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*Measurement of the Left-Right Asymmetry
in Pion-Proton Radiative Exchange and
Charge Exchange Scattering from
301 to 625 MeV/c on
a Transversely Polarized Target*

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ABSTRACT OF THE DISSERTATION

Measurement of the Left-Right Asymmetry in Pion-Proton Radiative Exchange and Charge Exchange Scattering from 301 to 625 MeV/c on a Transversely Polarized Target

by

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The left-right asymmetry A_N in $\pi^-p \rightarrow \gamma n$ has been measured at $p_\pi=301, 316, 427, 471, 547, 586$, and 625 MeV/c using a transversely polarized target. The final-state neutron and gamma were detected in coincidence by two sets of matching neutron and gamma detectors at gamma angles centered around 90° and 110° c.m. A gamma detector consisted of an array of 15 counters, each was a $15 \times 15 \times 25$ cm³ block of lead-glass. A neutron detector consisted of 15 counters also, each one was a cylindrical plastic scintillator 7.6 cm in diameter and 45.7 cm long. The A_N results are compared with the predictions from the most recent single-pion photoproduction partial-wave analysis by Arai and Fujii. The agreement is poor, casting doubt on the correctness of the value for the radiative-decay amplitude of the neutral Roper resonance now in use. A comparison is made with the 90° recoil proton polarization data of the inverse reaction derived from γd scattering, there are substantial discrepancies.

Charge exchange ($\pi^-p \rightarrow \pi^0n$) events were the major yield in this experiment. Very precise values of the charge exchange analyzing power were obtained with an error of typically 3%. The charge exchange results are compared with the predictions from recent πN partial wave analyses. At the lower incident energies little difference is seen between the VPI, Karlsruhe-Helsinki, and CMU-LBL analyses, and there is excellent agreement with our experiment. From the onset of the Roper resonance the VPI solution is strongly favored.

CHAPTER ONE

THEORY

Many physicists firmly believe that Quantum Chromodynamics (QCD) is the fundamental theory of strong interactions. The basic ingredients of QCD are spin $1/2$ Fermi fields called quarks and spin 1 gauge fields called gluons. The symmetry group associated with QCD is the $SU(3)$ of 'color,' the quarks carry one of three 'color' charges and the quark-quark interactions are mediated by the exchange of eight 'colored' gluons. The color charge and gluons play the same role as the electric charge and photon in quantum electrodynamics (QED), but unlike the photon the gluons carry color which gives rise to self-interactions. According to QCD all strongly interacting particles are bound states of confined quarks and gluons. Confinement means that only color singlet or colorless states are observed in nature, and no isolated quarks or gluons are seen. The simplest color-singlet quark combinations that can be formed from three colors are qqq and $q\bar{q}$ states; these are identified with half integer spin baryons and integral spin mesons. Historically, it was the observed multiplet structures of the low-lying baryons and mesons that motivated Gell-Mann and Zweig to introduce the quark model in 1964 following the success of the $SU(3)$ model. So far the observed spectrum of hadrons is well accounted for using simple qqq and $q\bar{q}$ quark configurations. However, more complicated quark-gluon combinations, such as pure gluon states gg

or ggg , or hybrid baryon states q^3g , or $q^4\bar{q}$ baryon or $q^2\bar{q}^2$ meson states, have been suggested and a few possible candidates have been identified. Missing quark or quark-gluon states have an important implication for QCD since the confinement mechanism is not understood at present. The formidable mathematical problems associated with the non-perturbative nature of QCD represent a huge obstacle in the theoretical study of the bound states. The one non-perturbative QCD treatment available at present is the lattice gauge technique which has been applied to only a small number of problems with limited success.

Many models of hadrons with "QCD-inspired" ideas have been introduced to explain the hadron resonance spectrum. One important new assumption to the simple quark model has been the inclusion of one-gluon-exchange interactions¹ as the dominant short range potential. Gluon exchanges between a pair of quarks give rise to spin-spin and spin-orbit forces. Indeed, using only the spin-spin interactions in the non-relativistic harmonic oscillator Hamiltonian, the quark model of Isgur, Karl, and Koniuk² has had phenomenal success in predicting the masses as well as the decay amplitudes of a large number of baryon resonances. The hadronic and radiative decay amplitudes are perhaps more important than the masses in revealing the underlying quark-gluon dynamics since the decay widths probe directly the internal structures of the hadrons.

The properties of each resonance, such as the mass, width, spin and isospin quantum numbers, and the decay amplitudes, are all determined from Partial Wave Analyses (PWAs). The inputs to the PWAs are the measured differential and polarization cross sections from

various scattering experiments. Pion-nucleon scattering and photoproduction experiments are very important in studies of the N^* and Δ resonances.

This thesis is a detailed report on the measurement of the left-right asymmetry in the reactions $\pi^-p \rightarrow \gamma n$ (radiative "exchange" or REX) and $\pi^-p \rightarrow \pi^0 n$ (charge exchange or CEX) using a transversely polarized target in the energy region of the P_{33} $\Delta(1232)$ and P_{11} $N^*(1440)$ resonances. The present measurements are part of the program by the UCLA Particle Physics group to measure the complete set of πN scattering parameters in this energy region. The past experiments include measurements of the differential cross section and the left-right asymmetry in $\pi^\pm p$ elastic and π^-p charge exchange scattering. Recently the A and R spin rotation parameters in $\pi^\pm p$ elastic reactions have been measured. These measurements provide new data that are important for future partial wave analyses. The data help to discriminate among various available PWAs, which in turn are used to test theoretical models of the strong interaction.

Currently there are three detailed πN PWAs available, namely the analyses by the Karlsruhe-Helsinki³ (K-H), Carnegie-Mellon University-Lawrence Berkeley Laboratory⁴ (C-L), and Virginia Polytechnic Institute⁵ (VPI) groups. An important difference between these PWAs concerns the properties of the $P_{11}(1440)$ or Roper resonance: C-L concluded that the P_{11} is broad, $\Gamma \sim 340$ MeV; K-H determined it to be narrow, $\Gamma \sim 135$ MeV; VPI has reported two nearby poles in the complex plane at $(1359-100i)$ and $(1410-80i)$ MeV. The difference in these PWAs comes mainly from the following: 1) the handling of some conflicting

data sets in the data bank; 2) the use of theoretical constraints. The K-H and C-L analyses found unique results for the phases due to constraints derived from Mandelstam's two-variable analyticity. The VPI analysis is based on an empirical parametrization which is not unique and an approximation is used for the inelastic processes; 3) the energy region covered by K-H is almost double that of VPI. Clearly, it is important to see which PWA, if any, is the right one.

The tantalizing finding of the VPI FP84 πN PWA update⁵ which appeared in 1985 is the existence of the two nearby poles in the P_{11} wave in the vicinity of the Roper resonance. Such a double pole could be the first hint for the existence of two nearby P_{11} resonances. But, both Hhler of the K-H group and Cutkosky of C-L are not convinced of the need for two independent resonances to fit the data in this energy region. Hhler⁶ has reported that a fit of the K-H partial waves using the VPI formula also leads to two poles. Cutkosky⁷ has stressed that a single multi-channel resonance often has poles on several Riemann sheets; this rule applies to the Roper resonance because of the opening of the $\pi\Delta$ channel at 497 MeV/c. It is interesting that only the P_{11} wave appears to have two nearby poles. However, if the Roper is indeed a double resonance, some modifications to the simple quark shell model are necessary to account for the second P_{11} resonance. One of the most interesting possibilities is the existence of a hybrid baryon, a new type of matter made of three quarks and a gluon.^{8,9} The lightest hybrid baryon is predicted to have the quantum numbers and mass of the Roper resonance.^{10,11} A unique identification of one of the two Roper resonances as a hybrid baryon may be possible via a detailed study of the radiative-decay mode of the charged and neutral states.¹² The

charged hybrid P_{11} cannot decay electromagnetically, $P_{11}^+(\text{hybrid}) \nrightarrow p\gamma$, since it belongs to a 70-plet of flavor-spin SU(6). This was first pointed out by Barnes and Close.¹³ On the other hand, the neutral decay, $P_{11}^0 \rightarrow n\gamma$, is allowed. Thus, if π^+ or π^0 photoproduction on a proton shows only one P_{11} resonance, while in π^- photoproduction on a neutron or its inverse reaction, radiative capture, there are two P_{11} resonances as in π^-p elastic scattering, then this would be a good indication that the one P_{11} resonance not seen in $P_{11}^+ \rightarrow p\gamma$ is a hybrid state.

Present theories are guided largely by the mass values. Listed in Table 1.1 are the predictions for the mass of the two lowest P_{11} resonances from several recent models, including potential,² bag,¹⁴⁻¹⁷ color flux-tube,¹⁸ Skyrmion,^{19,20} and hybrid models.^{10,11} The radiative decay amplitudes for the Δ and N^* resonances are determined from partial-wave analyses of single-pion photoproduction. All photoproduction analyses rely heavily on nN PWAs for information about the existence, masses, and widths of the resonances. The radiative decay of the Roper resonance is especially attractive as a testing ground for various quark-model calculations. Conveniently, most models are divergent in the predictions of the magnitude and even the sign of the $P_{11}(1440)$ radiative decay amplitude as is illustrated in Table 1.2.

In the remainder of this chapter, the pion-nucleon scattering and the single-pion photoproduction formalism are discussed in detail. The equations obtained from the scattering formalism are used in the partial wave analyses.

Table 1.1 The masses of the two lowest energy P_{11} resonances predicted by recent models.

Mass (MeV)		Model	Authors
1405	1705	Harmonic Oscil. Pot.	Isgur-Karl ²
1543	1646	MIT-bag, Radial Excit.	Bowler-Hey ¹⁴
1410	1603	MIT-bag, Breathing Mode	DeGrand-Rebbi ¹⁵
1416	1617	Perturbed MIT-bag	Close-Horgan ¹⁶
1418	1533	Cloudy Bag	Umland et al. ¹⁷
1580		Colored Flux Tubes	Carlson et al. ¹⁸
*	1427	Skyrmion	Mattis-Karliner ¹⁹
	1575	q^3g - hybrid	Barnes-Close ¹⁰
1436	1565	q^3g - hybrid	Golowich et al. ¹¹

* No mass value listed for the lowest P_{11} state.

Table 1.2 Quark model and partial wave analysis (PWA) predictions of the radiative-decay amplitude (in $10^{-3} \text{ GeV}^{-1/2}$) of the neutral Roper resonance.

Quark Model	$A_{1/2}^n$	PWA	$A_{1/2}^n$
Copley et al. ²¹	-20	Metcalf-Walker ²⁶	-43±35
Feynman et al. ²²	-18	Knies et al. ²⁷	0
Kubota-Ohta ²³	+4	Noelle ²⁸	+63
Sugimoto-Toya ²⁴	+50	Barbour et al. ²⁹	+59
Koniuk-Isgur ²	+16	Arai-Fujii ³⁰	+23±9
Murthy-Bhaduri ²⁵	+24	Crawford-Morton ³¹	+56±15

SECTION 1.1 PION-NUCLEON SCATTERING FORMALISM

The scattering amplitudes, which are the central subject of interest in scattering problems, for a quantum system going from an initial state $|i\rangle$ to a final state $|f\rangle$ are generated by the scattering operator S ; the S-matrix elements are written as $S_{fi} = \langle f|S|i\rangle$. Once the S-matrix elements are known, all experimental quantities such as the differential cross sections and polarizations can be expressed in terms of the scattering amplitudes. Of course, at present there is no theory of the strong interaction which can be used to construct the S-matrix like it is done in Quantum Electrodynamics. Without a full theory, various conservation laws such as the ones for energy-momentum, angular momentum, parity, and isospin are used to restrict the possible form of the S-matrix elements.

Closely related to the S-matrix is the transition or T-matrix which is obtained from the S-matrix by subtracting out the uninteresting scattering in which the initial and the final states are identical. The T-matrix and the S-matrix are related by

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^4(P_f - P_i) T_{fi}, \quad (1.1)$$

where P_i and P_f are the total four-momenta of the initial and the final state, respectively. The four-dimensional Dirac delta function expressing conservation of energy-momentum has been explicitly exhibited. The standard expression for the differential cross section for a two-body reaction in the center-of-mass is

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \left(\frac{p_f}{p_i}\right) |T_{fi}|^2, \quad (1.2)$$

where s is the square of the total c.m. energy, and $p_i = |\vec{p}_i|$ and $p_f = |\vec{p}_f|$. It is convenient to introduce the scattering amplitude

$F(s, \theta, \phi)$, which is defined so that its square is equal to the differential cross section

$$\frac{d\sigma}{d\Omega} = |F(s, \theta, \phi)|^2. \quad (1.3)$$

Comparing eqs. 1.2 and 1.3, one sees that F is related to the T-matrix by

$$F(s, \theta, \phi) = \frac{1}{8\pi/s} \left(\frac{p_f}{p_i} \right)^{1/2} T_{fi}(s, \theta, \phi). \quad (1.4)$$

Concerning the construction of the initial and final states, it is generally assumed that well before and after the scattering the particles are free and noninteracting. This allows the initial and the final states to be built from the free particle plane wave solutions, which can be labelled by the momentum, spin, and other internal quantum numbers of the particles involved.

Consider now the elastic scattering of a spin-0 pion by a spin-1/2 nucleon. The initial and the final state of the πN system are conveniently labelled by the initial and the final momentum of the pion, $\vec{k}_i$ and $\vec{k}_f$, and the spin states of the nucleon using the spinors χ_i and χ_f , so that $|i\rangle = |\vec{k}_i, \chi_i\rangle$ and $|f\rangle = |\vec{k}_f, \chi_f\rangle$ or simply $|\chi_i\rangle$ and $|\chi_f\rangle$ since $|\vec{k}_i| = |\vec{k}_f|$ for two-body elastic scattering. Accounting for the two possible initial and final nucleon spin states (m_s (or ν) = $\pm 1/2$), the four scattering amplitudes are compactly written using a 2×2 matrix whose columns and rows refer to the initial and final spins, respectively. The four matrix elements are reduced to two independent ones as a consequence of parity invariance.

The most general form of the πN scattering matrix M that is invariant under parity is given by

$$M = f(\theta) + g(\theta)\vec{\sigma} \cdot \hat{n},^* \quad (1.5)$$

where f and g are two complex amplitudes, $\vec{\sigma}$ are the three Pauli matrices, and $\hat{n}$ is the unit vector normal to the scattering plane,

$$\hat{n} = \frac{\vec{k}_i \times \vec{k}_f}{|\vec{k}_i \times \vec{k}_f|}. \quad (1.6)$$

The complete determination of the f and g amplitudes apart from an overall phase requires three measurements, namely the unpolarized differential cross section $(d\sigma/d\Omega)_{\text{unpol}}$, the left-right asymmetry A_N using a transversely polarized target, and the A and R spin rotation parameters using a polarized (in the scattering plane) proton target. The relations between the experimental quantities and f and g , which are called the spin-nonflip and spin-flip amplitudes, respectively, are given below.

The differential cross section for scattering from the initial state $|X_i\rangle$ to the final state $|X_f\rangle$ is given by

$$\frac{d\sigma}{d\Omega}(X_i \rightarrow X_f) = |\langle X_f | M | X_i \rangle|^2 = |\langle X_f | f + g\vec{\sigma} \cdot \hat{n} | X_i \rangle|^2. \quad (1.7)$$

If the final spin orientation is not measured, the differential cross section (from a given initial spin direction) is obtained by summing over the final spin states, thus

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \sum_{X_f} |\langle X_f | M | X_i \rangle|^2 = \sum_{X_f} \langle X_i | M^\dagger | X_f \rangle \langle X_f | M | X_i \rangle \\ &= \langle X_i | M^\dagger M | X_i \rangle = \langle X_i | (f^* + g^* \vec{\sigma} \cdot \hat{n})(f + g \vec{\sigma} \cdot \hat{n}) | X_i \rangle \\ &= |f|^2 + |g|^2 + \langle X_i | (f^* g + f g^*) \vec{\sigma} \cdot \hat{n} | X_i \rangle, \end{aligned} \quad (1.8)$$

where in the last step, the identity

*Different authors use different forms for M . For example, Sakurai³² uses $M=f(\theta)+ig(\theta)\vec{\sigma} \cdot \hat{n}$, and Williams³³ uses $M=f(\theta)-ig(\theta)\vec{\sigma} \cdot \hat{n}$.

$$(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{b}) = \vec{a} \cdot \vec{b} + i\vec{\sigma} \cdot (\vec{a} \times \vec{b}),$$

was used to evaluate $(\vec{\sigma} \cdot \hat{n})(\vec{\sigma} \cdot \hat{n})$ which is $\hat{n} \cdot \hat{n} = 1$. The initial state $|X_i\rangle$ is normalized so that $\langle X_i | X_i \rangle = 1$. The initial polarization vector is defined by

$$\vec{P}_i = \frac{\langle X_i | \vec{\sigma} | X_i \rangle}{\langle X_i | X_i \rangle} = \langle X_i | \vec{\sigma} | X_i \rangle. \quad (1.9)$$

The differential cross section is then given by

$$\frac{d\sigma}{d\Omega} = |f|^2 + |g|^2 + (f^*g + fg^*)\vec{P}_i \cdot \hat{n}. \quad (1.10)$$

Due to the $(f^*g + fg^*)\vec{P}_i \cdot \hat{n} = 2\text{Re}(f^*g)\vec{P}_i \cdot \hat{n}$ term, the cross section generally exhibits a left-right asymmetry. If the system is completely unpolarized in the initial state, then $\vec{P}_i = 0$ and the unpolarized differential cross section is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = |f|^2 + |g|^2. \quad (1.11)$$

The polarization of the scattered particles $\vec{P}_f$ can be expressed in terms of the initial polarization vector $\vec{P}_i$. Using the fact that the final state $|X_f\rangle$ is related to the initial state $|X_i\rangle$ through the scattering matrix: $|X_f\rangle = M|X_i\rangle$, the scattered particle polarization is given by

$$\vec{P}_f = \frac{\langle X_f | \vec{\sigma} | X_f \rangle}{\langle X_f | X_f \rangle} = \frac{\langle X_i | M^\dagger \vec{\sigma} M | X_i \rangle}{\langle X_i | M^\dagger M | X_i \rangle}. \quad (1.12)$$

Inserting $f + g\vec{\sigma} \cdot \hat{n}$ for M , one obtains

$$\vec{P}_f = \frac{(|f|^2 - |g|^2)\vec{P}_i + (2\text{Re}(f^*g) + 2|g|^2\vec{P}_i \cdot \hat{n})\hat{n} + 2\text{Im}(f^*g)\vec{P}_i \times \hat{n}}{|f|^2 + |g|^2 + 2\text{Re}(f^*g)\vec{P}_i \cdot \hat{n}}. \quad (1.13)$$

If the incident beam is unpolarized so that $\vec{P}_i = 0$,

$$\vec{P}_f = \frac{2\text{Re}(f^*g)}{|f|^2 + |g|^2} \hat{n}. \quad (1.14)$$

The initially unpolarized beam acquires a polarization along $\hat{n}$ by the scattering.

So far, the differential cross section and the recoil polarization for scattering from completely polarized or unpolarized initial states have been considered. In order to describe partially polarized states, the density matrix formalism is used. The initial spin density matrix is given by

$$\rho_i = \frac{1}{2}(1 + \vec{P}_i \cdot \vec{\sigma}) \quad (1.15)$$

where the average polarization of the incident beam is given by the polarization vector

$$\vec{P}_i = \text{Tr}(\rho_i \vec{\sigma}). \quad (1.16)$$

The differential cross section for scattering from an initial state with polarization $\vec{P}_i$ is given by

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \text{Tr}(M\rho_i M^\dagger) = \text{Tr}[(f+g\vec{\sigma} \cdot \hat{n})\frac{1}{2}(1+\vec{P}_i \cdot \vec{\sigma})(f^*+g^*\vec{\sigma} \cdot \hat{n})] \\ &= |f|^2 + |g|^2 + (f^*g + fg^*)\vec{P}_i \cdot \hat{n}. \end{aligned} \quad (1.17)$$

This result has exactly the same form as eq. 1.10. The expression for the scattered nucleon polarization is

$$\begin{aligned} \vec{P}_f &= \frac{\text{Tr}(M\rho_i M^\dagger \vec{\sigma})}{\text{Tr}(M\rho_i M^\dagger)} \\ &= \frac{(|f|^2 - |g|^2)\vec{P}_i + (2\text{Re}(f^*g) + 2|g|^2\vec{P}_i \cdot \hat{n})\hat{n} + 2\text{Im}(f^*g)\vec{P}_i \times \hat{n}}{|f|^2 + |g|^2 + 2\text{Re}(f^*g)\vec{P}_i \cdot \hat{n}}, \end{aligned} \quad (1.18)$$

this again has the same form as the fully polarized case given by eq. 1.13.

The expression for the differential cross section, eq. 1.17 may be

written in another form which will be convenient in the discussion of the left-right asymmetry from a transversely polarized target.

Factoring out the $(|f|^2 + |g|^2)$ term, $\frac{d\sigma}{d\Omega}$ becomes

$$\frac{d\sigma}{d\Omega} = (|f|^2 + |g|^2) \left[1 + \frac{(f^*g + fg^*)}{|f|^2 + |g|^2} \vec{P}_1 \cdot \hat{n} \right].$$

Since the unpolarized differential cross section $(\frac{d\sigma}{d\Omega})_{\text{unpol}} = |f|^2 + |g|^2 = I_0(\theta)$, and defining A_N by

$$A_N(\theta) = \frac{(f^*g + fg^*)}{|f|^2 + |g|^2} = \frac{2\text{Re}(f^*g)}{|f|^2 + |g|^2}, \quad (1.19)$$

the differential cross section may be written as

$$\frac{d\sigma}{d\Omega} = I_0(\theta) [1 + A_N(\theta) \vec{P}_1 \cdot \hat{n}]. \quad (1.20)$$

A_N is called the analyzing power of the reaction. The analyzing power A_N may be determined by measuring the cross sections for scattering to the left and right using a transversely polarized target. The initial polarization vector $\vec{P}_1$ is normal to the scattering plane; the left and right refers to $\vec{k}_f$ being left or right of $\vec{k}_i$ with the same opening angle, so that the scattering to the left gives $\vec{P}_1 \cdot \hat{n} = +P_1$ and the scattering to the right gives $\vec{P}_1 \cdot \hat{n} = -P_1$. The left-right asymmetry ϵ is given by

$$\epsilon = \frac{(\frac{d\sigma}{d\Omega})_L - (\frac{d\sigma}{d\Omega})_R}{(\frac{d\sigma}{d\Omega})_L + (\frac{d\sigma}{d\Omega})_R} = A_N P_1. \quad (1.21)$$

Because $A_N = \frac{\epsilon}{P_1}$ is obtained by a measurement of the left-right scattering asymmetry, A_N is also often referred to as the left-right asymmetry.

The A and R spin rotation parameters are measured from a polarized proton target with the polarization vector lying in the scattering plane. In this case since $\vec{P}_1 \cdot \hat{n} = 0$, the cross section is equal to the

unpolarized differential cross section, namely $\frac{d\sigma}{d\Omega} = |f|^2 + |g|^2 = I_0(\vartheta)$. The expression (eq. 1.18) for the recoil polarization becomes

$$I_0 \vec{P}_f = (|f|^2 - |g|^2) \vec{P}_i + 2\text{Re}(f^*g) \hat{n} + 2\text{Im}(f^*g) \vec{P}_i \times \hat{n}. \quad (1.22)$$

Consider the projection of the recoil polarization $\vec{P}_f$ along $\hat{s}$, where the unit vector $\hat{s}$ is defined by $\hat{s} = \hat{k}_f \times \hat{n}$ ($\hat{k}_f$ is the unit vector along the initial momentum $\vec{k}_f$). (The unit vectors $\hat{k}_f$, $\hat{n}$, and $\hat{s}$ form a right-handed coordinate system.) The expression for $\vec{P}_f \cdot \hat{s}$ is obtained by resolving $\hat{k}_f$ and $\hat{s}$ in terms of the initial coordinate system given by three unit vectors $\hat{k}_i$, $\hat{n}$, and $\hat{m} = \hat{k}_i \times \hat{n}$. The component of $\vec{P}_f$ in the direction of $\hat{s}$ is given by

$$\vec{P}_f \cdot \hat{s} = A \vec{P}_i \cdot \hat{k}_i + R \vec{P}_i \cdot \hat{m} \quad (1.23)$$

where the coefficients A and R are defined by

$$I_0 A = (|f|^2 - |g|^2) \sin\theta + 2\text{Im}(f^*g) \cos\theta \quad (1.24)$$

$$I_0 R = (|f|^2 - |g|^2) \cos\theta - 2\text{Im}(f^*g) \sin\theta. \quad (1.25)$$

A and R can be determined by measuring the up-down asymmetry in a second scattering. Suppose the up-down asymmetry produced in the second scattering is ϵ_A when the target is polarized parallel to $\hat{k}_i$, and ϵ_R when the target is polarized parallel to $\hat{m}$. The A and R spin rotation parameters are determined from the asymmetries as they are related by

$$\epsilon_A = P_C A P_i$$

$$\epsilon_R = P_C R P_i$$

where P_C is the analyzing power of the second scattering, which is equal to the polarization that would be produced from an unpolarized beam.

It is noted that while a measurement of the analyzing power A_N determines $\text{Re}(f^*g)$, a measurement of A and R determines $\text{Im}(f^*g)$. The

parameters A_N , A , and R are not all independent, in fact, they are related by

$$A_N^2 + A^2 + R^2 = 1. \quad (1.26)$$

The polarization measurements are important in resolving the ambiguities, such as the Minami and the Yang ambiguities, inherent in knowing only the differential cross section values.

Partial wave expansion of f and g is accomplished by expanding the initial and the final plane wave solutions in terms of the spherical harmonics, which describe the orbital angular momentum of the system, and adding the spin of the nucleon to each orbital angular momentum to give the total angular momentum of the system. For a given value of the orbital momentum l , the total angular momentum j can be $l+1/2$ or $l-1/2$ (except for $l=0$); conversely, for a given j , $l=j\pm 1/2$. Total angular momentum conservation restricts the final state having the same j value as the initial state, and parity conservation limits the final orbital momentum to be equal to the initial orbital momentum.

For each l , two partial wave amplitudes contribute, T_{l+} for $j=l+1/2$ and T_{l-} for $j=l-1/2$. $T_{l\pm}$ has the form

$$T_{l\pm} = \frac{\eta_{l\pm} e^{2i\delta_{l\pm}} - 1}{2i} \quad (1.27)$$

where $\eta_{l\pm}$ is the elasticity with the condition $0 \leq \eta_{l\pm} \leq 1$, and $\delta_{l\pm}$ is the phase shift of the $l_{\pm}$ wave due to the interaction. The plots of $T_{l\pm}$ are the familiar Argand diagrams. The partial wave expansions of f and g are

$$f(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} [(\frac{1}{2}l+1)T_{l+} + \frac{1}{2}lT_{l-}] P_l(\cos\theta), \quad (1.28)$$

$$g(\theta) = \frac{1}{k} \sum_{l=1}^{\infty} [T_{l+} - T_{l-}] \sin\theta P_l'(\cos\theta), \quad (1.29)$$

where $P_l(\cos\theta)$ are the Legendre polynomials and $P_l'(\cos\theta) = \frac{d}{d\cos\theta} P_l(\cos\theta)$.

SECTION 1.2 PION PHOTOPRODUCTION AND RADIATIVE DECAY OF N^* RESONANCES

Photoproduction of pions from nucleons is possible for incident photons of $E_\gamma > 145$ MeV through the reactions:

Below 2 GeV the reactions proceed largely through formation of intermediate resonance states. The single-pion photoproduction measurements are analyzed in terms of the helicity or equivalently the multipole amplitudes.

In the helicity formalism, the spin of a particle is quantized along the direction of motion of the particle. The helicity operator Λ is given by

$$\Lambda = \frac{\vec{J} \cdot \vec{P}}{|\vec{P}|}\tag{1.31}$$

where $\vec{J}$ is the total angular momentum, and $\vec{P}$ is the linear momentum operator. Because the orbital angular momentum has no component in the direction of the motion, the eigenvalues λ of Λ are the projections of the spin of the particle along the momentum $\vec{p}$. A particle with nonzero mass and spin s has $2s+1$ possible values of λ , $-s \leq \lambda \leq s$. A zero mass

particle has just two helicity states with $\lambda=\pm s$. Denoting a single particle state by: $|\vec{p};\lambda\rangle$, a two-particle state is obtained by a direct product of two single particle states

$$|\vec{p}_1, \vec{p}_2; \lambda_1, \lambda_2\rangle = |\vec{p}_1; \lambda_1\rangle |\vec{p}_2; \lambda_2\rangle. \quad (1.32)$$

Since $\vec{p}_1 = -\vec{p}_2 = \vec{p}$ in the center-of-mass frame, the two particle state can be written as $N|p\theta\phi\lambda_1\lambda_2\rangle$, where N is a kinematical normalization factor, and p is the magnitude of $\vec{p}$, and the angles (θ, ϕ) give the direction of $\vec{p}$.

Consider now the interaction of particles 1 and 2, with masses m_1 and m_2 and spins s_1 and s_2 , becoming particles 3 and 4, with masses m_3 and m_4 and spins s_3 and s_4 . The T-matrix element for the transition from $\vec{p}_i (=p_i \hat{z})$ to $\vec{p}_f (=p_f, \theta, \phi)$ is given by

$$T_{fi} = \langle f|T|i\rangle = \frac{(2\pi)^2 4\sqrt{s}}{\sqrt{(p_i p_f)}} \langle p_f \theta \phi \lambda_3 \lambda_4 | T | p_i 0 0 \lambda_1 \lambda_2 \rangle, \quad (1.33)$$

where the λ 's are the helicities of the particles. The partial wave expansion of T_{fi} is obtained by expanding the plane wave helicity states in terms of the total angular momentum states. Because the helicity λ is invariant under rotations, it is possible to construct angular momentum helicity states having definite values of the quantum numbers j and m . Using the rotation matrices

$D_{m',m}^j(\alpha\beta\gamma) = e^{-i(\alpha m' + \beta m)} d_{m',m}^j(\beta)$, the plane wave helicity states and the angular momentum helicity states are related by

$$|\theta\phi\lambda_1\lambda_2\rangle = \sum_{j,m} \left(\frac{2j+1}{4\pi}\right)^{1/2} D_{m\lambda}^j(\phi, \theta, -\phi) |jm\lambda_1\lambda_2\rangle \quad (1.34)$$

$$\text{and } |jm\lambda_1\lambda_2\rangle = \left(\frac{2j+1}{4\pi}\right)^{1/2} \int d\phi \int \sin\theta d\theta [D_{m\lambda}^{j*}(\phi, \theta, -\phi)] |\theta, \phi, \lambda_1\lambda_2\rangle, \quad (1.35)$$

where $\lambda = \lambda_1 - \lambda_2$ is the total helicity of the two-particle state.

Substituting eq. 1.34 into eq. 1.33, and assuming that the interaction is invariant under parity and rotational transformations, one obtains

$$T_{fi} = \frac{8\pi/s}{\sqrt{(p_i p_f)}} \sum_j \left[(2j+1) \langle \lambda_3 \lambda_4 | T^j | \lambda_1 \lambda_2 \rangle e^{i(\lambda_1 - \lambda_f)\phi} d_{\lambda_1 \lambda_f}^j(\theta) \right]. \quad (1.36)$$

The differential cross section is given by

$$\frac{d\sigma}{d\Omega}(\lambda_1 \lambda_2 \rightarrow \lambda_3 \lambda_4) = \frac{1}{64\pi^2 s} \left(\frac{p_f}{p_i} \right) |T_{fi}|^2. \quad (1.37)$$

If the polarization of the interacting particles is not considered, the cross section is averaged over the initial spin states and summed over the final spin states,

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{unpol}} = \frac{1}{64\pi^2 s} \left(\frac{p_f}{p_i} \right) |T_{fi}|^2_{\text{avg}}, \quad (1.38)$$

where $|T_{fi}|^2_{\text{avg}}$ is given by

$$|T_{fi}|^2_{\text{avg}} = \frac{1}{(2s_1+1)(2s_2+1)} \sum_{\text{spins}} |T_{fi}|^2. \quad (1.39)$$

For a massless particle of spin s , the spin multiplicity factor $(2s+1)$ is replaced by 2.

The scattering amplitude $f_{\mu\lambda}(\theta, \phi)$ (with $\lambda = \lambda_1 - \lambda_2$ and $\mu = \lambda_3 - \lambda_4$) is defined by

$$\begin{aligned} f_{\mu\lambda}(\theta, \phi) &= \frac{1}{8\pi/s} \left(\frac{p_f}{p_i} \right)^{1/2} T_{fi} \\ &= \frac{1}{p_i} \sum_j (2j+1) \langle \mu | T^j | \lambda \rangle e^{i(\lambda - \mu)\phi} d_{\lambda\mu}^j(\theta) \\ &= \sum_j f_{\mu\lambda}^j (2j+1) d_{\lambda\mu}^j(\theta) e^{i(\lambda - \mu)\phi} \end{aligned} \quad (1.40)$$

where in the last step the quantity $(p_i)^{-1} \langle \mu | T^j | \lambda \rangle$ has been replaced by the partial wave amplitude $f_{\mu\lambda}^j$. The differential cross section is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{1}{(2s_1+1)(2s_2+1)} \sum_{\lambda,\mu} |f_{\mu\lambda}|^2. \quad (1.41)$$

$$= \frac{1}{(2s_1+1)(2s_2+1)} \sum_{\lambda,\mu} \left| \sum_j (2j+1) f_{\mu\lambda}^j d_{\lambda\mu}^j(\theta) \right|^2. \quad (1.42)$$

Integrating over $d\Omega$ and using the orthogonality of $d_{\lambda\mu}^j(\theta)$ the total cross section is

$$\sigma_T = \frac{1}{(2s_1+1)(2s_2+1)} \sum_{\lambda,\mu} \sum_j 4\pi(2j+1) |f_{\mu\lambda}^j|^2. \quad (1.43)$$

As a consequence of parity symmetry, the helicity amplitudes $f_{\mu\lambda}$ are not all independent. Half of the amplitudes are related to the other half by the relation

$$\langle -\mu | T^J | -\lambda \rangle = \eta_1 \eta_2 \eta_3 \eta_4 (-1)^{s_1+s_2-s_3-s_4} \langle \mu | T^J | \lambda \rangle \quad (1.44)$$

where η_i and s_i are the intrinsic parity and the spin of the i th particle. In particular for $\gamma N \rightarrow \pi N$,

$$\langle -\mu | T^J | -\lambda \rangle = -\langle \mu | T^J | \lambda \rangle. \quad (1.45)$$

Applying the helicity formalism to the single-pion photoproduction reactions, there are eight helicity amplitudes to consider as the helicity of the initial γN system, $\lambda = \lambda_\gamma - \lambda_N$, can assume the values $\pm 3/2$ and $\pm 1/2$, and the final πN helicity is $\mu = -\lambda_N = \pm 1/2$. The eight helicity amplitudes are reduced to half as the four with $\lambda_\gamma = -1$ are simply related to the four with $\lambda_\gamma = +1$ by parity symmetry; applying eq. 1.45 to eq. 1.40, one obtains

$$f_{-\mu-\lambda} = -e^{i(\lambda-\mu)(\pi-2\phi)} f_{\mu\lambda}, \quad (1.46)$$

which at $\phi=0$ becomes

$$f_{-\mu-\lambda} = -(-1)^{\lambda-\mu} f_{\mu\lambda}. \quad (1.47)$$

Choosing the four helicity amplitudes for $\lambda_\gamma = +1$ to be independent and labelling them as H_1 to H_4 , the 2×4 $f_{\mu\lambda}(\theta, \phi=0)$ matrix is given in Table 1.3. Except for the overall phase, seven independent measurements are

needed to completely determine the four complex amplitudes. Some of the required measurements and the relations to the helicity amplitudes are given below.

The differential cross section for either photon helicity is the same, using eq. 1.41,

$$\frac{d\sigma}{d\Omega}(\Theta) = \frac{1}{2}(|H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2) = \sigma(\Theta). \quad (1.48)$$

The recoil nucleon polarization along the normal to the scattering plane (in the direction $\vec{k}_\gamma \times \vec{k}_\pi$) is

$$P(\Theta) = -\frac{1}{\sigma(\Theta)} \text{Im}(H_1 H_3^* + H_2 H_4^*). \quad (1.49)$$

This result holds for either photon helicity.

The differential cross sections for photons polarized perpendicular and parallel to the scattering plane are given by

$$\sigma_{\perp}(\Theta) = \frac{1}{2}(|H_1 + H_4|^2 + |H_2 - H_3|^2) \quad (1.50)$$

$$\sigma_{\parallel}(\Theta) = \frac{1}{2}(|H_1 - H_4|^2 + |H_2 + H_3|^2). \quad (1.51)$$

The asymmetry using polarized photons is

$$\Sigma(\Theta) = \frac{\sigma_{\perp} - \sigma_{\parallel}}{\sigma_{\perp} + \sigma_{\parallel}} = \frac{1}{\sigma(\Theta)} \text{Re}(H_1 H_4^* - H_2 H_3^*). \quad (1.52)$$

Table 1.3 Helicity amplitudes $f_{\mu\lambda}(\Theta, \phi=0)$.

		$\lambda_\gamma=1$		$\lambda_\gamma=-1$	
		$\lambda=3/2$	$1/2$	$-1/2$	$-3/2$
$\mu=$	$1/2$	H_1	H_2	H_4	$-H_3$
	$-1/2$	H_3	H_4	$-H_2$	H_1

If the σ_+ and σ_- are the differential cross sections for target nucleons polarized "up" and "down" in the direction normal to the scattering plane, the polarized target asymmetry is given by

$$T(\theta) = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{1}{\sigma(\theta)} \text{Im}(H_1 H_2^* + H_3 H_4^*). \quad (1.53)$$

The partial wave amplitudes $f_{\lambda\mu}^j$, which were defined in eq. 1.40, refer to states of definite j but of mixed parity. Final states of definite parity are formed by adding and subtracting the final states having opposite helicity, μ and $-\mu$. The parity conserving helicity amplitudes are given by:

$$\begin{aligned} A_{\ell+} &= -\frac{1}{\sqrt{2}} \left(f_{1/2,1/2}^j + f_{-1/2,1/2}^j \right) \\ A_{(\ell+1)-} &= \frac{1}{\sqrt{2}} \left(f_{1/2,1/2}^j - f_{-1/2,1/2}^j \right) \\ B_{\ell+} &= \frac{1}{\sqrt{2}} \left(f_{1/2,3/2}^j + f_{-1/2,3/2}^j \right), \quad \ell \geq 1 \\ B_{(\ell+1)-} &= -\frac{1}{\sqrt{2}} \left(f_{1/2,3/2}^j - f_{-1/2,3/2}^j \right), \quad \ell \geq 1 \end{aligned} \quad (1.54)$$

where $j=\ell+1/2$, and the subscripts ℓ and $(\ell+1)$ on the A's and B's denote the orbital angular momentum of the pion in the final state; for example the $A_{(\ell+1)-}$ amplitude contributes to the $j=(\ell+1)-1/2=\ell+1/2$ and $(\ell+1)$ orbital angular momentum state of the the πN system which has parity $-(-1)^{\ell+1}$. The $A(B)_{\ell+}$ and $A(B)_{(\ell+1)-}$ amplitudes have opposite parity. The helicity amplitudes H_1 to H_4 can now be written in terms of the A and B amplitudes:

$$H_1(\theta) = f_{1/2,3/2}(\theta, 0) = \sum_{\ell=0}^{\infty} -(B_{\ell+} - B_{(\ell+1)-}) \sqrt{2}(\ell+1) d_{3/2,1/2}^{\ell+1/2}(\theta)$$

$$\begin{aligned}
&= \sum_{\ell=0} (B_{\ell+} - B_{(\ell+1)-}) \frac{\sqrt{2} \sin \theta \cos(\theta/2)}{\sqrt{\ell(\ell+2)}} (P_{\ell}''(\cos \theta) - P_{\ell+1}''(\cos \theta)) \\
H_2(\theta) &= f_{1/2, 1/2}(\theta, 0) = \sum_{\ell=0} -(A_{\ell+} - A_{(\ell+1)-}) \sqrt{2}(\ell+1) d_{1/2, 1/2}^{\ell+1/2}(\theta) \\
&= \sum_{\ell=0} (A_{\ell+} - A_{(\ell+1)-}) \sqrt{2} \cos(\theta/2) (P_{\ell}'(\cos \theta) - P_{\ell+1}'(\cos \theta)) \\
H_3(\theta) &= f_{-1/2, 3/2}(\theta, 0) = \sum_{\ell=0} (B_{\ell+} + B_{(\ell+1)-}) \sqrt{2}(\ell+1) d_{3/2, -1/2}^{\ell+1/2}(\theta) \\
&= \sum_{\ell=0} (B_{\ell+} + B_{(\ell+1)-}) \frac{\sqrt{2} \sin \theta \sin(\theta/2)}{\sqrt{\ell(\ell+2)}} (P_{\ell}''(\cos \theta) + P_{\ell+1}''(\cos \theta)) \\
H_4(\theta) &= f_{-1/2, 1/2}(\theta, 0) = \sum_{\ell=0} -(A_{\ell+} + A_{(\ell+1)-}) \sqrt{2}(\ell+1) d_{1/2, -1/2}^{\ell+1/2}(\theta) \\
&= \sum_{\ell=0} (A_{\ell+} - A_{(\ell+1)-}) \sqrt{2} \sin(\theta/2) (P_{\ell}'(\cos \theta) + P_{\ell+1}'(\cos \theta)).
\end{aligned} \tag{1.55}$$

The total cross section is given by (using eq. 1.43)

$$\sigma_T = 4\pi \sum_{\ell} (\ell+1) [|A_{\ell+}|^2 + |A_{(\ell+1)-}|^2 + |B_{\ell+}|^2 + |B_{(\ell+1)-}|^2]. \tag{1.56}$$

Table 1.4 The contribution of the multipoles EL and ML to photoproduction.

Photon State	Total Angular Momentum j	Parity	The Final π -N state	Angular Distribution
E1	1/2	-	$s_{1/2}$	1
	3/2	-	$d_{3/2}$	$2+3\sin^2 \theta$
M1	1/2	+	$p_{1/2}$	1
	3/2	+	$p_{3/2}$	$2+3\sin^2 \theta$
E2	3/2	+	$p_{3/2}$	$1+\cos^2 \theta$
	5/2	+	$f_{5/2}$	$1+6\cos^2 \theta - 5\cos^4 \theta$
M2	3/2	-	$d_{3/2}$	$1+\cos^2 \theta$
	5/2	-	$d_{5/2}$	$1+6\cos^2 \theta - 5\cos^4 \theta$

An alternative description of photoproduction can be obtained by expanding the incoming photon wave into eigenstates of the total angular momentum L of the photon called electric and magnetic multipoles, EL and ML . $L=1$ is called a dipole and $L=2$ a quadrupole transition. The electric and magnetic $2L$ poles are differentiated by the parity, EL has parity $(-1)^L$ and ML has $(-1)^{L+1}$. The minimum value of L is 1 since the photon has spin 1. Adding the photon angular momentum L to the spin of the nucleon, the total angular momentum j can be $j=L+1/2$ or $L-1/2$. Various multipole transitions and the final state angular distributions in the c.m. are listed in Table 1.4. Note that a multipole transition does not have a unique angular distribution. For example, both the $E1 \rightarrow d_{3/2}$ and $M1 \rightarrow p_{3/2}$ transitions give the same $2+3\sin^2\theta$ distribution. This is an example of the ambiguities inherent in analyses of the pion photoproduction experiments.

The multipole amplitudes are equivalent to the helicity amplitudes. The relations between the two types of amplitudes may be obtained as follows. The initial γN state can be described in terms of states $|j,m;L;\ell'\rangle$ in which the orbital angular momentum ℓ' and the total angular momentum of the photon L have a definite value. The connection between the helicity states $|\lambda\rangle$ ($=|jm;\lambda\rangle$) and the $|j,m;L;\ell'\rangle$ states is obtained by adding the spin 1 of the photon to the orbital angular momentum ℓ' to give L , and then adding the spin 1/2 of the nucleon to obtain the total angular momentum j . Using the Clebsch-Gordon coefficients $C_{j_1 j_2}(j,m;m_1,m_2)$, the expansion is given by

$$|j,m;\lambda\rangle = \sum_{L,\ell'} \left(\frac{2\ell'+1}{2j+1} \right)^{1/2} C_{L1/2}(j,\lambda;\lambda_1,\lambda_2) C_{\ell',1}(L,\lambda_1;0,\lambda_1) |j,m;L;\ell'\rangle. \quad (1.57)$$

Summing over ℓ' which can assume the values $L-1$, L , and $L+1$ and evaluating the $C_{\ell,1}(L,\lambda_1;0,\lambda_1)$ coefficients, eq. 1.57 becomes

$$|j,m;\lambda\rangle = \sum_L \left(\frac{L(L+1)(2L+1)}{2(2j+1)} \right)^{1/2} C_{L1/2}(j,\lambda;\lambda_1,\lambda_2) \left[\frac{-1}{\sqrt{L(L+1)}} |j,m;L;\ell'=L\rangle \right. \\ \left. + \frac{1}{\sqrt{(L+1)(2L+1)}} |j,m;L;\ell'=L+1\rangle + \frac{1}{\sqrt{L(2L+1)}} |j,m;L;\ell'=L-1\rangle \right]. \quad (1.58)$$

The parity of the photon states is given by $-(-1)^{\ell'}$, the extra minus sign comes from the photon having a negative intrinsic parity. Thus, the first state in the bracket which has parity $(-1)^{L+1}$ since $\ell'=L$ is hence identified with the magnetic multipole ML . The two other states have parity $(-1)^L$ since $\ell'=L\pm 1$, they are identified with the electric multipole EL . Defining the magnetic and the electric multipole states $|j,m;ML\rangle$ and $|j,m;EL\rangle$ by

$$|j,m;ML\rangle = \frac{1}{\sqrt{L(L+1)}} |j,m;L;\ell'=L\rangle \quad (1.59)$$

$$|j,m;EL\rangle = \frac{1}{\sqrt{(2L+1)}} \left[\frac{1}{\sqrt{(L+1)}} |j,m;L;\ell'=L+1\rangle + \frac{1}{\sqrt{L}} |j,m;L;\ell'=L-1\rangle \right], \quad (1.60)$$

eq. 1.58 can be written as

$$|j,m;\lambda\rangle = \sum_L \left(\frac{L(L+1)(2L+1)}{2(2j+1)} \right)^{1/2} C_{L1/2}(j,\lambda;\lambda_1,\lambda_2) \left[-\lambda_1 |j,m;ML\rangle + |j,m;EL\rangle \right]. \quad (1.61)$$

Finally summing over $L=j-1/2$ and $j+1/2$, one obtains

$$|j,m;\lambda\rangle = \frac{1}{4} \left\{ \begin{array}{l} \lambda_2=+1/2, [(j-1/2)(j+\lambda)]^{1/2} \\ \lambda_2=-1/2, [(j-1/2)(j-\lambda)]^{1/2} \end{array} \right\} \left[-\lambda_1 |j,m;ML+\rangle + |j,m;EL+\rangle \right] \\ + \frac{1}{4} \left\{ \begin{array}{l} \lambda_2=1/2, -[(j+3/2)(j-\lambda+1)]^{1/2} \\ \lambda_2=-1/2, [(j+3/2)(j+\lambda+1)]^{1/2} \end{array} \right\} \left[-\lambda_1 |j,m;ML-\rangle + |j,m;EL-\rangle \right], \quad (1.62)$$

where the "+" in $|EL\pm\rangle$ and $|ML\pm\rangle$ is used to denote that $j=L\pm 1/2$. For the final πN state, the helicity states $|j,m;\lambda\rangle$ can be expressed in

terms of the states $|j, m; \ell\rangle$ in which the orbital angular momentum ℓ is diagonal:

$$|j, m; \mu\rangle = \sum_{\ell} \left[\frac{(2\ell+1)}{(2j+1)} \right]^{1/2} c_{\ell 1/2}(j, \mu; 0, \mu) |j, m; \ell\rangle. \quad (1.63)$$

Summing over $\ell=j-1/2$ and $j+1/2$, the last equation becomes

$$|j, m; \mu\rangle = \frac{1}{\sqrt{2}} |j, m; \ell+\rangle - \frac{2\mu}{\sqrt{2}} |j, m; \ell-\rangle. \quad (1.64)$$

where $\ell\pm$ is used to signify $j=\ell\pm 1/2$. Interestingly, the last equation reveals that the helicity state $|j, m; \mu\rangle$ is not a parity eigenstate.

The partial wave helicity amplitudes $f_{\mu\lambda}^j$ can now be expressed in terms of the multipole amplitudes. Using eqs. 1.62 and 1.64 to calculate $f_{\mu\lambda}^j = (p_i)^{-1} \langle j, m; \mu | T^j | j, m; \lambda \rangle$, one obtains

$$\begin{aligned} f_{1/2, 3/2}^{\ell+1/2} &= \frac{\sqrt{\ell(\ell+2)}}{2\sqrt{2}} \left[E_{\ell+} - M_{\ell+} - E_{(\ell+1)-} - M_{(\ell+1)-} \right] \\ f_{1/2, 1/2}^{\ell+1/2} &= \frac{1}{2\sqrt{2}} \left[(\ell+2)(M_{(\ell+1)-} - E_{\ell+}) - \ell(M_{\ell+} + E_{(\ell+1)-}) \right] \\ f_{1/2, -1/2}^{\ell+1/2} &= \frac{1}{2\sqrt{2}} \left[\ell(M_{\ell+} - E_{(\ell+1)-}) + (\ell+2)(M_{(\ell+1)-} + E_{\ell+}) \right] \\ f_{1/2, -3/2}^{\ell+1/2} &= \frac{\sqrt{\ell(\ell+2)}}{2\sqrt{2}} \left[M_{\ell+} - E_{\ell+} - E_{(\ell+1)-} - M_{(\ell+1)-} \right], \end{aligned} \quad (1.65)$$

where j is equal to $\ell+1/2$, and the nonvanishing multipole amplitudes

$M_{\ell+}$, $E_{\ell+}$, $M_{(\ell+1)-}$, and $E_{(\ell+1)-}$ are defined by:

$$\begin{aligned} M_{\ell+} &= (p_i)^{-1} \langle j, m; \ell+ | T^j | j, m; M_{\ell+} \rangle \\ E_{\ell+} &= -(p_i)^{-1} \langle j, m; \ell+ | T^j | j, m; E_{\ell+} \rangle \\ M_{(\ell+1)-} &= (p_i)^{-1} \langle j, m; \ell- | T^j | j, m; M_{\ell-} \rangle \\ E_{(\ell+1)-} &= (p_i)^{-1} \langle j, m; \ell- | T^j | j, m; E_{\ell-} \rangle. \end{aligned} \quad (1.66)$$

The subscripts on the multipoles indicate the orbital angular momentum l of the pion in the final state.

Finally, using eqs. 1.54 and 1.65, the electric and magnetic multipoles are expressed in terms of the partial wave amplitudes A and B by:

$$\begin{aligned}
 (l+1)E_{l+} &= A_{l+} + \left(\frac{l}{l+2} \right)^{1/2} B_{l+} \\
 (l+1)M_{l+} &= A_{l+} - \left(\frac{l+2}{l} \right)^{1/2} B_{l+} \\
 (l+1)E_{(l+1)-} &= -A_{(l+1)-} + \left(\frac{l+2}{l} \right)^{1/2} B_{(l+1)-} \\
 (l+1)M_{(l+1)-} &= A_{(l+1)-} + \left(\frac{l}{l+2} \right)^{1/2} B_{(l+1)-}.
 \end{aligned} \tag{1.67}$$

There are three methods for determining the individual multipoles based on:

- (1) The simple isobar-model in which the partial waves are parametrized as a smooth background to which Breit-Wigner resonances are added.
- (2) Fixed- t dispersion relations; this technique uses the apparent resonance dominance of the photoproduction amplitudes to get a relatively simple parametrization of the imaginary parts. Fixed- t dispersion relations are then used to calculate the real parts.
- (3) Energy independent phase shift analyses in which the partial waves are evaluated from fits to sets of single energy values. It is necessary to use Watson's theorem to fix the complex phases of many of the partial waves for getting a unique solution. Due to the onset of the inelasticity, this becomes difficult above the first resonance region.

The basic assumption in the quark model for the radiative

transition $N^* \leftrightarrow N\gamma$ is that a single quark can absorb or emit a photon leading to excitation or deexcitation of the system. The nonrelativistic form of the interaction is

$$H_{int} = \sum_j \vec{J}_j \cdot \vec{A}(\vec{r}_j) = \sum_j q_j \left[-ig\vec{\sigma}^j \cdot \vec{k} \times \vec{A} + (\vec{p}^j + \vec{p}'^j) \cdot \vec{A} \right] \frac{e}{2m_q} \quad (1.68)$$

where eq_j is the charge of the j th quark; $\vec{p}^j$ and $\vec{p}'^j$ are the initial and final momenta, respectively, of the quark emitting a photon of momentum $\vec{k}$; $geq_1/2m_q$ represents the magnetic moment of a bound quark (g would be 1 for a pure Dirac particle); and m_q is the effective mass of the bound quark; $ge/2m_q$ is set equal to the proton's magnetic moment μ . The radiative widths may be determined by taking the matrix elements of this interaction between an N^* resonance state and the nucleon. The radiative decay width Γ_γ of a N^* resonance of spin J is given by

$$\Gamma_\gamma = \frac{k^2}{4\pi} \frac{m_N}{m_R} \frac{8}{2J+1} \left\{ |A_{1/2}|^2 + |A_{3/2}|^2 \right\} \quad (1.69)$$

where m_N and m_R are the mass of the nucleon and the N^* resonance, respectively, and the helicity amplitudes $A_{1/2}$ and $A_{3/2}$ are defined by

$$A_{3/2} = \langle N; m_z = +1/2 | H_{int} | N^*; m_z = 3/2 \rangle \quad (1.70)$$

$$A_{1/2} = \langle N; m_z = -1/2 | H_{int} | N^*; m_z = 1/2 \rangle. \quad (1.71)$$

Ignoring nonresonant background within the partial wave of the resonance, the contribution of the resonance to the integrated single-pion photoproduction cross section is

$$\begin{aligned} \sigma_T &= (2J+1) \frac{\pi}{k^2} \frac{\Gamma_\gamma \Gamma_\pi}{\Gamma^2} \\ &= \frac{m_N}{m_R} \frac{\Gamma_\pi}{\Gamma} \frac{2}{\Gamma} \left\{ |A_{1/2}|^2 + |A_{3/2}|^2 \right\} \end{aligned} \quad (1.72)$$

where Γ_π is the partial width for the resonance decay into πN .

Comparing eq. 1.72 with eq. 1.56, for the simple case of resonance dominance,

$$A_{\ell+}(\sqrt{s}=m_R) = \left[\frac{1}{\pi} \frac{\Gamma_\pi}{\Gamma} \frac{1}{2J+1} \frac{m_N}{m_R} \right]^{1/2} A_{1/2} \quad (1.73)$$

$$B_{\ell+}(\sqrt{s}=m_R) = \left[\frac{1}{\pi} \frac{\Gamma_\pi}{\Gamma} \frac{1}{2J+1} \frac{m_N}{m_R} \right]^{1/2} A_{3/2}. \quad (1.74)$$

Similar relations for $A_{(\ell+1)-}$ and $B_{(\ell+1)-}$ are obtained for a resonance with spin $J=\ell+1/2$ and parity of $(-1)^{\ell+1}$.

The radiative excitation of the $P_{33} \Delta(1232)$ resonance proceeds mainly via the magnetic M_{1+} transition; the electric E_{1+} is small but not zero; values for the ratio R of the electric quadrupole to the magnetic dipole amplitude obtained in different multipole analyses are listed in Table 1.5, the Particle Data Group quotes $R=(-1.3\pm0.5)\%$.³⁵ The value of R provides an important test on various models of hadrons as it is a measure of the tensor components of the effective forces between quarks. In the simple nonrelativistic quark model, both the nucleon and the $\Delta(1232)$ are s -states of three quarks and belong to the $[56, 0^+]$ multiplet. The transition $\Delta(1232) \leftrightarrow N\gamma$ is assumed to proceed simply via a quark spin-flip as given by the $\vec{\sigma}$ term in eq. 1.68. The naive spherical quark models predict zero for R . On the other hand, in the quark model of Isgur, Karl, and Koniuk³⁶, the color hyperfine interaction from the one-gluon-exchange potential produces d -waves in the nucleon and the $\Delta(1232)$ yielding $R=-0.7\%$. A recent calculation by K. Bermuth et al.³⁷ in the context of the cloudy bag model gives R values in the range of -1.8% to -3.3% for a bag radius of 1 to 0.6 fm.

Table 1.5 Ratio of the electric quadrupole to the magnetic dipole amplitude for the $\Delta(1232) \rightarrow N\gamma$ transition.

PWA	$A_{3/2}$	$A_{1/2}$	E_{1+}/M_{1+} (%)
Barbour et al. ²⁹	-271 ± 10	-142 ± 7	-2.37
Arai-Fujii ³⁰	-264 ± 2	-247 ± 1	-0.90
Crawford-Morton ³¹	-247 ± 10	-136 ± 6	-1.2
Davidson et al. ³⁴	-229 ± 6	-125 ± 3	-1.5 ± 0.2

CHAPTER TWO

EXPERIMENT

The experiment was performed in the east cave of the Pion and Particle Physics (P^3) channel at the Clinton P. Anderson Meson Facility (LAMPF) of the Los Alamos National Laboratory. The work covers the energy region of the $P_{33}(1232)$ and the $P_{11}(1440)$ (the Roper) resonances using seven pion beam momenta ranging from 301 to 625 MeV/c. The experimental setup is shown in Fig. 2.1. The major components of the setup are a polarized proton target, beam monitors, and two sets of neutron and gamma detectors. Not shown in the figure are the extensive shielding arrangement. Concrete blocks were located about the neutron detectors, and there was even a concrete roof to reduce the sources of background due to backscattering and skyshine.

The left-right asymmetry is given by

$$\epsilon = \frac{(\frac{d\sigma}{d\Omega})_L - (\frac{d\sigma}{d\Omega})_R}{(\frac{d\sigma}{d\Omega})_L + (\frac{d\sigma}{d\Omega})_R}$$

where $(\frac{d\sigma}{d\Omega})_L$ and $(\frac{d\sigma}{d\Omega})_R$ are the differential cross sections at the same scattering angle θ in the left- and right-hand directions with respect to the incident beam looking downstream. In this experiment the proton target is polarized perpendicular to the scattering plane.

Operationally, instead of moving the detectors left and right or setting up two identical left and right detectors, the target

Fig. 2.1 Schematic of the experimental setup. S_1 and S_2 are the beam-defining scintillation counters. N_A and N_B are two sets of 15 element neutron detectors, and G_A and G_B are two sets of the corresponding 15 element gamma detectors. VN_A and VN_B are charged particle veto counters in front of the neutron detectors, and VG_A and VG_B are veto counters for the gamma detectors. The 0° line is used for reference. The actual pion beam is bent by the magnetic field of the polarized target.

polarization is reversed to measure the left and the right cross sections. The target polarization reversal is equivalent to a 180 degree rotation about the incident beam axis, such that pions scattering to the right on a proton with spin 'up' is the same as scattering to the left with spin 'down'. Hence the left-right asymmetry is equivalent to the 'up'-'down' difference. Conveniently, since the two cross sections are measured with the same set of detectors, some multiplicative factors such as the detector efficiency and the detector solid angle cancel. Thus, only the number of good events observed with the 'up' and 'down' target polarization and the relative beam normalization between the two runs are needed for the determination of the left-right asymmetry. It is important to keep the experimental condition between the 'up' and the 'down' runs as close as possible. Reversing the target polarization every few hours is one of the ways to ensure that the data are taken under identical conditions.

SECTION 2.1 PION BEAMS

The heart of LAMPF is a half-mile-long linear accelerator (linac) that provides proton beams with energies up to 800 MeV. The maximum current is 1 mA. Three particle beams are available at LAMPF: H^+ (proton), H^- (hydrogen atom with an extra electron), and P^- (polarized H^-). The H^+ beam and either the H^- or P^- beam are simultaneously accelerated in the three-stage linac.

The first stage of the accelerator contains three injector systems, one for each particle type. Each injector has an ion source to produce positive or negative hydrogen ions and a Cockcroft-Walton

Fig. 2.2 LAMPF experimental areas. (From LAMPF handbook)

accelerator which accelerates the ions to 750 KeV. The H^+ ion source is pulsed at 120 Hz (8.33 msec period), each pulse is 0.57 msec long, giving a macro duty cycle of 6.84%. In the second stage, the beams are accelerated from 750 KeV to 100 MeV in a 62 meter Alvarez type drift tube linac operating at 201.25 MHz. The 201 MHz radio-frequency gives the microstructure of the beam; each of the 0.57 msec long macro pulses consists of 0.25 nsec micro-buckets every 5 nsec. The third stage of the accelerator is the side-coupled cavity waveguide linac; this section operating at 805 MHz accelerates the beams to 800 MeV. At the end of the linac, the beams enter a beam switchyard where they are magnetically separated and directed down four main beam lines leading to the experimental areas A, B, C, and the Los Alamos Neutron Scattering Center. The LAMPF experimental areas are shown in Fig. 2.2. Most of H^+ beam goes straight for use in Area A.

The 800 MeV H^+ beam impinging on the A-2 production target (6 cm graphite) produces positive and negative pions which are channeled into the Pion and Particle Physics (P^3) experimental areas. The P^3 channel provides a high intensity pion beam with momenta from 190 to 730 MeV/c with a 0.25 to 1% momentum bite. The P^3 channel consists of three bending magnets (dipoles), sixteen quadrupole magnets, and various horizontal and vertical adjustable slits to control the intensity and a degrader/absorber to cleanse the beam. The P^3 channel elements are shown in Fig. 2.3. The channel views the A-2 target at 20° and the channel's solid angle is defined by the horizontally and vertically adjustable slits, MS01 and MS02. The pion beam produced in the A-2 target is collected by the first quadrupole doublet and momentum analyzed by the first bending magnet BM01. The beam then passes

- LEGEND -
 QD • QUAD MAGNET
 MS • MOVEABLE JAWS
 BM • BENDING MAGNET
 BL • BEAM PLUG
 SV • VACUUM VALVE
 QM • QUAD MAGNET

Fig. 2.3 The Pion and Particle Physics (P^3) channel. (From LAMPF handbook)

through another quadrupole doublet and is brought to a momentum dispersed focus midway between the first and the second dipole, where a set of horizontal slits, MS03 A and B, define the channel's momentum acceptance. At this point a proton degrader of variable thickness may be inserted to provide a momentum separation between the π^+ and proton, which initially have the same momentum. The second bending magnet BM02 translates this momentum separation into a spatial separation; the degraded protons having lower momentum than the central pions are removed from the beam as they are bent into the proton absorber at the entrance to the third bending magnet. The degrader is not used for the π^- beam. After passing through a second set of doublet-dipole(BM02)-doublet combination, a second focus is formed in the center of the third bending magnet BM03. This magnet steers the beam to the east or west output leg, each consisting of four quadrupole magnets. The present experiment was staged in the east cave, and the last quadrupoles QM13 to QM16 provided the final beam focus on the proton target located 6 meters from the last quadrupole in the channel.

The magnet settings were determined by running a TRANSPORT beam optics program, which gave the magnetic field values for the desired beam tune for the nominal value of 350 MeV/c. A second program SHUNT using the TRANSPORT output scaled the momentum and calculated the channel magnet shunt settings for the scaled magnetic field values. Fine tuning was accomplished with the aid of two Beam Profile Monitors (BPMs). A beam profile monitor is a small multi-wire proportional chamber with x- and y-planes, each with 64 wires 2 mm apart. For each plane, 64 wires each with a current-to-voltage converter-amplifier followed by a temporary memory are sequentially scanned by an analog multiplexer and the resulting wave form is displayed on an oscilloscope

to give a histogram of the beam profile. BPM 1 was stationed 1.5 meters upstream of the target, and BPM 2 was 1.5 meters downstream of the target. BPM 2 was used only during beam tune (with the polarized target magnet off). With the help of the beam profile monitors, the waist of the beam was brought to the center of the target by adjusting the last four quadrupoles QM13 to QM16. The beam spot at the target was approximately 2 cm in diameter as measured with polaroid film placed against the target.

The polarized proton target magnet produced a 25 kGauss field over the volume of the target. Two small steering magnets located at the end of the channel were used to compensate for the bending of the beam in the target magnetic field. The X-steering magnet was used to shift the beam horizontally, and the Y-steering magnet corrected for any vertical missteering of the beam. The target magnetic field caused the beam to pass through the target at some angle with respect to the undeflected beam direction (when the target magnet is off). The true incident beam angle is important in determining the correct scattering angles. A summary of the beam characteristics is given in Table 2.1.

Several beam monitors were used to monitor the beam rate and beam drift during the experiment. The BPM 1 was always in the beam, the X and Y oscilloscope display of the beam profile allowed for a visual monitoring of the beam. Two muon telescopes each consisting of two small counters were installed above and below the beam pipe. The telescopes were sensitive to the muons from pions decaying in flight, and so the muon counters were useful as a relative beam monitor. The most important beam monitor was a pair of thin scintillation counters, S1 and S2, which were in the beam 60 cm upstream of the target, see

Table 2.1 Summary of beam characteristics.

p_{π} (MeV/c)	$\Delta p/p$ (%FWHM)	Rate ($10^6 \pi^-/\text{sec}$)	Duty Factor (%)	Inc Beam Angle (degrees)
301	0.25	0.38	6.84	27.4
316	0.25	0.38	6.84	26.6
427	0.30	0.95	10.24	19.5
471	0.40	0.88	10.24	17.7
547	0.50	0.87	10.24	15.3
586	0.65	0.58	6.82	14.3
625	1.00	0.88	10.24	13.4
625	0.70	0.57	6.82	13.4

Fig. 2.1. The S1 and S2 counters each measured 5 cm wide, 25 cm long, and 3 mm thick; the area of the counter was large enough to contain the beam. The counters had 50 cm long light guides and iron pipes around the photomultiplier tubes to shield the tubes from the target magnetic field. The coincidence between S1 and S2 gave the absolute pion rate listed in Table 2.1; the pion rate was limited by the counting rate of the S1 and S2 counters. The S1·S2 coincidence was used in the event trigger electronics. The requirement that each event had to have a good beam signal greatly reduced the accidental trigger rate. Since the asymmetry measurement is a relative measurement between 'up' and 'down' target polarization, the absolute S1·S2 efficiency is unimportant as long as the counters remain equally efficient. At the beam exit about 5 meters from the target, four large counters were used to monitor the beam rate and the beam drift from run to run. The dimensions of the counters are $76 \times 17 \times 0.6 \text{ cm}^3$. The two vertical ones

were in front and the two horizontal ones in the back. The beam rate was monitored by counting the coincidence between the front and the back counters. Also the comparison of the counts in the left and the right or the up and down counters were useful in monitoring the beam drift. The muon telescope counters and the beam exit counters were effective in monitoring the performance of the S1 and S2 counters.

SECTION 2.2 POLARIZED PROTON TARGET

The polarized proton target (PPT) was the most complex piece of apparatus in the experiment. It consisted of eight sub-systems: magnet, cryostat, ^4He system, ^3He system, thermometric system, target material, microwave electronics used for polarizing the protons, and nuclear magnetic resonance (NMR) electronics used for measuring the degree of polarization. The various target elements are shown in Fig. 2.4. A more extensive description may be found in the thesis of Tippens,³⁸ and also in the polarized proton target handbook.³⁹

To polarize the protons, the magnetic field $\vec{B}$ is introduced to split the two energy levels:

$$E_+ = -\mu_p B \text{ for proton spin parallel to } \vec{B}$$

$$\text{and } E_- = +\mu_p B \text{ for proton spin anti-parallel to } \vec{B},$$

where $\mu_p = 8.8043 \times 10^{-18} \text{ MeV Gauss}^{-1}$ is the magnetic moment of the proton. In thermal equilibrium, the two energy levels are populated according to Boltzmann statistics,

$$\langle + \rangle / \langle - \rangle = \exp(2\mu_p B / kT),$$

where k is the Boltzmann constant ($k = 8.6174 \times 10^{-11} \text{ MeV } ^\circ\text{K}^{-1}$), and T the

Fig. 2.4 Elements of the polarized proton target. (From PPT handbook)

temperature in degrees Kelvin. The "+" and "-" stand for the spin parallel and anti-parallel states, respectively. The polarization is

$$\frac{\langle + \rangle - \langle - \rangle}{\langle + \rangle + \langle - \rangle} = \tanh \left(\frac{\mu_p B}{kT} \right).$$

After inserting typical values for B (25 kGauss) and T (0.5 °K), one gets only 0.51% for the proton polarization. For electrons, however, because of their much larger magnetic dipole moment ($\mu_e = 5.7951 \times 10^{-15}$ MeV Gauss⁻¹), a similar calculation using the same values for B and T yields an electron polarization of 99.76%. The target polarization is enhanced with the Dynamic Nuclear Polarization technique using microwaves to transfer the polarization from the electrons to the protons. Bombardment with microwaves of the right frequency,

$$h\nu = 2\mu_e B \pm 2\mu_p B, \text{ (or } \nu = 70.06 \text{ GHz} \pm 106.4 \text{ MHz),}$$

simultaneously flips an electron and a proton spin. The electron quickly realigns and is ready to flip another proton. The proton, however, cannot flip back unless it interacts with the atomic lattice, so it stays put typically for several hours. Continued application of the microwaves can lead to an enhanced proton polarization comparable to the electron polarization. This process may be visualized with the aid of Fig. 2.5(a) where the four energy levels of an isolated electron-proton pair in a strong magnetic field are shown. In the 25 kGauss field at 1/2 °K, the electrons are almost completely polarized, and hence without the microwaves the electron-proton pairs occupy the two lowest states $|S_e, S_p\rangle = |-\rightarrow\rangle$ and $|--\rangle$ with nearly equal ratio (recall the 0.51% polarization for the protons). As shown in Fig. 2.5(b), the microwaves of $\nu = (70.04 - 0.106)$ or $(70.04 + 0.106)$ GHz induce $|--\rangle \leftrightarrow |++\rangle$ or $|-\rightarrow\rangle \leftrightarrow |+\rightarrow\rangle$ transitions. The electron-proton dipole-dipole interaction is responsible for the simultaneous spin flip of electron and proton.

Fig. 2.5 (a) Energy levels of an isolated electron-proton pair in a strong magnetic field. (b) Transitions induced by microwaves of the resonant frequencies.

In order to polarize the protons parallel to $\vec{B}$, the target is exposed to (70.04-0.106) GHz microwaves to induce the $|--\rangle \rightarrow |++\rangle$ transition to flip a proton spin up. The $| -+\rangle \leftrightarrow |+-\rangle$ transitions can not occur because they are off-resonance of the driving frequency. When the electron-lattice relaxation rate ($1/T_e = 10^3 \text{ sec}^{-1}$) is faster than the $|--\rangle \leftrightarrow |++\rangle$ transition rate which is determined by the microwave power, the electron can quickly realign from up to down to form a $| -+\rangle$ or a $|--\rangle$ state. The latter state can participate in another $|--\rangle \rightarrow |++\rangle$ transition. Continuing in this manner, a high nuclear polarization comparable to the electronic polarization can be achieved. The anti-parallel polarization can be similarly reached by bombarding with (70.04+0.106) GHz microwaves.

The polarized target magnet (PTM) was a conventional iron-core C frame magnet which produced a 25 kGauss field uniform to within $\Delta B/B = 0.02\%$ over the target volume. Because the frequency of the

microwaves used to polarize the target depends directly on the magnetic field, the uniformity of the magnetic field is important in ensuring optimal resonance condition over the entire target volume. The target magnet was oriented such that the incident beam passed through a 13 cm diameter cylindrical hole in the back of the yoke; due to the physical size of the return yoke the maximum accessible angle was 155°. The field map of the target magnet is shown in Fig. 2.6. The field map shows that the magnetic field in the 13 cm hole is small but has opposite sign with respect to the 25 kGauss field in the center of the target.

Fig. 2.6 Field map of the polarized target magnet.

The target cryostat housed the target cell and the ^3He - ^4He refrigeration system used to cool the target down to $1/2^\circ\text{K}$. The cryostat vacuum jacket was maintained below 10^{-6} torr in order to adequately insulate the cryogenic systems from the outside.

The ^3He system provided a bath of liquid ^3He at a temperature below 0.5°K to cool the target material. ^3He is expensive therefore it flowed in a closed loop. ^3He was supplied at a pressure somewhat less than an atmosphere, it was precooled by thermal contact with the outflowing ^4He gas, and liquified by thermal contact with the 2°K bath of liquid ^4He . The liquid ^3He flowed through a counter-current heat exchanger where it was cooled to about 1°K by the return ^3He gas stream from the target. The ^3He was further cooled to 0.5°K as it was pumped out at 100 torr or lower through a 12 inch diameter pipe to the ^3He pumps. The exhaust ^3He gas was cleaned by passing through an oil mist eliminator and liquid nitrogen trap as part of the recycling operation.

The ^4He system provided a bath of liquid ^4He (the evaporator) at a temperature of about 2°K to liquify the ^3He . Liquid ^4He at a temperature of 4°K was drawn from a dewar through a double walled evacuated fill tube into the cryostat. The ^4He liquid flowed through the separator valve in the separator whose main function was to separate the gas that was boiled off in the fill tube and to provide a reservoir of liquid to feed the evaporator. The separated gas was used to cool the heat shields and the first ^3He - ^4He heat exchanger before leaving the cryostat. Liquid ^4He flowed from the separator through the evaporator valve into the evaporator tank which operated at 2°K .

Evaporation of ^4He in the evaporator liquified ^3He gas. Gas from the evaporator flowed through the second ^3He - ^4He heat exchanger which cooled the ^3He gas to a temperature just above the liquification point. The ^4He gas was then pumped out of the cryostat through the evaporator pump-out line and passed to the ^4He pump.

Three types of thermometers, the vapor pressure of ^3He and ^4He , thermocouples, and carbon resistors, were used to monitor the cryostat performance and to measure the absolute target temperature during the calibration of the NMR system based on thermal equilibrium measurements. The thermocouples were used only during the initial cool-down. Three types of carbon resistance thermometers were used: 200 Ω , 1 watt resistors in the ^4He region, 100 Ω resistors in the ^3He region, and one 10 Ω resistor capable of measuring down to 0.4 $^\circ\text{K}$ near the target cell. At low temperatures both the thermal and electrical conductivities decrease (the resistance roughly follows the relationship: $\log R \propto 1/T$), so that the problem of Joule heating becomes critical. The absolute temperature T (in $^\circ\text{K}$) of the target near 1 $^\circ\text{K}$ was determined from the ^3He vapor pressure P (in torr) using the empirical formula

$$T = 0.7086 + 0.0435 \times P - 0.0010 \times P^2 \quad (0.71 \text{ } ^\circ\text{K} < T < 1.14 \text{ } ^\circ\text{K}).$$

The target material was ethylene glycol ($\text{C}_2\text{H}_4(\text{OH})_2$), except for the 301 and 316 MeV/c data runs, where 1,2-propanedial ($\text{C}_3\text{H}_6(\text{OH})_2$) was used. Both ethylene glycol and 1,2-propanediol were doped with Cr(V) to form paramagnetic complexes, they were prepared in the form of beads about 1 mm in diameter to improve thermal contact with the ^3He bath for dissipating the heat of the absorbed microwave radiation. The density

of 1,2-propanediol is 1.036 gm/cc and 1.109 gm/cc for ethylene glycol. The packing fraction of the beads is $2/3$, and so the partial density of free hydrogen for both target materials is about 0.07 gm/cc. The target beads were loaded in a cylindrical copper cavity with a 40 cc inner teflon target holding cell, a top and bottom flattened sphere 2.8 cm high and 4.6 cm in diameter.

The 70 GHz microwaves that were used to transfer the electron polarization to the protons were supplied by a watercooled 400 mW klystron and transmitted to the target through a waveguide. An appropriate change in the microwave frequency reversed the target polarization. The frequency may be measured by either of the two wavemeters, which are adjustable resonance cavities. The microwave power to the target was adjusted using an attenuator. Too little power will not saturate the required transition, and too much power will heat the beads reducing the electron polarization, which in turn will reduce the proton polarization.

The NMR (Nuclear Magnetic Resonance) technique is used to monitor and measure the target polarization which is the relative population of the two proton energy levels ($\pm\mu_p B$). The target is irradiated with radio frequency (RF) photons of energy $2\mu_p B$ (106.4 MHz) from the NMR coil in the target to induce the transition between the two energy levels. If the populations of the two levels are equal (zero polarization) the number of transitions up and down are equal so that no power is taken from or given to the RF field. If the lower energy level has a larger population (target polarized parallel to $\vec{B}$), then more upward than downward transitions are induced, thereby absorbing

power from the RF field and giving a dip in the RF signal as shown in Fig. 2.7(a). On the other hand, if the higher energy level has a larger population (target polarized anti-parallel to $\vec{B}$), the resulting net downward transitions emit energy to the RF field giving a hump in the RF signal as shown in Fig. 2.7(b).

The NMR electronics consists of an RF oscillator which drives two identical circuits one of which includes the NMR coil and the other a dummy used for background subtraction. The difference in signal between the two circuits is amplified by a differential amplifier and analyzed by a transient waveform recorder. The digitized NMR signal is then passed to the computer. The NMR curve is generated by sweeping the RF frequencies between 106.1 to 106.7 MHz. Since the NMR measurement obviously depolarizes the target, it is necessary to limit the RF power used so as to reduce the amount of depolarization to an insignificant level.

The target polarization is determined by measuring the size or the

Fig. 2.7 NMR power absorption curves. (a) Positive polarization absorb energy from RF field, and (b) negative polarization emit energy to RF field.

area of the NMR absorption curve which to the first order is proportional to the polarization. The proportionality factor is determined by measuring the signal area for a known polarization, namely at Thermal Equilibrium (TE). The measurement is carried out at a relatively high temperature of 1 °K so that the spin lattice relaxation time is not excessively long, and Boltzmann statistics can be used to determine the 'natural' polarization. The polarization at 1 °K with a 25 kGauss B field is 0.26% without the microwaves. The enhanced polarization P is determined from the relation

$$\frac{P}{A} = \frac{P_0}{A_0}$$

where A is the integrated area of the enhanced NMR signal, P_0 is the thermal equilibrium polarization and A_0 the corresponding area of the NMR signal. In the case of enhanced polarized signal the area is readily measured as the NMR signal is much larger than the background, see Fig. 2.8(a). The background is determined by fitting a smooth quadratic function to the flat regions on either sides of the NMR peak; the resulting background fit is the dashed line in the figure. The typical error in determining the area A is less than 0.5%. The accuracy of the target polarization is limited by the accuracy of the TE measurement. The basic difficulty is due to the smallness of the TE signal. This can be seen by comparing the 0.26% polarization of the TE signal with the 80% for the enhanced signal. In order to see the small TE signal, the gain of the NMR electronics is increased from 50 to 1000, a factor of 20 more than the enhanced signal setting. Also, in order to reduce the random noise in the NMR signal, 256 RF measurements are made and the average is used in the TE signal area determination. A typical thermal equilibrium signal is shown in Fig. 2.8(b). The TE

Fig. 2.8 Typical spectra produced by the NMR system. Dashed curves represent the background. (a) Enhanced spectrum for an 80% polarized target. (b) Typical thermal equilibrium signal.

signal area is determined after subtracting the baseline (the dashed line in the figure); it is obtained by varying the target magnetic field to move the nuclear resonance frequency outside the RF frequency range of the sweep. Both lower and higher magnetic field values are used to obtain the baseline measurement. As with the TE signal measurement, the baseline is the average of 256 measurements. A set of 12 to 20 TE peak signal minus the baseline is used to determine the average area of the TE signal. The standard deviation of the mean is 2 to 3%. After including the errors due to the nonlinearity of the NMR amplifier and the absolute target temperature used in the P_0 calculation, the systematic uncertainty in the measurement of the target polarization is estimated to be about 4%.

During the experiment, the target polarization was measured every five minutes, and the thermal equilibrium calibrations were performed every few days. The average up and down polarizations at all the experimental momenta are listed in Table 2.2. The fluctuation in the polarization from run to run is much smaller than the 4% systematic uncertainty, and hence it is not given in the table.

SECTION 2.3 DETECTORS

The objective of the experiment was to measure the left-right asymmetry in $\pi^-p \rightarrow \gamma n$. The experiment was designed to take full advantage of the two-body nature of the REX reaction. Since the kinematics are those of a two-body reaction, the energies of the final state gamma and neutron in each counter are well-defined and this translates to nice peaks in the energy or time-of-flight spectra

Table 2.2 Average target polarization.

P_n (MeV/c)	Ave $\uparrow$ Pol (%)	Ave $\downarrow$ Pol (%)
301	0.78	0.79
316	0.78	0.79
427	0.80	0.78
471	0.80	0.79
547	0.79	0.78
586	0.81	0.80
625	0.80	0.80
625	0.79	0.79

signalling the REX events. The two-body geometry of the emitted gamma and neutron is crucial in separating the REX from the CEX events which have identical neutral final state particles. The CEX reaction has a cross section 30 to 100 times larger than REX. The π^0 decay promptly yields two back-to-back gammas in the π^0 rest frame. These gammas are folded forward by the Lorentz transformation to the laboratory frame. A complete interception of CEX gammas requires a detector with a large area. However, the gammas from the REX reaction are spatially defined by the two-body kinematics. Thus the combined REX and CEX gamma event distribution in a large detector can be imagined as a REX peak superimposed on a broad distribution of CEX events. So with a careful placement of neutron and gamma counters, the REX events can be separated from the predominant CEX.

The gamma and neutron were detected in coincidence. Two angular regions were measured simultaneously using two matched neutron and

Fig. 2.9 (a) Schematic trajectory of the 471 MeV/c negative pion beam in the 25 kGauss field of the target magnet. (b) Laboratory production angle of the REX gamma and neutron for 471 MeV/c pions at 90° c.m.

gamma detector pairs. The main photon angles studied were centered around 90° and 110° center-of-mass. The angular placement of the gamma and neutron detectors were complicated because of the bending of the incident beam in the 25 kGauss field of the target magnet. The direction of the magnetic field pointed down throughout the experiment and caused the negative pion beams to be deflected to the right on passing through the target, see Fig. 2.9(a). Since the magnitude of the bending depends on the momentum of the beam, it is convenient to measure all laboratory angles with respect to the undeflected beam line (the 0° in the figure). The angle of the beam at the point of interaction in the target is called the incident beam angle θ_{inc} ; the central θ_{inc} ranged from 13.4° for the 625 MeV/c beam to 27.4° for the

301 MeV/c beam. Depending on the interaction point in the 40 cc target, the incident angle can vary as much as $\pm 2^\circ$ from the central value. The central incident beam angle was calculated by tracing the trajectory of the beam starting from about 2 meters upstream of the target and through the 13 cm hole in the target magnet return yoke and then to the 25 kGauss field at the center of the target. The beam was traced using 1 cm steps, a field map is shown in Fig. 2.6. The energy loss as the beam traversed through various medium, such as air, scintillators, and the target material, was included in the calculation of the beam trajectory. The calculation showed that the beam needed to be slightly deflected to the left of the center of the beam pipe in order to hit the center of the target, this was accomplished with the aid of the X-steering magnet. The actual laboratory scattering angle is the difference in angle between the detector floor angle and the incident beam angle. Inversely, the detector floor angle is determined by adding to or subtracting from the incident beam angle the scattering angle under study. For example, a REX measurement at 90° c.m. with a 471 MeV/c beam which has the central $\theta_{inc}=17.7$ degrees corresponds to the laboratory scattering angle of 44.9° for the neutron and 70.7° for the gamma. The neutron detector is set at $\theta_n^{DET}=44.9^\circ+17.7^\circ=62.6^\circ$ to the right of the beam, and the gamma detector at $\theta_\gamma^{DET}=70.7^\circ-17.7^\circ=53.0^\circ$ to the left of the beam as shown in Fig. 2.9(b). The neutron and gamma detector angle and radial distance from the target at all the measured points are listed in Table 2.3. The neutron detector defined the solid angle. Both the REX and CEX reactions use the same neutron laboratory scattering angle, but the center-of-mass scattering angles calculated from the neutron laboratory scattering angle are slightly different for the REX and the CEX reactions.

Table 2.3 The detector positions.

Pion Beam		n Detector		γ Detector		Scattering Angles		
P_{π} (MeV/c)	Θ_{inc} (deg)	R_n (cm)	Θ_n (deg)	R_{γ} (cm)	Θ_n (deg)	Θ_n^{LAB} (deg)	$\tilde{\Theta}_{\gamma}^{REX}$ (deg)	$\tilde{\Theta}_{\pi^0}^{CEX}$ (deg)
301	27.4	250	74.0	250	48.8	46.6	91.6	85.3
		250	101.0	250	12.9	73.6	47.0	32.5
316	26.6	250	74.0	250	48.8	47.4	89.5	83.6
		250	101.0	250	12.9	74.4	44.9	30.8
427	19.5	250	54.5	225	72.7	35.0	109.9	107.4
		250	64.8	250	52.5	45.3	90.2	86.7
471	17.7	250	52.2	225	73.2	34.5	110.0	107.9
		250	62.6	250	53.0	44.9	89.9	86.9
547	15.3	250	49.2	225	73.2	33.9	109.9	108.3
		250	59.4	250	53.4	44.1	90.0	87.7
586	14.3	250	47.9	225	73.2	33.6	110.0	108.6
		250	58.0	265	53.4	43.8	90.0	88.0
625	13.4	275	41.8	225	84.1	28.4	120.0	118.9
		250	56.8	270	53.4	53.4	90.0	88.2
		250	49.2	225	67.9	35.8	105.0	103.6
		250	64.8	310	40.2	51.4	74.8	72.5

SECTION 2.3.1 GAMMA DETECTORS

Gammas were detected in two multiscaler gamma detectors, each consisting of fifteen counters arranged in a 5 (vertical) by 3 (horizontal) rectangular array. Each counter was a 15×15 cm², 25 cm-deep lead-glass block optically isolated from its neighbors using aluminum foil, it was viewed by a 12.7 cm photomultiplier tube, either

an Amperex XP2041 or a RCA 4522. A set of 15 counters was tightly packed inside a $53 \times 81 \times 43$ cm³ aluminum box with interior μ -metal shielding, and placed in a 1.3 cm thick open-ended steel box which helped to shield the photomultiplier tubes from the magnetic field of the polarized target magnet. The iron box reduced the stray field to 5 - 10 Gauss at 1.8 meters from the target with the target magnet at 25 kGauss. In front of each detector were four double-ended charged particle veto counters. Each veto counter measured 81.3 cm high, 14.6 cm wide, and 1.3 cm thick. Horizontally the four counters covered 53 cm with 2 cm overlap between counters. The extra 4 cm veto counter coverage on either side of the gamma counters was needed to veto the charged particles coming in at steeper angles after being bent in the target magnetic field which extended out to about 50 cm from the center of the target. Fig. 2.10 shows the front and top view of the 15 lead-glass counters and the four veto counters in front of the gamma counters. The gamma counters were calibrated with a 150 MeV electron beam. Using a diffused electron beam across the front face of the gamma counters, the counter voltages were adjusted such that the electron pulse height peak from all the gamma counters was in the same channel. The 6 radiation length thick lead-glass Cerenkov counters were highly efficient in detecting gammas. From a separate study with an electron beam, it was found that electron initiated electromagnetic showers were usually contained in one counter except when electrons entered within 1 cm of the counter's edge. For the purpose of determining the REX gamma geometry, containment of the shower in one gamma counter is important. Single-gamma counter trigger events comprised 85% of the total events, and of the multiple gamma counter hit events, more than 90% were two hit events. For the two gamma hit

FRONT VIEW

TOP VIEW

Fig. 2.10 The fifteen element gamma detector with charged particle veto counters in front.

events the signals almost always came from two neighboring counters; it seems likely that these events are due to gammas striking near the edge of a counter and the gamma energy being shared by two adjacent counters. Only the single or the two-neighbor hit events were processed. For the two gamma hit events, the gamma pulse height information was used in resolving the gamma geometry. The energy resolution $\Delta E/E$ of the gamma counters was about 25%.

The gamma detectors sat on adjustable aluminum tables. The distance from the target could be set easily as the tables rolled on rails which ran radially from the target. The angular alignment was a little more difficult as the detectors were quite heavy. A rail switchyard system with a short inner switching track which rolled on the floor was used to move the detector to the desired angle where it was placed on the long rail on which the detector normally rested. The short and long rails pivoted independently with respect to the center of the target.

SECTION 2.3.2 NEUTRON DETECTORS

Neutrons were detected in two multiscaler neutron detectors, each consisting of 15 counters. Each counter was a cylindrical NE110 plastic scintillator 7.6 cm in diameter and 45.7 cm long, viewed by a 5 cm RCA 8575 photomultiplier tube. Shorter counters (19.1 cm) were used for the 301 and 316 MeV/c measurement at 50° c.m. for better timing resolution. The long counters were about 40% efficient for detecting 150 MeV neutrons and the short counters about 20%. Each set of 15 neutron counters was mounted using two aluminum support plates with

holes bored out to hold the counters at each ends. The holes in the plates were used to align the neutron counters. In front of each 15 counter set was a large double-ended charged particle veto counter measuring 62 cm high and 43 cm wide and 1.3 cm thick. Fig 2.11 shows the front and top view of a neutron counter set with the veto counter in front. The neutron counters were calibrated every few days using a ^{60}Co source by adjusting the counter voltages to set the middle of the Compton edge at 50 mV, which was the neutron discriminator threshold level. This effectively corresponds to a 4 MeV neutron.

The method for clean REX to CEX event separation was briefly discussed previously. The strategy involves exploiting the two-body REX geometry; for each neutron counter the REX gammas go to one of the 15 gamma counters, whereas the CEX gammas illuminate the entire front face of all 15 gamma counters. The neutron counters were arranged such that each counter was matched with one gamma counter as dictated by the REX kinematics. Fig. 2.12 shows the front of a gamma detector and the matched neutron counter array. The one-to-one neutron-to-gamma counter arrangement required using several different sets of neutron counter support plates with differing hole spacing for different incident energies and scattering angles. The radial distance of the detectors, between 2 and 3 meters, was chosen on the basis of a Monte Carlo study in which the REX to CEX ratio for the matched neutron-gamma counter pairs was optimized.

FRONT VIEW

TOP VIEW

Fig. 2.11 The fifteen element neutron detector with a large charged particle veto counter in front.

Fig. 2.12 One-to-one neutron-to-gamma counter matching for REX.

SECTION 2.4 ELECTRONICS

An event trigger was initiated by a three fold coincidence of a beam, a gamma, and a neutron signal. Two completely independent sets of electronics were used for each of the two neutron-gamma detectors pairs. The two sets of electronics were very similar and consisted of identical NIM modules. Because of the difference in the energy of the detected particles and the different types of photomultiplier tubes used in the gamma counters, the two sets of electronics differed in the discriminator threshold settings for the gammas, in timing delays, and

in neutron ADC gate width. A detailed description of the gamma, the neutron, and the master event coincidence electronic circuit is given in the following subsections.

SECTION 2.4.1 GAMMA COUNTER ELECTRONICS

Signals from the gamma detectors were processed as diagrammed in Fig. 2.13. The negative analog pulse from the gamma counters G1 to G15 was split into two equal pulses using a passive 1:1 splitter. One of the two pulses from the splitter was sent to an Analog-to-Digital Converter (ADC) CAMAC unit which measures the integrated charge (commonly refer to as the pulse height) of the input pulse. The G1-15 pulses going to the ADC units were delayed about 100 nsec so that they would arrive at the same time as the ADC gate signal which was generated after the master event coincidence had been made. The other pulse from the splitter went into a Constant Fraction Discriminator (CFD). The CFD threshold for the gamma pulses were set at -100 to -150 mV. The time delay between the input and the output pulse from the CFD is fixed regardless of the size of the input pulse; it gives a small jitter. The output pulses were used as the stop signal for the Time-to-Digital Converter (TDC) CAMAC unit and to activate a scaler for monitoring the singles rate. The output pulses from the 15 CFDs of both of the gamma detectors were fanned together to form ΣG ,

$$\Sigma G = G1 + G2 + \cdots + G15,$$

where "+" stands for the logical OR of the signals.

The 8 signals from the charge particle veto counters after passing through a discriminator triggered a fast two-fold coincidence between

Fig. 2.13 Gamma Electronics.

the up and down ends of the veto counters; the coincidence eliminated spurious signals due to tube noise. The veto counters were scaled to monitor the singles rates. The logic pulses produced from the up-down coincidences for each of the veto counters were logically ORed to form ΣVG ,

$$\Sigma VG = (VG1 \cdot VG2) + \dots + (VG7 \cdot VG8).$$

Finally, a NIM logic pulse signalling a Good Gamma in the gamma detector, given by the two-fold coincidence between the ΣG and logical NOT of the ΣVG , was sent to the master event coincidence and event trigger.

SECTION 2.4.2 NEUTRON COUNTER ELECTRONICS

Each neutron detector generated 15 individual neutron counter signals ($N1 - N15$) and 2 signals from the large double-ended charged particle veto counter ($VN1$ and $VN2$). The neutron counter electronics is shown in Fig. 2.14. The two signals from the veto counter passed a discriminator; the logic pulses from the discriminators were put into a coincidence. The output pulse from $VN = VN1 \cdot VN2$ which indicated a charged particle vetoed charged particle signals in the neutron counters. The negative analog pulse from the neutron counters was split into three using a passive 1:1:2 splitter. The 1/2 size pulse of each neutron counter was sent to a CF Discriminator with a 50 mV threshold. The output logic pulses from the CFD were connected to a scaler and a TDC CAMAC unit to measure the neutron time-of-flight from the target to the counter. The CFD logic pulses for all 15 neutron counters were logically ORed to form the ΣN ,

Fig. 2.14 Neutron Electronics.

$$\Sigma N = N1 + N2 + \dots + N15.$$

One of the 1/4 pulses was sent to an ADC CAMAC unit after about 100 nsec time delay using long low loss cables. The 1/4 pulses of all neutron counter were fanned together and used for vetoing some very large neutron counter signals from particles depositing a large energy in the neutron counters. Charged particle signals in the neutron counters were vetoed by the VN veto signals, but some very large pulses retriggered the CFD units at later time, 150 to 400 nsec after the initial trigger, when these double triggers would not be vetoed by the VN signals. So the purpose of the large neutron pulse veto circuit was to discriminate against the very large neutron pulses. The discriminator threshold was set at 300 mV, it generated a 400 nsec logic pulse which would veto the accidental retriggers which otherwise would pass as a good neutron signal. The NOT logic pulse from the large neutron (LN) pulse discriminator, after about 100 nsec delay, was put into a three-fold coincidence with the ΣN and the NOT of the VN pulses. The Good Neutron signal generated from the coincidence was then sent to the master event coincidence circuit.

SECTION 2.4.3 THE MASTER EVENT

The last component in a good event trigger was the Good Beam signal obtained by a coincidence between the S1 and S2 beam counters, see Fig. 2.15. The coincidence eliminated randoms and noise from the beam counters which were counting close to the limit. The logic pulses from the S1-S2 coincidence were used to drive a scaler and to generate a stop for a TDC CAMAC unit.

Fig. 2.15 Event Trigger Electronics.

A three-fold coincidence among a Good Beam, a Good Gamma, and a Good Neutron signalled a good event, see Fig. 2.15. The Good Gamma determined the time of the event; operationally, a very narrow (less than 5 nsec) Good Gamma logic pulse was used in the coincidence with the wider neutron and beam signals. The beam signal was about 20 nsec. Because the time difference between the prompt signals (gammas detected in neutron counters) and CEX neutrons ranged from 20 to 90 nanoseconds, the Good Neutron logic pulses were limited to one and half to twice the prompt and CEX neutron time difference.

The next logic level, which occurred before the event trigger signal was sent to the LAMPF Trigger Module which started the data acquisition, involved the event inhibit and the event busy. The Run Gate signal was generated by the LAMPF Gate Generator which signalled the readiness for data acquisition. The Run Gate was turned on at the start of a data run and turned off at the finish of a run. The Beam Gate which was given by the accelerator indicated the status of the beam. The beam gate signal was on during a beam macropulse. The coincidence of the Good Event signal and the Beam-Run signal further suppressed accidental event triggers. The output of the last coincidence started the gate generator which gave the event busy or the deadtime. The stop for the Event Busy was given by the output register which sent a logic pulse to the gate generator when the CAMAC read for the triggered event was finished. After the start of Event Busy, an event busy pulse was fed back to the coincidence of Good Event and Beam and Run Gates to suppress triggering on a new event until the current event had been processed. Lastly, the coincidence of Event Busy and NOT Computer Busy was used to generate the Event Trigger logic pulse

for the trigger. If the computer was busy, this placed the current event on hold until the computer was ready to process the event.

The TDC Starts for the start of time-of-flight measurements and the ADC Gates for the pulse height measurements were made at the same time as the start of the Event Busy. Because the neutron analog pulses were wider than the gamma pulses, wider Neutron ADC gates were generated with a logic unit.

SECTION 2.5 DATA AQUISITION

The LAMPF standard Q Data Acquisition software running on a DEC PDP 11/45 computer with the RSX-11D operating system were used to collect the data. The raw experimental informations from the ADC, TDC and scaler CAMAC modules were read by a microprogrammable branch driver (MBD) and transfered to the computer. The MED was programmed to carry out specific CAMAC operations in response to different "Event" triggers. For instance, when the main gamma-neutron coincidence events, called "Event 10" and "Event 11", were triggered, the MBD read the ADC and TDC CAMAC modules with the pulse height and time-of-flight information from 15 gamma and 15 neutron counters and the two TOF information from the S1-S2 and the 201 MHz RF signals. The CAMAC modules were reset after each read for the next event. The collected 62 16 bit data were passed to the computer, which recorded the data on magnetic tapes for later off-line analysis. Some of the raw data were processed in the "May Process" mode, where the computer processed the data only if it was not busy, so that fewer events would be lost due to computer dead time. The on-line display was used to monitor the

quality of the data and to ensure proper functioning of the counters and the other experimental apparatus. Another event type was the scaler read called "Event 6" and "Event 14"; they were triggered every 10 seconds to read 124 data words. "Events 9, 16, 17, and 18" were triggered every 5 minutes to read the 1024 channel NMR spectrum to measure the target polarization.

CHAPTER THREE

ANALYSIS

In the experiment we detected a neutron and a gamma in coincidence. These two particles can come from the following reactions:

- 1) $\pi^- + p \rightarrow \gamma + n$, radiative exchange (REX), our primary reaction.
- 2) $\pi^- + p \rightarrow \pi^0 + n \rightarrow 2\gamma + n$, charge exchange (CEX), our secondary reaction.
- 3) $\pi^- + X \rightarrow \gamma + n + Y$, a background, where X is an element other than hydrogen in the target and Y is the residual nucleus.
- 4) $\pi^- + X \rightarrow \pi^0 + n + Y \rightarrow 2\gamma + n + Y$, a background, where X and Y are defined as above.

The clearest experimental signal for REX is the sharp peak in the neutron time-of-flight (TOF) spectrum. At a given incident pion energy and angle, the energy of the emitted REX neutron is well determined from two-body kinematics. The energy of the neutron cannot be measured directly as it is neutral; it is only seen by detecting the recoiling proton or the alpha particle from the breakup of carbon in a plastic scintillator. The energy of the detected proton or the alpha particle is not directly correlated with that of the neutron since neither the scattering angle nor the mass of the residual nucleus is known. The energy of the neutron is determined by measuring its time of flight.

The REX neutrons having a well-defined energy show up as a sharp peak in the TOF spectrum. The neutrons from the background reaction, which are generally less energetic than the REX neutrons, are produced with a wide range of energies, and hence much of the background events are rejected with tight cuts on the neutron TOF spectra. The remaining background events under the REX peak except CEX are subtracted using the 'dummy' target data.

The CEX reaction yields two particles in the final state, namely π^0 and neutron. The π^0 promptly decays into two gammas and one of the two is detected in the experiment. At each angle, the CEX neutrons having a well-defined energy manifest themselves as a sharp peak in the neutron TOF spectrum. The energy of the detected CEX gamma depends on the opening angle of the gamma with respect to the parent π^0 direction; the maximum energy gammas are produced along the π^0 direction. The energies of the neutrons and gammas from the REX and CEX reactions are compared at each of the experimental data points in Table 3.1.

The major challenge in the data analysis is the clean separation between the REX and the CEX events. For most of our detector positions, the neutron time-of-flight information is of little help in the REX to CEX event separation because the REX to CEX neutron TOF difference is less than 1 nsec, which results in a substantial overlap of the two TOF peaks (see the last column of Table 3.1). Also, the gamma pulse height information is insufficient to discriminate between the REX and the CEX gammas because of the energy overlap between the REX and the CEX gammas and the poor energy resolution, about 20%, from the lead-glass gamma detector. CEX with a cross section that is 30 to 100 times larger than REX represents a potentially dangerous polarization-dependent background to REX.

Table 3.1 Energies of the neutrons and gammas from REX and CEX for the experimental data points. All the quantities are given in the laboratory frame. Δ_t represents the TOF difference per 1 meter between the REX and the CEX neutrons.

Inc	neutron	REX			CEX					Δ_t
		T_n	Θ_γ	T_γ	T_n	Θ_{π^0}	T_{π^0}	$T_\gamma^{\max}$	$T_\gamma^{\min}$	
p_π	Θ_n	MeV	deg	MeV	MeV	deg	MeV	MeV	MeV	ns/m
301	46.6	65.4	77.8	265.1	51.5	69.8	144.0	261.6	139.5	1.08
	73.6	20.6	37.7	309.9	8.8	25.0	186.8	306.9	160.9	8.37
316	47.4	67.8	75.3	276.4	54.0	67.7	155.2	273.6	145.1	1.01
	74.4	20.2	35.6	324.0	8.6	23.5	200.6	321.4	167.8	8.54
427	35.0	141.6	92.1	306.7	127.9	87.6	185.4	305.4	160.2	0.29
	45.3	106.1	72.3	342.2	92.8	67.4	220.6	342.2	177.8	0.45
471	34.5	163.0	90.8	326.8	149.5	86.9	205.3	326.3	170.1	0.23
	44.9	121.4	70.7	368.5	108.2	66.5	246.6	369.2	190.8	0.36
547	33.9	201.5	88.5	361.6	188.3	85.2	239.8	362.2	187.4	0.15
	44.1	150.4	68.7	412.7	137.7	65.3	290.5	414.4	212.7	0.24
586	33.6	222.3	87.5	378.6	209.4	84.5	256.6	379.6	195.8	0.13
	43.8	165.8	67.7	435.2	153.2	64.7	312.8	437.4	223.9	0.20
625	28.4	271.8	97.4	367.2	258.8	94.7	245.2	367.8	190.1	0.09
	35.8	228.2	81.2	410.8	215.4	78.5	288.6	412.5	211.8	0.12
	43.4	181.3	66.7	457.6	169.0	64.0	335.0	460.1	235.0	0.17
	51.4	133.9	53.4	505.1	121.8	50.6	382.2	508.2	258.6	0.27

The key to solving the REX to CEX event separation is the geometry of the two-body final state of REX as compared to the three-body of the CEX reaction. As discussed in the Chapter 2 under the experimental arrangement of the neutron counters, each neutron counter was uniquely matched with one gamma counter as dictated by the REX kinematics. From

this one-to-one matching, only one of the 15 gamma counters for each neutron counter is expected to yield REX events and the rest only CEX events. The events in the matched gamma counter are not all REX but contaminated with CEX events which must be subtracted. The magnitude of the CEX contamination is extrapolated using the event distribution from the other counters which contain only CEX events. For this process, it is useful to picture the event distribution from the 15 gamma counters with respect to one neutron counter at a time, where the event distribution shows a REX enhancement in one matched counter above a broad background of CEX events in all 15 counters.

Once the REX and the CEX events are separately extracted, the yield of good events from the spin-up and the spin-down data, minus the background, are used in the determination of the left-right asymmetry, which is a relative measurement. Many factors such as the acceptance and the efficiency of the counters cancel out if the conditions remain the same for the up and the down data, simplifying the calculation of the asymmetry. The data were taken in spin pairs, such as up-down, down-up, up-down, etc., reversing the target polarization every other 4 to 5 hour runs in order to minimize the effects of a long term drift in the experimental apparatus and to ensure that the up and the down data are taken under as similar conditions as possible.

SECTION 3.1 GOOD EVENT EXTRACTION

The REX and the CEX events are extracted from the neutron time-of-flight spectra after the GAMMA TIMING CORRECTION and the RF TIMING CUT AND CORRECTION procedures, discussed in the following

subsections, are used to "clean" up the TOF spectra. In these correction procedures, several passes are made through the recorded data. The experimentally measured quantities such as the neutron TOF and the gamma pulse height are histogrammed for each pass. The accumulated histograms, especially the neutron TOF spectra, are examined to study the effects of the last pass and to determine the new sets of cuts and parameters for the subsequent passes.

SECTION 3.1.1 GAMMA TIMING CORRECTION

All timing information was measured relative to the triggering gamma counter. In the experiment the 15 gamma signals fanned together and they were aligned to a few nanoseconds of each other. The gamma timing starts were realigned in the replay, this process of aligning the timing signals is called the Gamma Timing Correction. Clearly, this process is especially important for CEX, since the CEX gamma can go to any one of the 15 counters and any imperfection in the counter timing alignment will result in broad TOF peaks.

The $S1 \cdot S2$ signal was used in the gamma counter timing alignment. Recall that the $S1$ and $S2$ are the beam monitors stationed just upstream of the target, thus $S1 \cdot S2$ relates the TOF of the gamma from the target to the triggering gamma counter. The $S1 \cdot S2$ signal was part of the event trigger and hence common to all events. The TOF of the $S1 \cdot S2$ signal with respect to each of the 15 gamma counter timing starts (triggers) revealed how well the gamma counters are aligned in time and how much of the timing offsets are needed to correct for any misalignment. The $S1 \cdot S2$ TOF was histogrammed in 15 spectra for each of

the 15 gamma triggers and the centroid of the resulting S1·S2 TOF peaks were determined. In Fig. 3.1(a), the S1·S2 spectra with respect to the triggers from 3 of the 15 gamma counters, namely G3, G4, and G6, are plotted together. From the figure, it was determined that the G4 and G6 counters are misaligned by about 1.5 nsec earlier and later, respectively, than the G3 counter, and the G4 and G6 counters require, respectively, +1.5 and -1.5 nsec timing offsets to bring the S1·S2 TOF peaks together, see Fig. 3.1(b).

The centroids of the S1·S2 peaks were determined to 0.2 nsec accuracy. From the average value of the 15 centroids, the timing offsets were assigned to each of the gamma counters. Generally the timing offsets ranged from -1.5 to +1.5 nsec. The timing offsets or the timing correction parameters were used in subsequent passes where for each event an appropriate timing correction parameter was added to all timing information according to the identity of the triggering gamma counter. The effects of the gamma timing correction on a neutron TOF spectrum are demonstrated in Fig. 3.2; the CEX peak is definitely sharper after the gamma timing correction.

SECTION 3.1.2 RF TIMING CUT AND CORRECTION

The accidentals made up a few percent of the triggers and much of these were removed by requiring all the events to have a good beam signal, namely S1·S2 as well as a good RF timing signal. The 201 MHz pulsed time structure of the main proton beam gave the secondary pion beam the same time structure, where the pions were bunched every 5 nsec in 0.5 nsec wide buckets. A spectrum of the RF timing signal is shown

Fig. 3.1 S1-S2 timing spectra with respect to 3 different timing starts given by the G3, G4, and G6 gamma counters. (a) A misalignment of the gamma counters is reflected in the different positions of the S1-S2 TOF peaks. (b) After the Gamma Timing Correction the S1-S2 peaks line up.

Fig. 3.2 Typical neutron time-of-flight spectrum, (a) before, and (b) after the Gamma Timing Correction.

in Fig. 3.3(a). The spectrum displays the 201 MHz RF time structure, where the peaks come every 5 nsec. Due to prescaling where every tenth signal was sampled there are 10 complete or 9 complete and 2 partial peaks in the RF timing spectra. The peaks are identified with the beam buckets.

The demand that each trigger was due to a real beam particle implied that each RF timing signal belongs to one of the peaks in the RF timing spectrum. Cuts were applied to the RF timing signal to accept only those events which fall inside a 3.0 nsec window centered about the centroid of a RF peak, see Fig. 3.3(b). The events that fell outside the RF windows were rejected, since it was ambiguous to determine to which of two consecutive beam buckets the RF signal belonged, and hence it is more likely that they are accidentals. In Fig. 3.4, two neutron TOF spectra of those events which were accepted and rejected by the RF timing cuts are compared. Unlike the RF-accepted neutron TOF spectrum which shows two sharp peaks characteristic of the neutron TOF spectra, the RF-rejected TOF spectrum does not exhibit any peak at all, which supports the assumption that the rejected events are accidentals.

About 13% of the events were rejected from the 110° c.m. data as a result of the RF timing cut, whereas only about 5% was rejected from the 90° data. For the 110° c.m. data, the neutron detectors were stationed very close to the exiting beam which produced more background.

The size of the RF peak widths, which are somewhat larger than the 0.5 nsec width of the beam bucket, is due to microscopic timing jitter in the gamma counters. The timing jitter is due to different gamma conversion points in the triggering gamma counter and different light

Fig. 3.3 (a) RF timing spectrum showing RF peaks every 5 nsec. (b) A RF spectrum after the RF timing cut. The accidental triggers are removed.

Fig. 3.4 (a) Neutron TOF spectrum of the RF-accepted events.
(b) A neutron TOF spectrum of the RF-rejected events.

paths to the photomultiplier tube at the back of the counter. The timing jitter was corrected using the timing difference between the RF time and the centroid of the nearest RF peak. This RF timing correction was accomplished as follows: From the RF timing spectrum, the positions of all 10 or 11 centroids of the RF peaks were determined with a resolution of 0.2 nsec. For each event, which passed the RF timing cut such that the RF timing was within 1.5 nsec of some RF peak, the time difference between the centroid of the RF peak and the RF time was added to the neutron TOF and to the S1-S2 timing. This microscopic timing correction did have some modest smoothing and sharpening effects on a neutron TOF spectrum as one can see in Fig. 3.5.

In summary, the timing correction procedure consisted of aligning the gamma counters in time using the S1-S2 timing signals, and correcting for the microscopic timing jitters using the RF timing signals.

SECTION 3.1.3 GAMMA PULSE HEIGHT CUT

Gammas were detected in lead-glass Cerenkov counters with an energy resolution of about 25%. Although the energy resolution was insufficient to distinguish the REX gammas from the CEX gammas, which were generally less energetic than the REX gammas, the pulse height cuts on the gammas were very effective in reducing the background events.

The gamma pulse height cuts were determined by the following procedures: A pass through the data is made applying the CEX neutron TOF cuts, picking out the events from the CEX TOF peaks. The selected

Fig. 3.5 Neutron TOF spectrum, (a) before, and (b) after the RF Timing Correction.

events are CEX enhanced, a resulting gamma pulse height spectrum with a broad CEX peak is shown in Fig. 3.6(b). The broadness of the CEX peak is partly due to the 20% energy resolution of the counter, but mainly due to the spread in the CEX gamma energy. Most of the background gammas have lower energies than the CEX gammas, and this is apparent from comparing the gamma pulse height spectra before and after the CEX neutron TOF cuts, see Fig. 3.6. In this comparison, it is clear that the gammas from the background reactions mostly populate the low end of the energy spectrum, and as the energy increase the number of the background events decreases rapidly. However, there is a long flat background tail which persists to the highest energy. Since the asymmetry is a relative measurement between spin-up and spin-down scattering, the exact value of the pulse height cut is irrelevant as long as the experimental conditions remain the same. Because of the abundance of the CEX events, 70 to 80% of the events from the CEX peaks including the higher energy CEX gammas were kept and the rest was rejected along with the large number of low energy background events. The effects of the gamma pulse height cuts on a neutron TOF spectrum is shown in Fig. 3.7. Many of the gamma prompt events and 50 to 60% of the background events in the CEX peak region and to the right side of the CEX peak are removed as a result of the gamma pulse height cut.

SECTION 3.1.4 NEUTRON TIME-OF-FLIGHT CUT

The neutron time-of-flight spectrum after application of the timing correction and the gamma pulse height cuts is shown in Fig. 3.7(b). The neutron TOF spectrum displays the gamma prompt peak

Fig. 3.6 Gamma pulse height spectrum, (a) all events, (b) a subset of CEX enhanced events. The arrow indicates the lower limit of the gamma pulse height cut.

Fig. 3.7 Neutron TOF spectrum, (a) before, and (b) after the gamma pulse height cut. The REX and the CEX events are extracted using a tight cut about the sharp REX-CEX composite peak.

and the prominent CEX neutron peak above the broad background. The prompt peak, which comes earlier than the CEX peak, is the result of gammas being detected by the neutron counters. Except at 301 and 316 MeV/c where separate REX and CEX peaks are seen, e.g. Figs. 3.8 and 3.9, at the higher energies of the experiment the REX peak is hidden under the dominant CEX peak. The slightly more energetic REX neutrons populate the early time part of the CEX peak, see Fig. 3.9.

The width of the CEX neutron TOF peak ranged from 2.5 nsec for 625 MeV/c and 3.5 nsec for 427 MeV/c to 16 nsec for 301 MeV/c at 50° c.m.

Fig. 3.8 Neutron TOF spectrum at 316 MeV/c, 50° c.m. Due to the differences in the time-of-flight, distinct REX and CEX peaks are seen. The REX and CEX events are independently extracted with separate TOF cuts as indicated by the arrows about the REX and CEX peaks.

At 301 MeV/c, as a result of the bending of the low energy incident π^- beam in the 25 kGauss magnetic field of the polarized target, there is about 4° variation in the scattering angle depending on the interaction point in the target. The 4° spread in scattering angle (about 32.5° c.m.) gives the CEX neutron a velocity (v/c) range of 0.12 to 0.15, which gives the TOF difference of 13 nsec for a 235 cm flight path to the neutron counter.

The REX and CEX events were extracted by applying cuts about the prominent peaks in the neutron TOF spectra. At 301 and 316 MeV/c where

Fig. 3.9 Neutron TOF spectrum at 316 MeV/c, 90° c.m. Although the REX and the CEX peaks overlap somewhat, separate REX and CEX time-of-flight cuts are made as indicated by the arrows. The TOF spectrum of the events from the REX enhanced matched gamma counter is represented by the heavy solid bars.

the difference in the neutron TOF between REX and CEX gives rise to two distinct REX and CEX peaks, two separate sets of REX and CEX cuts were applied to extract the good events; for example in Figs. 3.8 and 3.9, the arrows about the REX and the CEX peaks indicate the limits of the REX and the CEX cuts. At the higher energies of the experiment, the REX and the CEX events were extracted together with cuts about the composite REX and CEX TOF peaks. Because the small number of the REX neutrons occupy the early part of the REX-CEX peak, the left limits were applied loosely, a few channels before the rapidly increasing left edge of the CEX peak, as shown in Fig. 3.7(b).

Again because the asymmetry measurements are relative, the exact placement of the cuts are irrelevant, but for each neutron counter the very same cuts were applied to the spin-up, the spin-down, and the background runs in order to ensure the same efficiency and acceptance for all the runs. In fact the asymmetry should be independent of any specific sets of cuts used and this was studied using several different TOF cuts, this is detailed in Sections 3.3 and 3.4. Especially for the composite REX-CEX TOF peaks, narrow TOF cuts of varying widths were used to remove some fraction of the CEX events while retaining most of the REX events knowing that the REX neutrons come slightly early in time than the CEX neutrons.

In summary, at 301 and 316 MeV/c the REX and the CEX events were separately extracted using the neutrons TOF cuts. At the higher energies, the REX and the CEX events had to be selected together due to insufficient difference in the neutron TOF between the two reactions. Later, the REX and the CEX events were separated using the geometry of the gammas in relation to the neutrons.

SECTION 3.1.5 BACKGROUND SUBTRACTION

Pions that are scattered by the nonhydrogenic parts of the polarized target, the carbon and the oxygen atoms in the target material, the target cell, the steel container, and ^3He contribute to the background. Much of the background events were removed by the neutron TOF and the gamma pulse height cuts. The remaining background events were subtracted using the 'dummy' target data obtained in separate runs with teflon (C_2F_4) beads as target in place of the ethylene glycol ($\text{C}_2\text{H}_6\text{O}_2$) beads. For the 301 and 316 MeV/c data, 1,2-propanedial ($\text{C}_3\text{H}_8\text{O}_2$) was used as the polarized proton target and carbon beads were used as the background target.

The solid curve in Fig. 3.10(a) is a neutron TOF spectrum using the teflon target, while the bar graph is from the polarized target. The composite REX and CEX peak sits atop a broad background which is removed by subtracting the normalized TOF spectrum taken with the background target; after the background subtraction, only the REX and CEX events remain as one can see from Fig. 3.10(b).

In the asymmetry measurement, only the relative normalization between the spin-up, the spin-down, and the background runs are needed; the background run is essentially treated as if it is another target spin configuration. For convenience, the spin-down and the background runs are normalized to the spin-up runs. The spin-down to spin-up normalization is given by the ratio of the number of incident beam particles obtained from the S1-S2 beam monitor in the spin-up and spin-down runs.

The background normalization was accomplished by comparing the height of the neutron TOF spectra from the background target runs to

Fig. 3.10 Neutron TOF spectra. (a) Polarized target data (bar graph) and the 'dummy' background target (solid curve). (b) Difference between the polarized target and the normalized background data. This is a bin-by-bin subtraction of the bar and solid graphs of figure (a).

the polarized target runs in the region of the broad background away from the CEX peak, such that the broad background would be exactly cancelled by the normalized spectra from the background runs. By normalizing with respect to the neutron TOF spectra, many factors such as differences in target density and atomic makeup and beam between the background and the polarized runs are automatically included, since the TOF spectra reflect the actual experimental conditions of the runs. Two multiplicative factors were used to normalize the background. One was simply the beam normalization factor between the background run and the polarized target run, and the other was the background target density factor (BTDF) which relates the differences in the density of the background target with the polarized proton target. By separating out the beam normalization, one single BTDF can be used for the 427 to 625 MeV/c runs which used ethylene glycol/teflon targets, and another BTDF value for the 301 and 316 MeV/c propanediol/carbon target runs. The BTDF was empirically determined by comparing the number of events in the off-time region from the background and the polarized target data; for the 427 to 625 MeV/c data, BTDF of 0.80 ± 0.04 was obtained, and at 301 and 316 MeV/c 0.86 ± 0.05 was found. The BTDF values are the average of the individual BTDF obtained for each neutron counters, the error in BTDF is the standard deviation of the mean. The background density factor being less than 1 indicates that the background target contains more of the nonhydrogenic materials which produce the background events than the polarized target. The subtracted neutron TOF spectrum obtained by bin-by-bin subtraction of a normalized background target spectrum from a polarized target spectrum is shown in Fig. 3.10(b). As one can see from the figure, the off-time region to the right of the CEX peak is flat and the number of events

fluctuates about zero, indicating that the background was correctly subtracted, the peak contains only the REX and the CEX events.

SECTION 3.2 STATISTICAL AND SYSTEMATIC UNCERTAINTIES

The left-right asymmetry, A_N , is given by the expression

$$A_N = \frac{1}{P_t} \frac{N\uparrow - N\downarrow}{N\uparrow + N\downarrow - 2B},$$

where P_t is the known target polarization, $N\uparrow$ is the yield of good events from the spin-up target, and the $N\downarrow$ is the normalized yield from the spin-down target, and the B is the normalized background yield.

The observed REX or CEX asymmetry at each neutron counter is calculated with the yields from the neutron TOF spectra with tight cuts about the sharp peaks which most clearly identify the events as REX or CEX. Each neutron detector is divided into 3 contiguous angular bins as defined by 3 columns of 5 neutron counters each.

The uncertainty in the asymmetry is calculated using the usual propagation of errors method. A_N being a function of the independent variables P_t , $N\uparrow$, $N\downarrow$, and B , the uncertainty in A_N is given by

$$\sigma_{A_N} = \left[\left(\frac{\partial A_N}{\partial P_t} \right)^2 \sigma_{P_t}^2 + \left(\frac{\partial A_N}{\partial N\uparrow} \right)^2 \sigma_{N\uparrow}^2 + \left(\frac{\partial A_N}{\partial N\downarrow} \right)^2 \sigma_{N\downarrow}^2 + \left(\frac{\partial A_N}{\partial B} \right)^2 \sigma_B^2 \right]^{1/2},$$

$$\text{where } \frac{\partial A_N}{\partial P_t} = \frac{A_N}{P_t},$$

$$\frac{\partial A_N}{\partial N\uparrow} = \frac{1}{P_t} \frac{2(N\downarrow - B)}{(N\uparrow + N\downarrow - 2B)^2},$$

$$\frac{\partial A_N}{\partial N\downarrow} = -\frac{1}{P_t} \frac{2(N\uparrow - B)}{(N\uparrow + N\downarrow - 2B)^2},$$

$$\frac{\partial A_N}{\partial B} = \frac{1}{P_t} \frac{2(N\uparrow - N\downarrow)}{(N\uparrow + N\downarrow - 2B)^2},$$

$$\text{so that } \sigma_{A_N} = \left[A_N^2 \left(\frac{\sigma_{P_t}}{P_t} \right)^2 + \frac{A_N^2}{4P_t(N\uparrow - N\downarrow)^2} \left\{ (N\downarrow - B)^2 \sigma_{N\uparrow}^2 + (N\uparrow - B)^2 \sigma_{N\downarrow}^2 + (N\uparrow - N\downarrow)^2 \sigma_B^2 \right\} \right]^{1/2}.$$

In the last equation, the first term in the right hand side represents the systematic error in the experiment coming from the 4% uncertainty in the calibration of the NMR system which measured the target polarization. The rest of the contributions to σ_{A_N} are due to statistical fluctuation; using the Poisson probability function, the variance of $N\uparrow$, $N\downarrow$ and B are,

$$\begin{aligned} \sigma_{N\uparrow}^2 &= N\uparrow, \\ \sigma_{N\downarrow}^2 &= N\downarrow, \\ \text{and } \sigma_B^2 &= B. \end{aligned}$$

Because the uncertainty in the polarization scales the asymmetry values by $(1 \pm \Delta P_t / P_t)$, only the statistical errors are given.

SECTION 3.3 REX EVENTS

The two-body final state of the REX reaction allows us to impose angular correlation and the coplanarity requirements between the detected gamma and neutron for extracting the REX events; these requirements were incorporated in the one-to-one neutron-to-gamma counter arrangement used in the experiment. The REX gamma is kinematically restricted to just one gamma counter for a neutron detected at a particular neutron counter. The CEX gammas illuminate the entire front face of the 15 gamma counter array. Thus for each

Fig. 3.11 A 2-dimensional neutron-gamma counter relative event density distribution. The completely shaded diagonal elements represent the events from the matched neutron-gamma counter pairs. The partially shaded elements are the events from the neighboring gamma counters.

neutron counter, the one matched gamma counter yields the REX events along with some "background" CEX events which need to be subtracted.

A scatter plot displaying the event distribution for all neutron-gamma counter combinations was constructed using the extracted good events after the application of the neutron TOF and the gamma pulse height cuts. The counters were numbered such that neutron counter 1 corresponded to gamma counter 1, etc. according to the neutron to gamma counter matching as prescribed by the REX kinematics. Fig. 3.11 displays a two-dimensional (2D) histogram of the number of events from each neutron-gamma counter combination; the REX events

ideally lie only on the diagonal which represent the matched counter pairs, whereas the CEX events are distributed over the entire histogram. Because the REX neutron-gamma counter matching was not perfect, there is a tiny REX yield in adjacent counters.

Only the events that lie on the diagonal of the 2D histogram were used in the REX analysis. A 2D histogram of the CEX event distribution was produced by a Monte Carlo code. From this CEX distribution, the number of background CEX events in the diagonal elements of the 2D histogram was determined by extrapolating the number of CEX events from the nondiagonal and nonadjacent counters. For example, shown in

Fig. 3.12 A sample event distribution in the 15 gamma counters for one (counter 10) of the neutron counters. The solid curve is the Monte Carlo generated CEX distribution normalized to fit the data. The shaded boxes represent the REX events above the background CEX in the coplanar and the adjacent counters.

Fig. 3.12 is the event distribution for neutron counter 10 at 625 MeV/c 90° c.m.; the spin-up and the spin-down distributions have been added. The nonhydrogenic background was first subtracted leaving only the REX and the CEX events. The Monte Carlo generated CEX distribution was normalized to fit the noncoplanar and nonadjacent data; in Fig. 3.12, the matched gamma counter 10 and the adjacent counters 5 and 15 were not used in the fitting. As one can see from the figure, there is good agreement between the experiment and the Monte Carlo generated CEX event distribution. From the figure, one can see that out of 275 events in the matched counter, about 90 events are CEX events and the rest are REX.

	N=1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
G															
1	158	72	43	44	34	50	87	45	41	22	51	55	55	23	32
2	77	142	61	60	39	53	96	60	41	37	62	67	56	39	49
3	57	56	128	86	56	48	74	77	68	54	49	41	60	51	43
4	46	59	49	124	55	47	77	50	74	64	48	52	73	67	69
5	36	49	42	63	138	28	38	39	48	88	13	35	55	56	78
6	94	71	53	39	25	141	68	70	45	22	76	81	88	38	36
7	86	104	61	48	37	78	168	79	58	50	74	64	105	64	52
8	58	55	64	64	58	48	102	137	68	78	64	79	104	71	75
9	60	61	55	87	58	41	61	89	138	96	41	64	83	100	79
10	30	38	44	67	91	27	44	52	77	150	22	54	74	91	107
11	65	53	48	38	20	98	85	52	40	23	186	102	85	47	33
12	56	71	47	63	39	78	95	69	58	46	89	175	119	55	65
13	47	55	42	53	47	41	88	95	88	64	66	79	184	94	67
14	30	40	40	76	49	45	61	84	92	79	55	69	101	154	110
15	28	44	56	66	64	31	58	53	74	115	33	61	96	90	189

Fig. 3.13 N_{ij} matrix for the 625 MeV/c, 90° data with neutron TOF cut 'B'.

The REX asymmetry is calculated using a modified version of the expression for A_N , which explicitly shows the subtraction of the CEX contamination in the coplanar neutron-gamma counter using the Monte Carlo generated CEX event distribution. The asymmetry observed in neutron counter i is given by;

$$A_N^{\text{REX}}(i) = \frac{1}{P_t} \frac{N_{\uparrow}^{\text{REX}}(i) - N_{\downarrow}^{\text{REX}}(i)}{N_{\uparrow}^{\text{REX}}(i) + N_{\downarrow}^{\text{REX}}(i)}$$

$$\text{where } N_{\uparrow}^{\text{REX}}(i) = N_{\uparrow ii} - B_{ii} - \frac{1}{J} \sum_j f_{ij} [N_{\uparrow ij} - B_{ij}],$$

$$\text{and } N_{\downarrow}^{\text{REX}}(i) = N_{\downarrow ii} - B_{ii} - \frac{1}{J} \sum_j f_{ij} [N_{\downarrow ij} - B_{ij}],$$

$N_{\uparrow ij}$, $N_{\downarrow ij}$ and B_{ij} denote the number of spin-up, spin-down and background events, respectively, detected in neutron counter i and gamma counter j . An example of a $N_{\uparrow ij}$ matrix is displayed in Fig. 3.13. The true number of REX events is determined by subtracting the nonhydrogenic background events and the CEX contamination from the REX candidate events in the matched coplanar counter. The contamination of the CEX peak is determined by the sum term in the equation which gives the average number of CEX events as extrapolated from the number of CEX events, $[N_{ij} - B_{ij}]$, in the noncoplanar and the nonadjacent counters using the Monte Carlo CEX event distribution factor, f_{ij} , which relates the number of CEX events expected in gamma counter j to the matched gamma counter i with neutrons detected at counter i . The factor f_{ij} is given by the ratio of the CEX event numbers in the matched gamma counter i with respect to the gamma counter j as determined from the Monte Carlo CEX event distribution; a sample Monte Carlo CEX event matrix is shown in Fig. 3.14.

	N=1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
G															
1	1210	1103	946	750	543	1038	1011	848	684	479	831	783	686	527	396
2	1231	1222	1240	963	745	1030	1044	1051	860	716	806	837	784	681	535
3	970	1239	1221	1239	970	870	1046	1051	1046	870	701	814	841	814	701
4	745	963	1123	1222	1231	716	860	1047	1044	1030	535	681	810	837	806
5	543	750	993	1103	1210	479	684	791	1011	1038	396	527	673	783	831
6	1294	1186	953	755	522	1270	1177	980	774	536	1096	1047	881	712	509
7	1318	1303	1286	970	787	1293	1290	1297	988	772	1078	1090	1084	892	731
8	976	1300	1311	1300	976	997	1293	1291	1293	997	912	1093	1104	1093	912
9	787	970	1200	1303	1318	772	988	1170	1290	1293	731	892	1081	1090	1078
10	522	755	1012	1186	1294	536	774	1051	1177	1270	509	712	850	1047	1096
11	1173	1121	921	688	535	1354	1241	973	774	540	1312	1229	994	782	543
12	1127	1201	1126	953	606	1362	1358	1329	973	821	1329	1342	1330	1021	785
13	964	1132	1211	1132	964	1011	1361	1380	1361	1011	1034	1331	1347	1331	1034
14	686	953	1142	1201	1127	821	973	1263	1358	1362	785	1021	1243	1342	1329
15	535	688	906	1121	1173	540	774	1028	1241	1354	543	782	1055	1229	1312

Fig. 3.14 A Monte Carlo generated CEX event distribution matrix at 625 MeV/c, 90° .

For every data set, three REX asymmetry values were calculated using three different neutron TOF cuts. For the 12 data points at 625 MeV/c ranging in $\cos\theta_\gamma$ from 0.32 to -0.55, three REX asymmetry values were calculated for TOF cut 'A', 'B', and 'C', see Table 3.2. The TOF cut 'A' is the widest with a width of 2.5 ns, cut 'B' is slightly narrower with a width of 2.0 ns and cut 'C' is the narrowest with a width of 1.5 ns. The narrow TOF cuts are used to remove a fraction of the CEX events while retaining most of the REX events thus improving the REX to CEX event ratio. The average REX to CEX event ratio at 625 MeV/c with the TOF cut 'A' is 2.2 (1.3 at the angular region about $\cos\theta = 0.26$) and the ratio improves to 3.5 (1.9 at $\cos\theta = 0.26$) for the TOF cut 'C'. Even though the cut 'C' has the best REX to CEX event ratio,

usually the compromise cut 'B' is preferred since compared to cut 'A' there are fewer background CEX events, and because the cut 'C' with its lack of CEX events causes the CEX event extrapolation and the subtraction of CEX contamination process to become less reliable than the cut 'B'. As one can see from the table, the asymmetry values are very consistent for different TOF cuts.

The greatest difficulty in obtaining the REX asymmetry is the reliable subtraction of the CEX contamination from the good events to yield the true number of REX events. The CEX events represent a large polarization dependent background, especially dangerous at those points

Table 3.2 625 MeV/c REX asymmetry values from three different neutron TOF cuts.

$\tilde{\theta}_Y$	Cut 'A'		Cut 'B'		Cut 'C'		Events in Cut 'B'		
	A_N	σ_{A_N}	A_N	σ_{A_N}	A_N	σ_{A_N}	REX	CEX	Backgrd
71.4	-.23	.06	-.21	.06	-.22	.06	962	676	63
74.8	-.11	.07	-.13	.07	-.14	.06	836	612	74
78.1	-.14	.06	-.17	.06	-.17	.06	873	490	55
86.4	-.12	.06	-.14	.05	-.20	.05	982	500	47
90.0	-.19	.06	-.17	.06	-.19	.05	885	420	30
93.6	-.27	.06	-.21	.06	-.23	.05	873	372	56
101.2	-.16	.06	-.19	.06	-.21	.06	841	351	53
105.0	-.26	.06	-.25	.06	-.24	.06	677	231	45
108.7	-.14	.06	-.14	.06	-.16	.06	758	241	35
116.5	-.22	.07	-.20	.06	-.21	.07	559	155	23
120.0	-.22	.07	-.23	.07	-.19	.07	432	90	9
123.3	-.20	.07	-.21	.07	-.19	.07	497	97	23

where the REX and the CEX asymmetries differ in sign. The reliability of the CEX subtraction and the effects on the REX asymmetry values were examined by varying the number of CEX events by $\pm 10\%$. Recall that the number of the CEX events in the matched counter is determined by fitting the Monte Carlo generated CEX event distribution to the noncoplanar and the nonadjacent data. For example, shown in Fig. 3.15 is an event distribution for neutron counter 10 at 625 MeV/c 90° , where the spin-up and -down events were added and the nonhydrogenic background subtracted leaving only the REX and the CEX events; REX yielding counter #10 and the left-right adjacent counters 5 and 15 and the up adjacent counter 9 were not used in the fitting. The fitted Monte Carlo generated CEX distribution is represented by the solid curve and the dashed curves above and below mark the $\pm 10\%$ deviation from the best fit. The normalization factor used in fitting the Monte Carlo CEX event distribution to the experimental data is precisely the mean of the ratios of the experimental numbers of events with respect to the Monte Carlo numbers of events found in the noncoplanar and nonadjacent counters. The uncertainty in the mean was determined to be 5 to 10%. The effect on the REX asymmetry values due to this 10% uncertainty in the Monte Carlo normalization was explored by recalculating the asymmetries with a $\pm 10\%$ variation in the number of CEX events obtained from the best fit. The 625 MeV/c REX asymmetry values for the 10% variation in the CEX event numbers are listed in Table 3.3. As one can see in the table, the asymmetries are consistent within the calculated statistical errors.

The REX asymmetries were not affected by the uncertainty in the background normalization due to the very small number of background

Fig. 3.15 Event distribution in the gamma counters for neutron counter #10 at 625 MeV/c, 90°. The solid curve is the Monte Carlo generated CEX distribution normalized to fit the CEX data, and the dashed curves represent the $\pm 10\%$ uncertainty in the normalization.

Table 3.3 625 MeV/c REX asymmetry values showing the effect of varying the number of background CEX events by $\pm 10\%$.

$\tilde{\theta}_Y$	10% less CEX		The best fit		10% more CEX	
	A_N	σ_{A_N}	A_N	σ_{A_N}	A_N	σ_{A_N}
71.4	-.14	.06	-.21	.06	-.29	.07
74.8	-.06	.06	-.13	.07	-.21	.07
78.1	-.11	.06	-.17	.06	-.23	.06
86.4	-.10	.05	-.14	.05	-.19	.06
90.0	-.14	.05	-.17	.06	-.22	.06
93.6	-.18	.05	-.21	.06	-.25	.06
101.2	-.17	.05	-.19	.06	-.22	.06
105.0	-.23	.06	-.25	.06	-.27	.06
108.7	-.12	.05	-.14	.06	-.15	.06
116.5	-.19	.06	-.20	.06	-.22	.07
120.0	-.21	.07	-.23	.07	-.24	.07
123.3	-.19	.06	-.21	.07	-.22	.07

events remaining after the application of neutron TOF and gamma pulse height cuts.

SECTION 3.4 CEX EVENTS

The CEX events were selected from the neutron TOF spectra by applying tight cuts about the CEX neutron peak. The selected events are mostly CEX events with a small number of REX events.

In calculating the CEX asymmetries, one must use extra care not to include any REX events especially at those points where the CEX and the REX asymmetries differ in sign. With our experimental arrangement of the counters, the coplanar neutron-gamma counter pairs contain more than 80% of all REX events with the rest distributed in the adjacent (neighboring) gamma counters. Among the adjacent counters, the left and right adjacent counters have more REX events than the up and down adjacent counters. In calculating the final CEX asymmetry values, only the events from the noncoplanar and the nonadjacent counters were used. To investigate the size of the statistical errors and to see the influence the REX events have on the CEX asymmetries, additional events in the adjacent and the coplanar counter were considered. The numbers in the first line of Table 3.4 were obtained using the noncoplanar, nonadjacent events, the numbers in the second line were obtained by adding adjacent events to the events used in the first line and the third line numbers were obtained by adding coplanar events to the events used in the second line. The smaller size of the errors in the second and the third line reflects the increased number of CEX events from including the adjacent and the coplanar counters. The REX asymmetries are negative at these angles at 625 MeV/c and a definite REX influence is evident in comparing the asymmetry values from the first and the third lines of each angle.

Table 3.4 CEX asymmetries and the number of events at 625 MeV/c when the REX contaminated adjacent and the coplanar counters are included.

$\tilde{\theta}_{\pi^0}$	$A_N \pm \sigma_{A_N}$	Number of Events			
		CEX $\uparrow$	CEX $\downarrow$	Backgrd	
68.9	1.056 ± 0.017	5460	482	698	
	1.039 ± 0.014	7470	723	855	add adj.
	0.929 ± 0.013	8628	1314	930	add copl.
72.5	1.015 ± 0.017	5394	585	630	
	0.985 ± 0.013	8010	990	891	add adj.
	0.901 ± 0.012	9040	1514	968	add copl.
75.9	0.998 ± 0.018	5284	612	713	
	0.960 ± 0.015	6918	932	857	add adj.
	0.865 ± 0.013	7864	1465	919	add copl.
99.7	0.403 ± 0.032	1740	893	764	
	0.404 ± 0.026	2465	1270	945	add adj.
	0.311 ± 0.021	3117	1888	1025	add copl.
103.6	0.308 ± 0.033	1373	836	563	
	0.331 ± 0.027	2099	1224	830	add adj.
	0.238 ± 0.023	2596	1772	907	add copl.
107.4	0.307 ± 0.032	1537	938	633	
	0.268 ± 0.026	2107	1368	792	add adj.
	0.201 ± 0.022	2627	1903	843	add copl.

The dependence of the asymmetry values on a particular set of cuts such as the cuts on the neutron time-of-flight were studied. Because the asymmetry is a relative measurement between the spin-up and the spin-down target, the asymmetry should be independent of any specific

sets of cuts applied to the data. Three sets of asymmetry values were calculated at 586 MeV/c by varying the width of the neutron time-of-flight cuts, see Table 3.5. The three time-of-flight cuts include a 0.5 nanoseconds narrower and wider window than the optimum cut (with window about 2.5 nsec) which contains most of the CEX neutron peak. From the table, the 'narrow' and the 'wide' cuts give slightly larger errors than the 'optimum' cut, because the 'narrow' cut gives fewer CEX events than the 'optimum' and the 'wide' cut gives more background than the 'optimum' cut. But all three time-of-flight cuts give consistent results.

Finally, the uncertainty in the background normalization averaged about 5% for the experiment. In all cases, the error in the

Table 3.5 Dependence of the 586 MeV/c CEX asymmetry values on three different neutron TOF cuts.

$\tilde{\theta}_{\pi^0}$	$A_N \pm \sigma_{A_N}$	Number of Events			Different TOF cuts
		CEX $\uparrow$	CEX $\downarrow$	Backgrd	
84.3	0.862 ± 0.024	3327	598	570	'wide'
	0.876 ± 0.023	3148	540	472	'optimum'
	0.872 ± 0.026	2334	404	340	'narrow'
88.0	0.785 ± 0.026	2802	627	574	'wide'
	0.792 ± 0.025	2684	595	461	'optimum'
	0.804 ± 0.027	2183	467	332	'narrow'
91.5	0.675 ± 0.024	2917	861	639	'wide'
	0.677 ± 0.023	2802	823	475	'optimum'
	0.665 ± 0.026	2135	647	324	'narrow'

asymmetries due the 5% uncertainty in the background normalization was found to be less than the magnitude of the statistical error. For example at 547 MeV/c, the asymmetries were calculated using 5% variation in the background normalization, see Table 3.6. As one can see from the table, the asymmetry values are consistent with respect to the uncertainty in the background normalization.

Table 3.6 Effect on the 547 MeV/c CEX asymmetry values from varying the background normalization by $\pm 5\%$.

$\tilde{\Theta}_{\pi^0}$	$A_N \pm \sigma_{A_N}$	Number of Events			Background Norm. change
		CEX $\uparrow$	CEX $\downarrow$	Backgrd	
83.8	0.894 ± 0.019	3914	684	515	-5%
	0.908 ± 0.020	3878	648	551	0%
	0.922 ± 0.021	3842	612	587	+5%
87.7	0.866 ± 0.019	3854	730	522	-5%
	0.880 ± 0.020	3818	695	558	0%
	0.894 ± 0.021	3781	658	595	+5%
91.6	0.718 ± 0.019	3744	1045	501	-5%
	0.728 ± 0.019	3709	1010	536	0%
	0.739 ± 0.020	3674	975	571	+5%

CHAPTER FOUR

RESULTS AND CONCLUSIONS

SECTION 4.1 REX RESULTS

The results for the left-right asymmetry parameter, A_N , in $\pi^- p \rightarrow \gamma n$ at 301, 316, 427, 471, 547, 586, and 625 MeV/c are listed in Table 4.1. The errors given in the table represent the statistical errors, the 4% systematic uncertainty in the absolute calibration of the target polarization is not included.

SECTION 4.1.1 DISCUSSION OF THE RESULTS

The results listed in Table 4.1 are plotted in Figs. 4.1(a) to (g) together with the the predictions from the most recent energy-dependent partial wave analysis (PWA) of single pion photoproduction of Arai and Fujii.³⁰ At all measured momenta, the asymmetry distribution shows no dramatic variations with angle; the asymmetry values are generally decreasing with energy in this energy region. At 301 and 316 MeV/c, the measured asymmetry values are between 0.4 to 0.8 and there is good agreement with the Arai and Fujii predictions. However from 427 to 625 MeV/c, the agreement is poor and this leads us to question their numerical value for the radiative decay amplitude of the Roper

Table 4.1 Results for the left-right asymmetry parameter, A_N , in $\pi^- p \rightarrow \gamma n$ measured with a transversely polarized target.

p_π MeV/c	$\tilde{\theta}_\gamma$ deg	$\cos\tilde{\theta}_\gamma$	A_N	σ_{A_N}	p_π MeV/c	$\tilde{\theta}_\gamma$ deg	$\cos\tilde{\theta}_\gamma$	A_N	σ_{A_N}
301	51.9	0.62	0.59	0.11	316	44.9	0.71	0.51	0.09
	88.0	0.04	0.62	0.10		49.8	0.65	0.56	0.11
	91.6	-.03	0.62	0.12		85.9	0.07	0.74	0.08
	95.2	-.09	0.54	0.17		89.5	0.01	0.71	0.09
						93.1	-.05	0.68	0.07
427	86.5	0.06	0.44	0.06	471	86.2	0.07	0.41	0.08
	90.2	0.00	0.48	0.06		89.9	0.00	0.40	0.07
	93.8	-.07	0.48	0.06		93.6	-.06	0.32	0.07
	106.1	-.28	0.55	0.07		106.3	-.28	0.25	0.07
	109.9	-.34	0.43	0.08		110.0	-.34	0.46	0.09
	113.4	-.40	0.48	0.08		113.6	-.40	0.43	0.08
547	86.2	0.07	0.08	0.08	586	86.4	0.06	-.04	0.08
	90.0	0.00	0.03	0.08		90.0	0.00	0.00	0.08
	93.7	-.06	0.04	0.07		93.5	-.06	0.00	0.08
	106.2	-.28	0.14	0.07		106.2	-.28	0.00	0.08
	109.9	-.34	0.21	0.08		110.0	-.34	-.04	0.08
	113.5	-.40	0.01	0.08		113.6	-.40	-.03	0.08
625	71.4	0.32	-.21	0.08	625	101.2	-.19	-.19	0.06
	74.8	0.26	-.13	0.08		105.0	-.26	-.25	0.06
	78.1	0.21	-.17	0.08		108.7	-.32	-.14	0.06
	86.4	0.06	-.14	0.07		116.5	-.45	-.20	0.07
	90.0	0.00	-.17	0.07		120.0	-.50	-.23	0.07
	93.6	-.06	-.21	0.06		123.3	-.55	-.21	0.07

Fig. 4.1 The left-right asymmetry A_N measured in $\pi^-p \rightarrow \gamma n$ using a transversely polarized target. The solid curves are the predictions from the single pion photoproduction PWA of Arai and Fujii (Ref. 30). (a) $A_N(\pi^-p \rightarrow \gamma n)$ at $p_\pi=301$ MeV/c.

Fig. 4.1(b) $A_N(\pi^- p \rightarrow \gamma n)$ at $p_{\pi^-} = 316$ MeV/c. The chain-dot curve is the prediction by Berends (Ref. 41).

Fig. 4.1(c) $A_N(\pi^- p \rightarrow \gamma n)$ at $p_\pi = 427$ MeV/c. The open circles are the data of Alder et al. (Ref. 40).

Fig. 4.1(d) $A_N(\pi^- p \rightarrow \gamma n)$ at $p_\pi = 471$ MeV/c. The dash curve is the prediction by Noelle (Ref. 28).

Fig. 4.1(e) $A_N(\pi^- p \rightarrow \gamma n)$ at $p_\pi = 547$ MeV/c.

Fig. 4.1(f) $A_N(\pi^- p \rightarrow \gamma n)$ at $p_\pi = 586$ MeV/c. The dash curve is the prediction by Noelle (Ref. 28) at $p_\pi = 575$ MeV/c.

Fig. 4.1(g) $A_N(\pi^- p \rightarrow \gamma n)$ at $p_\pi=625$ MeV/c.

resonance, $A_{1/2}^n = +23 \pm 9 \times 10^{-3} \text{ GeV}^{-1/2}$. Comparison is also made with the prediction of a 1977 PWA by Noelle²⁸ in Figs. 4.1(d) and (f); in figure (f) our 586 MeV/c results are compared with their 575 MeV/c evaluation. The agreement with our data is quite good.

The only other existing measurements of the left-right asymmetry are a few points at 427 MeV/c by a SIN group.⁴⁰ Within their sizable error, there is good agreement with our results as shown in Fig. 4.1(c).

As a consequence of time reversal invariance, A_N must be the same in sign and magnitude as the recoil proton polarization, P , measured in π^- photoproduction from a neutron target. Results obtained with a deuterium target, mainly at 90° , are shown in Fig. 4.2 and are compared with our data. Because presently there is no reason to doubt the applicability of time reversal invariance in this process, we conclude from the poor agreement between our data and that of the inverse reaction measured with a deuterium target, that the deuterium corrections are not fully understood. Hence, the values of the radiative decay amplitude of the neutral resonances in the literature which are all based on deuterium target measurements deserve fresh experimental study.

Fig. 4.2 Asymmetry A_N at 90° c.m. obtained in this experiment compared to the recoil proton polarization P measured in π^- photoproduction using a deuterium target by Beneventano et al. (Ref. 42), J. Kenemuth and P.C. Stein (Ref. 43), and H. Takeda et al. (Ref. 44).

SECTION 4.2 CEX RESULTS

The results for the left-right asymmetry, A_N , in $\pi^- p \rightarrow \pi^0 n$ at $p_\pi = 301, 316, 427, 471, 547, 586$, and 625 MeV/c are listed in Table 4.2. The errors given in the table are the combination of the statistical uncertainty and the 5% uncertainty in the background subtraction, but without the 4% systematic uncertainty in the absolute value of the target polarization.

SECTION 4.2.1 COMPARISONS WITH THE PARTIAL WAVE ANALYSES

Present CEX data are compared with the predictions of recent partial wave analysis by the Karlsruhe-Helsinki (K-H),³ the Carnegie Mellon University-Lawrence Berkeley Laboratory (C-L),⁴ and the Virginia Polytechnique Institute (VPI)⁵ group in Figs. 4.3(a) to (g). None of the aforementioned PWAs include our results in their data base.

The experiment can be divided into three energy regions:

a) The measurements at 301 and 316 MeV/c cover the area just around the peak of the $\Delta(1232)$ resonance. The asymmetry curves predicted by K-H and VPI have a similar overall shape, see Figs. 4.3(a) and (b). K-H predicts the asymmetry to increase from 0 to a maximum of 0.7 at $\cos\theta$ of about 0.1, and to decrease slowly back to 0. VPI predicts a steeper initial rise and slightly larger A_N values than K-H for $\cos\theta$ greater than 0.4. For $\cos\theta$ less than 0.4, VPI predicts slightly smaller A_N values than K-H. Both the K-H and the VPI PWA are in acceptable agreement with our results, but overall, our data show a

Table 4.2 Results for the left-right asymmetry parameter, A_N , in $\pi^- p \rightarrow \pi^0 n$ measured with a transversely polarized target.

p_π MeV/c	$\tilde{\theta}_{\pi^0}$ deg	$\cos\tilde{\theta}_{\pi^0}$	A_N	σ_{A_N}	p_π MeV/c	$\tilde{\theta}_{\pi^0}$ deg	$\cos\tilde{\theta}_{\pi^0}$	A_N	σ_{A_N}
301	32.5	0.84	0.251	0.029	316	30.8	0.86	0.247	0.025
	38.8	0.78	0.302	0.025		37.1	0.80	0.257	0.022
	81.4	0.15	0.678	0.049		79.6	0.18	0.675	0.038
	85.3	0.08	0.730	0.042		83.6	0.11	0.702	0.036
	89.3	0.01	0.686	0.040		87.6	0.04	0.684	0.032
427	82.7	0.13	0.922	0.024	471	83.0	0.12	1.010	0.027
	86.7	0.06	0.909	0.023		86.9	0.05	0.959	0.028
	90.5	-.01	0.892	0.023		90.8	-.01	0.904	0.026
	103.5	-.23	0.313	0.035		104.1	-.24	0.339	0.039
	107.4	-.30	0.138	0.034		107.9	-.31	0.098	0.043
	111.1	-.36	-.033	0.028		111.6	-.37	-.071	0.036
547	83.8	0.11	0.908	0.021	586	84.3	0.10	0.876	0.024
	87.7	0.04	0.880	0.021		88.0	0.03	0.792	0.026
	91.6	-.03	0.728	0.020		91.5	-.03	0.677	0.024
	104.4	-.25	0.334	0.028		104.7	-.25	0.289	0.034
	108.3	-.31	0.239	0.031		108.5	-.32	0.313	0.038
	112.0	-.37	0.208	0.027		112.3	-.38	0.249	0.032
625	68.9	0.36	1.056	0.020	625	99.7	-.17	0.390	0.032
	72.5	0.30	1.015	0.020		103.6	-.24	0.298	0.033
	75.9	0.24	0.998	0.020		107.4	-.30	0.298	0.032
	84.5	0.10	0.778	0.024		115.4	-.43	0.344	0.045
	88.2	0.03	0.696	0.025		118.9	-.48	0.499	0.058
	91.8	-.03	0.589	0.023		122.3	-.53	0.512	0.052

Fig. 4.3 The left-right asymmetry A_N in $\pi^- p \rightarrow \pi^0 n$ measured with a transversely polarized target. The A_N results are compared with the PWA predictions by the K-H (Ref. 3), C-L (Ref. 4), and VPI (Ref. 5) groups. (a) $A_N(\pi^- p \rightarrow \pi^0 n)$ at $p_\pi=301$ MeV/c.

Fig. 4.3(b) $A_N(\pi^- p \rightarrow \pi^0 n)$ at $p_\pi = 316$ MeV/c.

Fig. 4.3(c) $A_N(\pi^-p \rightarrow \pi^0n)$ at $p_{\pi^-}=427$ MeV/c. The open triangles are the data of Alder et al. (Ref. 40).

Fig. 4.3(d) $A_N(\pi^- p \rightarrow \pi^0 n)$ at $p_\pi = 471$ MeV/c.

Fig. 4.3(e) $A_N(\pi^- p \rightarrow \pi^0 n)$ at $p_\pi = 547$ MeV/c.

Fig. 4.3(f) $A_N(\pi^- p \rightarrow \pi^0 n)$ at $p_\pi = 586$ MeV/c.

Fig. 4.3(g) $A_N(\pi^-p \rightarrow \pi^0n)$ at $p_\pi=625$ MeV/c.

slight preference for K-H over VPI due to the better agreement with K-H in the forward angle.

b) The measurements at 427 and 471 MeV/c are in a region where there is no πN resonance. The asymmetry curves predicted by K-H, C-L, and VPI are very similar, see Figs. 4.3(c) and (d). The asymmetry curves are characterized by large asymmetry values approaching 1 near $\cos\theta=0.1$, and a subsequent rapid decrease which leads to a negative peak at $\cos\theta=-0.7$. The asymmetry values are negative for $\cos\theta$ less than -0.3 ; the 471 MeV/c data shows a more pronounced negative dip than the 427 MeV/c data. Our data show a very slight preference for K-H and C-L over VPI at 427 MeV/c; the VPI predictions give slightly larger A_N than our data at $\cos\theta$ from -0.2 to -0.4 . Our data at 427 MeV/c are compared with the results of Alder et al.,⁴⁰ see Fig. 4.3(c), and there is good agreement. At 471 MeV/c, our data show a slight preference for VPI and C-L over K-H, where the K-H predictions at $\cos\theta$ of -0.2 to -0.4 give slightly larger asymmetry values than our results.

c) The region of the greatest interest is from 547 to 625 MeV/c. As shown in Figs. 4.3(e), (f) and (g), the three PWA predictions agree well with each other in the forward angles, but diverge badly in the backward angles. In the forward angular region, A_N increases with angle and attains the maximum value of 1 near $\cos\theta=0.3$; the VPI solution peaks at a slightly smaller angle than K-H and C-L, especially at 625 MeV/c. At $\cos\theta$ from 0.2 to -0.8 , the three solutions diverge with VPI predicting lower asymmetry values than the other two solutions; at extreme backward angles, all three solutions converge again. Our data overwhelmingly favor VPI and their lower minimum in the backward angles. At 625 MeV/c, the agreement with the VPI predictions would be even better if their asymmetry peak at $\cos\theta$ of 0.4

could be shifted slightly to a larger angle, or if our target polarization is increased by 4%, thereby slightly lowering the asymmetry values of those large A_N data at $\cos\theta$ of 0.4 to 0.0.

To make a numerical comparison between our data and the PWAs, we calculated the quantity

$$\chi^2 = \frac{1}{N} \frac{\sum (A_N - A_N^{\text{PWA}})^2}{\sigma_{A_N}^2},$$

where A_N^{PWA} is the asymmetry predicted from the partial wave analysis. The results which are given in Table 4.3 are calculated with and without the $\pm 4\%$ systematic uncertainty in A_N .

Our results are compared with the recent data of Wightman et al.,⁴⁵ see Figs. 4.4.(a)-(c). Within their larger errors there is good agreement with our data.

In conclusion, our $A_N(\text{CEX})$ results agree well with the PWA predictions by the VPI group at all seven incident beam momenta. It is

Table 4.3 A numerical comparison of χ^2 of our data with various PWA predictions.

P_π MeV/c	K-H			C-L			VPI		
	-4%	0%	+4%	-4%	0%	+4%	-4%	0%	+4%
301	0.3	0.2	0.7				3.7	3.7	4.3
316	0.5	1.5	3.4				5.3	6.4	8.2
427	1.5	0.9	2.8	3.1	1.6	2.5	7.3	4.8	4.9
471	6.3	4.8	5.4	3.1	2.1	3.1	3.4	0.8	0.5
547	38.	46.	55.	26.	34.	44.	3.5	1.5	2.3
586	31.	36.	43.	66.	76.	87.	5.6	2.2	0.7
625	17.	17.	19.	35.	35.	39.	12.	5.4	1.7

Fig. 4.4 Comparison of our CEX results with the data of Wightman et al. (Ref. 45) (a) at 547, (b) at 586, and (c) at 625 MeV/c.

easy to blame the P_{11} wave for the disagreement among the PWAs because of the different values quoted for the P_{11} parameters. However, it has been pointed out by Hohler⁴⁶ that in the energy region of this experiment a clear difference is in the values for the S_{11} wave. This is illustrated in Fig. 4.5(a), which gives the amplitude of the S_{11} and P_{11} as obtained by the PWAs. So on the basis of the excellent agreement of our A_N results with the predictions of the VPI PWA, we favor their values for the S_{11} amplitude. As Fig. 4.5(b) shows, the three PWA groups obtain very similar values for the P_{11} amplitude up to $W=1600$ MeV. The conflicting values for the mass, width, and pole position (or positions) of the Roper resonance are due to the differences at higher energies $W>1600$ MeV, and different theoretical interpretations. Further experimental works, including the inelastic reactions such as $\pi^-p \rightarrow \gamma n$, $\pi^-p \rightarrow \pi\pi N$, and $\pi^-p \rightarrow \eta n$, are needed to find the detailed structures of the P_{11} resonance.

Fig. 4.5 (a) S_{11} and (b) P_{11} scattering amplitudes predicted from the πN PWAs by the K-H (Ref. 3), C-L (Ref. 4), and VPI (Ref. 5) groups.

SECTION 4.2.2 TEST OF ISOSPIN INVARIANCE WITH THE TRIANGLE INEQUALITIES OF THE TRANSVERSITY CROSS SECTION

Isospin invariance in the strong interaction is thought to be broken at the few percent level due to the differences in the mass and electric charge of the up and down quarks. Combining pions which have $I=1$ with nucleons which have $I=1/2$, the πN system can have $I=1/2$ or $I=3/2$ for its total isospin. In terms of the total isospin vector $|I, I_z\rangle$, the pion-nucleon states are given by

$$\begin{aligned} |\pi^+ p\rangle &= |3/2, 3/2\rangle \\ |\pi^- p\rangle &= \sqrt{\frac{1}{3}}|3/2, -1/2\rangle - \sqrt{\frac{2}{3}}|1/2, -1/2\rangle \\ |\pi^0 n\rangle &= \sqrt{\frac{2}{3}}|3/2, -1/2\rangle + \sqrt{\frac{1}{3}}|1/2, -1/2\rangle. \end{aligned}$$

Using isospin invariance, the reaction amplitudes are written

$$\begin{aligned} a^+ &= \langle \pi^+ p | H_S | \pi^+ p \rangle = T_{3/2} \\ a^- &= \langle \pi^- p | H_S | \pi^- p \rangle = \frac{1}{3}T_{3/2} + \frac{2}{3}T_{1/2} \\ a^0 &= \langle \pi^0 n | H_S | \pi^- p \rangle = \frac{\sqrt{2}}{3}(T_{3/2} - T_{1/2}) \end{aligned}$$

where $T_{2/3} = \langle 3/2 | H_S | 3/2 \rangle$ and $T_{1/2} = \langle 1/2 | H_S | 1/2 \rangle$. The amplitudes a^+ , a^- , and a^0 are related by $\sqrt{2}a^0 = a^+ - a^-$ which expresses a triangle in the complex plane, and this leads to the triangle inequalities for unpolarized cross sections

$$\frac{1}{2}(\sqrt{d\sigma^+} - \sqrt{d\sigma^-})^2 \leq d\sigma^0 \leq \frac{1}{2}(\sqrt{d\sigma^+} + \sqrt{d\sigma^-})^2,$$

where $\sigma^+ = \sigma(\pi^+ p \rightarrow \pi^+ p)$, $\sigma^- = \sigma(\pi^- p \rightarrow \pi^- p)$, and $\sigma^0 = \sigma(\pi^- p \rightarrow \pi^0 n)$.

A more sensitive test can be obtained with the aid of the transversity cross sections. They are formed by combining the analyzing power with the differential cross section to give

$$d\Sigma_{\uparrow} = d\sigma(1+A_N) \text{ and } d\Sigma_{\downarrow} = d\sigma(1-A_N).$$

The up and down transversity cross sections are the cross sections obtained with a polarized target with the polarization direction up and

down; both transversity cross sections independently obey the triangle inequalities:

$$\frac{1}{2}(\sqrt{d\Sigma_{\uparrow}^{+}} - \sqrt{d\Sigma_{\uparrow}^{-}})^2 \leq d\Sigma_{\uparrow}^0 \leq \frac{1}{2}(\sqrt{d\Sigma_{\uparrow}^{+}} + \sqrt{d\Sigma_{\uparrow}^{-}})^2$$

and

$$\frac{1}{2}(\sqrt{d\Sigma_{\downarrow}^{+}} - \sqrt{d\Sigma_{\downarrow}^{-}})^2 \leq d\Sigma_{\downarrow}^0 \leq \frac{1}{2}(\sqrt{d\Sigma_{\downarrow}^{+}} + \sqrt{d\Sigma_{\downarrow}^{-}})^2.$$

The CEX transversity cross sections are given in Table 4.4, they are obtained using the unpolarized cross section data of Borchers et al.⁴⁷ The cross section data are first fitted to Legendre polynomials, which are then used to determine the cross section at the angles of the present experiment as shown in Fig. 4.6 at 471 MeV/c. The π^+p elastic transversity cross sections are similarly obtained using the data of Sadler et al.⁴⁸ for the cross section and the data of Mokhtari et al.⁴⁹ for the analyzing power. Figs. 4.7 and 4.8 show the fitted elastic up and down transversity cross sections at 471 MeV/c. The upper and lower bounds on $d\Sigma_{\uparrow}^0$ and $d\Sigma_{\downarrow}^0$ are obtained using the fitted elastic transversity cross sections.

The triangle inequalities using the transversity cross sections are tested at 471, 547, and 625 MeV/c, see Figs. 4.9(a) to (e). At 547 and 625 MeV/c, the data of Wightman et al.⁴⁵ have been added. No evidence for isospin violation is seen.

Table 4.4 π^-p Charge Exchange Transversity Cross Sections.

p_π (MeV/c)	$\cos\tilde{\theta}_{\pi^0}$	$d\sigma^0/d\Omega$ (mb/sr)		$d\Sigma_\uparrow^0$ (mb/sr)		$d\Sigma_\downarrow^0$ (mb/sr)	
471	0.12	0.67 ± 0.10		1.34 ± 0.23		0.00 ± 0.02	
	0.05	0.56	0.08	1.10	0.22	0.02	0.02
	-0.01	0.49	0.07	0.94	0.14	0.05	0.02
	-0.24	0.32	0.05	0.43	0.07	0.21	0.03
	-0.31	0.29	0.04	0.32	0.06	0.27	0.05
	-0.37	0.28	0.04	0.26	0.05	0.29	0.06
547	0.11	0.51 ± 0.07		0.98 ± 0.15		0.05 ± 0.01	
	0.04	0.45	0.06	0.84	0.14	0.05	0.01
	-0.03	0.39	0.06	0.68	0.10	0.11	0.02
	-0.25	0.28	0.05	0.37	0.06	0.19	0.03
	-0.31	0.25	0.04	0.31	0.05	0.19	0.03
	-0.37	0.23	0.04	0.28	0.05	0.18	0.03
625	0.36	0.67 ± 0.09		1.38 ± 0.20		0.00 ± 0.01	
	0.30	0.58	0.08	1.17	0.19	0.00	0.01
	0.24	0.51	0.07	1.02	0.18	0.00	0.01
	0.10	0.40	0.06	0.70	0.12	0.09	0.02
	0.03	0.35	0.05	0.60	0.09	0.11	0.02
	-0.03	0.32	0.05	0.51	0.08	0.13	0.02
	-0.17	0.25	0.04	0.35	0.06	0.15	0.03
	-0.24	0.22	0.03	0.28	0.04	0.15	0.02
	-0.30	0.19	0.03	0.24	0.04	0.13	0.02
	-0.43	0.13	0.02	0.18	0.03	0.09	0.02
	-0.48	0.11	0.02	0.17	0.03	0.06	0.01
	-0.53	0.10	0.02	0.15	0.03	0.05	0.01

Fig. 4.6 $\pi^- p$ charge exchange differential cross section data of Borcharding et al. (Ref. 47) at 471 MeV/c. The solid curve is the best fit through the data. The open boxes are the cross section values evaluated at the angles of this experiment.

Fig. 4.7 π^+p elastic differential and transversity cross sections at 471 MeV/c. The closed circles are the data of Sadler et al. (Ref. 48), the solid curve is the best fit through the data. The A_N data of Mokhtari et al. (Ref. 49) are then used to determine the up and down transversity cross sections represented by the up and down open triangles. The dash and the chain-dot curves are the best fit through the data.

Fig. 4.8 π^-p elastic differential and transversity cross sections at 471 MeV/c. See the figure caption for Fig. 4.7.

Fig. 4.9 Test of isospin invariance using the triangle inequalities on the CEX transversity cross sections. The vertical axis shows $d\sigma/d\Omega(\pi^-p \rightarrow \pi^0 n)$. The shaded area is forbidden if isospin is conserved. The error bars on the bounds indicate the uncertainties on the isospin bounds. Our CEX data are the solid squares, and the open circles at 547 and 625 MeV/c are the data of Wightman et al. (Ref. 45). (a) The transversity up limits and cross section at $p_\pi = 471$ MeV/c.

Fig. 4.9(b) The transversity down at $p_{\pi}=471$ MeV/c.

Fig. 4.9(c) The transversity up at $p_\pi=547 \text{ MeV/c}$.

Fig. 4.9(d) The transversity down at $p_\pi=547 \text{ MeV/c}$.

Fig. 4.9(e) The transversity up at $p_{\pi}=625 \text{ MeV/c}$.

Fig. 4.9(f) The transversity down at $p_\pi=625$ MeV/c.

APPENDIX A

A METHOD FOR THE DETERMINATION OF THE REX CROSS SECTION

Besides the REX and CEX asymmetry results, an interesting quantity that can be extracted from the experiment is the differential cross section for REX. Using known values of the CEX cross section and the experimental ratio between REX and CEX events together with the experimental CEX acceptance, REX cross section values with 10 to 20% uncertainty can be obtained. In principle, one could test time reversal invariance by comparing the REX cross sections with the values from the inverse reaction, π^- photoproduction on a neutron, via detailed balance. The photon-induced reactions on neutrons use a deuterium target, which require deuterium corrections for the effects of Pauli blocking, Fermi motion, nuclear binding, and off-mass-shell amplitudes. In view of the primitive state of the deuterium correction, our evaluation of the REX cross section should be used to test the applicability of the deuterium correction rather than as a test of time reversal invariance.

The REX cross sections are determined by the following procedure. The experimental event rate for a given reaction at a detector subtending the solid angle $d\Omega$ is given by

$$dN = \epsilon \eta F N_s \sigma(\theta, \phi) d\Omega$$

where F is the beam flux, N_s the number of independent scattering

centers intercepted by the beam, $\sigma(\theta, \phi) = \frac{d\sigma}{d\Omega}$ the differential cross section for the reaction, and ϵ and η the acceptance and the efficiency of the detector, respectively. Recall that in the experiment the final-state gamma and neutron were detected in coincidence. The solid angle is determined by the neutron counters, and the acceptance ϵ has been separated out to account for the gamma acceptance. For the REX gammas, the ϵ_{REX} is close to one, and for CEX ϵ_{CEX} is about 25% as determined from Monte Carlo studies. The ratio of the event rates for REX and CEX is then given by

$$\frac{dN_{\text{REX}}}{dN_{\text{CEX}}} = \frac{\epsilon_{\text{REX}} \left(\frac{d\sigma}{d\Omega}(\text{REX}) \right)^{\text{LAB}}}{\epsilon_{\text{CEX}} \left(\frac{d\sigma}{d\Omega}(\text{CEX}) \right)^{\text{LAB}}}$$

where the F , N_s , $(d\Omega)^{\text{LAB}}$, and η factors cancel because the REX and CEX events were detected simultaneously. Using the Jacobian factors, $JAC(\text{REX})$ and $JAC(\text{CEX})$, which transform the c.m. to the lab frame cross section values, the c.m. REX cross section is related to the c.m. CEX cross section by

$$\frac{d\sigma}{d\Omega}(\text{REX}) = \frac{d\sigma}{d\Omega}(\text{CEX}) \times \frac{JAC(\text{REX})}{JAC(\text{CEX})} \times \frac{dN_{\text{REX}}}{dN_{\text{CEX}}} \times \frac{\epsilon_{\text{CEX}}}{\epsilon_{\text{REX}}}.$$

The ratio of the CEX to REX cross section is called the Panofsky ratio. The $\frac{d\sigma}{d\Omega}(\text{CEX})$ at the present experimental angles (see Table 4.4) are obtained using the data of Borchertding.⁴⁷ The ϵ_{CEX} values are obtained from Monte Carlo studies which determined the fraction of the total number of the CEX gammas intercepted by the gamma counters for neutrons detected at a particular neutron counter.

The ϵ_{REX} and the REX and CEX event numbers depend on the software cuts, such as the neutron TOF and the gamma pulse height cuts, used to extract the good events. Depending of the exact method used, the

estimate for ϵ_{REX} is $85 \pm 10\%$, this is the largest source of error in determining the REX cross section. Using the 586 MeV/c data, the dependence of the software cuts has been investigated by varying the size of the cuts. Three different gamma pulse height (ADC) and three neutron time-of-flight (TOF) cuts have been used. The event yields are then used to calculate the REX cross sections as listed in Table A.1. Increasing the cut on the gamma ADC initially helps to eliminate the background events, but soon begins to remove some good CEX events. The three γ cuts have less effect on the generally more energetic REX gammas. The Low γ ADC cut removes 10 to 20% of the total CEX gammas, however, because the experiment requires a clear gamma geometry in order to distinguish between the REX and CEX events, No γ ADC cut case also throws out 5 to 10% of the CEX and REX gammas which fired more than one gamma counter. The Mid γ ADC cut refers to the middle ADC cut level between the Low and the No cut. The calculated REX cross section values are consistent for all three γ ADC cuts within the large errors. Also, three different neutron TOF cuts have been tried with the Low γ ADC cut. The middle TOF cuts extract most of the CEX events, the wider and the narrower TOF cuts are used as comparison. From the table, the narrow cut clearly removes more CEX event than REX, which results in larger REX cross section values. The other two TOF cuts give very similar results. Much more work is needed to reduce the size of the errors in the REX cross sections.

The precise determination of the REX cross section is not the objective of this work, a mere illustration of our method for determining the REX values is given. Very preliminary REX results are listed in Table A.2. Also listed in the table are the cross sections

Table A.1 REX differential cross sections at 586 MeV/c. The cross section (in $\mu\text{b}/\text{sr}$) has been calculated for three different gamma ADC cuts and also for three different neutron TOF cuts.

$\cos\tilde{\theta}_\gamma$	No γ ADC	Mid γ ADC	Low γ ADC
0.06	12.2 ± 2.9	11.6 ± 2.3	12.2 ± 2.0
0.00	9.2 ± 2.6	9.5 ± 2.8	10.9 ± 2.1
-.06	9.0 ± 1.6	8.7 ± 1.9	10.0 ± 2.2

$\cos\tilde{\theta}_\gamma$	Wide n TOF	Mid n TOF	Narrow n TOF
0.06	11.4 ± 2.1	12.2 ± 2.0	16.3 ± 1.9
0.00	10.7 ± 1.9	10.9 ± 2.1	13.1 ± 2.1
-.06	9.9 ± 2.1	10.0 ± 2.2	12.8 ± 2.3

for π^- photoproduction on a neutron,⁵⁰ and for REX (the last column) which was calculated using detailed balance,

$$\frac{d\sigma}{d\Omega}(\gamma n \rightarrow \pi^- p) = \frac{d\sigma}{d\Omega}(\gamma n \rightarrow \pi^- p) \times 2 \frac{p_\gamma^2}{p_\pi^2}$$

where p_γ and p_π are the c.m. momenta of gamma and π respectively. The agreement of the REX cross section values with the values obtained through detailed balance suggests that the deuterium correction procedures is valid to 20%.

Table A.2 Illustration of the application of our method for the REX cross section determination. Preliminary REX cross sections are given in the third column. The fourth column gives the π^- photoproduction cross sections. The fifth column is the REX cross section calculated from column four using detailed balance.

p_π (MeV/c)	$\tilde{\theta}_\gamma$ (deg.)	$\frac{d\sigma}{d\Omega}(\pi^-p \rightarrow \gamma n)$ ($\mu\text{b/sr}$)	$\frac{d\sigma}{d\Omega}(\gamma n \rightarrow \pi^- p)$ ($\mu\text{b/sr}$)	$\frac{d\sigma}{d\Omega}(\gamma n \leftarrow \pi^- p)$ ($\mu\text{b/sr}$)
427	90	18. $\pm$ 3.	9.3 $\pm$.7	21.4 $\pm$ 1.6
	110	12. 3.	7.5 .5	17.3 1.2
471	90	17. 3.	6.7 .5	15.1 1.1
	110	13. 3.	6.8 .5	15.3 1.2
547	90	12. 3.	7.0 .6	15.4 1.3
	110	10. 3.	6.2 .6	13.6 1.3
586	90	10. 3.	6.3 .4	13.7 0.9
	110	9. 3.	5.2 .5	11.3 1.1
625	75	15. 3.	7.0 .5	15.1 1.1
	90	15. 3.	5.9 .5	12.7 1.1
	105	14. 3.	5.7 .5	12.3 1.1
	120	9. 3.	6.1 .5	13.2 1.1

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