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Charged Pion Production from Neutron-Proton Collisions at 790 MeV

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CHARGED PION PRODUCTION FROM NEUTRON-PROTON
COLLISIONS AT 790 MeV

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ABSTRACT

The two reactions $np \rightarrow nn\pi^+$ and $np \rightarrow pp\pi^-$ were studied at 790 MeV (incident neutron energy). Pion spectra were measured at 10 different angles with a multiwire proportional chamber spectrometer. Approximately 100,000 events were analyzed. The angular distribution of pions in the np center of momentum system ($\frac{d\sigma}{d\Omega^*}$) was given by $[(123.1 \pm 2.7) + (88.3 \pm 4.9)\cos^2(\theta^*)] \frac{\mu b}{sr}$. The cross-section $\sigma(np \rightarrow NN\pi^\pm)$ was determined to be $1.92 \pm .20$ mb by integrating $\frac{d\sigma}{d\Omega^*}$ over all angles. The partial cross-section for pion production from T=0 np interactions (σ_{01}) was found to be $.1_{-.1}^{+.5}$ mb by using the relation $\sigma_{01} = 2\sigma(np \rightarrow NN\pi^\pm) - \sigma(pp \rightarrow pp\pi^0)$. Stronger indications of non-resonant pion production were given by (1) the presence of asymmetries between the positive and negative pion spectra and (2) a comparison of the data with an isobar model calculation.

CHAPTER I

INTRODUCTION

Pion production in nucleon-nucleon collisions has been the subject of considerable interest in nuclear and particle physics for many years. A number of experiments have been conducted, spanning the energy region from threshold to many GeV. However, the bulk of the experimental data has come from pp collisions, due to the difficulty in obtaining intense and monoenergetic neutron beams.

This paper describes an experiment performed at the Los Alamos Meson Physics Facility, in which a 790 MeV neutron beam (~ 18 MeV FWHM) was used to initiate the following reactions:

$$np \rightarrow nn\pi^+$$

$$np \rightarrow pp\pi^- .$$

Pion spectra were measured at ten different angles with a multiwire proportional chamber spectrometer. Approximately 100,000 pion events were analyzed.

Previous experiments concerning these two reactions in the medium energy range have been conducted (Table 1). However, the neutron beams utilized in these experiments typically had a FWHM which was 10 - 20% of the quoted energy. In addition, most if not all of the measurements have suffered from poor statistical accuracy.

Studies involving neutron-proton reactions are necessary in order to provide information on the initial $T=0$ nucleon-nucleon isotopic spin state. Unlike pp and nn systems, which are required to be in a $T=1$ state, an np system can be in either a $T=0$ or $T=1$ state. It is possible to isolate effects due to the $T=0$ interaction by appropriately combining

Table 1.

A summary of some previous experiments

Reaction	Neutron Beam Energy	Number of Events, Quantities Measured	Reference
$np \rightarrow pp\pi^-$ $np \rightarrow nn\pi^+$	400 MeV	$\sim 400, \sigma, \frac{d\sigma}{d\Omega}(\theta), \frac{d^2\sigma}{d\Omega dE}$	1
$np \rightarrow pp\pi^-$	$\sim 290 - 440$ MeV	4079, σ versus neutron energy	2
$np \rightarrow pp\pi^-$ $np \rightarrow nn\pi^+$	585 MeV	unknown number of events $\sigma, \frac{d\sigma}{d\Omega}(\theta), \frac{d^2\sigma}{d\Omega dE}$	3
$np \rightarrow pp\pi^-$ $np \rightarrow nn\pi^+$	600 MeV	unknown number of events $\sigma, \frac{d\sigma}{d\Omega}(\theta)$	4
$np \rightarrow pp\pi^-$	290 - 970 MeV (effective energy = 780 MeV)	1721, $\sigma, \frac{d\sigma}{d\Omega}(\theta), \frac{d^2\sigma}{d\Omega dE}$, angular correlations between pp and $p\pi^-$	5

np and pp (or nn) data. This was first demonstrated by Watson, Brueckner, Gell-Mann, and Rosenfeld.^{6,7,8} Using the laws of conservation of isotopic spin, parity, and angular momentum, they showed that the total cross-sections for all single pion production reactions at a given energy could be described by four independent partial cross-sections (Table 2). Each partial cross-section has two subscripts which identify, in order, the isotopic spins of the initial and final state nucleons. It should be noted that the neglect of Coulomb effects and the π^0 - $\pi^\pm$ and n-p mass differences introduce small uncertainties in the relationships given in Table 2. However, if these effects are ignored, the partial cross-section associated with the T=0 nucleon-nucleon initial state (σ_{01}) is given by $2\sigma(np \rightarrow np\pi^0) - \sigma(pp \rightarrow np\pi^+) + \sigma(pp \rightarrow pp\pi^0)$ or $2\sigma(np \rightarrow NN\pi^\pm) - \sigma(pp \rightarrow pp\pi^0)$.

A phenomenological model developed by Mandelstam⁹ has been used to describe single pion production in the region from threshold to about 800 MeV. The three basic assumptions of this model are as follows:

- (1) The outgoing pion and one of the nucleons form the $\Delta(1236)$ resonant state.
- (2) Pions are produced only from low angular momentum states of the nucleon-nucleon system.
- (3) The production matrix elements are independent of energy, except for factors involving the final state nucleon-nucleon and pion-nucleon interactions.

The $\Delta(1236)$ resonant state mentioned in the first assumption has an isotopic spin of $\frac{3}{2}$. Production of the $\Delta(1236)$ can therefore come only from colliding nucleons in a T=1 state, since isotopic spin is conserved in any reaction involving strong interaction forces. Thus assumption 1

Table 2.

Single pion production total cross-sections expressed in terms of partial cross-sections

Reaction	Total Cross-Section
$pp \rightarrow d\pi^+$	$\sigma_{10}(d)$
$pp \rightarrow pn\pi^+$	$\sigma_{10}(np) + \sigma_{11}$
$pp \rightarrow pp\pi^0$	σ_{11}
$np \rightarrow nn\pi^+$	$\frac{1}{2}(\sigma_{11} + \sigma_{01})$
$np \rightarrow pp\pi^-$	$\frac{1}{2}(\sigma_{11} + \sigma_{01})$
$np \rightarrow d\pi^0$	$\frac{1}{2} \sigma_{10}(d)$
$np \rightarrow np\pi^0$	$\frac{1}{2}(\sigma_{10}(np) + \sigma_{01})$
$nn \rightarrow d\pi^-$	$\sigma_{10}(d)$
$nn \rightarrow np\pi^-$	$\sigma_{10}(np) + \sigma_{11}$
$nn \rightarrow nn\pi^0$	σ_{11}

$$\sigma_{10} = \sigma_{10}(d) + \sigma_{10}(np)$$

is in effect stating that the cross-section σ_{01} is zero. However, results from some of the experiments listed in Table I have indicated that σ_{01} is a significant fraction of the total $(np \rightarrow NN\pi^{\pm})$ cross-section, although the errors are generally quite large. Mandelstam's second assumption fails beyond 700 - 800 MeV, as too many partial waves become involved.

The one pion exchange model^{10,11} has been used in the region from about 800 - 1500 MeV. Here it is assumed that pion production proceeds via peripheral collisions between the two nucleons. The neglect of the nucleon-nucleon final state interaction apparently restricts this model to energies greater than about 800 MeV.

Stephenson, Gibbs, and Gibson^{12,13} have recently formulated a phenomenological description of pion production which includes an NN final state interaction and S and P wave pion-nucleon phase-shifts. Their model has been applied with good success to 0° neutron spectra from the reaction $pp \rightarrow np\pi^{+}$ at 647, 771, and 805 MeV incident proton energies.¹⁴

The present data provide important tests of pion production theories, such as those just described. This should contribute to a better understanding of the strong interaction forces between nucleons.

CHAPTER II

EXPERIMENTAL METHOD

A. OUTLINE OF THE EXPERIMENT

This experiment was performed at the Nucleon Physics Laboratory (NPL) located at the Los Alamos Meson Physics Facility (LAMPF). LAMPF was designed to accelerate 200-800 MeV H^+ and H^- ions simultaneously, with average intensities of 1 ma and 100 μ a, respectively. Details on the construction and operation of LAMPF can be found elsewhere (for example, see Reference 15).

The Nucleon Physics Laboratory (also known as Area B) and other experimental areas at LAMPF are shown in Figure 1. An 800 MeV proton beam was focused onto a liquid deuterium target (LD_2), where neutrons were produced via the reaction $pd \rightarrow nX$. Bending magnets located behind the LD_2 target diverted the remaining protons into a beam stop.

The proton beam was delivered in 500 μ s macropulses, with a frequency of 120 pulses/sec. Each macropulse had an RF micropulse structure which consisted of $\frac{1}{4}$ ns FWHM bursts every 5 ns. The significance of the micropulse structure will be discussed in the next section. Beam currents up to about 6 μ a were utilized in this experiment. At 6 μ a, around 25 watts of beam heating power were dissipated in the liquid deuterium (areal density ≈ 1.80 gm/cm²). However, this was easily handled by the LD_2 system, which was designed to remove over 200 watts. (Further details on the construction and operation of the LD_2 target system can be found in Reference 16.)

Neutrons produced at 0° with respect to the incident proton beam were selected by means of the collimator shown in Figure 2. This

Figure 1.

Schematic drawing of the experimental areas at LAMPF

Figure 2.

Schematic view of the Nucleon Physics Laboratory

collimator, 3.66 m in length, was made from a 155 mm gun barrel surrounded by large amounts of concrete and steel shielding. Annular lead and steel inserts placed in the gun barrel reduced the collimator diameter to 2.54 cm (1 in.). A lead plug of 6.35 cm (2-1/2 in.) thickness was placed near the downstream end of the collimator in order to reduce the flux of beam associated gamma rays. (Gamma rays can be formed, for example, from the reactions $pd \rightarrow ppn\pi^0$, $\pi^0 \rightarrow \gamma\gamma$). As will be explained in a later section, positrons and electrons from gamma-ray conversions created an undesirable background. The lead had essentially no effect on the shape of the 0° neutron spectrum, although it did reduce the intensity somewhat.

After exiting the collimator, the neutrons passed through a sweep magnet (M1), which served to remove residual charged particles in the beam. Located downstream from M1 was a liquid hydrogen target (LH_2), in which the np interactions took place. The liquid hydrogen was contained in a cylindrical mylar flask 13.2 cm by 10.15 cm, and had an areal density of $.94 \text{ gm/cm}^2$. The liquid hydrogen density varied by about 1% (rms) because of pressure fluctuations in the target flask. Further details on the LH_2 target are given in Reference 16.

Pions and other charged particles produced in the LH_2 target were momentum analyzed as a function of angle with a multiwire proportional spectrometer. This spectrometer will be described in Section C.

B. NEUTRON BEAM ENERGY SPECTRUM

The 0° degree neutron beam spectrum produced through the reaction $pd \rightarrow nx$ at 800 MeV incident proton energy is shown in Figure 3. The narrow peak close to the incident proton energy is due to quasi-elastic charge-exchange scattering involving the incident proton and the

Figure 3.

0° neutron spectrum from the reaction $pd \rightarrow nx$ at 800 MeV incident proton energy

FWHM of peak ≈ 18 MeV

Flux of high energy neutrons incident on LH_2 target/ μ a of proton beam $\approx 9 \times 10^5$ /sec

deuterium neutron. Most of the pions detected in this experiment were produced by neutrons in this peak.

The neutron spectrum was inferred by measuring the proton energy spectrum from the reaction $np \rightarrow np$. This was done in an earlier spectrometer experiment at NPL.^{16,17} The position of the high energy peak centroid was found to be about 10 MeV below the incident proton energy. This 10 MeV shift included the effects of the deuteron break-up process as well as beam proton energy losses in the LD_2 target. Elastic np data taken during the present experiment also indicated a shift between the centroid location and proton beam energy (790 ± 5 MeV versus 800 MeV). Therefore, the effective neutron beam energy was chosen to be 790 MeV.

The lower energy neutrons constituted an undesirable background, since they could also produce pions in the LH_2 target. If the beam bursts had been spaced far enough apart, pions produced by neutrons of different energies would have come at unique times with respect to the beam RF structure. (Pion RF times were measured by a scintillation counter located immediately behind the LH_2 target.) The occurrence of a burst every 5 ns allowed neutrons of different energies associated with different micropulses to arrive at the LH_2 target with the same RF time, thus producing pions with the same RF time. Because of this fact, and a finite RF timing resolution, pions produced by neutrons around 500 MeV were indistinguishable from those produced by neutrons in the 790 MeV peak. However, the overall effects of the 500 MeV neutrons were small, for the following reasons:

1. The total flux of these neutrons (roughly 460-530 MeV) was about 50% of that in the 790 MeV peak.

2. The ratio $\frac{\sigma(np \rightarrow NN\pi^{\pm})_{500 \text{ MeV}}}{\sigma(np \rightarrow NN\pi^{\pm})_{790 \text{ MeV}}}$ was estimated from the present and

existing data to be about .35.

3. A large part of the pion energy spectrum region at 790 MeV was unavailable to pions produced by the 500 MeV neutrons.

Corrections applied to the data in order to account for pions created by the 500 MeV neutrons will be discussed in Chapter 3.

The horizontal neutron beam profile at the LH₂ target is shown in Figure 4. This graph was constructed by extending the trajectories of spectrometer events back to the LH₂ target midplane. The vertical distribution was almost identical.

C. SPECTROMETER

The multiwire proportional chamber (MWPC) spectrometer used in this experiment has been the subject of several detailed accounts.^{17,18,19,20} The following description is therefore brief wherever possible.

Figure 5 shows the spectrometer system layout. S1, a scintillation counter, determined timing with respect to the beam RF structure and, in conjunction with counter S2, provided particle time-of-flight (TOF) information. Bending magnet M2 was outfitted with a special current reversing switch which allowed the detection of both positive and negative particles. Four wire chambers W1-W4, each containing one vertical and one horizontal set of sensing wires, were used to determine particle trajectories for later use in momentum calculations. A gas threshold Cerenkov counter (C), was located between W1 and W2 to tag relativistic electrons. Helium bags were placed in the magnet gap, and between W3 and W4, in order to reduce the effects of multiple Coulomb scattering. Previous experiments performed with the spectrometer utilized an

Figure 4.

Horizontal neutron beam profile at the LH_2 target

anti-counter immediately downstream of the bending magnet in order to veto spectrometer events initiated by charged particles in the neutron beam. However, this counter was found to be unnecessary, and it was therefore eliminated.

The signature of a valid event (EVENT) was defined to be the coincidence $S1 \cdot (W1X \cdot W2X \cdot W3X \cdot W4X)^{3/4} \cdot (W1Y \cdot W2Y \cdot W3Y \cdot W4Y)^{3/4} \cdot S2L \cdot S2R$. Here X and Y refer to the horizontal and vertical position determining signal planes, respectively. S2L and S2R correspond to the phototubes on each end of S2. Note that only three out of four X and/or Y signal planes were required to signal an event. This is because 3X and 3Y coordinates, along with a magnetic field map, contained enough information for particle momentum determinations.

The magnet, wire chambers, and scintillation counters S1 and S2 were mounted on a stand which could be pivoted about a point directly beneath the LH_2 target (see Figure 6). This permitted measurements to be made as a function of angle.

1. Bending Magnet

The bending magnet used in the spectrometer was an H frame magnet with "pancake" coils. The distance between the pole faces was 15.2 cm (6 in.), the pole faces themselves being 91.4 cm (36 in.) in length and 45.7 cm (18 in.) wide. At the maximum current of 800 amperes, the field strength in the center of the magnet was approximately 18 kilogauss.

The magnetic field in the magnet was mapped at four currents - 800, 600, 400, and 200 amperes. This mapping was done in a three-dimensional Cartesian coordinate system with steps of 2.54 cm in each direction. The resulting data were stored on computer tape for later

Figure 6.

A photograph of the spectrometer
The Cerenkov counter is not shown.

use in momentum calculations. In addition, the integral $\int |B_y| dz$ was measured at the currents shown in Figure 7, where B_y refers to the vertical component of the magnetic field. The coordinate z was measured along the symmetry axis through the magnet gap (from W2-W3), parallel to the pole faces. The overall uncertainty in the magnetic field measurements was believed to be about .7%.

In order to detect pions at 10 different angles, a total of 17 different magnet currents below 200 amps was used. Field maps were generated by first determining the integral $\int B_y dz$ from interpolation (Figure 7), and then scaling the 200 amp field map by the ratio

$$\frac{\int B_y dz}{\int B_y dz_{200 \text{ amps}}} .$$

2. Wire Chambers

The four Charpak-type multiwire proportional chambers W1-W4 each contained two signal planes, one with horizontal and one with vertical sense wires. Each signal plane was positioned between a set of two high voltage planes. Sense wires were 20 μ diameter gold-plated tungsten wire, with a 2 mm spacing. Signals from the individual sense wires were read into combination amplifier-logic cards. Each card was capable of servicing eight adjacent wires.

The first three chambers W1-W3 were identical, each containing 96 horizontal and 192 vertical signal wires. However, since W1 and W2 were located close to the LH₂ target, not all of the wires in these chambers were made active. The last chamber (W4) contained 192 horizontal and 320 vertical wires.

The overall efficiency of each signal plane for detecting charged particles was typically over 99%.

Figure 7.

Bending power and magnetization curves for the spectrometer bending magnet

3. Cerenkov Counter

The construction of the gas-threshold Cerenkov counter is shown in Figure 8. The horizontal cylinder was filled with isobutane gas at atmospheric pressure. Cerenkov light could be produced in the isobutane (refractive index ≈ 1.001 at STP) by electrons and pions having momenta greater than about 10 MeV/c and 3000 MeV/c, respectively. However, in this experiment, the maximum pion momentum was approximately 600 MeV/c, far below the 3000 MeV/c threshold. An elliptical plane mirror reflected the Cerenkov light into a special light collecting funnel. Light gathered by this funnel was subsequently focused onto a phototube face.

In order not to seriously limit the spectrometer solid angle acceptance, the counter was constructed from 25.4 cm (10 in.) inside diameter tubing. Isobutane gas was brought in through the bottom and discharged through the top via an oil-filled bubbler. Black polyethylene windows (opaque to light) were located at either end of the tube. The elliptically-shaped mirror consisted of aluminized mylar stretched over an aluminum frame. Light reflected by this mirror passed through a transparent mylar window which served to isolate the isobutane from the light funnel and phototube. This window was made necessary because of the difficulty in making a gas-tight seal at the funnel-phototube interface. Unfortunately, it caused a loss in the amount of light reaching the phototube.

The funnel was designed²¹ to collect light with a maximum divergence of 3° . This allowed electrons to be detected over the entire spectrometer geometric acceptance region. Light reflected from the funnel formed a 3.8 cm (1.5 in.) diameter circle at the phototube face,

Figure 8.
Schematic diagram of the Cerenkov counter

with a 90° divergence. The funnel was machined from Plexiglas, and the inside surface was coated with aluminum.

The phototube used to detect the Cerenkov light was an RCA 31000Q, with a 5.1 cm (2 in.) diameter curved quartz window. This tube is similar to an RCA 8850, except that the 8850 has a flat glass face. The quartz window of the 31000Q was desirable since it could transmit ultraviolet Cerenkov light.

Due to the importance of the Cerenkov counter in tagging electrons, extensive tests were conducted in order to determine its efficiency. The counter was taken to the Lawrence Berkeley Laboratory 184 in. Cyclotron, where a 180 MeV/c negative particle beam was available. This beam was composed mainly of pions, although 10 - 15% of the particles were electrons.

The experimental set-up used in the tests is shown in Figure 9. Pions and electrons were separable by their times-of-flight between the scintillation counters S_a and S_d . Two small scintillation detectors S_b and S_c (each approximately 6 cm²) were used to map the efficiency over the surface of the counter. The electron efficiency of the counter was defined to be

$$\frac{\sum_i [(S_a \cdot S_d)_e \cdot S_b \cdot S_c \cdot C]_i}{\sum_j [(S_a \cdot S_d)_e \cdot S_b \cdot S_c]_j},$$

where $(S_a \cdot S_d)_e$ refers to an electron (determined by its time-of-flight), and C refers to a Cerenkov counter signal. During the initial phase of the tests it became apparent that the phototube (a 31000Q) was not functioning properly. Due to the difficulty in obtaining an immediate replacement, an 8850 was used as a substitute.

Figure 9.

Experimental set-up for testing the Cerenkov counter
at the LBL 184" Cyclotron

Figure 10a shows the counter efficiency as S_b and S_c were moved together across the counter face. The loss in efficiency at the center was probably caused by the flat face of the 8850. A large angle reflection from the funnel was needed for light near the center to strike the phototube; however, such rays could graze the flat face and not actually enter the tube. The curved face of the 31000Q was better suited for collecting such light. The response of the Cerenkov counter to rotation about an axis perpendicular to the beam is shown in Figure 10b. (The counter and beam axes coincided at 0°). The efficiency was constant over $\pm 3^\circ$ as planned, with the exception of a possible 1° misalignment.

Prior to taking data at LAMPF, the counter was installed in the spectrometer and tested with a new 31000Q phototube. The lead plug was left out of the neutron collimator, so that a large flux of gamma rays was present. Lead shot was placed immediately ahead of S1 in order to convert some of the gamma rays into electron-positron pairs. The polarity of the spectrometer magnet was chosen such that negative particles were detected. This meant that virtually all of the spectrometer events were electrons. The Cerenkov counter efficiency was determined by the ratio $\sum_i (C \cdot e)_i / \sum_j e_j$, where e represents a spectrometer event (electron). This ratio was measured as a function of momentum and angle across the face of the counter. Absolute efficiencies of about .75 were obtained, essentially independent of angle or momentum. (It was inferred from the presence of two peaks in the S1 pulse height distribution that some of the events which passed through the Cerenkov counter were actually electron-positron pairs. This meant that the spectrometer test gave an upper limit on the absolute efficiency for detecting a single electron. However, it was

a) Cerenkov counter efficiency versus the horizontal distance from the beam axis to the counter axis

b) Response of the Cerenkov counter to rotation about an axis perpendicular to the beam

Figure 10.

Some results from the Cerenkov counter tests at LBL

determined from the results of a separate test with cosmic ray muons that the efficiency for detecting single particles was ~.7 - .8.)

Isobutane gas was utilized in the Cerenkov counter because of its poor scintillation properties. However, knock-on electrons produced in the isobutane by a passing particle could occasionally produce enough light to trigger the phototube. The efficiency for detecting pions in this manner was estimated during the Berkeley tests by measuring the quantity

$$\frac{\sum_i [(S_a \cdot S_d)_\pi \cdot S_b \cdot S_c \cdot C]_i}{\sum_j [(S_a \cdot S_d)_\pi \cdot S_b \cdot S_c]_j}$$

This number was found to be no greater than .01.

4. Data Collection System

Schematic diagrams of the spectrometer data collection system and electronics are shown in Figures 11 and 12, respectively. After a valid event (or MASTER GATE) occurred, the following sequence of steps took place:

- (1) The triggering system was inhibited from accepting any more events. This was accomplished by setting the inhibit flip-flop (INHIBIT FF).
- (2) A start pulse was generated for the time-to-amplitude converters (TAC's).
- (3) Conversion of the analog-to-digital converters (ADC's) was begun.
- (4) An EVENT signal was sent to the scratch pad memory (SPM)²² a buffer which stored incoming wire chamber data.

Figure 11.

Spectrometer data collection system

Figure 12.

Spectrometer electronics diagram

- (5) The SPM sent an ENCODE ENABLE signal to the wire chambers, in order to initiate the transmission of the hit-wire information.

The ADC information was read into the micro-programmed branch driver (MBD),²³ an interface between the CAMAC electronics and the PDP-11 computer. During the same time, wire chamber information was stored in the SPM. After these steps had taken place, the MBD removed the event INHIBIT (thus freeing the system for another event). The SPM data were subsequently sent to the MBD, which was capable of storing information on several events. The MBD transferred its contents to the PDP-11 computer between beam macropulses.

A special data-handling program²⁴ was responsible for writing the data onto magnetic tape as well as providing on-line analysis of some of the incoming events. For example, the program used the horizontal bend angle made by particles through the spectrometer in order to estimate their momenta. Histogramming and plotting routines were available to display the results of such analysis. Figure 13 shows an on-line plot of particle momentum versus TOF. It can be seen that π 's and e's (left), protons (middle), and deuterons (top, right) formed distinct contours. More will be said about this in the next chapter. (The actual pion data were taken with magnet currents considerably lower than the current used to collect the events in Figure 13. At the low "pion data" currents, deuterons were not able to pass through the spectrometer, since their momenta were too high. However, Figure 13 accurately illustrates the separation of pions and protons.)

TOF

Figure 13.

On-line plot of particle momentum versus TOF

D. BEAM AND HYDROGEN TARGET MONITOR SYSTEMS

1. Beam Monitors

Two different systems were used to monitor beam conditions in this experiment. One of these was a toroidal current integrator (CI) which measured the flux of protons incident upon the LD₂ target. This monitor had an overall accuracy of about 5%.

A neutron flux monitor was located at the downstream end of the collimator (Figure 14). This system consisted of an anti-counter (A), a polyethylene (CH₂) radiator, and two identical range telescopes L (left) and R (right). The anti-counter was used to discriminate against charged particles incident on the converter. A left (right) neutron monitor count was recorded if the coincidence requirement $\bar{A} \cdot L1 \cdot L2$ ($\bar{A} \cdot R1 \cdot R2$) was satisfied. Copper absorbers were located between the two scintillators in each telescope in order to reduce the chance of detecting low energy neutrons coming from the collimator walls.

During part of this experiment, one of the counters in the right telescope turned out to be unreliable. Therefore, only the left telescope was used to monitor the neutron flux.

2. Hydrogen Target Monitor

As is shown in Figure 14, the hydrogen target monitor system (H) consisted of four scintillation counters H1, H2, H3, and H4. These four counters formed a telescope oriented at 45° with respect to the neutron beam line. Range material (copper) was located between H2 and H3, and between H3 and H4. The coincidence requirement $H1 \cdot H2 \cdot H3 \cdot \bar{H4}$ could be met by elastically scattered protons corresponding to neutron energies between about 350 and 900 MeV. The hydrogen monitor's function was to provide a check on the condition of the LH₂ target

Figure 14.

Neutron flux and LH₂ monitor systems

E. ELECTRONICS DEAD TIME CORRECTIONS

All of the electronics were gated on only during the 500 μ s beam macropulse. This insured that events were accepted and that the monitors were scaled only when the beam was on. However, as explained in the previous section, the electronics were gated off (inhibited from accepting events) while the data acquisition system was busy processing an event. A dead time was introduced and, therefore, had to be taken into account. This was achieved by scaling all the monitors (except the current integrator) with the data acquisition INHIBIT in effect. Monitors treated in this manner are referred to as gated (with the subscript g). The monitors were also scaled without the INHIBIT, and are referred to as ungated (with the subscript u). It was, therefore, possible to calculate the live time from the quantities $\frac{L_g}{L_u}$, $\frac{H_g}{H_u}$, etc.

A special type of dead time was associated with the neutron flux monitor system. Each anti-counter event prevented the monitor system from scaling during a time interval determined by the width of the anti-counter's discriminator signal (20 ns). Corrections for this effect (up to ~2%) were made. A 2% dead time corresponded to an anti-counter event rate of 1×10^6 /sec.

F. DATA

The data were collected in 1975 during October and November. Both positive and negative pions were detected at the following spectrometer (lab) angles: 0° , 8° , 16° , 24° , 30° , 36° , 42° , 48° , 56° , and 72° . The data at each angle were broken up into a series of LH₂ full and empty "runs". Multiple Coulomb scattering and pion decays made measurements much below 200 MeV/c impractical.

It had been previously determined from a preliminary phase of this experiment that the negative pions had a considerably larger background/foreground ratio ($\frac{LH_2 \text{ empty}}{LH_2 \text{ full}}$) than did the positive pions. Thus, more negative than positive pion data were collected. The difference in the background subtraction between the two types of pions can be qualitatively explained by considering the nucleons in complex nuclei (such as carbon, which was present in the LH_2 target flask) to be independent of one another. Beam neutrons, colliding with neutrons in such nuclei, can only produce negative pions, though both negative and positive pions are produced in np collisions. At 0° , where S1 was directly in the beam, the ratio ($\frac{LH_2 \text{ empty}}{LH_2 \text{ full}}$) was about .35 for negative pions and about .15 for positive pions. These numbers dropped by factors of ~1.5-3 at the larger angles.

Protons from elastic np collisions were detected at all angles except 72° . This was done in order to determine a normalization for the pion data. This normalization will be discussed in the following chapter.

CHAPTER III

DATA ANALYSIS AND RESULTS

A. OUTLINE OF DATA ANALYSIS

The first step in the data analysis was to determine for each event various quantities of interest, such as its momentum, polar scattering angle, azimuthal scattering angle, etc. A Fortran program called Tewa, run on the LASL CDC 7600 computers, was used for this purpose. Tewa analyzed the raw data and wrote onto magnetic tape an array of physical quantities for each event. Data output by Tewa were referred to as G-data. The G-data were subsequently processed with a plotting and histogramming program named Kiowa.²⁵ Particle identification, background subtractions, cross-section calculations, and most corrections to the data were done in Kiowa.

B. TEWA

The analysis of a given event by Tewa began with the decoding of all analogue and wire word information associated with that event. Polar and azimuthal angles θ_s and ϕ_s , based in the spectrometer coordinate system shown in Figure 15, were calculated from the hit wire positions in the first two chambers. If these angles were within acceptable limits (to be discussed later), a momentum calculation was performed. This calculation was begun by estimating the particle's momentum as shown in Equation 1. Here k is a constant and α is the horizontal bend angle made by the particle through the spectrometer.

$$p \approx \frac{k \int B_y dz}{\alpha} \quad (\text{Eq. 1})$$

Figure 15.

Spectrometer coordinate system

The straight line segments (or tracks) drawn between the W_1 - W_2 and W_3 - W_4 hit wire positions approached each other most closely near the magnet symmetry plane.

This estimate of p (usually good to within 5%) was used as a starting point in a Runge-Kutta solution to the equation of motion $\frac{d^2\vec{r}}{d\ell^2} = q \frac{\vec{u} \times \vec{B}}{p}$. Here $d\ell$ is a path element of the particle in the magnetic field, q is its charge, and $\vec{u}$ is a unit vector along the particle's direction of motion. The trajectory of the particle $\vec{r}$ was numerically integrated through the field starting from a plane halfway between the first and second wire chambers (Figure 15). The calculation was continued until a plane halfway between the last two chambers was reached. By comparing the measured and calculated trajectory information, a χ^2 (goodness of fit) was determined. Minimization of this χ^2 led to small changes in the momentum and event position coordinates. Note that only horizontal trajectories were fit with a χ^2 minimization procedure to the magnetic field. This was because vertical displacements caused by the magnetic field were very small (~ 5 mrad) and could be neglected.

The preceding method of momentum determination was used for events which were categorized as perfect eight plane events. These events had one cluster of hit wires in each of the eight signal planes. (A cluster is defined as a group of one or more adjacent hit wires.) If there happened to be two or more wires in a cluster, the center of the wire group was used as the effective hit position. Usually around 80% of the events were perfect eight plane events.

For events having a single cluster in six or seven planes (one X and/or one Y plane missing), a special reconstruction process was used. This process was based on the fact that the incoming and outgoing tracks made by an event approached each other most closely, on the average, at a Z-coordinate (ZLOC) near the XY magnet symmetry plane (Figure 15). The reconstruction of a missing coordinate in a given track (called track_m)

was achieved by first finding the intersection point of the opposite track with the plane $Z = ZLOC$. The $X(Y)$ value of this point and the one $X(Y)$ value in track_m were then used to determine the missing $X(Y)$. Note that both one X and one Y coordinate could be reconstructed by this method for a given event. $ZLOC$ was calculated as part of the G-data for all perfect eight plane events, and an average value of $ZLOC$ (used in the reconstruction process) was determined as a function of the magnet current. However, the variations in $ZLOC$ with current were very small (usually millimeters).

Another class of events occurred, namely those with two or more clusters in one or more planes (multiple cluster events). Some of these events could be analyzed by ignoring information in the multiple cluster planes and proceeding as in the case for six and seven plane events. However, this method of analysis was not always possible. A special version of Tewa was available which could track all possible trajectory combinations through the magnet and determine the one with the smallest χ^2 . Some of the data were analyzed with this version of Tewa in order to estimate the number of pion events that had not been analyzed by the original Tewa version. For the positive pion data, the fractional increase in the number of pions was about .02. The multiple clustering in this case was probably caused by delta rays and noisy wires. However, a substantial fractional increase (up to about .2) in the number of pions was found in some of the negative pion data taken at the smaller laboratory angles. Many of these additional events had multiple clusters in the first two wire chambers and a relatively large S1 pulse height. Physically, this corresponded to the situation where the pion and one or both of the protons from the reaction $np \rightarrow pp\pi^-$ moved close enough

together to pass through S1 and the first two wire chambers. The pulse height in S1 was thus larger than that produced by a single pion. The bending magnet separated the particles and ultimately only the pion completed the event trigger logic (by passing through W3, W4, and S2).

C. REMOVAL OF STRUCTURE ASSOCIATED WITH THE 2 mm SIGNAL WIRE SPACING

Due to the discrete nature of the coordinate information associated with each event (individual hit-wires occurring at fixed intervals), a granular structure appeared in histograms of θ_s , ϕ_s , α , β (vertical bend angle), and p . In order to correct this problem, it was assumed that the charged particle detection efficiency of each signal wire was unity within ± 1 mm of the wire and 0 outside this region. In Kiowa the hit positions were, therefore, distributed uniformly within this 2 mm interval by computer generated random numbers. The angular quantities were then recalculated from the new (randomized) hit positions. A randomized momentum p_r was calculated from the original momentum p and the original and randomized horizontal bend angles α and α_r through the relation

$$p_r = \frac{\alpha_r}{\alpha} p.$$

D. SPECTROMETER ACCEPTANCE REGION

The spectrometer geometric acceptance region was limited vertically by the magnet gap and horizontally by the finite width of the wire chambers and the Cerenkov counter. In order to prevent the possibility of particles striking the magnet pole faces, events were rejected if they did not obey the relation $\tan(\theta_s) \sin(\theta_s) \leq .02$. The horizontal limits of the acceptance region as imposed by the wire chambers were empirically determined by examining plots of θ_s versus the chamber hit-wire locations for proton events. This was done with different momentum bands and several magnet currents. Figure 16 shows the resulting horizontal acceptance

Figure 16.

Spectrometer acceptance region

The region enclosed by the solid lines was used in the data analysis.

region. The subscript SL (SR) on the angle θ_{SL} (θ_{SR}) represents the scattering of a particle towards the left (right) side of the incident neutron beam direction. The magnet current dependence has been removed by plotting θ_S versus the ratio $P / \int B_y dz$. In order to avoid striking the Cerenkov counter, θ_{SL} and θ_{SR} were limited to about .05 rad.

The effects of multiple Coulomb scattering or the acceptance region were also studied. It was determined that such scattering could be safely neglected above a pion momentum of about 180 MeV/c (see Bjork, Reference 16, pp. 132-137).

At each spectrometer current and angle setting, the spectrometer acceptance region was transformed into the laboratory coordinate system (Figure 17). Polar lab angles θ_L and θ_R , corresponding to θ_{SL} and θ_{SR} , were calculated as a function of momentum. If the spectrometer azimuthal acceptance had not been restricted, the effective solid angle $\Delta\Omega$ for detecting a particle would have been the entire solid angle between θ_L and θ_R . In general, however, particles could be detected only over a fraction ϵ of the azimuthal angle. The quantity ϵ was a function of a particle's lab angle θ . Thus, in order to properly calculate cross-section information, it was necessary to give each event a "solid angle weight". This weight (W) is shown in Equation 2.

$$W(\theta, p) = \frac{1}{\epsilon(\theta) \cdot \Delta\Omega(\theta_L(p), \theta_R(p))} \quad (\text{Eq. 2})$$

E. PARTICLE IDENTIFICATION

The mass of each particle was determined from its momentum and TOF as shown in Equation 3. Here c is the speed of light and L is the distance between S1 and S2.

Figure 17.

Relationship between the laboratory angles (θ, ϕ) and the spectrometer angles (θ_s, ϕ_s)

$$m = p \sqrt{\frac{(\text{TOF})^2}{L^2} - \frac{1}{c^2}} \quad (\text{Eq. 3})$$

As a group, pions and electrons were easily separated on this basis from protons, which were present at positive magnet currents. (Here particles referred to as pions also included muons. This will be discussed in the next section.) At negative magnet currents, only the pion-electron group was present.

A very small fraction (< 1%) of the events were mistimed, and this resulted in erroneous mass calculations. However, pions and electrons could also be distinguished from protons on the basis of pulse height information. Thus, mistimed pions (outside of the pion mass loci) were identified by appropriate cuts placed on the S1 and S2 pulse heights. Mistimed protons were treated in a similar manner.

It was determined that most six plane events at positive currents were high momentum protons which passed through the first two chambers and then scattered off the frame of W3 into W4. For the sake of convenience, all six plane events were rejected by Kiowa. The resulting number of pions lost was negligible.

Electrons and pions were not separable by either pulse height or mass information. Thus, since the Cerenkov counter efficiency was less than unity, some of the electrons were clearly indistinguishable from pions. As it turned out, however, this was not a problem. At lab angles other than 0°, usually less than 3% of the pion-electron group triggered the Cerenkov counter. At 0°, there was a considerable electron background produced from the conversions of gamma rays associated with the incident neutron beam. (The lead plug in the collimator was used to reduce the

intensity of these gamma rays.) However, these electrons could be distinguished from pions by their RF times. This is illustrated in Figure 18a. From the distance between the LD₂ target and S1 (11.25 m), one would expect a difference of about 7 ns between the two peaks. This shows up as a 2 ns difference because of the fact that an RF burst occurred every 5 ns. RF cuts (1.9 ns in width) were applied to the data not only to remove the electrons at 0°, but also to ensure that the pions were indeed produced by 790 MeV neutrons (see Figures 18b and 19). An error (estimated to be about 5%) was introduced into the final results, since the exact placement and width of the RF cuts were somewhat arbitrary.

F. DECAY CORRECTION

1. Decay Correction Method for Perfect Eight Plane Events

Since pions are unstable particles, it was necessary to investigate the effect this had on the data and to introduce appropriate corrections. The mean lifetime of a pion is 2.60×10^{-8} sec, and most (over 99.97%) of the decays proceed as follows: $\pi^{\pm} \rightarrow \mu^{\pm} + \nu$. The fraction of pions which were able to survive the flight path through the spectrometer is shown as a function of momentum in Figure 20.

It was possible for a pion and its decay muon to satisfy collectively the spectrometer trigger logic and thus appear as a valid event. If it had been possible to identify and thus eliminate all such events (called muon events), the decay correction could have been based entirely on Figure 20. Unfortunately, pion and muon events were not separable by mass or pulse height information. Therefore, a Monte-Carlo program was written in order to study the effects of pion decays in the spectrometer. This program indicated that the probability of detecting a muon event was dependent upon the pion's momentum, azimuthal and polar angles, and the

Figure 18.

RF histograms of negative particles at 0° and 16°
with momenta between ~ 235 and 375 MeV/c

The RF spectra of the highest momentum pions at each angle showed the least contamination from the lower energy beam neutrons (compare Figures 18b and 19).

Figure 19.

RF histogram of negative pions having momenta
between ~375 and 575 MeV/c at 16°

Figure 20.

The fraction of pions surviving the flight path through the spectrometer as a function of momentum

magnet current. The code also demonstrated that many muon events should have abnormal values of ZLOC and/or β (vertical bend angle). This fact was qualitatively confirmed by studying the data. For non-decaying particles (such as protons) the ZLOC and β distributions were determined mainly by multiple Coulomb scattering. However, a pion-muon event ZLOC or β distribution was determined by pion decays as well as multiple Coulomb scattering. The effects of multiple Coulomb scattering on a particle are a function of pv , where v is the particle's velocity. Thus, by histogramming ZLOC and β for "pions" and protons having the same value of pv , the effects due to muon events became apparent. Figures 21 and 22 show such histograms for "pions" and protons with $pv \approx 355$ MeV. Perfect eight plane events were chosen in order to avoid reconstruction of missing hit-wire information. It can be seen in both sets of histograms that the pion data exhibit long tails on either side of the peak. The events in these tail regions must, therefore, have been muon events.

The quantities ZLOC and β are in a sense orthogonal, since ZLOC is most sensitive to horizontal displacements, while β is by definition a measure of the vertical displacement. Thus, in order to remove as many muon events as possible from the data, restrictions were placed on both ZLOC and β . Appropriate limits were determined from proton data at several different momenta. A Gaussian function was fit to each proton distribution and the standard deviations σ_{ZLOC} and σ_{β} were plotted as a function of pv . Each set of points was then connected with a smooth curve (Figure 23). For each "pion" Kiowa calculated σ_{ZLOC} and σ_{β} , depending upon its value of pv . Cuts were made on the basis of a quantity K which is given in Equation 4.

Figure 23.

σ_{ZLOC} and σ_{β} for protons as a function of pv

$$K = \sqrt{\left[\frac{(ZLOC - ZLOC_0)}{\sigma_{ZLOC}} \right]^2 + \left[\frac{(\beta - \beta_0)}{\sigma_\beta} \right]^2} \quad (\text{Eq. 4})$$

Here $ZLOC_0$ and β_0 refer to the centroids of the two distributions. $ZLOC_0$ and β_0 were determined prior to processing the final data by examining pion and proton histograms. An event was rejected by Kiowa if K was greater than 3.2. Since the $ZLOC$ and β distributions were essentially independent and Gaussian, this meant that less than 1% of the genuine pion events were rejected (assuming no errors in σ_{ZLOC} , σ_β , $ZLOC_0$, and β_0). Typically, 15% of the "pions" had a value of K greater than 3.2.

The fraction of muon events remaining after the cuts were applied was estimated (from the Monte-Carlo program) to vary between about .02 to .09 as a function of momentum. These unidentifiable muon events caused slight distortions in the pion spectral shapes, since the momentum calculated by Tewa for a muon event was not always close to the momentum of the original pion. However, results from the Monte-Carlo program indicated that this distortion was negligible.

All pion events surviving the $ZLOC$ and β cuts were given a decay weight factor $D(p)$ in order to correct for pion decays. $D(p)$ was determined from Figure 20 and the fraction of unidentifiable muon events.

2. Decay Correction for Non-Perfect Events

The preceding method of correcting for pion decays was applied to perfect eight plane events. Other types of events (such as seven plane and multiple cluster events) required reconstruction of hit-wire positions and thus did not always contain enough information for a calculation of β and $ZLOC$. These events were treated in the following manner:

a. Cuts were applied as in the case of the perfect eight plane events and each remaining event was given the decay weight $D(p)$.

b. The ratio $r = \left(\frac{\text{total number of cut perfect eight plane events}}{\text{total number of perfect eight plane events}} \right)$ was calculated. This was done at each spectrometer angle and magnet current.

c. The data at a given angle and magnet current were multiplied by a correction factor such that the ratio $\left(\frac{\text{total number of cut events}}{\text{total number of events}} \right)$ was made effectively equal to r . This correction factor was always very close to one.

The overall error in the decay correction was estimated to be about 5%, partly because of uncertainties in $\sigma_{ZLOC}(pv)$ and $\sigma_{\beta}(pv)$.

G. ENERGY LOSS CORRECTION

The momenta calculated by Tewa were subject to small corrections because of energy losses incurred by particles as they passed through the LH_2 target and the spectrometer. Energy losses in the outgoing track (between W3 and W4) did not affect the momentum determined by Tewa. However, losses occurring before the downstream end of the magnet affected the bend angle made by a particle, and consequently its "Tewa" momentum.

The energy lost by pions was estimated from the Bethe equation.²⁶ Each material between the center of the LH_2 target and the downstream edge of W2 was taken into account. Energy losses in the magnet gap were determined to be negligible. For LH_2 full runs, the energy loss correction shifted the Tewa momenta upward by ~ 3 MeV/c at 600 MeV/c to ~ 5 MeV/c at 180 MeV/c. The corrections were much smaller for LH_2 empty runs.

H. INELASTIC AND ELASTIC NUCLEAR SCATTERING CORRECTIONS

The fraction of pions absorbed in the spectrometer was estimated as a function of pion momentum from data given in the literature. Between

~ 1-2% of the negative pions and less than 1% of the positive pions were lost in this manner. Many of the negative pions were lost during LH₂ full runs because of absorption by protons.

Elastic nuclear scattering by negative pions from nuclei in the LH₂ target and spectrometer proved to be negligible. However, the relatively large cross-section for π^+p elastic scattering caused up to about 6% of the positive pions to undergo scattering in the liquid hydrogen. A Monte-Carlo program was used to study the effects of such scattering. Results from this program indicated that the majority of the pions scattered were effectively lost from the spectrometer. Appropriate corrections were introduced as a function of pion momentum.

I. MOMENTUM RESOLUTION

The momentum resolution of the spectrometer was determined by the following factors:

- (1) The finite wire spacing in the wire chamber signal planes
- (2) Multiple Coulomb and nuclear scattering
- (3) Wire chamber misalignment
- (4) Uncertainties associated with the spectrometer magnetic field measurements
- (5) Energy loss variations in the LH₂ target

The finite signal wire spacing introduced an uncertainty $\Delta\alpha$ in the horizontal bend angle made by a particle, and consequently its momentum, since $\frac{\Delta p}{p}$ was approximately equal to $\frac{\Delta\alpha}{\alpha}$. The quantity $\Delta\alpha$ was calculated from the rms deviation d of a track about a hit-wire and the distance l between W1-W2 and W3-W4. This is illustrated in Equation 5.

$$\Delta\alpha = \sqrt{\left(\frac{\sqrt{2}d}{\ell}\right)^2 + \left(\frac{\sqrt{2}d}{\ell}\right)^2} = \frac{2d}{\ell} = \frac{2d}{1000 \text{ mm}} \quad (\text{Eq. 5})$$

Here $\frac{\sqrt{2}d}{\ell}$ represents the angular uncertainty associated with either the incoming or outgoing track. The quantity d was estimated to be $\sim \frac{1 \text{ mm}}{\sqrt{3}}$ based on the signal wire response assumed in Section C. Thus, the fractional momentum uncertainty $\frac{\Delta p}{p}$ was given by $\sim \frac{1.2 \text{ mrad}}{\alpha}$ or $\sim .003$ for typical bend angles (22°).

The Rossi-Griesen formula²⁷ (Equation 6) was applied in order to estimate the loss in resolution caused by multiple Coulomb scattering.

$$\sqrt{\langle\theta^2\rangle} = \left(\frac{15 \text{ MeV}}{\text{pv}}\right) \sqrt{\frac{t}{L_{\text{rad}}}} \quad (\text{Eq. 6})$$

Here t and L_{rad} are, respectively, the thickness and radiation length of the medium traversed by a particle. The rms projected scattering angle $\sqrt{\langle\theta^2\rangle}$ was calculated for each material in the spectrometer between the first signal plane in W1 and the last signal plane in W4. These values of $\sqrt{\langle\theta^2\rangle}$ were added in quadrature, yielding a net uncertainty in the horizontal bend angle. The fractional momentum uncertainty ranged from .007 at 600 MeV/c to about .03 at 180 MeV/c. Nuclear scattering effects were determined to be negligible.

Uncertainties in the wire chamber alignment were estimated to introduce an error of about 1 mrad in α (corresponding to $\frac{\Delta p}{p} = \sim .003$). The magnetic fields were known to be about .7%, resulting in the same momentum uncertainty. Variations in the amount of energy lost by pions in the LH₂ target (when full) caused fractional momentum uncertainties of .003 at 600 MeV/c and .015 at 180 MeV/c.

The resolution of the spectrometer, calculated by adding all the preceding momentum uncertainties in quadrature, was slightly under 10 MeV/c at all pion momenta.

The incident neutron beam, as previously discussed, was not mono-energetic. Therefore, a pion spectrum measured by the spectrometer was really a superposition of spectra created by neutrons of slightly different energies. The resulting loss in momentum resolution was estimated by calculating the maximum pion momentum kinematically possible at various lab angles for neutrons having energies of 781 and 799 MeV. These energies correspond to $\frac{1}{2}$ FWHM on either side of the 790 MeV neutron peak. The resulting pion momentum spread varied from about 16 MeV/c at 0° to 7 MeV/c at 72° .

The overall momentum resolution was determined by adding the spectrometer resolution in quadrature with the resolution loss caused by the neutron beam. It varied between about 10 to 18 MeV/c, depending upon the lab angle.

J. ANGULAR RESOLUTION

Several factors limited the spectrometer angular resolution. These are listed below:

- (1) The finite signal wire spacing in W1 and W2
- (2) Misalignment of W1 and W2
- (3) Multiple Coulomb and nuclear scattering

The finite signal wire spacing and possible misalignment of W1 and W2 together caused uncertainties of ~ 1 mrad in the scattering angles. Losses in the angular resolution caused by multiple Coulomb scattering in the LH_2 target, S1, W1, and the Cerenkov counter were estimated from the Rossi-Griesen formula to vary from ~ 4 mrad at 600 MeV/c to ~ 14 mrad

at 180 MeV/c. Nuclear scattering effects were negligible. The spectrometer angular resolution, as determined by items 1 - 3, was thus ~ 4 mrad at 600 MeV/c and ~ 14 mrad at 180 MeV/c.

The absolute or overall angular resolution was dependent upon the possible misalignment of the spectrometer with respect to the incident beam direction as well as the spectrometer resolution. However, the estimated alignment error of 1 mrad raised the overall angular resolution only slightly above the spectrometer resolution.

K. NORMALIZATION

In order to normalize the data, it was necessary to determine the number of 790 MeV neutrons incident on the LH_2 target for each left neutron monitor count ($\frac{nm}{L}$). This was done by detecting protons from the reaction $np \rightarrow np$ and comparing the proton yields to a previously normalized set of 790 MeV data²⁸, referred to as the "N" data. A two step process was used to obtain the "N" data normalization. First, deuterons from the reaction $np \rightarrow \pi^0 d$ were detected as a function of angle along with protons from the elastic np reaction. The deuteron yields were subsequently compared with data on the reaction $pp \rightarrow \pi^+ d$ by means of the relation given in Equation 7.

$$\frac{d\sigma}{d\Omega} (np \rightarrow \pi^0 d) = \frac{1}{2} \frac{d\sigma}{d\Omega} (pp \rightarrow \pi^+ d) \quad (\text{Eq. 7})$$

Cross-section information on the pp reaction is known to about 5%. However, the relationship given in Equation 7 has an uncertainty of about 5%, due to the possible break-up of isotopic spin invariance by Coulomb interactions. The absolute error in the normalization of the "N" data was estimated to be 7% by adding the two 5% uncertainties in quadrature.

Three different sets of elastic np data taken during present experiment were analyzed for normalization purposes - two at 0° , and one at 36° .

These data sets were taken at different times, and therefore provided a check on the stability of the system. The values of $\frac{nn}{L}$ from these three data sets are given in Table 3.

Table 3.

Neutron flux monitor calibration results

Lab Angle	$\frac{nn}{L} \pm \text{Statistical Error}$
0°	773 $\pm$ 12
0°	744 $\pm$ 10
36°	741 $\pm$ 27

The apparent discrepancy between the two 0° data sets may be due to misalignment(s) of the spectrometer, since the np elastic cross-section around 0° is sensitive to small changes in angle. In addition, a possible 1% variation in the LH₂ target density added an additional uncertainty in the value of $\frac{nn}{L}$ for each data set. The average of all three values of $\frac{nn}{L}$ (denoted by $\overline{\frac{nn}{L}}$) was used to normalize the pion data. The absolute error associated with $\overline{\frac{nn}{L}}$ (~7%) was determined almost entirely by the uncertainty in the normalization of the "N" data.

Although np elastic data were taken at nine angles, it was decided that the analysis of three data sets was sufficient. Almost all of the data taking was done under stable conditions as demonstrated by the constancy of the ratios L_u/CI , L_g/H_g , etc. (For example, differences in the ratio $(L_g/H_g)_{LH_2 \text{ full}}$ from run to run were usually consistent with those expected from LH₂ target density variations and statistical errors associated with H_g and L_g .) In addition, the pion yields from two sets of 42° data taken at different times were the same within statistical errors when normalized to L_g .

1. DETERMINATION OF PION KINETIC ENERGY SPECTRA

In order to provide a comparison to the results obtained by Dzhelepov et al.,³ pion kinetic energy spectra, rather than momentum spectra, were generated. Two types of kinetic energy spectra were produced - one in the laboratory coordinate system, and the other in the colliding np center of momentum system (CMS). The CMS spectra are useful for theoretical interpretations.

As noted previously in section I, the momentum resolution varied between about 10 - 18 MeV/c. However, the pions were grouped in 20 MeV/c wide bins for convenience. The laboratory double differential cross-section $\frac{d^2\sigma}{d\Omega dT}$ was determined at kinetic energies ($\bar{T}$) corresponding to the center of each momentum bin. This was done in a direct manner. Let F and B represent the number of neutrons incident upon the LH₂ target at a given magnet current and angle for all of the LH₂ target full and empty data, respectively. In addition, let W_i be the solid angle weight associated with each event, and D_i be its decay correction factor. Then $\frac{d^2\sigma}{d\Omega dT}$ was given by Equation 8, where ΔT is the kinetic energy width of the momentum bin being considered, and ρ is the LH₂ target density (5.59×10^{23} nuclei/cm²). Individual events in the foreground and background are represented by j and k, respectively.

$$\frac{d^2\sigma}{d\Omega dT} = \frac{1}{\Delta T \rho} \left[\frac{1}{F} \sum_j D_j W_j - \frac{1}{B} \sum_k D_k W_k \right] \quad (\text{Eq. 8})$$

The statistical error E associated with each value of $\frac{d^2\sigma}{d\Omega dT}$ was calculated as shown in Equation 9. The statistical errors in F and B (typically less than .1%) were neglected in the calculation of E, since $E/\frac{d^2\sigma}{d\Omega dT}$ was always greater than about .05.

$$E = \frac{1}{\Delta T \rho} \sqrt{\frac{1}{F^2} \sum_j (D_j W_j)^2 + \frac{1}{B^2} \sum_k (D_k W_k)^2} \quad (\text{Eq. 9})$$

The effects caused by absorption and elastic scattering were taken into account by multiplying $\frac{d^2\sigma}{d\Omega dT}$ and E by a correction factor f(T). (Differences in absorption and elastic scattering between LH₂ full and empty runs were included in f(T) by using information on the ratio $\frac{\text{LH}_2 \text{ empty}}{\text{LH}_2 \text{ full}}$). For all of the positive pion data, and some of the negative pion data ($\geq 42^\circ$), the multiple cluster corrections were also included in f(T). However, for the remainder of the negative pion data (where the multiple cluster effects were considerably larger), the corrections were made by actually combining the multiple cluster and regular Tewa data. This was done as shown in Equations 10 and 11. Here F_c and B_c represent the number of neutrons in the foreground and background runs analyzed with the multiple cluster version of Tewa. Also, the events in these runs are devoted by j⁻ (foreground) and k⁻ (background). The ratios $\frac{F_c}{F}$ and $\frac{B_c}{B}$ were typically between .5 and 1.

$$\begin{aligned} \frac{d^2\sigma}{d\Omega dT} = \frac{1}{\Delta T \rho} \left[\frac{1}{F} \sum_j D_j W_j + \frac{1}{F_c} \sum_{j^-} D_{j^-} W_{j^-} - \frac{1}{B} \sum_k D_k W_k \right. \\ \left. - \frac{1}{B_c} \sum_{k^-} D_{k^-} W_{k^-} \right] \quad (\text{Eq. 10}) \end{aligned}$$

$$\begin{aligned} E = \frac{1}{\Delta T \rho} \sqrt{\frac{1}{F^2} \sum_j (D_j W_j)^2 + \frac{1}{F_c^2} \sum_{j^-} (D_{j^-} W_{j^-})^2 + \frac{1}{B^2} \sum_k (D_k W_k)^2} \\ + \frac{1}{B_c^2} \sum_{k^-} (D_{k^-} W_{k^-})^2 \quad (\text{Eq. 11}) \end{aligned}$$

The average laboratory angle $\bar{\theta}$ associated with each momentum bin was determined from the average of the upper and lower angular limits of the bin. The angular width of the bins varied between about $2^\circ - 3^\circ$. CMS double differential cross-sections and statistical errors were calculated from the relations

$$\frac{d^2\sigma}{d\Omega^*dT^*}(\bar{\theta}^*, \bar{T}^*) = J(\bar{\theta}, \bar{T}) \frac{d^2\sigma}{d\Omega dT}(\bar{\theta}, \bar{T}) \text{ and } E^*(\bar{\theta}^*, \bar{T}^*) = J(\bar{\theta}, \bar{T}) E(\bar{\theta}, \bar{T}).$$

Here $J(\bar{\theta}, \bar{T})$ is the Jacobian transformation factor ($= \frac{p^*}{p}$). The quantities $\bar{\theta}^*$ and $\bar{T}^*$ (which correspond to $\bar{\theta}$ and $\bar{T}$) were calculated from the appropriate kinematical relationships.

Before presenting the data, it is necessary to discuss the effects of two additional systematic errors in the data. These are listed below.

- (1) The pion yields from beam neutrons between about 460 to 530 MeV (mentioned previously in Chapter I).
- (2) Double pion production.

In order to estimate the effects of the low energy beam neutrons, it was assumed that the neutrons all had an energy of 500 MeV, and that the pion spectra in the CMS₅₀₀ system were identical at all CMS₅₀₀ angles. (Here the subscript 500 is used to distinguish the CMS system at 500 MeV from the one at 790 MeV.) The shape of the spectra was approximated by 3-body Lorentz invariant phase space integrated over all angles $(\int \frac{d^2\sigma d\Omega^*}{d\Omega^*dT^*} \text{ phase space})$. The cross-section $\sigma(np \rightarrow NN\pi^+)$ at 500 MeV was determined to be roughly .7 mb from a linear interpolation between the data points given in References 1, 3 and 4. The results of these approximations indicated that the pion yields at 0° (lab) varied between .8 to .4 $\frac{\mu\text{b}}{\text{sr-MeV}}$ in the kinetic energy interval from about 90 MeV to 210 MeV ($\sim 180 - 320$ MeV/c). At the larger angles, the contributions were smaller (see

Appendix 2). All the cross-section estimates were subtracted from the data. A 75% error was placed on these estimates because of the nature of the assumptions which were made.

Double pion production can occur around 600 MeV (incident neutron energy). Charged pions can be created in the following np reactions:

$$np \rightarrow \pi^+ \pi^- np$$

$$np \rightarrow \pi^+ \pi^- d$$

$$np \rightarrow \pi^+ \pi^0 nn$$

$$np \rightarrow \pi^- \pi^0 pp .$$

Very little information on these reactions below 1 GeV exists; however, the total cross-section for the first reaction is roughly .05 mb at 790 MeV.⁵

The reaction $pp \rightarrow \pi^- \pi^+ pp$ has been studied at 730 MeV.²⁹ Negative pions from this reaction were found to be concentrated at the smaller lab angles. By using these results as well as information given by Lock and Measday³⁰, it was estimated that double pion production reactions could contribute up to $\sim .2 - .4 \frac{\mu b}{sr-MeV}$ between 100 MeV and 150 MeV (195 - 255 MeV/c) at 0° and 8°, and less than $.2 \frac{\mu b}{sr-MeV}$ at the larger angles (and lower energies). No attempt was made to subtract these numbers from the data since the corrections were so poorly known. In any case, the overall effects of double pion production were probably significantly less than the effects of the 500 MeV neutrons.

M. PION KINETIC ENERGY SPECTRA

On the following pages all 20 CMS spectra are shown, along with 6 laboratory spectra (Figures 24 - 36). The cross-sections and errors displayed in these figures are tabulated in Appendix 1.

Figure 24.

Pion CMS kinetic energy spectra at $\theta^* = 4^\circ$

Figure 25.

Pion CMS kinetic energy spectra at $\theta^* = 16^\circ$

Figure 26.

Pion CMS kinetic energy spectra at $\theta^* = 31^\circ$

Figure 27.

Pion CMS kinetic energy spectra at $\theta^* = 46^\circ$

Figure 28.

Pion CMS kinetic energy spectra at $\theta^* = 57^\circ$

Figure 29.

Pion CMS kinetic energy spectra at $\theta^* = 67^\circ$

Figure 30.

Pion CMS kinetic energy spectra at $\theta^* = 76^\circ$

Figure 31.

Pion CMS kinetic energy spectra at $\theta^* = 85^\circ$

Figure 32.

Pion CMS kinetic energy spectra at $\theta^* = 95^\circ$

Figure 33.

Pion CMS kinetic energy spectra at $\theta^* = 113^\circ$

Figure 34.

Pion lab kinetic energy spectra at 0° (spectrometer angle)

Figure 35.

Pion lab kinetic energy spectra at 36° (spectrometer angle)

Figure 36.

Pion lab kinetic energy spectra at 72° (spectrometer angle)

It can be seen in Appendix I that the cross-section measurements associated with a given spectrum do not correspond to the same angle in the laboratory or CMS system. However, this was not believed to be a problem, since the spectra did not vary rapidly with a change in angle. The effective angle quoted with each CMS spectrum was determined from an averaging process that will be described in the next section.

The smooth curves drawn through many of the CMS spectra represent a least squares fit of $\frac{d\sigma}{d\Omega^*dT^*}$ to the function $F(T^*)$ given in Equation 12.

$$\frac{d^2\sigma(T^*)}{d\Omega^*dT^*} = F(T^*) = AT^{*5/2} (B-T^*)^2 \quad (\text{Eq. 12})$$

Here A and B are adjustable coefficients that varied from spectra to spectra. The average chi-squared per degree of freedom (χ^2) of all the fits was 1.03. The curves serve as a guide to the eye and, as will be explained in the next section, were used to estimate the area underneath each spectrum. Unfortunately, no satisfactory fitting function could be found for most of the remaining spectra ($4^\circ \rightarrow 46^\circ$ CMS).

The maximum pion CMS kinetic energy possible from a 790 MeV neutron is about 195 MeV (195 MeV for positive pions and 197 MeV for negative pions). However, it can be seen that the spectra often appear to reach a "cutoff" at energies slightly greater than $\sim 195 - 200$ MeV. This was probably caused by the presence of beam neutrons having energies between 790 - 800 MeV as well as the finite pion momentum resolution. Non-zero values of $\frac{d^2\sigma}{d\Omega^*dT^*}$ were occasionally found well past 200 MeV; however, these were believed to have been caused by muon events (momentum miscalculations by Tewa) and were therefore disregarded.

The 4° negative pion spectrum does not seem to approach the kinematic limit as smoothly as the other spectra. This may be due to the formation of a final state interaction of the two nucleons (protons). The lack of such a "peaking" in the positive pion data is contrary to the predictions of Phillips.³¹ He concluded that the cross-section for the nn final state interaction should be greater than that for the pp interaction.

N. DETERMINATION OF $\frac{d\sigma}{d\Omega^*}(\theta^*)$ AND $\sigma(np \rightarrow nn\pi^+)$

The integral $\int \frac{d^2\sigma dT^*}{d\Omega^* dT^*} (= \frac{d\sigma}{d\Omega^*})$ was evaluated for each CMS spectrum by combining the area underneath the data points with an extrapolation to zero kinetic energy. This was done in order to determine the differential cross-section angular distribution $\frac{d\sigma}{d\Omega^*}(\theta^*)$.

The area underneath each set of data points (S) was calculated as shown in Equation 13.

$$S = \left[\frac{d^2\sigma(T_1^*)}{d\Omega^* dT^*} \right] (T_2^* - T_1^*) + \left[\frac{d^2\sigma(T_n^*)}{d\Omega^* dT^*} \right] (T_n^* - T_{n-1}^*) + \sum_{i=2}^{n-1} \left[\frac{d^2\sigma(T_i^*)}{d\Omega^* dT^*} \right] \frac{(T_{i+1}^* - T_{i-1}^*)}{2}, \quad T_1 < T_2 < \dots < T_n \quad (\text{Eq. 13})$$

An error ΔS associated with S was determined as shown in Equation 14.

$$\Delta S = \sqrt{[E_1^*(T_1^*)]^2 \cdot [T_2^* - T_1^*]^2 + [E_n^*(T_n^*)]^2 \cdot [T_n^* - T_{n-1}^*]^2 + \sum_{i=2}^{n-1} [E_i^*(T_i^*)]^2 \cdot \left[\frac{T_{i+1}^* - T_{i-1}^*}{2} \right]^2} \quad (\text{Eq. 14})$$

The quantity E^* represents the statistical error E^* at those points for which the 500 MeV neutron correction was unnecessary. For the remaining

points E^* is just E^* added in quadrature with the 500 MeV neutron correction error.

The area underneath the missing portion of each spectrum fitted to the function $F(T^*)$ was estimated by evaluating the integral

$$I = \int_0^{T_m^*} F(T^*) dT^*, \text{ where } T_m^* = T_1^* - \frac{(T_2^* - T_1^*)}{2}.$$

An error of .5I was assigned to I, based on comparisons to areas underneath other possible extrapolations. The missing area in each of the remaining spectra (I') was estimated to be

$$\frac{1}{2} \int_0^{T_m^*} G(T^*) dT^*,$$

where $G(T^*)$ is the straight line between the points (0,0) and

$$\left(\frac{1}{3} [T_1^* + T_2^* + T_3^*], \frac{1}{3} \left[\frac{d^2\sigma(T_1^*)}{d\Omega^*dT^*} + \frac{d^2\sigma(T_2^*)}{d\Omega^*dT^*} + \frac{d^2\sigma(T_3^*)}{d\Omega^*dT^*} \right] \right).$$

An uncertainty of .5I' was assigned to I'.

The differential cross-section $\frac{d\sigma}{d\Omega^*}$ at each angle was given by S+I or S+I'. All the values of $\frac{d\sigma}{d\Omega^*}$ are listed in Table 4, along with S, ΔS , I or I', and an overall error $\Delta \frac{d\sigma}{d\Omega^*}$. This error consisted of three separate errors added in quadrature - ΔS , .5I or .5I', and $.01 \frac{d\sigma}{d\Omega^*}$. The last error was necessary in order to allow for possible LH₂ target density variations. Additional errors, such as those inherent in the decay correction and RF cuts, were not included in $\Delta \frac{d\sigma}{d\Omega^*}$.

The effective angle θ^* quoted for each of the CMS spectra was determined as shown in Equations 15 and 16. Here + and - refer to the positive and negative pion data at a given spectrometer angle.

Table 4.

Differential cross-section results

Positive pion data (+)

Negative pion data (-)

(all quantities except θ^* in $\frac{\mu\text{b}}{\text{sr}}$)

$\theta^*(^\circ)$	$\frac{d\sigma_+}{d\Omega^*}$	$\Delta\frac{d\sigma_+}{d\Omega^*}$	S_+	ΔS_+	I_+ (or $I_+^{\sim}$)
4	226	9	226	9	<.5
16	202	8	202	8	<.5
31	190	6	185	5	5
46	158	6	154	5	4
57	160	7	152	5	8
67	139	6	132	5	7
76	126	7	118	5	8
85	127	7	117	5	10
95	119	8	107	5	12
113	125	19	88	3	37
$\theta^*(^\circ)$	$\frac{d\sigma_-}{d\Omega^*}$	$\Delta\frac{d\sigma_-}{d\Omega^*}$	S_-	ΔS_-	I_- (or $I_-^{\sim}$)
4	219	8	219	8	<.5
16	190	8	189	8	1
31	190	6	185	5	5
46	158	6	152	5	6
57	150	6	143	5	7
67	136	6	130	5	6
76	127	7	119	5	8
85	130	7	121	5	9
95	131	9	117	5	14
113	138	22	94	3	44

$$\theta_{\pm}^* = \left\{ \left[\frac{d^2\sigma_{\pm}(T_1^*)}{d\Omega^*dT^*} \right] (T_2^* - T_1^*)\theta_1^* + \left[\frac{d^2\sigma_{\pm}(T_n^*)}{d\Omega^*dT^*} \right] (T_n^* - T_{n-1}^*)\theta_n^* + \sum_{i=2}^{n-1} \left[\frac{d^2\sigma_{\pm}(T_i^*)}{d\Omega^*dT^*} \right] \frac{(T_{i+1}^* - T_{i-1}^*)\theta_i^*}{2} \right\} / S_{\pm} \quad (\text{Eq. 15})$$

$$\theta^* = \frac{\theta_+^* + \theta_-^*}{2} \quad (\text{Eq. 16})$$

At any given lab angle θ_+^* and θ_-^* differed by less than one degree. The angle θ^* was estimated to have an uncertainty of $1^\circ - 2^\circ$.

The angular variation of the differential cross-sections $\frac{d\sigma_+}{d\Omega^*}$ and $\frac{d\sigma_-}{d\Omega^*}$ can be expressed as shown in Equations 17 and 18. (As before, + and - refer to the positive and negative pion data, respectively.)

$$\frac{d\sigma_+}{d\Omega^*}(\theta^*) = \sum_n a_n^+ \cos^n(\theta^*) \quad (\text{Eq. 17})$$

$$\frac{d\sigma_-}{d\Omega^*}(\theta^*) = \sum_n a_n^- \cos^n(\theta^*) \quad (\text{Eq. 18})$$

Charge symmetry requires that $a_n^+ = a_n^-$ for even n , and $a_n^+ = -a_n^-$ for odd n . (Here it is assumed that effects caused by Coulomb interactions in the final state $pp\pi^-$ can be ignored.) Therefore, by forming the quantities shown in Equations 19 and 20, it is possible to calculate separately the coefficients with even and odd indices (see Table 5).

$$\frac{1}{2} \left[\frac{d\sigma_+}{d\Omega^*}(\theta^*) + \frac{d\sigma_-}{d\Omega^*}(\theta^*) \right] = \sum_{n=0,1,2,\dots} a_{2n} \cos^{2n}(\theta^*).$$

$$a_{2n} = a_{2n}^+ = a_{2n}^- \quad (\text{Eq. 19})$$

Table 5.

The sums and differences of $\frac{1}{2} \left[\frac{d\sigma_+}{d\Omega^*}(\theta^*) \right]$ and $\frac{1}{2} \left[\frac{d\sigma_-}{d\Omega^*}(\theta^*) \right]$

(All quantities except θ^* in $\frac{\mu\text{b}}{\text{sr}}$)

$\theta^*(^\circ)$	$\frac{1}{2} \left[\frac{d\sigma_+}{d\Omega^*}(\theta^*) + \frac{d\sigma_-}{d\Omega^*}(\theta^*) \right] \pm \text{ERROR}$	$\frac{1}{2} \left[\frac{d\sigma_+}{d\Omega^*}(\theta^*) - \frac{d\sigma_-}{d\Omega^*}(\theta^*) \right] \pm \text{ERROR}$
4	222 ± 6	4 ± 6
16	196 ± 6	6 ± 6
31	190 ± 4	0 ± 4
46	158 ± 4	0 ± 4
57	155 ± 5	5 ± 5
67	138 ± 4	2 ± 4
76	126 ± 5	0 ± 5
85	128 ± 5	-2 ± 5
95	125 ± 6	-6 ± 6
113	132 ± 15	-6 ± 15

$$\text{ERROR} = \frac{1}{2} \sqrt{\left[\left(\frac{d\sigma_+}{d\Omega^*}(\theta^*) \right)^2 + \left(\frac{d\sigma_-}{d\Omega^*}(\theta^*) \right)^2 \right]}$$

$$\frac{1}{2} \left[\frac{d\sigma^+}{d\Omega^*}(\theta^*) - \frac{d\sigma^-}{d\Omega^*}(\theta^*) \right] = \sum_{n=0,1,2,\dots} a_{2n+1} \cos^{2n+1}(\theta^*).$$

$$a_{2n+1} = a_{2n+1}^+ = -a_{2n+1}^- \quad (\text{Eq. 20})$$

The data can also be expressed in the form shown in Equation 21 (see Table 6).

$$\left(\frac{d\sigma^+}{d\Omega^*}(\theta^*) , \frac{d\sigma^-}{d\Omega^*}(180^\circ - \theta^*) \right) = \sum_{n=0,1,2,\dots} a_n \cos^n(\theta^*), \quad (\text{Eq. 21})$$

$$a_n = a_n^+ = a_n^- \text{ for even } n$$

$$a_n = a_n^+ = -a_n^- \text{ for odd } n$$

The computer program Curfit³² was used in order to fit the data to Equations 17, 18, 19, 20, and 21. The data and some of the least squares fits are shown in Figures 37 through 39. It can be seen from Table 7 that $\frac{d\sigma}{d\Omega^*}(\theta^*)$ was probably best represented by the polynomial $a_0 + a_2 \cos^2(\theta^*)$. It was impossible to determine conclusively if any odd powers of $\cos(\theta^*)$ were present, since the differences between $\frac{d\sigma^+}{d\Omega^*}(\theta^*)$ and $\frac{d\sigma^-}{d\Omega^*}(\theta^*)$ were small compared to the errors involved. (The question of asymmetry will be discussed more fully in section P.) The best estimate of $\frac{d\sigma}{d\Omega^*}(\theta^*)$ was chosen to be $[(123.1 \pm 2.7) + (88.3 \pm 4.9) \cos^2(\theta^*)] \frac{\mu\text{b}}{\text{sr}}$, as determined from Equation 21. This choice was somewhat arbitrary; however, the coefficients a_0 and a_2 obtained from Equations 17, 18, and 19 were not much different.

The total cross-sections (σ) given in Table 7 were calculated by integrating the polynomials $a_0 + a_2 \cos^2(\theta^*)$ over all angles (Equation 22).

Table 6.

The differential cross-section $\left[\frac{d\sigma^+}{d\Omega^*}(\theta^*) , \frac{d\sigma^-}{d\Omega^*}(180^\circ - \theta^*) \right]$

$\theta^*(^\circ)$	$\left[\frac{d\sigma^+}{d\Omega^*}(\theta^*) , \frac{d\sigma^-}{d\Omega^*}(180^\circ - \theta^*) \right] \left(\frac{\mu\text{b}}{\text{sr}} \right)$	
4	226 ± 9	$\left(= \frac{d\sigma^+}{d\Omega^*}(4^\circ) \right)$
16	202 ± 8	$\left(= \frac{d\sigma^+}{d\Omega^*}(16^\circ) \right)$
31	190 ± 6	$\left(= \frac{d\sigma^+}{d\Omega^*}(31^\circ) \right)$
46	158 ± 6	$\left(= \frac{d\sigma^+}{d\Omega^*}(46^\circ) \right)$
57	160 ± 7	$\left(= \frac{d\sigma^+}{d\Omega^*}(57^\circ) \right)$
67	138 ± 11	$\left(= \frac{1}{2} \left[\frac{d\sigma^+}{d\Omega^*}(67^\circ) + \frac{d\sigma^-}{d\Omega^*}(113^\circ) \right] \pm \text{ERROR} \right)$
76	126 ± 7	$\left(= \frac{d\sigma^+}{d\Omega^*}(76^\circ) \right)$

Table 6. (Continued)

The differential cross-section $\left[\frac{d\sigma}{d\Omega^*}^+(\theta^*) , \frac{d\sigma}{d\Omega^*}^-(180^\circ - \theta^*) \right]$

$\theta^*(^\circ)$	$\left[\frac{d\sigma}{d\Omega^*}^+(\theta^*) , \frac{d\sigma}{d\Omega^*}^-(180^\circ - \theta^*) \right] \left(\frac{\mu b}{sr} \right)$	
85	129 ± 6	$\left(= \frac{1}{2} \left[\frac{d\sigma}{d\Omega^*}^+(85^\circ) + \frac{d\sigma}{d\Omega^*}^-(95^\circ) \right] \pm \text{ERROR} \right)$
95	124 ± 5	$\left(= \frac{1}{2} \left[\frac{d\sigma}{d\Omega^*}^+(95^\circ) + \frac{d\sigma}{d\Omega^*}^-(85^\circ) \right] \pm \text{ERROR} \right)$
104	127 ± 7	$\left(= \frac{d\sigma}{d\Omega^*}^-(76^\circ) \right)$
113	130 ± 10	$\left(= \frac{1}{2} \left[\frac{d\sigma}{d\Omega^*}^+(113^\circ) + \frac{d\sigma}{d\Omega^*}^-(67^\circ) \right] \pm \text{ERROR} \right)$
123	150 ± 6	$\left(= \frac{d\sigma}{d\Omega^*}^-(57^\circ) \right)$
134	158 ± 6	$\left(= \frac{d\sigma}{d\Omega^*}^-(46^\circ) \right)$
149	190 ± 6	$\left(= \frac{d\sigma}{d\Omega^*}^-(31^\circ) \right)$

Table 6. (Continued)

The differential cross-section $\left[\frac{d\sigma_+}{d\Omega^*}(\theta^*) , \frac{d\sigma_-}{d\Omega^*}(180^\circ - \theta^*) \right]$

$\theta^*(^\circ)$	$\left[\frac{d\sigma_+}{d\Omega^*}(\theta^*) , \frac{d\sigma_-}{d\Omega^*}(180^\circ - \theta^*) \right] \left(\frac{\mu b}{sr} \right)$
164	$190 \pm 8 \quad \left(= \frac{d\sigma_-}{d\Omega^*}(16^\circ) \right)$
176	$219 \pm 8 \quad \left(= \frac{d\sigma_-}{d\Omega^*}(4^\circ) \right)$

$$\text{ERROR} = \frac{1}{2} \sqrt{\left(\Delta \frac{d\sigma_+}{d\Omega^*} \right)^2 + \left(\Delta \frac{d\sigma_-}{d\Omega^*} \right)^2}$$

Figure 37.

The angular variations of $\frac{d\sigma_+}{d\Omega^*}$ and $\frac{d\sigma_-}{d\Omega^*}$

Figure 38.

The angular variations of $\frac{1}{2} \left[\frac{d\sigma_+}{d\Omega^*} + \frac{d\sigma_-}{d\Omega^*} \right]$ and $\frac{1}{2} \left[\frac{d\sigma_+}{d\Omega^*} - \frac{d\sigma_-}{d\Omega^*} \right]$

Figure 39.

The angular variation of $\left[\frac{d\sigma^+}{d\Omega^*}(\theta^*) , \frac{d\sigma^-}{d\Omega^*}(180^\circ - \theta^*) \right]$

The curve represents the function

$$\left[\frac{d\sigma^+}{d\Omega^*}(\theta^*) , \frac{d\sigma^-}{d\Omega^*}(180^\circ - \theta^*) \right] = [123.1 + 88.3 \cos^2(\theta^*)] \frac{\mu b}{sr} .$$

Table 7.

A summary of the coefficients of polynomials fitted to $\frac{d\sigma_+}{d\Omega^*}(\theta^*)$, $\frac{d\sigma_-}{d\Omega^*}(\theta^*)$, etc.

($a_0, a_1, a_2, a_3, a_4, {}^\dagger a_{02}$ in $\frac{\mu\text{b}}{\text{sr}}$)

Equation #					
17	$a_0 = 122.7 \pm 3.4$	$a_2 = 92.0 \pm 6.8$	$a_{02} = 4.2$	$\chi^2 = .97$	$\sigma = 1.93 \pm .07 \text{ mb}$
18	$a_0 = 124.9 \pm 3.5$	$a_2 = 83.0 \pm 6.6$	$a_{02} = 4.2$	$\chi^2 = .87$	$\sigma = 1.92 \pm .07 \text{ mb}$
21	$a_0 = 123.1 \pm 2.7$	$a_2 = 88.3 \pm 4.9$	$a_{02} = 3.2$	$\chi^2 = 1.04$	$\sigma = 1.92 \pm .05 \text{ m'}$
19	$a_0 = 123.7 \pm 2.4$	$a_2 = 87.6 \pm 4.7$	$a_{02} = 3.0$	$\chi^2 = 1.48$	$\sigma = 1.92 \pm .05 \text{ mb}$
19	$a_0 = 126.0 \pm 3.0$	$a_2 = 65.3 \pm 17.7$	$a_4 = 23.9 \pm 18.3$	$\chi^2 = 1.45$	
21	$a_0 = 125.1 \pm 3.2$	$a_2 = 68.7 \pm 17.9$	$a_4 = 20.9 \pm 18.4$	$\chi^2 = 1.02$	
21	$a_0 = 123.1 \pm 2.7$	$a_1 = 2.7 \pm 2.6$	$a_2 = 88.3 \pm 4.9$	$\chi^2 = 1.04$	
20	$a_1 = 2.8 \pm 2.5$			$\chi^2 = .30$	
20	$a_3 = 3.6 \pm 3.5$			$\chi^2 = .32$	

† The quantity a_{02} is the covariance between a_0 and a_2 .

(Note that only even powers of $\cos(\theta^*)$ can contribute to the total cross-section.)

$$\sigma = \int_0^{2\pi} d\phi^* \int_0^\pi (a_0 + a_2 \cos^2(\theta^*)) \sin(\theta^*) d\theta^* = 4\pi \left[a_0 + \frac{a_2}{3} \right] \quad (\text{Eq. 22})$$

The errors associated with each value of σ (as given in Table 7) were calculated as shown in Equation 23. Here a_{02} is the covariance between a_0 and a_2 .

$$\Delta\sigma = 4\pi \sqrt{(\Delta a_0)^2 + \frac{(\Delta a_2)^2}{9} + \frac{2(a_{02})^2}{3}} \quad (\text{Eq. 23})$$

It can be seen that all the values of σ are in excellent agreement. The total cross-section obtained from the "best estimate" of $\frac{d\sigma}{d\Omega^*}(\theta^*)$ (1.92 mb) was chosen as the best value of $\sigma(\text{np} \rightarrow \text{NN}\pi^\pm)$. The absolute error in $\sigma(\text{np} \rightarrow \text{NN}\pi^\pm)$ was estimated to be .20 mb by adding the RF cut, decay correction, and normalization uncertainties (5%, 5%, 7%) in quadrature with $\frac{\Delta\sigma}{\sigma}$.

0. DETERMINATION OF σ_{01} AND $\frac{d\sigma_{01}}{d\Omega^*}(\theta^*)$

It was mentioned previously in Chapter I that the cross-section σ_{01} is given by $2\sigma(\text{np} \rightarrow \text{NN}\pi^\pm) - \sigma_{11}$ ($\sigma_{11} = \sigma(\text{pp} \rightarrow \text{pp}\pi^0)$). No experiments have been performed to determine the value of σ_{11} at 790 MeV. However, there are three measurements between 735 and 970 MeV. These are listed in Table 8. The cross-section σ_{11} at 790 MeV was estimated to be $3.7 \pm .3$ mb from linear interpolations of both σ_{11} and $\Delta\sigma_{11}$ between the 735 and 970 MeV points. By using this estimate, the cross-section σ_{01} was determined to be $.1_{-.1}^{+.5}$ mb. This result is compared to a previous compilation of σ_{01} data³⁰ in Table 9 and Figure 40.

Table 8.

Measurements of σ_{11} between 735 and 970 MeV

T_p (MeV)	$\sigma_{11} \pm \Delta\sigma_{11}$ (mb)	Reference
735	$3.46 \pm .25$	33
925	6 ± 2	34
970	$4.3 \pm .5$	35

Table 9.

A summary of data concerning the reaction $np \rightarrow NN\pi^\pm$

Neutron energy (MeV)	440	585	600	780	790
$\sigma(np \rightarrow NN\pi^\pm)$ (mb)	$.187 \pm .015$	$1.3 \pm .2$	$1.68 \pm .17$	~ 2.4	$1.92 \pm .20$
σ_{11} (mb)	$.200 \pm .020$	$1.72 \pm .12$	$2.00 \pm .12$	~ 3.6	$3.7 \pm .3$
σ_{01} (mb)	$.174 \pm .035$	$.9 \pm .4$	$1.36 \pm .36$	$1.2^{+1.8}_{-.9}$	$.1^{+.5}_{-.1}$
$\sigma(T=1)$ (mb)	~ 3	~ 10	~ 11	~ 20	~ 20
$\sigma(T=0)$ (mb)	$.52 \pm .10$	2.7 ± 1.2	4.1 ± 1.2	$3.6^{+5.4}_{-2.7}$	$.3^{+1.5}_{-.3}$

Table 9 also lists the total cross-sections for pion production from the interaction of nucleons in the $T=0$ and $T=1$ isotopic spin states ($\sigma(T=0)$, $\sigma(T=1)$). These cross-sections were evaluated as shown in Equations 24 and 25.³⁰

$$\sigma(T=0) = 3\sigma_{01} \quad (\text{Eq. 24})$$

$$\sigma(T=1) = \sigma_{10}(d) + \sigma_{10}(np) + 2\sigma_{11} \quad (\text{Eq. 25})$$

Figure 40.

The partial cross-section σ_{01} as a function of energy

It can be seen that the ratio $\frac{\sigma(T=0)}{\sigma(T=1)}$ is always much less than 1, and thus pions are produced preferentially from the T=1 nucleon-nucleon state.

It was possible to estimate the angular variation of $\frac{d\sigma_{01}}{d\Omega^*}$ by using the relationship given in Equation 26.³⁶

$$\frac{d\sigma_{01}}{d\Omega^*}(\theta^*) = \left[\frac{d\sigma_+}{d\Omega^*}(\theta^*) + \frac{d\sigma_-}{d\Omega^*}(\theta^*) - \frac{d\sigma_{11}}{d\Omega^*}(\theta^*) \right] \quad (\text{Eq. 26})$$

The angular distribution $\frac{d\sigma_{11}}{d\Omega^*}(\theta^*)$ was found by Cence et al.³³ to have the form $[230 + 82\cos^2(\theta^*) + 92\cos^4(\theta^*)] \frac{\mu\text{b}}{\text{sr}}$ at 735 MeV. Use of this function in Equation 26 would have resulted in a negative (and thus unphysical) coefficient of $\cos^4(\theta^*)$, since the present data had no such term. However, Cence et al. found that the function $[218 + 173\cos^2(\theta^*)] \frac{\mu\text{b}}{\text{sr}}$ represented their data nearly as well as the polynomial containing the $\cos^4(\theta^*)$ term. The coefficients of this function (218 and 173) were increased by the ratio

$$\frac{\sigma_{11}^{790 \text{ MeV}}}{\sigma_{11}^{735 \text{ MeV}}} \left(\approx \frac{3.7}{3.46} \right)$$

and subsequently used in Equation 26. The resulting expression for $\frac{d\sigma_{01}}{d\Omega^*}(\theta^*)$ is shown in Equation 27.

$$\frac{d\sigma_{01}}{d\Omega^*}(\theta^*) \approx \left[10^{+50}_{-10} + (0^{+20}) \cos^2(\theta^*) \right] \frac{\mu\text{b}}{\text{sr}} \quad (\text{Eq. 27})$$

The errors on the coefficients of $\frac{d\sigma_{01}}{d\Omega^*}(\theta^*)$ were calculated from the uncertainty in the cross-section σ_{01} .

P. ASYMMETRY

Differences between the spectra of positive and negative pions can arise from the following effects:

- (1) Charge and mass differences between the final states $nn\pi^+$ and $pp\pi^-$
- (2) Interference between T=0 and T=1 strong interaction amplitudes^a

(Asymmetries can also be caused by polarization of the beam and/or target.

However, no such polarizations were present in this experiment.)

In order to test for asymmetries, the cross-sections $\frac{d^2\sigma^+}{d\Omega^*dT^*}$ and $\frac{d^2\sigma^-}{d\Omega^*dT^*}$ were compared in those regions unaffected by the 500 MeV neutrons.

(It was previously determined that little if any asymmetry existed between $\frac{d\sigma^+}{d\Omega^*}$ and $\frac{d\sigma^-}{d\Omega^*}$. However, this result is somewhat inconclusive, because of uncertainties associated with the 500 MeV neutron correction and extrapolations of $\frac{d^2\sigma}{d\Omega^*dT^*}$ to zero kinetic energy.) A comparison of $\frac{d^2\sigma^+}{d\Omega^*dT^*}$ and $\frac{d^2\sigma^-}{d\Omega^*dT^*}$ was done by generating the "difference spectra" shown in Figures 41 - 45. The elements of each spectrum ($DS(T_i^*)$) were calculated as shown in Equation 28.

$$DS(T_i^*) = \frac{\frac{d^2\sigma^+}{d\Omega^*dT^*}(T_i^*) - \frac{d^2\sigma^-}{d\Omega^*dT^*}(T_i^*)}{\left(\frac{d\sigma^+}{d\Omega^*} + \frac{d\sigma^-}{d\Omega^*}\right)} \quad (\text{Eq. 28})$$

The factor $\left(\frac{d\sigma^+}{d\Omega^*} + \frac{d\sigma^-}{d\Omega^*}\right)$ in the denominator was used to remove the overall normalization error. The uncertainty (ΔDS) associated with each value of DS was calculated as shown in Equation 29.

Figure 41.

Difference spectra at 4° and 16° (CMS)

Figure 42.

Difference spectra at 31° and 46° (CMS)

Figure 43.

Difference spectra at 57° and 67° (CMS)

Figure 44.

Difference spectra at 76° and 85° (CMS)

Figure 45.

Difference spectra at 95° and 113° (CMS)

$$\Delta DS = \frac{\sqrt{E_+'^2 + \left(.01 \frac{d^2\sigma_+}{d\Omega^*dT^*}\right)^2 + E_-'^2 + \left(.01 \frac{d^2\sigma_-}{d\Omega^*dT^*}\right)^2}}{\left(\frac{d\sigma_+}{d\Omega^*} + \frac{d\sigma_-}{d\Omega^*}\right)} \quad (\text{Eq. 29})$$

The factors $\left(.01 \frac{d^2\sigma_{\pm}}{d\Omega^*dT^*}\right)$ were present in order to allow for LH₂ target density variations, although in general these factors were negligible compared to $E_{\pm}'$. In order to make a quantitative statement about possible asymmetries, the expression given in Equation 30 was evaluated for each difference spectra.

$$R = \frac{\sum_i^n \left[\frac{DS(T_i^*)}{\Delta DS(T_i^*)} \right]^2}{n} \quad (\text{Eq. 30})$$

If asymmetries were to exist at a given angle, the quantity R would be expected (on the average) to be greater than ~1. Table 10 lists the values of R. Only those points unaffected by the 500 MeV neutron correction were included in the evaluation of R. These points are located to the right of the dashed lines in Figures 41 through 45.

It is evident from Table 10 and Figures 41 through 45 that asymmetries are present in the data. (Asymmetries were present through all stages of the data analysis). In most of the spectra from 4° - 85° CMS, the negative pion yields are predominant in the region from about 160 to 200 MeV. On the other hand, the positive pions are predominant below about 160 MeV. This pattern appears to be reversed in the spectra at 95° and 113°. It is difficult to explain the size of the asymmetries (often > 10%) solely from charge and mass differences between the final states $pp\pi^-$ and $nn\pi^+$. Thus it seems reasonable to conclude that at least some of the asymmetry is caused by interfering T=0 and T=1 amplitudes.

Table 10.

R (asymmetry parameter) versus θ^*

$\theta^*(^\circ)$	R
4	3.08
16	1.11
31	4.66
46	3.66
57	2.20
67	1.27
76	.84
85	1.01
95	1.41
113	2.85

It is interesting to note that the negative pion data appear to have little if any asymmetry when reflected about 90° , whereas the positive data seems to be asymmetric. This is shown in Figures 46 and 47, where the CMS spectra $(67_+^\circ, 113_+^\circ)$, $(67_-^\circ, 113_-^\circ)$, $(85_+^\circ, 95_+^\circ)$, and $(85_-^\circ, 95_-^\circ)$ are compared. The quantity DS' was calculated as shown in Equation 31. Here the subscript 1 (2) refers to data at 67° or 85° (113° or 95°). The error bars and cross-section values were determined from the curves $(F(T^*))$ fit to the data.

$$DS'(T^*) = \frac{\left. \frac{d^2\sigma}{d\Omega^*dT^*}(T^*) \right|_1 - \left. \frac{d^2\sigma}{d\Omega^*dT^*}(T^*) \right|_2}{\left. \frac{d\sigma}{d\Omega^*} \right|_1 + \left. \frac{d\sigma}{d\Omega^*} \right|_2} \quad (\text{Eq. 31})$$

The difference in the behavior of the negative and positive pions may be caused by Coulomb effects in the final state $pp\pi^-$.

Figure 46.

Difference Spectra for $(67^{\circ}_{+}, 113^{\circ}_{+})$ and $(67^{\circ}_{-}, 113^{\circ}_{-})$

Figure 47.

Difference Spectra for $(85^{\circ}_+, 95^{\circ}_+)$ and $(85^{\circ}_-, 95^{\circ}_-)$

CHAPTER IV

DISCUSSION AND INTERPRETATION OF RESULTS

A. COMPARISON OF THE PRESENT AND PREVIOUS DATA

It is of interest to compare the present results to those of previous np experiments. Dzhelepov et al.³ found that the peaks of their CMS kinetic energy spectra were located at about $.60 T_{\max}^*$, where $T_{\max}^*$ is the maximum pion kinetic energy. The peaks of the spectra in the present experiment appear to vary between about .55 to .65 $T_{\max}^*$. Yodh¹ found substantial asymmetries in the spectra of positive and negative pions around 400 MeV. Dzhelepov et al. also found asymmetries, although these were attributed to the effects of low energy beam neutrons (~ 400 MeV).

Dzhelepov et al. and Kazarinov et al.⁴ found the pion angular distributions $\frac{d\sigma}{d\Omega^*}(\theta^*)$ and $\frac{d\sigma_{01}}{d\Omega^*}(\theta^*)$ to be of the form $a_0 + a_2 \cos^2(\theta^*)$. This is in agreement with the present data. Yodh¹ and Handler² found $\frac{d\sigma}{d\Omega^*}(\theta^*)$ to vary as $a_0 + a_1 \cos(\theta^*) + a_2 \cos^2(\theta^*)$. Unfortunately, not enough data was gathered by Rushbrooke et al.⁵ for an accurate determination of $\frac{d\sigma}{d\Omega^*}(\theta^*)$. The values of σ_{01} and $\sigma(np \rightarrow NN\pi^+)$ are compared in Table 9 and Figure 40.

B. COMPARISON OF THE PRESENT DATA TO AN ISOBAR MODEL CALCULATION

It was concluded by Cence et al.³³ that the majority of pions produced from the reaction $pp \rightarrow pp\pi^0$ at 735 MeV came from decaying isobars. Cence et al. based their conclusion on the good agreement of their data with the Lindenbaum and Sternheimer isobar model.³⁷ The major assumptions of this model are listed below.

- (1) All pion production proceeds via the creation of an isobar

- (2) The isobar and remaining nucleon do not interact after the initial energy and momentum transfer
- (3) Interactions between the isobar decay products and the other nucleon can be ignored

A comparison of the isobar model to the present data was expected to show the effects of $T=0$ pion production, based on the results of the Cence experiment. Such a comparison is most easily accomplished by calculating the quantity $\frac{d\sigma}{dT^*}$ (as was done by Cence et al.). This cross-section is independent of the isobar angular distribution and any isobar polarization.

In order to determine $\frac{d\sigma}{dT^*}$ from the present data, the integral $\int \frac{d^2\sigma d\Omega_i^*}{d\Omega^* dT^*}$ was approximated as shown in Equation 32. The values of $\frac{d^2\sigma}{d\Omega^* dT^*}$ were calculated at predetermined kinetic energies (T_i^*) by linear interpolation between adjacent cross-sections located in the same spectrum. The effective angles θ_i^* were also calculated by linear interpolation. The indices $j = 1-8$ represent, in order, the spectra from $4^\circ - 85^\circ$.

$$\frac{d\sigma}{dT^*}(T_i^*) = \sum_{\substack{+, - \\ j=1}}^8 \frac{d^2\sigma}{d\Omega^* dT^*}(T_{ij}^*, \theta_{ij}^*) \Delta\Omega_{ij} \quad (\text{Eq. 32})$$

$$T_1^* \rightarrow T_9^* = (70, 85, 100, 115, 130, 145, 160, 175, 190) \text{ MeV}$$

$$\Delta\Omega_{i1} = 2\pi \left[1 - \cos\left(\frac{\theta_{i1}^* + \theta_{i2}^*}{2}\right) \right]$$

$$\Delta\Omega_{in} = 2\pi \left[\cos\left(\frac{\theta_{i(n-1)}^* + \theta_{in}^*}{2}\right) - \cos\left(\frac{\theta_{in}^* + \theta_{i(n+1)}^*}{2}\right) \right] \quad n = 2, 7$$

$$\Delta\Omega_{i8} = 2\pi \cos\left(\frac{\theta_{i7}^* + \theta_{i8}^*}{2}\right)$$

Figure 48 shows $\frac{d\sigma}{dT^*}$ along with an isobar model calculation, and 3-body Lorentz invariant phase space. Error bars on the data points reflect

Figure 48.

Comparison of the data with an isobar model prediction and phase space

The errors on the 160, 175, and 190 MeV data points are too small to be shown.

the uncertainties associated with $\frac{d^2\sigma}{d\Omega^*dT^*}(\theta^*, T^*)$ (see Appendix 1). The dotted line includes extrapolated data (see section N of Chapter III). Both the isobar and phase space curves were generated with the Berkeley program Sage^{38, 39}, and are normalized to the same total cross-section as the data. The Sage isobar calculation was used to represent the predictions of the Lindenbaum and Sternheimer model. The isobar mass (μ) was distributed according to the Breit-Wigner function BW(μ) given in Equation 33.

$$BW(\mu) = \frac{1}{\left[\left(\frac{2(\mu - \mu_0)}{\Gamma} \right)^2 + 1 \right]} \quad (Eq. 33)$$

$\mu_0 = 1232 \text{ MeV}, \Gamma = 115 \text{ MeV}$

It seems reasonable to conclude that the shift of data with respect to the isobar curve is mainly due to $T=0$ pion production. This statement can be substantiated by comparing the locations of $\left(\frac{d\sigma}{dT^*} \right)_{\max}$ in the present and Cence data ($.6T^*_{\max}$ versus $\sim .7T^*_{\max}$). Dzhelepov et al.³ also determined that their pion spectra were shifted towards lower energies when compared to π^0 spectra.

C. CONCLUSION

It was hypothesized from asymmetries between the positive and negative pion data that the cross-section σ_{01} is non-zero at 790 MeV. Additional evidence for non-resonant pion production was found by comparing the data to an isobar model prediction and a π^0 spectrum. Lack of information on the cross-section $\sigma(pp \rightarrow pp\pi^0)$ at 790 MeV and a 7% normalization error in the present data contributed to a large uncertainty in the actual

value of σ_{01} .^{*} However, it was still possible to show that pions are produced preferentially from the $T=1$ nucleon-nucleon state.

^{*}An experiment has been planned at LAMPF to determine the cross-section $\sigma(pp \rightarrow pp\pi^0)$ at 800 MeV.⁴⁰ This measurement, if of sufficient accuracy (~5%), will reduce the error in σ_{01} . In addition, future studies of the reactions $pp \rightarrow d\pi^+$ and $np \rightarrow d\pi^0$ should improve the overall normalization of the present data, and thus further reduce the uncertainty in σ_{01} .

APPENDIX I

Tabulated Data

All the data presented in Figures 24 through 36 are given in this appendix. The quantities $\Delta \frac{d^2\sigma}{d\Omega dT}$ and $\Delta \frac{d^2\sigma}{d\Omega^* dT^*}$ are errors associated with the double-differential cross-sections $\frac{d^2\sigma}{d\Omega dT}$ and $\frac{d^2\sigma}{d\Omega^* dT^*}$. The cross-sections denoted by the symbol (+) have errors which were calculated as shown below.

$$\Delta \frac{d^2\sigma}{d\Omega dT} = \sqrt{E^2 + (\Delta C_{500})^2}$$

$$\Delta \frac{d^2\sigma}{d\Omega^* dT^*} = J \Delta \frac{d^2\sigma}{d\Omega dT}$$

ΔC_{500} = 500 MeV neutron correction
uncertainty (=75%)

All the remaining cross-sections are given with their statistical errors (E, E*).

The units associated with each quantity are listed below.

Quantity	Units
$\bar{p}$	MeV/c
$\bar{T}, \bar{T}^*$	MeV
$\bar{\theta}, \bar{\theta}^*$	deg.
$\frac{d^2\sigma}{d\Omega dT}, \Delta \frac{d^2\sigma}{d\Omega dT}, \frac{d^2\sigma}{d\Omega^* dT^*}, \Delta \frac{d^2\sigma}{d\Omega^* dT^*}$	$\frac{\mu b}{sr-MeV}$

Spectrometer angle = 0°

$\bar{P}$	$\bar{T}$	$\bar{\theta}$	$\frac{d^2\sigma_+}{d\Omega dT}$	$\frac{\Delta d^2\sigma_+}{d\Omega dT}$	$\frac{d^2\sigma_-}{d\Omega dT}$	$\frac{\Delta d^2\sigma_-}{d\Omega dT}$	$\bar{T}^*$	$\bar{\theta}^*$	$\frac{d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{\Delta d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{d^2\sigma_-}{d\Omega^*dT^*}$	$\frac{\Delta d^2\sigma_-}{d\Omega^*dT^*}$
185	92	2.2	+ .09	$\begin{smallmatrix} +.66 \\ -.09 \end{smallmatrix}$	+ .15	$\begin{smallmatrix} +.66 \\ -.15 \end{smallmatrix}$	17	5.8	+ .03	$\begin{smallmatrix} +.25 \\ -.03 \end{smallmatrix}$	+ .06	$\begin{smallmatrix} +.25 \\ -.06 \end{smallmatrix}$
205	108	2.2	+ .36	$\begin{smallmatrix} +.75 \\ -.36 \end{smallmatrix}$	+ .66	$\begin{smallmatrix} +.71 \\ -.66 \end{smallmatrix}$	23	5.4	+ .15	$\begin{smallmatrix} +.31 \\ -.15 \end{smallmatrix}$	+ .27	$\begin{smallmatrix} +.29 \\ -.27 \end{smallmatrix}$
225	125	1.6	+ .65	$\begin{smallmatrix} +.72 \\ -.65 \end{smallmatrix}$	+ .65	$\begin{smallmatrix} +.81 \\ -.65 \end{smallmatrix}$	30	3.7	+ .28	$\begin{smallmatrix} +.31 \\ -.28 \end{smallmatrix}$	+ .28	$\begin{smallmatrix} +.35 \\ -.28 \end{smallmatrix}$
245	142	1.5	+1.04	.66	+1.56	.66	38	3.4	+ .46	.29	+ .70	.29
265	160	1.9	+1.26	.68	+1.88	.60	46	4.1	+ .58	.31	+ .86	.28
285	178	2.2	+1.38	.55	+2.23	.52	54	4.7	+ .65	.26	+ 1.05	.24
305	196	2.2	+2.00	.42	+2.50	.40	63	4.6	+ .96	.20	+ 1.20	.19
325	214	2.2	3.21	.35	2.48	.27	71	4.5	1.56	.17	1.20	.13
345	233	1.5	3.00	.41	3.07	.33	80	3.0	1.48	.20	1.51	.16
365	251	1.5	3.63	.43	3.00	.32	90	3.0	1.81	.21	1.49	.16
385	270	1.5	3.53	.50	2.88	.36	99	3.0	1.77	.25	1.45	.18
405	289	1.6	3.39	.45	3.15	.35	108	3.2	1.71	.23	1.59	.18
425	308	2.0	4.29	.45	3.03	.31	118	3.9	2.18	.23	1.54	.16
445	327	2.1	4.69	.44	4.07	.29	128	4.1	2.41	.23	2.08	.15
465		2.2	4.07	.40	3.59	.31	138	4.3	2.10	.21	1.85	.16
485		2.2	4.20	.42	3.45	.27	148	4.3	2.17	.21	1.79	.14
505	384	2.2	2.69	.31	2.89	.27	158	4.2	1.39	.16	1.50	.14

Spectrometer angle = 0° (Continued)

$\bar{P}$	$\bar{T}$	$\bar{\theta}$	$\frac{d^2\sigma_+}{d\Omega dT}$	$\frac{\Delta d^2\sigma_+}{d\Omega dT}$	$\frac{d^2\sigma_-}{d\Omega dT}$	$\frac{\Delta d^2\sigma_-}{d\Omega dT}$	$\bar{T}^*$	$\bar{\theta}^*$	$\frac{d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{\Delta d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{d^2\sigma_-}{d\Omega^*dT^*}$	$\frac{\Delta d^2\sigma_-}{d\Omega^*dT^*}$
525	404	2.0	2.27	.32	2.10	.26	168	3.8	1.18	.17	1.09	.13
545	423	1.5	1.19	.26	1.84	.23	178	2.9	.62	.13	.96	.12
565	442	1.5	.60	.22	.84	.19	188	2.9	.32	.11	.44	.10
585	462	1.5	.02	.02	.67	.14	198	2.9	.01	.01	.35	.07

Spectrometer Angle = 8°

$\bar{P}$	$\bar{T}$	$\bar{\theta}$	$\frac{d^2\sigma_+}{d\Omega dT}$	$\frac{\Delta d^2\sigma_+}{d\Omega dT}$	$\frac{d^2\sigma_-}{d\Omega dT}$	$\frac{\Delta d^2\sigma_-}{d\Omega dT}$	$\bar{T}^*$	$\bar{\theta}^*$	$\frac{d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{\Delta d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{d^2\sigma_-}{d\Omega^*dT^*}$	$\frac{\Delta d^2\sigma_-}{d\Omega^*dT^*}$
185	92	7.9	+ .63	.60	+ .90	.60	18	20.5	+ .25	.24	+ .35	.24
205	108	8.8	+ .18	+ .67 - .18	+ .42	+ .64 - .42	25	21.3	+ .08	+ .28 - .08	+ .18	+ .27 - .18
225	125	9.4	+ .64	+ .72 - .64	+ .69	.64	32	21.6	+ .28	+ .32 - .28	+ .31	.28
245	142	6.7	+1.32	.67	+1.36	.63	39	14.9	+ .60	.30	+ .62	.29
265	160	7.2	+ .98	.60	+1.99	.59	47	15.6	+ .46	.28	+ .93	.28
285	178	8.0	+1.41	.50	+1.93	.48	56	16.9	+ .67	.24	+ .92	.23
305	196	8.6	+1.62	.36	+1.99	.33	65	17.8	+ .79	.18	+ .97	.16
325	214	9.1	2.71	.29	2.58	.25	74	18.6	1.35	.14	1.28	.13
345	233	9.4	3.03	.33	2.84	.27	83	18.9	1.53	.16	1.43	.13
365	251	9.4	3.11	.35	2.71	.25	93	18.7	1.58	.18	1.38	.13
385	270	6.7	3.13	.39	±2.77	.43	101	13.3	1.59	.20	±1.41	.22
405	289	6.8	4.01	.45	±2.73	.43	110	13.4	2.05	.23	±1.40	.22
425	308	7.4	4.11	.41	±2.77	.40	120	14.5	2.12	.21	±1.43	.21
445	327	7.8	3.40	.36	±2.72	.37	130	15.2	1.77	.19	±1.41	.19
465	346	8.3	3.25	.36	±3.00	.38	141	16.0	1.70	.19	±1.57	.20
485	365	8.6	3.26	.32	±2.93	.37	151	16.5	1.71	.17	±1.54	.19
505	384	9.0	2.31	.27	±2.50	.33	161	17.2	1.22	.14	±1.32	.17
525	404	7.2	1.63	.25	±1.40	.24	172	17.6	.86	.13	±.74	.13

Spectrometer Angle = 8° (Continued)

$\bar{P}$	$\bar{T}$	$\bar{\theta}$	$\frac{d^2\sigma_+}{d\Omega dT}$	$\Delta \frac{d^2\sigma_+}{d\Omega dT}$	$\frac{d^2\sigma_-}{d\Omega dT}$	$\Delta \frac{d^2\sigma_-}{d\Omega dT}$	$\bar{T}^*$	$\bar{\theta}^*$	$\frac{d^2\sigma_+}{d\Omega^*dT^*}$	$\Delta \frac{d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{d^2\sigma_-}{d\Omega^*dT^*}$	$\Delta \frac{d^2\sigma_-}{d\Omega^*dT^*}$
545	423	9.4	.63	.16	$\ddagger$.65	.19	182	17.9	.33	.08	$\ddagger$.35	.10
565	442	9.4	.29	.13	$\ddagger$.68	.16	193	17.8	.16	.07	$\ddagger$.	.09
585	462	9.4	.11	.07	$\ddagger$.06	$^{+.11}_{-.06}$	203	17.8	.06	.03	$\ddagger$.03	$^{+.06}_{-.03}$

$\ddagger$ The statistical errors given with these cross-sections were increased from their original values because of possible spectrometer or beam instabilities.

Spectrometer angle = 16°

$\bar{P}$	$\bar{T}$	$\bar{\theta}$	$\frac{d^2\sigma_+}{d\Omega dT}$	$\Delta \frac{d^2\sigma_+}{d\Omega dT}$	$\frac{d^2\sigma_-}{d\Omega dT}$	$\Delta \frac{d^2\sigma_-}{d\Omega dT}$	$\bar{T}^*$	$\bar{\theta}^*$	$\frac{d^2\sigma_+}{d\Omega^*dT^*}$	$\Delta \frac{d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{d^2\sigma_-}{d\Omega^*dT^*}$	$\Delta \frac{d^2\sigma_-}{d\Omega^*dT^*}$
245	142	14.7	+1.35	.57	+1.19	.55	43	32.0	+ .65	.27	+ .57	.26
265	160	15.2	+1.33	.52	+1.41	.51	52	32.2	+ .66	.26	+ .69	.25
285	178	16.0	+1.52	.41	+1.96	.40	61	33.1	+ .77	.21	+ .99	.20
305	196	16.6	+1.95	.25	+1.89	.24	71	33.7	+1.00	.13	+ .97	.12
325	214	17.0	2.55	.18	2.60	.16	80	34.0	1.33	.09	1.36	.08
345	233	17.4	2.77	.21	2.65	.17	90	34.3	1.47	.11	1.41	.09
365	251	17.4	2.76	.20	2.78	.15	100	34.0	1.47	.11	1.48	.08
385	270	14.7	3.31	.25	2.74	.18	107	28.7	1.75	.13	1.45	.09
405	289	14.8	3.34	.25	3.32	.19	117	28.7	1.77	.13	1.76	.10
425	308	15.4	3.51	.23	3.18	.16	128	29.7	1.88	.12	1.71	.09
445	327	15.8	3.51	.22	2.89	.15	139	30.2	1.90	.12	1.56	.08
465	346	16.2	3.03	.19	2.79	.14	149	30.8	1.65	.11	1.52	.07
485	365	16.6	2.18	.17	2.22	.12	160	31.4	1.19	.09	1.22	.07
505	384	16.9	1.42	.14	1.71	.10	171	31.9	.78	.08	.94	.06
525	404	17.2	.36	.08	1.05	.09	183	32.3	.20	.04	.58	.05
545	423	17.4	.16	.06	.47	.07	194	32.6	.09	.03	.26	.04
565	442	17.4	.06	.03	.03	+ .04 - .03	204	32.5	.03	.02	.02	.02

Spectrometer angle = 24°

$\bar{P}$	$\bar{T}$	$\bar{J}$	$\frac{d^2\sigma_+}{d\Omega dT}$	$\frac{d^2\sigma_+}{\Delta d\Omega dT}$	$\frac{d^2\sigma_-}{d\Omega dT}$	$\frac{d^2\sigma_-}{\Delta d\Omega dT}$	$\frac{\bar{T}^*}{\bar{\theta}^*}$	$\frac{d^2\sigma_+}{d\Omega^* dT^*}$	$\frac{d^2\sigma_+}{\Delta d\Omega^* dT^*}$	$\frac{d^2\sigma_-}{d\Omega^* dT^*}$	$\frac{d^2\sigma_-}{\Delta d\Omega^* dT^*}$
235	134	22.8	† .68	.52	†1.20	.52	46	† .35	.27	† .62	.27
255	151	23.5	†1.19	.48	†1.09	.47	55	† .63	.26	† .58	.25
275	169	24.2	†1.35	.34	†1.76	.34	65	† .74	.19	† .96	.19
295	187	24.8	1.96	.17	2.17	.14	76	1.09	.10	1.21	.08
315	205	25.3	2.06	.20	2.01	.16	86	1.16	.11	1.14	.09
335	223	25.4	2.45	.21	2.52	.17	97	1.40	.12	1.44	.09
355	242	22.7	2.68	.22	2.25	.13	103	1.50	.12	1.26	.07
375	261	22.8	3.00	.25	2.34	.13	113	1.68	.14	1.31	.07
395	279	23.4	2.88	.22	2.40	.10	125	1.63	.12	1.36	.06
415	298	23.9	2.55	.19	2.14	.10	136	1.46	.11	1.23	.06
435	317	24.4	2.21	.16	2.11	.09	148	1.28	.09	1.22	.05
455	336	24.8	1.59	.15	1.61	.08	160	.93	.09	.93	.05
475	356	25.0	.95	.11	1.04	.07	171	.56	.07	.61	.04
495	375	25.4	.35	.07	.69	.06	183	.21	.04	.41	.04
515	394	25.4	.04	+.05 -.04	.32	.05	195	.03	.03	.19	.03

Spectrometer angle = 30°

$\bar{P}$	$\bar{T}$	$\bar{\theta}$	$\frac{d^2\sigma_+}{d\Omega dT}$	$\frac{d^2\sigma_+}{\Delta\Omega dT}$	$\frac{d^2\sigma_-}{d\Omega dT}$	$\frac{d^2\sigma_-}{\Delta\Omega dT}$	$\bar{P}^*$	$\bar{\theta}^*$	$\frac{d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{d^2\sigma_+}{\Delta\Omega^*dT^*}$	$\frac{d^2\sigma_-}{d\Omega^*dT^*}$	$\frac{d^2\sigma_-}{\Delta\Omega^*dT^*}$
235	134	29.2	+ .88	.48	+1.04	.47	53	59.6	+ .50	.27	+ .59	.27
255	151	30.0	+1.46	.36	+1.18	.36	64	59.7	+ .85	.21	+ .68	.21
275	169	30.7	+1.81	.21	+1.63	.19	75	59.8	+1.07	.12	+ .96	.11
295	187	31.2	2.11	.15	2.23	.12	86	59.8	1.27	.09	1.33	.07
315	205	31.4	1.88	.16	2.14	.13	97	59.4	1.14	.10	1.29	.08
335	223	28.7	2.48	.19	2.16	.17	102	54.5	1.46	.11	1.27	.10
355	242	28.8	2.67	.17	2.50	.14	113	54.2	1.59	.10	1.49	.08
375	261	29.4	2.46	.19	2.44	.12	125	54.8	1.48	.11	1.47	.07
395	279	30.0	2.38	.13	1.81	.13	138	55.5	1.45	.08	1.10	.08
415	298	30.4	2.09	.13	1.69	.10	150	55.8	1.28	.08	1.03	.06
435	317	30.8	1.29	.11	1.29	.09	163	56.2	.80	.07	.80	.06
455	336	31.2	.79	.11	.88	.08	175	56.6	.49	.07	.54	.05
475	356	31.4	.33	.06	.54	.08	188	56.8	.21	.04	.34	.05
495	375	31.4	.09	.03	.03	+ .08 - .03	199	56.6	.06	.02	.02	+ .05 - .02

Spectrometer angle = 36°

$\bar{P}$	$\bar{T}$	$\bar{\theta}$	$\frac{d^2\sigma_+}{d\Omega dT}$	$\frac{d^2\sigma_+}{\Delta d\Omega dT}$	$\frac{d^2\sigma_-}{d\Omega dT}$	$\frac{d^2\sigma_-}{\Delta d\Omega dT}$	$\bar{T}^*$	$\bar{\theta}^*$	$\frac{d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{d^2\sigma_+}{\Delta d\Omega^*dT^*}$	$\frac{d^2\sigma_-}{d\Omega^*dT^*}$	$\frac{d^2\sigma_-}{\Delta d\Omega^*dT^*}$
215	117	35.0	+ .74	.46	+ .78	.47	52	70.5	+ .45	.28	+ .47	.29
235	134	35.9	+1.07	.39	+ .87	.39	63	70.2	+ .67	.24	+ .54	.24
255	151	36.	+1.36	.21	+1.60	.22	74	70.0	+ .86	.13	+1.02	.14
275	169	37.2	1.83	.12	1.82	.12	86	69.8	1.18	.08	1.17	.08
295	187	37.4	1.91	.14	1.85	.13	97	69.3	1.24	.09	1.20	.09
315	205	34.7	2.19	.14	2.12	.13	103	64.6	1.38	.09	1.33	.08
335	223	34.8	2.11	.13	2.02	.13	115	64.2	1.34	.08	1.28	.08
355	242	35.6	1.92	.13	1.93	.11	128	64.9	1.23	.08	1.24	.07
375	261	36.0	1.97	.11	1.65	.10	141	65.2	1.28	.07	1.07	.07
395	279	36.5	1.51	.10	1.37	.08	154	65.6	.99	.06	.89	.05
415	298	36.9	.82	.08	.92	.08	167	65.9	.54	.05	.60	.05
435	317	37.3	.37	.06	.56	.06	180	66.2	.24	.04	.37	.04
455	336	37.4	.08	.02	.10	.06	193	66.2	.05	.02	.07	.04

Spectrometer angle = 42°

$\bar{P}$	$\bar{T}$	$\bar{\theta}$	$\frac{d^2\sigma_+}{d\Omega dT}$	$\frac{\Delta d^2\sigma_+}{d\Omega dT}$	$\frac{d^2\sigma_-}{d\Omega dT}$	$\frac{\Delta d^2\sigma_-}{d\Omega dT}$	$\bar{T}^*$	$\bar{\theta}^*$	$\frac{d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{\Delta d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{d^2\sigma_-}{d\Omega^*dT^*}$	$\frac{\Delta d^2\sigma_-}{d\Omega^*dT^*}$
205	108	41.0	† .65	.42	† .28	.43	56	80.1	† .43	.28	† .19	.29
225	125	41.8	†1.13	.35	† .92	.35	67	79.4	† .77	.24	† .62	.24
245	142	42.6	†1.29	.19	†1.29	.18	80	79.1	† .89	.13	† .89	.12
265	160	43.2	1.65	.13	1.71	.12	92	78.8	1.15	.09	1.20	.08
285	178	43.4	1.73	.12	2.10	.12	104	78.2	1.22	.08	1.47	.09
305	196	40.7	1.84	.14	1.81	.11	110	73.7	1.25	.10	1.23	.07
325	214	41.0	1.70	.13	1.72	.10	123	73.6	1.16	.09	1.18	.07
345	233	41.6	1.57	.11	1.66	.10	137	73.9	1.09	.08	1.15	.07
365	251	42.2	1.25	.10	1.19	.08	151	74.3	.87	.07	.83	.06
385	270	42.7	.82	.08	.88	.07	165	74.7	.58	.06	.62	.05
405	289	43.0	.40	.06	.47	.05	179	74.8	.29	.04	.33	.04
425	308	43.4	.11	.03	.19	.04	193	75.1	.08	.02	.13	.03

Spectrometer angle = 48°

$\bar{P}$	$\bar{T}$	$\bar{\theta}$	$\frac{d^2\sigma_+}{d\Omega dT}$	$\frac{\Delta d^2\sigma_+}{d\Omega dT}$	$\frac{d^2\sigma_-}{d\Omega dT}$	$\frac{\Delta d^2\sigma_-}{d\Omega dT}$	$\bar{T}^*$	$\bar{\theta}^*$	$\frac{d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{\Delta d^2\sigma_+}{d\Omega^*dT^*}$	$\frac{d^2\sigma_-}{d\Omega^*dT^*}$	$\frac{\Delta d^2\sigma_-}{d\Omega^*dT^*}$
195	100	47.3	† .57	.37	† .75	.37	60	89.2	† .42	.27	† .55	.27
215	117	48.3	† 1.12	.26	† 1.09	.26	73	88.5	† .84	.19	† .81	.19
235	134	49.0	1.61	.12	1.51	.15	86	87.9	1.22	.09	1.14	.11
255	151	49.4	1.61	.13	1.58	.17	99	87.3	1.22	.10	1.20	.13
275	169	46.7	1.81	.14	1.70	.12	106	83.0	1.32	.10	1.25	.09
295	187	46.8	1.65	.11	1.57	.11	118	82.3	1.21	.08	1.16	.08
315	205	47.6	1.45	.12	1.58	.11	133	82.7	1.08	.09	1.18	.08
335	223	48.2	1.18	.09	1.30	.08	148	83.0	.88	.07	.98	.06
355	242	48.7	.81	.07	.80	.07	163	83.2	.61	.05	.60	.05
375	261	49.1	.36	.05	.51	.06	178	83.3	.27	.04	.39	.04
395	279	49.4	.07	.07	.16	.04	193	83.4	.05	.05	.13	.03

Spectrometer angle = 50°

$\bar{P}$	$\bar{T}$	$\bar{\theta}$	$\frac{d^2\sigma_+}{d\Omega dT}$	$\Delta \frac{d^2\sigma_+}{d\Omega dT}$	$\frac{d^2\sigma_-}{d\Omega dT}$	$\Delta \frac{d^2\sigma_-}{d\Omega dT}$	$\bar{T}^*$	$\bar{\theta}^*$	$\frac{d^2\sigma_+}{d\Omega^* dT^*}$	$\Delta \frac{d^2\sigma_+}{d\Omega^* dT^*}$	$\frac{d^2\sigma_-}{d\Omega^* dT^*}$	$\Delta \frac{d^2\sigma_-}{d\Omega^* dT^*}$
185	92	55.3	† .87	.31	† .95	.32	68	99.3	† .72	.26	† .79	.27
205	108	56.4	†1.06	.18	†1.05	.19	82	98.6	† .89	.15	† .88	.16
225	125	57.1	1.39	.11	1.46	.11	97	97.9	1.18	.09	1.24	.09
245	142	57.4	1.38	.14	1.45	.14	111	97.1	1.17	.12	1.23	.12
265	160	54.7	1.46	.11	1.65	.10	118	93.1	1.19	.09	1.35	.08
285	178	55.1	1.13	.12	1.47	.08	133	92.8	.93	.10	1.20	.07
305	196	55.8	1.01	.08	1.07	.07	149	93.0	.83	.07	.89	.05
325	214	56.4	.56	.08	.67	.05	165	93.2	.47	.07	.56	.04
345	233	56.9	.36	.05	.29	.04	182	93.3	.30	.04	.25	.04
365	251	57.3	.12	.03	.05	.03	198	93.4	.10	.02	.05	.03

Spectrometer angle = 72°

$\bar{P}$	$\bar{T}$	$\bar{\theta}$	$\frac{d^2\sigma_+}{d\Omega dT}$	$\Delta \frac{d^2\sigma_+}{d\Omega dT}$	$\frac{d^2\sigma_-}{d\Omega dT}$	$\Delta \frac{d^2\sigma_-}{d\Omega dT}$	$\bar{T}^*$	$\bar{\theta}^*$	$\frac{d^2\sigma_+}{d\Omega^* dT^*}$	$\Delta \frac{d^2\sigma_+}{d\Omega^* dT^*}$	$\frac{d^2\sigma_-}{d\Omega^* dT^*}$	$\Delta \frac{d^2\sigma_-}{d\Omega^* dT^*}$
185	92	72.7	1.14	.09	1.30	.09	101	115.6	1.20	.10	1.38	.10
205	108	73.3	1.07	.11	1.34	.10	118	114.8	1.13	.12	1.41	.11
225	125	70.7	1.04	.08	1.18	.07	128	111.4	1.06	.08	1.19	.07
245	142	71.4	.99	.07	.92	.07	146	111.1	1.00	.07	.94	.07
265	160	72.1	.64	.05	.65	.05	165	111.1	.65	.05	.66	.05
285	178	72.8	.29	.03	.25	.03	184	111.2	.30	.03	.25	.03
305	196	73.2	.07	.02	-	-	203	111.1	.07	.02	-	-

APPENDIX II

500 MeV Neutron Correction

This appendix lists all the double differential cross-sections $(\frac{d^2\sigma}{d\Omega dT})_{500}$ which were subtracted from the data. The units associated with each quantity are given below.

Quantity	Units
$\bar{p}$	MeV/c
$\bar{T}$	MeV
$\frac{d^2\sigma}{d\Omega dT}$	$\frac{\mu b}{sr-MeV}$

$\bar{p}$	$\bar{T}$	$\frac{d^2\sigma}{d\Omega dT}_{500}$	$\bar{p}$	$\bar{T}$	$\frac{d^2\sigma}{d\Omega dT}_{500}$
Spectrometer angle = 0°			Spectrometer angle = 8°		
185	92	.75	185	92	.70
205	108	.80	205	108	.75
225	125	.80	225	125	.75
245	142	.75	245	142	.75
265	160	.70	265	160	.70
285	178	.60	285	178	.55
305	196	.40	305	196	.30
Spectrometer angle = 16°			Spectrometer angle = 24°		
245	142	.70	235	134	.65
265	160	.65	255	151	.60
285	178	.50	275	169	.40
305	196	.25			
Spectrometer angle = 30°			Spectrometer angle = 36°		
235	134	.60	215	117	.60
255	151	.45	235	134	.50
275	169	.20	255	151	.25
Spectrometer angle = 42°			Spectrometer angle = 48°		
205	108	.55	195	100	.45
225	125	.45	215	117	.30
245	142	.20			
Spectrometer angle = 56°			Spectrometer angle = 72°		
185	92	.40	no corrections made		
205	108	.20			

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