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A Two-Dimensional Relaxation Program for Systems with Inhomogeneous Permeability

W. E. Deeds
C. V. Dodd

OAK RIDGE NATIONAL LABORATORY

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JUNE 1976

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A TWO-DIMENSIONAL RELAXATION PROGRAM FOR SYSTEMS
WITH INHOMOGENEOUS PERMEABILITY*

W. E. Deeds[†] and C. V. Dodd

ABSTRACT

A computer program is presented and explained which will solve axially symmetric eddy-current problems, including those with permeable media present. An unusual feature is the combination of coarse and fine lattices, which permit rapid relaxation of coarse lattices in homogeneous regions to be combined with more accurate calculations using a fine lattice in the transition regions where the permeability is varying. Although the actual program listed is restricted to coils encircling a coaxial metal rod, the program can be modified to include any axially symmetric configuration.

INTRODUCTION

Relaxation methods have been used for solving many types of problems for many years. They have been used in eddy-current calculations at Oak Ridge National Laboratory since 1963.^{1,2,3} The earlier calculations gave good results, except when ferromagnetic materials were present, in which case the large permeability changes at the boundaries gave unreliable results.

*Funding provided by the Superconducting Magnet Development Program, Thermonuclear Division, Oak Ridge National Laboratory.

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¹C. V. Dodd, "Eddy-Current Impedance Calculated by a Relaxation Method," pp. 300-314 in *Proceedings of the Symposium on Physics and Nondestructive Testing*, Southwest Research Institute, San Antonio, Texas, 1963.

²C. V. Dodd, *Solutions to Electromagnetic Induction Problems*, ORNL-TM-1842 (June 1967). Ph.D. Thesis, The University of Tennessee.

³J. Nelson Tunstall and C. V. Dodd III, *A Computer Program to Solve Eddy-Current Problems*, K-1740 (April 8, 1968).

There are two possible ways of treating such large permeability changes. One way is to treat the boundary points as a separate set, satisfying conditions different from the points inside the homogeneous regions. The other way is to introduce an extrafine lattice in the transition region, so that the change of the permeability in one lattice spacing remains small. The former method appears simpler at first, but the boundary conditions become quite complicated in three dimensions, since the normal and tangential components of the fields are treated differently, and the slope of the boundary is important. Since three-dimensional calculations were a further objective, the second approach seemed better. The extrafine lattice in the transition regions must satisfy the equations for an inhomogeneous medium, while the various homogeneous regions can use the equations for homogeneous media and have relatively coarse lattices.

The coarse lattices in the homogeneous regions must overlap the fine lattices sufficiently to allow them to affect each other as they are relaxed alternately, using as boundary conditions the values computed for the other lattice in the previous iteration. At points in the fine lattice boundary which are located between the coarse lattice boundary points, interpolated values of the vector potential can be used.

In order to conserve memory storage capacity in the computer, a relaxation procedure was used in which the newly computed value of the vector potential at a point immediately replaced the value from the previous iteration and was used in subsequent calculations in the current stage of the iteration. This made it unnecessary to store two complete sets of values for the lattices, one for the previous iteration and one for the present iteration. This method gives an unsymmetrical approach to the converged values, but it converges at least as fast as the (not overrelaxed) conventional system, and it does not appear to cause any instability or oscillation.

Another change in the present program from those previously used at Oak Ridge National Laboratory is in the use of "symmetrical" derivatives in the finite difference approximations, rather than the unsymmetrical ones used previously. This did not appear to make any difference in the speed of convergence or in any of the results, but the equations are more symmetrical.

The results obtained using this program for a coil encircling a rod, as shown in Fig. 1, have been compared with the results of an analytical solution which has been experimentally verified to within about 1%. Using a moderately fine mesh, the program runs for about 5 min on an IBM 360 model 91, is within about 10% of the analytical solution when the rod has unit permeability, and is within about 30% when the rod has a permeability of 4.

THEORY

The vector Helmholtz equation for the vector potential in the Coulomb gauge ($\nabla \cdot \vec{A} = 0$) is

$$\nabla^2 \vec{A} - \mu\sigma \frac{\partial \vec{A}}{\partial t} - \mu\epsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J} + \mu \nabla \left(\frac{1}{\mu} \right) \times (\nabla \times \vec{A}) . \quad (1)$$

If we neglect the displacement current term at induction frequencies and assume sinusoidal time-dependence of the driving current and vector potential, then (1) reduces to

$$\nabla^2 \vec{A} = j\omega\mu\sigma \vec{A} - \mu \vec{J}_{\text{ext}} + \mu \nabla \left(\frac{1}{\mu} \right) \times (\nabla \times \vec{A}) . \quad (2)$$

If the driving current density $\vec{J}_{\text{ext}}$ has only a θ component in cylindrical coordinates and all conductors are symmetrical about the axis, then $\vec{A}$ also has only a single component, $A_\theta \equiv A$, which satisfies the scalar equation

$$\begin{aligned} \frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial z^2} - \frac{A}{r^2} + \mu \left[\frac{\partial(1/\mu)}{\partial r} \left(\frac{1}{r} \frac{\partial r A}{\partial r} \right) + \frac{\partial(1/\mu)}{\partial z} \frac{\partial A}{\partial z} \right] \\ = j\omega\mu\sigma A - \mu J_\theta . \end{aligned} \quad (3)$$

The finite difference approximations to the various derivatives are as follows:

Fig. 1. Relaxation lattice for encircling coil.

$$\frac{\partial^2 A}{\partial r^2} \doteq \frac{A_{r+a,z} + A_{r-a,z} - 2A_{r,z}}{a^2}, \quad (4)$$

$$\frac{1}{r} \frac{\partial A}{\partial r} \doteq \frac{A_{r+a,z} - A_{r-a,z}}{2ar}, \quad (5)$$

$$\frac{\partial^2 A}{\partial z^2} \doteq \frac{A_{r,z+a} + A_{r,z-a} - 2A_{r,z}}{a^2}, \quad (6)$$

$$\mu \frac{\partial(1/\mu)}{\partial r} \doteq \left(\frac{\mu_{r,z}}{\mu_{r+a,z}} - \frac{\mu_{r,z}}{\mu_{r-a,z}} \right) / 2a, \quad (7)$$

$$\mu \frac{\partial(1/\mu)}{\partial z} \doteq \left(\frac{\mu_{r,z}}{\mu_{r,z+a}} - \frac{\mu_{r,z}}{\mu_{r,z-a}} \right) / 2a, \quad (8)$$

$$\frac{1}{r} \frac{\partial(rA)}{\partial r} \doteq \frac{(r+a)A_{r+a,z} - (r-a)A_{r-a,z}}{2ar}, \quad (9)$$

$$\frac{\partial A}{\partial z} \doteq \frac{A_{r,z+a} - A_{r,z-a}}{2a}. \quad (10)$$

When these approximations are put into Eq. (3) and the result is solved for A at (r,z) , we get the relaxation algorithm for an inhomogeneous medium:

$$\begin{aligned}
A_{r,z} = & \left\{ \left[1 + \frac{a}{2r} + \left(\frac{r+a}{4r} \right) \left(\frac{\mu_{r,z}}{\mu_{r+a,z}} - \frac{\mu_{r,z}}{\mu_{r-a,z}} \right) \right] A_{r+a,z} \right. \\
& + \left[1 - \frac{a}{2r} - \left(\frac{r-a}{4r} \right) \left(\frac{\mu_{r,z}}{\mu_{r+a,z}} - \frac{\mu_{r,z}}{\mu_{r-a,z}} \right) \right] A_{r-a,z} \\
& + \left[1 + \frac{1}{4} \left(\frac{\mu_{r,z}}{\mu_{r,z+a}} - \frac{\mu_{r,z}}{\mu_{r,z-a}} \right) \right] A_{r,z+a} \\
& \left. + \left[1 - \frac{1}{4} \left(\frac{\mu_{r,z}}{\mu_{r,z+a}} - \frac{\mu_{r,z}}{\mu_{r,z-a}} \right) \right] A_{r,z-a} + a^2 \mu_{r,z} J_{r,z} \right\} \\
& \div [4 + (a/r)^2 + j\omega\mu_{r,z}\sigma_{r,z}a^2] . \quad (11)
\end{aligned}$$

In a homogeneous medium, the permeabilities are all the same, and Eq.

(11) reduces to

$$\begin{aligned}
A_{r,z} = & \{ [1 + (a/2r)] A_{r+a,z} + [1 - (a/2r)] A_{r-a,z} + A_{r,z+a} + A_{r,z-a} \\
& + a^2 \mu_{r,z} J_{r,z} \} \div [4 + (a/r)^2 + j\omega\mu_{r,z}\sigma_{r,z}a^2] . \quad (12)
\end{aligned}$$

It is evident from Eq. (12) that $a^2\mu J$ must have the same dimensions as the vector potential, A . Since a is the lattice spacing and J is the driving current density, we see that a^2J is the driving current through one square of the lattice. If N_q is the number of turns in the coil and I_q is the current in each turn, then

$$I_0 = N_q I_q = \text{total current in the coil.}$$

If n is the number of lattice spacings in the mean coil radius, $\bar{R}$, then $n = \bar{R}/a$. If $R_2 - R_1 = (r_2 - r_1)/\bar{R}$ = normalized radial dimension of

the coil, and if $L_2 - L_1 = (\ell_2 - \ell_1)/\bar{R}$ = normalized axial length of the coil, then

$$n^2(R_2 - R_1)(L_2 - L_1) = \left(\frac{\bar{R}}{a}\right)^2 \left(\frac{r_2 - r_1}{\bar{R}}\right) \left(\frac{\ell_2 - \ell_1}{\bar{R}}\right) = \left(\frac{r_2 - r_1}{a}\right) \left(\frac{\ell_2 - \ell_1}{a}\right)$$

= number of lattice points in coil cross section .

Therefore

$$A \propto \mu a^2 J = \frac{\mu N_q I_q}{n^2(R_2 - R_1)(L_2 - L_1)} . \quad (13)$$

Equation (13) is the relationship by which the vector potential varies with the various scaling parameters: permeability μ , coil turns N_q , coil current I_q , coil mean radius in lattice spacings n , and the normalized cross-sectional area $(R_2 - R_1)(L_2 - L_1)$.

The other quantities, such as forces, heating, etc., which depend upon the vector potential, have their scaling factors given elsewhere.⁴

Equations (4)-(10) are called symmetrical derivatives, because they are centered at the point (r, z) , in contrast to a one-sided derivative, such as

$$\left(\frac{\partial A}{\partial r}\right)_{\text{one-sided}} = \frac{A_{r+a, z} - A_{r, z}}{a} , \quad (14)$$

which gives slightly different equations in place of (11) and (12). The differences are generally small, except near the axis or at boundaries where A changes rapidly.

⁴ C. V. Dodd and W. E. Deeds, *Scaling Relations for Eddy Current Phenomena*, ORNL-5077 (November 1975).

The computer program uses Eq. (12) for the coarse lattice in the homogeneous regions, such as the metals, coils, and air, and uses Eq. (11) for the fine lattices at the boundaries between homogeneous regions, where the values of μ and σ are changing. The program alternately steps through the coarse and fine lattices, using the values just computed for the other lattice as boundary values in the relaxation of the lattice being computed.

With modifications, the program can be used for any axially symmetric distribution of coils and conductors. The running time is of the order of a few minutes on an IBM 360 model 91 for an array of about 10,000 lattice points, roughly half in the coarse lattice and half in the fine lattice. The layout of the coarse and fine lattices is shown in Fig. 2; the variation of the conductivity with the radius is shown at the bottom.

The quantity SUM is the sum of the absolute values of the changes of vector potential from one iteration to the next and thus is a measure of the convergence of the vector potential to the limiting values. However, the value of the vector potential at each point may oscillate before settling down to a monotonic approach. In Figs. 3 and 4 are shown curves of the variation with the number of iterations of the magnitude and phase of the vector potential at the coarse lattice points marked with circles in Fig. 2. Note that the points nearer the coil source receive the signal sooner and converge faster. The signal will propagate through the lattice at the rate of one lattice spacing per iteration. Hence, it will take of the order of 150 iterations for the signal to reach the boundaries and start back. Since a number of reflections back and forth are required to reach a steady state, we see that it is not surprising that the number of iterations required is large.

DESCRIPTION OF THE PROGRAM

There is a main control program, which defines and initializes several quantities used in the calculations and controls the switching from coarse to fine lattices and back, as well as to the output subroutine. The first subroutine, called only once, is named LATT. It sets up the coarse and fine lattices and initializes the values of the vector potential at all lattice points to zero.

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Fig. 2. Part of fine and coarse lattices.

Fig. 3. Convergence of magnitude of vector potential.

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Fig. 4. Convergence of phase of vector potential.

The COARSE subroutine sets the values of the vector potential at points which are one coarse lattice spacing inside the transition regions (in the fine lattice) equal to the value of the vector potential obtained at that point in the previous relaxation of the fine lattice. This is zero at the start, of course, but gradually builds up through successive iterations. Using these initialized values as fixed boundary values for the coarse lattice relaxation, plus zero values of the vector potential on the symmetry axis and around the outside of the whole lattice, the coarse lattice is relaxed. Each region with different properties, such as the metal, the coil, or air, is treated separately, of course. The quantity called CUR = $\mu / (\text{cross-sectional area of the coil in lattice spacings})$ can be seen to be equal to the factor $\mu \alpha^2 J$ in Eq. (13) if the driving current I is 1 A. First we note that H in the program has the same lattice spacing as the α in Eq. (13). Then

$$\mu \alpha^2 J = \mu H^2 J = \mu \frac{I}{(r_2 - r_1/H)(\ell_2 - \ell_1/H)} = \frac{\mu I}{n^2(R_2 - R_1)(L_2 - L_1)} = I * \text{CUR} ,$$

which becomes CUR when $I = 1$. The actual driving current I has been factored out as one of the scaling parameters. The coarse lattice points on the initialized boundary points are not changed. They are four fine lattice steps inside the fine lattice. At the end of each relaxation calculation, the changes are calculated in the values of the real and imaginary parts of the vector potential from the previous iteration, the values are "over-relaxed" by the constant CC, and the changes are added to the "SUM," which measures the progress toward convergence.

Then the main control program switches to the FINE subroutine, in which the fine lattice boundaries are initialized, and then the rest of the fine lattice is relaxed. The values of the vector potential at the boundaries are set equal to the values obtained for the coarse lattice in the previous stage. This is done directly for those fine lattice points which coincide with coarse lattice points, but values for fine lattice points between the coarse lattice points are obtained by a linear interpolation. Then the real and imaginary parts of the vector

potential are relaxed and overrelaxed at each point in the fine lattice, except at the fixed boundary values, which overlap into the coarse lattice two coarse lattice steps.

When the process has converged sufficiently, as determined by the decrease in the value of SUM or by the limit imposed on the number of iterations, the program switches to the OUTPUT subroutine, which prints out the real and imaginary parts of the vector potential at various points in the coarse and fine lattices, near the region of the driving coil.

DETAILED DESCRIPTION OF THE RELAX.F4 PROGRAM

Main Control Routine

The COMMON block specifications may need to be changed for different problems. At present the program is written for a cylindrical metal rod encircled by a rectangular cross-section coil in air, as shown in Fig. 1. $A(M,N)$ and $B(M,N)$ are the real and imaginary parts of the vector potential in the coarse lattice, with 60 lattice indices along the radial direction in cylindrical coordinates and 101 lattice indices along the axial direction. $AF(M,N)$ and $BF(M,N)$ are the corresponding quantities for the fine lattice, which at present has 13 radial steps and 301 axial steps (three times as fine as the coarse lattice). $MAT(M,N)$ and $MATF(M,N)$ are matrices whose elements denote the type of material at the point (M,N) in the coarse and fine lattices respectively. $PERR(N)$, $PERZ(N)$, and $COND(N)$ specify combinations of the permeability along radial and z directions and conductivity values at various points in the fine lattice.

One may also want to change the data given in line 1300, in order to keep track of the problem being solved.

The radius in the coarse lattice is $r = (M - 1)\alpha$, where M goes from 1 to 60 and α is the lattice spacing. We shall use just the index M without the α to express radial distances.

Various constants are defined as follows:

IB = boundary radius + 1 (in number of lattice spacings)
 CT = convergence test constant for "SUM"
 F = frequency in hertz
 $W = \mu_0 \omega = (4\pi \times 10^{-7}) \times (2\pi F)$
 UM = permeability of the metal
 $CONM$ = conductivity of the metal
 CC = over-relaxation constant ($1 \leq CC \leq 2$)
 $RBAR$ = mean coil radius in meters
 IR = number of lattice spacings in $RBAR + 1$
 RNS = mean coil radius in lattice spacings
 H = lattice spacing in meters
 $COILPT$ = number of lattice points in the coil
 $L2$ = normalized z -coordinate of one end of coil
 $L1$ = normalized z -coordinate of other end of coil
 $R1$ = normalized inner coil radius
 $R2$ = normalized outer coil radius
 $CUR = \mu_0 / \text{cross-sectional area of coil (in lattice squares)}$
 IZ = z coordinate of coil center
 IT = number of iterations
 NW = number of iterations since last printout
 $IMIN$ = minimum number of iterations
 PO = factor used in three-dimensional calculations only
 $X2 = \omega \mu_0 H^2$

The program then produces a printout of some of the input parameters.

LATT Subroutine

The LATT subroutine (lines 27,000 through 35,500) first assigns numbers to the various regions in the problem in the form of a matrix $MAT(M,N)$: 0 represents the axis and the outer boundaries (which are never changed from zero vector potential); 1 represents the transition region (from metal to air); 2 represents the air region; 3 represents the coil region; and 4 represents the central metal rod. A map of the coarse lattice is then printed (lines 31,600-32,400).

The fine lattice, described by $MATF(M,N)$, is set up in a similar manner and is printed out in lines 34,300-35,100. The radial extent is from $M = 1$ to 13, or 12 lattice spacings, while the axial extent is from $N = 1$ to 301, so that the fine lattice spacings are one-third the size of the coarse lattice spacings.

$PERR(M) = (\nu_M/\nu_{M+1} - \nu_M/\nu_{M-1})/4$ is a function used in the relaxation calculations for a region with inhomogeneous permeability.

$COND(M) = \omega\mu\sigma(H/3)^2$, where $H/3$ is the fine lattice spacing and $\sigma = \sigma(M)$ is a function of the radius M .

COARSE Subroutine

The convergence measure, "SUM," is first initialized to zero; then there is a DO loop which steps through the coarse lattice in the radial direction for all points inside the outer boundaries, which are left at zero vector potential. Then several quantities are defined which are used in the relaxation calculations:

$RI = a/r$ = inverse of radius in lattice spacings

$RH = RI/2$

$X1 = 4 + (a/r)^2$

$X3 = 1 + (a/2r)$

$X4 = 1 - (a/2r)$

$I1$ = radial index in the fine lattice which corresponds to the value of M in the coarse lattice

$I2 = 4$ = value of the axial index in the fine lattice where the initialization process starts, corresponding to the value $N = 2$ in the coarse lattice; this is just inside the lower boundary, which is never changed from zero vector potential

The DO loop in line 11,600 causes the program to step through the coarse lattice in the axial direction for all points inside the boundary.

If the point is in the transition region [$MAT(M,N) = 1$], according to the test in line 11,800, the relaxation process is bypassed, and the

values of the $R\mathcal{L}$ and Im parts are initialized from the fine lattice in lines 13,500 and 13,600.

Otherwise, the $R\mathcal{L}$ and Im parts of $A(R,Z)$ are calculated by the relaxation algorithm, using the expressions for $X5$ and $X6$. Lines 14,200-14,400 make the calculation for air; lines 14,800-15,000 for the coil region; and lines 15,400-15,600 for the metal. The values found are overrelaxed in lines 16,200-16,500, and the convergence measure "SUM" is calculated in lines 16,600-16,700.

FINE Subroutine

The fine lattice boundary values are initialized in lines 19,500-20,800. The values at points coinciding with coarse lattice points are calculated in lines 19,500-20,100. The $R\mathcal{L}$ and Im parts at fine lattice points between the coarse lattice boundary points are set equal to values linearly interpolated between the values at the neighboring coarse lattice points.

In lines 22,600-23,200 the relaxation formulas for an inhomogeneous medium are applied and overrelaxed. The various quantities used in the relaxation are defined in lines 21,100-22,200. The only one requiring further explanation is AOR , which is the reciprocal of the radius in fine lattice spacings. The radius must be measured from the central axis of the problem, not from $M = 1$ in the fine lattice, and therefore r is a fairly large number in terms of fine lattice spacings.

OUTPUT Subroutine

In lines 25,000-25,400 are instructions to print the vector potential in the coarse lattice for radii out to $M = 30$ and, for Z values from the center of the coil (where $N = 51$) down to $N = 40$. It is understood that the converged values are symmetrical about the coil center in this particular problem.

In lines 26,000-26,400 are instructions to print the corresponding $R\mathcal{L}$ and Im parts at fine lattice points from the coil midposition ($K = 151$) to $K = 140$ for all values of the radial index J (which goes from $J = 1$ to 13).

DISCUSSION OF MODIFICATIONS TO THE PROGRAM

Several versions of the program have been tried with different values of the permeability of the central metal conductor, from $\mu = 1$ to $\mu = 200$.

A comparison of the accurate ENDFTL potentials and the RELAX results for a metal of unit permeability is shown in Table 1. The magnitudes are within 10% and the phases are within 1° of the correct values at the lattice points with values of $M = 7, 11, 15, 19$ and $N = 41, 45, 49$. Some of the disagreement may be caused by inexact matching of the ENDFTL and RELAX points of evaluation.

An extrafine lattice, twice as fine as the usual fine lattice, has also been tried for the case when the metal permeability is $\mu = 4$. When compared with the "exact" values of the vector potential obtained from the ENDFTL program, the results are as shown in Table 2. The extrafine lattice gave magnitude errors only about one-third as large as those for the regular fine lattice, and the phases were about 7° closer to the ENDFTL values. Thus it is evident that the extrafine lattice is a considerable improvement, and the lattice should perhaps be even finer. However, this is somewhat discouraging, since there are already almost as many points in the fine lattice as in the coarse lattice, and $\mu = 4$ is a rather small permeability for a ferromagnetic medium. Presumably, for a larger permeability, an excessively fine lattice would be required to make the derivatives remain sufficiently small. From this standpoint, it now seems preferable to have only one layer of boundary points between media and use a separate set of boundary conditions for them, rather than using a "smooth" transition layer of inhomogeneous material to keep the derivatives small. The reason we used the transition layer approach here was that we had the ultimate aim of making three-dimensional relaxations, in which the boundary conditions are coupled and quite complex, so that it still seems that the inhomogeneous transition layer approach would be preferable for that sort of problem.

Probably some of the PRINT statements could be omitted. Among these are the ones for printing the map of the fine lattice. And the ones for printing IT and SUM could be moved from lines 7000-7100 to

Table 1. Comparison of ENDFTL and RELAX results for metal of unit permeability

(All magnitudes should be multiplied by 10^{-8} ; phases are in degrees)

Z	R	7			11			15			19		
		ENDFTL	RELAX	Error	ENDFTL	RELAX	Error	ENDFTL	RELAX	Error	ENDFTL	RELAX	Error
41	Mag.	5.816	5.234	-10%	9.783	8.315	-9.89%	13.822	12.424	-10.11%	17.737	15.964	-10.00%
	Ph.	-15.33	-14.38	+0.95°	-13.87	-12.98	+0.89°	-11.75	-10.90	+0.85°	-9.02	-8.18	+0.84°
45	Mag.	6.678	6.085	-8.88%	11.425	10.437	-8.65%	16.648	15.232	-8.50%	22.513	20.761	-7.78%
	Ph.	-14.66	-13.79	+0.87°	-13.10	-12.23	+0.87°	-10.81	-10.02	+0.79°	-7.89	-7.11	+0.78°
49	Mag.	7.1965	6.602	-8.26%	12.456	11.444	-8.13%	18.589	17.135	-7.82%	26.477	24.655	-6.88%
	Ph.	-14.30	-13.41	+0.89°	-12.66	-11.85	+0.81°	-10.24	-9.51	+0.73°	-7.12	-6.43	+0.69°

Table 2. Comparison of vector potential obtained with ENDFTL and with RELAX
(Fine lattice version of 4 Nov. 1975 and extrafine lattice version of 13 Nov. 1975)
(All magnitudes should be multiplied by 10^{-8} ; $\mu = 4$; phases are in degrees)

Z	R	7					11					15					19				
		ENDFTL	RELAX fine	Error	Extra- fine	Error	ENDFTL	RELAX fine	Error	Extra- fine	Error	ENDFTL	RELAX fine	Error	Extra- fine	Error	ENDFTL	RELAX fine	Error	Extra- fine	Error
41	Mag.	6.15	11.42	+85.9%	7.90	+28%	10.22	19.33	+89.2%	13.37	+30.8%	14.23	27.66	+94.4%	19.24	+35.2%	18.18	36.66	+101.7%	25.45	+40.0%
	Ph.	-4.19	-44.89	-40.70°	-37.96	-33.77°	-3.81	-39.12	-35.31°	-32.32	-28.51°	-3.24	-30.64	-27.38°	-23.35	-20.55°	-2.52	-19.77	-17.25°	-12.97	-10.45°
45	Mag.	8.45	13.01	+53.9%	9.34	+10.5%	14.10	22.32	+26.1%	16.23	+14.9%	19.70	32.85	+66.7%	24.12	+22.4%	25.34	45.57	+79.8%	33.70	+33.0%
	Ph.	-3.87	-43.54	-39.67°	-36.85	-32.98°	-3.54	-37.54	-34.00°	-30.30	-27.36°	-0.29	-28.75	-28.46°	-22.40	-22.11°	-2.22	-17.75	-15.54°	-11.73	-9.51°
49	Mag.	6.16	14.00	+127.2%	10.22	+65.9%	17.70	24.16	+24.5%	17.93	+1.3%	25.02	36.45	+45.7%	27.43	+9.6%	32.67	52.82	+61.7%	40.72	+24.6%
	Ph.	-4.19	-42.63	-38.44°	-36.24	-32.05°	-3.30	-36.59	-33.29°	-30.20	-26.9°	-0.27	-27.62	-27.35°	-21.51	-21.34°	-1.98	-16.27	-14.29°	-10.77	-8.79°

lines 24,725-24,750, with the print number changed from 60, so that the number of iterations and the SUM would only be printed at the end of every 500 iterations.

In order to produce more rapid convergence, the overrelaxation constant, *CC*, could be increased from 1.0 to 1.5 or 1.75. However, care must be exercised, since the immediate replacement of the old value with the new, relaxed value makes instability and oscillation somewhat more likely than with the usual relaxation procedure.

If a different geometry is desired, the COMMON block statements will need to be changed, as well as the LATT subroutine, the initializations in the COARSE and FINE subroutines, and the PRINT ranges in the OUTPUT subroutine. In fact, only the relaxation and overrelaxation algorithms can be carried over with any confidence, although the method of shifting back and forth to the fine lattice will still be valid and although it might be changed to shifting between boundary and nonboundary points.

USING THE PROGRAM ON A TIME-SHARING COMPUTER

The RELAX.F4 file may be read off the magnetic tape in the MONITOR mode with the command

```
FILE R, D622, RELAX.F4
```

If desired, it can then be left stored on the user's disk file. When sufficient time has elapsed for copying the file from the tape, then the commands

```
EDIT RELAX.F4
```

```
.P100:5500
```

will cause the first 55 lines to be printed.

The input parameters are mostly located in the MAIN control routine and are explained in the comment line just preceding the parameter input. In the EDIT mode, one can just replace the data line by typing

```
.R(line number)(carriage return)
```

and then retyping the line with the correct data after the Teletype reprints the line number. Remember that there are six spaces after the line number before the parameter is given.

The dates in lines 200 and 1300 should probably be changed, so as to keep track of the version being run.

If the metal permeability, UM , is changed in line 2400, the DATA statement in line 27,900 should also be changed. It should start with four values equal to UM and end with four values of 1.0, with five intermediate values in between. For example, if $UM = 4.$, the DATA statement might be

```
DATA V/4*4., 3.5, 3., 2.5, 2., 1.5, 4*1./
```

If the fine lattice spacing is changed from its present one-third coarse lattice value, the number of intermediate data values must be increased. For example, if an extrafine lattice, twice as fine as at present, is used, there should be 7 values equal to UM , followed by 11 intermediate values, and ending with 7 values equal to 1., making a total of 25 data values instead of the present 13.

Similarly, if the conductivity of the metal is changed from $CONM = 2.8E6$ in line 2600, the conductivity data in line 28,000 should also be changed, starting with 4 values equal to $CONM$, followed by 5 intermediate values, and ending with 4 values of zero. As before, if a different fine lattice spacing is used, a different number of data values will be required.

If it is not desired to have printouts every 500 iterations, but only at the end, then the NW statements can be deleted (lines 5300, 7500, 7600, 7700, 7800) with the commands

```
D5300
```

```
D7500:7800
```

The number of iterations can be changed by altering $IMIN$ in line 5500. It should probably be at least 2500 (see Figs. 3 and 4), unless CC is greater than 1.

If one wants the results to be printed out on the Teletype terminal, instead of the line printer, one can change all the PRINT statements to TYPE. This is most easily done with TECO.

The program is set up for a rectangular coil centered at $[(IR-1)a, IZa]$ encircling a metal rod of radius $(IB-1)a$, relative permeability UM , and conductivity $CONM$. For any other geometry, extensive modifications to the LATT subroutine, the COMMON blocks, and the initialization procedures will be necessary.

Given in the appendix is a printout of a sample run for a central rod of relative permeability $UM = 4$, conductivity $CONM = 2.8 \times 10^6$ mhos/m, and radius of 20 lattice spacings, encircled by a coil of square cross section (three lattice numbers on a side), centered at a radius of 25 lattice spacings (radial index = 26). The rest of the region is assumed to be air; the transition region extends from radial index 19 to 21, and the fine lattice has spacings one-third as large as the coarse lattice. The coarse and fine lattice maps are printed out immediately after the program, followed by the values of SUM at various iterations.

The $R\bar{L}$ and Im parts of the vector potential in the coarse and fine lattices near the coil are also printed out after every 500 iterations. The values of M , the radial index, are given at the left side and increase downward, while the axial index, N , is given at the top and increases to the right. The coil is centered at $M = 27$, $N = 51$ in the coarse lattice. The points in the coarse lattice which are compared with the ENDFTL values are located at $(M,N) = (7,41), (7,45), (7,49), (11,41), (11,45), (11,49), (15,41), (15,45), (15,49), (19,41), (19,45), (19,49)$. The first point is the one farthest from the coil; the last is nearest the coil; these are the points for which data are plotted in Figs. 3 and 4. At each of the points, the upper number is the real part of the vector potential, and the lower number is the imaginary part, in mksA units.

For determining eddy-current heating, forces, etc., only the values inside the metal are important, and they can be used to calculate other electromagnetic quantities of interest, as explained in an earlier report.⁵

⁵C. V. Dodd and W. E. Deeds, *Scaling Relations for Eddy Current Phenomena*, ORNL-5077 (November 1975).

As stated previously, the purpose of writing this two-dimensional relaxation program was to test the method of alternating between coarse and fine lattice relaxations for use in a three-dimensional unsymmetrical situation. In order to test the method, the program was written to duplicate a symmetrical configuration whose solution was accurately known. Therefore, the program has been specialized to the cylindrically symmetric encircling coil. If a more complicated, but still axially symmetric, problem is to be solved, almost the entire program needs to be rewritten. The relaxation calculations in the coarse and fine lattices will still be valid, but all of the initializing routines will probably need modification, and it is impossible to say in general whether it would be easier to initialize individual lattice points or to write subroutines to initialize various groups of points, as has been done here.

After all editing changes have been made, one can save them by typing

Q

and a carriage return. Then one can exit and remove the line numbers by typing

B

followed by a carriage return.

To run the program on the 360/91 from a Teletype terminal, one needs to type a job card, say JOB.JCL as follows:

```
//(job name) (space) JOB (space) (charge No.), '(bin designation)'
/*ROUTE (25 spaces) PRINT (space) LOCAL
/**CLASS (space) CPU91 = 10M, REGION = 360K
//(space) EXEC (space) FORTHCLG, PARM.FORT = "XREF," PARM.GO =
    'DUMP = I'
//FORT.SYSIN (space) DD (space)* = RELAX.F4
/*
//
ENDINPUT
*BR:JOB.JCL
```

After the computer returns with an EXIT and a period, then type

IBM (space) JOB.JCL

After the terminal stops responding to this, type

TO (space) DISPATCHER

Then the computer will come back with a number of questions to which you should type the answers, such as

JOB NAME: D

REGION SIZE: 360

PHONE NUMBER:

:

SEND IT: YES

Now you can finally sign off, if you wish.

ACKNOWLEDGMENTS

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00100 C RELAX.F4, A 2-D AXISYMMETRIC RELAXATION PROGRAM
00200 C VERSION OF 11 DECEMBER 1975
00300 C
00400 C *** MAIN CONTROL ROUTINE
00500 C
00600 C REAL L1,L2
00700 C COMMON//A(60,101),B(60,101),AF(13,301),BF(13,301),
00800 C 1AA,AAF,AC,BB,BBF,BC,IB,H,IR,IZ,S,SUM,W,UM,CONM,CC,F,
00900 C 2NW,CUR,X2,PERR(20),PERZ(20),
01000 C 3COND(20),MAT(60,101),MATF(13,301)
01100 C
01200 C PRINT 2
01300 C 2 FORMAT(' RELAX 11 DECEMBER 1975 ')
01400 C IB=1+(BOUNDARY RADIUS IN LATTICE SPACINGS)
01500 C IB=21
01600 C CT=CONVERGENCE TEST CONSTANT FOR "SUM"
01700 C CT=100.
01800 C F=FREQUENCY IN HERTZ
01900 C F=1.E3
02000 C PI=3.1415926536
02100 C W=MU ZERO*OMEGA=(4PI*E-7)*(2PI*F)
02200 C W=8.*PI*PI*1.E-7*F
02300 C UM=PERMEABILITY OF THE METAL
02400 C UM=1.
02500 C CONM=CONDUCTIVITY OF THE METAL
02600 C CONM=2.8E6
02700 C CC=OVER-RELAXATION CONSTANT(1<CC<2)
02800 C CC=1.
02900 C RBAR=MEAN COIL RADIUS IN METERS
03000 C RBAR=.01
03100 C IR=1+(NUMBER OF LATTICE SPACINGS IN RBAR)
03200 C IR=27
03300 C RNS=MEAN COIL RADIUS IN LATTICE SPACINGS
03400 C RNS=FLOAT(IR-1)
03500 C H=LATTICE SPACING IN METERS
03600 C H=RBAR/RNS
03700 C COILPT=NUMBER OF LATTICE POINTS IN THE COIL
03800 C COILPT=9.0
03900 C L2=NORMALIZED Z-COORDINATE OF ONE END OF COIL
04000 C L2=.11538
04100 C L1=NORMALIZED Z-COORDINATE OF OTHER END OF COIL
04200 C L1=0.0
04300 C R1=NORMALIZED INNER COIL RADIUS
04400 C R1=.94231
04500 C R2=NORMALIZED OUTER COIL RADIUS
04600 C R2=1.05769
04700 C CUR=MU ZERO/CROSS-SECT. AREA OF COIL IN LATTICE SQUARES
04800 C CUR=1.25664E-6/(RNS*RNS*(R2-R1)*(L2-L1))
04900 C IZ=Z-COORDINATE OF COIL CENTER
05000 C IZ=51

```



```

05100 C      IT=NUMBER OF ITERATIONS, INITIALIZED TO 1
05200      IT=1
05300      NW=1
05400 C      IMIN=MINIMUM NUMBER OF ITERATIONS
05500      IMIN=3000
05600 C      X2=OMEGA*MU*SIGMA*H|2
05700      X2=W*UM*CONM*H*H
05800 C
05900      PRINT 5,UM,CONM,CC,F
06000      5 FORMAT('MU=',F6.2,',COND=',E8.2,',CC=',F4.2,',F=',F6.1)
06100 C
06200      INITIALIZING VARIABLES
06300 C
06400      CALL LATT
06500 C
06600      50 CALL COARSE
06700 C
06800      55 CALL FINE
06900 C
07000      PRINT 60,IT,SUM
07100      60 FORMAT(' ITER=',I5.5X,'SUM=',E10.4)
07200 C
07300      NUMBER OF ITERATION IS INCREASED BY ONE
07400      IT=IT+1
07500      NW=NW+1
07600      IF(NW.LT.500)GO TO 50
07700      CALL OUTPUT
07800      NW=0
07900 C
08000 C      IF LESS THAN IMIN ITERATIONS, DOES NOT TEST CONVERGENCE
08100      IF(IT.LT.IMIN)GO TO 50
08200 C
08300 C      IF NOT CONVERGED,REITERATE
08400      IF(SUM.GT.CT)GO TO 50
08500 C
08600      STOP
08700      END
08800 C
08900 C      *****
09000 C
09100      SUBROUTINE COARSE
09200 C
09300      COMMON//A(60,101),B(60,101),AF(13,301),BF(13,301),
09400      1AA,AAF,AC,BB,BBF,BC,IB,H,IR,IZ,S,SUM,W,UM,CONM,CC,F,
09500      2NW,CUR,X2,PERR(20),PERZ(20),
09600      3COND(20),MAT(60,101),MATF(13,301)
09700 C
09800 C      SUM IS INITIALIZED
09900      SUM=0.0
10000 C

```

```

10100 C STEP THROUGH LATTICE IN R & Z DIRECTIONS RESPECTIVELY
10200 C
10300 DO 60 M=2,59
10400 C RI=INVERSE OF RADIUS IN LATTICE SPACINGS
10500 RI=1./FLOAT(M-1)
10600 RH=RI*.5
10700 C X1,X3,AND X4 ARE QUANTITIES USED IN CALCULATION
10800 X1=4.+RI*RI
10900 X3=1.+RH
11000 X4=1.-RH
11100 C I1=FINE LATTICE RADIUS CORRESP.TO M IN COARSE LATTICE
11200 I1=3*(M-I8)+7
11300 C I2=Z-COORDINATE IN FINE LATTICE,INITIALIZED AT 4
11400 I2=4
11500 C
11600 DO 55 N=2,100
11700 C TEST TO DETERMINE REGION
11800 IF(MAT(M,N).EQ .1)GO TO 10
11900 C
12000 C X5&X6 ARE RL & IM PARTS OF A(R,Z) AVERAGED OVER
12100 C THE FOUR NEAREST NEIGHBORS. THIS FACTOR APPEARS IN
12200 C ALL REGIONS.
12300 C
12400 X5=X3*A(M+1,N)+X4*A(M-1,N)+A(M,N+1)+A(M,N-1)
12500 X6=X3*B(M+1,N)+X4*B(M-1,N)+B(M,N+1)+B(M,N-1)
12600 C
12700 C TESTS IF IN METAL,AIR,COIL, OR BOUNDARY
12800 C
12900 MGO=MAT(M,N)
13000 GO TO(10,20,30,40),MGO
13100 C
13200 C POINT IN FINE LATTICE:VALUE TRANSFERRED TO COARSE LATT
13300 C
13400 10 CONTINUE
13500 A(M,N)=AF(I1,I2)
13600 B(M,N)=BF(I1,I2)
13700 I2=I2+3
13800 GO TO 55
13900 C
14000 C IN AIR
14100 C
14200 20 AC=X5/X1
14300 BC=X6/X1
14400 GO TO 50
14500 C
14600 C IN THE COIL: CUR=H|2*U*J CONDUCTIVITY ASSUMED 0
14700 C
14800 30 AC=(CUR+X5)/X1
14900 BC=X6/X1
15000 GO TO 50

```

```

15100 C
15200 C      IN THE METAL
15300 C
15400 40 DEN=1./(X1*X1+X2*X2)
15500 AC=(X5*X1+X6*X2)*DEN
15600 BC=(X6*X1-X5*X2)*DEN
15700 C
15800 C      OVER-RELAXATION IS PERFORMED AND CHANGE IN LATTICE
15900 C      VALUES FROM PREVIOUS RELAXATION IS CALCULATED.
16000 C      CONVERGENCE MEASURE "SUM" IS INCREMENTED
16100 C
16200 50 AC=AC-A(M,N)
16300 BC=BC-B(M,N)
16400 A(M,N)=A(M,N)+CC*AC
16500 B(M,N)=B(M,N)+CC*BC
16600 S=ABS(AC)+ABS(BC)
16700 SUM=SUM+S
16800 55 CONTINUE
16900 60 CONTINUE
17000 C
17100 RETURN
17200 END
17300 C
17400 C      *****
17500 C
17600 SUBROUTINE FINE
17700 C
17800 COMMON/A(60,101),B(60,101),AF(13,301),BF(13,301),
17900 1AA,AAF,AC,BB,BBF,BC,IB,H,IR,IZ,S,SUM,W,UM,CONM,CC,F,
18000 2NW,CUR,X2,PERR(20),PERZ(20),
18100 3COND(20),MAT(60,101),MATF(13,301)
18200 DATA W1,W2 /.6666667,.3333333/
18300 C
18400 C      STEPPING THROUGH FINE LATTICE
18500 C
18600 DO 50 M=1,13
18700 C
18800 C      IF ON FINE LATTICE BOUNDARIES,INITIALIZE FROM COARSE LATT
18900 C      IF(MATF(M,2).EQ.0)GO TO 10
19000 C      OTHERWISE, PERFORM RELAXATION
19100 GO TO 30
19200 C
19300 C      INITIALIZE FROM COARSE LATTICE
19400 C
19500 10 DO 20 N=1,298,3
19600 C      I2=COARSE LATTICE Z CORRESP.TO FINE LATTICE N
19700 I2=(N+2)/3
19800 C      I1=COARSE LATTICE R CORRESP.TO FINE LATTICE M
19900 I1=(IB+(M-7)/3)
20000 AF(M,N)=A(I1,I2)

```

```

20100      BF(M,N)=B(I1,I2)
20200      C      INTERMEDIATE FINE LATTICE VALUES INTERPOLATED
20300      C      BETWEEN COARSE LATTICE POINTS
20400      AF(M,N+1)=W1*A(I1,I2)+W2*A(I1,I2+1)
20500      BF(M,N+1)=W1*B(I1,I2)+W2*B(I1,I2+1)
20600      AF(M,N+2)=W2*A(I1,I2)+W1*A(I1,I2+1)
20700      BF(M,N+2)=W2*B(I1,I2)+W1*B(I1,I2+1)
20800      20 CONTINUE
20900      GO TO 50
21000      30 CONTINUE
21100      C      AOR=1/R IN FINE LATTICE SPACINGS FROM THE AXIS
21200      AOR=1./FLOAT(3*IB-10+M)
21300      C      X1,X3,X4,DENF ARE PARAMETERS USED IN RELAXATION CALC.
21400      X1=4.+AOR*AOR
21500      X3=1.+5*AOR+(1.+AOR)*PERR(M)
21600      X4=1.-5*AOR-(1.-AOR)*PERR(M)
21700      DENF=1./(X1*X1+COND(M)*COND(M))
21800      X1=X1*DENF
21900      CF=COND(M)*DENF
22000      DO 50 N=2,300
22100      X5=X3*AF(M+1,N)+X4*AF(M-1,N)+AF(M,N+1)+AF(M,N-1)
22200      X6=X3*BF(M+1,N)+X4*BF(M-1,N)+BF(M,N+1)+BF(M,N-1)
22300      C
22400      C      FINE LATTICE RELAXATION IS PERFORMED
22500      C
22600      AAF=X5*X1+X6*CF
22700      BBF=X6*X1-X5*CF
22800      C
22900      C      OVER-RELAXATION IS PERFORMED
23000      C
23100      AF(M,N)=AF(M,N)+CC*(AAF-AF(M,N))
23200      BF(M,N)=BF(M,N)+CC*(BBF-BF(M,N))
23300      C
23400      50 CONTINUE
23500      C
23600      RETURN
23700      END
23800      C
23900      C      *****
24000      C
24100      SUBROUTINE OUTPUT
24200      C
24300      COMMON//A(60,101),B(60,101),AF(13,301),BF(13,301),
24400      1AA,AAF,AC,BH,BHF,BC,IB,H,IP,I2,S,SUM,W,UM,CUNM,CC,F,
24500      2NW,CUR,X2,PERR(20),PERZ(20),
24600      3COND(20),MAT(60,101),MATF(13,301)
24700      C
24800      C      COARSE LATTICE VALUES NEAR COIL ARE PRINTED
24900      C
25000      PRINT 1,(N,N=40,51)

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```

25100      1 FORMAT(2X,12I10)
25200      DO 30 M=2,30
25300      PRINT 10,M,(A(M,N),N=40,51),(B(M,N),N=40,51)
25400      10 FORMAT('0',12,2X,12(E9.3,1X)/5X,12(E9.3,1X))
25500      20 CONTINUE
25600      30 CONTINUE
25700      C
25800      C      FINE LATTICE VALUES NEAR COIL ARE PRINTED
25900      C
26000      PRINT 1,(K,K=140,151)
26100      DO 60 J=1,13
26200      PRINT 40,J,(AF(J,K),K=140,151),(BF(J,K),K=140,151)
26300      40 FORMAT('0',12,2X,12(E9.3,1X)/5X,12(E9.3,1X))
26400      50 CONTINUE
26500      60 CONTINUE
26600      C
26700      RETURN
26800      END
26900      C
27000      C      *****
27100      C
27200      SUBROUTINE LATT
27300      C
27400      COMMON/A(60,101),S(60,101),AF(13,301),BF(13,301),
27500      1AA,AAF,AC,BB,BBF,BC,IB,H,IR,IZ,S,SUM,W,UM,CONM,CC,F,
27600      2NW,CUR,X2,PERR(20),PERZ(20),
27700      3COND(20),MAT(60,101),MATF(13,301)
27800      DIMENSION U(13),CON(13)
27900      DATA U/13*1./
28000      DATA CON/4*2.8E6,2.34E6,1.87E6,1.4E6,9.3E5,4.6E5,4*0./
28100      C
28200      C      SET UP COARSE LATTICE
28300      C      MAT(M,N) HAS THE VALUE 0 ON THE AXIS AND AROUND THE
28400      C      OUTER BOUNDARIES, WHERE THE VECTOR POTENTIAL IS ALWAYS
28500      C      KEPT ZERO; MAT(M,N)=1 FOR THE TRANSITION REGION;
28600      C      MAT(M,N)=2 IN THE AIR REGION; MAT(M,N)=3 IN THE COIL
28700      C      REGION; AND MAT(M,N)=4 IN THE CENTRAL METAL PCO
28800      C
28900      DO 20 M=1,60
29000      DO 20 N=1,101
29100      MAT(M,N)=0
29200      20 CONTINUE
29300      DO 30 N=2,100
29400      DO 25 M=2,19
29500      MAT(M,N)=4
29600      25 CONTINUE
29700      DO 26 M=20,22
29800      MAT(M,N)=1
29900      26 CONTINUE
30000      DO 30 M=23,60

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30100      MAT(M,N)=2
30200      30 CONTINUE
30300      MAT(26,50)=3
30400      MAT(27,50)=3
30500      MAT(28,50)=3
30600      MAT(26,51)=3
30700      MAT(27,51)=3
30800      MAT(28,51)=3
30900      MAT(26,52)=3
31000      MAT(27,52)=3
31100      MAT(28,52)=3
31200      C
31300      C      SET COARSE LATTICE VALUES TO 0 AND PRINT COARSE MAP
31400      C      SET UP FINE LATTICE
31500      C
31600      PRINT 60,(M,M=1,60)
31700      DO 40 N=1,101
31800      PRINT 50,N,(MAT(M,N),M=1,60)
31900      DO 40 M=1,60
32000      A(M,N)=0.
32100      B(M,N)=0.
32200      40 CONTINUE
32300      50 FORMAT(1X,13,60I2)
32400      60 FORMAT(4X,60I2)
32500      C
32600      C      SET UP FINE LATTICE
32700      C
32800      DO 100 N=1,301
32900      M=1
33000      MATF(M,N)=0
33100      M=13
33200      MATF(M,N)=0
33300      DO 100 M=2,12
33400      MATF(M,N)=M
33500      100 CONTINUE
33600      DO 110 M=2,12
33700      C      PERR(M)=PERMEABILITY FACTOR IN RELAXATION FORMULA
33800      PERR(M)=(U(M)/U(M+1)-U(M)/U(M-1))*25
33900      C      COND(M)=OMEGA*MU*SIGMA*(FINE LATTICE SPACING SQUARED)
34000      COND(M)=W*CON(M)*U(M)*(H*.3333333)**2
34100      110 CONTINUE
34200      C
34300      C      SET FINE LATTICE VALUES TO 0 AND PRINT FINE MAP
34400      C
34500      PRINT 60,(M,M=1,13)
34600      DO 120 N=1,301
34700      PRINT 50,N,(MATF(M,N),M=1,13)
34800      DO 120 M=1,13
34900      AF(M,N)=0.
35000      BF(M,N)=0.
35100      120 CONTINUE
35200      C
35300      C
35400      RETURN
35500      END

```


231	N	1	6	7	8	91011112	C
232	N	2	6	7	8	91011112	C
233	N	3	6	7	8	91011112	C
234	N	4	6	7	8	91011112	C
235	N	5	6	7	8	91011112	C
236	N	6	6	7	8	91011112	C
237	N	7	6	7	8	91011112	C
238	N	8	6	7	8	91011112	C
239	N	9	6	7	8	91011112	C
240	N	10	6	7	8	91011112	C
241	N	11	6	7	8	91011112	C
242	N	12	6	7	8	91011112	C
243	N	13	6	7	8	91011112	C
244	N	14	6	7	8	91011112	C
245	N	15	6	7	8	91011112	C
246	N	16	6	7	8	91011112	C
247	N	17	6	7	8	91011112	C
248	N	18	6	7	8	91011112	C
249	N	19	6	7	8	91011112	C
250	N	20	6	7	8	91011112	C
251	N	21	6	7	8	91011112	C
252	N	22	6	7	8	91011112	C
253	N	23	6	7	8	91011112	C
254	N	24	6	7	8	91011112	C
255	N	25	6	7	8	91011112	C
256	N	26	6	7	8	91011112	C
257	N	27	6	7	8	91011112	C
258	N	28	6	7	8	91011112	C
259	N	29	6	7	8	91011112	C
260	N	30	6	7	8	91011112	C
261	N	31	6	7	8	91011112	C
262	N	32	6	7	8	91011112	C
263	N	33	6	7	8	91011112	C
264	N	34	6	7	8	91011112	C
265	N	35	6	7	8	91011112	C
266	N	36	6	7	8	91011112	C
267	N	37	6	7	8	91011112	C
268	N	38	6	7	8	91011112	C
269	N	39	6	7	8	91011112	C
270	N	40	6	7	8	91011112	C
271	N	41	6	7	8	91011112	C
272	N	42	6	7	8	91011112	C
273	N	43	6	7	8	91011112	C
274	N	44	6	7	8	91011112	C
275	N	45	6	7	8	91011112	C
276	N	46	6	7	8	91011112	C
277	N	47	6	7	8	91011112	C
278	N	48	6	7	8	91011112	C
279	N	49	6	7	8	91011112	C
280	N	50	6	7	8	91011112	C
281	N	51	6	7	8	91011112	C
282	N	52	6	7	8	91011112	C
283	N	53	6	7	8	91011112	C
284	N	54	6	7	8	91011112	C
285	N	55	6	7	8	91011112	C
286	N	56	6	7	8	91011112	C
287	N	57	6	7	8	91011112	C
288	N	58	6	7	8	91011112	C
289	N	59	6	7	8	91011112	C
290	N	60	6	7	8	91011112	C
291	N	61	6	7	8	91011112	C
292	N	62	6	7	8	91011112	C
293	N	63	6	7	8	91011112	C
294	N	64	6	7	8	91011112	C
295	N	65	6	7	8	91011112	C
296	N	66	6	7	8	91011112	C
297	N	67	6	7	8	91011112	C
298							

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