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APPLICATION OF STRESS CORROSION TO GEOTHERMAL RESERVOIRS

by

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ABSTRACT

There are several alternative equations which describe slow crack growth by stress corrosion. Presently available data suggest that an alternative form may be preferable to the form which is most often used, but the issue cannot be clearly decided.

Presently available stress corrosion data on glasses and ceramics suggest that rocks in a proposed geothermal reservoir will crack readily over long time periods, thus seriously limiting the operation of this type of power source. However, in situ hydrofracturing measurements together with a theoretical treatment suggest that such a reservoir will contain a relatively high pressure over a long period of time without further cracking. Further experimentation is desirable to measure directly the critical stresses for crack growth rates on the order of 10^{-7} m/sec.

I. EQUATIONS FOR SLOW CRACK GROWTH

There are several alternative expressions for rate-controlled crack growth. Each of these provides a useful tool for interpreting experimental data over some range of experimental conditions. The Stuart and Anderson (1953) equation, derived from reaction rate theory and accounting for both the forward and reverse chemical reactions, can provide an accurate representation of experimental data over a wide range of applied stress, while the Charles and Hillig (1962, 1965) equation does not incorporate the reverse reaction and is valid only at relatively high applied stress. At higher stresses, the major difference between the two theories involves the two alternative energy terms through which the crack tip stress may enter into the equation. (1) The strain potential energy $1/2(\sigma^2/E)V_m$ at the crack tip and (2) the change in activation energy with tension, σV^* , where $V^* = \frac{\partial F^*}{\partial \sigma}$ is called the activation volume.

Stuart and Anderson considered only (1), while the usual form of the Charles and Hillig theory (Wiederhorn, 1967) considers only (2).

Because we do not know exactly what is happening on a molecular level at the crack tip, we cannot be sure whether (1), (2), or some other expression or combination of expressions should be included in the crack equation. I will show below, however, that an interpretation of the presently available experimental evidence seems to support the use of the first term.

The Stuart and Anderson expression for crack velocity, v , (see also Anderson and Perkins, 1975) can be easily modified to include both terms (1) and (2).

$$v = \frac{\lambda dN}{dt} = \frac{\lambda N k T}{h} \exp\left[\frac{(-F^* + \sigma V^* + SE)}{kT}\right] \sinh\left(\frac{PE - SE}{2kT}\right) \quad (1)$$

where F^* is the activation-free energy, SE is the surface energy per molecule for the creation of the solid-fluid interface, PE is the strain energy per molecule at the crack tip, N is the number of molecules exposed at the crack tip, λ is the length of crack growth per molecule removed, and σ is the stress at the crack tip.

$$PE = \frac{1}{2} \frac{\sigma^2}{E} V_m \quad (2)$$

$$SE = \frac{\gamma_s}{\rho} V_m \quad (3)$$

$$XN = V_m^{1/3} \quad (4)$$

where E is Young's modulus, ρ is the crack tip radius, γ_s is the surface energy (surface tension) for the liquid-solid interface, and V_m is the volume per molecule of the solid.

In addition, it is useful to substitute the stress intensity factor, $K_I = \frac{\sigma}{2} \sqrt{\pi\rho}$, for the stress at the crack tip, and we get, at high stresses,

$$v = \frac{\lambda NkT}{h} \exp(-F^* + \frac{2V^*}{\sqrt{\pi\rho}} K_I + \frac{2V_m}{E\pi\rho} K_I^2) / kT \quad (5)$$

The available data on slow crack growth in glasses do not permit the unambiguous determination of whether the argument of the exponential should be linear or quadratic in K_I , although it has become traditional to keep only the linear term. Wiederhorn and Bolz (1970) have determined the stress dependence of the crack velocity for six glasses. From Eq. (5), their parameter $b = \frac{1}{kT} \frac{\partial \ln v}{\partial K_I}$ is given by

$$b = \frac{2V^*}{\sqrt{\pi\rho}} + \frac{4V_m}{E\pi\rho} K_I \quad (6)$$

Since V_m , E, and K_I can be measured, we can determine ρ from b if we assume that only the second term in (6) is important. Alternatively, if we assume that only the first term in (6) is important, and assume $\rho = 15 \cdot 10^{-10}$ m, which is a reasonable value, we can calculate V^* . The measured values of b and the calculated values of ρ range from 10^1 to $20 \cdot 10^{-10}$ m and are certainly reasonable, because this is only slightly larger than the minimum crack tip radius which would permit water molecules to adhere to the crack surface and also leave room for other molecules to flow to the crack tip. While the values of V^*/V_m seem reasonable, it is possible that V^* is really much smaller. The conclusion to be drawn from Table I is that the third term in the exponential in Eq. (5) is probably important, while it is not clear whether or not the second term is important. This is in disagreement with the current practice of ignoring the third term and keeping the second term.

TABLE I

Glass	b (m ^{2.5})	ρ (10 ⁻¹⁰ m)	V^*/V_m
Silica	3.59 E-25	10.0	.27
Aluminosilicate I	2.29 E-25	15.9	.17
Aluminosilicate II	2.72 E-25	13.2	.20
Borosilicate	3.32 E-25	10.8	.25
Lead-Alkali	2.39 E-25	15.0	.18
Soda-Lime	1.83 E-25	19.6	.14

II. APPLICATION OF STRESS CORROSION THEORY TO DRY ROCK GEOTHERMAL RESERVOIRS

The LASL project for the development of Dry Rock Geothermal Reservoirs involves the creation of a large underground crack by hydrofracturing, and the circulation of water within the crack while the crack is held open by water pressure which exceeds the local horizontal stress in the rock. For a given crack size, the volume of the crack will increase linearly with the differential pressure P' , where $P' = P - S$, and P is the fluid pressure and S is the horizontal stress in the rock. The volume of the crack, and hence the flow of water and energy yield, are thus limited by the maximum fluid pressure which the crack can contain without growing appreciably (hundreds of meters) in size during the proposed life of the geothermal well (tens of years). Thus P' cannot exceed for very long the stress which leads to crack propagation at rates exceeding about 10 m/yr (about 3×10^{-7} m/sec).

The critical parameter which determines crack growth is the stress intensity factor, K_I , which depends on both P' and on the radius of the (circular) crack, a.

$$K_I = P' \sqrt{\frac{2}{\pi}} \sqrt{a} \quad (7)$$

Hydrofracturing experiments *in situ* resulted in fracturing at values of $K_{I}^{hf} \approx 2.9 \text{ MN/m}^{3/2}$.

Stress intensity factors lower than this value may still lead to slow crack propagation in excess of 3×10^{-7} m/sec. There are at least two ways to use presently available information to estimate the stress intensity factors which will be sufficiently low to ensure that slow crack propagation will not destroy a dry rock geothermal well.

One way is to merely look at the results of stress corrosion experiments on glasses and ceramics and to extrapolate the results to the conditions of the dry geothermal well. It is reasonable to assume that crack propagation in granitic rocks is intermediate between glass and alumina (unfortunately there are no data available on stress corrosion in rock-forming minerals). This type of analysis suggests that K_I^s , the critical value of K_I at which $v = 10^{-7}$ m/sec lies between $0.2 \text{ MN/m}^{3/2}$ and $1.5 \text{ MN/m}^{3/2}$ in the environment of a geothermal well ($T = 196^\circ\text{C}$). If this is true, this places a serious constraint upon the extractor of energy from dry geothermal reservoirs.

A more sophisticated estimate of the pressure which can be contained for a long period of time in a dry geothermal reservoir can be made from an interpretation of existing hydrofracturing data. Data on the pressure and pumping rate during hydrofracturing can lead to values of velocity and K_I which may be extrapolated to lower rates of crack growth. The following equations are relevant:

$$V = \frac{4\pi}{3} (1 - \nu^2) \frac{P'}{E} R^3 \quad (8)$$

where V is the volume of the crack. Inserting Eq. (7),

$$V = \frac{4\pi}{3} \sqrt{\frac{\pi}{2}} (1 - \nu^2) \frac{K_I}{E} R^{2.5} \quad (9)$$

$$\frac{dR}{dt} = \frac{dR}{dV} \frac{dV}{dt} \quad (10)$$

$$\frac{dV}{dR} = \frac{1}{dR/dV} = 2.5 \frac{V}{R} = \frac{10\pi}{3} \sqrt{\frac{\pi}{2}} (1 - \nu^2) \frac{K_I}{E} R^{1.5} \quad (11)$$

The relevant data for granite and granodiorite rocks are $E = 83 \text{ GPa}$, $\nu = 0.22$. The data on $\frac{dV}{dt}$ for in situ hydrofracturing are difficult to interpret, partly because of the high permeability of the rocks. The data used in the following analysis are probably not the best available and are intended to be illustrative. In a test well, the pumping rate was about $6 \cdot 10^{-4} \text{ m}^3/\text{sec}$ (10 gallons per minute), the crack radius was about 10 meters, and the pump pressure led to a calculated K_I of $2.9 \text{ MN/m}^{3/2}$ based on Eq. (7). From Eqs. (10) and (11),

$\frac{dR}{dt} = 4.46 \cdot 10^{-2} \text{ m/sec}$. It is necessary to extrapolate this velocity over seven orders of magnitude to determine the stress intensity factor which permits the long term operation of a geothermal well. Going back to Eq. (5), we will assume that the

second term in the exponential can be ignored.

Then,

$$\frac{v}{v_0} = \exp\left[\frac{2V_m}{E\pi\rho} (K_I^2 - K_{I_0}^2)/kT\right]$$

and

$$K_I^2 = K_{I_0}^2 + \frac{E\pi}{2V_m} \rho kT \ln v/v_0 \quad (12)$$

Assuming $70 < E < 90 \text{ GPa}$, $10 < \rho < 25 \cdot 10^{-10} \text{ m}$, and $1.75 < V_m < 4.0 \cdot 10^{-29} \text{ m}^3/\text{molecule}$ at $T = 197^\circ\text{C}$, the factor $\frac{E\pi\rho kT}{2V_m}$ lying between $1.78 \cdot 10^{10}$ and $1.30 \cdot 10^{11}$.

If we set $v/v_0 = 10^{-7}$ and $K_{I_0} = 2.9 \text{ MN/m}^{3/2}$, we

get the result that for slow crack propagation $2.50 < K_I < 2.85 \text{ MN/m}^{3/2}$. The assumption of this particular functional form for the velocity-stress equation leads to the conclusion that the stress which can safely be contained in a geothermal reservoir is only slightly lower than the stress which causes hydrofracturing.

There are three major assumptions in the above conclusions. (1) I have assumed that Eq. (5) describes the data adequately, with the second term in the exponential being negligibly small. The justification for using this equation in preference to another equation is rather weak. (2) It is possible that at lower crack velocities an alternative cracking mechanism such as intergranular corrosion is operational. (3) While the above conclusions are not strongly dependent upon any of the parameters entering into Eq. (12), it is possible that large errors in the estimated values for ρ and V_m could affect the results.

There is, therefore, still a great deal of uncertainty concerning whether or not stress corrosion will be a serious constraint on the operation of a dry rock geothermal reservoir.

It is highly desirable to obtain data concerning slow cracking ($\sim 10^{-7} \text{ m/sec}$) in natural rock specimens in an aqueous environment. Although it is most desirable to obtain these data under conditions closely approximating a geothermal reservoir (circulating water under pressure), data on rocks at low pressure exposed to water or water vapor will help lessen the uncertainty in the present analysis.

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