NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

INTENSITY AND ENERGY SPREAD AT INJECTION IN PROTON SYNCHROTRONS*

J. Glaus

Brookhaven National Laboratory
Upton, New York 11973

MASTER

CONF-770213--80

Summary

Injection and capture in strong focusing proton synchrotrons often involves multiturn stacking in horizontal betatron phase space followed by some approximation of adiabatic capture. The energy spread in the coasting beam is usually relatively small so that the bunches are tight. Local space charge densities may be sufficiently high temporarily to limit intensity. An increase in the energy spread before capture, obtained by a dynamic variation of the energy in the beam to be injected as is done in some weak focusing installations rather than by an increase of its energy spread, should help to alleviate this limitation.

Introduction

The intensity achievable early in the accelerating cycle of a proton synchrotron may be limited by any of a number of factors. Among them are the characteristics of the beam to be injected, the methods of injection and capture and the characteristics of the accelerator in terms of closed orbit position and shape and the widths and locations of various betatron stopbands. Many of these parameters are adjustable and methods for adjusting them for maximum intensity have been developed. If one calculates the magnitude and shape of the self-field of such maximized beams, using observed azimuthal and cross sectional dimensions and distributions and taking account of the boundaries formed by vacuum chamber wall and magnet pole faces, one may find that they are large enough to affect the transverse focusing system significantly, reducing its linear focusing gradients and introducing higher order terms, all as functions of the azimuthal position in the bunch. The effect is strongest near the center of the bunch where the charge density is highest and more pronounced in the direction of the smallest dimension of the vacuum chamber and/or beam cross section. For particles with small betatron amplitudes one may find peak tune shifts in excess of 0.3; the magnitude of the shift decreases with increasing amplitude due to the nonlinearity of the self-field. Taking theoretical shapes and distributions, derived from the results of emittance measurements in the beam to be injected and knowledge of the injection process, one may find shifts that are larger than that by a factor 2-3. One also observes that the beam blows up in the direction of the largest tune shift, filling the available aperture more or less irrespective of the details of the injection process and that beam is being lost continuously for a long time, though at a continuously decreasing rate. Thus, one gains the impression that the distribution in the bunches adjusts itself continuously by diffusion-like processes, losing particles while doing so, until a semi-stable condition is reached. At that time the competition between amplitude growth due to a multiplicity of mechanisms and their decrease due to acceleration has been decided in favor of the latter.

The main reason for the growth must be that of the repeated and more or less random passage through stopbands and resonances that were imperfectly compensated, that is forced by the synchrotron motion.¹ The particles oscillate about the stable fixed points azimuthally and in the process meet with continuously changing transverse focusing conditions induced by the

non-uniform charge distribution in the bunches, which itself changes gradually in time. If the loss is associated with the non-uniformity of the charge distribution and the presence of stopbands, it seems desirable for operation at high intensities to compensate the stopbands as well as possible, which increases the allowable peak charge density and decreases the need to accelerate fast, to populate the bunches as uniformly as possible to the allowable density and to make the bunch volume as large as possible.

Uniformly Charged Buckets

If we disregard the vertical betatron motion we are left with a four-dimensional phase space that encompasses the horizontal betatron motion and the synchrotron motion. If one disregards dispersion, the projection of a distribution in that four space onto two-dimensional real space yields the distribution in that space, i.e. the horizontal plane. It is well known that a uniform distribution on a closed surface in four space corresponds with a uniform distribution inside a closed contour in two space, e.g. a uniformly populated ellipsoidal shell corresponds with a uniform density inside an ellipse. Thus if all particles have Hamiltonians $H(x, x', \varphi, \Delta E)$ of the same value and are uniformly distributed the bunches in (x, φ) will be uniformly populated. Although this may seem a somewhat unphysical situation it may be approximated by the application of a suitable strategy for injection and capture. According to that strategy, one stacks in betatron phase space, using a conventional multiturn injection scheme but contrary to convention one varies the energy of the injected beam, correlating the instantaneous values of energy offset and betatron amplitude in such a way that particles with large offsets have small amplitudes and vice versa. Injection is to be followed by some approximation of adiabatic capture in such a way that the particles with the largest amplitudes become synchronous. Since in true adiabatic capture there is a one-to-one relationship between the energy offset from the synchronous value before and the value of the Hamiltonian in synchrotron phase space after capture, it follows that particles with large synchrotron amplitudes have small betatron amplitudes while particles with large betatron amplitudes have small synchrotron amplitudes in bunches obtained in this manner.

It may be useful to review the process in real space, approximating it with a sequence of discrete steps. Ideally the first batch of particles is injected with zero coherent betatron amplitude, the closed orbit (c.o.) being coaxial with the axis of the incoming beam. After one revolution this batch is terminated because the c.o. moves one step away towards its nominal position in the center of the vacuum chamber in order that the injected batch may survive. After the c.o. has reached its nominal position this batch forms a nearly monoenergetic uniform ring of charge of small radial extent coaxial with it. The adiabatic capture process breaks that ring up in identical bunches, with the same cross section as that of the original ring if dispersion is disregarded. The length of each bunch is determined by the difference between the energy of its particles at injection and the synchronous energy. The larger the difference, the closer to the separatrix the particles move in synchrotron space and the longer the bunches are. The longitudinal distribution in each bunch is no longer

* Work performed under the auspices of the U.S. Energy Research and Development Administration.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

uniform but $1/\cos$ -like with a density that is low in the middle and increasing towards the ends. Only the incoherent energy spread in the beam before capture begins and the lack of exact adiabaticity during capture prevents the density in the ends from becoming infinite. The next batch goes through the same process as the previous one. The differences are that the betatron amplitudes of its particles range from a minimum that is equal or larger than the maximum of the previous batch, to a new maximum, and that the energy is closer to the synchronous energy. After capture these particles form bunches that are wider and shorter than the ones formed from the previous batch. Again, the longitudinal extent is $1/\cos$ -like, but over a shorter length. In this way the bunches formed from successive batches are increasingly wider and shorter, each with the $1/\cos$ -like distribution, radially as well as azimuthally, until the last ones that are very wide and have zero length, because they were injected with synchronous energy. Since all the bunches of different batches superimpose in real space the total bunch is elliptical in horizontal projection and uniformly populated, longitudinally as well as radially, provided that the magnitude of the step in the c.o. is correctly related to the step number.

In the real world the c.o. position and the particle energy are continuous functions of time, but this does not effect the final result. The initial and final values of c.o. position and energy, and their relationship, are easily established. The variable that remains to be determined is their time dependence. In first approximation that dependence should be linear because then equal surface elements $dx d\phi$ in the bunches will receive linac beam during equal time intervals. The resulting distribution will be uniform if equal time intervals of linac beam correspond with equal amounts of charge. In practice the amount of charge accepted per unit time is directly proportional with the partial acceptance which varies with betatron amplitude. Thus, ideally the time dependence of closed orbit motion and of energy offset should be determined by the behavior of the partial acceptance.

Dispersion

Particles with different energies have differently located closed orbits at any instant. This dispersion has been excluded from the previous considerations. It tends to decrease the charge density in the physical centers of the bunches and to increase it near the edges in the central region. The severity of the effect depends upon the injection energy, measured as a fraction of the transition energy and locally upon the value of the X_p function.

Application to the AGS

The AGS at BNL is one of the accelerators whose intensity is limited by incoherent space charge effects as described² and, therefore, a candidate for the modification suggested. It is injected with a nominal energy of 200 MeV, during 200 μ sec (42 rev) maximum. Injection takes place by means of a dynamic and local deflection of the closed orbit; the inflector is located 5 cm outboard of the undeflected closed orbit at a point where the amplitude function $\beta_{hor} = 16$ m and $X_p = 1.87$ m. There is an approximation of adiabatic capture and the rate of change of the guiding field B is low during the first 10-15 msec of the accelerating cycle (at present 4.7 kG/sec). The peak energy gain per turn can nominally be as high as a 384 keV although its actual value is somewhat less than half that. Accelerated intensities of 1.05×10^{13} protons per pulse have been obtained without modulation of the injection energy. From the average radius ($R = 128.458$ m), the

radius of curvature in the guiding field magnets magnets ($\rho = 85.344$ m), the injection energy and B , we obtain for the synchronous energy gain per turn near injection 32.375 keV, achievable with (e.g.) a stable phase angle $\phi_s = 10^\circ$ and a peak energy gain per turn $\Delta T = 186.5$ keV, or $\phi_s = 5^\circ$ and $\Delta T = 371.5$ keV. Application of Cole and Morton's tables³ returns phase spreads in the bucket of 1.50π , resp. 1.65π rad. and peak energy errors in the buckets of 2, resp. 3.1 MeV, corresponding with energy spreads of ± 1.03 , resp. ± 1.73 MeV in the coasting beam. These numbers imply a harmonic number $h = 12$. If we take the inflector as the limiting aperture stop the maximum betatron amplitude for particles of synchronous energy is 5 cm at that point. The inflector is located in a region where $\beta = \beta_{AV}$, radially as well as axially, so we take these 5 cm as the average half width of the beam. Disregarding the nonlinearity in the synchrotron motion and the dispersion we approximate the actual bunches with bunches that are elliptical in the equilibrium plane with half-axes of 25×0.05 m² resp. 27×0.05 m² and areas of 3.96 resp. 4.35 m². Assuming an average height of 6 cm (which requires a free vertical aperture of 7 cm at the vertical β_{max} points) the volumes of individual bunches become 0.23 resp. 0.26 m³. In order to find the possible density we assume the bunches uniformly populated with density ρ and calculate first the vertical component of the beam self-field, assuming the bunches to have infinite dimensions in the horizontal plane and then the vertical betatron tune shift per unit density. We obtained

$$\frac{\partial \nu_y}{\partial n} = - \frac{1}{2\nu_y} \frac{e}{\epsilon_0 E_0} \frac{R^2}{\beta^2 \gamma^3} = - 3.2 \times 10^{-14}$$

with n the number density ($\rho = ne$), e the electronic charge, E_0 the rest energy, R the accelerator radius, β and γ the usual relativistic parameters and $\nu_y \approx 8.5$ the nominal vertical betatron frequency. Combining this with the volume of the twelve bunches we find that the vertical betatron frequency changes with the total intensity as $\partial \nu_y / \partial N \approx 1.16 \times 10^{-14}$. One concludes that in these bunches the vertical tune shifts are only of the order of 0.1 per 10^{13} particles. This suggests that intensities of 2×10^{13} protons per pulse might be within fairly easy reach with this method even after all the approximations in this representation have been taken into account. An additional advantage is that the net capture efficiency, i.e. intensity accelerator/intensity in coasting beam is very high: no particles are lost out of betatron phase space due to increasing energy errors and, therefore, decreasing acceptances during capture.

Final Remarks

We have shown that the energy spread in the coasting beam is an important parameter for high intensity operation of a proton synchrotron and indicated means for maximizing the intensity. The potential benefits of this approach (less loss and higher intensity) seem to warrant further investigation. Theoretical work, computer simulation and direct experimentation are all possible and desirable. Direct experimentation may involve tracing the development of a single batch as described above. This can be done by injecting a short pulse (one or a few revolutions) at fixed energy followed by capturing while observing beam position, current and profile as functions of time. One expects to see the characteristic $1/\cos$ -like azimuthal distribution after capture and may measure bunch length as function of injection energy (at fixed frequency of the starting oscillator). The result should be sensitive to the incoherent energy spread in the injected beam and to details in the capturing process, in particular

References

to its departure from adiabaticity. The coherent betatron amplitude of the beam sample can be varied independently by changing the timing of this sequence with respect of the magnet cycle and the results may be observed on the profile monitor. It should show a $1/\cos$ -like distribution also, modified by the dispersion at the location of the profile monitor and the magnitude and distribution of the incoherent betatron amplitudes in terms of the coherent one.

1. R. Chasman, A. Garren, R.L. Gluckstern, F.E. Mills, "Resonance Crossing in a Proton Synchrotron", Proc. IV Int. Conf. on H.E. Accel., Stanford, p. 604, 1974.
2. J. Claus, "Incoherent Transverse Space Charge Effects During Injection, Rf Capture and Early Acceleration in the AGS", IEEE Trans. Nucl. Sci. Vol. NS-22, No. 3, June 1975.
3. F.T. Cole and P.L. Morton, "Areas and Bunching Factors of Partially Filled Buckets", UCID 10130, Sept. 21, 1964.