SEARCH FOR BACKWARDS PRODUCED EXOTIC MESON RESONANCES
AND STUDY OF PARTICLE SPECTRA AND MASS

DISTRIBUTIONS IN THE REACTION

$$\pi^+ + p \rightarrow n \text{ (forward)} + X^{++}$$

AT 8.4 GeV/c^{*†}

Technical Report

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ABSTRACT

Results are reported from a hybrid experiment designed to search for backwards produced $I = 2$ exotic meson resonances by investigating the backward scattering reactions:

$$\pi^+ + p \rightarrow n \text{ (forward)} + \begin{cases} \pi^+ \pi^+ \\ \pi^+ \pi^+ \pi^+ \pi^- \\ \pi^+ \pi^+ \pi^+ \pi^+ \pi^- \pi^- \\ p \bar{p} \pi^+ \pi^+ \end{cases}$$

at an incident pion momentum of 8.4 GeV/c. The SLAC 15" rapid cycling bubble chamber, operating at 20 cycles per second, was triggered by detecting fast forward neutrons in an optical spark chamber and scintillation counter array downstream. Following two-component duality arguments, Jacob and Weyers¹ and Rosner² have suggested backward scattering processes as a favored mode for exotic meson resonance production. Upper limits of the order of one microbarn are reported for the production of narrow exotic meson resonances with masses ≤ 3 GeV and which could decay into even G-parity states: $\pi^+ \pi^+$, $\pi^+ \pi^+ \pi^+ \pi^-$, $\pi^+ \pi^+ \pi^+ \pi^+ \pi^- \pi^-$, and $p \bar{p} \pi^+ \pi^+$. The mass resolution in the above channels varied from 5 to 30 MeV over the mass range for the most restrictive sample of events. The statistical significance of the data corresponds to 30 events per microbarn. Evidence for forward N^* and ρ^0 production are also reported. The main features of the data can be simulated by a Monte Carlo model which incorporates Lorentz-invariant phase space modified by exponential

transverse momentum damping on each final state particle and, in addition, takes into account the geometrical constraints and trigger requirements of the experiment.

1. H. Jacob and J. Weyers, Nuovo Cimento, LXIX, 521 (1970).
2. J. Rosner, Phys. Rev. Letters 23, 266 (1969).

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CHAPTER I

A REVIEW OF EXOTIC MESON RESONANCES

A. Introduction

Exotic Mesons and Baryons

Exotic hadrons are best described in the context of the quark model^{1,2,3} for elementary particles. According to this model the basis vectors for the first and second fundamental representation of SU(3) form the triplet of (p,n,λ) quarks and the corresponding triplet of antiquarks ($\bar{p}, \bar{n}, \bar{\lambda}$). All mesonic states are then quark-antiquark ($\bar{q}q$) bound states belonging to a singlet or octet corresponding to the 1 or 8 irreducible representations obtained from the decomposition

$$3 \otimes \bar{3} = 1 \oplus 8 \quad (1a)$$

All baryonic states are (qqq) or ($\bar{q}\bar{q}\bar{q}$) bound states belonging to a singlet, octet or decuplet corresponding to the 1, 8, or 10 irreducible representations obtained from the decomposition

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10 \quad (1b)$$

Rosner⁴ classified exotic hadrons into three broad types.

1. TYPE-I EXOTICS: Any state which cannot be described by the above model (with approximately harmonic qq and $\bar{q}q$ forces) is called a type-I exotic. For example, an isospin $I = 2$, doubly-charged meson would be exotic. Similarly an $I = 1$, $Y = 2$, K^+p resonance would be an example of an exotic baryon. Existence of $Z^{\star 0}$ or $K^+p(K^0n)$ resonance has been reported but there is still considerable controversy regarding its true resonant nature.

2. TYPE-II EXOTICS: A $\bar{q}q$ state may have intrinsic spin $S = 0$ or 1, parity $P = (-)^{L+1}$ where L is orbital angular momentum; the charge conjugation quantum number $C = (-)^{L+S}$ for the neutral states. The spin of the hadron corresponding to the $\bar{q}q$ state is given by $\bar{J} = \bar{L} + \bar{S}$ such that for $S = 0$, $\bar{J} = \bar{L}$ and for $S = 1$, $J = L-1, L, L+1$. Now $J = 0$ for (i) $S = 0, L = 0$ and (ii) $S = 1, L = 1$; hence for $J = 0$, the allowed values for J^{PC} are 0^{-+} and 0^{++} . For $S = 0$, $J(\text{odd})$ corresponds to $L(\text{odd})$ so that only $J^{PC} = (\text{odd})^{\pm\pm}$ are allowed. Similarly it may be shown that $J^{PC} = (\text{even})^{-+}, (\text{even})^{\pm\pm}$ are also allowed. We may now write down allowed values of J^{PC} :

$$J^{PC} = 0^{-+}, 0^{++}, (\text{odd})^{\pm\pm}, (\text{even})^{\pm\pm} \quad (2a)$$

Type-II exotic mesons then correspond to the forbidden values of J^{PC} , that is

$$J^{PC} = 0^{--}, 0^{+-}, (\text{odd})^{-+}, (\text{even})^{+-} \quad (2b)$$

3. TYPE-III EXOTICS: Low lying states with allowed values for I, Y and J^{PC} but whose existence would require serious distortion of harmonic $\bar{q}q$ forces are included in this category.

B. Models for Exotic Mesons

Allowed Decay Modes of Exotic Mesons Based on the Simple Quark Model

The lowest lying exotic mesons would be $qq\bar{q}\bar{q}$ bound states with $L = 0$ just as the lowest lying non-exotic mesons are $\bar{q}q$ bound states with $L = 0$ and 1. On the basis of

experimentally observed decay modes of mesons and baryons and allowed MMM and MBB couplings, Zweig^{7,8,9} and Freund¹⁰ have proposed a set of rules for drawing quark diagrams for allowed couplings.

Rule 1: Mesons will be represented as quark-antiquark lines, for example

Figure 1a

Baryons and antibaryons will be represented as three quark and three antiquark lines respectively, for example

Figure 1b

Rule 2: At any reaction vertex, each of the three hadrons must exchange at least one quark line with each of the remaining two hadrons. We write down allowed diagrams for MMM and MBB couplings.

MMM:

Figure 2a e.g. $\rho^0 \rightarrow \pi^+ \pi^-$

Figure 2b e.g. $N^{*+} \rightarrow \pi^+ n$

Rule 3: No quark line is connected with an antiquark line from the same hadron. The following diagrams are not allowed:

e.g. $M_4 \rightarrow \pi^+ \pi^+$

Figure 2c

According to rule 3, the decay $\varphi \rightarrow \rho^- \pi^+$ is forbidden. Similarly the decay $f' \rightarrow \pi^+ \pi^-$ is not allowed. If φ is really a pure $\bar{\lambda}\lambda$ state, the rate for the decay process $\varphi \rightarrow \rho^- \pi^+$ is a direct measurement of the violation of rule 3, since it would require the annihilation of $\bar{\lambda}\lambda$ in φ . However, the decay $\varphi \rightarrow \rho^- \pi^+$ may proceed through small admixtures of $\bar{n}n$ or $\bar{p}p$ states or through rescattering of intermediate $\bar{K}K$ states as in $\varphi \rightarrow K^+ K^- \rightarrow \pi^+ \rho^-$ according to the diagram below:

Figure 2d

The rate for the process $\varphi \rightarrow \pi^+ \pi^- \pi^0$ is suppressed almost an order of magnitude relative to that for $\omega \rightarrow \pi^+ \pi^- \pi^0$, showing that contributions from second order diagrams such as figure 2d must be small.

A very important consequence of rules 2 and 3 is that if exotic mesons ($qq\bar{q}\bar{q}$) are represented by two quark and two antiquark lines, their decay into mesons is suppressed to the same extent as $\varphi \rightarrow \pi^+ \pi^-$ is forbidden. See Figure 2c. However the coupling $M_4 \rightarrow \bar{B}B$ does not violate any of the above rules.

Fig. 2e

Thus, if exotic mesons exist they would tend to couple more strongly to $B\bar{B}$ systems rather than MM systems. For example, the decays

$$M_4^{++} \text{ (exotic)} \rightarrow \pi^+ \pi^+, \pi^+ \pi^+ \pi^+ \pi^-, \rho^+ \pi^+, \text{ etc.} \quad (3a)$$

are not allowed according to Figure 2c. Examples of allowed decays are:

$$M_4^{++} \text{ (exotic)} \rightarrow \Delta^{++} \bar{\Delta}^0, \Delta^{++} \bar{n}, \Delta^{++} \bar{\Delta}^-, \Delta^+ \bar{p}, \text{ etc.} \quad (3b)$$

An exotic meson may decay into another exotic meson and other non-exotic mesons. See Figure 2f.

Fig. 2f

Processes such as $M_4 \rightarrow \bar{B}B \rightarrow \pi^+ \pi^+$ do not make substantial contributions for the same reason as apply to $\phi \rightarrow \bar{K}K \rightarrow \pi^+ \rho^-$. Contributions from diagrams such as shown in Figure 2g must be small.

Fig. 2g

Freund¹⁴ arrives at similar conclusion about allowed and forbidden couplings of exotic mesons to $\bar{B}B$ and MM systems using various kinds of "rubber band" and "string" models for hadrons.^{12,13}

Decay Widths for Exotic Mesons

If exotic meson masses are below the baryon-antibaryon threshold, it can decay only into multimeson final states. According to the coupling rules discussed exotic mesons couple only weakly to mesons; they must be narrow resonances.

However, there are several competing effects: the suppression due to the selection rules, the availability of many multimeson channels and lots of phase space. Depending upon how strong the selection rules are, the exotic meson resonance may be quite broad.

If, however, exotic meson masses are above baryon-anti-baryon thresholds, the availability of many channels and strong coupling to $B\bar{B}$ systems suggest very broad resonant width.

Masses for Lowest-Lying Exotic Mesons

Any prediction on exotic meson masses must necessarily be strongly model-dependent. Considering mesons as $\bar{q}q$ harmonic oscillators with force constant ω and baryons as three qq harmonic oscillators, Faiman¹⁵ obtains a parametrization of the ρ - A_2 and Δ_δ - Δ_β Regge Trajectories as

$$M_n^2 = 2\mu^2 + (n + 3/2) \omega \sqrt{2} \quad (4a)$$

$$M_n^2 = 3\mu^2 + (n + 6/2) \Omega \sqrt{3} \quad (4b)$$

with

$$\mu^2 = -.5 \text{ GeV}^2, \omega \sqrt{2} = \Omega \sqrt{3} = 1.05 \text{ GeV}^2 \text{ and}$$

where n corresponds to the principal quantum number ($n = 0, 1, 2, \dots$). Similarly considering exotic mesons as $(q q \bar{q} \bar{q})$ or four ω oscillators and two Ω oscillators, he obtains a parametrization for the leading strangeness = 0 exotic meson trajectory as

$$M_{n, n'}^2 = 4\mu^2 + (n + 3/2) 2\omega + (n' + 6/2) \sqrt{2(\omega^2 + \Omega^2)} \quad (4c)$$

Using the parameter values obtained for (4a) and (4b), and $n = n' = 0$, he obtains for the least massive member of the trajectory the mass value of 2075 MeV.

Freund¹⁴ using the expression

$$M_N^2 = a + bN \quad (5)$$

for the mass of an N quark-antiquark body and approximating $M_2^2 \approx m_p^2$ and $M_3^2 = m_\Delta^2$ (1236), predicts the lowest-lying exotic mesons to be at least 1600 MeV in mass. Arndt, et al.¹⁷ place the lower limit for exotic meson mass as 1500 MeV.

Rosner¹⁶ notes that meson-meson and meson-baryon systems if allowed to resonate according to the quark model do so at least once between threshold and a low momentum p_O^* in the center of mass; p_O^* is of the order of 350 MeV/c and 250 MeV/c for MM and MB systems respectively. Rosner has a simple optical interpretation for this: two particles A and B begin forming resonances with one another at a certain well-defined relative distance given by

$$p_O^{AB} (R_A + R_B) = \bar{L} \quad (6)$$

where R_i is the radius of particle i and $\bar{L}$ is some average orbital angular momentum between A and B. Using $p_O^{MB} = 250$ MeV/c, one gets from eq. (6)

$$R_M + R_B = .8f \quad (7)$$

and using $p_O^{MM} = 350$ MeV/c one then gets

$$R_M/R_B \approx 5/9 \quad (8)$$

For low $\bar{L}$ values ($\bar{L} = 1$) and using eqs. (7) and (8), one gets the value $p_0^{\bar{B}\bar{B}} = 200 \text{ MeV/c}$ for $\bar{B}\bar{B}$ systems. On the basis of this model, Rosner predicts for the masses of exotic baryon-antibaryon resonances the mass ranges:

$$\begin{aligned} 2175 \text{ MeV} &\leq M[(\bar{\Delta} N) I=2] \leq 2215 \text{ MeV} \\ 2225 \text{ MeV} &\leq M[(\bar{Y}_1^* N)_{I=3/2}] \leq 2265 \text{ MeV} \end{aligned} \quad (9)$$

C. Duality Diagrams and Allowed Processes for Exotic Meson Production

Rules for Drawing Duality Diagrams

Two component duality, absence of exotic couplings to meson-meson systems and the assumption that s or t exotic channels have zero contributions to the respective scattering amplitudes find a very pictorial representation in terms of duality diagrams for scattering amplitudes. Rosner²⁷ and Harari²⁸ have written down the rules for drawing duality diagrams. These rules are presented below as extensions of rules 1, 2 and 3 given in section B.

Rule 4: If all two-body scattering diagrams are to satisfy duality, they must be planar, i. e., no quark or antiquark lines are allowed to cross each other.

Rule 5: Only non-exotic exchanges are to be considered; in any $B = 0(s, t, u)$ -channel it should be possible to "cut" the diagram into two by cutting a quark and an antiquark line, or in any $B = 1$ channel by cutting three quark or three antiquark lines.

We now consider various mechanisms for exotic meson production and see whether they are allowed or forbidden by the rules for drawing duality diagrams.

D. Production of Strangeness=0, Isospin=2, Exotic Mesons (X_2)

Consider the general production reaction:

$$A + B \rightarrow X_2 + C \quad (10)$$

The initial state $AB = \pi N, KN, \bar{N}N$, will be discussed individually.

(a) N reactions: Reactions for the production of different charge states of $I=2$, exotic meson X_2 are written down below:

$$\begin{aligned} \text{(i)} \quad & \pi^+ + p \rightarrow (n, N^{*0}) + X_2^{++} \\ \text{(ii)} \quad & \pi^+ + p \rightarrow (p, N^{*+}) + X_2^+ \\ \text{(iii)} \quad & \pi^+ + n \rightarrow (n, N^{*0}) + X_2^+ \\ \text{(iv)} \quad & \pi^+ + n \rightarrow (p, N^{*+}) + X_2^0 \\ \text{(v)} \quad & \pi^- + p \rightarrow (p, N^{*+}) + X_2^- \\ \text{(vi)} \quad & \pi^- + n \rightarrow (n, N^{*0}) + X_2^- \\ \text{(vii)} \quad & \pi^- + n \rightarrow (p, N^{*+}) + X_2^{--} \end{aligned} \quad (11)$$

The observed final states in any of the above reactions will depend on possible decay modes for the exotic meson X_2 , for example

$$X_2^{++} \rightarrow \begin{cases} \pi^+\pi^+, \pi^+\pi^+\pi^-\pi^-, \rho^+\pi^+, \text{ etc. (even G states)} \\ \pi^+\pi^+\pi^0, \rho^+\pi^+\pi^0, \text{ etc. (odd G states)} \\ K^+\bar{K}^0\pi^+, \text{ etc.} \\ \Delta^{++}\bar{n}, \Delta^+\bar{\Delta}^-, p\bar{\Delta}^-, \text{ etc.} \\ \Sigma^+\bar{\Lambda}_0\pi^+, \Sigma^+\bar{\Sigma}^-, \text{ etc.} \end{cases} \quad (12)$$

In the reactions considered above, the exotic meson X_2 may be produced either in the backward or forward direction in the center of mass. Fig 3(a) and 3(b) are duality diagrams corresponding to forward and backward production of exotic meson, X_2 , in the reactions

Figure 3(a) 'Forward Production of X_2^{++} '

Figure 3(b) 'Backward Production of X_2^{++} '

Forward production of X_2^{++} as shown in Figure 3(a) involves the exchange of $I = 1, 2, 3$ meson trajectories. Since $I = 2, 3$ exchanges are forbidden, the reaction should be able to proceed through $I = 1$ meson trajectory exchange. However, the duality diagram shows a $q \bar{q} q \bar{q}$ exchange in the t -channel and is, therefore, a forbidden diagram. Clearly forward production of X_2^{++} is not favored by duality diagram rules.

Backward production of X_2^{++} as shown in figure 3(b) proceeds through the exchange of $I = 3/2$ baryon trajectory and is an 'allowed' diagram. Figure 3(c) represents the duality diagram corresponding to the backward production of non-exotic mesons such as ρ, A_1, A_2 , etc. in $\pi^- + p \rightarrow p + (\rho^-, A_1^-, A_2^-, \text{etc.})$.

Figure 3(c) 'Backward Production of ρ^- '

A comparison of figures 3(b) and 3(c) show that both diagrams are allowed and are similar in other ways. Both involve $\bar{\Delta}$ trajectory exchange and apart from small kinematical factors, differ only in the coupling at the lower vertex. Jacob and Weyer²⁹, therefore, argue that the ratio for the production

cross-section for these processes must be of the order:

$$R \approx \frac{g_{\Delta NX_2}}{g_{\Delta NM}}$$

and using symmetry arguments show that $R \approx 1$. It is quite clear that based on the models under consideration, backward production of exotic mesons appears more favored than forward production and in the πN reactions considered, the exotic meson X_2^{++} is produced at a $\bar{\Delta} N$ vertex. We have already argued that an exotic meson will couple more readily to $\bar{B}B$ systems than to MM systems.

Backward Production of Non-Exotic Mesons

TABLE 1

$\sigma(\mu b)$: Cross Section for Backward Processes

$$\pi^- + p \rightarrow p_{\text{forward}} + (\pi^-, \rho^-, A_1^-, A_2^-)$$

P_{lab} GeV/c	$u(\text{beam to } p_{\text{forward}})$ (GeV) ²	π^-	ρ^-	A_1^-	A_2^-
8	$-.40 \leq u \leq +.12$	$1.38 \pm .14$	$1.86 \pm .02$	$.50^{+.05}_{-.03}$	$.50^{+.05}_{-.03}$
16	$-.73 \leq u \leq -.10$	$.24 \pm .03$	$.32 \pm .04$	$.35^{+.30}_{-.28}$	$.21 \pm .07$

Table 1 shows cross-sections for backwards produced π^- , ρ^- , A_1^- , A_2^- as reported by Anderson, et al.⁴² from the missing mass experiment

$$\pi^- + p \rightarrow p_{\text{fwd}} + M_X^- \quad (14)$$

at π^- incident momenta of 8 and 16 GeV/c respectively. However, a streamer chamber version of the above experiment at an incident π^- momentum of 8 GeV/c performed by the Illinois-Argonne Group⁴³ report no convincing evidence for A_1^- or A_2^- production in the reaction

$$\pi^- + p \rightarrow p_{\text{fwd}} + \pi^+ + \pi^+ + \pi^- \quad (15)$$

The statistical significance of their data corresponds to 1000 events per microbarn and they state upper limit for A_1^- A_2^- production as

$$\begin{aligned} \sigma(A_1) &< .58 \mu\text{b} \\ \sigma(A_2) &< .32 \mu\text{b} \end{aligned} \quad (16)$$

However, they report strong backward production of f^0 and ρ^0 in association with forward N^* and Δ 's (1520 MeV and 1670 MeV) according to the reaction:

$$\pi^- + p \rightarrow (N^{*0}, \Delta^0) + (f^0, \rho^0) \quad (17)$$

$$\begin{array}{c} \downarrow \qquad \qquad \downarrow \\ p_{\text{fwd}} \quad \pi^- \quad \pi^+ \pi^- \end{array}$$

Backward production of f^0 and ρ^0 requires $\begin{smallmatrix} B = -1 \\ I = 1/2, 3/2 \end{smallmatrix}$ exchange while A_1^- and A_2^- backward production requires only $\begin{smallmatrix} B = -1 \\ I = 3/2 \end{smallmatrix}$ exchange. The low production upper limit on A_1 and A_2 may be interpreted to mean that $I = 3/2$ baryon trajectory exchange is suppressed relative to $I = 1/2$ baryon trajectory exchange. Such a suppression would affect exotic meson ($I=2$) production in the backward direction, corresponding to reactions shown in equations (11) and (13a), since backward production in all

these reactions involve only $I = 3/2$ baryon trajectory exchange.

One may, in general, conclude from the above discussion that cross sections for the backward productions of exotic mesons in processes considered in equation (11) are of the order of one microbarn.

(b) KN reactions: Non strange exotic mesons may also be produced in KN reactions in association with strange baryons, for example:

The duality diagrams show that a reaction is equivalent to the annihilation of at least one quark-antiquark pair in all channels. Clearly then no K^+p , K^+n can proceed according to the diagram rules since there is no way the λ - or p -quark of K^+ can annihilate with any quark in the incident target nucleon.

'Backward Production
of X_2^{--} ' Fig. 4(a)
'allowed'

'Forward Production
of X_2^{--} ' Fig. 4(b)
'forbidden'

Figures 4(a) and 4(b) correspond to backward and forward production respectively of exotic mesons in the reaction:

Forward production is again forbidden while backward production is favored. Figure 4(c) corresponds to backward production nonexotic mesons (π^- , ρ^- , A_1^- , A_2^- , etc.) in the reaction:

'Forward Production of Non-Exotic Mesons'

Figure 4(c)

Comparison of Figures 4(a) and 4(c) suggests that the ratio of the rates of the processes considered in equations 19(a) and (b), apart from kinematical factors, may be expressed as

$$R = \frac{g_{\Delta N_8 M}^2}{g_{\Delta N_8 M_4}^2} \quad (20)$$

where M_4 indicates an exotic meson with $q \bar{q} \bar{q} \bar{q}$ content. It may be mentioned that processes in equations 19(a) and (b) both involve the exchange of $I = 3/2$ baryon trajectory exchange.

(c) $\bar{N}N$ reactions: The different charge states of the exotic meson X_2 with $I = 2$ and $S = 0$ may also be produced in $\bar{p}p$ and $\bar{p}n$ reactions, for example:

and

Figure 5(a) 'Backward Production of Non-Exotic Mesons'

Figure 5(b)

'Backward Production of X_2^{--} '

Figures 5(a) and 5(b) are duality diagrams corresponding to the respective processes

$$\bar{p} + p \rightarrow \pi^+ + (\pi^-, \rho^-, \text{etc.}) \quad (22a)$$

$$\bar{p} + n \rightarrow \pi^+ + X_2^{--} \quad (22b)$$

The duality diagram corresponding to the production of exotic meson production is forbidden because in the s-channel, there are two quarks and two antiquarks. It may, however, be pointed out that $|\bar{p}\rangle |n\rangle$ couples to isospin equal to 1 state according to isospin addition rules. In fact, the reactions (22a) and (22b) are similar to each other. Both proceed through Δ -exchange and differ only at the lower vertex as shown in figures 5(a) and 5(b). Assuming that ρ and ω trajectories dominate odd-signature amplitudes at high energies and that there are no dibaryon resonances, Lipkin³⁰ derives a new finite energy sum rule which relates the resonance contribution at the lower vertex in figure 5(a) to that in figure 5(b) as

$$\int_0^N A^R(\bar{p} \Delta^-) v^n dv = \frac{1-\epsilon}{1+\epsilon} \int_0^N A^R(\bar{p} \Delta^{++}) v^n dv$$

E. Production of Strange Exotic Mesons.

Some reactions for the production of strange exotic mesons are discussed below.

(a) πN reactions: Strangeness = 1, isospin = 3/2 exotic mesons are produced in the following reactions:

Strangeness = 2 and isospin = 2 exotic mesons are produced in the following reactions:

Both multimeson and baryon-antibaryon decay modes should be accessible. Some possible decay modes are written down below:

$$\begin{aligned}
K_{S=1, I=3/2}^{++} &\rightarrow K^+\pi^+, K^+K^+K^-\pi^+, \text{ etc.} \\
&\rightarrow K^{*+}\pi^+, \text{ etc.} \\
&\rightarrow K^{*+}\pi^+\pi^0, \pi^+\pi^+K^0, \text{ etc.} \\
&\rightarrow \Sigma^-p, \bar{\Sigma}^0p\pi^+, \bar{\Sigma}^0\Delta^{++}, \text{ etc.} \quad (24a)
\end{aligned}$$

and

$$\begin{aligned}
K_{S=2, I=2}^{+++} &\rightarrow K^+K^+\pi^+, K^0K^0\pi^+\pi^+\pi^+, \text{ etc.} \\
&\rightarrow \pi^+\pi^+\pi^+, \pi^+\pi^+\pi^+\pi^0, \text{ etc.} \\
&\rightarrow \bar{\Sigma}^-K^+p, \text{ etc.} \quad (24b)
\end{aligned}$$

(b) KN reactions: Strangeness = 1, isospin = 3/2 exotics may also be produced in KN reactions, for example:

$$\begin{aligned}
\text{(i)} \quad &K^+ + p \rightarrow n + K_{3/2}^{++} \\
\text{(ii)} \quad &K^+ + p \rightarrow p + K_{3/2}^+ \\
\text{(iii)} \quad &K^+ + n \rightarrow n + K_{3/2}^+ \\
\text{(iv)} \quad &K^- + n \rightarrow \Xi^0 + K_{3/2}^- \\
\text{(v)} \quad &K^- + p \rightarrow \Xi^0 + K_{3/2}^0 \quad (25a)
\end{aligned}$$

The corresponding reactions for strangeness = 2, isospin = 2 exotic mesons are:

$$\begin{aligned}
\text{(i)} \quad &K^+ + p \rightarrow \Sigma^- + K_2^{+++} \\
\text{(ii)} \quad &\quad \rightarrow \Sigma^0 + K_2^{++} \\
\text{(iii)} \quad &\quad \rightarrow \Sigma^+ + K_2^+ \\
\text{(iv)} \quad &K^+ + n \rightarrow \Sigma^+ + K_2^0 \\
\text{(v)} \quad &K^+ + n \rightarrow \Sigma^- + K_2^{++} \quad (25b)
\end{aligned}$$

F. Status of Experimental Search for Exotic Mesons

Most past searches for exotic mesons have been in forward production processes and multimeson modes of exotic meson

decay have been studied. Typical upper limits for the production of exotic mesons in forward processes are of the order of 2 - 10 μb . Previous reports of exotic meson resonances in $\rho^-\pi^-$ (1320) and $(K\pi\pi)^{--}$ (1170) were not confirmed by higher statistics experiments. Evidence for an exotic meson with mass around 500 MeV with statistical significance of 3 - 5 standard deviations has been reported by three independent sources^{31,40,41}.

Search for Exotic Mesons in π^-d Reactions

Cohen^{33,34}, et al. report upper limits of 2 - 6 μb for the forward production resonances in $I = 2$ multimeson channels from π^-d reactions at incident π^- momentum of 7 GeV/c. Table 2 gives the details of the individual channels investigated.

TABLE 2

Cross Sections and $I = 2$ Resonance Production
Upper Limit from Cohen^{33,34}

Reaction	total (μb)	$I=2$ Upper Limit (μb)
$\pi^-d \rightarrow p_S p \pi^- \pi^-$ ($G=+$)	462 ± 49	5
$\pi^-d \rightarrow p_S p \pi^- \pi^- \pi^0$ ($G=-$)	383 ± 56	6
$\pi^-d \rightarrow p_S p \pi^- \pi^- + N \pi^0$ ($N \geq 2$)	566 ± 113	6
$\pi^-d \rightarrow p_S p \pi^+ \pi^- \pi^- \pi^-$ ($G=+$)	168 ± 21	2
$\pi^-d \rightarrow p_S p \pi^+ \pi^- \pi^- \pi^- \pi^0$ ($G=-$)	161 ± 28	2
$\pi^-d \rightarrow p_S p \pi^+ \pi^- \pi^- \pi^- + N \pi^0$ ($N \geq 2$)	203 ± 51	6

A resonance at 1320 MeV was reported in $\pi^- \rho^-$ spectrum by Vanderhagen³⁵ in the reaction

$$\pi^- d \rightarrow p_s p \pi^- \pi^- \pi^0 \quad (26)$$

at π^- momentum of 5 GeV/c. However, Katz³⁵, et al. report no significant structure in $\pi^- \rho^-$ spectrum for the above reaction at incident π^- momentum of 7 GeV/c. Summing up all events corresponding to the reaction $\pi^- d \rightarrow p_s p \pi^- \pi^- \pi^0$, for incident π^- momentum of 7.0, 5.1, 3.7 and 3.2 GeV/c, Katz obtains a low-order polynomial plus resonance fit to the experimental data with the resonance center at 1250 ± 30 MeV. However, the event excess (1150 to 1350 MeV) is equivalent to two σ -deviation enhancement above the polynomial fit to $\pi^- \rho^-$ mass spectrum. The upper limit for the production of this resonance is placed at $11 \pm 3 \mu\text{b}$ at an incident π^- momentum of 7 GeV/c.

Evidence for an Exotic Meson around 500 MeV

Abramov⁴⁰, et al. report a resonance enhancement of 3 - 4 σ -deviation statistical significance at mass squared about $.25 (\text{GeV})^2$ in the X^- system for the small momentum transfer process

$$\pi^- d \rightarrow n_s p X^- \quad (27)$$

at incident π^- momentum of 1.47 and 1.24 GeV/c. An upper limit of 80 MeV was placed for the width of the resonance. Table 3 gives the cross section for resonance production in the X^- system.

TABLE 3

Cross Section for Resonance Production in X^- from Reference 40

P_{π^-} GeV/c	Events over background	angular acceptance	M_{X^2} (GeV) ²	$\sigma_{b/str}$
1.47	80 ± 25	$-1 < \cos \theta^* < -.9$	.25	11.5 ± 3.5
1.24	104 ± 27		.26	20.0 ± 5.2

Buhl³¹ et al. studied the process

$$\begin{aligned}
 \pi^- + p &\rightarrow p + X^- \\
 &\quad \downarrow \\
 &\quad (i) \quad \pi^- \pi^0 \\
 &\quad (ii) \quad \pi^- \pi^- \pi^+ \\
 &\quad (iii) \quad \pi^- \pi^- \pi^+ \pi^0
 \end{aligned} \tag{28}$$

where the outgoing proton is fast forward.

Search for Exotic Mesons in K^-d , K^-p Reactions

Strange and non-strange exotic channels are accessible in K^-N reactions. Giacomelli³⁷, et al. conclude from a search of exotic states produced in quasi two-body K^-d reactions at incident K^- momentum of 3 GeV/c that if exotic states of relatively low mass exist and have widths of about 100 MeV their production cross-sections are between one to two orders of magnitude smaller than those for non-exotic resonances of the same strangeness and baryonic number. They report no structure in $K^{*-}(890)\pi^-$, $K^-\pi^-$, $K^-\pi^-\pi^0$, $\bar{K}^0\pi^-\pi^-$ spectrum produced in the reaction

(strangeness = -1) system. A similar missing mass analysis for the X^{--} (strangeness = 0) system off identifiable Σ^+ 's shows no structure in the X^{--} system. They place an upper limit of $10\mu\text{b}$ at the 90% confidence level for exotic meson production in the X^{--} system in reaction (30b).

Stone³² reports on the production of strangeness = 2 exotic mesons in the reactions

$$(i) \quad K^+ + p \rightarrow X_{S=2}^+ + \Sigma^+ \quad (31a)$$

$$(ii) \quad K^+ + p \rightarrow X_{S=2}^{++} + \Lambda^0 \quad (31b)$$

$$(iii) \quad K^+ + p \rightarrow X_{S=2}^{+++} + \Sigma^- \quad (31c)$$

from K^+p bubble chamber experiment at K^+ incident momentum of 12 GeV/c. Some structure around $M \approx 2880$ MeV in the compiled spectrum is reported. An upper limit of $10\mu\text{b}$ is set for the production of $S=2$ exotic mesons in the above reactions.

Exotic Search in $\bar{p}p$ Reactions

French⁴¹ reports a 3 - 5 standard deviation signal in $\pi^+\pi^+$, $\pi^+\pi^-$, $\pi^+\pi^0$ invariant mass plot at .53 GeV from $\bar{p}p$ reactions at 3.6 and 5.7 GeV/c. He looked at mass spectra for three separate groups of reactions and found mass enhancements of 3 - 5 σ statistical significance at .53 GeV for each group; the summed spectra of all the different channels had a 4.7 σ signal at .53 GeV. The width of the resonance was placed at 20 MeV. The reactions investigated are given below:

$$(i) \quad \left. \begin{aligned} \bar{p}p &\rightarrow \pi^+\pi^+\pi^-\pi^-\pi^0 \\ &\rightarrow \pi^+\pi^+\pi^-\pi^- \\ &\rightarrow \pi^+\pi^+\pi^-\pi^-X_0 \end{aligned} \right\} \quad \begin{array}{l} 13023 \text{ events} \\ \\ \end{array} \quad (32a)$$

$$\begin{array}{ll}
 \text{(ii)} & \begin{array}{l} \bar{p}p \rightarrow 3\pi^+ 3\pi^- \\ \rightarrow 3\pi^+ 3\pi^-\pi^0 \end{array} \left. \vphantom{\begin{array}{l} \bar{p}p \rightarrow 3\pi^+ 3\pi^- \\ \rightarrow 3\pi^+ 3\pi^-\pi^0 \end{array}} \right\} 12807 \text{ events}
 \end{array} \quad (32b)$$

$$\begin{array}{ll}
 \text{(iii)} & \begin{array}{l} \bar{p}p \rightarrow p\bar{n}\pi^+\pi^-\pi^- \\ \rightarrow \bar{p}n\pi^-\pi^+\pi^+ \\ \rightarrow \bar{K}K\pi\pi (m=3,4,5) \end{array} \left. \vphantom{\begin{array}{l} \bar{p}p \rightarrow p\bar{n}\pi^+\pi^-\pi^- \\ \rightarrow \bar{p}n\pi^-\pi^+\pi^+ \\ \rightarrow \bar{K}K\pi\pi (m=3,4,5) \end{array}} \right\} 14266 \text{ events}
 \end{array} \quad (32c)$$

It may be mentioned that a 3.7σ signal at .53 GeV has also been observed in $\pi^+\pi^-$ and $\pi^+\pi^0$ mass spectra from the Harvard³⁸ data on the reaction

$$\pi^- + p \rightarrow p\pi^+\pi^-\pi^-\pi^0 \quad (33)$$

Evidence for Exotic State at 1500 MeV

Arndt³⁶, et al. argue that $I^G(J^P)=2^+(2^+)$ is the most likely state in which to find an exotic meson. Palou and Ynderain, using the Froissart-Gribov representation to calculate the $\pi\pi$ low energy parameters, found that the effective range and shape parameter for the $I^G_{J^P} = 2^+(2^+)$ state have the same sign and approximately the same magnitude as for the $I^G_{J^P} = 0^+(2^+)$ state. Arndt, et al. attribute this surprising result to a $I^G_{J^P} = 2^+(2^+)$ resonance at ~ 1500 MeV.

Backwards-Produced Strange Exotic Meson in π^+p Interactions at 15 GeV/c

C. Baltay⁴⁵, et al. have carried out a search for strange ($I = 3/2$) exotic mesons using the reaction

$$\pi^+ + p \rightarrow \Lambda(\text{forward}) + K^{++} \quad (34)$$

where the incident π^+ momentum is 15 GeV/c. The strange,

exotic system K^{++} is produced backwards, opposite a forward produced Λ^0 that is detected in the SLAC 82" bubble chamber. The K^{++} is produced at a $\Sigma^- p$ vertex and is therefore important according to the Rosner prescription²³. Furthermore, the K^{++} system is the missing mass off of the forward detected Λ^0 . The authors place 95% confidence level upper limits of $1.7\mu\text{b}$ on the production of exotic resonances with masses $< 2.8\text{ GeV}$ and $2.6\mu\text{b}$ with masses between 2.8 GeV and 4 GeV .

G. Two-Component Duality and the Absence of Exotic Resonances

In this section the salient features of two-component duality¹⁸⁻²² are outlined. The success of the model in the case of meson-meson and meson-baryon scattering is discussed in relation to the model's failure in the case of baryon-antibaryon scattering.

The Usual Duality Framework

The original duality conjecture due to Dolen, Horn, and Schmid¹⁸ may be stated as follows:

"Regge poles and resonances are two alternative descriptions for scattering amplitudes." The two descriptions are equivalent and complete by themselves; the specific usefulness of one or the other picture depends on the physical simplicity of the case under study. Thus for example, the Regge picture is more simple at high energies while the resonance picture is more convenient at low energies. The two pictures are related to each other through the finite energy sum rules (FESR) relations:

$$\int_0^N \text{Im} v^n \text{Im} A(v, t) = \sum_i \beta_i(t) \frac{\alpha_i(t) + n + 1}{\alpha_i(t) + n + 1} \quad (35)$$

where α_i are the Regge slope parameters and β_i are the Regge residue functions and $v = (s - u)/4M$. N is an arbitrary parameter and $A(v, t)$ is the invariant two body scattering amplitude.

The resonance dominance assumption which states that the imaginary part of the scattering amplitude is dominated by s -channel resonances in a localized sense (a resonance contribution dominates in the immediate vicinity of its energy), converts the FESR relation into a powerful tool in strong interactions.

The two-component duality model due to Harari and Freund^{18d, 20a, b} may be stated as follows:

"Two-body hadronic scattering amplitudes can be described as the sum of two components. In the s -channel picture, the first component is the sum of s -channel resonance contributions while the second component is made of the non-resonant background. In the t -channel picture, the first component is made up from the sum of t -channel Regge trajectory contributions while the second component is the Pomeranchuk trajectory contribution alone."

The real essence of the Harari-Freund conjecture is the unique association of the Pomeranchuk contribution with the s -channel non-resonant background. This also means that the sum of the s -channel resonances must then be completely balanced by the sum of the t -channel Regge contributions. In the light of the two-component model, the FESR relation is now written as

$$\int_0^N v^n \text{Im } A_B(v, t) dv = \beta_p(t) \frac{\alpha_p(t) + n + 1}{\alpha_p(t) + n + 1} \quad (36a)$$

and

$$\int_0^N v^n \text{Im } A_{\text{res}}(v, t) dv = \sum_{i \neq p} \beta_i(t) \frac{\alpha_i(t) + n + 1}{\alpha_i(t) + n + 1} \quad (36b)$$

where A_B is the non-resonant background term and A_{res} includes the s-channel resonances. Two very important results follow from this:

(a) Amplitudes in which the Pomeron is not allowed (e.g. charge or strangeness exchange reactions) are entirely described as a sum of s-channel resonances.

(b) Amplitudes for which no s-channel resonances are observed (e.g. K^+p , pp , pn , $\pi^+\pi^+$, etc.) are completely described by the Pomeron exchange part of the amplitude. Therefore, at high energies the total cross-sections for such processes must be flat as opposed to the total cross-sections for such processes as K^-p , π^+n , π^-n , $\bar{p}p$, etc., which allow s-channel resonances and show an energy-dependence in their cross-sections.

Absence of Exotic Resonances and the Duality Bootstrap

The duality framework and the assumption that neither exotic resonances or trajectories exist, lead to very powerful constraints and relations among the allowed Regge trajectory parameters. They lead to idea of exchange degeneracy and the correct singlet-octet mixing angles in the case of meson-meson scattering. However, the beautiful self-consistency of the model breaks down in the case baryon-antibaryon scattering. Let us first consider various cases of meson-meson scattering.

(a) $\pi\pi$ scattering: In the case of $\pi\pi$ scattering, the allowed trajectories have $I = 0$ and 1 and $G = +$. Hence only the ρ and f (or P') contribute. The ρ contributes with opposite signatures in the case of $\pi^+\pi^-$ and $\pi^+\pi^+$ scattering. Thus

$$\begin{aligned} A_{\pi^+\pi^-} &= \rho_{\pi\pi} + f_{\pi\pi} \\ A_{\pi^+\pi^+} &= -\rho_{\pi\pi} + f_{\pi\pi} \end{aligned} \quad (37)$$

Since we require that $A_{\pi^+\pi^+} = 0$ (s -channel $I=2$) we get

$$\rho_{\pi\pi} = f_{\pi\pi} \quad (38)$$

which implies

$$\begin{aligned} \alpha_\rho(t) &= \alpha_f(t) \\ \beta_{\rho\pi\pi}(t) &= \beta_{f\pi\pi}(t) \end{aligned} \quad (39)$$

Therefore, trajectories which couple to pions must occur as degenerate isoscalar-isovector pairs with even G-parity.

(b) $K\pi$ scattering: The case for $K\pi$ scattering is similar to $\pi\pi$ scattering. We may write

$$\begin{aligned} A_{K^+\pi^+} &= f_{K\pi} - \rho_{K\pi} \\ A_{K^+\pi^-} &= f_{K\pi} + \rho_{K\pi} \end{aligned} \quad (40)$$

Requiring the exotic amplitude $A_{K^+\pi^+} = 0$, we get the new relation

$$\beta_{\rho K\pi}(t) = \beta_{f K\pi}(t) \quad (41)$$

(c) KK scattering: Here the allowed trajectories have $I=0,1$ as before but both G-parity states are allowed as K has no

G-parity. There are now four independent amplitudes in this case:

$$\begin{aligned}
 A_{K^+K^-} &= f_{KK} + A_{2KK} + \omega_{KK} + \rho_{KK} \\
 A_{K^+K^+} &= f_{KK} + A_{2KK} - \omega_{KK} - \rho_{KK} \\
 A_{K^+K^0} &= f_{KK} - A_{2KK} - \omega_{KK} + \rho_{KK} \\
 A_{K^+\overline{K}^0} &= f_{KK} - A_{2KK} + \omega_{KK} - \rho_{KK}
 \end{aligned} \tag{42}$$

Again placing the exotic amplitudes $A_{K^+K^+} = A_{K^+K^0} = 0$, we get

$$\begin{aligned}
 \alpha_{A_2} &= \alpha_\rho, \quad \alpha_f = \alpha_\omega \\
 \beta_{fKK} &= \beta_{\omega KK}, \quad \beta_{A_2KK} = \beta_{\rho KK}
 \end{aligned} \tag{43}$$

Using factorization for the Regge residue functions $\beta_{\rho K\pi}$, $\beta_{fK\pi}$, β_{fKK} , and $\beta_{\rho KK}$, we may derive the relations:

$$\beta_{fKK} \cdot \beta_{f\pi\pi} = \beta_{fK\pi}^2, \quad \beta_{\rho KK} \beta_{\rho\pi\pi} = \beta_{\rho K\pi}^2 \tag{44a}$$

But since $\beta_{\rho K\pi} = \beta_{fK\pi}$ and $\beta_{f\pi\pi} = \beta_{\rho\pi\pi}$, we get $\beta_{fKK} = \beta_{\rho KK}$ and therefore

$$\beta_{A_2KK} = \beta_{\rho KK} = \beta_{fKK} = \beta_{\omega KK} \tag{44b}$$

Trajectories which couple to both kaons and pions must occur as degenerate quartets.

However, this quartet degeneracy gives rise to a vanishing amplitude for the process $K^+\overline{K}^0$. This can be avoided if we assume there exists an extra pair of degenerate trajectories which couple to kaons and not to pions. A pair is needed so

that they will contribute with opposite phases in K^+K^0 and K^+K^+ scattering, giving zero for these amplitudes as before. If these were allowed to couple to pions, we would get sextet degeneracy instead. This would not solve the case of the vanishing KK amplitude. The existence of φ and f' which couple predominantly to KK but not too strongly to pions, is a triumph of the model.

The quartet degeneracy of ρ , A_2 , ω and f with respect to the Regge trajectory parameters holds approximately as seen from fits to available experimental data. The φ and f' trajectories are also degenerate approximately with respect to each other but not with respect to the above quartet. It may be mentioned that φ and f' are both mixtures of SU(3) singlet and octet representations. Requiring that these specific combinations of singlet and octet give rise to zero coupling with pions gives $\tan^2 \theta = 1/2$ for singlet-octet mixing angle θ as shown by Chiu and Finkelstein.²² The relation is identical to mixing relation obtained from the quark model and is considered to be one of the major triumphs of the Harari-Freund conjecture. Indeed it is no minor success that one can arrive at such a set of consistent relations for the case of meson-meson scattering.

The Baryon-Antibaryon Catastrophe

If we consider baryon-baryon reactions, we find that all trajectories coupling to baryons must occur in degenerate pairs of opposite charge conjugation. This follows from the fact that all BB reactions are exotic in the s-channel (baryon

number = 2) so that all trajectories allowed in the t-channel must cancel in pairs to give vanishing resonance amplitudes in the s-channel.

Baryon-antibaryon reactions may be divided into three basic groups depending on whether octets or decuplets are involved. Thus we have (a) octet-octet or B_8 - B_8 scattering, (b) octet-decuplet or B_8 - Δ_{10} scattering and (c) decuplet-decuplet or Δ_{10} - $\bar{\Delta}_{10}$ scattering. Let us first consider octet-octet baryon-antibaryon scattering. The situation is very similar to octet-octet meson-meson scattering. $\Sigma\bar{\Sigma}$, $\Sigma\bar{N}$, and $N\bar{N}$ in BB case correspond respectively to $\pi\pi$, $K\pi$, $K\bar{K}$ in the case of meson-meson scattering. However, in the case of $\Sigma\bar{\Sigma}$ scattering as opposed to $\pi\pi$ scattering, since the baryons are not definite G-parity state, all four trajectories are allowed. We write down various s-channel scattering amplitudes below:

$$\begin{aligned}
 A_{\Sigma^+\Sigma^+} &= \rho_{\Sigma\Sigma} + A_{2\Sigma\Sigma} + f_{\Sigma\Sigma} + \omega_{\Sigma\Sigma} \\
 0 = A_{\Sigma^+\Sigma^+} &= -\rho_{\Sigma\Sigma} + A_{2\Sigma\Sigma} + f_{\Sigma\Sigma} - \omega_{\Sigma\Sigma} \\
 0 = A_{\Sigma^+\Sigma^-} &= \rho_{\Sigma\Sigma} - A_{2\Sigma\Sigma} + f_{\Sigma\Sigma} - \omega_{\Sigma\Sigma} \\
 0 = A_{\Sigma^+\Sigma^-} &= -\rho_{\Sigma\Sigma} - A_{2\Sigma\Sigma} + f_{\Sigma\Sigma} + \omega_{\Sigma\Sigma} \\
 0 = A_{\Sigma^+\Sigma^0} &= \rho_{\Sigma\Sigma} + A_{2\Sigma\Sigma} - f_{\Sigma\Sigma} - \omega_{\Sigma\Sigma} \\
 A_{\Sigma^+\Sigma^0} &= -\rho_{\Sigma\Sigma} + A_{2\Sigma\Sigma} - f_{\Sigma\Sigma} + \omega_{\Sigma\Sigma} \quad (45)
 \end{aligned}$$

Requiring all exotic amplitudes to be equal to zero, we find that the four trajectories satisfy strong degeneracy. We have

$$\rho_{\Sigma\Sigma} = A_{2\Sigma\Sigma} = f_{\Sigma\Sigma} = \omega_{\Sigma\Sigma} \quad (46)$$

A similar set of equations may be written down for the $\Sigma\bar{N}$ scattering. Again equating all exotic s-channel amplitudes to be zero, that is

$$A_{\Sigma^+p} = A_{\Sigma^+n} = A_{\Sigma^+\bar{n}} = 0$$

we get four-fold degeneracy, expressed as

$$\rho_{\Sigma N} = A_{2\Sigma N} = f_{\Sigma N} = \omega_{\Sigma N} \quad (47)$$

There are four s-channel amplitudes for NN scattering. The corresponding Regge contributions may be written down as

$$\begin{aligned} A_{p\bar{p}} &= \rho_{NN} + A_{2NN} + \omega_{NN} + f_{NN} \\ 0 &= A_{pp} = -\rho_{NN} + A_{2NN} - \omega_{NN} + f_{NN} \\ 0 &= A_{pn} = \rho_{NN} - A_{2NN} - \omega_{NN} + f_{NN} \\ A_{p\bar{n}} &= -\rho_{NN} - A_{2NN} + \omega_{NN} - f_{NN} \end{aligned} \quad (48)$$

there are two exotic amplitudes in this case. We get pairwise degeneracy.

$$\rho_{NN} = A_{2NN}, \quad \omega_{NN} = f_{NN} \quad (49)$$

However, if now assume factorization for Regge residues, and use the degeneracy relations from $\Sigma\Sigma$ and ΣN scattering, we also get quartet degeneracy for trajectories coupling to $N\bar{N}$.

The trouble with the above scheme may now be pointed out.

The quartet degeneracy relations give vanishing amplitudes for $\Sigma^+\bar{\Sigma}^0$ and $p\bar{n}$ scattering. This is a failure of the model. One may save the model yet by assuming pairs of degenerate trajec-

tories which couple only to $\Sigma\Sigma$ and NN separately. These trajectories are peculiar and have never been used in fits to experimental data.

Global Duality, Absence of Exotics and the $B\bar{B}$ Catastrophe

Lipkin²⁵ has shown that global duality and the absence of exotic assumption are sufficient conditions to lead to the decoupling of the isovector trajectories for $\Delta\bar{\Delta}$ scattering and for the case of all $B\bar{B}$ charge exchange reactions with exotic s-channel quantum numbers. Lipkin makes the following argument. Assumptions required: (a) FESR may be written down for the scattering amplitudes. (b) No $I \geq 2$ trajectories may be exchanged in the t-channel. (c) If both s and u channel quantum numbers are exotic the left hand side of FESR will be equal to zero. Consider the charge exchange reaction

The corresponding u-channel reaction is

The t-channel reaction is

Fig. 6

Both the s-channel and u-channel quantum numbers are exotic; hence the l.h.s. of FESR is equal to zero. Now we may write FESR separately for amplitudes even or odd under crossing ($s \leftrightarrow u$ or $v \leftrightarrow -v$). Only even(odd) signature trajectories contribute to amplitudes even(odd) under crossing.^{18c} Therefore

$$\int_0^N v^n \operatorname{Im} A^{(-)}(v, t) dv = \beta_{\rho}^{\Delta\bar{\Delta}} \frac{N \alpha_{\rho}(t) + n + 1}{\alpha_{\rho}(t) + n + 1} \quad (51a)$$

$$\int_0^N v^n \operatorname{Im} A^{(+)}(v, t) dv = \beta_{A_2}^{\Delta\bar{\Delta}} \frac{N \alpha_{A_2}(t) + n + 1}{\alpha_{A_2}(t) + n + 1} \quad (51b)$$

Since the left hand side is zero we get

$$\beta_{\rho}^{\Delta\bar{\Delta}} = \beta_{A_2}^{\Delta\bar{\Delta}} = 0 \quad (52)$$

Both ρ and A_2 are decoupled from $\Delta\bar{\Delta}$ individually. The scattering amplitude for the process $\Delta\bar{\Delta} \rightarrow \Delta\bar{\Delta}$ therefore vanishes. The same argument is valid for the following charge exchange reactions with exotic s and u channel quantum numbers.

$$\begin{aligned} p + \bar{\Delta}^- &\rightarrow \Delta^{++} + \bar{\Delta}^0 \\ \Delta^+ + \bar{\Delta}^- &\rightarrow \Delta^{++} + \bar{n} \\ p + \bar{\Delta}^- &\rightarrow \Delta^{++} + \bar{n} \\ p + \bar{\Sigma}^- &\rightarrow \Delta^{++} + \bar{\lambda} \\ p + \bar{\Xi}^- &\rightarrow \Delta^{++} + \bar{\Xi}^0 \\ p + \bar{\Xi}^0 &\rightarrow n + \bar{\Xi}^- \end{aligned} \quad (53)$$

If we now consider $\Delta^+ \bar{\Delta}^- \rightarrow \Delta^+ \bar{\Delta}^-$, we find that both isoscalar and isovector exchanges are allowed. However, we have already seen that isovector trajectories do not couple to $\Delta\bar{\Delta}$,

and hence we have only isoscalar exchange to contend with. Here we have to make Harari-Freund conjecture about the separation of the Pomeron and background parts from both sides of FESR. Assuming this, we have the same problem of the decoupling of the isoscalar trajectories from $\Delta\bar{\Delta}$ as in the case of the isovector trajectories.

H. SU(3), Crossing Symmetry and Absence of Exotic Resonances

According to the crossing symmetry for invariant amplitudes, the invariant amplitude is assumed to be an analytic function of the Lorentz invariant s , t , and u such that when the amplitude is analytically continued to values of s , t , and u corresponding to the physical process in one of the cross-channels, it is just the amplitude for that crossed-channel. If for example we define²¹

$$\begin{aligned} A + B &\rightarrow C + D \text{ (s-channel)} \\ A + \bar{C} &\rightarrow \bar{B} + D \text{ (t-channel)} \\ A + \bar{D} &\rightarrow \bar{B} + C \text{ (u-channel)} \end{aligned} \quad (54)$$

Then the crossing symmetry relation is

$$\langle CD | m(s, t, u) | AB \rangle = \langle \bar{B}D | m(t, s, u) | A\bar{C} \rangle = \langle \bar{B}C | m(u, s, t) | A\bar{D} \rangle \quad (55)$$

If the S-matrix is invariant under certain group transformations, then the scattering amplitude may be expanded into eigenamplitudes of that group. If the S-matrix is invariant under SU(3) transformation, then the scattering amplitude may

be expanded into eigenamplitudes $A_s(I)$, $A_t(I)$ or $A_u(I)$, depending on the channel in which the expansion is performed. I labels the $SU(3)$ representations. If crossing symmetry is to hold, then the eigenamplitudes in one channel must be linearly related to the eigenamplitudes in other channels. Defining the s-t and s-u crossing matrices as X_{st} and X_{su} , we get

$$A_s(I) = \sum_{I'} (X_{st})_{II'} A_t(I') \quad (56a)$$

$$A_s(I) = \sum_{I'} (X_{su})_{II'} A_u(I') \quad (56b)$$

We now write down the various $SU(3)$ eigenamplitudes involved in octet-octet, octet-decuplet and decuplet-decuplet scattering from the decompositions

$$\underline{8} \otimes \underline{8} = \underline{1} \oplus \underline{8}_F \oplus \underline{8}_D \oplus \underline{10} \oplus \underline{\overline{10}} \oplus \underline{27} \quad (57)$$

$$\underline{8} \otimes \underline{\overline{10}} = \underline{8} \oplus \underline{\overline{10}} \oplus \underline{27} \oplus \underline{35} \quad (58)$$

$$\underline{10} \otimes \underline{\overline{10}} = \underline{1} \oplus \underline{8} \oplus \underline{27} \oplus \underline{64} \quad (59)$$

Meson-Meson Scattering²⁴

In this case for octet-octet scattering we have five independent amplitudes in both the s and t channels. They are $A(\underline{1})$, $A(\underline{8}_D)$, $A(\underline{8}_F)$, $A(\underline{10}, \underline{\overline{10}})$ and $A(\underline{27})$. Charge conjugation invariance makes $A(\underline{10})$ and $A(\underline{\overline{10}})$ equal. Absence of exotic resonances or trajectories gives the relation

$$A_t(\underline{10}) = A_t(\underline{27}) = 0$$

$$\begin{aligned}
0 = A_S(\underline{10}) &= X(\underline{10}, \underline{1}) A_t(\underline{1}) + X(\underline{10}, \underline{8}_F) A_t(\underline{8}_F) \\
&\quad + X(\underline{10}, \underline{8}_D) A_t(\underline{8}_D) \\
0 = A_S(\underline{27}) &= X(\underline{27}, \underline{1}) A_t(\underline{1}) + X(\underline{27}, \underline{8}_F) A_t(\underline{8}_F) \\
&\quad + X(\underline{27}, \underline{8}_D) A_t(\underline{8}_D) \quad (60)
\end{aligned}$$

We have two homogeneous linear relations between three SU(3) t-channel amplitudes. They must all be related to each other within a multiplicative factor. We may express this as

$$A_t(\underline{1}) = \alpha A_t(\underline{8}_F) = \beta A_t(\underline{8}_D) \quad (61)$$

where α and β are some parameters related to the s-t crossing matrix. This actually is a powerful result. It means that all processes such as

$$\begin{aligned}
\pi K &\rightarrow \pi K \\
K\eta &\rightarrow K\eta \\
\pi\pi &\rightarrow K\bar{K}
\end{aligned} \quad (62)$$

are related to each other by a single parameter.

Another very important result that we have automatically obtained is the tacit requirement that t-channel trajectories occur as SU(3) singlet and octet. If for example we set $A(\underline{1}) = 0$, we now have two linear homogeneous relations between two t-channel amplitudes, which can be satisfied only if both of them vanish separately.

Baryon-Antibaryon Scattering

For octet-octet scattering, only F-type of coupling is allowed. Therefore, there are two independent amplitudes in

the t-channel, namely $A_t(\underline{1})$ and $A_t(\underline{8}_f)$. The same is true for allowed s-channel amplitudes. Requiring that $A_s(\underline{10})$ and $A_s(\underline{27})$ vanish gives two linear homogeneous relations between two t-channel amplitudes which again gives the trivial result that all $B\bar{B}$ octet-octet amplitudes vanish separately.

For octet-decuplet scattering for example, $p\bar{\Delta} \rightarrow p\bar{\Delta}$, we allow only one amplitude in the s-channel, i.e. $A_s(8)$ while there are two amplitudes in the t-channel namely $A_t(\underline{1})$ and $A_t(\underline{8}_f)$. Requiring $A_s(\underline{10})$ and $A_s(\underline{27})$ and $A_s(\underline{35})$ to be zero gives the trivial result that all amplitudes be equal to zero.

For decuplet-decuplet scattering for example $\Delta\bar{\Delta} \rightarrow \Delta\bar{\Delta}$, we have a similar situation. We have two allowed amplitudes in the t-channel namely $A_t(\underline{1})$ and $A_t(\underline{8}_f)$ and the vanishing of the exotic amplitudes $A_s(\underline{27})$ and $A_s(\underline{64})$. This can be satisfied only if all amplitudes vanish.

Physics Justification for an Exotic Meson Search Experiment

Two-component duality and the absence of exotic mesons gives consistent results in the case of meson-meson scattering. The result that meson trajectories occur as isoscalar and isovector nonets and the $\omega - \phi$ and $f - f'$ mixing angle are in direct agreement with quark model predictions. However, as shown in the preceeding sections, the two-component duality assumption together with no exotics assumption leads to trivial results in the case of baryon-antibaryon scattering. It gives vanishing amplitudes for baryon-antibaryon scattering.

Rosner²³ suggest a scheme to salvage two-component duality. He makes the assumption that exotic mesons exist and couple only to baryon-antibaryons and the couplings to meson channels is still zero. This way the meson-meson scattering argument made above remains intact and valid as before. However, now a consistent solution exists for the baryon-antibaryon case. The most obvious place then to look for exotic mesons is $B\bar{B}$ scattering. $N\bar{N}$ has no exotic channel. $\Delta\bar{\Delta}$ is not easily accesible to experiment. This leaves $N\bar{\Delta}$ as the potential candidate. There are no $\bar{\Delta}$ beams. Hence virtual production of $\bar{\Delta}$ at baryon vertices is the ideal place to look for exotic mesons.

In this thesis we report the experimental results for exotic meson research using the reaction

$$\pi^+ + p \rightarrow n_{\text{fwd}} + X_{I=2}^{++} \quad (63)$$

The incident momentum is 8.4 GeV/c. The exotic ($I = 2$) system is produced at $p\bar{\Delta}$ vertex as shown in the figure.

However, in the experiment reported in this thesis, the dominant accesible channels for the decay of the X^{++} system are in fact $\pi^+\pi^+$, $\pi^+\pi^+\pi^-\pi^-$, and $\pi^+\pi^+\pi^+\pi^-\pi^-$. The only BB channel accesible is $\pi^+\pi^+\pi^-$. But there is too little phase space for decay into this channel and hence this decay mode is suppressed. There are a very few candidate events in this channel.

It may be pointed out that Kugler²⁶ argues that if we abandon the factorization hypothesis for Regge residues, the

problem with octet-octet $B\bar{B}$ scattering goes away. He believes that duality result that $\Delta\bar{\Delta}$ scattering amplitude vanishes is essentially valid. The reaction rate for such processes must be vanishingly small. Another solution of course is to abandon duality completely. The experimental discovery of exotics would therefore lead to serious remodeling for two-component duality. There are other arguments which support duality arguments even if exotics do not exist.

REFERENCES

Quark Model

1. Kokkedee, J. J., The Quark Model. New York, Amsterdam:
W. A. Benjamin,
2. Lichtenberg, D. B., Unitary Symmetry and Elementary
Particles. Academic Press
3. Lipkin, H. J., Quark Model for Pedestrians.

Review Articles

4. Rosner, J. L., "Review of Exotic Mesons", Experimental
Meson Spectroscopy, edited by Charles Baltay and Arthur
H. Rosenfeld. N. Y., London: Columbia Press, 1970, p. 499.
5. Cohen, D., Review of Exotic Mesons. Columbia University,
Print 73-36-49.-
6. Brabson, B. B., Exotics, Baryon Exchange and Charm.
William and Mary, Conference Proceedings on Meson
Spectroscopy.

Selection Rules for Hadron Couplings

7. Mandula, J., et al., Phy. Rev. Letters 22 (1969), 1147.
8. Zweig, G., CERN Report, CERN-TH-402 (1964).
9. Horn, D., Lipkin, H. J., Meshkov, S., Phy. Rev. Letters
17 (1966), 1200.
10. Freund, P. G. O., Waltz, R., Rosner, S. J., Nucl. Phys.
B13 (1969), 237.
11. Rosner, J. L., "Sharp States in the Spectrum of Low
Lying Mesons," Phys. Letters, 33B (1970), 493.
12. Mandelstam, S., Phys. Rev. D1 (1970), 1720.

13. Frye, G., Lee, C. W., Susskind, L., Nuovo Cimento, 69A (1970), 497.
14. Freund, P. G. O., Baryon Resonances-73, Purdue University Conference Report.

Masses of Exotic Mesons

15. Faiman, D., Baryon Resonances - 73, Purdue University Conference Report.
16. Rosner, J. L., Compulsory Resonance Formation, Print 72-1985, Minnesota.
17. Arndt, R. A., and Roper, L. D., Virginia Tech., Print 74-0936, 1974.

Duality

18. Dolen, R., Horn, D., and Schmid, C., "Finite Energy Sum Rules and their Application to N Charge Exchange", Phy. Rev. 166 (1968), 1768.
- 18b. Dolen, R., and Horn, D., "Finite Energy Sum Rules--Use and Interpretation", Proceedings of the Schlading School, 1969, p. 125.
- 18c. Jackson, J. D., "Models for High Energy Processes", Rev. Mod. Phy., Vol 42, No. 1, p. 35.
- 18d. Freund, P. G. O., "Finite Energy Sum Rules and Bootstraps", Phys. Rev. Letters, 20 (1968), 235.
- 19a. Harari, H., "Duality and Hadron Dynamics", Print 70-280 Weizman Institute of Science, Rehovot, Israel.
- 19b. Jacob, M., "Duality in Strong Interactions", Proceedings of the Schlading School, 1969, p. 214.

- 19c. Schmid, C., "Duality and Exchange Degeneracy", Subnuclear Phenomenon, Proceedings of the Erice School, 1969, p. 108.
- 19d. Kugler, M., "Duality", Proceedings of the Schlading School, 1970, p. 443.
- 20a. Harari, H., "Pomeranchuk Trajectory and its Relation to Low Energy Scattering Amplitudes", Phy. Rev. Letters 20 (1968), 1395.
- 20b. Harari, H., and Zarmi, Y., "Duality and Pomeranchuk Singularity", Phys. Rev. 187 (1969), 187.
- 21. Rebbi, C., Slansky, R., "SU(2) and SU(3) Crossing Matrix", Rev. Mod. Phys., Vol. 42, No. 1, p. 35.
- 22. Chiu, and Finkelstein, , Phys. Letters 27B, (1968), 576.

Existence of Exotic Mesons and $\bar{B}B$ Catastrophe

- 23. Rosner, J. L., "Possibility of Baryon-Antibaryon Enhancements with Unusual Quantum Numbers", Phy. Rev. Letters 21 (1968), 950.
- 24. Lipkin, H. J., "Exchange Degeneracy, Flat Total Cross Sections and the Absence of Exotic Resonances", Nucl. Phys. B9 (1969), 349.
- 25. Lipkin, H. J., "A No-Go Theorem for Duality and Baryon-Antibaryon Scattering", Phys. Letters 32B (1970), 301.
- 26. Kugler, M., " $\bar{B}B$ Reactions, Duality and Factorization", Phys. Letters 32B (1970), 107.

Exotic Meson Production and Duality Diagram Constraints

- 27. Harari, H., "Duality Diagrams", Phy. Rev. Letters 22 (1969), 562.

28. Rosner, J. L., "Graphical Form of Duality", Phys. Rev. Letters 22 (1969), 689.
29. Jacob, M. and Weyers, J., "In Search of Exotic States", 69A (1970), 521, 70A (1970), 285(E).
30. Lipkin, H. J., "Experimental Tests for Exotic States in Baryon-Antibaryon System", Phys. Rev. D7 (1973), 2262.

Experimental Data on Exotic Meson Production

31. Buhl, T., Cline, D., and Terrel, R., "Search for Exotic Mesons Produced by Baryon Exchange and $\bar{B}B$ Coupling to Exotic Channels," Nucl. Phys. B37 (1972), 421.
32. Stone, S. L., "Strange Particle and B^- Meson Production in 12.7 GeV/c K^+p Interactions", Ph.D. Thesis, Rochester University, 1972.
33. Cohen, D. H., "Study of π^- Interactions in Deuterium at 7 GeV/c", Ph.D. Thesis, University of Rochester, 1973.
34. Cohen, D. H., Ferbel, T., Slattery, P., and Werner, B., "Search for $I=2$ Multipion Resonances", Nucl. Phys. B53 (1973), 1.
35. Katz, W. H., Ferbel, T., et al., "Searches for $\pi^- \rho^-$ Resonances Produced in 7 GeV/c $\pi^- d$ Interactions", Phys. Letters 31B (1970), 329.
36. Arndt, R. A., and Roper, L. D., "Possibility of an Exotic $I=2$ Non-strange Meson", Virginia Tech., Print 74-0936.
37. Giacomelli, G., et al., "Search for Exotic Resonances in $K^- d$ Interactions at 3 GeV/c", Phys. Letters 33B (1970), 373.
38. Sistern, L. K., et al., Nucl. Phys. B48 (1972), 493.

39. Davis, R., et al., "Search for Exotic States Produced in K^-d Interactions at 5.5 GeV/c", University of Kansas, Lawrence and Argonne National Lab., Print 74.
40. Abramov, B. M., et al., "New Data on Existence of Peak at $M_X \approx 0.5$ GeV in Missing Mass Spectrum of $\pi^-d \rightarrow p + N_S + X$ ", Institute for Theoretical and Experimental Physics, Print 885, 1974.
41. French, B. R., "Experimental Data Suggestive of Low Cross Section Small Width Effects in $\bar{p}p$ Interactions", submitted to 2nd Aix-en-Provence Conference on Elementary Particles (1973).
42. Anderson, E. W., et al., " π^-p Elastic Scattering Near 180° at 8 and 16 GeV/c", Phys. Rev. Letters 20 (1968), 1529; "Observation of a Backward Peak in $\pi^-p \rightarrow p\rho^-$ at 8 and 16 GeV/c", Phys. Rev. Letters 22 (1969), 102; "Observation of A_1^- , A_2^- and higher Mass Bosons Produced Near 180° in $\pi^-p \rightarrow p$ (Missing Mass) $^-$ at 16 GeV/c", Phys. Rev. Letters 22 (1969), 1390.
43. Abashian, A., et al., "Search for Backward A_1 Production in π^-p Reactions at 8 GeV/c", University of Illinois, Urbana, Argonne National Labs, and Iowa State, Print COO-1195-1974.
44. Treille, D., " $\pi^\pm p \rightarrow BX^\pm$ at 9 and 12 GeV/c",
 $\quad \quad \quad \hookrightarrow L, p_f, \lambda, \Sigma^+, N^{*+}$
 CERN-ETH-KARLSRUHE-SACLAY, Private Communication.
45. Baltay, C., et al., "Search for Exotics Produced Opposite a Forward Λ in 15 GeV/c πp Interactions", Columbia University Report 75-0254.

CHAPTER II

DESCRIPTION OF THE EXPERIMENT

A. Experimental Layout

In this experiment, we have studied the following reactions:

$$\pi^+ p \rightarrow X^{++} + \left\{ \begin{array}{c} n \\ \bar{n} \\ K_L^0 \\ \bar{K}_L^0 \end{array} \right\} \text{ fast and forward}$$

The X^{++} represents a doubly-charged system of slow secondary particles.

The SLAC 15" rapid-cycling bubble chamber, operating at 20 cycles per second was exposed to a beam of positive pions with momentum 8.4 GeV/c. The slow doubly-charged system X^{++} was observed in the bubble chamber and at the same time, a steel plate optical spark chamber and scintillation counter system was used to detect the fast neutron in a forward 9.5° half-angle square cone. Reactions with fast antineutrons, K_L^0 , $\bar{K}_L^0$ were also obtained because the forward neutral detector could not differentiate between the neutron and K_L^0 , $\bar{K}_L^0$, or $\bar{n}$.

Fig. 1 refers to the experimental set-up. The incident π^+ beam, defined by scintillation counters B_1 and B_2 , passed through .03" of stainless steel in the vacuum tank beam window, 9.5" of vacuum, .05" steel-wall of the liquid hydrogen bath tank, 3.5" of liquid hydrogen; .031" of stainless steel and .041" of copper plating of the chamber bellows before entering the fiducial hydrogen volume.

FIGURE 7

For reactions of interest, there are one or more neutral particles in the forward $\pm 9.5^\circ$ square cone. In the case of events where one or more charged particles accompanied the neutral particle in the forward $\pm 9.5^\circ$ cone, the particle shower from the interaction of the neutral particles in the spark chamber plates would have been very difficult to differentiate from showers arising from the interaction of the charged particles in the plates, therefore such events were excluded by including an anti-counter package. It would be extremely difficult to see neutrals and charged particles at the same time in forward 9.5° cone. Scintillation counters A_C and A_Y , which subtended a $\pm 9.5^\circ$ square cone at the bubble chamber center were used to anti-out events with charged particles in the above forward cone. Forward π^0 's were eliminated by using two 1/2" thick lead plates, (with 1" x 6" hole for the beam) which converted the γ -rays from $\pi^0 \rightarrow 2\gamma$ decays into e^+e^- pairs, which in their turn anti-out essentially all events with forward π^0 's.

F is a four gap foil chamber while S1, S2, S3 and S4 are 6-gap steel-plate optical spark chambers. N_1 , N_2 , N_3 and N_4 are scintillation counters. Neutral particles, interacting in the steel-plates of one of the spark chambers S1, S2, S3 or S4 produced charged particles which in their turn triggered one or more of the scintillation counters N_1 , N_2 , N_3 or N_4 . Since we were interested in fast neutrons, we required that charged secondaries from neutral particle-steel interaction traverse at least two consecutive counters. Counters N_1 - N_3 have holes to allow the beam to pass through. All spark chambers F,

S1-S4 have their central region deadened, with 1" x 6" holes, so the beam particles did not interact in them. Neutral-particle induced showers originated beyond the foil chamber. Thus, they could be differentiated during the scanning process from charged tracks originating further upstream (in the bubble chamber or chamber walls). $\bar{A}_b$ shadowed the hole in the spark chambers and counters and acted as a beam anti-counter.

The Trigger

The trigger used was:

$$B_1 \cdot B_2 \cdot \bar{A}_c \cdot \bar{A}_y \cdot (N_1 N_2 + N_2 N_3 + N_3 N_4) \cdot \bar{A}_b$$

Whenever the trigger was satisfied, the spark chambers fired in response to a signal from the fast logic; the spark chamber and bubble chamber flash-lights then received the signal to flash and joint spark chamber and bubble chamber pictures of each event were taken.

B. Description of the Beam

The pion (π^+) beam used for this experiment was produced by directing a 20.8 GeV primary electron beam onto a beryllium target. The secondary pion beam, after momentum selection, was then shaped before being directed onto the R.C.B.C. fiducial target volume.

Targeting

Fig. 8⁴⁶ describes the production and shaping of the beam. The primary electron beam was split into beams A, B, C using a set of pulsed magnets. At pulsed magnets PM30 and PM31,

FIGURE 8

beam B was further split into three beams: BL18, BL19 and BL20. The central beam BL20 was then directed by steering magnets AP30, AP31 onto the copper-beryllium target for BL20 and BL14. The secondary beam entered BL20 at a production angle of 1.0° and was monitored by TV-viewing of the aluminum oxide screen PR-35. The primary electron beam was dissipated in a water-cooled ball-dump with a 300 MW power-rating. Toroids I30, I33, I40 were used to measure beam current.

Electron Beam and Target Properties

<u>Electron Beam</u>		<u>Target</u>	
Energy	≤ 20.8 GeV	Material	beryllium (plus Cu)
Current	0-55 mA	Dimension	
Pulse Width	$1.6 \mu s$	z (along beam)	12"
		x	75"
Repetition Rate	1-180 pps	y	.188"
Spot Size at TC31	2.5×1.0 mm (FW at 10%M)	Power Rating	300 KW
		Production Angle	1°

Beam Tuning

i) Target to F_2 :

a) The F_2 -collimator width was set to .25". A 1-2 mm spot-size diameter electron beam was centered on the target using steering magnets AP30 and AP31 and the scintillating screen PR-35.

b) The F_1 vertical-collimator was set at gap-center 0.05" and gap width 0.25" while the F_1 horizontal collimator was

set at gap-center 0.0" and gap full width .17" which corresponded to a momentum bite ($\Delta p/p$) of 1% at focus F1.

c) All beamline 20 quadrupoles and dipoles were set to their nominal starting values (calculated from transport Bdl values and magnetization curves⁴⁶). Vertical centering of the beam at focus F2 was done by sweeping 20D2.1 to maximize $S_1 S_2$ /pulse. Notice Scintillation Counters S_1 and S_2 in Fig. 2. 20D2 was used for horizontal centering of the beam.

d) Quadrupoles 20Q2 and 20Q5, 20Q1, 20Q6, 20Q3 and 20Q4 were then tuned in their respective order of decreasing sensitivity (as above) to maximize $S_1 S_2$ /pulse.

Focus F2-Focus F4

a) Starting with the nominal settings for quadrupoles and dipoles, horizontal centering of the beam was done by sweeping 20D3 to maximize S_3 /pulse.

b) Final targeting of the beam into the rapid-cycling bubble chamber was done by sweeping 21D1, 21D1.1, 21D2 to maximize $B_1 B_2$ /pulse.

c) Further shaping of the beam was done by running the quadrupole in this section.

Beam Contaminations

A Freon-13 differential Cerenkov Counter at focus F2 between counters S_1 and S_2 was used to analyze the beam for contamination (Fig. 9a). The following break-down was obtained.

FIGURE 9a

FIGURE 9b

$$\pi^+ \approx 93\%$$

$$K^+ \approx 3\%$$

$$p \approx 4\%$$

$$\mu^+ \leq .1\%$$

both RF-separators off

RF-Separators

RFS₁ and RFS₂, two traveling wave separators between F1 and F2, were used to separate the p and K⁺ component of the beam from the unseparated beam.

Fig. 10 shows the relative deflection of the particles π^+ , K⁺, p at different momenta. At 8.4 GeV/c, it may be seen that for RFS₁, both π^+ and p deflection with respect to K⁺ is about .6". Therefore, RFS₁ was used to separate the K⁺ - contamination from the beam. Further, one sees that for RFS₂ the p deflection is about .8" with respect to the π^+ deflection. Thus RFS₁ and RFS₂ were used together to give a relatively pure pion beam.

The RF-separators were tuned by synchronizing the 2 μ S RF pulse with the 1.6 μ s accelerator pulse and by varying the phase to maximize S₁S₂/pulse. The phase ϕ was then changed to $\phi + 180$ and deflecting magnets 20D1.1 and 20D2.1 were used to steer the beam back to F2.

Fig. 9b shows a Cerenkov mass analysis of the beam with both the separators set as above. The beam was 99.5% pion; K⁺, p contamination being less than .5%. The separators were monitored continuously as they had a tendency to drift very sensitively with time and temperature.

BEAMLINE 20: NET DEFLECTION

FIGURE 10

Production of Proton Beam

About 4% of the unseparated beam was proton. RFS1 and RFS2 were used to extract the proton component of the beam. The Cerenkov counter was set at 131 lbs/inch² which is the sensitive pressure for proton-detection and with RFS1 off, the phase Φ_2 of RFS2 was varied to maximize S_3C/S_3 . The phase was then changed to $\Phi_2 + 180$ and 20D2.1 used to steer the beam back onto the focus. A Cerenkov analysis of the beam obtained showed a pion contamination of the order of 1%.

C. Rapid-Cycling Bubble Chamber

The main chamber body is an inverted single convolution bellows, 15.125 inches in diameter and 5.5 inches deep. The bottom surface of the chamber is held fixed, while the upper surface is attached to an armature which is located in the gap of a fixed-field d.c. shaker magnet. A 20 cycles per second A.C. current through the armature causes it to oscillate up and down in the fixed magnetic field of the shaker magnet, thereby also causing the chamber to pulse at 20 cycles per second (Fig. 11).

The bellows wall is .031" stainless steel with .014" copper plating. The surrounding liquid hydrogen bath is 3.5" wide and the wall of the containing tank is .050" stainless steel. Then there is an insulating 9.5" wide annular vacuum, the wall of the vacuum tank being .125" stainless steel. However, the vacuum tank has a 1.5" x 5" beam window which is only .030" thick.

FIGURE 12

FIGURE 13

The maximum usable upstream window (beam window) and the maximum usable downstream window (physics window) extends $\pm 60^\circ$ horizontally and $\pm 9.5^\circ$ vertically about the beam direction (Fig. 12).

Chamber Optics

The chamber uses bright-field illumination. The upper surface is covered with Scotchlite (trade-mark) while it is illuminated and photographed from below. A single-strip, 3-view (Berkeley format), 35mm camera, adapted from the Berkeley 25" hydrogen bubble chamber was used to photograph the events. The camera has a wind-time of 200-300 ms.

Analyzing Magnet

A superconducting Helmholtz pair, thermally insulated from the liquid hydrogen and maintained at cryogenic temperatures by a liquid helium bath provided the momentum analyzing magnetic field in the bubble chamber. The cylindrically symmetric field varied from 17.4 kg to 18.1 kg. The trajectory of a charged track (π^+) at two different momenta, 5 GeV/c and 8.4 GeV/c is shown in Fig. 13.

D. Description of Neutron Detector

The neutron detector consisted of the anticounter package followed by a foil-spark chamber and an alternating array of four steel-plate spark chambers and four scintillation counters. The foil chamber together with the rest of the steel-plate spark chambers had 1" x 6" central holes to allow the

FIGURE 14

NEUTRON DETECTOR

beam to go through without interacting in the neutron-detector. Fig. 13 shows the deflected beam trajectory in the magnetic field of the RCBC superconducting momentum-analyzing magnet. All counters and chambers were off-set from the true beam line so as to have the deflected beam centered on the beam-hole in the neutron-detector. The spark chambers were aligned in such a way that the camera may view all of them directly.

Anticounter Package

The anticounter package consisted of scintillation counters A_C and A_γ and two 1/2" lead plates squished between them as shown in Fig. 14. Each counter consisted of two 1/4" x 13" x 13" scintillator plates. Light from the two plates was collected by means of adiabatic light pipes and focused onto the receiving surface of a 56 AVP photomultiplier tube. Zener diodes were used to maintain a constant voltage on the last few dynodes of the photomultiplier tube. The photomultiplier tubes were shielded from the magnetic field of the RCBC superconducting magnet by using concentric soft-iron cylinders. Half inch thick lead plates with 1" x 6" central holes (for the beam) were squished between the counters A_C and A_γ as shown in the figure. Thus counter A_γ worked both as an anti-charge and anti-gamma counter. Both counters A_C and A_γ subtended a $\pm 9.5^\circ$ square-cone at the center of the RCBC. The purpose of the lead plates was to eliminate events with fast forward π^0 's decaying into 2γ 's, or γ , etc. One inch of lead corresponds to 2.54 cm/.58 cm radiation lengths,

thus $\approx 99\%$ of all γ 's were expected to convert in the lead. The interaction length of neutrons in lead is 18.2 cm, thus $\approx 13\%$ of forward neutrons were expected to interact in the lead.

Foil Chamber and Steel-Plate Chambers

The foil chamber consisted of four $3/8$ " gaps. The plates were made by stretching 1 mil thick aluminum over $1/2$ " thick and 1" wide lucite frames. There were five such plates corresponding to $10 \times (.001") = .01"$ of aluminum or .0006 neutron interaction lengths. A mixture of Neon (90%) and Helium (10%) was circulated through all chamber gaps at slight overpressure. The probability of a neutral particle interacting in the foil chamber being $\approx .001\%$, only charged particles would leave sparks along their tracks.

A shower that originated from a neutral particle interaction would very rarely originate in the foil chamber. Thus, charged tracks arising from interactions further upstream were differentiable from charged track showers induced by neutral particle interaction.

Spark chambers SC1, SC2, SC3, and SC4 were identical and consisted of six $3/8$ " gaps and five $7/16$ " steel plates besides the last two plates which were made from $1/4$ " aluminum plates (see Fig. 14). The gaps were formed by sandwiching $3/8$ " thick and 1" wide lucite frames between the steel plates. Mylar tape was used as isolation against edge sparks while RTV was used as sealant. A 90% Neon and 10% Helium mixture was circulated under a slight overpressure through the gaps.

A liquid nitrogen purifier unit was used to maintain the oxygen level in the spark chamber gas below .1%. The active area of all the spark chambers was about 18" x 18". The last two aluminum plates in each chamber corresponded to $\approx .029$ neutron interaction lengths, the probability of a neutral particle interaction in the last two plates was therefore $\approx 3\%$. This minimized the possibility of neutral particle showers originating in the last two plates of the chambers where it was difficult to locate the vertex of the showers. All chambers had a 1" x 6" central hole to allow the beam to pass through without interacting.

Neutron Counters

Counters N_1 , N_2 , N_3 and N_4 , sandwiched between the spark chambers as shown in Fig. 14, formed the neutron trigger counters. Their dimensions are given in Appendix VI, Table 19. These counters shadowed the anticounter package and subtended a $\pm 9.5^\circ$ square cone at the RCBC center. All counters were viewed through adiabatic light-pipes and coupled to 56 AVP photomultipliers. The tubes were shielded by two concentric soft-iron cylinders. All counters except N_4 had 1" x 6" central holes for the beam. Neutral particles which interacted in the steel plates produced 1, 2, 3, ≥ 3 -track charged showers which when energetic enough passed through at least two of the counters and caused a trigger.

Anticounters A_b

Scintillation counter A_b (1/4" x 1" x 6") which shadowed

FIGURE 15

the beam-hole in the neutral-detector was used as a veto in the trigger.

E. Description of Electronics and Logic System

Beam Electronics

The incident pion beam was defined by the coincidence B_1B_2 (Fig. 15). The signals from B_1 and B_2 were narrowed down by using 1 ns shorted cable slips and fed into separate 200 Megahertz discriminators. To filter out electronic noise, the discriminator threshold was set at 100 mV and output pulse width was set at 12 ns. The discriminators were gated on only during the 1.5 μ s beam spill. The average B_1B_2 count per pulse was 10, so that the average time separation between particles came to 150 ns. However, to avoid fake triggers arising from B_1 and B_2 having consecutive pulses within a pulse width of each other, a pile-up gate was set up. Coincidence B_1B_2 was formed and displayed on a scaler. A signal from B_1 was used to trigger a gate. One signal from the gate was delayed with respect to B_1B_2 while B_1B_2 was delayed with respect to another signal from the gate, and the gate width was set at 20 ns. This was done by putting B_1B_2 in anti coincidence with the advanced and delayed signals from the B_1 -triggered gate. To avoid spark chamber noise pickup in the circuit, all counter discriminators were inhibited for the remainder of the beam burst after each trigger.

Anti-Counter Electronics

The anti-counter system consisted of three counters, A_D , A_C and A_Y . The signal pulses from A_C and A_Y were reduced in

ANTI-COUNTER ELECTRONICS

70

FIGURE 16

width from 30-40 ns to 20-30 ns. Both A_C and A_Y were sent to separate updating discriminators; the discriminator threshold was set at 200 mV while the output width was set at 28 ns. The discriminators were gated on only during the beam spill and inhibited after each trigger for the rest of the burst. A_b signal was shaped and discriminated in the same way as A_C and A_Y signals. The coincidences $B_1 B_2 \bar{A}_b \bar{A}_C \bar{A}_Y$, $B_1 \cdot B_2 \cdot A_b \cdot A_C$; $B_1 \cdot B_2 \cdot A_b \cdot A_Y$, $B_1 \cdot B_2 \cdot A_b$, $B_1 \cdot B_2 \cdot \bar{A}_b$, $A_C \cdot A_Y$, $\bar{A}_C \cdot \bar{A}_Y$ were formed as shown in Fig. 16 and displayed on scalers. $B_1 B_2 A_b A_C$ and $B_1 \cdot B_2 \cdot A_b \cdot A_Y$ were used to monitor A_C and A_Y efficiency during the experiment. $B_1 \cdot B_2 \cdot \bar{A}_b$ gave a first order estimate of the $\pi^+ p$ cross section at 8.4 GeV/c. $B_1 \cdot B_2 \cdot \bar{A}_b \cdot \bar{A}_C \cdot \bar{A}_Y$ gave a measure of the cross section for no forward charged tracks in the $\pm 9.5^\circ$ forward square cone. $A_C \cdot A_Y$ was used to monitor beam halo outside the shadow of $B_1 \cdot B_2$.

All counters had an effective dead time equal to the output width of its corresponding discriminator. In the configuration shown in Fig. 14 for the lead sheets, both counters A_C and A_Y acted as anti-charge counters while A_Y alone acted as anti-gamma counter. Counter A_C and A_Y efficiencies during the experimental run averaged about 98% for each counter.

Neutron Detector Logic

Signals from counters N_1 , N_2 , N_3 and N_4 were clipped and sent to separate discriminators. The discriminator thresholds were set at 100 mV and the output pulse widths were set at 20 ns. All discriminators were gated on only during the beam spill and inhibited after each trigger. Coincidences $N_1 \cdot N_2$,

NEUTRON DETECTOR ELECTRONICS

FIGURE 17

$N_2 \cdot N_3$, $N_3 \cdot N_4$ were formed. One set of signals from these coincidences was fed to an OR unit to form the added signal $N_1 \cdot N_2 + N_2 \cdot N_3 + N_3 \cdot N_4$, which was then sent to form the master trigger.

In order to help in the neutral interaction vertex location, a record of the $N_i N_j$ pair coincidences which fired during the trigger was kept by using a gated-latch and data-board system. See Fig. 17. Each pair-coincidence when in time with a master trigger strobe signal turned on an R-S flip-flop. The signal from the flip-flop, in its turn, opened a transistor switch for a period determined by the flip-flop width setting. This switch-on-or-off information was directly relayed to a relay-controlled light on a data-board inside the optical tent area. To prevent noise signals reflected back along the transmission lines to the data-board from resetting the gated latch before the gated-latch could be photographed, the gated latch information on each line was fed to the data-board through individual optical isolators. During the experiment two gated latches were used in parallel to keep track of each other. Counter efficiencies during the experiment remained above 99%.

Master Trigger

The master trigger was formed from three basic signals:

- i) $B_1 \cdot B_2 \cdot \bar{A}_b$, which said an interaction has occurred in the bubble chamber; ii) $\bar{A}_c \bar{A}_\gamma$, which said a reaction with no forward charged tracks or gammas in the $\pm 9 \frac{1}{2}^\circ$ forward square cone had occurred; iii) and $N = N_1 N_2 + N_2 N_3 + N_3 N_4$, which said

FORMATION OF MASTER TRIGGER

FIGURE 18

that a neutral particle in the forward cone had interacted in one of the steel plates of the spark chambers and produced charged particles which were energetic enough to transverse at least two of the counters N_1-N_4 . The complete master trigger used is:

$$MT = B_1 \cdot B_2 \cdot \bar{A}_b \cdot \bar{A}_c \cdot \bar{A}_y (N_1 N_2 + N_2 N_3 + N_3 N_4) \quad (64)$$

It was formed by feeding signals $B_1 \cdot B_2$, $\bar{A}_b$, $\bar{A}_c$, $\bar{A}_y$ and N into a master coincidence unit. See Fig. 18.

The Master Trigger was then set to trigger a gate whose output width was fixed at 1.2 seconds. The logic output from the main gate was then used to inhibit the main machine gate for 1.2 seconds. The delayed output from the gate was used to keep the main machine gate inhibited for a further period of approximately 0.1 seconds to cover its own dead time. The machine gate was operated by an advance pulse from the accelerator. The accelerator prepulse which was $1\mu s$ wide and +10V in height was used to open a $10\mu sec$ gate enabling all discriminators in the trigger circuit during the beam spill. The machine prepulse arrived $2.5\mu s$ before the start of the beam spill. After each trigger, as already discussed, the main gate was inhibited for approximately 1.3 seconds.

The logic output of the machine-pulse gate was scaled and gave the total pulse count. The gate output from the machine-pulse gate which was set at $10\mu\text{s}$ was fed through a fan-out to the B_1 , B_2 , A_c , A_y , A_b , N_1 , N_2 , N_3 and N_4 discriminators so that these discriminators were on only during the beam spill.

To ensure that the master trigger would not pick-up any noise from the system, a three-fold coincidence was formed among (i) the direct master trigger, (ii) the 1.2 second wide logic output of the trigger gate (delayed by 150-200 ns by the rise-time of the gate) and (iii) the 1.2 second delayed (-) negative signal from the trigger gate. Thus for approximately 1.3 seconds after the trigger was formed, noise-pickup would be vetoed out.

The clean master-trigger from the three-fold coincidence unit was then fed to separate fan-outs. One fan-out was then used to send out inhibits to all individual counter discriminators while the second fan-out was used to send signals to: (i) spark chamber trigger pulsers which in their turn fired the spark chambers, (ii) the gated latch and dual coincidence unit, (iii) the TTL converter and (iv) bubble chamber trigger gate. The master trigger was also scaled.

FIGURE 19

FIGURE 20

SLOW LOGIC

The TTL converter was used to convert the NIM logic level signal to 1.5v signals with variable widths. A 5 μ s wide TTL signal was divided into seven separate signals using a fan-out and used to trigger the flash-fiducials, while a second 100ms wide TTL signal was used to turn on the spark chamber data box strobe input. Another TTL signal, delayed by the width of the flash-fiducial and data box signal was then sent to the spark chamber camera to wind up for the next picture, the wind-time of the camera was ~500ms. After a delay of 100ms, the gated-latch was reset by a TTL signal.

The b.c. trigger signal was sent to the RCBC control panel where it was used to advance the RCBC data box and flash the RCBC camera lamps. The RCBC data box and spark chamber data box advances were synchronized to keep spark chamber and bubble chamber films from the same event tagged with the same roll frame number.

The relative timings of various signals are shown in Fig. 19. Fig. 20 shows the general layout of the slow logic.

F. Discussion of Trigger Rates

Reaction Rates

The total cross-section is defined as follows:

$$\sigma_T = \frac{N_{\text{events}}}{n_h \times N_{\text{beam}} \times l_{\text{fid}}}$$

where

n_h = number of target particles per unit volume

N_{events} = number of interactions

N_{beam} = total number of beam particles

l_{fid} = fiducial length

In order to calculate an approximate reaction rate, it is required to find the total equivalent length of liquid hydrogen in the beam-path. There was 15.125 inches of liquid hydrogen in the bubble chamber and 7" of liquid hydrogen from the surrounding liquid hydrogen bath. Thus there was 22.125" or 56.2 cm of liquid hydrogen in the beam path. Further the bubble chamber beam window (.50" of steel), the hydrogen bath tank (.045" of steel) and the bubble chamber bellows (.030" steel) contributed $2(.05 + .045 + .03) = .250$ " of steel in the beam path.

If .250" of steel is equivalent to L " of liquid hydrogen, one gets

$$\begin{aligned} .250 \times \frac{\rho_{\text{Fe}}}{A_{\text{Fe}}} \times N_{\text{Avag}} \times (N_p + N_n)_{\text{Fe}} \\ = L_h \times \frac{\rho_h}{A_h} \times N_{\text{Avag}} \times (N_p + N_n)_h \end{aligned}$$

where ρ_{Fe} and ρ_h are steel and liquid hydrogen densities and $(N_p + N_n)_x$ is the number of nucleons per atom of X . Since $(N_p + N_n)_h = 1$ and $(N_p + N_n)_{\text{Fe}} \approx A_{\text{Fe}}$ where A_{Fe} is gram-atomic weight of steel and A_h that of hydrogen, we get

$$\begin{aligned} L_h &= (.250" \times \frac{\rho_{\text{Fe}}}{\rho_h} \times 2.54) \text{ cm} \\ &= (.250" \times \frac{7.87}{.06} \times 2.54) = 83 \text{ cm} \end{aligned}$$

The density for scintillator is 1.032. Thus the scintillator for the beam counters (.125") is equivalent to 5.5 centimeters.

The total equivalent amount of hydrogen in the beam trajectory is $L_{eq} = 144.7$ centimeters. The rate of interactions in the bubble chamber expressed as $R_{int} = B_1 B_2 \bar{A}_b / B_1 B_2$ is given by:

$$R_{int} = \frac{B_1 B_2 \bar{A}_b}{B_1 B_2} = \sigma_T \times \rho_h \times N_a \times L_f$$

Since σ_T for π^+p interactions at 8.4 GeV/c is about 26 millibarn, the expected rate of interactions comes out to be about 13.6 per cent, that is the expected value of $B_1 B_2 \bar{A}_b / B_1 B_2$ is 13.6 per cent.

According to 7 GeV/c π^+p data from LRL (private communication), the cross-section for events with no charged tracks in the $\pm 9.5^\circ$ forward square cone is 5 millibarn. Thus for such events, the expected value of the ratio $B_1 B_2 \bar{A}_b \bar{A}_c \bar{A}_\gamma / B_1 B_2$ is 2.5 per cent.

Trigger Rates

During the experiment, several rates were scaled.

(i) The average trigger rate of $[B_1 B_2 \bar{A}_b \bar{A}_c \bar{A}_\gamma (N_1 N_2 + N_2 N_3 + N_3 N_4)] / B_1 B_2$ was 1/1438.

(ii) The ratio of $B_1 B_2$ to $B_1 B_2$ separated or unpiled was 86 per cent.

(iii) The average ratio of $A_c A_\gamma$ to unpiled $B_1 B_2$ was 125 per cent, showing a beam halo of 25 per cent during the experiment.

(iv) $B_1 B_2 \bar{A}_b$, $B_1 B_2 \bar{A}_b \bar{A}_c$ and $B_1 B_2 \bar{A}_b \bar{A}_\gamma$ were scaled to monitor A_c and A_γ efficiencies continuously during the experiment.

(v) $B_1 B_2 \bar{A}_b$ which measured the total $\pi^+ p$ cross-section approximately was recorded as $B_1 B_2 \bar{A}_b / B_1 B_2 = 11$ per cent. The expected value as discussed in the last section is about 13.6 per cent. The smaller observed value may be attributed to very forward elastic scattering, where the scattered π^+ hit A_b .

(vi) $B_1 B_2 \bar{A}_b \bar{A}_c \bar{A}_\gamma$ which measured the cross-section for events with no charged tracks in the forward $\pm 9.5^\circ$ square cone was found to be $B_1 B_2 \bar{A}_b \bar{A}_c \bar{A}_\gamma / B_1 B_2 = 1.2$ per cent. The expected value of the ratio is about 2.5 per cent.

The analysis presented in this report is based on approximately 74,000 pictures. In a conventional bubble chamber with an average of 10 tracks per picture, this corresponds to 7.4×10^6 tracks and for a fiducial length of 22.7 centimeters, the above data sample would conventionally correspond to a statistical level of 6.2 events per microbarn. However, because of the trigger preselection of interesting channels, the chamber was really sensitive to approximately 1.18×10^8 tracks which corresponds to a statistical level of approximately 93 events per microbarn. Therefore, the hybrid system in this case gives a factor of 15 improvement in the statistics of the data sample. The effective statistical level after taking into account neutral detector efficiencies and trigger requirements corresponds to approximately 30 events per microbarn (see Chapter 5).

REFERENCES

46. Winkelmann, F. C., "Design and Operation of SLAC Beamline 20-21-22-23," SLAC Report No. 160, April 1973.

DATA-REDUCTION FLOW CHART

A. Data Reduction Flow Chart

The various steps in the data reduction process are shown in the flow chart. The bubble chamber film was first scanned for interesting events in the fiducial volume and a list of roll-frame numbers of the interesting events was generated. The events were measured on image-plane digitizers which put out the raw coordinates on IBM cards. Program CTAPE then converted the coordinates to MKS units and transferred the data onto magnetic tape. Program TEDIT⁴⁹ was used to edit each event and bad events were remeasured. The raw data tape was then used as input to the track reconstruction program TVGP.⁴⁷ Events for which track reconstruction failed were remeasured.

The kinematic fitting program SQUAW⁴⁸ was used to obtain one-constrained fits to various reaction hypotheses with one missing neutral track. A spark chamber scanning list was generated. Clean events (criterion to be discussed later) were then measured as in the case of bubble chamber pictures. The neutral track information after geometrical reconstruction using program GEOM⁵⁰ was then merged with the 1C-fit information tape. This was done using program FIX.⁵¹ The merged tape contained information of the azimuthal and dip angles of the neutral track. The use of the kinematic program SQUAW with this added information gave rise to three-constrained fits to the various reaction hypotheses tried earlier. Thus a 3C-fit information tape was generated.

Data summary tapes of two, four, and six prong events were generated at the 1C and 3C fit level using the various BOX programs.

B. Bubble Chamber Scanning and Measuring

The entire sample of 120,000 pictures consisted of three categories:

1) Triggered Sample: This sample consisted of approximately 90,000 bubble chamber pictures with their corresponding spark chamber pictures. These pictures corresponded to those events which satisfied the trigger requirements (see chapter 2).

2) Untriggered Sample: This sample consisted of approximately 10,000 pictures. In this case the rapid cycling bubble chamber was operated on one cycle per second and bubble chamber and spark chamber pictures were taken for each chamber expansion. These pictures were used to obtain the setting error for the bubble chamber film.

3) pp Interactions Sample: About 15,000 pictures were taken with an incident proton beam at 8.4 GeV/c, while the trigger required a neutral interaction in the downstream spark chambers as before. The purpose of these pictures was to use the reaction

$$pp \rightarrow \Delta^{++} n_{\text{forward}} \quad (65)$$

with known cross section for later renormalization.

Straight Through Tracks

About 100 pictures were taken of straight through tracks in the bubble chamber with the momentum analyzing magnet turned off. The purpose of these pictures was to study the distortions in the track curvature due to optical effects.

Scanning

The analysis was based on approximately 76,500 triggered pictures. About 3.5% of these pictures were rejected because they had very poor optical quality or had overlapping frames in two of three views. Approximately 74,000 three view bubble chamber pictures were scanned for two, four, six and eight prong topologies with and without neutral V_0 's in a fiducial length of 22.7 centimeters. A scanning list was generated.

A second scan of portions of the film selected at random was carried out to calculate scanning efficiencies. These are discussed later in chapter 4.

The untriggered pictures were scanned for four-pronged events only. A beam track count every 100 frames was maintained to calculate the total beam flux. Details appear in Appendix IIA.

Measuring

Approximately 23,000 events in the fiducial volume were found for the triggered sample, that is, approximately one third of the pictures had measurable events. All events were measured on the Vanderbilt image-plane digitizers which were directly coupled to an IBM 1820 computer. The raw data on IBM cards was then transferred to magnetic tape using program CTAPE. Program TEDIT was used to edit the raw data tape. Only about one per cent of the events had to be remeasured at this level. All four pronged events in the untriggered sample were measured in the same way.

SLAC 15'' RCBC COORDINATE SYSTEM

FIGURE 21

C. Geometrical Reconstruction of Tracks in the Bubble Chamber

Coordinate System

Fig. 21 shows the coordinate system used throughout the analysis. The coordinate system origin is on the fiducial plane (inside surface of glass-window) so that all distances into the bubble chamber are measured along +z-direction. The three camera positions are also shown in the figure.

Bubble Chamber Optical Parameters

The optical system for the 15" RCBC consisted of

- (i) scotchlite spherical reflectors at the top of the chamber,
- (ii) a glass window with six fiducial marks on the inside surface,
- (iii) another quartz window, and
- (iv) three separate cameras to give stereo-viewing of the chamber.

The surveyor supplied engineering and optical parameters such as focal lengths of the camera lens, refractive indices of liquid hydrogen, glass and quartz, camera positions and distances of the fiducial plane from the film planes, were used by program KOPGEN⁵² to reconstruct measured interaction vertex points in three dimensions. The various parameters were varied within allowed errors to obtain consistent reconstruction of single points on the film in three dimensions separately for each camera. The final fitted values of the parameters are given in Table 12b, Appendix III.

Track Reconstruction

Each event was represented by three separate pictures corresponding to the three camera views. The measurement

FIGURE 22

information from all the three views for an event was fed as input to the track reconstruction or geometry program TVGP.⁴⁷ TVGP used the optical and engineering parameters given in Table 12b, Appendix III and event information from all the three views and computed detailed information for the complete description of the event. TVGP provided the values for the measured length, momentum, dip and azimuthal angles and errors on base measurements for each track at an interaction vertex.

Setting Error for Bubble Chamber Measurements

All four-pronged events from the untriggered sample of bubble chamber pictures were measured and processed through the geometry program TVGP. The reconstructed events were then fitted to the reaction hypothesis

$$\pi^+ + p \rightarrow \pi^+ \pi^+ \pi^- p$$

using the kinematic fitting program SQUAW.⁴⁸ Since the beam tracks were very straight, TVGP measurements of beam track momentum were unreliable and had large errors. Therefore, the beam track momentum in SQUAW was set as $8.4 \text{ GeV}/c \pm 100 \text{ MeV}/c$ based on the beam optics consideration. Four-constraint (4C) fits to the above reaction hypothesis were obtained. The setting error in the bubble chamber film measurements were varied until one obtained a reasonable flat distribution for the χ^2 -probability of the 4C fits to the above reaction hypothesis.

A setting error of 80 microns on the film measurement gave good χ^2 -probability distributions for the 4C-fits to the reaction hypothesis considered above. Fig. 22a gives the

FIGURE 22a

FIGURE 22b

FIGURE 22c

FIGURE 22d

FIGURE 23a

FIGURE 23b

FIGURE 23c

χ^2 -probability distribution plot. The χ^2 -probability distribution was also studied as a function of the central value of the beam momentum. Values varying from 8.1 GeV/c to 8.6 GeV/c were used. The trapped value 8.4 GeV/c was found to be consistent with a reasonably flat χ^2 -probability distribution for the four-prong 4C-fits. The fitted beam momentum for the four-prong, 4C-fit event sample is plotted in Fig. 22. The value of $8.42 \pm .080$ GeV/c was obtained as the best value for the average beam track momentum. For the chosen film setting error of 80 microns and trapped beam momentum central value of 8.4 GeV/c, the stretch functions corresponding to the (ϕ, λ, k) measurements for each track in the events from the 4C-fit sample appeared to approximately follow a Gaussian distribution with zero mean and unit root mean square deviation. Fig. 22(b), (c) and (d) show the stretch function plots corresponding to the (ϕ, λ, k) for the beam tracks.

Straight Track Reconstruction

The straight tracks correspond to bubble chamber pictures taken with the superconducting magnet turned off. Fig. 23(a), (b) and (c) show the TVGP measured dip and azimuthal angle and curvature, $k = 1/(p \cos \lambda)$ for the straight tracks. The azimuthal angle distribution of the straight tracks is centered about 359.882° , that is, it is .02 degrees or about 3.5 milliradians from the expected zero degree azimuthal distribution center. The dip angle distribution is centered at +3.5 milliradians whereas it was expected to be centered about -1.0 milliradian according to the beam optics design. Straight

tracks should have no curvature which requires the k-distribution to be centered at 0. However, the TVGP measured k-distribution is centered at $-2.2 \times 10^{-5} \text{ (GeV/c)}^{-1}$. This corresponds to the curvature of a track with momentum equal to $\sim 250 \text{ GeV/c}$ in an 18 kilogauss field. The distortion produced curvature for the straight tracks is comparable to distortions in other chambers, such as the SLAC 80" bubble chamber.

Geometrical Reconstruction of Tracks by TVGP

Using the optical parameters as given in Table 12b, Appendix III, a setting error of 80 microns on the measured coordinates, and the fixed value $8.42 \pm .080 \text{ GeV/c}$ for the beam track momentum, TVGP was used to calculate the momentum, dip and azimuthal angles and the errors on the above quantities for each track at an interaction vertex. Three different mass interpretations were tried for each track, namely, the pion, kaon and proton mass interpretation.

All events were processed through the program TVGP and a special tape containing only geometry information on each event was created.

D. One Constraint Kinematic Fits

All events were then fitted to various final state hypotheses using the Maryland version of SQUAW. Hypotheses with missing neutron, antineutron, K_0 and $\bar{K}_0$ were tried. The different hypotheses tried for the two, four and six prong topologies are listed below:

two prong topology

$$\begin{array}{l}
 \pi^+ p \rightarrow \pi^+ \pi^+ n \\
 \quad \quad \quad p \bar{p} n \\
 \quad \quad \quad K^+ p \bar{K}_0
 \end{array} \quad (66)$$

four prong topology

$$\begin{array}{l}
 \pi^+ p \rightarrow \pi^+ \pi^+ \pi^+ \pi^- n \\
 \quad \quad \quad \pi^+ \pi^+ p \bar{p} n \\
 \quad \quad \quad \pi^+ \pi^+ K^+ K^- n \\
 \quad \quad \quad \pi^+ \pi^- p \bar{p} n \\
 \quad \quad \quad \pi^+ \pi^- K^+ p \bar{K}_0 \\
 \quad \quad \quad \pi^+ \pi^+ K^- p K_0
 \end{array} \quad (67)$$

six prong topology

$$\begin{array}{l}
 \pi^+ p \rightarrow \pi^+ \pi^+ \pi^+ \pi^+ \pi^- n \\
 \quad \quad \quad \pi^+ \pi^+ \pi^+ K^+ K^- \pi^- n \\
 \quad \quad \quad \pi^+ \pi^+ \pi^+ \pi^- p \bar{p} n \\
 \quad \quad \quad \pi^+ \pi^+ \pi^- \pi^- p \bar{p} n \\
 \quad \quad \quad \pi^+ \pi^+ \pi^- \pi^- K^+ p \bar{K}_0 \\
 \quad \quad \quad \pi^+ \pi^+ \pi^+ \pi^- K^- p K_0
 \end{array} \quad (68)$$

At this point, since no information on the neutral tracks was supplied, only one constraint fits to the hypotheses were obtained. In the case of two prong events only 57% of the events had a 1C-fit to at least one of the hypotheses tried. Eighty per cent of the four prong events and 78% of the six prong events made a 1C-fit to one or more of the hypotheses tried.

E. Spark Chamber Scanning and Measuring

Scanning

Event selection for scanning of corresponding spark chamber pictures was done on the basis of the following cuts:

- i) The event made a one constraint fit to one or more of the reaction hypotheses tried.
- ii) The missing momentum minus the error in the missing for the fitted event exceeded 3 GeV/c.
- iii) The one constraint fit predicted dip and azimuthal angles (λ_n, φ_n) for the missing neutral satisfied the constraints:

$$\begin{aligned} |\lambda_n| &\leq 45^\circ \\ |\varphi_n| &\leq 45^\circ \end{aligned} \quad (69)$$

Forty-seven per cent of the two prong events which made 1C fit survived the above cuts. In the case of four and six prongs, 44% and 31% respectively, survived the cuts.

Fig. 24 shows the film format for the spark chamber pictures. The direct view appears below the stereo view. The data box with roll frame digit information and gated-latch information appears on top. The figure shows the digit identification. The gated latch was used to identify the neutron counter pairs which fired during the trigger. The gated latch information was used to facilitate the identification of neutral vertex from the rest of the charged background.

The identification of a neutral interaction vertex or neutral star was based on the following considerations:

FIGURE 24

1. All charged tracks originated upstream and therefore had continuations in the foil chamber.

2. Neutral interaction induced showers had their vertices in one of the steel plate spark chambers S1-S4 (see Fig. 14). Probability of neutral interaction in the foil chamber was very small.

3. Neutral stars were one, two, three or more pronged. They were differentiated from the charged background by checking to see if the interaction vertex was located outside the foil chamber.

4. To avoid confusing situations where fake neutral stars appeared because of overlap of tracks, the neutral stars were identified by one-to-one spark matching in both views.

The scanning procedures for the spark chamber pictures were defined as follows:

- i) All pictures were classified into three categories:
 - (a) pictures with a very confusing background;
 - (b) pictures with more than three possible neutral candidates;
 - (c) pictures with three or less easily identifiable neutral candidates.
- ii) Any neutral star had at least one track with four or more sparks in the direction of the track.
- iii) All neutral stars were matchable spark by spark in both views.

Measuring

During the first pass, only events in category (c) as defined above were measured, while others were labeled blank. The data box roll-frame and gated latch information was digitized as single points for each light that was one. The numerical conversion was inferred from the positions of the different lights. Only the neutral star vertices were measured in both the direct and stereo views. The location of the vertices was done by eye.

All information was transferred to magnetic tape from data cards with the help of modified versions of CTAPE and TEDIT. This raw data tape was used as input to the track reconstruction program GEOM.⁵⁰ About 80% of the spark chamber pictures had measurable neutral candidates.

F. Merging of Bubble Chamber and Spark Chamber Information

Program FIX inserted the neutral track information in the TVGP part of the event record. The insertion was carried out in the following way:

- i) The SQUAW part of the record was moved down by 76 words.
- ii) All track counters and block length counters were changed at appropriate places in the record.
- iii) The beginning point (average over the beginning points of the charged tracks in the bubble chamber) and end point (neutral star vertex coordinates) of the neutral track was entered. The length, azimuthal angle ϕ , and slope s of

the neutral track along with all error matrix elements were inserted in a separate track information block.

iv) K_0 and n mass interpretations were tried.

v) Up to five neutral tracks were inserted in TVGP part of the record.

A new tape with added neutral track information and one constrained fit information was generated.

G. Three Constraint Kinematic Fits

The TVGP record information together with the added neutral track information was used as input to the kinematic fitting program SQUAW. The reaction hypotheses tried were the same as those in section D. This time, since the direction of the neutral track was known, three constraint fits were obtained. If an event record had more than one neutral track information SQUAW attempted to make a fit for each neutral track separately. Only about one to three per cent of the events made three constraint fits to one or more of the reaction hypotheses tried.

Data Summary Tapes

All missing mass analysis was based on the TVGP output tapes and one constrained fit SQUAW tapes. There were a total of 11 1C-fit SQUAW tapes. Programs BOX21C, BOX41C and BOX61C were used to generate three data summary tapes for two, four, and six prong events respectively. Similar data summary tapes were created from the 3C-fit SQUAW tapes. Fig. 25 gives a detailed report of the data reduction and event filtering process.

EVENT SELECTION and FILTERING

(continued)

REFERENCES

47. TVGP. Three view geometry program. Programming Notes, University of Maryland, T. Day, Technical Report #649.
48. SQUAW. A kinematic reconstruction program. Programming Notes, University of Maryland, T. Day, Technical Report #649.
49. TEDIT. A machine measurement sorting and ordering program, by M. Webster, Vanderbilt University, Nashville, Tennessee.
50. GEOM. A vertex reconstruction program for optical two-view spark chamber film, adapted from the Minnesota Spark Chamber Reconstruction Program by Eric Petraske, Vanderbilt University.
51. FIX. A Fortran IV program for merging spark chamber and TVGP reconstructed track information, Sheldon Stone, Vanderbilt University.
52. KOPGEN. A program to obtain optimum values for bubble chamber optical parameters, Rochester University program.

CHAPTER IV

PRODUCTION OF EXOTIC MESON RESONANCES

A. Event Selection Procedures for the Final States $2\pi n$, $4\pi n$, $6\pi n$ and $p\bar{p}\pi^+\pi^+$

As discussed in chapter III, section D, the events were fitted to several different reaction hypotheses in the two, four and six prong channels. However, the reaction channels reported here are:

$$\pi + p \rightarrow X^{++}n_{\text{forward}}$$

- i) $\pi^+\pi^+$
- ii) $\pi^+\pi^+\pi^+\pi^-$
- iii) $\pi^+\pi^+\pi^+\pi^+\pi^-\pi^-$
- iv) $p\bar{p}\pi^+\pi^+$

In reactions (i) to (iii), only even G parity, multipion decay modes of the $I=2$, charge = +2, X^{++} system are accessible. The decay mode (iv) is important because theorists have argued that exotic meson resonances may couple more strongly to baryon-antibaryon final states rather than multimeson final states.

The process of selecting events corresponding to the final states of interest was carried out at three different levels of the data reduction scheme. The largest sample of data consisted of all those events which were successfully reconstructed by the geometry program TVGP. Missing mass analysis based on these events yielded the largest number of events in any final state. A second, more restricted selection of events in any

FIGURE 26a

FIGURE 26b

final state was based on the 1C-fit sample, which consisted of all events which had a successful one constraint fit to at least one of the reaction hypotheses tried by the kinematic fitting program SQUAW. The most reliable selection of events in any final state was based on the 3C-fit sample.

Missing Mass Analysis

All two prong events after geometric reconstruction were interpreted using three different sets of mass-interpretations for the two positively charged tracks in the bubble chamber, i.e., (a) $\pi^+\pi^+$, (b) K^+p , (c) pp . In each case, the missing mass squared as calculated from the bubble chamber measurements of the energy momentum four vectors of the charged tracks was plotted for the cases (i) missing momentum ≥ 3 GeV/c and (ii) missing momentum ≥ 6 GeV/c. The neutron signal corresponding to the $\pi^+\pi^+$ mass interpretation is seen clearly in Fig. 26a, as the neutron signal is an enhancement away from the general background. The $\bar{n}$ signal corresponding to the pp interpretation was also seen. However, the $\bar{K}_0$ signal corresponding to the interpretation K^+p did not separate from the background. The missing mass squared resolution of the neutron peak was the same for both the missing momentum cuts and was equal to 1 (GeV)^2 .

In a similar way, for all TVGP-reconstructed four prong events, the missing mass squared off of various mass hypotheses for the charged tracks in the bubble chamber were plotted for missing neutral momentum greater than 3 GeV/c and 6 GeV/c. The neutron signal corresponding to $\pi^+\pi^+\pi^+\pi^-$ interpretation

TABLE 4

Final State	Pn > GeV/c	Gaussian + 2nd Order Polynomial			Gaussian + 3rd Order Polynomial			Average Area $\frac{a + a'}{2}$	Average Events Above Polynomial $\frac{b + b'}{2}$
		Confid. Level	Gaussian Area (a)	Events Above Polynomial (b)	Confid. Level	Gaussian Area (a')	Events Above Polynomial (b')		
$\pi^+\pi^+_n$	3	0.65	139 ± 97 $\sigma = .70$ $\langle m^2 \rangle = 1.03$	145 ± 12	0.85	77 ± 27 $\sigma = .46$ $\langle m^2 \rangle = 1.23$	81 ± 9	108 ± 50	113 ± 8
	6	0.58	68 ± 26 $\sigma = .48$ $\langle m^2 \rangle = 1.07$	73 ± 9	0.64	55 ± 18 $\sigma = .41$ $\langle m^2 \rangle = 1.04$	63 ± 9	62 ± 16	68 ± 6
$\pi^+\pi^+\pi^+\pi^-_n$	3	0.03	104 ± 27 $\sigma = .21$ $\langle m^2 \rangle = 1.06$	108 ± 10	0.62	85 ± 18 $\sigma = .19$ $\langle m^2 \rangle = 1.04$	87 ± 9	94 ± 16	98 ± 7
	6	0.91	38 ± 7 $\sigma = .21$ $\langle m^2 \rangle = 1.00$	38 ± 6	0.95	37 ± 7 $\sigma = .21$ $\langle m^2 \rangle = .99$	37 ± 6	38 ± 5	38 ± 4
$\pi^+\pi^+\pi^+\pi^+\pi^-\pi^-_n$	3	.00	35 ± 21 $\sigma = .23$ $m^2 = 1.18$	47 ± 7	0.20	40 ± 20 $\sigma = .26$ $m^2 = 1.09$	56 ± 7	38 ± 15	52 ± 5
	6	---	---	---	---	---	---	---	---

was seen as shown in Fig. 26b. However, the neutron signals corresponding to the mass interpretations $\pi^+\pi^-\bar{p}p$ and $\pi^+\pi^+K^+K^-$ were not separable from the background. The missing mass squared resolution for neutron peak in this case was approximately $.8 \text{ (GeV)}^2$. The higher missing momentum cut gave better signal separation from the background.

For six prong events, the neutron signal corresponding to the mass interpretation $\pi^+\pi^+\pi^+\pi^-\pi^-\pi^-$ was seen for the missing momentum cut $\geq 3 \text{ GeV/c}$. There were no events for the missing momentum cut $\geq 6 \text{ GeV/c}$.

All missing mass squared plots were fitted to a sum of a third or fourth order polynomial and a Gaussian using a least squares fitting program. All fitting was attempted for a small mass squared interval to avoid very high order polynomial fits. Events in the signal peak were obtained from the fitted area of Gaussian and by counting the events in the signal peak which were above the polynomial background under the peak, where the polynomial was the one used in the fitting above.

The results of the fit for the channels $2\pi n$, $4\pi n$, and $6\pi n$ are given in table 4. The confidence level of the fits with the fourth order polynomial are consistently better than those of fits with the third order polynomial. The errors in the fitted areas of the Gaussian is also very large in the case of the fit with the second-order polynomial. In the case of the fit with the third-order polynomial, the number of events obtained from the fitted area of the Gaussian in any channel is comparable to the number of events obtained by

counting events in the signal peak above the polynomial background. The center of the fitted Gaussian is shifted to higher mass squared values rather than the expected $.883 \text{ GeV}^2$ for the neutron.

Analysis Based on 1C-Fit Sample

At the one-constraint fit level, each event was fitted to several different final state hypotheses, where in each case, the neutral track information was missing. The chisquare (χ^2) of the fit as calculated by SQUAW is a measure of the goodness of a fit. The goodness of a fit is also stated in terms of the confidence level of the fit which is the χ^2 probability (P_{χ^2}) of the fit. A fit to an event is classified as the best fit to the event if the χ^2 -probability of the fit is highest as compared to the χ^2 -probabilities of other fits to the event. In a similar way the next best fit has next highest χ^2 probability for the fit. Events are classified as unique or ambiguous events. An unique event makes a fit to only one of the final state hypotheses tried while an ambiguous event makes fits to several different final states at the same time. The ambiguity level of any event selection procedure is defined as the percentage of ambiguous events in the total sample of events assigned to a chosen final state. The event selection procedure was studied by assigning events to a chosen final state in five different ways. The ambiguity level of each selection procedure was then studied as a function of cuts on (i) the χ^2 probability of the fit, (ii) the missing momentum for the fit and (iii) the missing mass squared for

TABLE 5

$\pi^+ \pi^+ n$										
$P_n >$	fit const.	P_{X^2} >	M_{X^2} cut	N_{X^2}	per cent ambiguous events	N_B	per cent ambiguous events	N_B/NB	per cent ambiguous events	N_U
3GeV/c	1C	4%	none	464	53%	272	20%	239	10%	216
	1C	4%	$-1 < M_X^2 < 3$	217	32%	182	19%	165	10%	147
	3C	2%	none	101	3%	101	3%	98	0%	98
5GeV/c	1C	4%	$-1 < M_X^2 < 3$	142	13%	139	11%	134	8%	123
	1C	4%	none							
	3C	2%	none	88	0%	88	0%	88	0%	88

TABLE 6

$\pi^+ \pi^+ \pi^+ \pi^- n$										
Pn >	fit const.	P_{X^2}	M_X^2 cut	N_{X^2}	per cent ambiguous events	N_B	per cent ambiguous events	N_B/NB	per cent ambiguous events	N_U
3GeV/c	1C	6%		505	50%	354	28%	307	17%	255
	1C	6%	$-1 < M_X^2 < 3$	415	40%	337	27%	293	16%	246
	3C	5%	none	145	18%	142	16%			119
6GeV/c	1C	6%	none	121	16%	111	9%	107	6%	101
	1C	6%	$-1 < M_X^2 < 3$	99	6%	97	4%	95	2%	93

TABLE 7

$\pi^+ \pi^+ \pi^+ \pi^+ \pi^- \pi^- n$										
Pn >	fit const.	P_{X^2}	M_X^2 cut	N_{X^2}	per cent ambiguous events	N_B	per cent ambiguous events	N_B/NB	per cent ambiguous events	N_U
3GeV/c	1C	6%	none	125	28%	117	23%	106	15%	90
	1C	6%	$-1 < M_X^2 < 3$	120	27%	113	22%	103	14%	88
	3C	2%	none	40	7%	40	7%	39	5%	37

the fit. Four different categories of events for any chosen final state were obtained. Tables 5, 6 and 7 show the number of events of each category for the final states $2\pi n$, $4\pi n$ and $6\pi n$ respectively. The number of events in the various categories are defined as follows:

- A. N_{all} = This is the most inclusive category and includes any event which makes a fit to the chosen final state with or without fits to other final states.
- B. N_{χ^2} = This category is a subsample of the first category. A cut is placed on the χ^2 probability of the chosen final state to eliminate the low χ^2 probability peak in the χ^2 probability plot. Thus $N_{\chi^2} \leq N_{all}$.
- C. N_B = Only events which make best fits to the chosen final state are included in this category $N_B \leq N_{\chi^2}$.
- D. $N_{B/NB}$ = This category includes events which make best fits to the chosen final state and all those events for which the χ^2 probability of the best fit is at least ten times greater than the χ^2 probability of the next best fit.
- E. N_U = Each event makes an unique fit to the chosen final state.

In category B, for missing momentum ≥ 3 GeV/c there were 464, 505 and 125 events for the final states $\pi^+\pi^+n$, $\pi^+\pi^+\pi^+\pi^-n$ and $\pi^+\pi^+\pi^+\pi^+\pi^-\pi^-n$ respectively, where χ^2 probability cuts of 4% for the $\pi^+\pi^+n$ state and 6% for the $\pi^+\pi^+\pi^+\pi^-n$ and $\pi^+\pi^+\pi^+\pi^+\pi^-\pi^-n$ states were required.

At this level, approximately 50% of the events assigned to the $2\pi n$ and $4\pi n$ states were ambiguous while 30% of the

events corresponding to the $6\pi n$ state were ambiguous. The lower ambiguity level for the $6\pi n$ state is consistent with the fact that most of the charged tracks in the bubble chamber were slow and hence more accurately measured. A further cut on the missing mass squared corresponding to $-1 \leq M_X^2 \leq 3$ reduced the ambiguity level considerably for the $2\pi n$ and $4\pi n$ states.

Eliminating the events in the tail seemed to help since these events were probably very badly measured events. It may be pointed out that for missing momentum ≥ 6 GeV/c, the ambiguity level was of the order of 10% and 15% for the $2\pi n$ and $4\pi n$ states respectively and there were no events for the $6\pi n$ states. However, since the number of events was reduced by a factor of almost five, it appeared that a large percentage of good events was being eliminated by this cut.

In the best fit sample of category C, for missing momentum ≥ 3 GeV/c, 272, 354 and 117 events were obtained for the $2\pi n$, $4\pi n$ and $6\pi n$ states respectively. The ambiguity level in this sample was found to be about 15% to 20%. Introducing the missing mass squared cut did not seem to affect the ambiguity level noticeably. The missing momentum ≥ 6 GeV/c cut gave very reliable sample of events (ambiguity level $\sim 10\%$) for this category of events.

Further requiring that the chosen final state not only be the best fit but in cases where there were other final state fits, the χ^2 probability of the best fit be at least ten times the χ^2 probability of the next best fit. 239, 307 and 106 events were obtained for the $2\pi n$, $4\pi n$ and $6\pi n$ states.

The ambiguity level for these events was found to be of the order of 10% to 15%. Such ambiguity levels are not uncommon for bubble chamber data.

The most exclusive sample of events for any final state consisted of events in category E. All events in this category are unique fits to the chosen final state. Final selection procedures are discussed later.

Analysis Based on 3C-Fit Sample

The 3C-fit sample consisted of events which made three-constraint fits to one or more final state hypotheses tried. Selection procedures were the same as for the 1C-fit sample. The χ^2 probability cut for the $2\pi n$, $4\pi n$ and $6\pi n$ states were 2%, 5% and 2% respectively. For missing momentum ≥ 3 GeV/c, 101, 142 and 40 events were obtained for the $2\pi n$, $4\pi n$ and $6\pi n$ states respectively in category B. For the $2\pi n$ states all events were also events that made best fits to that final state. Only 3% of the events in this state were ambiguous. For the $4\pi n$ state most of the events were also best fits to that state. However, the ambiguity level in this case was of the order of 18%. For the $6\pi n$ state, most of the events were also unique fits to that final state. Only 7% of the events were ambiguous.

B. Monte Carlo Simulation of Events and Geometrical Triggering Efficiencies

A complete Monte Carlo simulation of the detection and triggering system used in the experiment was performed. The

data was simulated by generating Monte Carlo distributions of Lorentz-invariant phase space weighted by a desired matrix element squared, and requiring the generated events to satisfy the kinematic and geometrical constraints used in the experiment. Two different models for the matrix element squared were used:

1. Model I

$$|M|^2 \propto \pi_i \exp(-B p_{\perp i}^2) \quad (71a)$$

2. Model II

$$|M|^2 \propto \exp(Au) \pi_i \exp(-B p_{\perp i}^2) \quad (71b)$$

where u is the four momentum transfer squared from the beam to the forward neutron and the sum over the transverse momentum squared extends over all final state particles. For details of the final parameter values of the two models, see chapter VI.

Events were generated with cuts on the forward neutron momentum, i.e., $p_n \geq 3$ GeV/c. Then the geometrical constraints of the forward detecting system and trigger requirements were imposed on the generated event to see if it would give rise to a trigger. The triggering efficiencies were then calculated by comparing relevant plots before and after applying the geometry and trigger requirements. The trigger required a forward neutral particle in the $\pm 9.5^\circ$ forward square cone with no charged forward particles in the above cone and that the forward neutron was energetic enough to cross at least two of the neutron counters. Several good events were therefore lost due to the geometry and trigger requirements. Some of the

factors to which the loss of events may be attributed are discussed below:

(i) Charged Tracks in the $\pm 9.5^\circ$ Forward Square Cone

For events with charged tracks in the forward cone, the measured azimuthal and dip angles at the interaction vertex in the bubble chamber were swum to the anticounter position using the measured charged track momentum and a detailed knowledge of the magnetic field in the region. If an event had one or more charged tracks intersecting the anticounters even after bending through the magnetic field, the event was antied out. The percentage of events vetoed depended on the final state reaction. For the $2\pi n$ state, 15 % of the events with forward-incident neutrons was antied out by the anticounters. The percentage of vetoed events was 38% and 69% for the $4\pi n$ and $6\pi n$ states respectively.

(ii) Neutral Particle Passes through the Hole

All spark chambers had a 1" x 6" central hole to allow the beam to pass through without interacting. Therefore, events with very fast forward neutrons failed to trigger the system because neutrons passed through the hole in the lead plates, spark chambers and neutron counters N_1 through N_3 . 4.7, 3.7, 2.8 percentage of all forward neutron events were lost in the hole for the $2\pi n$, $4\pi n$, and $6\pi n$ states.

(iii) Attenuation in the Anti-Counter Package

All forward neutral particles had to traverse (a) .05" of steel of the b.c. bellows, (b) .045" of liquid hydrogen bath tank and (c) .030" of steel of the vacuum tank wall besides (d).5" of scintillator plastic of the counters A_c and

A_γ and (e) 1" of lead plates between these counters. Depending on the inelastic cross sections (neutron; Fe, Pb, Scintillator), ≈13% of all incident neutrons interacted producing charged particles which vetoed such events. Depending on the elastic cross section of neutron with steel, lead and scintillator, a very small percentage of the incident neutrons were elastically scattered out of the spark chambers and never had a chance to trigger the system.

(iv) Neutral Conversion in the Spark Chamber

The neutral detector system consisted of approximately 17.8cm of steel, 10.2cm of aluminum and 1.2cm of plastic scintillator. The conversion of neutrons in the detector therefore depended on the elastic and inelastic (neutron, Fe), (neutron, Al) and (neutron, plastic scintillator) cross section. Calculation of these cross sections are carried out in detail in appendix VI. Various factors contributed to the loss of events, namely; (a) not all incident neutrons interacted in the steel or aluminum plates of the spark chambers; (b) some neutrons suffered elastic scattering and were scattered out of the spark chambers; (c) and in some cases the charged secondaries were too slow to go through two consecutive counters, or (d) only neutrals were produced in the final state.

A very detailed study of all possible (neutron, nucleon) reactions to various final states using the Monte Carlo program NVERTEX and known cross sections for these reactions was carried out to obtain neutron conversion efficiency of 78%. Further details are discussed in chapter VIC.

TABLE 8

MONTE CARLO GEOMETRICAL TRIGGERING EFFICIENCIES

$$|M|^2 = e^{-15 \sum_i p_i^2} \quad p_n > 3 \text{ GeV}/c$$

Final State	$\epsilon_t\%$ $M_{X^{++}} < 1.5 \text{ GeV}$	$\epsilon_t\%$ $1.5 < M_{X^{++}} < 2.5$	$\epsilon_t\%$ $M_{X^{++}} > 2.5$	$\epsilon_t\%$ $M_{X^{++}} > \text{threshold}$ complete mass-spectrum
$\pi^+ \pi^+ n$	60	38	13	43
$\pi^+ \pi^+ \pi^+ \pi^- n$	34	34	27	30
$\pi^+ \pi^+ \pi^+ \pi^+ \pi^- \pi^- n$		20	19	19

Table 8 shows the Monte Carlo generated geometrical triggering efficiencies as a function of the invariant mass $M_{X^{++}}$ for decays into 2π , 4π , 6π states. The mass region was divided into three regions, namely: (a) $M_{X^{++}} < 1.5$ GeV, (b) $1.5 \text{ GeV} < M_{X^{++}} < 2.5 \text{ GeV}$, and (c) $M_{X^{++}} > 2.5 \text{ GeV}$. For $M_{X^{++}} \rightarrow \pi^+\pi^+$, the geometrical triggering efficiencies varied from 60% at low masses to about 13% at higher masses. For the decay mode, $M_{X^{++}} \rightarrow \pi^+\pi^+\pi^+\pi^-$, the triggering efficiency remained more or less constant, about 30%. Finally for $M_{X^{++}} \rightarrow \pi^+\pi^+\pi^+\pi^-\pi^-$, the triggering efficiency remained about 20%. The decreasing trigger efficiencies with increasing charge multiplicity was attributable mostly to forward charged particles hitting the anticounters.

C. Definitions of Various Useful Terms

(a) The microbarn cross section equivalent for a single interaction is defined as:

$$\sigma_1 = \frac{\sigma(\mu b)}{N_{\text{events}}} = \frac{10^{30}}{n_h \times l_f \times N_{\text{beam}}} \quad (73)$$

where

$$n_h = \frac{\rho_h \times N_{\text{Av}}}{M_{\text{H}_2}} \times n_{\text{atoms/molecule}}$$

and

ρ_h = density of liquid hydrogen

N_{Av} = 6.023×10^{23} molecules/mole

M_{H_2} = gram molecular weight of H_2

$n_{\text{atoms/molecule}}$ = hydrogen atoms in H_2 molecule

so that n_h is the density of hydrogen atoms or protons in the target. Further,

l_f = fiducial length in cm along the beam
observed interaction are measured.

N_{beam} = total effective beam flux, which produced
the observed triggers.

(b) The raw events per microbarn is defined as:

$$N_{\mu b}^{\text{raw}} = N_{\mu b}^r = \frac{l}{\sigma_l} = n_h \times l_f \times N_{\text{beam}} \times 10^{-30} \quad (74)$$

(c) The effective events per microbarn is then defined
as:

$$\begin{aligned} N_{\mu b}(\text{effective}) &= N_{\mu b}^e \\ &= N_{\mu b}^r \times \epsilon_{\text{attenuation}} \times \epsilon_{\text{flare}} \\ &\quad \times \epsilon_{\text{electronics}} \times \epsilon_{\text{b.c. scanning}} \\ &\quad \times \epsilon_{\text{TVGP failure}} \times \epsilon_{\text{M.C. geometrical}} \\ &\quad \quad \quad \text{trigger efficiency} \end{aligned} \quad (75)$$

where the various ϵ 's represent correction factors due to various sources of event losses. The individual corrections are discussed below.

(d) The channel cross section is then defined as:

$$\sigma(\mu b) = N_{\text{events}}(\text{channel}) / N_{\mu b}(\text{effective}) \quad (76)$$

D. Discussion of Corrections to the Raw Events per Microbarn

(a) Flare Corrections

About 6% of the events in the fiducial volume were lost because the event vertex was obscured in the flare on the film

from bubble chamber camera flash lights. The flare correction was obtained by plotting the x-vertex of all events in the fiducial length and counting the number of events in the hole below the smooth line for the expected vertex distribution. Thus, only 94% of the total number of events were actually observed, giving $\epsilon_{\text{flare}} = .94$.

(b) Bubble Chamber Scanning Efficiency

The bubble chamber scanning efficiencies were found to be topology dependent and relatively high because there were no more than three interactions per frame. Thus the pictures were relatively clean compared to conventional untriggered bubble chamber pictures. The details of the calculations of the efficiencies are given in appendix IIC, while their values are listed below.

<u>Topology</u>	<u>B.C. Scanning</u>
102: 2 prongs	.969
104: 4 prongs	.989
106: 6 prongs	.989
112: 2 prongs + V_0	.860

(c) Geometry of TVGP Failures

Less than 2% of the measured events were failed by the geometric reconstruction program TVGP. The failure rate was slightly higher for the V events.

<u>Topology</u>	<u>TVGP Failures Rates Percentage</u>
102	.8
104	.9
106	1.5
108	.5
112	1.2
114	1.1

(d) Special Geometry Failures

Ten to fifteen per cent of the events were poorly measured. Tracks which were very fast, or very short, or very sharply dipping had TVGP reconstructed momentum errors as large as the magnitude of the momentums themselves. Such tracks were designated as non-stopping two pointers, the measured curvature information was ignored; the angles were then calculated by regarding the track as a two-pointer and then approximating it by the last measured point and the vertex. Events with one badly measured track besides a missing neutral track failed to make 1C-fits to the final state hypotheses tried. They made 0C-fits while at the 3C level they made 2C-fits. The failure rates are tabulated below.

<u>Topology</u>	<u>Failure Rate</u>
102	.134
104	.114
106	.099

E. Calculation of Raw and Effective Events per Microbarn

Raw Number of Events per Microbarn

Using a fiducial length of 22.7 cm, a total effective beam flux of 1.111×10^8 tracks (see Appendix IV), and a proton density of $.37046 \times 10^{23}$ per cubic centimeter (based on liquid hydrogen density of .062 grams per cubic centimeter) and the definition

$$N_{\mu b}^r = \frac{1}{\sigma_l} = n_h \times l_f \times N_{\text{beam}} \times 10^{-30}$$

for the raw events per microbarn (see Chapter IV, section C), the number 93.34 events per microbarn was obtained for the sample of events being reported here.

Events per Microbarn (Corrected for Attenuation, Flare and Electronics)

Taking into account beam attenuation losses of 5% and flare correction losses of 6% and using electronic triggering efficiency of 98.5% one obtained the corrected value of 82.1 events per microbarn, i.e., $N_{\mu b}^r = 93.34 \times .95 \times .94 \times .985 = 82.1$ events/ μb .

Events per Microbarn (Corrected for Scanning and Geometry Failures)

Scanning efficiencies, TVGP reconstruction efficiencies and corrections for non-stopping two pointers are all topology dependent. Values of the events per microbarn corrected for these factors were obtained using the relation:

$$N_{\mu b}^{r''} = N_b^r \times \epsilon_{\text{scanning}} \times \epsilon_{\text{TVGP}} \times \epsilon_{2 \text{ pointers}}$$

and the values of the above ϵ 's as given in section D. The corrected values are tabulated below:

<u>Topology</u>	<u>Events per microbarn</u>
102	68.3
104	71.3
106	72.1
112	69.2

To obtain the effective events per microbarn for the various final state reactions, the last set of topological events per microbarn values have to be multiplied by the Monte Carlo generated geometrical triggering efficiencies for these final states. These values are discussed in the next chapter.

CHAPTER V

RESULTS FOR THE SEARCH FOR BACKWARDS-PRODUCED
EXOTIC MESON RESONANCES

A. Comparisons with Known Measurements

The Reaction $\pi^+ + p \rightarrow K^{*+}(890) + Y^{*+}(1385)$

The experiment was sensitive to the reaction:

The final state observed is $\pi^+ \pi^+ \Lambda^0 K^0$ where the Λ^0 was observed as neutral associated V^0 in the bubble chamber while the K_L^0 (from K^0) was observed as a fast forward neutral particle in the detector downstream. Such events were classified as belonging to 112 topology. The b.c. scanning efficiency of .87 for the 112's was lower than the efficiencies of events without associated V^0 because sometimes the event appeared as a four-prong event. TVGP reconstruction efficiency was high, only 1.2 per cent of the events were failed by TVGP.

The invariant mass of the oppositely charged tracks corresponding to the neutral V^0 in the bubble chamber was calculated and plotted from the measured energy-momentum four vectors for these tracks with three different sets of mass interpretations, namely: (i) $e^+ e^-$, (ii) $\pi^+ \pi^-$, (iii) $p \pi^-$.

Kinematic Program SQUAW was used to fit the TVGP reconstructed V^0 events to the reaction hypothesis:

where K^0 was designated as the missing neutral track. One constraint fits to the above reaction hypothesis were obtained. The invariant mass of $K^0\pi^+$ combination was plotted against the invariant mass of the $\pi^+\pi^-p$ combination and enhancements corresponding to the K^{*+} and Y^{*+} mass bands were observed. Requiring cuts on the K^{*+} and Y^{*+} mass distribution, 13 events were obtained for the final state $K^{*+}Y^{*+}$.

In chapter 4, the uncorrected statistical level of the experiment was calculated as 93.34 events per microbarn. The following corrections for loss of events due to various factors were made: (a) The bubble chamber scanning efficiency contributed a factor $\epsilon_{\text{scanning}} = .87$

(b) The TVGP reconstruction failure rate of 1.2% gave the factor $\epsilon_{\text{TVGP}} = .988$.

(c) Only K_L^0 's were detected in the spark chamber and corresponding to the $K_S^0 - K_L^0$ branching ratio of 50%, a factor $\epsilon_{K_S^0/K_L^0} = .5$ was needed.

(d) For the $\Lambda^0 \rightarrow p\pi^-$ branching ratio of 64.2% the $\epsilon_{\Lambda^0 \rightarrow p\pi^-} = .642$ was introduced.

(e) Finally the $K^{*+}(890) \rightarrow K^0\pi^+$ gave rise to the factor $\epsilon_{K^{*+} \rightarrow K^0\pi^+} = .667$.

Thus corrected for the above factors and using:

$$N_{\mu b}^{r'} = N_{\mu b}^r \times \epsilon_{\text{scanning}} \times \epsilon_{\text{TVGP}} \times \epsilon_{K^{*+} \rightarrow K^0\pi^+} \\ \times \epsilon_{K_S^0/K_L^0} \times \epsilon_{\Lambda^0 \rightarrow p\pi^-}$$

the corrected number of events per microbarn came to 17.189.

Using the Monte Carlo program NVERTEX, events were generated corresponding to the reaction under study and required to satisfy the geometrical and trigger constraints using the experimental-dependent program MCE82.⁵⁴ The low triggering efficiency of 7% was mostly due to the fact that the π^+ 's from the forward K^0 -decays vetoes the events. Thus, the effective statistical level for this sample corresponded to 1.203 events per microbarn.

The cross-section for the reaction $\pi^+ + p \rightarrow Y^{*+}(1385) K^{*+}(890)$ was, therefore, obtained as $10.8 \pm 5 \mu\text{b}$. This is in substantial agreement with previous measurements⁵³ of $10 \pm 4 \mu\text{b}$.

The Reaction $p + p \rightarrow \Delta^{++}(1236)n_{\text{fwd}}$

About 15000 pictures were taken with an incident proton beam at 8.4 GeV/c. The trigger requirements were identical to those in the case of the incident pion beam. The analysis presented here was based on a sample of 2527 events in a fiducial length defined as 25.4 cm and corresponded to a statistical level of 1.5 raw or uncorrected events per microbarn.

All analysis of the above sample was done at Purdue University using travelling microscopes for measurements and on-line geometry reconstructions with the Purdue version of TVGP. TVGP reconstruction efficiency was almost 100%. Fig. 27a shows a plot of the missing mass squared for a partial sample of the data, based on the mass interpretation of the tracks in the bubble chamber as a proton and positive pion. The neutron peak is easily identifiable for missing momentum

FIGURE 27

≥ 4 GeV/c. Fig. 27b shows the invariant mass plot for the unfitted data corresponding to the mass interpretation ($p\pi^+$) for the charged tracks in the bubble chamber. The $\Delta^{++}(1236)$ peak is very easily identified.

$$\begin{array}{lcl} \text{The reaction: } p + p \rightarrow \Delta^{++} n_{\text{fwd}} & & (79) \\ & \rightarrow p\pi^+ & \end{array}$$

was analyzed for comparison with previous measurements. All events were fitted to the above reaction hypothesis together with other possible hypotheses using the kinematic fitting program SQUAW. Since no neutral track information was provided at this point, only one constraint fits to various hypotheses tried were obtained. Event selection to the reaction hypothesis above was based on the following cuts or requirements: (i) The event made a one-constraint fit to the reaction hypothesis of interest with χ^2 -probability or confidence level of the fit $\geq 10\%$, (ii) The fitted missing momentum for the fit was ≥ 4 GeV/c, (iii) The invariant mass for the track combination ($p\pi^+$) was ≤ 1.475 GeV.

On the basis of the above selection procedure, 265 events were obtained corresponding to the reaction $p + p \rightarrow \Delta^{++} n_{\text{fwd}}$. Of these, about 80% were unique fits to the above hypothesis.

The statistical level of the raw data corresponded to 1.5 events per microbarn. A scanning efficiency factor of .95, a flare correction factor of .94, a factor of .994 for the fractional decay rate of $\Delta^{++} \rightarrow p\pi^+$, a χ^2 -probability cut correction factor of .9 and finally a Monte Carlo generated

geometrical and electronic triggering efficiency factor of .35 reduced the statistical level of the data to correspond effectively to .47 events per microbarn. Therefore, 265 events corresponded to 560 ± 280 microbarns. This may be compared to the previous measurement⁵⁵ of 675 ± 225 microbarns.

B. Search for Narrow Exotic Resonances in $X^{++} \rightarrow \pi^+\pi^+$,
 $\pi^+\pi^+\pi^+\pi^-$, or $\pi^+\pi^+\pi^+\pi^-\pi^-$

Invariant Mass Resolutions for Exotic Final States

The errors in the invariant masses for exotic meson channels $\pi^+\pi^+$, $\pi^+\pi^+\pi^+\pi^-$ and $\pi^+\pi^+\pi^+\pi^-\pi^-$ were calculated using SQUAW (see Appendix III), both at the one-constraint fit level and three-constraint fit level. The error in the invariant mass or the exotic channel mass resolution depended on the constraint class, the final state being considered and the mass region being studied. At the 3C-fit level, for the $\pi^+\pi^+$ state, the mass resolution varied from an average of 10 MeV for lower masses (<1500 MeV) to an average of 27 MeV for higher masses (>1500 MeV). In the lower mass region (<1500 MeV), the 1C-fit resolution was comparable with the 3C-fit resolution in the same region. The events in this region corresponded to very fast forward neutrons so that the charged tracks in the bubble chamber were slow and were, therefore, more accurately measurable. At higher masses (>2000 MeV), the 1C mass resolution was of the order of 120 MeV. The 3C mass resolution for the final state $\pi^+\pi^+\pi^+\pi^-$

FIGURE 28

remained about 25 MeV over the whole mass spectrum. The 1C mass resolution for this state was again comparable to the 3C mass resolution for lower mass regions (<1500 MeV). The 1C mass resolution for higher masses (>1500 MeV) averaged about 60 MeV. For $\pi^+\pi^+\pi^+\pi^-\pi^-$ state, the 3C mass resolution averaged about 18 MeV, while the 1C mass resolution averaged about 24 MeV.

Invariant Mass Distributions for the Multipion Exotic States

Multipion invariant mass distributions corresponding to the exotic meson resonance decays, $X^{++} \rightarrow \pi^+\pi^+$, $\pi^+\pi^+\pi^+\pi^-$, and $\pi^+\pi^+\pi^+\pi^-\pi^-$ were plotted from three different categories of events where the categories depended on the selection criteria used. The categories are discussed below:

(i) Inclusive Category of One Constraint Fits

Fig. 28, parts (a), (b) and (c) show the invariant mass distributions for the states $\pi^+\pi^+$, $\pi^+\pi^+\pi^+\pi^-$ and $\pi^+\pi^+\pi^+\pi^-\pi^-$ respectively and which are based on the one constraint fits to these states. The solid histogram in parts (a), (b) and (c) are based on the most inclusive selection procedure. This sample included any event which made a one constraint fit to the final state of interest with or without fits to other final states. This missing neutral momentum was required to be greater than or equal to 3 GeV/c and χ^2 probability of the fits were required to satisfy the cuts

- (a) χ^2 probability $\geq 4\%$ for $n_{\text{fwd}} \pi^+\pi^+$
- (b) χ^2 probability $\geq 6\%$ for $n_{\text{fwd}} \pi^+\pi^+\pi^+\pi^-$
- (c) χ^2 probability $\geq 6\%$ for $n_{\text{fwd}} \pi^+\pi^+\pi^+\pi^-\pi^-$

464, 505 and 125 events were obtained for the 2π , 4π , 6π states respectively according to this procedure.

As discussed in chapter IV, section A, about 50% of the events in any final state selected according to the above procedure made ambiguous fits, that is, an event made acceptable 1C fits to states other than the state being considered. Clearly this procedure of identifying an event as belonging to a given final state is not very satisfactory. However, if narrow resonances were being produced strongly, they would tend to populate a few bins while the invariant masses corresponding to misidentified events would be more dispersed over the total mass range. Consequently narrow resonances, being produced strongly would show up as clearly identifiable enhancements over the general background.

Invariant mass combination $n\pi^+$ from the final state $n_{\text{fwd}}\pi^+\pi^+$ where the events to this state were selected according to the above procedures, showed a clear enhancement corresponding to the $N^*(1688)$ resonance. See chapter 6, section E. It may be restated here that the invariant mass resolution of the 1C-fit sample is nearly as good as that of the 3C-fit sample. However, the 1C-fit sample has the added advantage of higher statistics.

No evidence for exotic resonance formation in the 2π , 4π , or 6π invariant mass distribution was seen. The upper solid curves in Fig. 28 are third or fourth order polynomial fits to the solid histograms. Confidence levels of 60% or greater for the polynomial fits to the above histograms were interpreted as quantitative evidence that there was no narrow (100 MeV)

exotic resonance formation in these channels. The highest deviation of the histogram from the smooth polynomial fit curve was less than three standard deviations.

(ii) Best Fit Category of One Constraint Fits

The dashed histograms in Fig. 28, parts (a), (b) and (c) correspond to the 2π , 4π and 6π exotic channels and are based on the best one constraint fit sample for which the event selection procedure required that (a) each event be an overall best fit to the final state of interest; (b) the χ^2 probability of the best fit was at least ten times the χ^2 probability of the next best fit; (c) the missing mass squared corresponding to the fit satisfied the cut, $-1 \text{ (GeV)}^2 \leq M_X^2 \leq 3 \text{ (GeV)}^2$; (d) the missing neutron momentum satisfied the cut, $P_n \geq 3 \text{ GeV/c}$; and (e) the cuts on the χ^2 -probability for the three exotic states under consideration satisfied the same cuts as the last category of events.

The above selection procedure is much more reliable than the last one. 165, 293 and 103 events corresponding to the 2π , 4π and 6π states respectively were obtained according to this procedure. The ambiguity level, or the per centage of events with ambiguous fits, is less than 15% at this level.

Again no evidence for exotic meson resonance formation was noticed. The lower smooth curves in Fig. 28 are third or fourth order polynomial fits to the dashed histograms and confidence levels of 60% or more for the polynomial fits to the data were interpreted to mean absence of exotic meson resonance formation in the 2π , 4π and 6π channels.

FIGURE 29

(iii) Three Constraints Fit Category

Fig. 29, parts (a), (b) and (c) show the invariant mass distributions corresponding to the 2π , 4π and 6π exotic channels, based on the 3C fit sample. This is the most exclusive selection procedure. In some cases, more than one fit to a given final state were obtained depending on which neutral coordinate was used for the fit. The selection procedure chose the best fit among the successful fits to the final state. Missing momentum greater than 3 GeV/c was required for the chosen fit. Two, five and two per cent cuts respectively were imposed on the χ^2 -probabilities of fits to the 2π , 4π and 6π states respectively. 101, 145 and 40 events were obtained for the 2π , 4π and 6π states respectively according to the above selection procedure. The ambiguity level for the 2π state was three per cent while that for 6π state was seven per cent and was about 18% for the 4π state. In this respect, the 3C fit category is comparable to the best fit 1C category.

The 3C fit sample is especially meaningful for understanding the shapes of different distributions. The smooth curves shown on these plots were obtained from a Monte Carlo simulation of the data (see chapter 6) where the Lorentz invariant phase space was modified by the squared matrix element

$$|M|^2 = \pi e^{-1.5 \sum_i p_{\perp i}^2} \quad (80)$$

The index i refers to each final state particle, including the neutron and $p_{\perp i}$ is the transverse momentum of particle i . The same matrix element was used for each final state. The good

agreement between the model and data is consistent with the absence of any noteworthy dynamical features in the mass spectra. Any deviation from the model would have been interpreted as indicative of quasi-many-body intermediate states in the above reactions.

C. Upper Limits on the Production Cross-Sections of Narrow Exotic Resonance into 2π , 4π , and 6π States

Four Standard Deviation Criterion

In three different categories of events corresponding to the 2π , 4π and 6π exotic channels, no evidence for resonance production was noted. Upper limits for exotic meson resonance production may be placed on the basis of any of the three selection procedures discussed in section B. The most liberal upper limits may be based on the most inclusive sample represented by the solid histograms in Fig. 28, parts (a), (b) and (c). Narrow resonance production in any of the channels considered above would appear as sharp deviations in the solid histograms from the smooth polynomial-fitted curve. The deviations may be positive or negative with respect to the smooth curve. Fluctuations in the data from the smooth polynomial fit at the level of $\pm$ four or fewer standard deviations are considered to be statistical in nature, not signifying any special dynamical feature in the data.

The upper limits reported here were computed from the smooth polynomial fits to the solid histograms in Fig. 28. The area under the smooth curve was constrained to equal the

area of the corresponding histogram during the fitting procedure. If n is the number of events in a 100 MeV bin under the smooth curve, and a number N is defined as:

$$N - 4\sqrt{N} = n \quad (81)$$

N may be interpreted as the expected number of events in the above bin such that a four standard deviation, downward, statistical fluctuation reduces the number of events in the bin to the observed number n under the smooth curve. Then $N - n$, the expected number of events in the bin above the smooth curve was defined as the upper limit for resonance production in the bin. The criterion used for calculating the upper limit for narrow exotic meson resonance production is arbitrary but since the systematic uncertainties are large (~ 50 per cent, discussed later), a liberal estimate of the upper limits is reasonable.

Monte Carlo Triggering Efficiencies and Events to Microbarn Conversion

To calculate the geometrical triggering efficiencies, the Monte Carlo program NVERTEX⁵⁶ was used together with the experiment-dependent program MCE82 which took into consideration the geometrical constraints and triggering requirements of the experiment. NVERTEX was required to generate events for the reaction

$$\pi^+ + p \rightarrow n + X^{++}$$

and then X^{++} was allowed to decay in several different ways depending on the exotic channels whose triggering efficiencies

FIGURE 30

were being calculated; for example:

$$\text{i) } X^{++} \rightarrow \pi^+ \pi^+ \quad (82a)$$

$$\text{ii) } X^{++} \rightarrow \pi^+ \pi^+ \pi^+ \pi^- \quad (82b)$$

$$\begin{aligned} \text{iii) } X^{++} &\rightarrow \pi^+ X'^+ \\ &\quad \downarrow \pi^+ X''^0 \\ &\quad \downarrow \pi^+ \pi^- \end{aligned} \quad (82c)$$

$$\text{iv) } X^{++} \rightarrow \pi^+ \pi^+ \pi^+ \pi^+ \pi^- \pi^- \quad (82d)$$

Various masses for the X^{++} were tried; uniform and spin-dependent decay distributions for the X^{++} were used and the effect of various intermediate states in the X^{++} decay was also studied. For details, see Appendix V.

The triggering efficiencies varied between 30 to 50 per cent over the complete mass spectrum and were more or less insensitive to the details of the decay and to the decay charge multiplicities.

Fig. 30, parts (a), (b) and (c) show the upper limits for exotic meson resonance production in channels $\pi^+ \pi^+$, $\pi^+ \pi^+ \pi^+ \pi^-$ and $\pi^+ \pi^+ \pi^+ \pi^+ \pi^- \pi^-$ respectively plotted as a function of the channel mass. For the 2π -exotic channel, the upper limit for narrow (100 MeV) exotic meson resonance was of the order of one microbarn per 100 MeV interval and remained nearly constant over the whole mass range (≤ 3000 MeV). In the case of the 4π -exotic channel, the upper limit increased for higher masses (~ 2000 MeV) and again started falling (> 2700 MeV), remaining of the order of 1.-2 microbarns per 100 MeV interval over the mass range accessible (≤ 3000 MeV). However, for the 6π -exotic channel, the upper limit for narrow exotic meson

resonance production increased from about .6 to 1.6 microbarns per 100 MeV interval over the mass range, 1.8 GeV to 3.05 GeV.

D. Cross Sections of the $(\pi^+\pi^+, \pi^+\pi^+\pi^-\pi^-, \pi^+\pi^+\pi^+\pi^-\pi^-)n_{\text{fwd}}$
Final States

Cross sections for the reactions

$$\pi^+ + p \rightarrow n_{\text{fwd}} + \begin{matrix} \pi^+\pi^+ \\ \pi^+\pi^+\pi^+\pi^- \\ \pi^+\pi^+\pi^+\pi^-\pi^- \end{matrix}$$

where the momentum of the forward neutron was required to be greater than or equal to 3 GeV/c, were calculated from the upper and lower polynomial fitted curves shown in Fig. 28. Thus, upper and lower limits for the cross sections to the reactions defined above were obtained. The higher values are certainly overestimates, particularly at large X^{++} masses; since at large masses, the resolution is poor and the probability of misinterpreting events is larger. The lower values are probably more realistic estimates of the cross-sections.

The accessible exotic channel mass region was divided into three mass regions, namely

- i) $M_{X^{++}} < 1.5 \text{ (GeV)}$
- ii) $1.5 \text{ (GeV)} \leq M_{X^{++}} \leq 2.5 \text{ (GeV)}$
- and iii) $M_{X^{++}} > 2.5 \text{ (GeV)}$

In chapter IV, section E, the topologically dependent events per microbarn equivalents were calculated. To obtain the finally corrected events per microbarn equivalents for the

TABLE 9

CORRECTED EVENTS PER MICROBARN
(INCLUDING MONTE CARLO TRIGGERING EFFICIENCIES)

Final State	$M_{X^{++}} < 1.5 \text{ (GeV)}$ Events/ μb	$1.5 \text{ (GeV)} < M_{X^{++}} < 2.5 \text{ (GeV/c)}$ Events/ μb	$M_{X^{++}} > 2.5 \text{ (GeV)}$ Events/ μb	Complete Spectrum Events/ μb
$\pi^+ \pi^+ n_{\text{fwd}}$	40.98	25.95	8.88	29.37
$\pi^+ \pi^+ \pi^+ \pi^- n_{\text{fwd}}$	24.24	24.24	19.25	21.39
$\pi^+ \pi^+ \pi^+ \pi^+ \pi^- \pi^- n_{\text{fwd}}$		14.00	13.70	13.70

TABLE 10

FINAL STATE CROSS SECTIONS

Final State	$M(X^{++}) \leq 1.5 \text{ GeV}$	Cross Section(μb)		Complete Mass Spectrum
		$1.5 \text{ GeV} < M(X^{++}) \leq 2.5 \text{ GeV}$	$2.5 \text{ GeV} > M(X^{++})$	
$\pi^+ \pi^+ n_{\text{fwd}}$	2.6 - 6.30	2.1 - 7.6	.7 - 3.3	5.7 - 16.4
$\pi^+ \pi^+ \pi^+ \pi^- n_{\text{fwd}}$	1.1 - 1.36	8.4 - 13.3	4.5 - 9.4	14.8 - 25.1
$\pi^+ \pi^+ \pi^+ \pi^+ \pi^- \pi^- n_{\text{fwd}}$		2.6 - 3.1	5.3 - 6.4	8.0 - 9.7

different exotic channels and different mass regions, the multiplication factor corresponding to the Monte Carlo generated triggering efficiencies for the above channels as a function of the mass regions are required. These are given in Table 8 and details of their calculation are discussed in chapter IV, section B. Table 9 gives the events per microbarn conversion factor corrected for the Monte Carlo triggering efficiencies. The statistical level of the data corresponds to about 30 events per microbarn for the $2\pi n$ channel, to about 22 events per microbarn for the $4\pi n$ channel and to about 14 events per microbarn for the $6\pi n$ channel.

Table 2 gives the different final state cross sections. The cross section for the final state $\pi^+\pi^+n_{fwd}$ is of the order of 10 microbarns, while that for the final state $\pi^+\pi^+\pi^+\pi^-n_{fwd}$ is of the order of 10 microbarns again. For the $2\pi n_{fwd}$ state, the cross section is made up mostly of contributions from the lower and intermediate mass regions, $M_{X^{++}} < 2.5$ (GeV); whereas for the $4\pi n_{fwd}$ state, most of the contribution to the total cross section (3 GeV/c) comes from the intermediate and higher mass region, 1.5 (GeV) $\leq M_{X^{++}} \leq 3.0$ (GeV). In the case of the $6\pi n$ state, 65 per cent of the contribution to the total cross section ($P_n \geq 3$ GeV/c) comes from the high mass region, 2.5 (GeV) $\leq M_{X^{++}} \leq 3.0$ (GeV).

E. Search for Exotic Resonances Decaying into Baryon-Antibaryon Final State ($\bar{p}p\pi^+\pi^+$)

In chapter 1, it was discussed that exotic mesons, if they existed, would decay more readily into baryon-antibaryon

states rather than multi-meson states. The decay channel $\bar{p}p\pi^+\pi^+$ is therefore of considerable interest. However, when the missing mass squared corresponding to the mass interpretation ($\bar{p}p\pi^+\pi^+$) for the charged tracks in the bubble chamber were plotted, no peak corresponding to the neutron signal was seen. Event selection at the 1C-fit level was carried on in the same way as for the all-meson decay channel. No events with unique fits to the final state $(\bar{p}p\pi^+\pi^+)n_{fwd}$ were found. All events were ambiguous with fits to final states with forward K^0 's or $\bar{K}^0$'s. None of the selection procedures at the 1C-fit level appeared reliable. Cross sections calculations were therefore based on the 3C-fit sample of the data.

At the three-constraint fit level, there were only 18 events which made a fit to the final state $\bar{p}p\pi^+\pi^+n_{fwd}$ with a kinematic χ^2 -probability greater than 4%. Of these events only 3 events made unique fits to the above final state. All other events made fits to other final states besides the state $\bar{p}p\pi^+\pi^+n_{fwd}$, for example, the final state $K^+K^-\pi^+\pi^+n_{fwd}$ or final states with forward K^0 's or $\bar{K}^0$'s. The invariant mass of the $(\bar{p}p\pi^+\pi^+)$ combination for the 18 candidates varied between 2.45 and 3.5 GeV and showed no indication of any mass enhancements which would correspond to exotic meson resonance production.

Upper limits for the production of exotic meson resonances decaying into the $\bar{p}p\pi^+\pi^+$ final state were calculated by first generating the Monte Carlo geometrical triggering efficiencies for the reaction hypothesis

$$\pi^+ + p \rightarrow X^{++} + n_{fwd} \quad (83)$$

$$\quad \quad \quad \downarrow$$

$$\quad \quad \quad \rightarrow \pi^+ \pi^+ \bar{p}p$$

using NVERTEX and the experiment-dependent program MCE82 (see Appendix V, Table 15). The average geometrical triggering efficiency came out to be about .16. According to chapter IV, section C, the statistical level for the four-prong events corresponded to 71.3 events per microbarn after applying geometrical triggering efficiency factor was reduced to 11.4 events per microbarn. Therefore, 18 events converted to 1.6 microbarns while 3 events converted to .26 microbarns. However, since these numbers were calculated from the 3C-fit sample of the data, the upper limits obtained may be low. In chapter III, section A, for the reaction hypothesis $\pi^+\pi^+\pi^+\pi^-n_{\text{fwd}}$ there were 145 events at the 3C-fit level corresponding to 415 events at the 1C-fit level, a reduction factor of .35 was involved in going from the 1C-fit level to the 3C-fit level. Thus, using the same reduction factor for the $\pi^+\pi^+\pi^-\bar{p}pn_{\text{fwd}}$ reaction hypothesis, the range of the upper limits of cross section for exotic meson resonances decaying into the final state $\bar{p}p\pi^+\pi^+$ were placed as .74 microbarns and 4.6 microbarns.

REFERENCES

53. Aderholz, et al., NPB 11 (1969), 259.
54. MCE82 A Monte-Carlo program which takes into account the geometrical constraints of the experimental set-up. Details are discussed in Chapter VIC.
55. Ginestet, et al., NPB 13 (1969), 283.
56. NVERTEX A Monte Carlo Program for generation of multi-vertex, multiprong events corresponding to high energy reactions. MIT Programming Notes, C. Bordner, A. Brenner, E. Ronat.

CHAPTER VI

STUDY OF PARTICLE SPECTRA AND CORRELATIONS
 IN THE REACTION $\pi^+ + p \rightarrow n(\text{forward}) + (2\pi, 4\pi, 6\pi)$
 AT INCIDENT π^+ MOMENTUM OF 8.4 GeV/c

A. Introduction

Particle Spectra and invariant mass distributions in the baryon exchange reactions

$$\pi^+ p \rightarrow \pi^+ \pi^+ n(\text{forward}) \quad (84a)$$

$$\rightarrow \pi^+ \pi^+ \pi^+ \pi^- n(\text{forward}) \quad (84b)$$

$$\rightarrow \pi^+ \pi^+ \pi^+ \pi^+ \pi^- \pi^- n(\text{forward}) \quad (84c)$$

at 8.4 GeV/c incident pion momentum have been studied and the comparison of the data with two different modified phase space models have been made. The experimental geometry requires the neutron production angle to be less than 9.5 degrees in the laboratory system. A further cut is placed on the forward neutron momentum at the analysis level. Only events with forward neutron momentum greater than 3 GeV/c are included in this study. Further, in order that the analysis reported in this chapter be representative of the reactions considered above, the most exclusive sample of the data is included in this study. In chapter IV, the different methods of assigning events to various final states is discussed in detail. Only those events which make a 3C-fit to a given final state are included. For the $2\pi n$ state, 97% of the events are unambiguous fits to that state, which means that event makes a 3C-fit to that state only. For the $4\pi n$ state, less than 18% of the

events are ambiguous; while in the case of $6\pi n$ state, only 7% of the events are ambiguous. In virtually all cases (>98%) where the event makes 3C-fits to states other than multipion-neutron state under consideration, the multipion-neutron fit has a higher χ^2 -probability for the fit than the χ^2 -probabilities for the other fits to the event.

The 3C-fit data used in this study consists of, after imposition of a cut on the χ^2 -probability to exclude interpretations with probability near zero, 101 events for the reaction $\pi^+ p \rightarrow \pi^+ \pi^+ n_{\text{forward}}$, 137 events for the reaction $\pi^+ p \rightarrow \pi^+ \pi^+ \pi^+ \pi^- n_{\text{forward}}$, and 40 events for the reaction $\pi^+ p \rightarrow \pi^+ \pi^+ \pi^+ \pi^+ \pi^- \pi^- n_{\text{forward}}$.

B. Monte Carlo Simulation of Data

The Pierls-Slansky Monte Carlo Event Generation program MCEG⁵⁷ was used to generate events corresponding to the reactions (84a), (84b) and (84c). Program MCEG throws events according to a Lorentz invariant phase space model, modified by a squared matrix element factor. The two models we discuss in this section correspond to two different choices for the matrix element weighting factor. The events thus generated are then checked to see if the forward neutron momentum is greater than 3 GeV/c and whether all produced tracks satisfy the geometrical constraints of the bubble chamber and forward neutron-detector system. This is done through a call to program MCE82 which we discuss in detail in the next section. The matrix elements corresponding to the two models studied are given below.

(i) Modified Phase Space Model I

In this model the phase space is weighted by the squared matrix element

$$|M|^2 \propto \pi \exp(-B p_{\perp i}^2) \quad (85)$$

The index i extends to all final state particles, including the neutron, and $p_{\perp i}$ is the transverse momentum of particle i . This form of $|M|^2$ was chosen as the simple matrix element which incorporates transverse momentum damping which is an universal feature of all high energy data. The value of the parameter B was determined by fitting the Monte Carlo generated smooth curves of $p_{\perp}^2$ and $p_{\parallel}$ distributions to the $p_{\perp}^2$ and $p_{\parallel}$ distributions for the real data. The same value, $B = 1.5 \text{ (GeV/c)}^{-2}$ gave good fits to the data for all three reactions studied. This model is referred to as MPSI and is represented as a solid curve in Figs. 31 through 41.

(ii) Modified Phase Space Model II

The matrix element in this case incorporates a u -dependence over and above the exponential $p_{\perp}^2$ dependence, where u is the squared four momentum transfer from the incident pion to the outgoing neutron. The purpose of the u -dependence is to study if better fits to the data are obtained, suggesting a baryon-exchange mechanism for the reactions. Thus

$$|M|^2 \propto \exp(Au) \pi \exp(-B p_{\perp j}^2) \quad (86)$$

The index j extends over all final state pions. The best values for the parameters A and B were obtained as in the last case of MPSI. For reaction (84a), the values of A and B are

$$\begin{aligned} A &= 1.2 \text{ (GeV/c)}^{-2} \\ B &= 1.0 \text{ (GeV/c)}^{-2} \end{aligned} \quad (87)$$

For reactions (84b) and (84c), the values of A and B are

$$A = B = 1.5 \text{ (GeV/c)}^{-2} \quad (88)$$

The modified phase space model, termed MPSII, is represented as a dashed curve in Figs. 31 through 41.

C. Program MCE82

Program MCE82 is called once for each Monte Carlo Event for which it is desired to simulate SLAC experiment E-82. The Monte Carlo program generates the four-vectors of two, four or six prong events according to Modified Lorentz Invariant Phase Space Models I or II. Only events with neutron momentum ≥ 3 GeV/c are accepted. Then program MCE82 is called to check if the generated event four-vectors satisfy the geometry and trigger requirements for SLAC experiment E-82. The trigger requires that events have no charged tracks hitting the anticounter and neutrons reaching the optical spark chamber system interact in one of the steel or aluminum plates to produce charged secondaries, energetic enough to cross at least two consecutive neutron counters. Program MCE82 operates through a series of calls to specific purpose subroutines, which are discussed below in some detail.

(i) Subroutine VERTEX(VTX): This subroutine chooses the interaction vertex location in the fiducial volume of the 15" RCBC. The x-coordinate of interaction vertex is distributed uniformly on the interval (-12.5 cm, 10.5 cm); while y and z-coordinates are chosen from Gaussian distributions with means and standard deviations of (-1.48 cm, 1.68 cm) and 7.45 cm, 0.32 cm) respectively.

(ii) Subroutine BMDIR (VTX, PHI, TANL): Given the location of the interaction vertex, (VIX), this subroutine determines the azimuth (ϕ) and the slope ($\tan \lambda$) of the beam at the interaction vertex. The slope ($\tan \lambda$) is chosen from a Gaussian with ($\mu = -.000652$, $\sigma = .00533$) while the azimuth ϕ is

obtained from:

$$\varphi(x) = \varphi_0(x=0) + x/R \quad (89)$$

where R = radius of curvature of the beam.

(iii) Logical Function XANTI (ICHG, XC, PC): This function returns the value .TRUE if the particle of charge ICHG with momentum PC at point XC in the RCBC intersect the anti-counter. This function calls the subroutine OUTFLD which propagates a singly charged particle inside a radial magnetic field whose magnetic induction vector is positive in the (-z) direction. The field is assumed to have no x or y component. The magnetic field is considered to be a series of 10 concentric rings centered at $x = y = 0$, and varying in outer radius from 21 centimeters to 96 centimeters. The magnetic induction vector varies from +17.77 kilogauss in the central ring to -2.37 kilogauss in the outermost ring. The particle is propagated through the magnetic field, using the value of the magnetic induction vector appropriate to the region in which the trajectory lies.

(iv) Subroutine NEUTRN (X, P, ITOWARD, IBEFORE, INSPKC, INHOLE): This subroutine determines if the neutron with momentum p at point x in the SLAC 15" RCBC has a direction which intersects with the fiducial volume of ITOWARD = .TRUE. in the affirmative case. The amount of stainless steel seen by the neutron while exciting from the RCBC, the amount of lead seen in the anti-counter and the total distance traveled by the neutron in the steel and aluminum plates are calculated. Then using the known interaction lengths of neutron in lead, aluminum and steel, the

subroutine returns `lBEFORE = .TRUE.` if the neutron interacts either in the RCBC or lead plate before reaching the optical spark chamber. The subroutine returns `INHOLE = .TRUE.` if the neutron goes through the hole in all the spark chambers, and returns `INSPKC = .TRUE.` if the neutron interacts either in the steel or aluminum plates of the optical spark chambers. Thus if `lTOWARD = TRUE'`, `lBEFORE = .FALSE.`, and `INHOLE = .FALSE.` and `INSPKC = .TRUE.`, we obtained a valid trigger. However, because of the trigger requirement that the charged secondaries from a neutron interaction be energetic enough to cross at least two consecutive counters, a correction for loss of events corresponding to the above effect besides elastic scattering, all neutral final state particles and back-scattering has to be included. This is considered in detail in the next section.

D. Neutron Conversion Efficiency in the Spark Chambers

Not all neutrons which interact in the spark chambers give rise to triggers. Triggers are lost for several reasons, for example:

(i) The charged secondaries are not energetic enough to cross two consecutive counters.

(ii) The secondaries at a neutron interaction vertex are all neutral.

(iii) Back-scattering in the first steel-plate spark chamber vetoes the event.

(iv) Some triggers are lost due to interactions in the chamber edges.

(v) If the charged secondaries at an interaction vertex are produced at very wide angles, they will fail to traverse two consecutive counters.

To take into account the above corrections, program NVERTEX was used to generate events corresponding to all possible neutron-induced inelastic final state channels such as:

$$\begin{array}{rcl}
 & & NN\pi \\
 nN \rightarrow & & NN\pi\pi \\
 & & NN\pi\pi
 \end{array} \quad (90)$$

where n stands for the incident neutrons and N stands for the target nucleon. Appropriate angular distributions at the final state production vertex was imposed, and the slowing of particles in different materials was taken into consideration. Finally, imposing the geometric and trigger constraints of the experimental set-up, the triggering efficiencies for the various final states were calculated. Details are shown in Tables 22 to 26.

Averaging over the momentum range 3 GeV/c to 7 GeV/c and using the known values of the cross sections for the various final states tried, it was found that only 78 per cent of neutron interactions in the spark chamber give rise to triggers. Twenty-two per cent of the events are lost due to effects considered above.

E. Comparison of Data with Models

Single Particle Spectra

Figure 31 displays the four momentum transfer squared from the beam to the neutron corresponding to the reactions (84a),

(84b) and (84c) respectively. The data show backward peaking characteristics of baryon exchange reactions. Some forward events are lost because neutrons pass through the beam hole in the neutron-detector without interacting. The peaking occurs at lower values of $-u$ for lower multiplicity events. The models are unable to reproduce the peaking very accurately. In general, the second model MPSII seems to reproduce the features of the data better than the model MPSI.

Figures 32 and 33 display the center-of-mass longitudinal momentum spectra and the center-of-mass production cosine spectra respectively for the produced particles for the three reactions studied. The neutron distributions become less forward peaked with increasing pion multiplicity for the final states. It may also be pointed out that the pion distribution in the center-of-mass approaches higher levels of isotropy with increasing multiplicity. Model MPSII again seems to fit the data fairly well and in general provides a better fit to the data than the model MPSI.

The transverse momentum squared spectra for produced pions shows some interesting features. Figures 34a and 34b display the transverse momentum squared spectra for the pions and neutron respectively, corresponding to the 2π , 4π and 6π final states respectively. Table 11 shows the average values of $p_{\perp}$ and $p_{\perp}^2$ for the three different final states. A comparison of Monte Carlo generated transverse momentum squared distributions with and without the geometrical constraints of the experiment shows that the transverse momentum squared

spectra is not significantly influenced or biased by the trigger requirements or geometrical constraints of the experiment. This means that the transverse momentum squared spectra is truly representative of the actual data sample. The mean value of $p_{\perp}$ appears to limit between .3 to .4 GeV/c^{58,59} independent of incident particle and energy in the case of pions produced in nucleon-nucleon collision and meson-nucleon collision. The transverse momentum squared spectra from our data differs significantly from pion transverse momentum squared spectra in the case of nucleon-nucleon and meson-nucleon scattering. The mean values for $p_{\perp}$ in our data is consistently higher than mean $p_{\perp}$ values for the same pion multiplicities in non-annihilation processes.^{59,60,61} Higher values for mean $p_{\perp}$ are consistent with baryon-antibaryon scattering.⁶⁰

According to Gregory, et al.,⁵⁹ the mean $p_{\perp}$ value appears to reach a limiting value of .8 GeV/c for $p\bar{p}$ annihilation into two pions. Our value of $.54 \pm .02$ for the mean $p_{\perp}$ of pions produced in reaction (84a) is not consistent with this limiting value but it is consistent with the value of $.52 \pm .06$ reported for mean $p_{\perp}$ of backwards-produced pions in the reaction $K^-p \rightarrow \lambda^0_{\text{forward}} \pi^+ \pi^-_{\text{backward}}$ at 8.25 GeV/c. Our mean $p_{\perp}$ value of $0.43 \pm .01$ for the four pion final state in reaction (84b) is relatively higher than the value .32 for the mean p of backwards-produced π^+ in the reaction $K^-p \rightarrow p \bar{K}^0 \pi^+ \pi^- \pi^0$ at 8.25 GeV/c.

Invariant Mass Distributions

Figure 37 displays the invariant two-pion mass distributions for all three reactions studied. There is no evidence

for resonance production in the $\pi^+\pi^+$ and $\pi^-\pi^-$ spectra from any of the three final states considered. Both modified phase space models seem to fit the mass spectra fairly well. There is, however, clear evidence for backwards ρ^0 production in $\pi^+\pi^-$ mass spectrum from the $(4\pi)n_{\text{fwd}}$ final state. The resonance production appears as a three standard deviation broad enhancement in the region of the ρ mass above the modified phase space model smooth curve fit to the data.

Figures 37 and 38 show the $n\pi^+$ and $n\pi^-$ mass spectra respectively from the various final states available. Evidence for some resonance enhancements in the mass region 1.55 - 1.75 GeV exists in the $n\pi^+$ spectrum corresponding to the $(2\pi)n_{\text{fwd}}$ final state. There is evidence for $\bar{\Delta}(1236)$ production in $n\pi^-$ spectrum corresponding to the $(4\pi)n_{\text{fwd}}$ final state. As shown in Figs. 39 and 40 there is no convincing evidence for three-body resonances in any of the reactions. It may be pointed out that the mass spectra are characterized by the absence of any strong features. Both the modified phase space models seem to reproduce the general features of the data fairly well except in those channels where there is evidence for resonance production.

F. Resonance Production

To calculate the cross section for resonance production, the most inclusive sample of reliably identified events were used. The argument for doing this is based on the assumption that the invariant mass of misidentified events would be

distributed smoothly and would not tend to populate narrow peaks so that only true narrow resonances would stand out as enhancements above the background. An event was assigned to a final state only if (i) it made a one constraint (1C) fit to that state and the χ^2 probability of the fit is greater than 4% in the case of $(2\pi)n_{\text{fwd}}$ final state, 6% in the case of $(4\pi)n_{\text{fwd}}$ and $(6\pi)n_{\text{fwd}}$ final states; (ii) the missing mass squared satisfied the cut $(-1.0 \leq MM^2 \leq 3.0 \text{ GeV}^2)$; (iii) and the χ^2 probability of the fit to which the event is assigned is greater than ten times the χ^2 probability of the next best fit.

Figure 41 displays the $n\pi^+$ and $n\pi^-$ mass spectra from the $(2\pi)n_{\text{fwd}}$ and $(4\pi)n_{\text{fwd}}$ respectively event samples selected according to the above criteria. There are 165 and 293 events in the two samples respectively. The curves superimposed on the data are fourth order polynomial fits (with confidence levels greater than 60 per cent) to the data and represent the non-resonant background. The events in the resonance peak are then obtained by counting the events in the peak above the smooth polynomial background. The events per microbarn conversion ratio used is the same as obtained for the overall or background reaction in which the resonance production occurs.

Reaction $\pi^+ p \rightarrow N^{*+} \pi^+$

This reaction is very similar to backward elastic p scattering. The two cross sections should be of the same order of magnitude when corrected for similar constraints. The cross section for the above reaction is $0.4 \pm .1$ microbarn where the

resonance region is defined by $1.55 \text{ (GeV)} \leq M_{n\pi^+} \leq 1.75 \text{ (GeV)}$. This may be compared to the total backward cross section $1.38 \pm 0.14 \mu\text{b}$ for π^-p scattering at 8 GeV/c .⁶² However, one must remember that only Δ -trajectory exchange is allowed in the case of π^-p scattering whereas both Δ and nucleon trajectories may be exchanged in the case of π^+p scattering. Only backward scattering is being considered.

Reactions $\pi^+p \rightarrow \Delta^-(1236) \pi^+\pi^+\pi^+$ and $\pi^+p \rightarrow n\pi^+\pi^+\rho^0$

Figures 41a and 41b respectively correspond to $n\pi^-$ and $\pi^+\pi^-$ mass spectra from the final state $(4\pi)n_{\text{fwd}}$. As in the case of the last reaction, using a similar procedure, lower limits of $0.8 \pm .25 \mu\text{b}$ and $0.6 \pm .1 \mu\text{b}$ are placed for the forward production of $\Delta^-(1236)$ and backward production of ρ^0 according to the reactions:

$$(i) \quad \pi^+p \rightarrow \Delta^-(1236) \pi^+\pi^+\pi^+ \quad (91)$$

$$\text{and} \quad (ii) \quad \pi^+p \rightarrow n\pi^+\pi^+\rho^0 \quad (92)$$

respectively. Our lower limit for backward production, that is, $0.6 \pm .1 \mu\text{b}$ may be compared with the value $1.86 \pm .2 \mu\text{b}$ for total backward cross section from $\rho^-(770)$ production in the reaction: $\pi^-p \rightarrow p_{\text{fwd}}\rho^-$ backward at 8 GeV/c .⁶²

REFERENCES

57. Slansky, R., Pierls, Yale University Report, March 1973.
58. Hoang, T. F., Nucl. Physics B38 (1972), 333.
59. Gregory, P. S., et al., Phys. Letters 43B (1973), 228.
60. Fry, J. R., et al., Phys. Letters 41B (1972), 539.
61. Hoang, T. F., et al., Nucl. Phys. B38 (1972), 62.
62. Anderson, E. W., et al., Phys. Rev. Letters 22 (1969), 102.
63. Orfanidis, S. J., Rittenberg, V., Nucl. Phys. B59 (1973), 570.
64. Hamer, C. J., Nuovo Cimento 12A (1972), 162.
65. Oh, B. Y., et al., Nucl. Phys. B63 (1973), 1.

TABLE 11 Average pion $p_{\perp}$ and $p_{\perp}^2$

Final State	$\langle p_{\perp} \rangle_{\pi}$	$\langle p_{\perp}^2 \rangle_{\pi}$
$\pi^+ \pi^+ n_{\text{forward}}$	$0.54 \pm .02 \text{ GeV/c}$	$0.39 \pm .03 (\text{GeV/c})^2$
$\pi^+ \pi^+ \pi^- n_{\text{fwd}}$	$0.43 \pm .01$	$0.25 \pm .01$
$\pi^+ \pi^+ \pi^+ \pi^- \pi^- n_{\text{fwd}}$	$0.33 \pm .01$	$0.16 \pm .01$

FIGURE 31

FIGURE 32a

FIGURE 32b

FIGURE 33a

FIGURE 33b

FIGURE 34b

FIGURE 35

FIGURE 36

FIGURE 37

FIGURE 38

FIGURE 39

FIGURE 40

CHAPTER VII

CONCLUSION

According to Rosner, the baryon-antibaryon catastrophe is the two-component duality model for hadronic scattering amplitudes may be avoided by assuming the existence of exotic mesons which couple only to baryon and antibaryon and whose couplings to mesons may still be zero. We have investigated the production of exotic meson resonances (isospin = 2) at the $p - \bar{\Delta}$ vertex in the backward scattering, baryon exchange process $\pi^+ + p \rightarrow n$ (forward) + X^{++} (backward) at incident pion momentum of 8.4 GeV/c. We have investigated even G parity decay modes $X^{++} \rightarrow \pi^+\pi^+$, $\pi^+\pi^+\pi^+\pi^-$, $\pi^+\pi^+\pi^+\pi^+\pi^-\pi^-$ and placed upper limits of the order of 1 - 2 microbarns for the production of narrow (100 MeV wide) exotic meson resonances with masses less than or equal to 3.0 GeV and which decay into even multiplicity pion states. The upper limit applies to production at a $p\bar{\Delta}$ vertex. It may be pointed out that the coupling at the production vertex is an allowed coupling according to the Zweig rule but the coupling to mesons at the decay vertex is forbidden according to the Zweig rule. We have investigated one baryon-antibaryon decay mode, namely $p\bar{p}\pi^+\pi^+$ and placed an upper limit of the order of 1 microbarn for decay into this channel. However, we had only three reliable events in the channel and the statistics is too poor to say anything conclusive. The question of exotic meson resonances being produced only in baryon-antibaryon scattering

and its detection in baryon-antibaryon decay modes is therefore largely an open one.

The single particle spectra of the baryon exchange reactions:

have been systematically studied. Except for the transverse momentum spectra, all distributions are influenced to some extent by the trigger requirements and the geometrical constraints of the experiment. The general features of the data can be reproduced fairly well with a Lorentz invariant phase space model, modified by a weighting factor which includes an exponential $p_{\perp}^2$ damping for all produced particles. A second model which includes an exponential u -dependence on the neutron over and above the exponential $p_{\perp}^2$ damping on the pions give a better fit to the data.

The cross section for the $(2\pi)n_{\text{fwd}}$ final state has an upper limit of 16.4 microbarns, that for the final state $(4\pi)n_{\text{fwd}}$ has an upper limit of 25.1 microbarns while the upper limit on the cross section for the final state $(6\pi)n_{\text{fwd}}$ is placed at 9.7 microbarns where the forward neutron momentum in all cases is 3.0 GeV/c. The mass spectra for the produced particles are characterized by the absence of convincing resonance features. Evidence for resonance structure in the mass range 1.55 - 1.75 MeV, corresponding to the $n\pi^+$ spectrum for the $(2\pi)n_{\text{fwd}}$ final state is very convincing. We place a

lower limit of $0.4 \pm .1$ microbarn for the baryon exchange process $\pi^+ p \rightarrow N^{*+} \pi^+$. Some evidence for forward $\Delta^-(1236)$ and backward ρ^0 production is also noted. Lower limits of $0.8 \pm .25$ microbarns and $0.6 \pm .1$ microbarns have been placed on the cross section for the reactions $\pi^+ p \rightarrow \Delta(1236)^- \pi^+ \pi^+ \pi^+$ and $\pi^+ p \rightarrow n \pi^+ \pi^+ \rho^0$ respectively.

An interesting feature of the data is the transverse momentum squared spectra for the backwards produced pions. It has already been pointed out that the transverse momentum squared spectra is not biased by the trigger requirements and geometrical constraints of the experiment. The $p_{\perp}^2$ distribution is more characteristic of annihilation processes. The mean $p_{\perp}$ values of $.54 \pm .02$ GeV/c, $0.43 \pm .01$ GeV/c and $0.33 \pm .01$ GeV/c for the pions from the two, four and six pion final states respectively are consistently higher than the limiting value of $0.3 - 0.4$ GeV/c for mean $p_{\perp}$ values of pions produced in meson-nucleon and nucleon-nucleon collisions. The higher values are consistent with baryon-antibaryon scattering and support the baryon-exchange nature of the reactions considered above.

APPENDIX I

RF Separators

Theory of Operation:

A rf-separator consists of cavity excited electromagnetically with radio frequency wave. The radio frequency of the separator, 2856 MHz is synchronized with the accelerator frequency. The electric vector of the radio wave points in the vertical direction and travels along the beam at nearly the same velocity as the beam ($\approx c$).

Particles with the same momentum and different masses have different velocities. Thus, particles of different masses produced in the interval $5 \times 10^{-12} \text{ sec}$. appear at the separator at different points on the sinusoidal RF wave form and therefore experience different electric field strengths. Let ϕ_1 be the phase of the field when m_1 arrives at the separator and ϕ_2 the phase of the field when m_2 arrives at the separator. Then

$$\phi_2 - \phi_1 = 2\pi\gamma\Delta t = \frac{2\pi c}{\lambda} \Delta t \text{ where}$$

γ is the frequency of the rf wave and Δt is the interval between the arrival of the two particles at the separator and c the speed of the e-m wave. Now

L = distance from target to separator

$$L = v_1 t = \frac{pc}{E_1} t$$

$$\text{or} \quad L(p^2 c^2 + m_1^2 c^4)^{1/2} = p c t$$

$$\text{or} \quad L c \left(1 + \frac{m_1^2 c^2}{2p^2}\right) = c t$$

$$L + L \frac{m_1^2 c^2}{2p^2} = t$$

and for particles with mass m_2

$$L + L \frac{m_2^2 c^2}{2p^2} = t + \Delta t$$

$$-\Delta t = L \frac{m_1^2 - m_2^2}{2p^2} c^2$$

Thus

$$\varphi = \frac{\pi L}{\lambda} c \frac{m_1^2 - m_2^2}{2p^2} c^2$$

The maximum impulse a particle will suffer is

$$D = q E_0 \Delta t = \Delta p_t; \quad q \equiv \text{charge}$$

Thus $\Delta\varphi$, the angular deflection of the particle is

$$\Delta\varphi = \frac{D}{p}$$

The corresponding deflection at focus F2 is

$$\delta = l_{\text{eff}} \Delta\varphi = l_{\text{eff}} \frac{D}{p}$$

where l_{eff} = distance from separator to focus 2.

$$\text{For RFS1} \quad \delta_{F_2}^1 = (3.45/p) \sin \alpha \quad (93a)$$

$$\text{For RFS2} \quad \delta_{F_2}^2 = (4.35/p) \sin \alpha \quad (93b)$$

where α is the separator phase with reference to the start of the cycle when the particle arrives:

(a)

(b)

$$\text{Thus } \delta = \underbrace{\delta_{F_2}^1 + \delta_{F_2}^2}_{\delta_o} = (3.45/p) (\sin \alpha + \sin(\varphi - \alpha)) \quad (94)$$

$(m_1) \quad (m_2)$

By choosing α , net relative deflections between the two particles may be varied between $-2 \delta_o \sin \varphi/2$ to $2 \delta_o \sin \varphi/2$. For details, see Winkelmann.⁶⁶

REFERENCES

66. Winkelmann, Fredrick C., Design and Operation of SLAC Beamline, 20-21-22 SLAC Report No. 160, April 1973.

APPENDIX II

A. Calculation of Total Beam Flux

Piled $B_1 B_2$: total number of beam particles. Some of these overlap each other as:

We set up electronics to ignore the beam tracks which follow within 20 nanoseconds of the previous one. To get the total number of unpiled or separated beam tracks the number actually used in our trigger, we have a conversion factor of .844.

$$\text{Unpiled } B_1 B_2 = \text{piled } B_1 B_2 (.844)$$

Now we find that a certain percentage of the triggers result in pictures that are optically too poor to be scanned. We get this ratio

$$= 1 - \frac{\text{total number of scan. frames}}{\text{total number of triggers}}$$

$$\epsilon_{\text{bad frames}} = 1 - \epsilon_{\text{scannable}}$$

$$\text{Good unpiled } B_1 B_2 = \text{piled } B_1 B_2 (\epsilon_{\text{pile up correction}}) (\epsilon_{\text{scannability correction}})$$

Then we have several rolls for which $B_1 B_2$ count has not been recorded. We get these in an average way by multiplying the average trigger rate by the number of such frames

$$\text{unrecorded } B_1 B_2 = \langle \text{trigger} \rangle \cdot \text{number of frames}$$

Thus the relevant beam flux for the experiment is:

$$\text{fluxc} = \text{corrected flux} = \text{piled count} * \epsilon_{\text{pile correction}}$$

$$* \epsilon_{\text{scannability correction}}$$

$$+ \langle \text{trigger rate} \rangle * \# \text{ frames for which } B_1 B_2 \text{ count missing.} \quad (95)$$

Useful Numbers

Number of scannable frames = 73882

average number of tracks per frame = 9.82018

average trigger rate = unpiled $B_1 B_2$ / MT = 1433.26

Flare corrected unpiled $B_1 B_2$ = 1.017041×10^8

Unpiled $B_1 B_2$ = 9.609032×10^7

Flux corrected for missing $B_1 B_2$ counts = 1.055183×10^8

Fluxc = actual experimental flux with all corrections
= 1.111321×10^8

B. Complete Event Type Breakdown

Total number of events = 22,999

102	9757
104	9288
106	2694
108	210
112	548
114	350
116	69
118	4
122	34
124	18
126	1

C. Calculation of Scanning Efficiency

Let N be the total number of events in the fiducial volume and E_i be the efficiency of the i th scan.

Scan Number	# Found in Scan	New Events in Scan	Total # Events Missing
1	$E_1 N = N_1$	$E_1 N$	$(1 - E_1) N$
2	$E_2 N = N_2$	$E_2 (1 - E_1) N$	$N - [E_1 N + E_2 (1 - E_1) N]$

If N_{12} is the number of events both scanners find, we have:

$$\begin{aligned}
 N_2 &= N_{12} + E_2 (1 - E_1) N \\
 &= N_{12} + \frac{N_2}{N} (1 - E_1) N \\
 E_1 &= \frac{N_{12}}{N_2} = \frac{\text{number of events common to both}}{\text{number of events second scanner finds in all}} \quad (96)
 \end{aligned}$$

D. Spark Chamber Scanning Efficiency

The following results were obtained by giving two scanners, an identical list of frames to scan. They were both instructed to follow the same scanning rules.

Scanner #	Total Events found N_i	Total # of found by both N_1 N_2	Events found by N_1 but not by N_2	Events found by N_2 but not by N_1
1	501 550	452	49	98

Thus the scanning efficiency $1 = \frac{N_1 \cap N_2}{N_2} = .822$

E. TVGP Failure Rates for All Topologies

Topology	# of Events	Failure rate percent
102	9757	.8
104	9288	.9
106	2694	1.5
108	210	.5
112	548	1.2
114	350	1.1
116	69	- -

APPENDIX III

A. 3C Mass Resolution of the Exotic Mass Combinations

All effective mass combinations for the exotic channels $\pi^+\pi^+$, $\pi^+\pi^+\pi^+\pi^-$, $\pi^+\pi^+\pi^+\pi^+\pi^-$ were plotted from the best 3C fits to these hypotheses. To calculate the errors of these effective mass calculations, we used SQUAW to fit the 2, 4, 6 - prongs to the hypotheses interpreted as two calls to subroutine RDY in ETYPE.⁶⁷

SQUAW not only calculated the invariant mass, but also the errors in the respective invariant masses. The averaged mass-resolution in different mass regions is given below. For 1C-resolution, we forced neutron as missing.

REFERENCES

67. See Ref. 48, Chapter 3.

TABLE 12a

3C AVERAGED MASS RESOLUTION FOR DIFFERENT MASS REGIONS

Final State		Mass	MeV 0 -500	MeV 500 -1000	MeV 1000 -1500	MeV 1500 -2000	MeV 2000 -2500	MeV 2500 -3000
$\pi^+ \pi^+ n$	3C	$\pi^+ \pi^+$	3	8	19	27	22	31
	1C		1	8	22.2	36	135	104
$\pi^+ \pi^+ \pi^+ \pi^- n$	3C	$\pi^+ \pi^+ \pi^+ \pi^-$			24	23	28	22
	1C				23	27	57	61
$\pi^+ \pi^+ \pi^+ \pi^+ \pi^- \pi^- n$	3C	$\pi^+ \pi^+ \pi^+ \pi^+ \pi^- \pi^-$					15	22
	1C					11	21	35

B. Optical Parameters of the Rapid Cycling Bubble Chamber

TABLE 12b

Coordinate System, see Figure 21Camera Positions

Camera	x-coordinate	y-coordinate
1.	25.182 (cm)	-25.183 (cm)
2.	25.183	25.183
3.	-25.183	25.183

Fiducial Positions

Fiducial	x-coordinate	y-coordinate
1.	.439 (cm)	.031 (cm)
2.	-12.240	-.086
3.	13.039	10.940
4.	13.146	.146
5.	.452	-12.688
6.	.443	12.724

<u>Glass Thickness</u>	9.196 cm	<u>Index</u>	1.5168
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<u>Z-Component of Field at (0, 0, 0)</u>	-17.354 Kilogauss
--	-------------------

<u>Hydrogen Index</u>	<u>Density of Liquid H₂</u>	<u>Measurement Error</u>
1.080	.0628 gm/cc	8 microns

Image Distance from Camera Nodal Point

<u>Camera 1</u>	<u>Camera 2</u>	<u>Camera 3</u>
9.644 (cm)	9.6171 (cm)	9.6038 (cm)

APPENDIX IV

Definition of Cross Section

During time t , number of events of the type under consideration for each target particle

$$\sigma = \frac{\text{Flux in time } t / \text{Area}}{\text{Area}}$$

$$(1) \quad \sigma = \frac{N_{\text{events}} / \left(\frac{(\rho_h a l_f)}{A_w} N_{\text{avag}} \right)}{N_{\text{beam}} / a} \text{ cm}^2 \quad (97)$$

where

ρ_h = mass density of liquid hydrogen

$a l_f$ = fiducial volume = v_f

A_w = gm. at. wt. or gm-mol-wt eq.

N_{avag} = Avagdro No., i.e. # of particles in a grm at. wt. or gm. mol. wt.

N_{beam} = Number of beam particles during the experiment

a = cross sectional area of beam

$$(2) \quad \sigma = \frac{N_{\text{events}} / (n_h v_f)}{N_{\text{beam}} / a} \text{ (cm}^2\text{)}$$

$$= \frac{N_{\text{events}} / (n_h v_f)}{N_{\text{beam}} / a} \times 10^{24} \text{ (barns)} \quad (98)$$

where n_h = particle density of liquid hydrogen.

$$(3) \quad \sigma = \frac{N_{\text{events}}}{n_h v_f / a N_{\text{beam}}} \times 10^{24} \text{ (barns)} \quad (99)$$

$$(4) \quad \sigma = \frac{N_{\text{events}}}{n_h l_f N_{\text{beam}}} \times 10^{24} \text{ (barns)} \quad (100)$$

$$(5) \text{ or } \sigma = \frac{N_{\text{events}}}{n_h L_{\text{equivalent}}} 10^{24} \text{ (barns)} \quad (101)$$

where

$$L_{\text{eq}} = l_f N_{\text{beam}} = \text{equivalent fiducial for all particles}$$

Barn and μ barn Equivalent for Each Event

$$\begin{aligned} (6) \quad \sigma_e^{\text{barns}} &= \text{cross section in barns for each event} \\ &= \frac{\sigma \text{ in barns for } N_{\text{events}}}{N_{\text{events}}} \\ &= \frac{\sigma_{N_{\text{events}}}^{\text{barns}}}{N_{\text{events}}} = \frac{10^{24}}{n_h l_f N_{\text{beam}}} \end{aligned} \quad (102)$$

$$\begin{aligned} (7) \quad \sigma_e^{\mu b} &= \text{cross section in } \mu b \text{ for each event} \\ &= \frac{\sigma \text{ in } \mu b \text{ for } N_{\text{events}}}{N_{\text{events}}} \\ &= \frac{10^{30}}{n_h l_f N_{\text{beam}}} \end{aligned} \quad (103)$$

Mean Path Length Equivalent for a $\mu b/\text{event}$

It is the total length that must be traversed by the beam particles before a cross section of 1 $\mu b/\text{ev}$ is obtained.

$$\begin{aligned} \sigma_{\mu b} &= \frac{N_{\text{event}} \times 10^{30}}{n_h \times L_{\text{eq}}} \\ \sigma_{\mu b/e} &= \frac{\sigma_{\mu b}}{N_{\text{event}}} = \left(\frac{\mu b}{\text{event}} \right) = \frac{10^{30}}{n_h \times L_{\text{eq}}} \end{aligned}$$

Let $\lambda_{\text{mpl}/\mu b e}$ be L_{eq} for $\sigma_{\mu b/e} = 1$.

(8) Therefore,

$$\lambda_{\text{mpl}/\mu b e} = 10^{30}/n_h \quad (104)$$

Events per μb

Total number of events for a reaction occurring at the level of $1 \mu b$.

$$\text{Events}/\mu b = 1/\sigma_{\mu b/e} = n_h \times L_{eq}/10^{30} = N_{\text{events}}/\sigma_{\mu b}$$

$$(9) \quad \text{Events}/\mu b = \frac{n_h \times l_f \times N_{\text{beam}}}{10^{30}} \quad (105)$$

$$(10) \quad \sigma(\mu b) = \sigma_{\mu b/e} N_{\text{events}} \quad (106)$$

$$(11) \quad \sigma(\mu b) = \frac{N_{\text{events}}}{(\text{Evt}/\mu b)} \quad (107)$$

(12) Attenuation of Beam Along Fiducial Length

$N(x) = N(x=0) \exp(-x/\lambda)$ where λ is the interaction length in the target and is given by $\lambda = (\text{Atomic weight})/(\sigma N_a \rho)$ where σ is the total cross section of the incident particle in the target and ρ is the density of the target. The total effective path length is therefore $L = N(x=0) \int_0^{L_f} \exp(-x/\lambda) dx$, which gives for the effective beam attenuation the result

$$\epsilon = \frac{\lambda}{L_f} (1 - e^{-L_f/\lambda})$$

where L_f is the measured fiducial length.

APPENDIX V

Study of Triggering Efficiencies for Exotic Resonance Production

If exotics were being produced, how would the geometry and trigger requirement bias their being seen. Monte-Carlo Event Generator NVERTEX⁶⁸ was used to generate events for all exotic topologies under study. The effect of the geometry was incorporated by using the program MCE82⁶⁹ and triggering efficiency with respect to events that were not antied out was also studied. Different masses were allowed for the exotics-- they were allowed to decay directly or through intermediate two-body quasi-states. Effect of spin for the exotics was also studied.

i) The overall triggering efficiency varied between 30 to 50% over the allowed mass spectrum for the exotics and appeared to be independent of the topology of the events. The efficiency fell off with increasing mass.

ii) The triggering efficiency with respect to the anti-counters appeared to vary between 100% at low mass to 60% at high mass.

REFERENCES

68. See Ref. 56, Chapter V.

69. See Chapter VI, Section C.

TABLE 13a

$$X^{++} \rightarrow \pi^+ \pi^+$$

Triggering Efficiency

t distribution

$$= e^{5t}$$

500 events thrown

(GeV) Mass X^{++}	$\epsilon = \frac{\text{\# of triggers}}{\text{\# of events}} \times 100$ $X^{++} \text{ Spin} =$ Distribution: Uniform	$\epsilon = \frac{\text{\# events not antied}}{\text{\# of events}} \times 100$ $X^{++} = \text{Spin}$ Distribution: Uniform
.5	44.6	100.0
1.0	50.0	97.2
1.5	43.0	95.0
2.0	40.4	90.6
2.5	42.6	85.6
3.0	31.2	64.4

REFERENCE

70. The distribution is not a very sensitive function of the t-distribution parameter which is chosen as 5 here to be consistent with other data in this region.

TABLE 13b

$$X^{++} \rightarrow \pi^+ \pi^+$$

Triggering Efficiency

X^{++} (AB) Mass GeV	$\epsilon_{\text{trigger}} \%$	$\epsilon_{\text{anti}} \%$
.500	59	99.4
1.000	51	99.0
1.500	53	92.0
2.000	47	80.0
2.500	39	67.0
3.000	23	42.0

TABLE 14a

$$X^{++} \rightarrow \pi^+ \pi^+ \pi^+ \pi^-$$

Triggering Efficiency

t distribution

$$= e^{5t}$$

 X^{++} Decay: Uniform

(GeV) Mass X^{++}	$\epsilon = \frac{\# \text{ triggers}}{\# \text{ events}}$ per cent	$\epsilon = \frac{\# \text{ events not anti}}{\# \text{ events}}$ per cent
.500		
1.000	54	97
1.500	49	89
2.000	45	78
2.500	42	72
3.000	30	52

TABLE 14b

Mass X^{++} (GeV)	Mass $A^+(BC)$ (GeV)	Mass $B^0(CD)$ (GeV)	$\epsilon_{\text{trigger}}$ per cent	ϵ_{anti} per cent
1.00	.75 .85	.50	50.80	94.20
		.50	50.40	
		.65	46.40	
1.50	1.25	.65	48.20	90.20
2.00	1.50	.50	49.20	80.60 82.60 83.00
		.75	48.00	
		1.00	49.00	
	1.25	.85	47.20	
	1.00	.50	49.40	
	.85	.65	48.20	
2.50	1.50	.50	45.60	76.00 73.40
		1.00	40.20	
		1.25	29.20	
	2.00	.50	42.80	
		1.00	44.20	
3.00	2.50	.65	30.20	48.20 48.00 53.80
		1.25	28.40	
	2.00	.65	30.20	
	1.50	.65	25.80	
	1.00	1.25 .65	29.20 29.60	

TABLE 15a

$$X^{++} \rightarrow \pi^+ \pi^+ \bar{p} p$$

Triggering Efficiency

X^{++} Mass (GeV)	$\epsilon_{\text{trigger}}$ per cent	ϵ_{anti} per cent
2.500	17	33
3.000	15	26
3.500	17	30

TABLE 15b

$$X^{++} \rightarrow \Delta^{++} \bar{\Delta}^0 \rightarrow \pi^+ p \pi^+ \bar{p}$$

Triggering Efficiency

X^{++} Mass (GeV)	$\epsilon_{\text{trigger}}$ per cent	ϵ_{anti} per cent
2.500	19	32
3.000	16	31

TABLE 16

$$X^{++} \rightarrow \pi^+ \pi^+ \pi^+ \pi^+ \pi^- \pi^-$$

Triggering Efficiency

X^{++} Mass (GeV)	$\epsilon_{\text{trigger}}$ per cent	$\epsilon_{\text{anti}} = \frac{\# \text{ events not anti}}{\# \text{ events}}$ per cent
1.000	50	95
1.500	50	86
2.000	42	75
2.500	35	60
3.000	19	36

APPENDIX VI

A. Calculation of Interaction Lengths from Known Cross Sections

Definitions:a. Mean Free Path of Interaction or Interaction Length

is defined as the average distance between interactions (elastic or inelastic). Therefore:

$$\lambda = \frac{L_{eq} \text{ (total equivalent length)}}{N_e \text{ (total number of events)}} \quad (108a)$$

since

$$\sigma = \frac{N_e}{\frac{\rho Na}{W_a} L_{eq}}, \text{ where}$$

ρ = density of target

Na = Avagadro's number

W_a = gram atomic weight

it follows that:

$$\lambda = \frac{W_a}{\sigma \rho Na} \quad (108b)$$

where $Na = 6.022 \times 10^{23}$ and $\sigma = 'X' 10^{24} \text{ cm}^2$, X in barns.

Thus corresponding to σ_{elastic} , $\sigma_{\text{inelastic}}$ and σ_{total} , three different types of interaction lengths, namely λ_{elastic} , $\lambda_{\text{inelastic}}$ and λ_{total} may be defined.

b. Probability of Interaction in Length L

From a knowledge of interaction lengths in different materials, the probability of interaction in a length L of a given material can be calculated. Let dN represent the number of interactions suffered by N particles per square centimeter in passing through a distance dL of matter, then

$$\frac{dN}{N} = -\sigma \times (\text{number of scattering centers in volume } dL(\text{cm}^3))$$

where σ is the cross section by definition,

$$\text{or } \frac{dN}{N} = -\sigma \times \frac{\rho Na}{Wa} dL$$

Integrating we obtain

$$N(L) = N_0 e^{-\frac{\sigma \rho Na}{Wa} L}$$

Since we have defined the interaction length

$$1/\lambda = \frac{\sigma \rho Na}{Wa}$$

$$N(L) = N_0(L=0) e^{-\frac{L}{\lambda}}$$

where L is the thickness of the material and N_0 is the incident flux. Thus the probability of interaction in a thickness L of a material with density ρ and gram atomic weight Wa is

$$P(L) = 1 - e^{-\frac{L}{\lambda}} \quad (109)$$

c. Calculation from Published Cross Sections

We may calculate the effective interaction length using the relation

$$\frac{L}{\lambda} = \frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2} + \frac{L_3}{\lambda_3} \quad (110)$$

where λ_1 = interaction length in steel
 λ_2 = interaction length in aluminum
 λ_3 = interaction length in scintillator

and L_1 = amount of steel in each module
 L_2 = amount of aluminum in each module
 L_3 = amount of scintillator between modules
 and $\lambda_i = \frac{A_i}{\sigma_i \rho_i N_O}$ where i refers to Fe, Al, Pb, etc.

Since we are talking of neutral conversion in the neutral detector, the cross sections refer to inelastic cross sections.

Interaction Lengths for Neutron Conversion in Different Material

A list of cross sections for neutrons incident on different materials with corresponding references is given in Table 17. Different incident neutron momentum range is covered. Fig. 42 shows (n, X) cross sections as a function of incident neutron momentum.

(n, steel) cross sections in the relevant momentum range (3 - 9 GeV) do not exist. The cross sections are not strongly momentum-dependent. In the momentum range 3 to 8 GeV/c, we plot the available cross sections as a function of atomic number A . We fit $\sigma_a(A) = (35 \pm 3)A^{(.72 \pm .02)}$ to the data points and obtain $\sigma_a(n, \text{Al})$, $\sigma_a(n, \text{Fe})$, and $\sigma_a(n, \text{Pb})$.

Royal E. Thern's thesis⁷³ refers to 3 GeV energy neutrons (2.8 GeV/c momentum). Our numbers 381, 644, and 1660 millibarns for (n, Al) , (n, Fe) and (n, Pb) respectively may be compared with Thern's thesis numbers of 568 and 1700 millibarns for (n, Fe) and (n, Pb) respectively. Our numbers are based on more recent data. The scintillator used was Pilot B, for which $H/C = 1.1$ was used. The cross section for (CH_α) was obtained by adding $\sigma(n, C)$ and $\alpha\sigma(n, H)$.

TABLE 17a

Reaction Cross Sections

For (nucleon, X) vs Incident Nucleon Momentum

Incident Momentum	1.70	2.14	3.00	3.60	4.44	5.36
Incident Particle	P	n	P	P	n	n
Measurement	σ_R	σ_{abs}	σ_{abs}	σ_{abs}	σ_{abs}	σ_{abs}
	mb	mb	mb	mb	mb	mb
^4He	111 $\pm$ 10					
9.01 ^9Be		187 $\pm$ 12	236 $\pm$ 4			
12.01 ^{12}C	258 $\pm$ 17	201 $\pm$ 13	260 $\pm$ 6	230 $\pm$ 12		218 $\pm$ 8
^{16}O	296 $\pm$ 50					
26.98 ^{26}Al		414 $\pm$ 23	503 $\pm$ 16		430 $\pm$ 56	
55.85 ^{55}Fe				690 $\pm$ 28		
63.54 ^{63}Cu		674 $\pm$ 34	914 $\pm$ 44		704 $\pm$ 146	638 $\pm$ 24
118.69 ^{118}Sn		1158 $\pm$ 63				
207.19 ^{207}Pb		1727 $\pm$ 45		1630 $\pm$ 75	1930 $\pm$ 306	1660 $\pm$ 90
208.98 ^{208}Bi		1739 $\pm$ 55				
238.03 ^{238}U		1887 $\pm$ 98				

TABLE 17b

Incident Momentum	5.86	9.19	19.30	24.00	24.20	28.00
Incident Particle	n	n	P	P	P	P
Measurement	σ_R	σ_{inel}	σ_{abs}	σ_{inel}	σ_{abs}	σ_{inel}
	mb	mb	mb	mb	mb	mb
${}^6\text{Li}$			194 $\pm$ 6			
${}^7\text{Li}$			208 $\pm$ 6			
9.01 ${}^9\text{Be}$			227 $\pm$ 5			
12.01 ${}^{12}\text{C}$	235 $\pm$ 16	218 $\pm$ 8	254 $\pm$ 6			218 $\pm$ 10
26.98 ${}^{27}\text{Al}$	381 $\pm$ 27	380 $\pm$ 13	472 $\pm$ 15			
55.85 ${}^{56}\text{Fe}$						666 $\pm$ ⁴⁴ ₂₂
63.54 ${}^{64}\text{Cu}$	586 $\pm$ 25	626 $\pm$ 29	850 $\pm$ 34	715 $\pm$ 20	740 $\pm$ 20	
112.40 ${}^{112}\text{Cd}$				1146 $\pm$ 40	1180 $\pm$ 25	
118.69 ${}^{119}\text{Sn}$		1218 $\pm$ 50				
207.19 ${}^{208}\text{Pb}$	1670 $\pm$ 79	1713 $\pm$ 66	1750 $\pm$ 126	1810 $\pm$ 60	1750 $\pm$ 30	1396 $\pm$ 16

FIGURE 42

TABLE 18

Material	Atomic Weight gms	Density $\frac{\text{gms}}{\text{cm}^3}$	σ_a (mb)	λ_a (cm)	σ_e (mb)	λ_e (cm)
H	1.01	.0708	30	789.5	10	2368.5
C	12.01	1.5000	231	57.6	122*	109.0
Scintillator	13.00	1.0320	263	79.5	133	157.3
Al	26.98	2.7000	381	43.6		
Fe	55.85	7.8700	644	18.3	578*	20.4
Pb	207.19	11.3900	1660	18.2	1100*	27.5

a = absorptive, e = elastic

*The elastic cross sections are quoted from Royal E. Thern's thesis which refer to 3 GeV energy neutrons (≈ 2.8 GeV/c momentum). Our numbers 381, 644 and 1660 millibarns for $\sigma(n, \text{Al})$, $\sigma(n, \text{Fe})$ and $\sigma(n, \text{Pb})$ respectively may be compared with Thern's thesis numbers of 568 and 1700 millibarns for $\sigma(n, \text{Fe})$ and $\sigma(n, \text{Pb})$ respectively. Our numbers are based on more recent data. The scintillator used was Pilot B, for which $H/C = 1.1$ was used. The cross section for (CH_α) was obtained by adding $\sigma(n, \text{C})$ and $\alpha \sigma(n, \text{H})$.

We are now in a position to calculate absorption and elastic interaction length equivalents for the anticounter package and neutron detector. See Table 19.

TABLE 19

ANTICOUNTER PACKAGE

(neutron as incident particle)

 $P_n \sim 3 - 10 \text{ GeV/c}$

Material	Thickness cm(L)	L/λ_a	L/λ_e
Scintillator	1.2400	.0160	.0052
Lead	2.5400	.1390	.0920

FOIL CHAMBER

Foil (Al)	.0254	.0006
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SINGLE STEEL PLATE CHAMBER

Stainless Steel	5.5500	.3030	.2720
Aluminum	1.2700	.0290	

NEUTRON COUNTER

Scintillator	.3100	.0039	.0019
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We are now in a position to calculate absorption and elastic interaction length equivalents for the anticounter package and neutron detector. See Table 19.

According to the trigger requirements, at least two of the counters ($N_1 - N_4$) [see Fig. 14] were required to fire, thus the last steel-plate spark chamber and counter did not contribute to the interaction equivalent for the neutron detector. Three steel-plate chambers and the three counters contributed 1.008 interaction or absorption lengths for neutron conversion, which translated to a raw neutron conversion efficiency of 63.5 per cent. In section C, it is shown that about 22 per cent of the neutron interactions did not give rise to triggers because of slow secondary charged tracks, all neutral-final-state particles and backward scattering. Thus the effective conversion efficiency for the neutron detector is about 50 per cent.

REFERENCES

71. A. Ashmore, PRL 5 (1960), 576; CERN SC 60-11.
72. J. H. Atkinson, PRL 4 (1959), 168.
73. J. H. Atkinson, PR 123 (1961), 1850.
74. P. H. Barrett, PR 114 (1959), 1374.
75. G. Bellettini, NP 79 (1966), 609.
76. T. Bowen, NC 9 (1958), 908.
77. T. Coor, PR 98 (1955), 1369.
78. G. J. Igo, NP B3 (1967), 181.
79. W. V. Jones, NC A8 (1972), 575.
80. M. J. Longo, PR 125 (1962), 701.
81. V. S. Pantuev, JEPT 15 (1962), 626.
82. R. N. Peacock, NC 22 (1961), 1290.
83. R. E. Thern, Ph.D. Thesis (1972) MIT (unpublished).

For compilation of total neutron-nucleus cross sections
see:

84. V. Franco, PR C6 (1972), 748.

Interaction Cross Section for K_L^0 Incident on Different Materials:

The data on $(K_L^0, \text{nucleus})$ inelastic cross sections are almost non-existent. Some data on $(K_L^0, \text{nucleus})$ cross section is available. Assuming that the momentum-dependence of the cross sections is very weak, the ratio of $\sigma_T(K_L^0, \text{nucleus})$ to $\sigma_T(n, \text{nucleus})$ is obtained and assuming that

$$\frac{\sigma_T(K_L^0, \text{nucleus})}{\sigma_T(n, \text{nucleus})} = \frac{\sigma_{\text{inelastic}}(K_L^0, \text{nucleus})}{\sigma_{\text{inelastic}}(n, \text{nucleus})}$$

the inelastic cross sections for K_L^0 incident on various materials is obtained from the knowledge of the above ratio and the $\sigma_{\text{inelastic}}(n, \text{nucleus})$ as obtained in the last section. See Tables 17 and 18.

TABLE 20

RATIO OF $\sigma_T(K_L^0, \text{nucleus})$ TO $\sigma_T(n, \text{nucleus})$

Target	Atomic Weight	$\sigma_T(K_L^0, \text{nucleus})$ $\langle p_{K_L^0} \rangle \approx 9 \text{ GeV/c}$		$\sigma_T(n, \text{nucleus})$ $\langle pn \rangle \approx 3.8 \text{ GeV/c}$		Ratio a/b
		(a)	(mb)	(b)	(mb)	
Carbon	12.01	191 $\pm$ 6	($\pm 3\%$)	369 $\pm$ 7		.52
Aluminum	26.98	387 $\pm$ 12	($\pm 3\%$)	666 $\pm$ 16		.58
Copper	63.54	843 $\pm$ 16	($\pm 2\%$)	1331 $\pm$ 17		.63
Lead	207.19	2314 $\pm$ 55	($\pm 2\%$)	3071 $\pm$ 79		.75

$\sigma_T(n, \text{nucleus})$ as a function of incident lab momentum is fairly flat in the region 2 - 10 GeV/c.

The value of the ratio $\sigma_T(K_L^0, \text{Fe}) / \sigma_T(n, \text{Fe})$ was found by plotting the above ratios as functions of atomic weight and obtaining the value corresponding to Fe from a smooth curve fit to the data points.

A straight line fit gives $R_{\text{Fe}} = .63$ (see Fig. 44)

REFERENCE

85. Lakin, et al., PL 31B (1970), 677.

FIGURE 44

TABLE 21

INTERACTION LENGTHS OF K_L^0 INCIDENT ON DIFFERENT MATERIALS

Material	Atomic Weight	inelastic millibarns	inelastic cm
Carbon	12.01	120	110.7
Aluminum	26.98	221	75.2
Steel	63.54	406	29.0
Lead	207.19	1245	24.3

TABLE 22

$$|M|^2 = e^{10t_\Delta}; f(\cos \theta_\Delta) \approx 1 + \cos^2 \theta_\Delta$$

TRIGGER EFFICIENCY: $nN \rightarrow NN$, 1000 NVERTEX

	Fraction of n-spark chamber interactions which trigger		
	$P_{inc}=3$	$P_{inc}=5$	$P_{inc}=7$
$nn \rightarrow n_s n_f \pi^0$			
$\Delta_s^0 n_f$	0.09	0.09	0.08
$n_s \Delta_f^0$	0.43	0.61	0.73
$nn \rightarrow p_s n_f \pi^-$			
$\Delta_s^0 n_f$	0.22	0.18	0.20
$p_s \Delta_f^-$	0.64	0.80	0.87
$nn \rightarrow n_s p_f \pi^-$			
$\Delta_s^- p_f$	0.93	0.93	0.96
$n_s \Delta_f^0$	0.94	0.95	0.95
$np \rightarrow n_s p_f \pi^0$			
$\Delta_s^0 p_f$	0.92	0.93	0.96
$n_s \Delta_f^+$	0.93	0.95	0.96
$np \rightarrow p_s n_f \pi^0$			
$\Delta_s^+ n_f$	0.14	0.13	0.12
$p_s \Delta_f^0$	0.43	0.61	0.73
$np \rightarrow n_s n_f \pi^+$			
$\Delta_s^+ n_f$	0.17	0.14	0.16
$n_s \Delta_f^+$	0.64	0.80	0.87
$np \rightarrow p_s p_f \pi^-$			
$\Delta_s^0 p_f$	0.93	0.93	0.96
$p_s \Delta_f^0$	0.94	0.95	0.95

TABLE 23

$$|M|^2 = e^{3t_{N_1}} / M^2(N_2 \pi) - M^2(N_1)$$

TRIGGER EFFICIENCY: $nN \rightarrow NN\pi$

	Fraction of n-spark chamber interactions which trigger		
	$P_{inc=3}$ (~500 events)	$P_{inc=5}$ (~700 events)	$P_{inc=7}$ (~700 events)
$nn \rightarrow (n_s \pi_s^0) n_f$	0.15	0.19	0.21
$n_s (n_f \pi_f^0)$	0.40	0.60	0.74
$nn \rightarrow (P \pi^-)_s n_f$	0.44	0.51	0.59
$P_s (n \pi^-)_f$	0.64	0.75	0.82
$nn \rightarrow (n \pi^-)_s P_f$	0.90	0.91	0.93
$n_s (P \pi^-)_f$	0.93	0.95	0.95
$np \rightarrow (n \pi^0)_s P_f$	0.91	0.91	0.93
$n_s (P \pi^0)_f$	0.90	0.93	0.95
$np \rightarrow (P \pi^0)_s n_f$	0.39	0.46	0.54
$P_s (n \pi^0)_f$	0.50	0.64	0.77
$np \rightarrow (n \pi^+)_s n_f$	0.21	0.24	0.26
$n_s (n \pi^+)_f$	0.56	0.71	0.79
$np \rightarrow (P \pi^-)_s P_f$	0.93	0.93	0.94
$P_s (P \pi^-)_f$	0.94	0.95	0.95

TABLE 24

$$|M|^2 = e^{\alpha(t_{N_1} + t_{N_2}) - 3 \sum_{\pi} p^2}$$

TRIGGER EFFICIENCY: $nN \rightarrow NN \pi \pi$

	Fraction of n-spark chamber interactions which trigger		
	$P_{inc}=3$ (~400 events)	$P_{inc}=5$ (~600 events)	$P_{inc}=7$ (~650 events)
$nn \rightarrow n_s n_f \pi^0 \pi^0$	0.44	0.63	0.74
$n_s n_f \pi^+ \pi^-$	0.72	0.77	0.82
$p_s n_f \pi^- \pi^0$	0.76	0.80	0.85
$n_s p_f \pi^- \pi^0$	0.95	0.94	0.95
$p_s p_f \pi^- \pi^-$	0.96	0.95	0.95
$np \rightarrow p_s n_f \pi^0 \pi^0$	0.65	0.74	0.81
$p_s n_f \pi^+ \pi^-$	0.84	0.85	0.88
$n_s n_f \pi^+ \pi^0$	0.59	0.71	0.78
$p_s p_f \pi^- \pi^0$	0.96	0.95	0.95
$n_s p_f \pi^0 \pi^0$	0.93	0.94	0.95
$n_s p_f \pi^+ \pi^-$	0.95	0.95	0.95

TABLE 25

$$|M|^2 = e^{2(t_{N_1} + t_{N_2}) - 3 \sum_{\pi} P^2}$$

TRIGGER EFFICIENCY: $nN \rightarrow NN \pi\pi\pi$

	Fraction of n-spark chamber events which trigger	
	P = 5 (~150 events)	P = 7 (~300 events)
$nn \rightarrow n_s n_f \pi^0 \pi^0 \pi^0$	0.74	0.85
$n_s n_f \pi^+ \pi^- \pi^0$	0.82	0.91
$P_s n_f \pi^- \pi^0 \pi^0$	0.89	0.94
$P_s n_f \pi^+ \pi^- \pi^-$	0.93	0.95
$n_s P_f \pi^- \pi^0 \pi^0$	0.94	0.95
$n_s P_f \pi^+ \pi^- \pi^-$	0.94	0.95
$P_s P_f \pi^- \pi^- \pi^0$	0.95	0.95
$nP \rightarrow P_s n_f \pi^0 \pi^0 \pi^0$	0.86	0.92
$P_s n_f \pi^+ \pi^- \pi^0$	0.91	0.95
$n_s n_f \pi^+ \pi^0 \pi^0$	0.78	0.88
$n_s n_f \pi^+ \pi^+ \pi^-$	0.86	0.93
$P_s P_f \pi^- \pi^0 \pi^0$	0.95	0.95
$P_s P_f \pi^+ \pi^- \pi^-$	0.95	0.95
$n_s P_f \pi^0 \pi^0 \pi^0$	0.93	0.95
$n_s P_f \pi^+ \pi^- \pi^0$	0.94	0.95

TABLE 26

SPARK CHAMBER TRIGGER EFFICIENCY (f)

Reaction		$P_{inc}=3 \text{ GeV/c}$		$P_{inc}=5 \text{ GeV/c}$		$P_{inc}=7 \text{ GeV/c}$		Average Efficiency
		$\sigma \text{ (nb)}$	f	$\sigma \text{ (nb)}$	f	$\sigma \text{ (nb)}$	f	
nn →	N Δ	7.2	0.67	4.5	0.73	2.5	0.84	
	NN*	11.4	0.60	5.2	0.60	4.5	0.65	
	NN $\pi\pi$	6.4	0.77	6.7	0.81	6.4	0.86	
	NN $\pi\pi\pi$			5.5	0.89	6.2	0.93	
	other	3.0	0.90	6.1	0.90	8.4	0.90	
nn →	inelastic trigger		0.69		0.81		0.85	0.78
nn →	neutrals	4.1	0.30	3.2	0.42	2.5	0.52	
nn →	charged trigger		0.65		0.76		0.80	0.74

Overall average:

nN → inelastic trigger f = 0.78
 nN → inelastic charged trigger f = 0.76
 nN → inelastic veto f = 0.03