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Power Threshold for Neutral Beam Current Drive

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Abstract

For fully noninductive current drive in tokamaks using neutral beams, there is a power and density threshold condition, setting a minimum value for $P^{3/2}/n^2$. If this condition is not met, a stationary state cannot occur, and a tokamak discharge will collapse. This is a consequence of the coupling between current and electron temperature, or between current drive efficiency and energy confinement time.

This note is intended to point out a potentially important consideration in the development of tokamaks based on fully noninductive current drive, particularly for designs that contemplate noninductive current buildup. The analysis has been motivated by the observation of slowly collapsing discharges in the DIII-D tokamak during fully noninductive neutral beam current drive (NBCD) experiments. Under some conditions in these discharges, after establishment of a state in which both the thermal energy and the toroidal current are provided by the neutral beams, both the current and the thermal energy decay with a characteristic time of a few seconds (see Figure 2). This appears to be a direct consequence of the coupling which results from the current-dependence of the energy confinement time and the dependence of the current drive efficiency on temperature. Because the threshold falls in the parameter range of interest for present experiments and future designs, it is important to take this phenomenon into account.

The important features of this process can be extracted from a zero-dimensional model. For simplicity, assume that the density $n = \langle n_e(r) \rangle$ is independent of time. Also take equal electron and ion temperatures, $\langle T_e \rangle = \langle T_i \rangle = T$. The thermal energy content of the plasma is determined by

$$3n \frac{dT}{dt} = -\frac{3nT}{\tau_E} + \frac{P}{eV} \quad (1)$$

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where τ_E is the global energy confinement time, V is the volume, and the deposited power is P . All units are SI, except that the temperature is measured in eV. The power P is assumed to go to both heating and current drive. The evolution of the plasma current is given by

$$\tau_{L/R} \frac{dI}{dt} = -I + \frac{P\eta_{CD}}{nR} \quad (2)$$

where $\tau_{L/R}$ is the magnetic time constant, R is the major radius, and η_{CD} is the current drive figure of merit ($= nI_{CD}R/P_{CD}$).

The magnetic time constant is the ratio of the inductance to resistance of the plasma, or approximately

$$\tau_{L/R} = k_{L/R} T^{3/2} \quad (3)$$

where

$$k_{L/R} \simeq \frac{\mu_0 \kappa a^2 (\ln \frac{8R}{a} - 2 + \frac{\ell_i}{2})}{2(\eta_0 T_0^{3/2})} \quad (4)$$

Here, κ is the plasma elongation, a is the minor radius, ℓ_i is the internal inductance, and η_0 is the resistivity at temperature T_0 . Typically, in large machines the inductance is of the order of a few microhenries, the resistance is a few times 10^{-8} ohms, and $\tau_{L/R}$ falls in the range of tens to hundreds of seconds.

For the energy confinement time there are a wide variety of possible choices. For this analysis we use Goldston's scaling¹ because it provides an adequate fit to large tokamak data, contains the important dependences on current and power, and has a simple form useful for analytic manipulation:

$$\tau_E = k_E \frac{I}{P^{1/2}} \quad (5)$$

where

$$k_E \simeq 3.3 \times 10^{-5} C_E \frac{\kappa^{1/2} A_i^{1/2} R^{1.75}}{a^{0.37}} \quad (6)$$

A_i is the plasma ion atomic mass, and C_E is a multiplier to account for possible confinement enhancement (the NBCE experiments on the DIII-D tokamak show $C_E \sim 1$). We note that the qualitative conclusions presented here regarding threshold behavior do not depend on the particular choice of confinement scaling rule. Any scaling relation for which τ_E improves with I and worsens with P will lead to similar behavior.

¹ R. J. Goldston, *Plasma Physics and Controlled Fusion* 26 87 (1984).

Typically, τ_E is a fraction of a second, much smaller than $\tau_{L/R}$, and so the temperature is always in quasi-equilibrium on the time scale of the current evolution:

$$\frac{3nT}{\tau_E} = \frac{P}{eV} \quad (7)$$

If, as with the Goldston scaling, τ_E is not an explicit function of temperature, this can be solved for

$$T = \frac{1}{3eV} \frac{P\tau_E}{n} \quad (8)$$

Neutral beams are the source for noninductive current drive in the cases discussed here. However, it should be noted that a power threshold will also occur for any other current drive scheme that shows similar dependence on electron temperature. The second term in Eqn. 2 is the beam-driven current, I_{CD} . This is obtained by averaging the local (one-dimensional) current over the plasma profiles. The local current is

$$J_{CD} = J_{circ} \left[1 - \frac{Z_b}{Z_{eff}} (1 - G) \right] \quad (9)$$

The first term is the circulating fast ion current, and the second is the electron response, including finite aspect ratio effects in $G(Z_b, \epsilon)$. Note that the electron correction depends on beam charge (Z_b), average ion charge (Z_{eff}), and inverse aspect ratio (ϵ), and can be treated as a simple multiplier of the fast ion current: $J_{CD} = J_{circ} F$. There is a weak dependence on temperature because the trapping factor G depends on the collisionality regime, but this is ignored. The factor G has been calculated by Start and Cordey² for the collisionless regime, and their numerical results are well approximated by $G \simeq (1.55 + 0.85/Z_{eff})\sqrt{\epsilon} - (0.20 + 1.55/Z_{eff})\epsilon$.

The fast ion current is

$$J_{circ} = eZ_b S_b \tau_s v_{b0} \xi_0 I(x, y) \quad (10)$$

where $S_b = P/eV E_{b0}$ is the source strength (E_{b0} is the injected beam energy), $\tau_s = [(4\pi\epsilon_0)^2 3m_e m_b v_{te}^3] / [16\sqrt{\pi} e^4 n_e Z_b^2 \ln \Lambda]$ is the slowing-down time, v_{b0} is the injected beam velocity, and ξ_0 is the cosine of the angle between the injected beam and the toroidal direction. The distribution of fast ions is accounted for in the integral

² D. F. H. Start and J. G. Cordey, *Phys. Fluids* 23 1477 (1980).

$I(x, y) = \frac{1}{6} \left(\frac{1+x^2}{x^3} \right)^y \int_0^x \left(\frac{u^3}{1+u^2} \right)^{y+1} du$. Mikkelsen and Singer³ give an approximate fit to this function: $I(x, y) \approx x^3 / [(4 + 3y) + (1.39 + 0.61y^{0.7})x^2 + x^3]$. The arguments are $x = v_{b0}/v_c$, where $v_c = v_{te}[(3\sqrt{\pi}/4)(m_e/m_b)(Z_{eff}/Z_2)]^{1/3}$ is the velocity at which equal momentum goes to electrons and ions, and $y = Z_2/3$. The mass-weighted charge factor is $Z_2 = (\sum n_i Z_i^2) / (m_b \sum n_i Z_i^2 / m_i)$.

Defining T_c as the temperature at which $v_c = v_{b0}$ gives $x = (T_c/T)^{1/2}$, with $T_c \approx 6.76 \times 10^{-2} A_b^{-1/3} (Z_2/Z_{eff})^{2/3} E_{b0}$. Putting all of this together, and averaging over the plasma cross-section gives

$$I_{CD} = k_{CD} \frac{PT^{3/2}}{n} \frac{(T_c/T)^{3/2}}{c_1 + c_2(T_c/T) + (T_c/T)^{3/2}} \quad (11)$$

with

$$k_{CD} = 1.39 \times 10^{18} \frac{A_b^{1/2} \xi_0 F}{Z_b \ln \Lambda} \frac{1}{RE_{b0}^{1/2}} \quad (12)$$

where $c_1 = 4 + 3y$ and $c_2 = 1.39 + 0.61y^{0.7}$. Substitution of I_{CD} in Eqn. 2 gives

$$k_{L/R} T^{3/2} \frac{dI}{dt} = -I \left[1 - k_{CD} \frac{PT^{3/2}}{In} \frac{1}{c_1(T/T_c)^{3/2} + c_2(T/T_c)^{1/2} + 1} \right] \quad (13)$$

The possibility of a threshold is clear from this expression. If the relationship between T and I (determined by the energy confinement time scaling) is such that there is a maximum in the second term in brackets, then a stationary state can only exist if the value of that maximum exceeds 1.

Define normalized current and temperature variables, $u = I/I_*$ and $v = T/T_*$, where

$$I_* = k_{CD} \frac{T_c^{3/2}}{c_1} \frac{P}{n} \quad \text{and} \quad T_* = \frac{k_E}{3eV} k_{CD} \frac{T_c^{3/2}}{c_1} \frac{P^{3/2}}{n^2} \quad (14)$$

Eqn. 8 gives the temperature in terms of the plasma current as simply

$$v = u \quad (15)$$

and the time dependence is given by

$$\frac{du}{dt} = -\frac{1}{u^{1/2}} + \frac{1}{u^{3/2} + \alpha u^{1/2} + \beta} \quad (16)$$

³ D. R. Mikkelsen and C. E. Singer, *Nucl. Technology/Fusion* 4 237 (1983).

where $\hat{t} = t/k_L/R T_*^{3/2}$, $\alpha = (c_2/c_1)(T_c/T_*)$, and $\beta = (1/c_1)(T_c/T_*)^{3/2}$.

First of all, consider possible stationary solutions to Eqn. 16. The right-hand side of this equation is sketched in Figure 1. Note first that, at low $P^{3/2}/n^2$ ($T_*/T_c \ll 1$) there are no stationary solutions. (dI/dt) is always negative, and the plasma collapses to zero current and temperature for any initial values. At higher $P^{3/2}/n^2$ there are two stationary solutions. Only the one at higher I and T is stable. If the initial current is below the lower stationary point, the plasma will collapse even if sufficient power is applied. Thus, there is a threshold condition on $P^{3/2}/n^2$ for the existence of a stationary state, and also a minimum initial current required for reaching this state via noninductive current build-up. The existence of a minimum current for any power level has important implications for the design of tokamaks that start up with no inductive drive.

Figure 1. Schematic of du/dt versus u (Equation 16) for low, threshold, and high values of $P^{3/2}/n^2$. Arrowheads indicate the direction in which the system state moves.

The steady-state solution of Eqn. 16 is given by the roots of

$$0 = u^{3/2} + (\alpha - 1)u^{1/2} + \beta \quad (17)$$

These roots can be found analytically. If the quantity $\nu = (\alpha - 1)^3/27 + \beta^2/4$ is positive, there is one real root which is non-physical ($u^{1/2} < 0$). If ν is negative, there are two positive real roots. The threshold condition is $\nu \leq 0$, giving $T_*/T_c \geq C_Z/c_1$

or

$$\frac{P^{3/2}}{n} \geq \frac{C_Z}{(k_E/3eV) k_{CD} T_c^{1/2}} \quad (18)$$

where $C_Z = c_2 + 3(c_1/4)^{1/3} = 1.39 + 0.61 (Z_2/3)^{0.7} + 3(1 + Z_2/4)^{1/3}$. In terms of the beam, plasma, and machine parameters, this becomes

$$\frac{P^{3/2}}{n^2} \geq 7.96 \times 10^{-31} \frac{Z_b}{A_b^{1/3} \xi_0} \left(\frac{Z_{eff}}{Z_2} \right)^{1/3} \frac{C_Z \ln \Lambda}{C_E A_i^{1/2} F} \kappa^{1/2} R^{0.25} a^{2.37} \quad (19)$$

Note that this condition depends most strongly on the size of the plasma, and is independent of beam energy. Well above the threshold, the stationary solution approaches $u = 1$, or

$$I = I_* = 2.44 \times 10^{16} \frac{E_{b0} \xi_0}{Z_b} \frac{Z_2}{Z_{eff}(4 + Z_2)} \frac{F}{\ln \Lambda} \frac{P}{nR} \quad (20)$$

$$T = T_* = 5.08 \times 10^{34} \frac{E_{b0} \xi_0}{Z_b} \frac{Z_2}{Z_{eff}(4 + Z_2)} \frac{F}{\ln \Lambda} \frac{P^{3/2}}{n^2 R V} \quad (21)$$

Figure 2 shows the time behavior of two NBCD discharges in DIII-D that are near the power threshold. In these discharges the target plasma is prepared inductively, but the inductive drive is removed at the same time the beams are turned on. Figure 2a shows the plasma current and 2b shows the time behavior of $P^{3/2}/n^2$. Shot 57456 is slightly above the threshold, and the current remains approximately constant from the end of the initial transient at 1250 ms to the decrease in injected neutral beam power at 2050 ms. The value of $P^{3/2}/n^2$ for shot 61539 is approximately two-thirds that of shot 57456, and the current decays throughout the beam pulse - slowly at first, then more rapidly as the temperature falls. The temperature shows a similar time dependence.

In Figure 3 we summarize the results of a number of fully noninductive NBCD attempts. The measured value of $P^{3/2}/n^2$ is plotted versus the calculated threshold value (Equation 19). The coefficient C_E varied between 0.6 and 1.2 for these plasmas, and was determined by fitting the measured confinement data to Equation 5.

This phenomenon may be important because the threshold conditions for power, density and current fall in the range of both present NBCD experiments and future large tokamak designs. Figure 4 shows the stable and unstable stationary conditions of plasma current versus applied power, for parameters characteristic of DIII-D and ITER. The corresponding temperature curves have qualitatively similar behavior,

Figure 2. Time history of (a) plasma current and (b) $P^{3/2}/n^2$ for two DIII-D discharges: 57456 (solid line) and 61539 (dashed line).

Figure 3. Experimental values of $P^{3/2}/n^2$ versus the threshold values predicted by Equation 19 for the experimental conditions.

except that the asymptotic dependence for large power is $T \propto P^{3/2}$, whereas the current follows $I \propto P$ (see Equations 20 and 21). For example, for ITER at $n = 6 \times 10^{19} \text{ m}^{-3}$, the minimum power is about 120 MW, and the current and temperature at the threshold are 13.8 MA and 11.9 keV respectively (this temperature does not depend on density).

Figure 4. Steady-state current versus power with density as a parameter for (a) DIII-D and (b) ITER conditions. The dashed line indicates the unstable branch. The parameters are for DIII-D: $a=0.63$, $R=1.65$, $\kappa=1.7$, $Z_{eff}=2.0$, $\ln \Lambda=18$, $C_E=2.0$, $A_i=2$, $A_b=2$, $\xi_0=0.71$, $E_{b0}=75$ keV; for ITER: $a=2.2$, $R=5.8$, $\kappa=1.8$, $Z_{eff}=1.5$, $\ln \Lambda=19$, $C_E=2.0$, $A_i=2.5$, $A_b=2$, $\xi_0=0.87$, $E_{b0}=1$ MeV.

An important element not included in this calculation is the bootstrap current. The bootstrap current is dependent on the details of the density and temperature profiles, as well as on the collisionality of the electrons. Thus it is awkward to include it in a zero dimensional formulation such as this. If included, a term proportional to T/I with a factor accounting for profiles and collisionality should be added to the right-hand side of Eqn. 2. The bootstrap effect will also appear as a positive term, proportional to $u^{-3/2}$ on the right-hand side of Eqn. 16, with a correction for collisionality which changes the dependence of this term to $u^{1/2}$ at small u . The behavior of this effect is qualitatively similar to the current drive term, and inclusion of the bootstrap current will not remove the threshold but will only modify the threshold conditions.

57456 61539

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TIME (msec)

TIME (sec)

$$\frac{C_z}{\left(\frac{k_{15}}{30V}\right) k_{CD} T_c^{1/2}} \times 10^{31}$$

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Fig 3

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