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RESOLVING BEAM TRANSPORT PROBLEMS IN ELECTROSTATIC ACCELERATORS

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ABSTRACT

This paper reviews problem areas in beam transmission which are frequently encountered during the design, operation and upgrading of electrostatic accelerators. Examples are provided of analytic procedures that clarify accelerator ion optics and lead to more effective beam transport. Suggestions are made for evaluating accelerator design with the goal of improved performance.

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1. Introduction

What constitutes a problem in beam transport through an electrostatic accelerator? Certainly anything that degrades or attenuates the beam between its creation at a source and demise at a target constitutes a problem. More insidiously, any transport deficiency that protracts or delays accelerator operation poses a problem. My intent in this presentation is not to solve specific problems in detail but rather to illustrate methods and concepts that have proved useful for resolving beam transport problems by reducing them to recognizable and soluble forms.

The present discussion is a continuation of work on this subject reported previously.¹⁻⁴⁾ Readers will find the earlier contributions helpful in describing methods and nomenclature presented here.

One beam transport problem which tends to dominate all others in electrostatic accelerators is that of getting the beam past the acceleration tube entrance without losing control. Parts of this (hopefully) familiar story are repeated in Section 2 to emphasize that this problem is always with us and to demonstrate again how potent is the lens effect at the entrance to an open acceleration tube.

During studies of the ORNL 25 MV tandem accelerator, efforts to state with simplicity the acceptance and emittance properties of electrostatic accelerators led to development of a general technique for determining phase space acceptance and emittance of beam transport systems.⁴⁾ Limited explorations using this technique indicate that it explains analytically many phenomena that previously have been understood (often vaguely) only through repetitious numerical calculations. Section 3 illustrates application of this technique to the analysis of beam injection into electrostatic acceleration tubes.

Day-to-day variations in transmission efficiency through an accelerator present a perplexing but probably unnecessary problem. Sometimes one operator is especially proficient in achieving good transmission while others struggle over much longer periods to achieve less. Although many factors may contribute, inadequate steering control always should be suspected. To paraphrase a familiar quotation, "Everybody talks about *beam steering* but nobody does anything about it." Section 4 is a light-hearted attempt to do something about making beam steering an accepted part of beam transport analysis.

2. Problems

Difficulties associated with the transport of charged particle beams through electrostatic accelerators originate primarily in a few regions where relatively low energy particles encounter large changes in electric fields. This occurs at the low-energy (LE) entrance to the accelerator and (for the higher charge states) again at the entrance to the high-energy (HE) stage of a tandem accelerator. Fringing fields extruded out from the acceleration tube create an aperture lens⁵⁾ capable of bending incident particles through very substantial angles. Although the remainder of each acceleration stage further influences beam behavior, the consequences for beam transport generally are much less than the impact of fringing fields at the entrance. Indeed, it is better to err by neglecting the whole of the remaining acceleration tube (treating it as empty space) than to discard the fringing fields at the entrance as a negligible "perturbation".

Figure 1 illustrates several ion optical properties of open-ended electrostatic acceleration tubes. Each of the four drawings depicts particle "rays" passing through a region in which the electric potential was determined by relaxing a rectangular mesh of 150 by 28 points. These examples are for purposes of illustration and do not correspond to any actual device. Particles enter at low energy from the left and are accelerated toward the right through two sections of simulated acceleration tubes. The horizontal axis represents the center of a beam of particles; the calculations are cylindrically symmetric about the beam axis. Equipotential lines drawn upward from the horizontal axis indicate the electrically graded region of acceleration. Equipotentials bulge outward through openings at the ends of the acceleration sections creating radially directed electric field components which profoundly influence beam behavior. (Shifts in positions of equipotential lines from case to case are an artifact of the calculational procedure and not relevant to this discussion.)

In Fig. 1a the overall voltage has been adjusted to cause rays originating from a point on axis at the extreme left to focus to a point on axis at the extreme right. A single-stage accelerator, or the LE stage of a tandem accelerator, might be operated in this way. The accelerator functions as a lens having its converging power concentrated just outside the acceleration tube entrance. Rays leaving the first acceleration section (left of center) diverge perceptibly in passing outward through the aperture lens formed there but the effect is now small because the particles have acquired considerable additional energy. Most of this divergence is cancelled at the entrance to the second acceleration section (right of center) where the aperture lens has essentially the same strength but opposite sense. (Near cancellation of aperture lens effects may be expected wherever gradient fluctuations occur between apertures of identical geometry provided the intervening separation does not permit the beam to change appreciably in size from one aperture to the next. Shapes of the apertures become relevant where monotonic changes in gradient take place, as discussed below.) Rays leaving the final acceleration section in Fig. 1a again diverge but the effect is nearly undetectable.

Figure 1b shows the consequences of a factor of four increase in voltage across the accelerator. The entrance lens responds as expected by shortening its focal length causing the beam to cross over inside the first acceleration section. The lens acts as if it were centered well in front of the accelerator.

Assuming output conditions corresponding to Fig. 1a are still desired, some change must now be made in the injected beam. One possibility is to move the source of beam along the axis as shown in Fig. 1c. Notice how different this makes the rays look compared to Fig. 1a. Such changes produce a real challenge in beam transport analysis. Having to move the object point for the accelerator causes a sizeable problem; when this is

done using lenses (not shown in Fig. 1c) the beam properties change, creating a second problem; even the identical source when relocated relative to the accelerator produces a different kind of transport, thus creating a third problem. Sometimes competing effects can be used advantageously to cancel each other but this seldom is easy to accomplish over the wide dynamic operating range required of electrostatic accelerators.

Single-stage accelerators frequently utilize variable extraction voltage or some form of "gap" lens to focus the beam. Fig. 1d illustrates how a gap lens may be used to recover the focus condition lost in Fig. 1b. A gap lens is weak intrinsically; its principal effect here is to reduce the strength of the fringing lens at the tube entrance by raising the injection energy thus lowering the energy gain of the accelerator. In fact, the ratio of final energy to injection energy (at the tube entrance) is lower in Fig. 1d than in Fig. 1a so that the (unwanted) focusing effect of the gap lens can be accommodated. This illustrates that the actual function of some beam transport elements may differ markedly from our mental images.

Because the entrance lens effect is so important to electrostatic accelerator beam optics, numerous efforts have been made to modify its properties. One approach is to keep the focal length constant for all accelerator voltages. This includes methods of electrically bypassing or "shorting" portions of the accelerator so that the gradient on unshorted sections remains nearly constant. The focal length of an aperture lens varies inversely with gradient change;⁵⁾ therefore, if the entrance section remains unshorted, its ion optical properties can be kept approximately constant by shorting other sections. Shorting devices are incorporated in accelerators built by National Electrostatics Corporation (NEC) for gradient control.⁶⁾ The shorting pattern used in these machines should be selected for optimum beam

transmission. Indiscriminate shorting of inclined-field type acceleration tubes would undoubtedly lead to disastrous consequences by imbalancing the effects of transverse electric forces; however, the problem is not insuperable if each tube section is designed to accept and restore the beam on axis so that shorting any one section does not significantly displace the beam from the axis.

An auxiliary voltage supply could be used to control the open entrance lens. HVEC once considered a low-gradient (weak lens) approach.⁷⁾ A high-gradient approach has been considered more recently at the University of Washington.⁸⁾ Both ideas require that appreciable energy be added to the injected beam as it traverses a preliminary portion of the accelerator in which the gradient is separately controlled so that subsequent gradient changes will have significantly diminished influence. The gradient in this initial section would then be adjusted to minimize variation of object position with terminal voltage.

Rather than battle the entrance lens problem, why not dispense with it entirely? HVEC has successfully eliminated nearly all the fringing field by introducing a "gridded lens" geometry in Model MP and subsequent tandem accelerators with similar modifications available for other models.⁹⁾ Fig. 2 illustrates some of the consequences. A semi-transparent, wire-mesh grid is stretched across the tube entrance constraining the electric field parallel to the axis in this region. An accelerating voltage is applied between this grid and an external electrode to produce a converging, gridded gap lens of adjustable focal length. Replacing the natural entrance lens with this controllable lens de-couples beam optics requirements of the accelerator from those of the injector and permits a long, fixed object distance. This stabilizes the optics over a wide range of voltages and reduces magnification at high terminal voltages. The grid intercepts some beam (typically under 10%) and, consequently, produces secondary

electrons which accelerate toward the terminal in a tandem accelerator. The mesh itself comprises miniature aperture lenses whose separate facets aberrate the passing beam somewhat. Nevertheless, a gridded lens of large diameter and long focal length is a comparatively benign beam transport element.

In Fig. 2a rays originate from a point on axis 70 mesh units off scale to the left (100 mesh units left of the grid). The gridded lens voltage was adjusted to focus these rays on axis at the far right (120 mesh units right of the grid). Fig. 2b shows the consequences of a 4-fold increase in accelerator voltage with no change at the gridded lens. Figure 4c shows the beam restored to its original focus by appropriate adjustment in lens voltage. Notice how closely rays in Figs. 2a and 2c resemble each other, in contrast to open tube conditions depicted in Figs. 1a and 1c. Operational experience verifies that the grid functions as intended and performs very well.

Figure 2d illustrates what happens if the acceleration tube has locally reduced gradient at the grid. Initially, the equipotentials remain flat but, in the region where the gradient rises to full value, equipotentials begin to curve (as seen in Fig. 2d) creating a lens weaker than at the open entrance but still contributing convergence to the beam (compare Figs. 2a and 2d which have the same gridded lens voltage). Modest changes in gradient are easily compensated by appropriate reductions in gridded lens voltage.

A new problem is created, however, if this or any other minor focusing action is not axially symmetric. Suppose, for example, that the tube electrodes have slot-shaped apertures rather than circles. Then these comparatively weak lens effects are no longer symmetric about the beam axis⁵⁾ and a situation arises analogous to having a beam pattern similar to Fig. 2a in the horizontal plane, say, and a pattern similar to Fig. 2d in the

vertical. The result is astigmatism at the stripper which the gridded lens cannot eliminate. The lesson here is that even modest focusing asymmetries within acceleration tubes can interfere with good beam transmission. Additional asymmetries must be introduced (e.g. using quadrupole lenses) to compensate and experience has shown this is a thorny problem to resolve.

So far, this description has concentrated on problems associated with single-stage accelerators, including the LE stage of tandem accelerators. Does this imply that if the beam somehow can be made to negotiate the LE portion and stripper of a tandem accelerator then the HE stage will take care of itself? Yes and no! If HE beam transport were not *laissez faire*, today's accelerators would behave poorly. As it is, ions stripped to low charge states by a gas stripper easily pass through the HE stage of contemporary tandem accelerators (which is all that is required) and present no problem. Difficulties in the HE stage begin to surface when heavy ions stripped to high charge states by solid foil strippers attempt to traverse long, small-aperture acceleration tubes. The differences arise because solid strippers scatter heavy ions to large angles while high charge states shorten the focal length of that pesky lens stationed at the entrance to the HE acceleration tube. This situation is exacerbated if tube apertures are not circular and further muddled when a second stripper is used. Obviously, current trends in heavy-ion research utilizing tandem accelerators insure continuity of beam transport problems (if not the beam itself) between LE and HE stages.

3. Perspectives

NOTE: Figures 3, 4 and 5 used for illustration in this section contain computational errors. Curves shown in Fig. 3 and the left half of Fig. 4 (exclusive of the diagonal line) are shifted left by very small amounts and slightly distorted. Except for the straight segment starting near the origin, curves in the right half of Fig. 4 and in Fig. 5 are shifted left, by amounts

comparable with divisions marked on the vertical scale, and also somewhat distorted; the straight segment starting near the origin should bend upward slightly at each end. Neither the general appearance of the graphs nor conclusions drawn in the text are affected by these errors.

The accelerated beam eventually strikes a target. This is a distinct physical object, something easy to visualize. The accelerator operator and his predecessor, the beam transport system designer, face an embarrassingly simple task: get maximum beam through various components and onto the target. But what strategy is optimum for each intermediate component? What property of a lens, a bending magnet, an acceleration tube, the whole accelerator, or just a length of pipe corresponds to a target? The targets in these cases are windows in phase space, combinations of constraints in beam size and angle. Analysis of these windows for various combinations of beam transport elements is a way of letting the accelerator tell the analyst how it will respond to the beam.

Each component of the beam transport system is characterized not only by its electromagnetic properties but also by physical size limitations. The beam passes through successive apertures which limit the maximum extent in size and angle that the beam may occupy in each part of the total system. Between any two apertures there is some well-defined quantity of beam (i.e. phase space area) that can be accepted into the first aperture, transmitted between apertures and then emitted unscathed from the second aperture. The phase space area (ellipse) accepted by apertures of radius r_1 and r_2 is given by⁴⁾

$$A_{12} = \pi r_1 r_2 / |a_{12}| ,$$

where a_{12} is a linear beam transport coefficient that describes how much displacement will occur at exit aperture r_2 for particles of a given angle entering at aperture r_1 .

Figure 3 illustrates some of the important phase space acceptance limitations of an idealized stage of electrostatic acceleration in combination with an adjustable injection lens. The four quadrants of Fig. 3

describe the acceptance of four different combinations of aperture constraints and operating conditions as a function of $R = (\text{Final energy}/\text{Initial energy})^{1/2}$. The diagonal line in the upper left quadrant represents the maximum acceptance of the acceleration tube (terminated in these examples by a small aperture representing, e.g., a stripper canal). Only beams having emittances below this line can possibly pass through with 100% transmission efficiency. Beams having emittances exactly on the line require perfect injection matching for 100% transmission; such conditions are represented schematically by solid curves in the inset drawing. Beams of larger emittance necessarily are attenuated regardless of how injected. Raising terminal voltage increases the accelerator's ability to accept beam. The acceptance formula for this diagonal line is⁴⁾

$$\mathcal{A}_{34} = \pi (r_3 r_4 / L_{34}) [L_2^1 (R+1)] ,$$

where $R = (\text{Final energy}/\text{Initial energy})^{1/2}$,

r_3 = entrance radius,

r_4 = exit (stripper) radius,

L_{34} = length of acceleration tube.

Minor drift distances and internal details have been ignored. Note that the entrance lens at r_3 is assumed not to change $\mathcal{A}_{34}$ despite its crucial importance to overall beam transport. As a consequence, $\mathcal{A}_{34}$ applies also to the gridded entrance type of acceleration tube (of uniform gradient) whereas the remainder of Fig. 3 does not.

The lower left quadrant of Fig. 3 displays acceptance $\mathcal{A}_{24}$ constrained by aperture r_2 at the external lens and aperture r_4 at the tube exit (stripper). In between lies the entrance lens at r_3 whose focal length varies approximately as^{4,5)}

$$f_3 = 4k L_{34} / (R^2 - 1),$$

where k = graded fraction of L_{34} .

$\mathcal{A}_{24}$ passes through a singularity when f_3 produces a focus between planes at r_2 and r_4 ($a_{12} \rightarrow 0$ defines a focus). Note that r_3 and any other apertures between r_2 and r_4 are here being ignored allowing rays to have slopes between $\pm\infty$. Like $\mathcal{A}_{34}$, acceptance $\mathcal{A}_{24}$ is independent of how the injection lens at r_2 is adjusted.

The upper right quadrant of Fig. 3 introduces the injection lens as an active, adjustable element. As the focal length of the injection lens is changed, the acceptance varies between apertures r_1 and r_3 and between apertures r_1 and r_4 . The curve labeled $\mathcal{A}_{134}$ results from the lens setting which gives maximum acceptance from r_1 through both r_3 and r_4 simultaneously. A cusp occurs in $\mathcal{A}_{134}$ where acceptance curves representing long and short focal length conditions cross each other. The cusp occupies a region where beam transmission will be poor regardless of how the injection lens is operated. Left of the cusp there is a short interval delineated by vertical marks in which the lens is required to diverge the beam (concave lens) for maximum acceptance. Although such negative lens operation is possible (e.g., a gridded einzel lens) it usually is not available.

The lower right quadrant of Fig. 3 demonstrates overall acceptance $\mathcal{A}_{14}$ from r_1 through r_4 without benefit of the injector lens at r_2 . Comparison of this curve to $\mathcal{A}_{134}$ above (obtained by using the lens to fullest advantage) dramatizes the power of an adjustable lens to enhance acceptance and transmission efficiency.

Figure 4 combines separate components of acceptance illustrated in Fig. 3 to present composite properties of this accelerator model. The dimensions correspond very roughly to the ORNL Model EN tandem accelerator: $r_1 = r_4 = 3$ mm, $r_3 = 12$ mm, $L_{13} = 3$ m, $L_{34} = 4$ m, $k = 0.8$; however, this is an illustrative exercise and not a serious attempt to analyze any particular

accelerator. Instead of holding r_2 fixed, a limiting angle of 10 mrad was assumed at r_1 , and r_2 computed according to $r_2 = [r_1^2 + (0.010 L_{12})^2]^{1/2}$; this limits the incident beam acceptance to about 30π mm mrad indicated by arrows at appropriate places on the ordinate. Three different lens positions were studied corresponding to $L_{12}/L_{23} = 3, 1$ and $1/3$ as indicated in Fig. 4. Horizontal scales are labeled directly as energy ratios.

Three drawings comprising the left half of Fig. 4 correspond to moving the injection lens toward (upper drawing) or away (lower drawing) from the accelerator. Segmented curves in each drawing correspond to limitations in acceptance imposed by the various apertures as illustrated previously in Fig. 3. Cusps occur in Fig. 4 as one combination of apertures after another cuts into the acceptance. The region above the arrows on each vertical scale is also excluded. The primary acceptance limit of the acceleration tube, $\mathcal{A}_{34}$, runs diagonally through each drawing. Projecting below this line are segments of $\mathcal{A}_{24}$ (revealed clearly in lower left) and an indentation from $\mathcal{A}_{134}$ (apparent to some extent in all curves). Where divergent (negative) injection lens operation is demanded along $\mathcal{A}_{134}$, the lens is turned off and acceptance $\mathcal{A}_{14}$ substituted.

From the curves on the left side of Fig. 4 it is evident that no single location of the injection lens is optimum for all accelerator voltages. (This is well known but is typical of beam transport problems that in the past have been difficult to approach analytically.) Placing the lens close to the acceleration tube (upper left) provides a good acceptance match at high terminal voltages and, perhaps as a mild surprise, also at very low terminal voltages; however, disaster lurks in between. Moving the lens away from the acceleration tube retracts the $\mathcal{A}_{134}$ dagger from the mid-section at the expense of encroachment by $\mathcal{A}_{24}$ first from the high-voltage extreme (center left) and finally from both sides (lower left).

The phase space acceptance window in acceleration tubes has been steadily closing down as tubes are made both longer and smaller in internal diameter. A way to open the window again is to insert adjustable lenses within tubes, either as an integral part of the electrostatic structure or in intermediate "dead" spaces between tube sections. Intermediate lenses improve acceptance and also cope remarkably well with the entrance lens problem already extensively discussed.

Drawings comprising the right half of Fig. 4 demonstrate changes in accelerator acceptance that occur when an intermediate lens is introduced within an acceleration stage. In these examples an adjustable thin lens is located in the center of the acceleration tube so that $L_{34} = L_{45} = 2m$, $r_4 = 12 \text{ mm}$, and $r_5 = 3mm$. The right half (with lens) and left half (without lens) of Fig. 4 are morphologically related. Acceptance $\mathcal{A}_{134}$ dominates the upper right curve while $\mathcal{A}_{24}$ encroaches from the right in both lower drawings. Acceptance $\mathcal{A}_{45}$ supercedes $\mathcal{A}_{34}$ as the fundamental tube (plus stripper) limit. The horizontal line connecting $\mathcal{A}_{45}$ and $\mathcal{A}_{24}$ in the lower right drawing is $\mathcal{A}_{23}$ which is too large to be limiting in the other parts of Fig. 4 (and still substantially exceeds the $30 \pi \text{ mm mrad}$ beam window, $\mathcal{A}_{12}$, marked by arrows).

Not only does the intermediate lens open the acceptance window, it also makes the acceptance acceptably uniform over the anticipated range of operating voltages and much more tolerant of the position of the external injection lens. With good transmission available at a variety of lens positions, internal and external lenses may be placed where desired to control other functions such as magnification.

What manner of change to the tube entrance is this? Where has such good transmission come from? This change is a true beam transport phenomenon that takes place in phase space, rather than a physical modification. Conventional beam transport analysis will show that an internal

lens permits the beam within the acceleration tube to diverge after passing the entrance lens. The stripper then "sees" through a different window in phase space at the tube entrance; it acquires a different perspective of the beam.

Figure 5 shows an acceptance diagram for the LE stage of the new 25 MV tandem accelerator now under construction at ORNL. The segmented curve in Fig. 5 was computed from the same basic model used for the right half of Fig. 4 with appropriate changes in dimensions: $k = 0.6$, $r_1 = 3$ mm, $r_2 = 14$ mm, $r_3 = r_4 = 12.7$ mm, $r_5 = 4.8$ mm, $L_{12} = 2.7$ m, $L_{23} = 2.1$ m, $L_{34} = 6.3$ m, $L_{45} = 12.6$ m. The incident beam window between r_1 and r_2 was halved by reducing the angular limit to 5 mrad while keeping $r_1 = 3$ mm. Generally, the acceptance of the 25 MV accelerator is about half that shown in Fig. 4 and this is reflected in the expanded vertical scale in Fig. 5. Nevertheless, the energy-normalized acceptance ($\text{mm mrad MeV}^{1/2}$) is comparable because beam energies in the 25 MV accelerator are everywhere about 4 times those of the EN-based model.

This section has emphasized the study of beam transport without the beam. Each beam transport component is constrained by its geometry and electromagnetic properties to certain beam preferences. Reduction of these preferences to a single variable, the phase space acceptance area, permits a wealth of analytic information easily to be gathered for each component and (with some skill) applied to systems containing many components. Each device has a harmony of its own and a message to tell -- if we will but listen.

4. Steering Theory

Beam steering surely remains the most neglected topic in the analysis of beam transport. Perhaps this proclamation of a "theory" of steering will awaken some -- and interest others -- in the need for more attention to this subject.

A steerer is a device that changes the angle of the beam. Steerers fall into two broad categories: controllable devices named "steerers" that are expected upon demand to steer the beam, and all other beam transport elements, by whatever name, that are not expected to steer the beam -- but do. Among the latter, adjustable lenses create significant operational difficulties because steering caused by the lens confuses the process of focusing when beam current passing through an aperture is the only measure of progress. Lens steering occurs when the beam enters the lens off-axis, a situation resulting from inadequate or imprecise steering farther upstream.

Controllable steerers usually are expected to move the beam transverse to the axis somewhere downstream of the steerer. Causally, however, if the beam is regarded conceptually as following a continuous, connected path, then the benefit may be envisioned as accruing either upstream or downstream to suit the problem. Suppose line segments corresponding to the motion of the beam are drawn to represent preferred initial and final beam directions projected onto each transverse plane. If these lines cross within the system, then for each plane a single steerer located at the crossing will provide the desired control. Two steerers for each plane located (within reason) anywhere in the system guarantee complete steering capability because a line (or curve) will connect the points at which steerers are located anywhere along the initial and final beam direction lines. Of course, internal obstructions may preclude some or all of the available beam paths necessitating the use of additional steerers.

Most beam transport systems are designed (in principle) not to require steerers. Where, then, should steerers be added in anticipation of correctional requirements? First, displacement planes where beam position is critical must be identified. Narrow apertures and slits obviously require the most precise beam positioning. Less precise beam centering may be important in certain larger apertures such as inside acceleration tubes.

Within lenses, beam position relative to an electromagnetic axis is of interest.

Upstream of each displacement plane exists an optimum steerer location. To obtain maximum lever arm, the steerer ideally should be as far upstream as practicable provided it does not precede another critical displacement plane. The displacement plane itself constitutes a steering node. The effect of a steerer increases linearly from zero as it is moved back from the plane. Lenses can produce additional upstream nodes (at foci). In free space, the effect of a steerer at the displacement plane varies linearly with the distance of the steerer from the local node for that plane regardless of what elements lie between the steerer and the displacement plane. The sense of the steerer inverts from one side to the other of the node. Often a node for one displacement plane is an optimum steerer location for another displacement plane.

A typical beam transport subsystem consists (functionally) of an aperture, some drift space, a lens, more space, and a second aperture. When the lens is in focus between apertures, the first aperture is located at a steering node relative to the second. Consider one transverse (e.g. vertical) plane. Beam passing through the first aperture is not necessarily aligned to the hypothetical axis centered in the lens. A steerer coincident with the first aperture (perhaps a little downstream or upstream) is ideally located for controlling beam position at the center of the lens but, because of the node, will exhibit negligible control of beam position at the second aperture. For beam control at the second aperture the optimum steerer location is essentially coincident with the lens, that is, as far as possible from either node but still between apertures. Note that two steerers are desired (in each plane) for this subsystem; however, they are not grouped as a pair because each steerer serves a well-defined, independent function.

The misconception that steerers function in pairs (allowing "dog-leg" steering) has led to some very poor choices of steerer location.

Because one optimum steerer location nearly always lies coincident with a lens, and also for economic reasons, quadrupole lenses sometimes are provided with unbalanced "dipole" excitations to produce both focusing and steering from one element. Elementary calculations¹⁰⁾ show that such "dipole" fields are quite nonuniform, varying about a factor of 2 across the aperture of the lens. As a consequence, focused beams will distort by amounts of order $d(r/R)^2$, where d is the amount the beam is steered and (r/R) is the ratio of beam radius to lens radius at the lens. To avoid severe aberration, this type of steering should be limited to moving the focused beam at most a few diameters.

Figure 6 illustrates a steering problem that can occur many ways but here is associated with inclined-field acceleration tubes. The vertical scale corresponds to vertical displacement of the beam centroid. The horizontal scale represents beam position in the HE stage of a Model MP tandem accelerator. Different tube designs are described in the upper and lower parts of the figure. Solid curves in Fig. 6 correspond to a charge 10 beam injected (from the left) perfectly on axis; both such beams encounter tube electrodes (near 15 mm displacement) at roughly the same place along the axis. Dot-dash curves correspond to initial steering applied at the arrows such that the emerging beam is centered at the exit of the final acceleration tube section. Dashed curves correspond to a beam which emerges perfectly on axis.

The tube design represented in Fig. 6 by the upper family of curves continuously deflects the beam in the vertical direction. Without steering, these acceleration tubes are functionally opaque to this beam. Steering at the terminal brings the beam (dot-dash curve) down to the axis but leaves an unavoidable residual angle that requires immediate correction by a second

steerer located near the accelerator exit. Persistent curvature forces the beam well off axis no matter how steered. Further reduction in average beam displacement could be achieved by installing a third steerer mid-way through this acceleration stage. Three steerers then would break the bow at its beginning, center and end.

The lower family of curves in Fig. 6 derive from a tube design which ordinarily transports the beam close to the axis, requiring little or no steering. An artificial beam excursion (solid curve) was created by shorting the last resistor in the first section of inclined electrodes. Naturally, the terminal steerer is very effective in restoring the beam nearly to coaxial geometry (dot-dash curve) because corrective steering is applied close to the single perturbation. In this case the beam is as well controlled by one steerer as the bowed beam (upper curves) can be controlled by two or even three steerers.

Steering is not intuitive. Without some (relatively simple) analysis neither the number of steerers nor their proper placement can be determined with certainty. Each steerer should have one well-defined task, to move the beam in a specified displacement plane. If beam motion in that plane can be monitored, then the steering adjustment is easily completed in seconds. Cantankerous controls are a common symptom of inadequate steering capacity. Good steering is a good investment.

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APPENDIX

Phase Space Acceptance Calculations

Phase space acceptance calculations mentioned in the text require, in the denominator, the a_{12} elements of standard 2×2 linear beam transport matrices.^{1,4)} The following matrices, when taken in various combinations, pertain to examples presented in Section 3:

$$\begin{array}{cccc}
 \text{Transport} & & \text{Acceleration} & \text{Intermediate} \\
 \text{Matrix} & & & \text{Lens} \\
 \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} & = & \begin{pmatrix} 1 & M_{45} \\ 0 & 1/R_{45} \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ -1/f_4 & 1 \end{pmatrix} \\
 & & & \text{Acceleration} \\
 & & & \begin{pmatrix} 1 & M_{34} \\ 0 & 1/R_{34} \end{pmatrix} \times
 \end{array}$$

$$\begin{array}{cccc}
 \text{Entrance} & & \text{Injector} & \\
 \text{Lens} & \text{Drift} & \text{Lens} & \text{Drift} \\
 \times \begin{pmatrix} 1 & 0 \\ -1/f_3 & 1 \end{pmatrix} & \begin{pmatrix} 1 & L_{23} \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix} & \begin{pmatrix} 1 & L_{12} \\ 0 & 1 \end{pmatrix} ,
 \end{array}$$

where

f_2 = focal length of injection lens,

$f_3 = 4kL/(R^2 - 1)$ = focal length of entrance lens,

f_4 = focal length of intermediate lens,

$M_{34} = 2L_{34}/(R_{34} + 1)$,

$M_{45} = 2L_{45}/(R_{45} + 1)$,

$R_{34} = [1 + (L_{45}/L_{34})(R^2 - 1)]^{1/2}$,

$R_{45} = R/R_{34}$,

$R = [\text{Final Energy}/\text{Initial Energy}]^{1/2}$.

Individual matrices not required for a specific calculation are omitted.

Products of the remaining matrices then lead to the following acceptances:

$$\begin{aligned}
 \mathcal{A}_{12} &= \pi r_1 r_2 / L_{12} , \\
 \mathcal{A}_{23} &= \pi r_2 r_3 / L_{23} , \\
 \mathcal{A}_{34} &= \pi (r_3 r_4 / L_{34}) \left[\frac{1}{2} (R_{34} + 1) \right], \\
 \mathcal{A}_{45} &= \pi (r_4 r_5 / L_{45}) \left[\frac{1}{2} (R + R_{34}) \right], \\
 \mathcal{A}_{24} &= \pi r_2 r_4 / |L_{23} + 2L_{34}(1 - L_{23}/f_3)/(R_{34} + 1)| , \\
 \mathcal{A}_{13}^* &= \pi r_1 r_3 / |L_{12} + L_{23} - L_{12}L_{23}/f_2| , \\
 \mathcal{A}_{14}^* &= \pi r_1 r_4 / |(1 - M_{34}/f_3)(L_{12} + L_{23} - L_{12}L_{23}/f_2) + M_{34}(1 - L_{12}/f_2)| , \\
 \mathcal{A}_{14} &= \pi r_1 r_4 / |(1 - M_{34}/f_3)(L_{12} + L_{23}) + M_{34}| , \\
 \mathcal{A}_{134} &= \text{Maximum } \mathcal{A}_{13}^*(f_2) \text{ and } \mathcal{A}_{14}^*(f_2) .
 \end{aligned}$$

Acceptance $\mathcal{A}_{45}$ has been reduced to the same energy realm as the others through the energy normalizing factor $R_{34} = (E_4/E_3)^{1/2}$. Values of $\mathcal{A}_{13}^*$ and $\mathcal{A}_{14}^*$ depend on f_2 ; $\mathcal{A}_{134}$ is determined by adjusting f_2 for best acceptance simultaneously in $\mathcal{A}_{13}^*(f_2)$ and $\mathcal{A}_{14}^*(f_2)$. Acceptance $\mathcal{A}_{14}$ corresponds to $\mathcal{A}_{14}^*(f_2 = \infty)$.

Beam Steering and Steering Nodes

The effect of a steerer in linear beam transport calculations may be included as follows:

Final Ray	Arbitrary Element(s)			Drift 2	Steerer	Drift 1	Initial Ray
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$$\begin{pmatrix} x \\ x' \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \ell_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \ell_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ 1 \end{pmatrix}$$

Here θ is the change in divergence (i.e. "angle") imparted locally to the beam by the steerer which is assumed (for convenience) to have negligible length. Drift lengths ℓ_1 and ℓ_2 are assumed to exist on each side of the steerer, followed by beam transport element(s) having arbitrary properties. The position of the final ray in the displacement plane is

$$x = a_{11}x_0 + [(\ell_1 + \ell_2) a_{11} + a_{12}] x'_0 + a_{13} + (a_{11}\ell_2 + a_{12})\theta .$$

Displacement in x changes linearly with θ . The steering effort (i.e. θ) required for a given displacement depends on a steering magnification term, $(a_{11}\ell_2 + a_{12})$.

At a steering node, magnification goes to zero, hence

$$(a_{11}\ell_2 + a_{12})_0 = 0,$$

or $(\ell_2)_0 = -a_{12}/a_{11}$, assuming $a_{11} \neq 0$,

where $(\ell_2)_0$ is the location of the steerer where it has no effect on beam displacement in the displacement plane. Now let

$$S = \ell_2 - (\ell_2)_0 ,$$

be the distance from the node to the steerer; then the equation for transverse motion in the displacement plane becomes

$$x = a_{11}x_0 + [(\ell_1 + \ell_2) a_{11} + a_{12}] x'_0 + a_{13} + a_{11} S \theta .$$

This equation shows that for a given applied steering angle, θ , the resulting motion in the displacement plane varies linearly with the distance, S , between steerer and node.

FIGURE CAPTIONS

- Fig. 1 Particle trajectories through open acceleration tube with natural focus (a), 4-fold voltage increase (b), 4-fold voltage and re-positioned source (c), 4-fold voltage and gap lens focus (d).
- Fig. 2 Particle trajectories through gridded-entrance acceleration tube with gap lens focus (a), 4-fold voltage increase (b), 4-fold voltage and refocused lens (c), original voltages and internal gradient step (d).
- Fig. 3 Component acceptances of electrostatic accelerator as function of energy ratio. Curve in each quadrant represents phase space acceptance as constrained by indicated apertures. Consult text regarding errors in this figure.
- Fig. 4 Limiting acceptances of electrostatic accelerator with external injection lens as function of energy ratio for three lens positions. Drawings on right include adjustable intermediate lens within acceleration tube. Consult text regarding errors in this figure.
- Fig. 5 Acceptance of simplified model of ORNL 25 MV accelerator as function of energy ratio. Consult text regarding errors in this figure.
- Fig. 6 Displacements of beams entering on axis (solid curves), steered to axis (dot-dash curves) and exiting on axis (dashed curves). Upper and lower drawings refer to different inclined-field tube designs for HE stage of MP accelerator.

FIGURE 1

FIGURE 2

FIGURE 3

FIGURE 4

ORNL-DWG 77-9483

FIGURE 5

FIGURE 6