigations set forth here are carried out, then numerical models should become a valuable licensing instrument in the next few years.

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