Quantum Effects of Pseudoparticle Size*

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Abstract

It is shown that the calculation of quantum effects associated with the classical four dimensional pseudoparticle can be carried to completion in the one loop, lowest order in source approximation, with size effects calculated exactly. The result is finite, and leaves symmetry breaking results intact. For the case of two flavors the theory reduces to a particularly simple form which suggests that the pseudoparticle acts as a source of zero mass bosons which in turn are made of fermion pairs.

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Recently, 't Hooft^{1,2} has calculated the quantum effects associated with the classical pseudoparticle solution found by Belavin, et al³ in non-abelian gauge theories. This pseudoparticle was conjectured by Polyakov⁴ to be the basis of the confinement mechanism of a color theory of strong interactions. Related vacuum tunneling effects have been stressed, as well, by Jackiw and Rebbi,⁵ and Callan, Dashen, and Gross.⁶

't Hooft's calculation of the generating functional

$$W = \langle 0 | 0 \rangle_{\text{out}}^{\text{in}} = \int D A D \bar{\Psi} D \Psi D \Phi \exp \left\{ \int d^4 x \mathcal{L} \right\} \quad (1)$$

proceeds via a perturbation expansion about the classical pseudoparticle solution keeping lowest order (quadratic) terms, i.e. calculating the one-loop contribution. The zero energy modes are handled by the collective coordinate formalism. They involve the position and size of the classical pseudoparticle solutions. Apart from a gauge and ghost fixing term and the scalar multiplets, the Lagrangian $\mathcal{L}$ describes N_f flavored color doublet fermions coupled color gauge invariantly (D_μ is the covariant derivative) to the triplet of color vector meson fields,

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a - \bar{\Psi} \not{x}_\mu D_\mu \Psi + \bar{\Psi}_s \not{x}_{st} \Psi_t. \quad (2)$$

Here a labels color isospin, and s, t are fermion flavor indices, suppressed in all but the arbitrary source term J_{st} . 't Hooft's calculation in Ref. 2 stops short of a generating functional with the collective pseudoparticle size coordinate ρ completely integrated; rather, he finds W for fixed ρ , obtains a corresponding effective Lagrangian (ρ fixed), and then integrates over ρ . In the unbroken color gauge theory interpretation of the effective Lagrangian he then gets an apparently intractable infrared divergence.

In this note we would like to point out that at least for $N_f = 2$ the generating functional can be computed exactly by performing the remaining ρ -integration explicitly. Then a corresponding effective Lagrangian can be constructed with some interesting properties discussed in the conclusion.

The functional integration that gives the 1-loop contribution proceeds as follows

$$\begin{aligned}
 W[J] &= e^{S_{cl}} \int DA_0 DA_{\neq 0} D\Phi D\psi D\bar{\psi} \exp \left\{ \int d^4x \mathcal{L}(A, \Phi) + \bar{\psi}(M+J)\psi \right\} \\
 &= \int d^4x \int_0^\infty d\rho \tilde{f}(\rho) \int D\psi D\bar{\psi} \exp \int d^4x \bar{\psi}(M+J)\psi \\
 &= \int d^4x \int_0^\infty d\rho \tilde{f}(\rho) \det(M(\rho, x) + J),
 \end{aligned} \tag{3}$$

where $M(\rho, x)$ is the differential operator obtained from (2) by expanding about the classical solution. The notation A_0 and $A_{\neq 0}$ indicates the separation of zero and non-zero eigenvalues in the gaussian integration, the former handled by the collective coordinate procedure. Now

$$\det[M(\rho, x) + J] \approx \pi_i E_i(\rho) \pi_k E_k^0(\rho, J), \tag{4}$$

where E_i 's are the eigenvalues of $M + J$, with $E_k^0(\rho, J=0) = 0$. With this separation 't Hooft² gets (u are some fixed spinors)

$$W[J] = \int d^4x \int_0^\infty d\rho \tilde{f}(\rho) \det_{st} \left[\int d^4z \frac{u_s^* J_{st}(z) u_t}{(\rho^2 + (x-z)^2)^3} \right]. \tag{5}$$

In the case of no scalar multiplets 't Hooft gives

$$f(g) = K e^{-\frac{8\pi}{g^2}} g^{-8} (\mu_0 g)^{\frac{7}{3}(1+N_f)} \mu_0^{5-3N_f}, \quad (6)$$

$$K = 2^{15+N_f} \pi^{6-2N_f} \exp[2N_f \alpha(1/2) - \alpha(1)], \quad (7)$$

where the α are numbers given in Ref. 2 and μ_0 is the renormalization subtraction point.

To continue further it is profitable to use Fourier transform on the source $J_{st}(x)$. For notational simplicity we write the formulae for the moment only for a diagonal source J . We define

$$J_i(z_i) = \int d^4 p_i e^{i p_i \cdot z_i} \tilde{J}_i(p_i). \quad (8)$$

Then after translating $z_i = x + w_i$ we obtain

$$W[\tilde{J}] = K \frac{e^{-8\pi^2/g^2}}{g^8} \mu_0^{\frac{2}{3}(11-N_f)} \frac{N_f}{\pi} \left[\int d^4 p_i u_\alpha^* \tilde{J}_i(p_i) u_\alpha \right] (2\pi)^4 \delta^4\left(\sum_{j=1}^{N_f} p_j\right) I, \quad (9)$$

where

$$I = \int_0^\infty dg g^{\frac{7}{3}(1+N_f)} \frac{N_f}{\pi} \int d^4 w_k \frac{e^{i p_k \cdot w_k}}{(g^2 + w_k^2)^3}. \quad (10)$$

Now the angular integration over any w_k can be done and we get

$$\int d^4 w_k \frac{e^{i p_k \cdot w_k}}{(g^2 + w_k^2)^3} = 2\pi^2 \int_0^\infty d|w_k| |w_k|^3 \frac{\sin(|p_k||w_k|)}{|p_k||w_k| (g^2 + w_k^2)^3}. \quad (11)$$

From this we see that there will be no angular correlation between the N_f momenta apart from momentum conservation, i.e. I is a function of $|p_k|$ only. For $|p_k| > 0$ I is a finite function bounded by e.g.

$$|I| < \left(\prod_{\ell=1}^{N_f} |p_\ell|^{-1} \right)^{\frac{10+N_f}{3N_f}} \frac{226N_f}{(10+N_f)(8N_f-10)} (2\pi)^{N_f} \quad (12)$$

For the case $N_f = 2$ we have $|p_1| = |p_2|$ and therefore by scaling the ρ and w_1, w_2 integrals we get,

$$\begin{aligned} W[\tilde{J}] &= K e^{-\frac{8\pi^2}{g^2} - 8} g^4 \pi^4 \mu_0^6 \int d^4 p_1 \int d^4 p_2 (2\pi)^4 \delta^4(p_1 + p_2) \\ &\quad \times \left[u_\alpha^* \tilde{J}_{11}(p_1) u^\alpha u_\alpha^* \tilde{J}_{22}(p_2) u^\alpha - u_\alpha^* \tilde{J}_{12}(p_1) u^\alpha u_\alpha^* \tilde{J}_{21}(p_2) u^\alpha \right] \\ &\quad \times |p_1|^{-2} |p_2|^{-2} \cdot C, \end{aligned} \quad (13)$$

where

$$C = \int_0^\infty ds \, s^7 \left[\int_0^\infty dt \frac{t^2 \sin t}{(t^2 + s^2)^3} \right]^2 \quad (14)$$

is a finite positive constant.

It is important to compare and contrast our result with 't Hooft's. All the chiral properties which arise from the determinant structure involving right-handed spinors u (and left-handed spinors when the parity reflected solution is added) remain the same. Thus conclusions regarding breakdown of $U(N^f) \times U(N^f)$ symmetry persist. Behavior of the theory with regard to the subtraction is also unaffected.

The interpretation of our result in terms of an effective Lagrangian differs from 't Hooft's in important and interesting aspects.

With pseudoparticle size effects exactly incorporated, the simple interpretation of the amplitude in terms of fermions propagating to the pseudoparticle fails in an essential way, i.e. the momentum dependence and corrections are changed. Rather, our final result for $N_f = 2$ has the form

$$\int d^4p \frac{K(p)K(-p)}{p^4} \quad (15)$$

describing 2 zero mass bosons propagating to the pseudoparticle instead of 2 fermion pairs. This can be interpreted as a propagator correction $\Delta m^2/p^4$ to a bare propagator $1/p^2$, with Δm^2 the mass shift due to the pseudoparticle interaction.

The parallelism between the Schwinger model and the four dimensional color gauge theory so often alluded to in these contexts⁷ is particularly evident here: the pseudoparticle seems to have the property of a source of bound fermion-anti fermion pairs, propagating like a zero mass excitation reminiscent of a Goldstone particle.

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