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NONLINEAR SATURATION OF THE
DISSIPATIVE TRAPPED-ELECTRON
INSTABILITY

BY

LIU CHEN, R. L. BERGER,
J. G. LOMINADZE, M. N. ROSENBLUTH,
AND P. H. RUTHERFORD

PLASMA PHYSICS
LABORATORY

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Liu Chen, R. L. Berger, J. G. Lominadze,
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Plasma Physics Laboratory, Princeton University
Princeton, N. J. 08540

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Liu Chen, R. L. Berger, J. G. Lominadze†,

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Plasma Physics Laboratory, Princeton University

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ABSTRACT

It is shown that trapped-electron induced scattering can be dominant over nonlinear ion Landau damping in the saturation of short-wavelength dispersive trapped-electron instabilities in tokamaks. Trapped-electron induced scattering transfers the wave energy to shorter wavelengths, where it can be dissipated by ion viscosity.

Dissipative trapped-electron instabilities are important in both current and future generations of tokamaks.¹ Although the linear theory of these modes has been studied in considerable detail, most treatments of the nonlinear evolution and saturation of the instabilities, and the associated anomalous transport, have been based on rather primitive models, such as (i) an ad hoc relation $\gamma \sim k_{\perp}^2 D$ to estimate the anomalous diffusion coefficient¹; (ii) a free-energy argument to estimate the saturation level²; or (iii) a one-dimensional resonant mode-coupling process.³ The approaches (i) and (ii) give upper bounds for the transport coefficients and are, presumably, to be employed only when there are no other mechanisms that give lower saturation levels. The approach (iii) is unsatisfactory for two reasons: first, the nonlinearities due to $\underline{E} \times \underline{B}$ convection are intrinsically two-dimensional; and, second, most of the unstable spectrum lies at short wavelengths^{2,3} $[k_{\perp}^2 \rho_i^2 (1 + T_e/T_i) \sim 1]$, where strong dispersion renders resonant mode-coupling ineffective.

In this letter, we present a new formulation of the nonlinear theory, which includes both two-dimensional nonlinearities and non-resonant mode-coupling interactions. We find that there are two effective, but competing, nonlinear mechanisms, namely trapped-electron induced scattering and nonlinear ion Landau damping.⁴ For typical tokamak parameters, our analysis demonstrates that trapped-electron induced scattering can dominate over nonlinear ion Landau damping and, in this case, the wave energy is scattered into shorter wavelengths, where dissipation due to ion viscosity would occur.

To simplify the analysis, we consider a plane slab geometry, taking $\underline{B}$ in the z -direction, and making the usual local WKB approximation for the perturbation fields. Thus, the effects of magnetic shear, toroidal geometry, and radial eigenmode structure are neglected. Although we ignore the destabilizing effect of electron ∇B -drift resonances⁵, our general formulation could easily be extended to include these terms.

The electron dynamics may be described by the drift-kinetic equation with an energy-dependent, number-conserving, Krook model to treat the collisional detrapping processes. Assuming $\omega \sim \nu_{\text{eff}} \ll \epsilon^{1/2} v_{Te}/qR$, where q is the safety factor, $\epsilon = r/R$, and $v_{Te} = (2T_e/m_e)^{1/2}$, we may average over the "bounce" motion of the trapped electrons. We obtain a trapped-electron perturbation given by

$$f_e^t(\underline{k}) = \Phi(\underline{k}) f_e^{(0)} + h_e(\underline{k}); \quad \Phi(\underline{k}) = e \phi(\underline{k})/T_e, \quad (1)$$

and, in linear order, we have

$$h_e^{(1)}(\underline{k}) = - \frac{\omega_{\underline{k}} - \omega_{*k}^T}{\omega_{\underline{k}} + i\nu_e(v)} \Phi(\underline{k}) f_e^{(0)}. \quad (2)$$

Here, $\nu_e(v) = \nu_{\text{eff}}/\bar{v}^3$, $\bar{v} = v/v_{Te}$, $\omega_{*k}^T = \omega_{*k} [1 - \eta_e (3/2 - \bar{v}^2)]$,

$\eta_e = d \ln T_e / d \ln n$, $\omega_{*k} = k_y c T_e / e B r_n$, and $r_n^{-1} = |d \ln n / dx|$.

In the case being considered, resonant-electron effects involving untrapped electrons are smaller than the trapped electron dissipative effects,⁶ so that the untrapped-electron perturbation $f_e^u(\underline{k})$ is given simply by

$f_e^u(\underline{k}) = \Phi(\underline{k}) f_e^{(0)}$. For the ions, trapping effects are negligible, assuming $\omega \gg \epsilon^{1/2} v_{Ti}/qR$, where $v_{Ti} = (2T_i/m_i)^{1/2}$. The ion dynamics may be described by the drift-kinetic equation generalized to the case $k_\perp \rho_i \sim 1$.

We obtain an ion perturbation given by

$$f_i(\underline{k}) = -\tau \Phi(\underline{k}) f_i^{(0)} + h_i(\underline{k}) \exp[ik_\perp \rho \sin(\theta_{\underline{k}} - \alpha_{\underline{k}})] \quad (3)$$

and, in linear order, we have

$$h_i^{(1)}(\underline{k}) = \frac{\tau \omega_{\underline{k}} + \omega_{\underline{k}}^*}{\omega_{\underline{k}} - k_\parallel v_\parallel} \Phi(\underline{k}) J_0(k_\perp \rho) f_i^{(0)} \quad (4)$$

where $\tau = T_e/T_i$, $\rho = v_\perp/\Omega_i$, $\Omega_i = eB/m_i c$, $\sin \theta_{\underline{k}} = v_y/v_\perp$, and $\sin \alpha_{\underline{k}} = k_y/k_\perp$.

From the quasi-neutrality condition $n_e^t + n_e^u - n_i = 0$, we obtain the dispersion relation

$$\begin{aligned} \mathcal{E}(\underline{k}) \equiv 1 + \tau + \left(\tau + \frac{\omega_{\underline{k}}^*}{\omega_{\underline{k}}} \right) \xi_{\underline{k}} Z(\xi_{\underline{k}}) \Gamma(b_{\underline{k}}) \\ - (2\epsilon)^{1/2} \left\langle \frac{\omega_{\underline{k}} - \omega_{\underline{k}}^*}{\omega_{\underline{k}} + i\nu_e(v)} \right\rangle = 0 \end{aligned} \quad (5)$$

Here, $\langle \rangle$ denotes an average over a Maxwellian distribution.

$\Gamma(b_{\underline{k}}) = I_0(b_{\underline{k}}) \exp(-b_{\underline{k}})$, $b_{\underline{k}} = k_\perp^2 \rho_i^2 / 2$, $\rho_i = v_{Ti}/\Omega_i$,

$\xi_{\underline{k}} = \omega_{\underline{k}} / |k_\parallel| v_{Ti}$, and Z is the plasma dispersion function. In the limit

$\xi_{\underline{k}} > 1$, we obtain

$$\omega_{\underline{k}} = \omega_{*k} \Gamma(b_{\underline{k}}) / H(b_{\underline{k}}), \quad (6)$$

$$\frac{\gamma_{\underline{k}}}{\omega_{\underline{k}}} = (2\epsilon)^{1/2} \left[\text{Im} \left\langle \frac{\omega_{\underline{k}} - \omega_{*k}^T}{\omega_{\underline{k}} + i\nu_{\text{eff}}/\bar{\nu}^{-3}} \right\rangle - \delta_{\underline{k}}^L - \delta_{\underline{k}}^V \right] / H(b_{\underline{k}}). \quad (7)$$

Here, $H(b_{\underline{k}}) = 1 + \tau - \tau \Gamma(b_{\underline{k}})$, and $\delta_{\underline{k}}^L = (\pi/2)^{1/2} (\tau + \omega_{*k}/\omega_{\underline{k}}) \Gamma(b_{\underline{k}})$

$\xi_{\underline{k}} \exp(-\xi_{\underline{k}}^2)$, a term describing the ion Landau damping. We have also

introduced an ion viscous damping $\delta_{\underline{k}}^V$; for $\omega_{\underline{k}} \gg \nu_{ii}$, this is given by⁷

$$\delta_{\underline{k}}^V = 7\tau b_{\underline{k}}^2 \nu_{ii} / 10\omega_{\underline{k}} \quad \text{for } b_{\underline{k}} < 1, \quad \text{and} \quad \delta_{\underline{k}}^V = 3(\pi+1)\tau b_{\underline{k}}^{1/2} \nu_{ii}^{1/2} / 8\pi^{1/2} \omega_{\underline{k}}$$

for $b_{\underline{k}} > 1$. In this treatment of the ions, we have neglected ion temperature gradients. For $\epsilon \sim 1/4$, $\eta_e \sim 1$, and $\tau \sim 1-2$, the maximum growth rate arising from the trapped-electron term is typically^{2,3,5} such that $\gamma_{\underline{k}}/\omega_{\underline{k}} \sim 10^{-1}$. Moreover, the modes with large growth rates form a fairly broad spectrum centered around $\tau b_{\underline{k}} \sim 1$ and, for typical parameters of present and future tokamaks, have $\omega_{\underline{k}} \gtrsim \nu_{\text{eff}}$.

We now consider the nonlinear evolution of the instability. Since the unstable modes are short-wavelength dispersive waves, for which the relevant mode-coupling processes are nonresonant, we must retain terms up to third order in the perturbations. However, $\gamma_{\underline{k}}/\omega_{\underline{k}}$ and ρ_i/r_n may both be treated as small parameters. The nonlinearities arise primarily

from nonlinear $\underline{E} \times \underline{B}$ convection terms in the drift-kinetic equations.

We may write the quantities $h_{e,i}$ appearing in Eqs. (1) and (3) as perturbation expansions $h_{e,i} = h_{e,i}^{(1)} + h_{e,i}^{(2)} + h_{e,i}^{(3)}$. In second order we have a set of virtual modes, with wave vector $\underline{q} = \underline{k} - \underline{k}'$ and frequencies $\omega_{\underline{q}} = \omega_{\underline{k}} - \omega_{\underline{k}'}$, where $\underline{k}$ and $\underline{k}'$ denote wave vectors of the linear normal modes. The perturbed distribution functions for these virtual modes are given, in terms of the second-order virtual-mode potential $\Phi(\underline{q})$, by expressions formally identical with those of Eqs. (1) - (4), together with explicitly nonlinear terms:

$$\begin{aligned} h_i^{(2)}(\underline{q}) &= \frac{i\Omega_i \rho_s^2}{\omega_{\underline{q}} - q_{\parallel} v_{\parallel}} \left[\underline{k} \times \underline{k}' \cdot \underline{e}_{\underline{z}} \Phi^*(\underline{k}') J_0(k'_{\perp} \rho) h_i^{(1)}(\underline{k}) + (\underline{k} \rightleftharpoons -\underline{k}') \right] \\ &= \frac{i\Omega_i \rho_s^2}{\omega_{\underline{q}} - q_{\parallel} v_{\parallel}} \left(\frac{\omega_{\underline{k}}^*}{\omega_{\underline{k}}} - \frac{\omega_{\underline{k}'}^*}{\omega_{\underline{k}'}} \right) \underline{k} \times \underline{k}' \cdot \underline{e}_{\underline{z}} \Phi^*(\underline{k}') \Phi(\underline{k}) J_0(k'_{\perp} \rho) J_0(k_{\perp} \rho) f_i^{(0)} \end{aligned} \quad (8)$$

$$\begin{aligned} h_e^{(2)}(\underline{q}) &= \frac{i\Omega_i \rho_s^2}{\omega_{\underline{q}} + i\nu_e(v)} \left[\underline{k} \times \underline{k}' \cdot \underline{e}_{\underline{z}} \Phi^*(\underline{k}') h_e^{(1)}(\underline{k}) + (\underline{k} \rightleftharpoons -\underline{k}') \right] \\ &= \frac{i\Omega_i \rho_s^2}{\omega_{\underline{q}} + i\nu_e(v)} \left(\frac{\omega_{\underline{k}}^T}{\omega_{\underline{k}}} - \frac{\omega_{\underline{k}'}^T}{\omega_{\underline{k}'}} \right) \underline{k} \times \underline{k}' \cdot \underline{e}_{\underline{z}} \Phi^*(\underline{k}') \Phi(\underline{k}) f_e^{(0)}, \end{aligned} \quad (9)$$

where $\rho_s = v_s / \Omega_i$ with $v_s = (T_e / m_i)^{1/2}$. In the ion term given in Eq. (8), we have assumed $\omega_{\underline{k}} \gg k_{\parallel} v_{\parallel}$ and $\omega_{\underline{k}'} \gg k'_{\parallel} v_{\parallel}$. In the electron term given in Eq. (9), we have taken note of the fact that the

dominant contribution to the velocity integral of Eq. (9) will arise from electrons with relatively high velocities v , such that $\omega_{\underline{k}}, \omega_{\underline{k}'} > \nu_e(v)$; on the other hand, the nonlinear interactions take place for $|\omega_{\underline{q}}| = |\omega_{\underline{k}} - \omega_{\underline{k}'}| \ll \omega_{\underline{k}}, \omega_{\underline{k}'}$. Accordingly, we have neglected $\nu_e(v)$ compared with $\omega_{\underline{k}}$ or $\omega_{\underline{k}'}$, but we have kept $\omega_{\underline{q}}$ compared with $\nu_e(v)$. It is a straightforward matter to integrate Eqs. (8) and (9) over velocities, and to substitute into the quasi-neutrality condition, to obtain the second-order virtual-mode potential $\Phi(\underline{q})$. In doing so, we again make use of the fact that the nonlinear interactions take place for $|\omega_{\underline{q}}| \ll \omega_{\underline{k}}, \omega_{\underline{k}'}$ and, accordingly, we set $\omega_{\underline{k}} = \omega_{\underline{k}'}$ in the factors in parenthesis on the right in Eqs. (8) and (9). We obtain

$$\varepsilon(\underline{q}) \Phi(\underline{q}) = - \frac{i\Omega_i \rho_s^2}{\omega_{\underline{k}}} \underline{k} \times \underline{k}' \cdot \underline{e}_{\underline{z}} \Phi^*(\underline{k}') \Phi(\underline{k}) [F_1(\underline{k}, \underline{k}') \chi_i(\underline{q}) + \chi_e(\underline{q})] \quad (10)$$

where $F_1(\underline{k}, \underline{k}') = \langle J_0(\underline{k}_\perp \rho) J_0(\underline{k}'_\perp \rho) J_0(\underline{q}_\perp \rho) \rangle$, and

$$\chi_i(\underline{q}) = \frac{\omega_{\underline{q}}^*}{\omega_{\underline{q}}} \xi_{\underline{q}} Z(\xi_{\underline{q}}); \quad \chi_e(\underline{q}) = (2\epsilon)^{1/2} \left\langle \frac{\omega_{\underline{q}}^*}{\omega_{\underline{q}} + i\nu_e(v)} \right\rangle, \quad (11)$$

$$\varepsilon(\underline{q}) \simeq \Gamma(\underline{h}_{\underline{q}}) \chi_i(\underline{q}) + \chi_e(\underline{q}). \quad (12)$$

The remainder of the calculation is fairly straightforward and follows standard procedures.⁴ We have third-order perturbations

$$h_i^{(3)}(\underline{k}) = - \frac{i \Omega_i \rho_s^2}{\omega_{\underline{k}} - k_{\parallel} v_{\parallel}} \sum_{\underline{k}'} \underline{k} \times \underline{k}' \cdot \underline{e}_{\underline{z}} [\Phi(\underline{k}') J_0(\underline{k}' \rho) h_i^{(2)}(\underline{q}) - \Phi(\underline{q}) J_0(\underline{q} \rho) h_i^{(1)}(\underline{k}')] \quad (13)$$

$$h_e^{(3)}(\underline{k}) = - \frac{i \Omega_i \rho_s^2}{\omega_{\underline{k}} + i \nu_e(v)} \sum_{\underline{k}'} \underline{k} \times \underline{k}' \cdot \underline{e}_{\underline{z}} [\Phi(\underline{k}') h_e^{(2)}(\underline{q}) - \Phi(\underline{q}) h_e^{(1)}(\underline{k}')] , \quad (14)$$

where, as before, $\underline{q} = \underline{k} - \underline{k}'$ denotes the wave vector of the second-order virtual mode. Using the quasi-neutrality condition to third order, and the random phase approximation, we obtain the wave kinetic equation:

$$\frac{1}{2} \frac{\partial N_{\underline{k}}}{\partial t} = \gamma_{\underline{k}} N_{\underline{k}} + \sum_{\underline{k}'} \beta(\underline{k}, \underline{k}') N_{\underline{k}} N_{\underline{k}'} . \quad (15)$$

Here, we have introduced the number of drift-wave plasmons, given by

$$N_{\underline{k}} = [\partial \mathcal{E}(\underline{k}) / \partial \omega_{\underline{k}}] | \Phi(\underline{k}) |^2 ,$$

where

$$\partial \mathcal{E}(\underline{k}) / \partial \omega_{\underline{k}} = H(b_{\underline{k}}) / \omega_{\underline{k}} .$$

The coupling coefficients are given by

$$\beta(\underline{k}, \underline{k}') = - \frac{\tau^2 \Omega_i^2 \underline{b}_{\underline{k}} \underline{b}_{\underline{k}'} \sin^2(\alpha_{\underline{k}} - \alpha_{\underline{k}'})}{H(\underline{b}_{\underline{k}}) H(\underline{b}_{\underline{k}'})} \quad (16)$$

$$\left(F(\underline{k}, \underline{k}') \operatorname{Im} \frac{\chi_i(\underline{q}) \chi_e^t(\underline{q})}{\mathcal{E}(\underline{q})} + G(\underline{k}, \underline{k}') \operatorname{Im} \frac{\chi_i(\underline{q})^2}{\mathcal{E}(\underline{q})} \right)$$

where

$$F(\underline{k}, \underline{k}') = \Gamma(\underline{b}_{\underline{q}}) - 2 \langle J_o(\underline{k}_{\perp} \rho) J_o(\underline{k}'_{\perp} \rho) J_o(\underline{q}_{\perp} \rho) \rangle + \langle J_o^2(\underline{k}_{\perp} \rho) J_o^2(\underline{k}'_{\perp} \rho) \rangle. \quad (17)$$

$$G(\underline{k}, \underline{k}') = \Gamma(\underline{b}_{\underline{q}}) \langle J_o^2(\underline{k}_{\perp} \rho) J_o^2(\underline{k}'_{\perp} \rho) \rangle - \langle J_o(\underline{k}_{\perp} \rho) J_o(\underline{k}'_{\perp} \rho) J_o(\underline{q}_{\perp} \rho) \rangle^2. \quad (18)$$

It should be noted that, although the quantities $\chi_e^t(\underline{k})$ and $\chi_e^t(\underline{k}')$ would formally be of order $\gamma_{\underline{k}}/\omega_{\underline{k}}$ and $\gamma_{\underline{k}'}/\omega_{\underline{k}'}$, the quantity $\chi_e^t(\underline{q})$ can be of order unity, since the wave vector $\underline{q}$ corresponds to a virtual mode, rather than a normal mode. As functions of $\omega_{\underline{q}}$, the functions $\chi_i(\underline{q})$, $\chi_e^t(\underline{q})$, and $\mathcal{E}(\underline{q})$ [defined in Eqs. (11) and (12)] will be sharply peaked around $\omega_{\underline{q}} \simeq 0$. As an approximation, to clarify the nature of the physical processes at work, we may replace the quantities in Eq. (16) by δ functions:

$$\begin{aligned} \operatorname{Im} \frac{\chi_i(\underline{q}) \chi_e^t(\underline{q})}{\mathcal{E}(\underline{q})} &= \delta(\omega_{\underline{k}} - \omega_{\underline{k}'}) \operatorname{Im} \int_{-\infty}^{\infty} \frac{\chi_i(\underline{q}) \chi_e^t(\underline{q})}{\mathcal{E}(\underline{q})} d\omega_{\underline{q}} \\ &= -\pi \delta(\omega_{\underline{k}} - \omega_{\underline{k}'}) \frac{(\omega_{*\underline{k}} - \omega_{*\underline{k}'}) (2\epsilon)^{1/2}}{\Gamma_{\underline{q}} - (2\epsilon)^{1/2}} \end{aligned} \quad (19)$$

$$\begin{aligned}
 \text{Im} \frac{\chi_i(\underline{q})^2}{\mathcal{E}(\underline{q})} &= \delta(\omega_{\underline{k}} - \omega_{\underline{k}'}) \text{Im} \int_{-\infty}^{\infty} \frac{\chi_i(\underline{q})^2}{\mathcal{E}(\underline{q})} d\omega_{\underline{q}} \\
 &= \pi \delta(\omega_{\underline{k}} - \omega_{\underline{k}'}) \frac{\omega_{\underline{k}}^* - \omega_{\underline{k}'}^*}{\Gamma_{\underline{q}} - (2\epsilon)^{1/2}}
 \end{aligned} \tag{20}$$

In Eqs. (10) and (20), the integrals over $\omega_{\underline{q}}$ have been performed by moving the contour to the semi-circle at infinity in the upper half-plane, and using the appropriate asymptotic forms of the functions defined in Eqs. (11) and (12).

In Eq. (16), the term in $G(\underline{k}, \underline{k}')$ corresponds to ion induced scattering⁴ (nonlinear ion Landau damping); the term in $F(\underline{k}, \underline{k}')$ is a new term, and corresponds to trapped-electron induced scattering. Since $\beta(\underline{k}, \underline{k}') = -\beta(\underline{k}', \underline{k})$, the total number of plasmons is conserved in both scattering processes. Both scattering processes are intrinsically two-dimensional. If the wave-vectors $\underline{k}$ and $\underline{k}'$ are in the same direction, then $\sin(\alpha_{\underline{k}} - \alpha_{\underline{k}'}) = 0$, with the result that $\beta(\underline{k}, \underline{k}') = 0$. We note, also, that $F(\underline{k}, \underline{k}') > 0$ and $G(\underline{k}, \underline{k}') > 0$. We see that the ion induced scattering contributes positively to $\beta(\underline{k}, \underline{k}')$ if $k_y < k'_y$; i.e., wave energy is transferred from short wavelengths to long wavelengths. On the other hand, the trapped-electron induced scattering contributes negatively to $\beta(\underline{k}, \underline{k}')$ if $k_y < k'_y$; i.e., wave energy is transferred from long wavelengths to short wavelengths. The relative importance of these two competing processes depends on the relative magnitudes of $F(\underline{k}, \underline{k}')$ and $G(\underline{k}, \underline{k}')$.

In the limit $b_{\underline{k}}, b_{\underline{k}'}, b_{\underline{q}} \ll 1$, we can easily show that $F(\underline{k}, \underline{k}') \simeq 2b_{\underline{k}} b_{\underline{k}'}$ $\cos^2(\alpha_{\underline{k}} - \alpha_{\underline{k}'})$ and $G(\underline{k}, \underline{k}') \simeq b_{\underline{k}} b_{\underline{k}'} \cos^2(\alpha_{\underline{k}} - \alpha_{\underline{k}'})$; in this case, trapped-electron induced scattering dominates whenever $\epsilon > 1/8$. In the limit $b_{\underline{k}}, b_{\underline{k}'}, b_{\underline{q}} > 1$, the expressions for F and G are more complicated, but it is clear that $G(\underline{k}, \underline{k}') < F(\underline{k}, \underline{k}') \simeq 1$; thus, in this case, trapped-electron induced scattering always dominates. Whenever trapped-electron induced scattering dominates, the virtual mode must have negative dissipation. On the other hand, our approximation is only valid if $\Gamma_{\underline{q}} > (2\epsilon)^{1/2}$; otherwise, the virtual mode could resonate with an unstable normal mode at very low frequency.⁸

Certain general conclusions can be drawn from Eq. (16). To simplify this discussion, we may restrict ourselves to the case $\tau \gg 1$ and $b_{\underline{k}} \ll 1$, such that $\tau b_{\underline{k}} \sim 1$. In this limit, the "small b " expansions of $F(\underline{k}, \underline{k}')$ and $G(\underline{k}, \underline{k}')$, given above, are valid; moreover $\Gamma_{\underline{q}} \simeq 1$, so that the denominators in Eqs. (19) and (20) remain positive. The dispersion relation is $\omega_{\underline{k}} \simeq \omega_{*\underline{k}} / (1 + \tau b_{\underline{k}})$ and the modes with largest frequencies, and largest growth rates,^{2,3} have $\tau b_{\underline{k}} \simeq 1$. In the δ -function approximation represented by Eqs. (19) and (20), a given mode $\underline{k}$ couples only with modes $\underline{k}'$ that have the same frequency. From the dispersion relation $\omega \simeq \omega_{*\underline{k}} / (1 + \tau b_{\underline{k}})$, we see that modes with a given ω lie on an ellipse in the $\underline{k}$ plane, centered on the k_y axis. Thus, a given mode couples only with other modes on the same ellipse. Trapped-electron induced scattering transfers the wave energy toward the largest k_y values, where it can be dissipated by ion collisional viscosity.

A rough estimate for the saturation amplitudes of the unstable modes can be obtained by balancing the growth rate with trapped-electron induced scattering, i.e., by setting $N_{\underline{k}} \sim \gamma_{\underline{k}} / \beta(\underline{k}, \underline{k}')$. Assuming $\epsilon \sim 1/4$, $\tau b_{\underline{k}} \sim 1$, and $\omega_{\underline{k}} \sim \omega_{*k} / 2$, we estimate that $\beta(\underline{k}, \underline{k}') \sim (\pi/20) \Omega_i^2 / \tau^2$, giving $|\tilde{n}_e / n_o| \simeq |\Phi_{\underline{k}}| \sim (\omega_{\underline{k}} N_{\underline{k}} / 2)^{1/2} \sim (\tau \rho_s / r_n) (\gamma_{\underline{k}} / \omega_{\underline{k}})^{1/2}$. In the quasilinear approximation, we can estimate the cross-field particle diffusion coefficient as $\bar{D} \simeq |k_y \phi_{\underline{k}} / B|^2 \gamma_{\underline{k}} / \omega_{\underline{k}}^2 \sim (27^2 \rho_s / r_n) (\gamma_{\underline{k}} / \omega_{\underline{k}})^2 D_B$, where $D_B = cT_e / eB$. For modes with similar wavelengths (i.e., $k_y \rho_s \sim k_{\perp} \rho_s \sim 1$) conventional estimates based on $D \sim \gamma_{\underline{k}} / k_{\perp}^2$ would give $D \sim (\rho_s / r_n) (\gamma_{\underline{k}} / \omega_{\underline{k}}) D_B$. Thus, for $\gamma_{\underline{k}} / \omega_{\underline{k}} \sim 10^{-1}$ (as is typical of these modes) and $\tau \sim 1-2$, our new estimate gives a slightly lower value for D than the conventional one. On the other hand, the largest contributions to $\gamma_{\underline{k}} / k_{\perp}^2$ arise from somewhat smaller $k_{\perp}$ -values [typically, $k_{\perp}^2 \rho_s^2 \sim 0.2$, giving $D \sim 0.2 (\rho_s / r_n) D_B$], for which trapped-electron induced scattering is a relatively ineffective saturation mechanism.

To determine the saturated spectrum of modes, a computational solution of Eq. (15) is needed. Resonant 3-wave coupling can also be included in such a treatment. This will be the subject of a future paper.

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† Permanent address: Academy of Sciences of the Georgian SSR Tbilisi, USSR; on a Scientific Exchange Visit under agreement between the National Academy of Sciences of the USA and the Academy of Sciences of the USSR.

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