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By

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SOME ISSUES IN HADRON PHYSICS*

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It is four years since the first results on high energy hadronic interactions were obtained in the 30 inch bubble chamber at Fermilab. These data and those from the CERN-ISR, which had begun producing only a short time earlier, have had a profound qualitative impact on our view of multiparticle dynamics. We now think routinely of three distinct reaction mechanisms.¹ Inelastic diffraction has been demonstrated to be an asymptotic phenomenon, accounting for one-third or more of the total inelastic cross-section. Production of hadrons with large transverse momenta was an unexpected and potentially profound discovery. The third reaction mechanism is variously termed "pionization" or "central production". It has to do with the population of the central region of rapidity space at small transverse momenta. Experimental studies of rapidity correlations in the central region have shown that the positive correlations among pions produced in this central region decrease rapidly in magnitude as the rapidity spacing between pions is increased.² This observation and the rough energy independence of the correlation function in the central region agree with predictions of models in which dominant short-range order is a basic ingredient.^{3,4} While no one model is singled out as "correct", a broad class of models is favored, and some models are clearly rejected.

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The classification of events into three categories is, of course, overly simplified. Diffractive events also yield pions in the central region, as do events in which a hadron is ejected at large p_T . Formally, moreover, unitarity connects the whole lot. Whether correlations are really only of short range character is also questionable. The relatively narrow range of available rapidity provides inescapable kinematic constraints. Surely the large diffractive sample provides a contribution with intrinsic long range correlations. Moreover, from a theoretical point of view, the models which predicted short-range correlation structure on the basis of short-range order have not yet been shown to accommodate naturally the rise with energy of the total cross-section nor the non-scaling rise with energy of the height of the central plateau in the single particle rapidity distribution.⁵ Long-range correlations may now be a theoretical necessity. While appearing to have selected a "right" class of models, the data from the ISR and from Fermilab have perhaps raised many more issues in small- p_T hadron physics than they have resolved.

Investigations in the Fermilab 30-inch chamber provided critical information on inelastic diffraction, in both inclusive and exclusive channels, and on hadron production in the central region. All of our knowledge of multiplicity distributions and a great deal of our insight into central region rapidity correlations are derived from the bubble chamber data. Although high-statistics counter experiments are essential for examining certain detailed features of diffraction, for example, the 4π solid

angle and momentum analysis capabilities of the bubble chamber seem well suited for the important higher multiplicity exclusive (or semi-inclusive) fraction of the total-cross section. We are still ignorant of the multi-dimensional structure of hadronic events.¹ A large sample of data is less important than clever methods of data analysis and display. Less charming than the current search for glamor and/or the hard-core, these investigations are nevertheless an inescapable prelude to further progress.

In the remaining paragraphs, I will touch on a few interesting unanswered questions concerning correlations in general and then describe a new method for extracting dynamical information from the invariant mass dependence of inclusive correlation functions. For most of these investigations, momentum analysis of final state hadrons is essential, and particle identification $\pi^0|\pi^\pm|K|\bar{p}$ is highly desirable. The increase in energy provided by the Doubler yields two advantages: a study of energy dependence of phenomena over a wider range of incident beam momenta, and an important increase of longitudinal phase space. The latter allows better isolation of the central region from the fragmentation and diffractive ends of the rapidity spectrum.

1. Correlation Studies

The reason for the study of correlations continues to be the development of greater insight into the nature of hadronic production mechanisms. Further, differences between the expectations of different theoretical frameworks often become apparent only at the level of two and more particle inclusive and semi-inclusive correlations.

1.1 In processes of the type $pp \rightarrow ppX$, how much correlation is observed between the Feynman x values of the two final protons? Is there much of a signal associated with the highly inelastic configuration in which both final state baryons have $|x| = 0$? If so, what does X do? When $|x| \gtrsim 0.9$ for both final protons, the events fall into the region of phase space associated with double-Pomeron exchange. There is also a background in this region from single-Pomeron diffraction dissociation. Energy dependence can be used to separate the two effects. Is there a discernable double-Pomeron signal, and is its cross-section consistent with predictions of exchange models? All of the above questions may also be asked for $hp \rightarrow hpX$ where h is any beam particle.

1.2 Single diffraction dissociation. More precise data are needed on the M_X^2 and t dependences of $d\sigma/dM_X^2 dt$ for $hp \rightarrow hX$, where h is any hadron. Nevertheless, suppose we isolate from the sample those events in which a Pomeron is exchanged at the h vertex. We can then investigate Pomeron-proton scattering as a function of M_X^2 and t . The study of correlations among particles in the missing mass system will elucidate the similarities and differences between hadron-hadron and Pomeron-hadron interactions.^{1,6}

1.3 Central Region. Information on correlations between particles other than pions is non-existent. What are the magnitude, range, and energy dependence of the correlations

between $K\bar{K}$, $K\pi$, πB , KB , and BB ? Are quantum numbers other than charge conserved locally in rapidity, as exchange models would prefer? How strong and what is the energy dependence of three and more particle correlation functions? In a detailed examination of semi-inclusive correlations, can one separate cleanly the purely momentum correlations from those of multiplicity origin? Neutral pion detection may be useful here. What is the cross-section for the production of various resonances, such as K^* , η , ω , ϕ , and A_2 , and for how much of the strength and azimuthal angle dependence of correlations are these resonances responsible?

2. Mass Dependence of Two-Pion Inclusive Correlation Functions.

In most investigations of inclusive correlations, the rapidity variable y is used. This procedure is well motivated when the only aim is to test the concept of short-range-order. Experimentally, in situations in which angles and not momenta are measured, it is the only available variable [$y = \log \tan(\theta/2)$]. For more detailed investigations of the correlations, rapidity may be seriously disadvantageous. Here I discuss inclusive correlations $C(M)$ as functions of the invariant mass M of the two particle system.⁷

There are evident arguments in favor of mass as a dynamical variable. Singularities such as resonances and thresholds appear at fixed values of mass. Bose-Einstein interference effects are expected to depend on the square of the difference of the four-vector momenta of identical bosons, $(p_1 - p_2)^2 = 4m_\pi^2 - M^2$.

Regge and duality properties of the four-to-four forward Mueller amplitude,⁴ which represents the correlation function, are functions of the two-particle invariant mass. To be sure, at large mass, rapidity separation and mass are related simply, because in this limit

$$\Delta y \sim \log M.$$

However, because the relationship between $C(\Delta y)$ and $C(M)$ involves an integral over the transverse momenta, possible important structure in the data at small M will be smeared when viewed in terms of Δy . The correlation function $C(\Delta y)$ is itself observed to be large at small $|\Delta y|$ (≈ 1). The interpretation of this structure in terms of resonances (or other forms of clustering) with relatively low mass may be more readily checked by examination of $C(M)$. Invariant mass has long been a useful variable in high energy investigations. Its disuse in high multiplicity processes is explained in part by the large combinatorial background present in the two particle density $(1/\sigma)d\sigma/dM$. The correlation function removes the uncorrelated background.

The definition of the two-particle inclusive correlation function is

$$C_2(p_1, p_2) \equiv \rho_2(p_1, p_2) - \rho_1(p_1)\rho_1(p_2). \quad (2.1)$$

In this expression, p_1 and p_2 are the momenta of the two hadrons. The two-particle density is

$$\rho_2(p_1, p_2) \equiv \frac{E_1 E_2}{\sigma} \frac{d^2 \sigma}{d^3 p_1 d^3 p_2}. \quad (2.2)$$

The single particle inclusive density is

$$\rho_1(p) = \frac{E}{\sigma} \frac{d\sigma}{d^3p} . \quad (2.3)$$

The cross-section σ appearing in Eqs.(2.2) and (2.3) is the inelastic cross-section.

The function $C_2(p_1, p_2)$ in Eq.(2.1) is a function of six independent variables. A common choice for these six is: total c.m. energy $\sqrt{s}$, the c.m. rapidities y_1 and y_2 of the two hadrons, the magnitudes p_{T1} and p_{T2} of the two transverse momenta, and ϕ the relative azimuthal angle between $\vec{p}_{T1}$ and $\vec{p}_{T2}$. In models with pure short range order, these six variables reduce to four: $\Delta y = (y_1 - y_2)$, p_{T1} , p_{T2} , and ϕ . Instead of Δy , I suggest using the invariant mass M of the two particle system as an independent variable.¹ Because interesting structure⁸ has been observed at small Δy in the ϕ distribution, I retain the ϕ variable, as well as p_{T1} and p_{T2} . To complete the set, one may add $\sqrt{s}$ and one other variable to locate the longitudinal position of the system with mass M . A possible choice is the rapidity $y_{\pi\pi}$ of the system. The relationship between M and Δy is

$$M^2 = 2m_\pi^2 - 2\vec{p}_{T1} \cdot \vec{p}_{T2} + 2m_{T1}m_{T2}\cosh(\Delta y) ,$$

$$\text{with } m_T^2 = m_\pi^2 + p_T^2 . \quad (2.4)$$

Equation (2.4) shows that

$$M^2 \sim \exp(|\Delta y|) ,$$

for large M and $|\Delta y|$. However, at small M or Δy , the relationship between these variables is influenced greatly by

the values of p_{T1} and p_{T2} . If the transverse momenta are integrated over, the effect is to smear possible structure in M or in Δy when viewed in terms of the other variable.

After integrating Eq. (2.1) over the p_T and ϕ dependences, and over the longitudinal position of the pair, we obtain a correlation distribution which is a function of M only (at fixed $\sqrt{s}$). We then have

$$C_2(M) = \rho_2(M) - \rho_1 \otimes \rho_1(M). \quad (2.5)$$

In Eq. (2.5), $\rho_2(M)$ is the usual differential cross-section $d\sigma/dM$, divided by σ_{inel} . The second term represents the combinatorial background which we subtract from ρ_2 to obtain $C_2(M)$. This product of single particle distribution at fixed pair mass is

$$\rho_1 \otimes \rho_1(M) = \int \frac{d^3 p_1}{E_1} \frac{d^3 p_2}{E_2} \delta(\sqrt{(p_1 + p_2)^2} - M) \rho_1(p_1) \rho_1(p_2). \quad (2.6)$$

In Ref. 7, a description is given of how the data for $\rho_1 \otimes \rho_1$ are expressed numerically as a function of M .

Because different effects influence the physics in the central portion and in the fragmentation ends of the rapidity spectrum, it would be useful to display $C(M)$ for values of $y_{\pi\pi}$ restricted to the central region, e.g. $|y_{\pi\pi}| < 2$. For reasons of simplicity and statistics, the data shown in Fig. 1 include pairs with all values of $y_{\pi\pi}$. They were obtained from the Argonne-Fermilab-Stony Brook analysis of an exposure of 205 GeV/c protons in the 30 inch hydrogen bubble chamber at Fermilab.⁹

Shown in Fig. 1 are the correlation functions $C^{+-}(M)$ and $C^{--}(M)$ for $(\pi^+\pi^-)$ and $(\pi^-\pi^-)$ pairs of pions, respectively, from $pp \rightarrow \pi^+\pi^-X$ and $pp \rightarrow \pi^-\pi^-X$. The mass distributions extend to very high values of M , but I concentrate here on the region $M \lesssim 2$ GeV. Strong positive correlations in the form of broad peaks near the two pion threshold are observed in both distributions. As M increases, $C_2^{--}(M)$ drops more rapidly than $C_2^{+-}(M)$. Above $M = 0.6$ GeV, $C_2^{--}(M)$ is consistent with zero. A ρ resonance signal is visible above the background in $C_2^{+-}(M)$.

The contrast between $(--)$ and $(+-)$ in Fig.1 may be compared with the similarity in shape² of $C_2^{+-}(\Delta y)$ and $C_2^{--}(\Delta y)$. In rapidity, the correlation functions differ to first approximation only in normalization, associated with the fact that

$$f_2^{+-} = f_2^{--} + \langle n_- \rangle.$$

The structure in $C_2^{--}(M)$ at small M in Fig.1 corresponds to the peak at $\Delta y = 0$ in $C_2^{--}(\Delta y)$. For $(+-)$, a much broader spectrum of mass values provides the peak in $C^{+-}(\Delta y)$ at $\Delta y = 0$.

Threshold enhancements are present in both $C_2^{+-}(M)$ and $C_2^{--}(M)$. They differ appreciably in overall magnitude, and to some extent in the details of their mass dependences. Some structure near threshold in both C_2^{+-} and C_2^{--} , as well as a broad background under the ρ in C_2^{+-} may be anticipated as a result of the decay of resonances with multiplicity $m > 2$, e.g. η , ω , A_2 , g . These decays produce correlated but generally non-resonant pairs of $\pi^+\pi^-$ and $\pi^-\pi^-$. Their effects are not removed from C_2 . This exclusive approach does not

explain naturally the fact that $C_2^{--}(M)$ is observed to decrease with M more rapidly than $C_2^{+-}(M)$. Indeed, the exclusive argument would suggest "backgrounds" of similar shape in both C_2^{+-} and C_2^{--} near, e.g., $M = 0.76$ GeV, the ρ location.

3. Mueller-Regge Interpretation of $C(M)$.

In the Mueller approach to inclusive correlations, the two-particle correlation function $C_2(p_1, p_2)$ is a properly defined discontinuity ("imaginary part") of a four-particle to four-particle forward scattering amplitude^{4,10}. At high energies, the Mueller-Regge approximation for $pp \rightarrow \pi\pi X$ is sketched in Fig.2(a). This graph applies to dipion production in the central region of rapidity space. The shaded oval represents a scattering amplitude from which the leading Pomeron exchange is excluded. It does not represent the $\pi\pi$ elastic amplitude. Although the Pomerons are each attached to the oval at zero four-vector momentum, the structure of the oval is still that of an amplitude with six external legs. Consequently, there is no general reason to suppose that the dipion mass dependence of Fig.2(a) should closely resemble that of elastic $\pi\pi$ scattering. In particular, in $C_2(M)$ for $pp \rightarrow \pi^+\pi^-X$, Fig.2(a) leads us to expect resonance signals at the $\rho, f, g, \dots$ positions, as well as a background. However, the relative contributions of the different resonant partial waves, and the resonance to background ratio need not be related simply to that measured in studies of $\pi^+\pi^-$ elastic scattering. The data in Fig.1 show in fact that the ρ resonance to background ratio in the M dependence of $pp \rightarrow \pi^+\pi^-X$ is much smaller than in the elastic $\pi^+\pi^-$ amplitude.

In an attempt to obtain a simple parametrization of the M dependence of $C_2(M)$, we study the behavior of Fig.2(a) expected at large M , where a Regge exchange approximation may be used. The resulting graphs are shown in Figs.2(b) and 2(c) for $pp \rightarrow \pi^+ \pi^- X$ and $pp \rightarrow \pi^- \pi^- X$, respectively. For $pp \rightarrow \pi^+ \pi^- X$, the leading exchanged Reggeon R in Fig.2(b) is the (ρ, f) pair, with intercept $\alpha_R(0) \approx 0.5$. For $pp \rightarrow \pi^- \pi^- X$, somewhat more discussion is necessary. In two-body phenomenology, duality arguments suggest that the Regge exchanges leading to an exotic system such as $\pi^- \pi^-$ occur in exchange degenerate pairs, and that their contributions cancel in the imaginary part of the scattering amplitude. Applying analogous arguments to $pp \rightarrow \pi^- \pi^- X$, we expect that the exchange labeled E in Fig.2(c) is either a low lying trajectory or cut (e.g. $\alpha_E(0) \lesssim 0$) or a normal trajectory (e.g. α_ρ) whose small coupling at $t=0$ is related to the deviation from exact exchange degeneracy.

Assuming that the exchanges in Figs.2(b) and 2(c) are factorizable, we obtain the following limiting expressions at large M :

$$C_2^{+-}(p_1, p_2) = \beta_{RP}(m_{T1}) \beta_{RP}(m_{T2}) (M^2)^{\alpha_R(0)-1} \quad (3.1)$$

$$\propto M^{-1}, \quad \text{as } M \rightarrow \infty.$$

$$C_2^{--}(p_1, p_2) = \beta_{EP}(m_{T1}) \beta_{EP}(m_{T2}) (M^2)^{\alpha_E(0)-1}. \quad (3.2)$$

The transverse mass m_T is defined in Eq.(2.4); β_{RP} and β_{EP} are vertex functions whose m_T dependence may be estimated from data on the single pion inclusive yield $\rho_1(p)$. In the

central region, the Mueller-Regge approach provides the expansion

$$\rho_1(p) = \beta_{PP}(m_T) + \beta_{RP}(m_T)s^{-\frac{1}{4}} \quad (3.3)$$

This equation shows that the vertex functions yield the transverse momentum damping of $\rho_1(p)$.

As is apparent from Fig.1, available data are concentrated at small M where Eqs.(3.1) and (3.2) are not obviously applicable. Inasmuch as we are working with the imaginary parts of amplitudes, we might hope nevertheless that the extrapolation of Eqs.(3.1) and (3.2) to small mass is meaningful in the sense of a dual average. Since the ρ signal is not prominent in $C_2^{+-}(M)$, the average is not difficult to obtain.

We wish to compare Eqs.(3.1) and (3.2) with the data at low M and to extract effective values for $\alpha_R(0)$ and $\alpha_E(0)$. The proper comparison should be done at fixed values of all kinematic variables other than M . Present statistics do not permit such a differential comparison. Integrating Eq.(3.1), we obtain

$$C_2^{+-}(M) = (M^2)^{\alpha_R(0)-1} \iint dp_1 dp_2 \beta_{RP}(m_{T1}) \beta_{RP}(m_{T2}) \delta(M - \sqrt{(p_1 + p_2)^2}) \quad (3.4)$$

Here dp represents the invariant integration element d^3p/E . The integrations over the p_T and ϕ variables and over the longitudinal position of the dipion pair introduce an added dependence on M through the delta function in Eq.(3.4).

As $M \rightarrow 2m_\pi$, the integral in Eq.(3.4) provides a threshold suppression factor proportional to $(M^2 - 4m_\pi^2)^{\frac{1}{2}}$. By contrast, at large M , the transverse momentum dependence of the vertex functions in Eq.(3.4) results in a function which falls rapidly

with M . In order to extract the effective Regge power $\alpha_R(0)$ from the data on $C_2^{+-}(M)$, it is first necessary to divide out these important threshold and large M factors. In the absence of detailed knowledge of $\beta_{RP}(m_T)$, this procedure is necessarily approximative.

Returning to Eq.(3.3), we note that at large s , the Mueller-Regge model provides the following prescription for the M dependence of the product $\rho_1^+ \otimes \rho_1^-$:

$$\rho_1^+ \otimes \rho_1^-(M) = \iint dp_1 dp_2 \beta_{PP}(m_{T1}) \beta_{PP}(m_{T2}) \delta(M - \sqrt{(p_1 + p_2)^2}) \quad (3.5)$$

This integral has exactly the same form as that in Eq.(3.4), except for the replacement of $\beta_{RP}(m_T)$ by $\beta_{PP}(m_T)$. If we assume that the m_T dependences of β_{RP} and β_{PP} are identical, we may divide Eq.(3.4) by Eq.(3.5) to obtain

$$R^{+-}(M) \equiv \frac{C^{+-}(M)}{\rho_1^+ \otimes \rho_1^-(M)} \propto (M^2)^{\alpha_R(0)-1} \quad (3.6)$$

Likewise,

$$R^{--}(M) \propto (M^2)^{\alpha_E(0)-1} \quad (3.7)$$

Alternatively, assuming that β_{RP} in Eq.(3.1) has the same m_T dependence as β_{EP} in Eq.(3.2), we may work directly with Eq.(3.4) to derive

$$C_2^{--}(M)/C_2^{+-}(M) \propto (M^2)^{\alpha_E(0)-\alpha_R(0)} \quad (3.8)$$

Motivated by Eq.(3.6), in Fig.3 we display $R^{+-}(M)$ on a logarithmic scale. The form of Eq.(3.6) requires that the

data points in Fig.3 should fall on one straight line. Quite obviously they do not. An excursion is visible in the ρ region, as expected. For M values below the ρ position a rough fit to the data suggests an effective trajectory with rather high intercept $\alpha(0) \approx 0.6$. For $M \gtrsim 1$ GeV, the intercept is much lower, $\alpha(0) \approx 0.35$. In view of the approximations made, it is hazardous to propose strong conclusions. However, we call attention to the fact that the Mueller-Regge expectation of $\alpha(0) \approx 0.5$ lies well within the range of values of $\alpha(0)$ which provide an acceptable average fit to the data in Fig.3.

In Fig.4, we display the ratio C_2^{--}/C_2^{+-} as suggested by Eq.(3.8). Here the statistics preclude an examination of the M dependence for $M \gtrsim 1$ GeV. For comparison, we show the M^{-2} dependence expected if $\alpha_R(0) - \alpha_E(0) = 1$ in Eq.(3.8). We observe that this form is consistent with the fall of C_2^{--}/C_2^{+-} . In the Mueller-Regge framework this result suggests that the process $pp \rightarrow \pi^- \pi^- X$ is mediated by a rather low lying exchange, with intercept $\alpha_E(0) \approx -0.5$. The duality expectation is borne out, in that the exchange degenerate pair of ρ and f with intercept $\alpha(0) \approx 0.5$ does not contribute to the M dependence of $pp \rightarrow \pi^- \pi^- X$. This result is an important verification that standard duality notions are applicable in inclusive processes. It stands in contrast to the list of embarrassing failures associated with early scaling criteria.¹⁰

In terms of rapidity, the Mueller-Regge expectation for the correlation function C^{--} is

$$C^{--}(\Delta y) \propto \exp[(\alpha_E(0) - \alpha_P(0))|\Delta y|] \quad (3.9)$$

The low intercept $\alpha_E(0) \approx -0.5$ provides a correlation length $\lambda^{--} \equiv (\alpha_P - \alpha_E)^{-1} \approx 2/3$, much shorter than the $(+-)$ value $\lambda^{+-} \approx 2$. The fact that $\lambda^{--} \approx \frac{1}{3} \lambda^{+-}$ is to be contrasted with expectations of cluster models¹¹ in which typically $\lambda^{--} \approx \lambda^{+-}$. Previously a small value of λ^{--} was suggested, based upon the observation⁸ of a sharp peak near $\Delta y = 0$ in the rapidity variation of the two dimensional distribution $C^{--}(\Delta y, \phi)$ for values of ϕ near zero. Our extraction of a small value of λ^{--} from the one-dimensional $C(M)$ is perhaps more direct. However, the two observations are surely related since small M and small $(\Delta y, \phi)$ are correlated kinematically.

Once this general consistency with Mueller-Regge expectations is established, the magnitudes of $C^{+-}(M)$ and $C^{--}(M)$ can be related to other observables, as is reported in Ref.7.

Obviously a great deal more can be done with correlation functions displayed as a function of M . Higher statistics data on $pp \rightarrow \pi\pi X$ will permit a more differential examination of $C(M)$, for example at fixed values of p_T and $y_{\pi\pi}$, and for $M > 1$ GeV, where the Regge approximations are more justified. Regarding the energy dependence of $C_2(M)$, we may remark that in all short-range order models, $C_2(p_1, p_2)$ is expected to approach an energy independent constant value as $s \rightarrow \infty$. This remains true if we integrate over all variables except M and $y_{\pi\pi}$. If the dipion system is distributed uniformly in rapidity, the integral over $y_{\pi\pi}$ is proportional to $\ln s$. Therefore, we may expect that $C_2^{\pm-}(M)/\ln s$ will become independent of s as $s \rightarrow \infty$. It would be useful to check this expectation with data over a range of incident momenta.

Particle identification in the final state (in the downstream spectrometer) will provide further opportunities. We may investigate the "rapidity range" of inclusive correlations as a function of the quantum numbers of the pair of final-state particles. Some possibilities and their expected correlation lengths are listed below.

Pair	$C(M) \propto M^{2\alpha_x - 2}$ α_x	Correlation Length $\lambda = 1/(1-\alpha_x)$
$\pi^+ \pi^-$	$\alpha_\rho(0) \approx 0.5$	2
$\pi^- \pi^-$	$\alpha_E(0) \approx -0.5$	2/3
$K^0 \pi^+$, $\bar{K}^0 \pi^-$	$\alpha_{K^*}(0) \approx 0.3$	$\sim 3/2$
$K^0 \pi^-$, $\bar{K}^0 \pi^+$ $K^+ \pi^+$, $K^- \pi^-$	$\alpha_E(0) \approx -0.5$	2/3
$K\bar{K}$, $B\bar{B}$, πp , $\bar{K}p$, $\Lambda\pi$	$\alpha_\rho(0) \approx 0.5$	2
$K^+ p$	$\alpha_E(0) \approx ?$	$< 2/3$
Λp , pp	$\alpha_E(0) \approx ?$	$\sim 2/3$

The systematics are such that exotic systems show threshold peaks in $C(M)$ which are sharper than for non-exotic combinations. Correspondingly, the range in Δy is also smaller. The possibility has been raised that quantum-mechanical statistical effects enhance the near threshold peak in the $\pi^- \pi^-$ distribution. Is a corresponding suppression visible for the di-fermion system pp when $M \rightarrow 2m_p$ or $\Delta y \rightarrow 0$?

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FIGURE CAPTIONS

- Fig. 1 Displayed as a function of dipion invariant mass are the correlation functions $C_2(M)$ for $\pi^+\pi^-$ pairs and $\pi^-\pi^-$ pairs from $pp \rightarrow \pi\pi X$ at 205 GeV/c. The nominal locations of the ρ ($M=0.76$ GeV) and f ($M=1.27$ GeV) are marked.
- Fig. 2 a) Mueller-Regge diagram for the inclusive reaction $pp \rightarrow \pi\pi X$ where the dipion system is produced in the central region of rapidity. The symbol P denotes the Pomeron. b) Mueller-Regge diagram for $pp \rightarrow \pi^+\pi^- X$ at large s and large mass M of the $\pi^+\pi^-$ pair. c) Mueller-Regge diagram for $pp \rightarrow \pi^-\pi^- X$ at large s and large mass M of the $\pi^-\pi^-$ pair.
- Fig. 3 The mass dependence of the ratio $R^{+-}(M) = C_2^{+-}(M)/\rho_1^+ \otimes \rho_1^-(M)$ for $pp \rightarrow \pi^+\pi^- X$ at 205 GeV/c. Two straight lines are drawn. They correspond to two choices of the parameter $\alpha(0)$ in the expression $R^{+-}(M) = g M^{2(\alpha(0)-1)}$ suggested by Mueller-Regge analysis.
- Fig. 4 The mass dependence of the ratio $C_2^{--}(M)/C_2^{+-}(M)$. The dashed line is drawn to show the M dependence expected if the ratio is proportional to M^{-2} . It is not a fit to the data.

Fig. 1

(a)

(b)

(c)

Fig. 2

Fig. 3

Fig. 4