NONTOPOLOGICAL SOLITONS

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I shall be describing mainly the work carried out over the last few years by T. D. Lee and a number of collaborators, including G. C. Wick, N. H. Christ, M. Margulies, A. Sirlin, and myself.

This work is guided by the premise that we already know enough basic physics to explain the extended structure of hadrons in terms of a conventional local field theory. On the evidence of (e, N) and (ν, N) reactions, we expect local field theory to hold good down to $\sim 10^{-16}$ cm. Therefore the hadrons extending over 10^{-13} should be bound states of a more elementary object. In view of the successes of group theory, we expect the elementary objects to be colored quarks, on the basis of deep inelastic e -scattering, the quarks should have no internal structure larger than 10^{-16} cm and can therefore be regarded as the elementary excitations of a Fermi field.

If everything is this easy, what stands in the way? Since the quarks have not been seen in isolation, they must be supposed either extremely massive or unstable; in the latter case they are at least as massive as the proton. Thus a three-quark nucleon or two-quark meson must be strongly bound, with relativistic effects prominent. It is the difficulty of handling bound states in relativistic quantum field theory that has hampered development of a simple conventional model of hadrons.

The study of solitons offers a new departure for this problem, by suggesting a different 0^{th} -order approximation from the one most often studied in the past. The principle is illustrated by the case of a quantum mechanical particle moving in two dimensions under the centrally symmetric and quasi-harmonic potential

$V = \frac{1}{2} r^2 [(1 - gr)^2 + \Delta^2 g^2]$. In Fig. 1 we graph the effective radial potential

$V_\ell = V + \ell^2/2r^2$ against r^2 for several values of ℓ . In general there may be two minima, one at small r^2 and one at large r^2 .

When ℓ exceeds a critical value ℓ_c , the absolute minimum of V_ℓ lies at the larger r^2 . The ground state wave function is then concentrated at this radius, which is very large if g is small. But the 0th-order of perturbation theory, which consists of taking $g = 0$, gives a ground state concentrated at the much smaller r^2 (minimum of dotted curve) and is therefore a very poor approximation. On the other hand, a very good approximation is obtained by using classical theory with the exact g . This gives a circular trajectory at the right radius, on which quantum fluctuations may be superimposed as corrections.

We note that as $g \rightarrow 0+$ at fixed Δ , $\ell_c \rightarrow \Delta^2$ and so if we make $\Delta^2 < 1$ the situation described above occurs even for small quantum numbers.

We postulate that a similar situation arises from the interaction between quarks and gluons; that even for low quark numbers, the quantum bound state is better approximated by a steady solution of the classical field equations than by the free-particle state of the quantum theory without interactions. The stable bound state of the classical theory is what we call a soliton.

Our solitons are stable only in isolation; they are what traditional usage has called "solitary waves". The traditional meaning of "soliton" is that of a configuration stable even against collisions; we propose to call such configurations "indestructible solitons", but do not anticipate any role for them in particle physics. We prefer to use the simpler term for the more applicable concept.

Our investigations are confined to nontopological solitons. This does not rule

out a theory with degenerate vacua; it does mean that in such a theory we assume a single choice of vacuum at large distances, independent of direction.

[Before these ideas are applied to a model of hadrons,] two questions should be investigated. First, one wishes to know how widespread is the occurrence of solitons (i.e., stable, localized solutions of field equations) in classical field theories, and what the characteristic features are. Second, one may ask whether the classical soliton is indeed a good approximation to the exact quantum ground state in those few limiting cases where the latter is known.

The first principle in looking for nontopological solitons in a classical theory of commuting fields is that stability cannot be achieved with completely time-independent solutions. This is because the energy could always be decreased by shrinking the volume of the soliton while maintaining the peak field amplitude. To achieve stable solitons, we introduce a conserved "charge" associated with a particle whose free mass we denote by m .

If the "charged" particle is a boson, we may call the corresponding field Φ . We pick a Lagrangian that depends on Φ only through $\Phi^\dagger \Phi$, and look for solutions in which $\Phi = e^{-i\omega t} \phi$, where ϕ and all other fields are time-independent. (More generally, we have $\partial \Phi / \partial t = \omega (\mathcal{M} \Phi)$ where $\mathcal{M}$ is a generator of some group on Φ that leaves the Lagrangian unchanged.) The "charge" Q is then defined by $Q = \frac{\partial L}{\partial \omega}$. Q is automatically conserved, and in the quantized theory it must be an integer. It plays the same role here as the angular momentum ℓ in the first-quantized illustration given previously. The energy E may now be minimized for fixed Q , since shrinking the volume would necessitate intensifying the field Φ . After minimization we have $\frac{dE}{dQ} = \omega$ when Q is taken as a continuous variable.

If the "charged" particle is a fermion, we denote its field by ψ and simply let $N = \int \psi^\dagger \psi$ be the conserved fermion number. N then plays the part of Q , and ϵ (the single-fermion energy in the presence of the soliton fields) plays that of ω . After arriving at the field equations we can formally allow N to vary continuously; then we have $\frac{dE}{dN} = \epsilon$.

The principal information we seek is the graph of E vs. Q (or N). This curve is to be compared with the straight line $E = Qm$, which represents the energy of an aggregate of free particles bearing the same total charge. For stable solitons to exist, the curve must lie below the straight line.

We recall that the slope of this curve is ω , and it is easily seen from the field equations at large distances that $\omega \leq m$. The behavior of E vs Q when $\xi \equiv \sqrt{m^2 - \omega^2} \ll 1$ depends only on the dimensionality of space and not on the particular model. It is generally true in this limit that $\phi \sim \xi$ and the radius $\sim \xi^{-1}$; hence $Q \sim \xi^2 \xi^{-d}$ in d space dimensions. The resulting curves for 1, 2, and 3 space dimensions are shown in Fig. 2; the low- ω part of the curve is omitted. In 1 or 2 dimensions, the curve can only lie under the straight line. In 3 dimensions the curve lies under the line when Q exceeds some value Q_S . In more detailed investigations we have found that the parameters can be chosen to make $Q_S < 1$ if desired.

As ω decreases to its lower limit $\omega_{\min}$, both the radius and Q become large. Here we encounter two kinds of behavior, depending on the specific model. In some cases $\omega_{\min} > 0$; the solution is then saturated in the sense that the amplitude of the charged field is roughly constant throughout the interior and does not

increase as the radius is made large. The energy then grows as $E \sim Q \omega_{\min}$. (See Fig. 3a.) In other cases $\omega_{\min} = 0$. This leads to an unsaturated solution in which the charged field grows without limit as the radius increases. Generally we then have $E \sim Q^k$, where $0 < k < 1$. (See Fig. 3b.)

In 1 dimension solitons can arise from a single field. We put

$$\mathcal{L} = \dot{\Phi}^\dagger \dot{\Phi} - \Phi^\dagger \Phi - U(\Phi^\dagger \Phi) + \text{counterterms},$$

$$m^2 = \left. \frac{dU(\phi^2)}{d\phi^2} \right|_{\phi=0},$$

$$\Phi = e^{-i\omega t} \phi(x), \quad \phi'^2 = U - \omega^2 \phi^2,$$

$$Q = \frac{\partial}{\partial \omega} \int \mathcal{L} dx = 2\omega \int \phi^2 dx, \quad E = \int (\phi'^2 + U + \omega^2 \phi^2) dx.$$

At the turning point $\phi' = 0$, we must have $U = \omega^2 \phi^2$. Therefore the curve $U = U(\phi^2)$ must intersect the straight line $U = \omega^2 \phi^2$. Since $\omega^2 < m^2 = U'(0)$, the curve $U(\phi^2)$ must have a portion concave down. Several soluble examples are known:

$$(A) \quad U = m^2 \phi^2 [1 - 2g^2 \phi^2 + g^4 (1 + \epsilon^2) \phi^4] \quad (i) \quad (\text{Fig. 4a})$$

Here the curve cannot intersect the straight line unless $\omega > \omega_{\min}$ where $\omega_{\min}^2$ is the slope of the tangent shown in Fig. (4ai). Hence the behavior is saturated at large Q (Fig. 4aii).

$$(B) \quad U = \frac{1}{4} \frac{m^2}{g^2} (\sqrt{1 + 8g^2 \phi^2} - 1) \quad (i) \quad (\text{Fig. 4b})$$

Here ω can be as close to 0 as desired (Fig. 4bi). The behavior at large Q is unsaturated with $E \sim Q^{3/4}$ (Fig. 4bii).

$$(C) \quad U = m^2 \phi^2 e^{-2g^2 \phi^2} \quad (\text{Fig. 4c})$$

Another unsaturated case, but here U decreases for large ϕ (Fig. 4ci) and hence the energy is concentrated at the boundary of the soliton. When $\omega \rightarrow 0$, the radius grows large but both Q and E remain finite (Fig. 4cii) — an exception possible only in 1 dimension.

In 3 dimensions we restrict ourselves to 4th-order polynomials for the sake of renormalization. This rules out positive definite functions $U(\phi^2)$ with a downward concavity as in the previous examples. But solitons are still possible with two fields.

We have studied two unsaturated examples:

$$(D) \quad \begin{aligned} \Phi &\rightarrow e^{-i\omega t} \phi, \quad \text{"charged" scalar boson,} \\ \chi &, \quad \text{neutral scalar boson,} \end{aligned}$$

$$\mathcal{L} = -\partial_\mu \Phi^\dagger \partial_\mu \Phi - \frac{1}{2}(\partial_\mu \chi)^2 - f^2 \chi^2 \Phi^\dagger \Phi - U(\chi) + \text{counterterms},^{(2)}$$

where U has a minimum at $\chi = \chi_{\text{vac}} = m/f$. We then have

$$Q = 2\omega \int \phi^2 d^3r,$$

$$E = \int [(\nabla \Phi)^2 + \frac{1}{2}(\nabla \chi)^2 + f^2 \chi^2 \phi^2 + U(\chi) + \omega^2 \phi^2] d^3r$$

and by minimizing E at fixed Q we obtain coupled nonlinear 2nd-order differential equations in ϕ and χ . The E - Q curve (Fig. 5) has the spike characteristic of 3 dimensions. The stability threshold Q_S can be made < 1 by suitably choosing the ratio of free masses. For large Q we have $E \sim Q^{3/4}$ unless U has a special form making the vacuum degenerate with a second minimum at $\chi = 0$. Then the energy is

surface-dominated and $E \sim Q^{2/3}$ at large Q .

- (E) ψ , "charged" spinor fermion
 σ , neutral scalar boson, (3), (4)

Here we adopt a slightly different formal approach. We start from the Hamiltonian density

$$\mathcal{H} = \mathcal{H}_\sigma + \sum_{k=1}^n \psi^{k\dagger} H_D \psi^k + \text{counterterms}$$

where

$$\mathcal{H}_\sigma = \frac{1}{2} \pi^2 + \frac{1}{2} (\nabla \sigma)^2 + U(\sigma), \quad \pi \text{ conjugate to } \sigma,$$

$$H_D = \frac{1}{i} \vec{\alpha} \cdot \vec{\nabla} + \beta (m + g\sigma)$$

and k is a "color" index which permits us to put up to n quarks in the same orbital.

We then approximate the whole Hamiltonian by the "quasiclassical" Hamiltonian

$$H_{\text{qcl}} = \int \left[\frac{1}{2} (\nabla \sigma_c)^2 + U(\sigma_c) \right] d^3r + \sum_k \sum_{\ell}^{\infty} \epsilon_{\ell} (a_{\ell}^{k\dagger} a_{\ell}^k + b_{\ell}^{k\dagger} b_{\ell}^k)$$

where the index ℓ ranges over the eigenfunctions of H_D with $\sigma = \sigma_c$, and ϵ_{ℓ} , a_{ℓ} , b_{ℓ} are the corresponding eigenvalues and annihilation operators for fermion and antifermion. Here σ_c is a c-number function, but the fermion field is still quantized (the a and b operators obey anticommutation rules).

The fermion number $N = \sum_k \sum_{\ell} (a_{\ell}^{k\dagger} a_{\ell}^k - b_{\ell}^{k\dagger} b_{\ell}^k)$ is conserved. We give it some fixed value $\leq n$; then the lowest eigenvalue of H_{qcl} for a given σ_c is

$$E = N\epsilon + \int \left[\frac{1}{2} (\nabla \sigma_c)^2 + U(\sigma_c) \right] d^3r$$

where ϵ is the lowest positive solution of

$$\epsilon \psi_c = \left[\frac{1}{i} \vec{\alpha} \cdot \vec{\nabla} + \beta (m + g \sigma_c) \right] \psi_c .$$

We minimize E with respect to σ_c and obtain

$$\nabla^2 \sigma_c - U'(\sigma_c) = g N \psi_c^\dagger \beta \psi_c .$$

The qualitative properties of this soliton are very similar to those of the previous example using only boson fields. In Fig. 5, simply replace Q by N .

We have also studied saturated examples in 3 dimensions.

(F) Same as (E) but with $N \gg n$.

This case has been studied by Lee and Wick⁽⁵⁾ and by Lee and Margulies⁽⁶⁾ as a model of nuclear matter. Here the nucleon density remains constant as N grows large, because of the Pauli principle. Both N and E , in that limit, are simply proportional to volume. (See Fig. 6.)

(G) $\vec{V}_\mu$, Yang-Mills vector field
 ϕ , Higgs scalar, isospinor.⁽⁷⁾

The Lagrangian has $SU_2 \otimes SU_2$ symmetry, with only the first group gauged. After the Higgs symmetry breaking, there remains one global SU_2 group under which ϕ is scalar and $\vec{V}_\mu$ is vector. Writing $\vec{V}_\mu = (\vec{V}_k, V_0)$, the time-dependence of the soliton is given by

$$\frac{\partial}{\partial t} \vec{V}_k = \vec{\omega} \times \vec{V}_k, \quad \frac{\partial}{\partial t} \vec{V}_0 = 0, \quad \frac{\partial}{\partial t} \phi = 0.$$

The "charged" field is thus $\vec{V}_k$, and the scalar "glue" is provided by ϕ . The 4th component $\vec{V}_0$ injects a new element: it mediates a self-repulsion of $\vec{V}_k$ which leads to saturation as in Fig. 6.

I now turn to the second question: how good is the quasiclassical approximation?

Let us define the binding energy $E_B = NE(1) - E(N)$ and the ratio $R = E_B(\text{exact})/E_B(\text{qcl})$ where $E_B(\text{qcl})$ is the value given by our soliton approximations.

For relativistic theories, exact results are known only when $g^2 \ll 1$. When $N \gg 1$, in all three cases (D), (E), and (F), we find $R = 1$. For $N = 2$, the exact result can be found by solving the Coulomb problem in center-of-mass coordinates, and $R = .768$.⁽³⁾

For nonrelativistic "charged" particles we find also $R = 1$ in the limit $g^2 \gg 1$ for any N ,⁽³⁾ as well as for $N \gg 1$ and any g^2 . Again, when $N = 2$ and $g^2 \ll 1$, we have $R = .768$.

Either large g^2 or large N tends to make the quantum corrections negligible, and even in the expected "worst" case $N = 2$, $g^2 \ll 1$, the approximation is not so bad. Thus we are encouraged to think that in a model of hadrons ($N = 2$ or 3 , $g^2 \gg 1$ but with relativistic binding) a quasiclassical treatment will be fairly accurate.

The hadron model we have in mind involves an $SU_3 \otimes SU_3$ half-gauged symmetry leading to global SU_3 after Higgs breaking, and four fields as shown in Table 1:

Table 1.

Field	Spin	SU_3^{gauged}	X	SU_3	$\rightarrow$	SU_3
ψ	$\frac{1}{2}$	3	X	1		3
σ	0	1	X	1		1
V_μ	1	8	X	1		8
ϕ	0	3	X	3		1, 1, 8

The ψ field represents the quark; its 3-fold symmetry is color. We have not yet injected flavor. The σ field is the scalar "glue". The vector field V_μ , as suggested by Nambu, mediates a repulsive interaction raising the energy of color non-singlets; that explains why they are not seen. The Higgs field ϕ is then needed to give the $\vec{V}_\mu$ a mass in the usual way. All the free masses are assumed large, so that the observed spectrum is merely that of the color-singlet bound states.

In the soliton approximation, a color-singlet state has $\vec{V}_\mu = 0$ everywhere. The ϕ field does not interact directly with ψ ($\phi^2 \psi^\dagger \psi$ is not renormalizable, while $\phi \psi^\dagger \psi$ breaks the pre-Higgs symmetry) and its effect on σ can presumably be incorporated into $U(\sigma)$. Thus the bound-state problem simplifies to that of example (E) above, with just ψ and σ , and $N = 2$ or 3 . It is the same model that led to the SLAC bag,⁽⁸⁾ and with different parameters was used by Creutz and Soh⁽⁹⁾ to give the MIT bag⁽¹⁰⁾ in a certain limit.

There are four main styles of bag, characterized by (a) either MIT- or SLAC-type quark wave function (see Fig. 7) and (b) either volume- or surface- dominated σ energy. See Table 2.

<u>Table 2.</u>	I	MIT - type quarks, $E_\sigma \sim \text{volume}$
	II	MIT - type quarks, $E_\sigma \sim \text{surface}$
	III	SLAC - type quarks, $E_\sigma \sim \text{volume}$
	IV	SLAC - type quarks, $E_\sigma \sim \text{surface}$

To obtain I or II from the field model, one sets (with Creutz and Soh) $m + g\sigma_0 = 0$ where $U(\sigma)$ has its second minimum at $\sigma = \sigma_0 < 0$. To obtain III or IV, one sets $m + g\sigma_0 < 0$. For I or III, one must have $U(\sigma_0) > U(0)$, for II or IV $U(\sigma_0) = U(0)$. The published MIT bag is I, the SLAC bag is IV.

If $m + g\sigma = 0$, before passing to the MIT limit, one can also obtain hybrid quark wave functions as in Fig. 8. These arise from a tendency for σ to overshoot the value σ_0 so as to give the quark the lower SLAC-type energy. This effect can be seen in the curves of Rafelski for the case $U \sim \sigma^2$. If an SU_3 flavor group with unbroken symmetry is introduced, the model gives 36 degenerate mesons, and 56 baryons.

Some preliminary numerical comparisons are shown in Table 3. (Some of these numbers are already known.) Letting

m = free quark mass

μ = free gluon mass

m_B = average (56) baryon mass
 $= 1.40 m_N$

B_- = $U(\sigma_0) \rightarrow B$ of MIT bag

B_+ = $\max U(\sigma)$ for $0 > \sigma > \sigma_0$

$$\xi = \left(\frac{m_B}{m}\right)^2, \quad \eta = \frac{m_B}{\mu}, \quad \zeta = \frac{B_-}{B_+},$$

we assume ξ, η, ζ all $\ll 1$ and obtain case

- I when $\xi \gg \eta \gg \zeta$
- II when $\xi \gg \zeta \gg \eta$
- III when $\zeta \gg (\xi, \eta)$
- IV when $\eta \gg (\xi, \zeta)$

Quantity	Expt.	I	II	III	IV
$m_N \langle r^2 \rangle^{\frac{1}{2}}$	3.86	4.25	4.78	2.86	3.21
$2m_N \mu_p$	2.79	2.36	2.66	1.90	2.14
μ_n / μ_p	- .685	$\longleftrightarrow$		$- 2/3$	$\longrightarrow$
g_Λ / g_V	1.2	$\longleftarrow 1.09 \longrightarrow$		$\longleftarrow 5/9 \longrightarrow$	

Table 3

The main result of the comparison is that all the numbers are in the general vicinity of experiment, although the MIT-type models seem somewhat closer. We mean to investigate the hybrid models as well.

Further questions: How do we calculate form factors from such a model?
How should we introduce flavor breaking? Does PCAC come in at all?

To conclude: This approach seems the most conservative of all avenues to the study of hadron structure. At the same time it incorporates most of the features definitely indicated by the research of the last fifteen years. Since a preliminary comparison with

13.

experiment leads to no glaring contradiction, the approach should be pursued until it leads either to a significant success or to an informative failure.

References

1. T. D. Lee, at the Symposium on Frontier Problems in High Energy Physics, Pisa, 1976 [Columbia University preprint No. CO-2271-76].
2. R. Friedberg, T. D. Lee and A. Sirlin, Phys.Rev. D13, 2739 (1976).
3. R. Friedberg and T. D. Lee, Phys.Rev. D15, 1694 (1977).
4. A. Nishimura, University of Tokyo preprint UT-275.
5. T. D. Lee and G. C. Wick, Phys.Rev. D9, 2291 (1974).
6. T. D. Lee and M. Margulies, Phys.Rev. D11, 1591 (1975).
7. R. Friedberg, T. D. Lee and A. Sirlin, Nucl.Phys. B115, 1, 32 (1976).
8. W. A. Bardeen, M. S. Chanowitz, S. D. Drell, M. Weinstein and T. M. Yan, Phys.Rev. D11, 1094 (1975).
9. M. Creutz and K. S. Soh, Phys.Rev. D12, 443 (1975).
10. A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys.Rev. D9, 3471 (1974).

FIGURE CAPTIONS

Figure 1 $V_\ell = V + \frac{\ell^2}{2r^2}$ against r^2 in simple 1st quantized example

Figure 2 E vs Q in upper range of ω

Figure 3 E vs Q in lower range of ω

Figure 4 Solitons in 1 dimension

Figure 5 Unsaturated 3-dimensional examples

Figure 6 Saturated 3-dimensional examples

Figure 7 (a) MIT-like Dirac wave function
(b) SLAC-like Dirac wave function

Figure 8 Hybrid Dirac wave function

FIG. 1

FIG. 2

FIG. 3

(a)

(b)

(c)

FIG. 4

FIG. 5

FIG. 6

7a

7b

8