DRIFT-CONVECTIVE CELLS IN A
NONUNIFORM PLASMA WITH
MAGNETIC SHEAR

BY

S. M. MAHAJAN, P. N. GUZDAR,
P. K. KAW AND C. OBERMAN

PLASMA PHYSICS
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and C. Oberman

Plasma Physics Laboratory
Princeton University
Princeton, New Jersey 08540

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Drift-Convective Cells in a Nonuniform

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S. M. Mahajan, P. N. Guzdar, P. K. Kaw

and C. Oberman

Plasma Physics Laboratory

Princeton University

Princeton, New Jersey 08540

ABSTRACT

We demonstrate the existence of near zero frequency growing modes driven by plasma density and temperature nonuniformities in a sheared magnetic field. These modes are viable candidates for explaining anomalous cross-field transport. We believe these modes have been observed in FM-1, when operated in the collisional limit.

It is well known that non-oscillatory potential fluctuations in a plasma are particularly suited to cause anomalous cross-field loss of particles and heat in a magnetically confined plasma. As an example, computational and analytical studies¹⁻³ have convincingly demonstrated that in 2-d strongly magnetized plasmas, the equilibrium fluctuation spectrum is dominated by long-lived macroscopic "convective cells" which $\vec{E} \times \vec{B}$ convect the plasma across the magnetic field and cause significant anomalous transport. It is, therefore, of importance to ascertain the possible existence of almost purely non-oscillatory modes in realistic 3-d systems with magnetic shear and plasma nonuniformity. In this letter, we report preliminary results of an investigation demonstrating the existence of such modes. These modes are the obvious extension of the 2-d convective cell to a 3-d nonuniform plasma. We call them "drift-convective cells."

It has been argued in the past^{4,5} that the introduction of magnetic shear should strongly suppress convective cells because it permits electrons to flow along the lines of force and short out the fields maintaining the cell. However, the argument ignores the role of physical processes inhibiting the parallel motion of electrons (e.g., collisions,⁶ magnetic mirroring⁷ etc.) Furthermore, the introduction of plasma nonuniformity and the associated available expansion free-energy changes the picture completely. A strong motivation for the present investigation came from recent microwave scattering experiments⁸ on FM-1 when operated in the collisional regime. These experiments show that in situations of strong shear ($L_s/L_n \lesssim 10$), coherent drift waves

are excited, whereas for weak shear ($L_s/L_n \sim 50$) the spectrum is peaked near $\omega \approx 0$. The conventional interpretation, that drift waves are strongly turbulent in the weak-shear limit and they nonlinearly pump zero-frequency quasimodes, is hardly convincing, since the power at drift frequency is observed to be extremely small. We show below that the drift-convective cell is a natural candidate to explain these observations.

We start our discussion with the derivation of local (shearless!) dispersion relation in a collisional nonuniform plasma.^{9,10} The fundamental equations are Braginskii fluid equations¹¹ for each species. We look for electrostatic low-frequency ($\omega \ll \omega_{ci}$) modes in a slab model (with $B_0 \hat{e}_z, \left\{ \frac{\nabla n_0}{\nabla T_0} \right\} \hat{e}_x$) with $k_\perp \rho_s < 1$ ($\rho_s^2 = T_e / M \omega_{ci}^2$). The following assumptions are made:

1. Electron inertia is neglected. Perpendicular electron motion is simply $\mathbf{v}_\perp = c(\mathbf{E} \times \mathbf{B})/B^2$. Parallel electron pressure and electric field are balanced by $e - i$ collisions. All electron Larmor radius effects are ignored.
2. Ion parallel motion is ignored. Ion polarization drift, FLR effects and perpendicular ion viscosity are retained.
3. Electron parallel thermal conductivity dominates the electron energy equation. Electron viscosity is neglected.
4. In the ion energy equation, ion perpendicular thermal conductivity is taken to be dominant.

After eliminating electron and ion velocities, linearizing the equations, expanding the fluctuating quantities as $f \sim \exp[ik_x x + ik_y y + ik_\parallel z - i\omega t]$ with the WKB assumption

$k_{xL_{n,T}} \gg 1$ and using the quasineutrality condition $\tilde{n}_e \approx \tilde{n}_i$, we obtain four equations between $\tilde{n}, \tilde{\phi}, \tilde{T}_e$ and $\tilde{T}_i$. Equating the determinant of coefficients to zero gives the dispersion relation (the four columns of the determinant refer respectively to coefficients of $\tilde{n}, \tilde{\phi}, \tilde{T}_e$ and $\tilde{T}_i$).

$$\begin{vmatrix} -i\omega + \eta & i\omega_* - \eta & (1 + \beta)\eta & 0 \\ -i\omega + \mu k_{\perp}^2 k_{\perp}^2 & i\omega_* - ik^2(\omega + \omega_{*nT}^i) + \mu k_{\perp}^2 k_{\perp}^2 & 0 & -(22/9)\mu k_{\perp}^2 k_{\perp}^2 \\ 0 & \omega_{*T}^i & 3i(m/M)v_{ei} & -(\omega + i\omega_0) \\ (1 + \beta)\eta & i(3/2)\omega_{*T}^e - (1 + \beta)\eta & -i(3/2)(\omega + i\omega_1) & -3m/Mv_{ei} \end{vmatrix} = 0 \quad (1)$$

where $\eta = k_{\parallel}^2 v_e^2 / 0.51 v_{ei}$, $\omega_{*n(T)}^{i,e} = k_y (cT_{i,e} / eB) n_0^{-1} (\partial n_0 / \partial x)$
 $[T_0^{-1} (\partial T_0 / \partial x)]$ are the various drift frequencies, $k^2 \equiv k_{\perp}^2 \rho_s^2$,
 $\beta = 0.71$, μ is the $\perp$ viscosity coefficient, $\omega_0 = k_{\perp}^2 \chi_{\perp}^i / n_0 + 3(m/M)v_{ei}$,
 and

$$\omega_1 = (1 + \beta)^2 \eta + (k_{\perp}^2 \chi_{\perp}^e / n_0) + k_{\parallel}^2 \chi_{\parallel}^e / n_0. \quad (2)$$

This dispersion relation has four roots. For $k_{\parallel} = 0$, these are (i) the ion flute mode with $\omega \approx -\omega_{*nT}^i$, (ii) the 2-d convective cell $\omega = -ik_{\perp}^2 \mu$, (iii) the ion thermal convective cell $\omega = -ik_{\perp}^2 \chi_{\perp}^i$.

and (iv) an electron-ion energy equipartition mode $\omega \approx -i(m/M)v_{ei}$. We now see what happens to the first two of these modes as a function of $k_{||}$. For $L_T = \infty$, the two roots are given by

$$\omega_+ \approx -\omega_{*i} - i\eta(1 + k^2)\omega_{*e}/k^2\omega_{*i}$$

$$\omega_- \approx [i(\eta - \mu k^2 k_{\perp}^2)\omega_{*e} - i\eta k^2 - \omega_{*i}][i(1 + k^2)\eta + k^2\omega_{*i}]^{-1}.$$

The ion flute mode ω_+ acquires large damping as $k_{||}(\eta)$ is increased. The ω_- mode suffers transition from a purely damped convective cell ($k_{||} = 0$) to a growing electron diamagnetic drift wave [$k_{||}^2 > k^2\omega_{*i}v_{ei}/v_e^2(1 + k^2)$]. In the intermediate domain of $k_{||}$ [$k_{||}^2 < k^2\omega_{*i}v_{ei}/v_e^2(1 + k^2)$], the ω_- mode takes the form

$$\omega_- = i(\eta - \mu k^2 k_{\perp}^2)(\omega_{*e}/\omega_{*i}k^2)[1 - i\eta(1 + k^2)/k^2\omega_{*i}]. \quad (4)$$

It is this almost purely growing mode, which is a natural extension of the 2-d convective cell in a nonuniform medium, that we call the drift-convective cell. In the local (shearless) approximation, this mode has also been discussed by Moiseev and Sagdeev.⁹ The ω_- mode may also be written in a simple form in the case $L_T \neq \infty$, $L_n = \infty$; thus the thermal drift-convective cell may be written as

$$\omega_- = i(1 + \beta)\eta(\omega_{*Te}/\omega_{*Ti}k^2)[1 - i\eta/k^2\omega_{*Ti}] - i\omega_1. \quad (5)$$

Note that the important physical processes for the drift-convective cell are parallel diffusive motion of electrons, $E \times B$ coupling to gradients $(\nabla n_0, \nabla T_0)$ and finite ion-temperature effects through FLR correction and/or ion-polarization drift term.

Although this letter is biased towards a discussion of the collisional plasma problem (because of FM-1 experiment), it should be emphasized that a collisionless analogue exists and may very well be a candidate for the observation of enhanced transport across field lines in higher temperature devices. It can be readily verified that in the limit $k_{||} v_e / \omega_{*i} < k_{\perp} \rho_s < 1$, the collisionless dispersion relation for drift waves,¹² yields

$$\omega \approx \frac{i}{\sqrt{2}} \left(\frac{\omega_{*e}}{\omega_{*i}} \right)^{1/2} \frac{k_{||} v_e}{k_{\perp} \rho_i} + \frac{k_{||}^2 v_e^2}{2k_{\perp}^2 \rho_i^2} \left(\frac{\omega_{*e} + \omega_{*i}}{\omega_{*i}^2} \right), \quad (6)$$

again an almost purely growing mode which may be called the collisionless drift-convective cell.

We now turn to the nonlocal problem, i.e., the formation of these modes in a plasma with magnetic shear. We concentrate here on the collisional plasma problem with $T_e = T_i$ and with density gradients serving as the free energy source. The nonlocal equations for the temperature gradient and collisionless problems form part of a longer paper, to be published shortly. The equation for the mode structure in a sheared magnetic field $B = B_0(\hat{e}_z + \hat{e}_y x/L_s)$ can be derived from a simplified version of (1) with the usual ansatz $k_{||} = k_y x/L_s$, $k^2 = k_y^2 - d^2/dx^2$, i.e.,

$$\frac{d^2 \phi}{dx^2} + \left(-k_y^2 - \frac{\alpha x^2}{x^2 - i\omega\delta} \right) \phi = 0 \quad (7)$$

where $\alpha = (\omega - 1)/(\omega + 1)$, $\delta = (m/M)\bar{v}\ell^2$, $\ell = L_S/L_n$, $\bar{v} = v_{ei}/\omega_*$ and in Eq. (7) and all the subsequent discussion, we have normalized x to ρ_i and time to ω_*^{-1} . We have neglected ion viscosity term in Eq. (6) since it only produces a weak threshold for the instability and influences the mode structure only very close to the mode rational surface. Multiplying Eq. (6) by ϕ^* , integrating over x and subtracting the corresponding conjugate equation generates the quadratic form [for $\phi(\pm\infty) \rightarrow 0$]: $\int_{-\infty}^{\infty} dx |\phi|^2 \times \text{Im}[\alpha x^2/(x^2 - i\omega\delta)] = 0$ where we have written $\omega = \Omega + i\gamma$. This demonstrates that unstable modes may exist, provided they satisfy the bounds

$$\gamma < \left(\frac{\Omega - \Omega^3}{2 + \Omega} \right)^{1/2}. \quad (8)$$

Most of the earlier analyses^{13,14} of equations, such as Eq. (6), were biased towards recovering the drift wave with $\Omega \approx 1$. We bias ourselves towards recovering the drift-convective cell with ($\Omega < \gamma \ll 1$). We solve Eq. (6) perturbatively with $\epsilon \sim \gamma \ll 1$ as the perturbation parameter. Thus, we formally write $\phi = \phi_0 + \phi_1$, $\alpha_0 = -1$, $\alpha_1 = 2i\gamma$, $\omega_0 = i\gamma$, $\omega_1 = \Omega$. The zeroth order equation, may now be written

$$(x^2 + \gamma\delta) \frac{d^2 \phi_0}{dx^2} + [(1 - k_y^2)x^2 - \gamma\delta k_y^2] \phi_0 = 0. \quad (9)$$

With the transformations $y^2 \delta \gamma = x^2$ and $\phi_0 = (y^2 + 1)^{1/2} \psi_0$, Eq. (9) goes over to the standard form for oblate spheroidal radial

eigenfunctions¹⁵

$$(y^2 + 1)\psi_0'' + 2y\psi_0' + \left(-\gamma\delta k_y^2 + \gamma\delta(1 - k_y^2)y^2 + \frac{1}{y^2 + 1} \right) \psi_0 = 0 . \quad (10)$$

In the standard notation for these functions (e.g., see Flammer¹⁶) we have $m = 1$, $\lambda_{mn} = \gamma\delta k_y^2$ and $c^2 = \gamma\delta(1 - k_y^2)$. The eigenvalue condition is a complicated relationship between λ_{mn} and c^2 ; in the asymptotic limit ($c^2 \gg 1$), it takes the simple form $\lambda_{1n} = -c^2 + 4cn$ which leads to the growth rate

$$\gamma = (16n^2/\delta)(1 - k_y^2)n \leq 1 \quad (11)$$

or in terms of physical quantities $\gamma_{\text{phys}} \sim 16n^2(1 - k_{\rho}^2)k_{\rho}^2 v_{\text{eth}}^2 / v_{\text{ei}} L_s^2$. A comparison of Eqs. (11) and (4) shows that the growth rate γ corresponds to the local growth rate at $x = 4nk_y \rho_s$. The condition $\gamma_{\text{phys}} < \omega_*$ implies $(L_s/L_n) > 4n(M/m)^{1/4} (k_{\rho} v_{\text{eth}}/v_{\text{ei}})^{1/2}$ which corresponds to weak and moderate shear limits in FM-1 when $k_{\rho} \sim 1$.

The eigenfunctions, which have unmixed parity and are well-behaved at the origin (i.e., $\psi = 0$ for odd symmetry or $\psi' = 0$ for even symmetry), lead to standing waves at large x . Mixed parity solutions will lead to partially standing waves. This points out the need for some reflection of waves at large x due to e.g. a sound wave turning point,¹⁴ a turning point^{17,18} due to dependence of α on x , existence of incoming waves from neighboring mode-rational surfaces¹⁹ etc.. The last of these is discussed in detail below. Interestingly enough, the assumption of almost standing wave solutions seems to be consistent with earlier

observations¹⁸ on FM-1. Proceeding further with an investigation of the spheroidal equation, we can readily obtain a first order perturbation to the eigenvalue by the usual quantum-mechanical perturbation theory. This gives

$$\Omega = 2\gamma^2 + \gamma\delta \int_{-\infty}^{+\infty} \frac{x^4 |\psi_0|^2 dx}{(x^2 + \gamma\delta)^2} \bigg/ \int_{-\infty}^{+\infty} \frac{x^2 |\psi_0|^2 dx}{(x^2 + \gamma\delta)^2} > 2\gamma^2 . \quad (12)$$

Note that Eqs. (11) and (12) are consistent with the bounds given above.

It is well known that the eigenmode problem of this kind discussed above leads to modes localized in x . To investigate modes, which span several rational surfaces, one can follow the analysis of Roberts and Taylor.¹⁹ It is particularly instructive to carry out such a calculation here since the single rational surface analysis given above requires standing waves far away. Following Ref. 19, we introduce the twisted coordinate system $\xi = x$, $\chi = y - xz/L_s$, $\zeta = z$. Making the ansatz $\phi = \phi(\zeta)\exp[ik_x\xi + ik_y\chi]$ we can show that the zeroth order equation for the quantity $\psi_0 = \phi_0(1 - \theta^2)^{1/2}$, where $\theta = k_x - k_y\zeta/L_s$, becomes

$$(1 - \theta^2) \frac{d^2\psi_0}{d\theta^2} - 2\theta \frac{d\psi_0}{d\theta} + \left(\gamma\delta k_y^2 + \gamma\delta(1 - k_y^2)\theta^2 - \frac{1}{1 - \theta^2} \right) \psi_0 = 0 . \quad (13)$$

This is the equation for angular oblate spheroidal functions and again leads to growing modes with growth rate given by Eq. (11). The growing mode now spans several rational surfaces radially and is standing in z ; the large radial extent enhances its importance for anomalous transport. The condition $|\theta| \leq 1$ puts some constraints

on the permissible n, m values in a toroidal geometry. We have also investigated the effect of nonuniformity of shear²⁰ on these modes. Details of this will be given in the longer paper.

To conclude we have shown that an obvious extension of a 2-d convective cell to a nonuniform plasma with magnetic shear is an almost purely growing drift-convective cell. This mode may have a large radial extent and thus lead to significant anomalous transport. The mechanism for the nonlinear saturation of these drift-convective cells is strongly suggested by the local dispersion relation, Eq. (3). Due to the enhanced level of fluctuations, there will be a strong eddy viscosity contribution to enhance μ (see Krommes and Oberman³) and hence make $\gamma_{NL} \rightarrow 0$. This will automatically lead to enhanced plasma diffusion (or thermal conduction in the case of thermal drift-convective cells). We further propose that these are the low-frequency modes ($\omega \approx 0$) that have been observed in FM-1 in the weak and middle shear situations. The absence of usual drift wave in these cases can be attributed to a nonlinear damping of drift waves by random potential fluctuations due to drift-convective cells.²¹ Finally, the interesting question of finite β effects on these modes will be reported in the forthcoming paper.

REFERENCES

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¹J. M. Dawson, H. Okuda and R. N. Carlile, Phys. Rev. Lett. 27, 491 (1971). See also H. Okuda and J. M. Dawson, Phys. Fluids 16, 408 (1973).

²J. B. Taylor and B. McNamara, Phys. Fluids 14, 1492 (1971).

³J. A. Krommes and C. Oberman, J. Plasma Phys. 16, 229 (1976).

⁴M. N. Rosenbluth and C. S. Liu, "in Proceedings of the 5th European Conference on Controlled Fusion and Plasma Physics," Grenoble, France, I, 12 (1972).

⁵J. B. Taylor, "in Proceedings of the 5th European Conference on Controlled Fusion and Plasma Physics," Grenoble, France, II, 83 (1972).

⁶H. Okuda and J. M. Dawson, Phys. Fluids 16, 1456 (1973).

⁷J. M. Dawson and T. Kamimura, Phys. Rev. Lett. 36, 313 (1976).

⁸M. Okabayashi and V. Arunasalam, Princeton University Plasma Physics Laboratory Rept. PPPL-1310.

⁹S. S. Moiseev and R. Z. Sagdeev, Sov. Phys. Tech. Phys. 9
196 (1964) [Zh. Tekh. Fiz. 34, 248 (1964)].

¹⁰B. Coppi, Phys. Lett., 14, 172 (1965).

¹¹V. I. Braginskii, Revs. of Plasma Phys. I, (M. A. Leontovich,
Ed.) New York, 214 (1975).

¹²B. B. Kadomtsev, Plasma Turbulence, (Academic Press) 88
(1965).

¹³B. Coppi, G. Laval, R. Pellat and M. N. Rosenbluth, Nucl.
Fusion 6, 261 (1966).

¹⁴L. D. Pearlstein and H. L. Berk, Phys. Rev. Lett. 23, 222
(1969). See also M. N. Rosenbluth and P. J. Catto, Nucl. Fusion
15, 573 (1975).

¹⁵M. Abramowitz and I. A. Stegun, N.B.S., Appl. Math. Series
55, 753 (1970).

¹⁶C. Flammer, "Spheroidal Wave Functions," Stanford Univ.
Press, Stanford, CA, (1957).

¹⁷N. A. Krall and M. N. Rosenbluth, Phys. Fluids 8, 1488
(1965).

¹⁸N. R. Sauthoff, Ph.D. Dissertation, Princeton University
Plasma Physics Laboratory (1975).

¹⁹K. V. Roberts and J. B. Taylor, Phys. Fluids 8, 315 (1965).

²⁰J. B. Taylor, in "Proceedings of 6th Conference on Plasma Physics and Nuclear Fusion Research," Bertchesgaden, Germany, D-10 (1976).

²¹P. N. Guzdar and P. K. Kaw, in "Proceedings of the Annual Controlled Fusion Theory Conference," San Diego, CA, G-6 (1977).