BEAM PARAMETERS FOR THE FERMILAB COLLIDER

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This note is a compendium of various formulae used to calculate beam size, luminosity, tune shift, etc., for the Fermilab Main-Ring-on-Doubler Collider. The results for specific cases are summarized in Table I. A recent CERN report by Guignard¹ may be consulted for a much more detailed set of storage ring formulae.

Transverse Beam Size

The beam width in each transverse dimension is determined by the emittance, ϵ , the area in phase space containing 95% of the beam. Assuming Gaussian distributions, the rms beam size is given by

$$\sigma^2 = \epsilon\beta/6\pi \quad (1)$$

where β is the usual amplitude function. The beam from the booster ($T = 8$ GeV) has an emittance $\epsilon = 1.5\pi$ mm mrad, in both the horizontal and vertical; if there is no dilution as the beam is accelerated, ϵ scales as

$$\epsilon = 13\pi/p \text{ mm mrad} \quad (2)$$

where the beam momentum p is in GeV/c. For β in units of meters we have

$$\sigma = 1.49 \sqrt{\beta/p} \text{ mm} \quad (3)$$

By repowering the closest three quadrupoles on each side of the interaction region Collins has shown² how the Main Ring beam can be focused down to $\beta^* = 2.5$ m, for momenta up to 150 GeV/c. More recently, D. E. Johnson³ has shown how to reduce β_{MR}^* down to 1 meter at 250 GeV and β_D^* down to 2.5 m at 1000 GeV. This would be accomplished with somewhat longer quadrupoles, requiring an additional ~ 2 m of space at each end of the long straight section,

and by repowering the closest four quadrupoles on each side of the long straight.

For the parameters presented at the 1977 Woods Hole Meeting

($\beta_{MR}^* = 2.5$ m, $\beta_D^* = 10$ m at 250 on 1000 GeV) the rms width of each beam would be 0.15 mm. Multiple scattering and instabilities presently cause some growth of the stored beam with time (factor of 2 in ~ 1000 sec at 100 GeV), but this should be reduced considerably as the Main Ring vacuum is improved and experience is gained in tuning the machine.

Transverse Momentum in the Beam

The rms spread of horizontal angles for beam particles is given by

$$\sigma_{x'}^2 = \epsilon / 6\pi\beta_x, \quad (1')$$

and similarly for the vertical. For the emittance discussed above this leads to

$$\sigma_{p_x} = 1.49 \sqrt{p/\beta_x} \text{ MeV/c} \quad (3')$$

for p in GeV/c and β_x in meters. For the cases shown in Table I $\sigma_{p_x} = 4$ to 20 MeV/c, and this small spread will not affect most experiments.

Momentum Spread

The momentum spread is determined by both the rf voltage per turn, V , and by the emittance in longitudinal phase space, ϵ_ℓ . This emittance is not well known; a value of 0.1 eV·sec is generally used. This leads to⁴

$$\sigma_p/p = 1.1\% \epsilon_\ell^{1/2} V^{1/4} / E^{3/4} \quad (4)$$

$$= 3.6 \times 10^{-3} V^{1/4} / E^{3/4}, \quad (5)$$

where V is in MV/turn and E is the beam energy in GeV. For $V = 3$ MV/turn,

the spread is about $\sigma_p/p = 10^{-4}$ at 150 GeV and less at higher energy. In the last two columns (F and G) in the table, 10 turn stacking in momentum space is assumed in the Doubler; assuming ϵ_L to be 20 times the single beam value leads to $\sigma_p/p = \pm 0.9 \times 10^{-4}$ at 1000 GeV, or $\sigma_p = \pm 90$ MeV.

The beam size gets contributions from both the betatron phase space discussed in the previous section and from the dispersion, η :

$$\sigma^2 = \sigma_\beta^2 + (\eta \sigma_p/p)^2 \quad (6)$$

It turns out that for the cases being considered the dispersion adds very little to the beam size. Even for 10 turn stacking and the standard intersection region ($\beta^* = 70$ m, $\eta^* = 2.3$ m), $\eta \sigma_p/p = \pm 0.21$ mm, boosting $\sigma_\beta = \pm 0.40$ mm to a total width of $\sigma = \pm 0.45$ mm.

Bunch Length

The bunch length also depends on the longitudinal emittance and rf voltage:

$$\sigma_L = 140 \epsilon_L^{1/2} / (VE)^{1/4} \quad (7)$$

$$= 44 / (VE)^{1/4} \text{ cm} \quad (8)$$

The dependence on V and E is quite slow, and ± 10 cm is typical for single turn injection; for 10 turn stacking this becomes ± 35 cm.

Recent Main Ring storage studies show a longitudinal instability which causes bunches to broaden to a full width at the base ($\sim 4\sigma$) of $\sim 5 \text{ nsec} = 150$ cm, roughly 4 times the value given by the calculation above. Presumably σ_p/p is also spread by a similar factor.

Luminosity

For head-on collisions with bunched beam⁵

$$\mathcal{L}_o = \frac{f N_1 N_2 / 2\pi}{[(\sigma_{x1}^2 + \sigma_{x2}^2)(\sigma_{z1}^2 + \sigma_{z2}^2)]^{\frac{1}{2}}} \quad (9)$$

where f = frequency of meetings of bunches = 53.1×10^6 /sec,

N_1 = number of protons/bunch in the first beam,

σ_{x1} = horizontal rms width of first beam, etc.

Assuming each beam to have 2×10^{13} protons spread out evenly over the 1113 rf buckets,

$$\mathcal{L}_o = \frac{0.273 \times 10^{30} \text{ cm}^{-2} \text{ sec}^{-1}}{[(\sigma_{x1}^2 + \sigma_{x2}^2)(\sigma_{z1}^2 + \sigma_{z2}^2)]^{\frac{1}{2}}} \quad (10)$$

when the σ 's are in units of mm. If the two beams are round ($\sigma_{x1} = \sigma_{z1} = \sigma_1$ and $\sigma_{x2} = \sigma_{z2} = \sigma_2$), then

$$\mathcal{L}_o = \frac{0.273 \times 10^{30} \text{ cm}^{-2} \text{ sec}^{-1}}{\sigma_1^2 + \sigma_2^2} \quad (11)$$

If the two beams are equal in size ($\sigma_{x1} = \sigma_{x2} = \sigma_x$ and $\sigma_{z1} = \sigma_{z2} = \sigma_z$), then

$$\mathcal{L}_o = \frac{0.273 \times 10^{30} \text{ cm}^{-2} \text{ sec}^{-1}}{2\sigma_x \sigma_z} \quad (12)$$

Note that $\mathcal{L}_o$ does not depend on the bunch length and is not directly affected by bunch lengthening.

The values for $\mathcal{L}_o$ shown in the table range from 10^{30} to $10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$.

These are values for freshly filled rings; the stored beams will not only lose particles as time goes on, but their phase space will be diluted by multiple scattering and possibly by beam instabilities. If the Main Ring beam is being ramped to supply beam to the fixed-target experiments and/or for cooling purposes, the duty factor will lower the average luminosity. Another factor lowering

the effective time-averaged luminosity will be the break down of experimental equipment which cannot be fixed until the next access time. On the other hand, one can expect increases in luminosity as the beam intensity goes up in the next few years and as work is done to understand and reduce the emittances. Also, two turn stacking may be feasible in the Main Ring.⁶

Variation of Luminosity with Crossing Angle

At present the transposed crossing geometry looks to be more attractive than the kissing geometry. The transposed geometry has a natural crossing angle $\alpha \approx 16$ mrad; if this angle is not reduced, luminosity will be lost, especially for those cases with high head-on luminosity. The luminosity for vertical crossing with half angle $\theta = \alpha/2$ can be obtained from the previous expressions for $\mathcal{L}_0$ by replacing σ_z with

$$\hat{\sigma}_z = (\sigma_z^2 + \sigma_\ell^2 \theta^2)^{\frac{1}{2}} \quad (13)$$

Thus, the ratio of luminosity at half-angle θ to the head-on case is given by⁵

$$\frac{\mathcal{L}}{\mathcal{L}_0} = \left[\frac{\sigma_{z1}^2 + \sigma_{z2}^2}{(\sigma_{z1}^2 + \sigma_{z2}^2) + (\sigma_{\ell 1}^2 + \sigma_{\ell 2}^2) \theta^2} \right]^{\frac{1}{2}} \quad (14)$$

This ratio has the general form

$$\frac{\mathcal{L}}{\mathcal{L}_0} = \frac{1}{[1 + \theta^2 / \Theta^2]^{\frac{1}{2}}} \quad , \quad (15)$$

where the scale is set by

$$\Theta = \left[\frac{\sigma_{z1}^2 + \sigma_{z2}^2}{\sigma_{\ell 1}^2 + \sigma_{\ell 2}^2} \right]^{\frac{1}{2}} \quad . \quad (16)$$

If the two beams have equal dimensions ($\sigma_{z1} = \sigma_{z2} = \sigma_z$ and $\sigma_{\ell 1} = \sigma_{\ell 2} = \sigma_\ell$), then

$$\Theta = \sigma_z / \sigma_\ell . \quad (17)$$

For $\alpha = \tilde{\alpha} \equiv 2\Theta$ (i. e., $\theta = \Theta$), $\mathcal{L} / \mathcal{L}_0 = 1/\sqrt{2}$. This becomes

$$\mathcal{L} / \mathcal{L}_0 \approx \tilde{\alpha} / \alpha \text{ for } \alpha \gg \tilde{\alpha} . \quad (18)$$

For $\epsilon_\ell = 0.1 \text{ eV} \cdot \text{sec}$, $\tilde{\alpha}$ varies from 0.9 to 7 mrad, as shown in Table I. Figure 1 explicitly shows the variation of $\mathcal{L}$ with angle calculated by T. L. Collins for the parameters shown in the table. The high luminosities deteriorate rapidly as α increases. If the longitudinal instability cannot be controlled, the values of $\tilde{\alpha}$ will be reduced by a factor of ~ 4 , making it even more urgent to be able to achieve near head-on collisions.

The beams will probably not be collided at exactly 0° since this would result in backgrounds from beam-beam collisions of adjacent bunches, as well as increase the beam-beam tune shift. Instead, a small angle of about 1 mrad will be introduced; this will cause a separation of 2.8 mm at the next meeting point, $\lambda_{\text{rf}}/2 = 2.8 \text{ m}$ from the central intersection point. For the parameters presented at the Woods Hole Meeting (column H of Table I), this separation corresponds to 12σ for the Main Ring beam and 18σ for the Doubler, where we have taken into account the growth in β away from the interaction point:

$$\beta(2.8 \text{ m}) = \beta^* [1 + (2.8/\beta^*)^2] .$$

For the Woods Hole case with $\epsilon_\ell = 0.1 \text{ eV} \cdot \text{sec}$ and $\alpha = 1 \text{ mrad}$, $\mathcal{L} / \mathcal{L}_0 = 0.96$; for bunch lengths 4 times the calculated values, this becomes $\mathcal{L} / \mathcal{L}_0 = 0.65$.

Length of the Luminous Region

For two beams with Gaussian distributions, the distribution of events

along the beam is also Gaussian. For head-on collisions the rms length of the event distribution is

$$\sigma_L = (\sigma_{L1}^2 + \sigma_{L2}^2)/2 \quad (19)$$

For single turn injection Table I shows $\sigma_L = 6$ to 8 cm, in which case 95% of the events will come from a region one foot long ($\pm 2\sigma$). The 10-turn stacking examples have $\sigma_L = \pm 18$ cm (28 inches for 95%). Experiments will generally be easier to build if the longitudinal instability can be overcome so that these lengths do not increase substantially.

For beams crossing at finite angles, the source length is reduced by the same factor as found for the luminosity

$$\sigma_L / (\sigma_L)_0 = \mathcal{L} / \mathcal{L}_0 \quad (20)$$

i.e., the central source brightness remains constant. For some experiments this reduction in source size may be advantageous and partially offset the misery of having lost luminosity.

Beam-Beam Tune Shift

Each time two bunches collide, each proton of one bunch receives a lateral impulse from the macroscopic ("space-charge") electromagnetic field generated by the other bunch. Expanding near the origin one can derive a tune-shift, $\Delta\nu$, caused by one beam on the other. The forces are nonlinear, however, and the actual tune shift varies with orbit, so $\Delta\nu$ is more properly thought of as a measure of the tune spread. For long term stability in proton machines, a limit of 0.005 is conventionally assumed, but this is probably very conservative.⁷

Sands gives⁸

$$\Delta\nu_{x1} = \frac{r_p}{2\pi} \frac{\beta_{x1}^*}{\gamma_1} \frac{N_2}{\sigma_{x2}(\sigma_{x2} + \sigma_{z2})} \quad (21)$$

where r_p = classical proton radius = $e^2 / M_p c^2 = 1.5347 \times 10^{-18}$ m,

N_2 = number of protons/bunch in beam 2,

$$\gamma_1 = E_1/M_p.$$

The equations for Δv_{x2} , Δv_{z1} and Δv_{z2} follow directly by transposition of symbols. For our standard case of 2×10^{13} protons evenly divided among 1113 bunches,

$$\Delta v_{x1} = 4.12 \times 10^{-3} \frac{\beta_{x1}^*}{E_1} \frac{1}{\sigma_{x2}(\sigma_{x2} + \sigma_{z2})}, \quad (22)$$

for β^* in meters, E in GeV and σ in mm. For round beams this becomes

$$\Delta v_1 = 2.06 \times 10^{-3} \beta_1^* / E_1 \sigma_2^2. \quad (23)$$

Using Eq. (3) this can be rewritten as

$$\Delta v_1 = 0.926 \times 10^{-3} \beta_1^* E_2 / \beta_2^* E_1. \quad (24)$$

Values for Δv are listed in Table I. The large value $\beta_D^* = 70\text{m}$ results in $\Delta v_D = 0.010$ for cases B and E and may limit the Main Ring intensity allowable, especially when the Doubler is running at lower energies. The very small intense beam of case G yields $\Delta v_{MR} = 0.012$ and may cause Main Ring instabilities. For the case presented at the Woods Hole Meeting, $\Delta v = 0.0009$ and should cause no problems.

If indeed the luminosity were to be limited by some maximum tune shift, it would be instructive to look at the ratio $\mathcal{L}/\Delta v$ in order to optimize $\mathcal{L}$ within this limit; for head-on collisions

$$\frac{\mathcal{L}}{\Delta v_{x1}} = \frac{f N_1 \gamma_1}{r_p \beta_{x1}^*} \left\{ \frac{\sigma_{x2}(\sigma_{x2} + \sigma_{z2})}{[(\sigma_{x1}^2 + \sigma_{x2}^2)(\sigma_{z1}^2 + \sigma_{z2}^2)]^{\frac{1}{2}}} \right\}. \quad (25)$$

For round beams

$$\frac{\mathcal{L}}{\Delta v_1} = 3.69 \times 10^{21} \frac{N_1 E_1}{\beta_1^*} \left\{ \frac{2\sigma_2^2}{\sigma_1^2 + \sigma_2^2} \right\}, \quad (26)$$

where N_1 is again the number of particles per bunch in beam 1, E is in GeV and β^* in meters.

For beams crossing at an angle $\alpha = 2\theta$, one should again replace σ_z with

$$\hat{\sigma}_z = (\sigma_z^2 + \sigma_\ell^2 \theta^2)^{\frac{1}{2}} \quad (13')$$

and the tune shift is less than that for the head-on condition. For $\theta \gg \Theta = \sigma_z / \sigma_\ell$, Δv_x has the same $1/\theta$ dependence as the luminosity, while $\Delta v_z \propto 1/\theta^2$.

Scaling

Many of the input numbers are presently uncertain and should be better measured, and if possible, improved upon. In some cases, such as beam intensity, such improvement is being actively pursued and will presumably have substantial enhancement over the values used here. While scaling with emittance or intensity can be obtained from the previous equations, we summarize the more important dependences here:

$$\begin{aligned} \sigma_x, \sigma_z &\propto \epsilon^{\frac{1}{2}} \\ \mathcal{L}, \Delta v &\propto 1/\epsilon \\ \sigma_p/p, \sigma_\ell, \sigma_L &\propto \epsilon_\ell^{\frac{1}{2}} \\ \Theta, \tilde{a} &\propto 1/\epsilon_\ell^{\frac{1}{2}} \\ \mathcal{L} &\propto N_{MR} N_D \\ \Delta v_D &\propto N_{MR} \\ \sigma_p/p &\propto V^{1/4} \\ \sigma_\ell &\propto 1/V^{1/4} \end{aligned}$$

If the emittance increases linearly with beam intensity, then

$$\mathcal{L}_0 \propto N^2 / \epsilon \propto N.$$

The dependence of $\mathcal{L}_0$ on β^* depends on both beams and is best understood by looking at specific examples such as those in Table I; the scaling goes as

$$\mathcal{L}_0 \propto 1 / \left(\frac{\beta_{MR}^*}{p_{MR}} + \frac{\beta_D^*}{p_D} \right) .$$

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Table I. Parameters for different modes of operation. For all modes $\beta_{MR}^* = 2.5$ m and 2×10^{13} protons are assumed in each ring (2×10^{14} in Doubler for modes F and G). Column H shows the case used in the Woods-Hole presentation; the luminosity for this case is similar to that shown in the figure for case D. Recent studies of the Main Ring as a storage device indicate that longitudinal instabilities presently increase the bunch lengths by a factor of ~ 4 over the values given here. While this would not affect the head-on luminosity $\mathcal{L}_0$, it would cause a factor of ~ 4 faster fall-off as the crossing angle α is increased ($\tilde{u} \sim 1/4$ of the values shown here).

		A	B	C	D	E	F	G	H
E_{MR}	(GeV)	150	150	400	400	400	400	400	250
E_D		400	1000	1000	400	1000	1000	1000	1000
β_D^*	(m)	5	70	70	5	5	70	5	10
σ_{MR}		0.19	0.19	0.12	0.12	0.12	0.12	0.12	0.15
σ_D	(mm)	0.17	0.40	0.40	0.17	0.11	0.40	0.11	0.15
V_{MR}	(MV/turn)	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
V_D		0.5	0.5	0.5	0.5	0.5	1.0	1.0	0.5
$(\sigma_p/p)_{MR}$	(10^{-5})	11.0	11.0	5.3	5.3	5.3	5.3	5.3	7.5
$(\sigma_p/p)_D$		3.4	1.7	1.7	3.4	1.7	9.0	9.0	1.7
$\sigma_{\ell MR}$		9.6	9.6	7.5	7.5	7.5	7.5	7.5	8.4
$\sigma_{\ell D}$	(cm)	11.7	9.3	9.3	11.7	9.3	35	35	9.3
σ_L	(cm)	7.6	6.7	6.0	6.9	6.0	18	18	6.3
$\tilde{\alpha} = 2\theta$	(mrad)	3.4	6.6	7.0	3.0	3.5	2.3	0.9	3.4
Δv_{MR}	(10^{-3})	1.2	0.2	0.1	0.5	1.2	0.8	12	0.9
Δv_D		0.7	3.9	10.4	1.8	0.7	10.4	0.7	0.9
$\sqrt{s}$	(GeV)	490	775	1265	800	1265	1265	1265	1000
$\mathcal{L}_0$	($10^{30} \text{ cm}^{-2} \text{ sec}^{-1}$)	4.2	1.4	1.6	6.6	11	15	110	6.2