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CONTINUOUS STRATIFIED SAMPLING
OF IN-PROCESS INVENTORY

Prepared by
C. D. Rinker

Published August 1976

MASTER

Prepared for the United States Energy
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**Kansas City
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Department 114

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CONTINUOUS STRATIFIED SAMPLING OF IN-PROCESS INVENTORY

BDX-613-1032, UNCLASSIFIED Special Report, Published August 1976

Prepared by C. D. Rinker, D/114

A stratified sampling plan replaces a yearly inventory of in-process product with resultant cost savings. An in-process inventory is physically sampled periodically to determine errors in the record inventory. New estimates of stratum and total population mean differences and variance of differences are obtained using exponential smoothing. Error rates are obtained in addition to the dollar magnitude of the difference. Decision criteria have been formulated for management use in detecting significant changes in record keeping errors. Sample computer programs for computation of stratum sample sizes are provided.

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THE BENDIX CORPORATION
KANSAS CITY DIVISION
P O BOX 1159
KANSAS CITY, MISSOURI 64141

A prime contractor for the United
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SUMMARY

A stratified sampling plan has been developed to replace a yearly physical inventory of several thousand products with wide variation in dollar value.

The results of a recent physical inventory were reviewed and used to stratify the total population and arrive at starting values for a sampling process. Improvements in sampling efficiency, recognized in prior work on stores inventories, led to the use of Neyman sampling theory. Formulae were developed sequentially for application to a computerized system. Various sampling rates were tested by a computer program which determined total sample size and allocated the total to strata based on the number in a stratum and the standard deviation of the stratum.

Three parameters which determine sampling plan characteristics were provided for use by management. One specifies the degree of closeness desired in the sample estimate and determines the total sample size for a given month. A second specifies when a sample estimate is considered different from past historical estimates and may be used to institute corrective action. A third specifies the weight given the sample estimate in determining a new population estimate and results in a slower or faster response to real changes in the population.

Calculations were performed using a simulation program to select the initial exponential smoothing constant. Further investigations of the impact of the smoothing constants on response time and variance of the estimate are planned.

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DISCUSSION

SCOPE AND PURPOSE

Bendix maintains a computer system which provides a record of each production job in process. Report points are defined for each job and information is collected and reported into the computer system at these points.

Historically, a physical inventory of all work in process has been conducted yearly to adjust material and labor ledger balances. The physical inventory is merely an audit of the automated record keeping system. An error-free automated system would eliminate the need for a physical inventory.

Rather than rely completely on the automated system, the error rate of record keeping may be determined by periodic sampling. The purpose of this study was to devise a sampling system which would estimate the automated record keeping error rate and determine if the error rate is within acceptable limits, and estimate the material dollar, labor dollar, and total dollar difference between physical inventory and record inventory.

PRIOR WORK

Sampling to determine dollar difference between physical inventory and record inventory has been accomplished for Central Stores and Test Equipment Stores. The methodology and results are given in a previous report* and were used as a starting point for the present study.

ACTIVITY

Purpose of Physical Inventory

Historically, a physical inventory has provided production management with the current information needed to establish future requirements of all items, to allocate component deliveries and shortages to production jobs, and to position manufacturing material in work at the current production level.

*C. D. Rinker, Economic Improvement of Inventory Sampling.

UNCLASSIFIED. Bendix Kansas City: BDX-613-885, April, 1973.

The same data provided accounting management with the information necessary to adjust the ledger balance for material and labor as a corrected start point for future transactions. It also allows adjustment for routine differences between inventories, including standard price changes for material and labor.

It is evident that the central issue in a physical inventory is the determination of the quantity of parts at a given point in the production process. Current production control data processing systems make it possible to obtain and verify the same data without performing the historical physical inventory process.

Definition of Information Available for Record Inventory

An Open Requirements (OR) System provides the total availability and requirement for each productive component. These data identify quantities located in "next assembly," in stores, in packaging, in hold categories, and in other applications. The record is adjusted for completions, stores issues, shipments, and scrap; it also presents meaningful information about the timing of future needs and about the quantity produced in the past.

A Material Authorization, Allocation, and Release System (MAARS) provides a record of components issued to production jobs for fabrication in next assembly. Components are assigned to specific jobs and the total of each part shown "in next assembly" of the OR system is supported by the detail to these jobs in the MAARS record. As each job is completed--with the information reconciled and the material processed to stores--the MAARS record is closed. The component record of the OR System will then show completions in next assembly.

The Work-In-Process (WIP) System is a record of the manufacturing for each production job. As each report point is completed, the information is compiled in the record and is used to follow the progress of scheduled assemblies and record the value of standard production labor in inventory. The Work-In-Process record of a production job is closed when all units concerned are processed to stores or are scrapped and the quantity involved is reconciled.

The quantity of accountable assemblies on each production job is identical in MAARS and the WIP System; the job is not closed until the two completion records are compatible.

All three systems have various computer "edits" which identify erroneous transactions and prevent the processing of incorrect data. Discrepancies

are resolved on a daily basis. Frequent audits, the surveillance associated with "edit" corrections, and normal maintenance of the system preserve the integrity of the records.

Replacing Physical Inventory With Record Inventory

The maintenance of current records, such as the OR System, MAARS, and the WIP System will save the production time now required to prepare for inventory, take inventory, and return to manufacturing processes. In addition, the support efforts associated with physical inventory can be reduced significantly.

It costs about \$120,000 annually to take the physical inventory. Not included are pre-inventory preparation costs such as grouping all related material, returning material not required in production to stores, identifying all production items, and segregating non-productive items, such as tooling and test equipment, which are not subject to this inventory. Nor does this cost include post inventory activity such as reorganizing the productive material, setting up the proper tooling for each production lot, and investigating improperly recorded information on inventory tags.

Since production inventory information normally involved in a physical inventory is maintained on a current basis, it is proposed that these records be accepted in lieu of shutting down production operations. Sampling techniques will be applied monthly to verify the accuracy of the records. Adjustments to the ledger balance will be made as discrepancies or changes are revealed, and a total inventory adjustment based on current records will be processed annually.

The sampling method will be operational by FY1974 and will verify items within stratified categories based on value. The sample size of each category will be statistically selected through consideration of historical variations within each group. Sampling will be done continuously and special emphasis will be placed on transaction processing and related cut-offs at the chosen annual total inventory adjustment period.

Historical statistics required to calculate the mean and standard deviations used to develop specific sampling techniques and sampling size were obtained from a comparison of the records and the actual inventory in December 1972.

Problem Definition

The sampling system was designed to estimate the automated record keeping error rate and determine if the error rate is within acceptable limits, and to estimate material dollars, labor dollars, and total dollars difference between physical inventory and record inventory.

A 95 percent confidence level has been specified for both items. Two statements will result:

- There is 95 percent confidence that the automated record keeping system differs (or is in error) from the physical record by less than k percent; and
- There is 95 percent confidence that the total dollar amount in physical inventory is equal to $T \pm d$ dollars. An equivalent statement is made for material and labor. The value, d, will be given by accounting in terms of a percentage which will be applied to total inventory dollars. Sample size requirements are a function of d.

Population Description

The population consists of in-process jobs, each with a unique identification. A job consists of a quantity of a particular part, representing the number of parts or assemblies which would go to stores if holds and scrap are not considered.

A job proceeds through a sequence of operations in the manufacturing area before reaching stores status. Each operation has a unique identifying number: Some numbers are identified as waybill points which are used for costing. Three costs are considered: material, labor, and total.

The reporting system may contain other report points, but all waybill points are reported.

Stratification of the Population

Each part or assembly has an inventory cost which represents the accumulated labor and material cost of the completed part when received in stores. It is the unit inventory total cost which is used to stratify the job population. Table 1 shows the ten strata that have been defined for initial planning.

Table 1. Strata Defined for Initial Planning

Strata	Unit Cost*
1	\$7300.00 or more
2	5500.00 to \$7299.99
3	3600.00 to 5499.99
4	1800.00 to 3599.99
5	750.00 to 1799.99
6	550.00 to 749.99
7	360.00 to 549.99
8	180.00 to 359.99
9	75.00 to 179.99
10	Less than 75.00
*Cost is based on inventory.	

Initial population statistics are taken from a 100 percent physical inventory which was compared to the record inventory. Statistics are shown in Tables 2, 3, and 4.

The Sample

It is planned that in-process jobs will be sampled monthly with the resulting statistics accumulated over 12 months. A 100 percent physical inventory has been conducted to provide starting values for population statistics. These values will be adjusted after each monthly sample.

Before a monthly sample is selected, the record of in-process jobs will be reviewed and the following statistics computed:

N_j = the number of jobs in each stratum j ,

V_i = the inventory value of job i ,

$N = \sum_j N_j$ = the total number of jobs in-process, and

$A = \frac{\sum_i V_i}{N}$ = the average job inventory value of all jobs in process.

Table 2. Total Dollar Difference Statistics

Strata	Jobs	Mean	Statistical Variance	Average Job Dollar Value
1	78	39.4675	24938.6	\$66854.90
2	24	0.4397	5.7	8853.05
3	125	13.9959	22388.5	14988.40
4	212	16.3852	16731.1	5546.08
5	572	12.2399	12656.8	2158.51
6	739	6.2961	6816.8	1844.67
7	321	-10.8590	18735.2	611.56
8	1043	17.1284	32325.7	678.60
9	1181	9.9667	3079.7	441.97
10	2385	14.4932	9229.2	135.42

It is assumed that the in-process job dollar differences are normally distributed. The desired confidence level is 95 percent. The total sample size required for 95 percent confidence is computed as follows:

$$n = \left(\frac{1.96 \hat{\sigma}_t}{AC} \right)^2$$

where

$\hat{\sigma}_t$ is the estimate of the standard deviation of total dollar difference,

A is the average job inventory value, and

C is a fraction denoting the desired precision of estimate.

Table 3. Labor Dollar Difference Statistics

Strata	Jobs	Mean	Statistical Variance	Average Job Dollar Value
1	78	-14.06	6400.5	\$ 4450.32
2	24	-0.11	0.3	934.76
3	125	2.77	218.0	1011.69
4	212	4.10	2615.1	394.91
5	572	0.29	1688.7	308.38
6	739	0.32	554.9	221.09
7	321	-8.72	11908.6	146.44
8	1043	2.31	1141.2	678.60
9	1181	1.77	371.6	92.06
10	2385	2.61	588.7	47.05

Table 4. Material Dollar Difference Statistics

Strata	Jobs	Mean	Statistical Variance	Average Job Dollar Value
1	78	53.53	35875.7	\$62404.60
2	24	0.55	5.4	7918.29
3	125	11.22	19872.1	13976.70
4	212	12.28	11677.8	5151.17
5	572	11.95	6971.6	1850.14
6	739	5.98	4816.0	1623.59
7	321	-2.14	2527.9	465.11
8	1043	14.82	30075.4	541.57
9	1181	8.20	2130.8	349.91
10	2385	11.88	6400.7	88.37

The total sample size, n , is then allocated to each stratum, j , as follows:

$$n_j = \frac{n N_j \hat{\sigma}_{tj}}{12 \sum_j N_j \hat{\sigma}_{tj}} .$$

Since the computed value of n_j may not be an integer the value is rounded up in all cases. For this particular application a minimum value of $n_j = 3$ has been chosen. The total sample size is then recomputed as:

$$n = \sum_j n_j$$

The samples, n_j , taken from each stratum will be random selections from the stratum population, N_j , which is large in comparison to the sample size.

Record Accuracy

Data compilation and computations for estimating record keeping error rate are explained below. There are six desired outputs for reporting each month:

- Percent of record errors in the current month,
- Standard deviation of record error rate in the current month,
- Notification of record errors in excess of cumulatively computed 95 percent confidence interval,
- Cumulative percent of record errors,
- Cumulative standard deviation of record error rate, and
- Cumulative 95 percent confidence interval of record accuracy.

The number of possible errors will vary for each in-process job. A count is kept of the number of items compared and the number of differences from record values for each job sampled.

Let

c_i = the total number of items compared over all sampled jobs during the current month i , and let

d_i = the total number of differences noted during the current month i .

Then

$c = \sum_{i=1}^{12} c_i$ = the total number of items compared for the latest 12 month period, and

$d = \sum_{i=1}^{12} d_i$ = total number of differences noted during the latest 12 month period.

The mean and standard deviation of the error rate will be computed each month as follows:

$f_i = \frac{d_i}{c_i}$ = fraction of record errors for the current month i ,

$p_i = 100 f_i$ = percentage of record errors for the current month i , and

$s_i = 100 \sqrt{\frac{f_i(1-f_i)}{c_i}}$ = standard deviation of record errors for the current month.

A cumulative mean error rate and standard deviation of error rate will be computed monthly as follows:

$f = \frac{d}{c}$ = fraction of record errors for the latest 12 month period,

$p = 100 f$ = percentage of record errors for the latest 12 month period, and

$s = 100 \sqrt{\frac{f(1-f)}{c}}$ = standard deviation of record errors for the latest 12 month period.

Management should be informed if there is an extreme departure from normal in the error rate. To detect a departure from normal the following are calculated:

$L = 100 - p - 1.65 s$ = cumulative lower 95 percent confidence limit on percent of record accuracy.

If $(L + p_i) > 100$, then the percent of errors observed during the current month, i , has deviated from normal to such an extent that corrective action should be taken.

Initial Values for Sampling

Initial values obtained from a 100 percent physical inventory are stored and updated from month to month. The values are the best estimates of population parameters. There are 13 stored values.

$\hat{\sigma}_t$ = estimate of total dollar difference standard deviation for the total population.

$\hat{\sigma}_m$ = estimate of material dollar difference standard deviation for the total population.

$\hat{\sigma}_l$ = estimate of labor dollar difference standard deviation for the total population.

$\hat{\sigma}_{tj}$ = estimate of total dollar difference standard deviation for stratum j .

$\hat{\sigma}_{mj}$ = estimate of material dollar difference standard deviation for stratum j .

$\hat{\sigma}_{lj}$ = estimate of labor dollar difference standard deviation for stratum j .

$\hat{\mu}_t$ = estimate of the mean value of total dollar difference for the total population.

$\hat{\mu}_m$ = estimate of the mean value of material dollar difference for the total population.

$\hat{\mu}_l$ = estimate of the mean value of labor dollar difference for the total population.

$\hat{\mu}_{tj}$ = estimate of the mean value of total dollar difference for stratum j .

$\hat{\mu}_{mj}$ = estimate of the mean value of material dollar difference for stratum j.

$\hat{\mu}_{lj}$ = estimate of the mean value of labor dollar difference for stratum j.

n_{ji} = sample size taken from stratum j in month i. Only the samples taken during the previous year are saved.

Estimating Mean Dollar Differences

Three values are obtained from each job in the sample.

v_{mjk} = material dollars difference of job k in stratum j.

v_{ljk} = labor dollars difference of job k in stratum j.

$v_{tjk} = v_{mjk} + v_{ljk}$ = total dollars difference of job k in stratum j.

For each stratum j, and for the total population, the sample means and extended value estimates are computed as shown below.

For each stratum j:

$$m_{mj} = \frac{\sum_k v_{mjk}}{n_j},$$

$$e_{mj} = N_j m_{mj},$$

$$m_{lj} = \frac{\sum_k v_{ljk}}{n_j},$$

$$e_{lj} = N_j m_{lj},$$

$$m_{tj} = m_{mj} + m_{lj}, \text{ and}$$

$$e_{tj} = N_j m_{tj}.$$

For the total population:

$$m_m = \frac{\sum_j \sum_k v_{mjk}}{n},$$

$$e_m = Nm_m,$$

$$m_l = \frac{\sum_j \sum_k v_{ljk}}{n},$$

$$e_l = Nm_l,$$

$$m_t = m_m + m_l, \text{ and}$$

$$e_t = Nm_t.$$

Figure 1 is an example of a group (stratum) statistical output from the sample. The sample output will be obtained for each stratum and for the total population.

The m values (sample means) are printed for items A, E, and I in Figure 1. The e values (extended dollars) are printed for items B, F, and J in Figure 1.

New estimates of the stratum and population means ($\hat{\mu}$) are obtained using a weighted average of the old $\hat{\mu}$ values and the sample means (m values) computed above; that is, single exponential smoothing with smoothing constant α . In the equations below the new estimates are on the left hand side and the old $\hat{\mu}$ values are on the right hand side. The new values of $\hat{\mu}$ are used in computing the new estimate of extended dollars E .

For each stratum j :

$$\hat{\mu}_{mj} = \alpha m_{mj} + (1-\alpha) \hat{\mu}_{mj},$$

$$E_{mj} = N_j \hat{\mu}_{mj},$$

$$\hat{\mu}_{lj} = \alpha m_{lj} + (1-\alpha) \hat{\mu}_{lj},$$

$$E_{1j} = N_j \hat{\mu}_{1j},$$

$$\hat{\mu}_{tj} = \alpha m_{tj} + (1-\alpha) \hat{\mu}_{1j}, \text{ and}$$

$$E_{tj} = N_j \hat{\mu}_{tj}.$$

GROUP \$2000 TO \$3000		
N _j = 1000		
n _j = 31		
(AA) n _{jT} = 432		
AVERAGE MATERIAL DOLLAR DIFFERENCE	PER JOB	EXTENDED VALUE
THIS MONTH	(A) \$3.72	(B) \$3720.
AVERAGE TO DATE	(C) \$2.56	(D) \$2560.
AVERAGE LABOR DOLLAR DIFFERENCE		
THIS MONTH	(E) \$1.16	(F) \$1160.
AVERAGE TO DATE	(G) \$.98	(H) \$ 980.
AVERAGE TOTAL DOLLAR DIFFERENCE		
THIS MONTH	(I) \$4.88	(J) \$4880.
AVERAGE TO DATE	(K) \$3.54	(L) \$3540.
AVERAGE STANDARD DEVIATION		
MATERIAL	(M) \$1.01	(N) \$1010.
LABOR	(O) \$.25	(P) \$ 250.
TOTAL	(Q) \$1.26	(R) \$1260.
95 PERCENT CONFIDENCE LEVEL		
MATERIAL	(S) \$3.61	(T) \$3610.
LABOR	(U) \$1.37	(V) \$1370.
TOTAL	(W) \$4.98	(X) \$4980.
(Y) MATERIAL DOLLAR DIFFERENCE THIS MONTH IS SIGNIFICANTLY GREATER THAN AVERAGE TO DATE.		

Figure 1. Typical Report Output

For the total population:

$$\hat{\mu}_m = \alpha m_m + (1-\alpha)\hat{\mu}_m.$$

$$E_m = N\hat{\mu}_m.$$

$$\hat{\mu}_1 = \alpha m_1 + (1-\alpha)\hat{\mu}_1.$$

$$E_1 = N\hat{\mu}_1.$$

$$\hat{\mu}_t = \alpha m_t + (1-\alpha)\hat{\mu}_t, \text{ and}$$

$$E_t = N\hat{\mu}_t.$$

The $\hat{\mu}$ values (estimates of the mean) are printed for items C, G, and K in Figure 1. The E values (extended dollars) are printed for Items D, H and L in Figure 1.

Variance of Dollar Differences

The variance of the sample values is obtained and used to adjust the prior estimate of population variance. The adjustment is a weighted average of the sample variance (s^2) and the prior estimate (σ^2) of the population variance; that is, single exponential smoothing with smoothing constant β .

For each stratum j:

$$s_{mj}^2 = \frac{n_j \sum_k v_{mjk}^2 - \left(\sum_k v_{mjk} \right)^2}{n_j (n_j - 1)},$$

$$s_{lj}^2 = \frac{n_j \sum_k v_{ljk}^2 - \left(\sum_k v_{ljk} \right)^2}{n_j (n_j - 1)}.$$

$$s_{tj}^2 = \frac{n_j \sum_k v_{tjk}^2 - \left(\sum_k v_{tjk} \right)^2}{n_j(n_j-1)},$$

$$\hat{\sigma}_{mj} = [\beta s_{mj}^2 + (1-\beta)\hat{\sigma}_{mj}^2]^{1/2},$$

$$\hat{\sigma}_{lj} = [\beta s_{lj}^2 + (1-\beta)\hat{\sigma}_{lj}^2]^{1/2}, \text{ and}$$

$$\hat{\sigma}_{tj} = [\beta s_{tj}^2 + (1-\beta)\hat{\sigma}_{tj}^2]^{1/2}.$$

The $\hat{\sigma}$ values on the right hand side are the old values and those on the left hand side are the latest estimate.

For the total population:

$$s_m^2 = \frac{n \sum_j \sum_k v_{mjk}^2 - \left(\sum_j \sum_k v_{mjk} \right)^2}{n(n-1)},$$

$$s_l^2 = \frac{n \sum_j \sum_k v_{lkj}^2 - \left(\sum_j \sum_k v_{lkj} \right)^2}{n(n-1)},$$

$$s_t^2 = \frac{n \sum_j \sum_k v_{tjk}^2 - \left(\sum_j \sum_k v_{tjk} \right)^2}{n(n-1)},$$

$$\hat{\sigma}_m = [\beta s_m^2 + (1-\beta)\hat{\sigma}_m^2]^{1/2},$$

$$\hat{\sigma}_l = [\beta s_l^2 + (1-\beta)\hat{\sigma}_l^2]^{1/2}, \text{ and}$$

$$\hat{\sigma}_t = [\beta s_t^2 + (1-\beta)\hat{\sigma}_t^2]^{1/2}.$$

The new $\hat{\sigma}$ values are printed as Items M, O, and Q of Figure 1.

Each $\hat{\sigma}$ is now multiplied by the corresponding value of N to obtain the extended dollar values, S. The S values are printed as Items N, P, and R of Figure 1. The calculations for S values follow.

For each stratum j:

$$S_{mj} = N_j \hat{\sigma}_{mj}$$

$$S_{lj} = N_j \hat{\sigma}_{lj}, \text{ and}$$

$$S_{tj} = N_j \hat{\sigma}_{tj}$$

For the total population:

$$S_m = N \hat{\sigma}_m$$

$$S_l = N \hat{\sigma}_l, \text{ and}$$

$$S_t = N \hat{\sigma}_t$$

Confidence Levels

The lower 95 percent confidence limits are computed for material, labor, and total dollar differences in each stratum and for the total population. The per job and extended dollar value confidence limits are computed and printed as Items S through X in Figure 1.

The value printed, say Item S = \$3.61, means there is 95 percent confidence that the average material dollar difference between record inventory and physical inventory is less than \$3.61 per job. The statement applies to stratum j. A similar output will give a dollar value for the total population.

Computations for the confidence limits are carried out as follows.

For each stratum j:

$$\text{Compute } n_{jT} = \sum_{k=1}^{12} n_{jk}$$

n_{jT} is the sum of the sample sizes taken from stratum j during the last 12 months. n_{jT} is printed as Item AA of Figure 1.

The 95 percent confidence limit values, γ , for the means, $\hat{\mu}$, of differences in unit (per job) total dollars, labor dollars, and material dollars are computed for each stratum j and for the population.

For each stratum j:

$$\gamma_{tj} = \hat{\mu}_{tj} + \frac{1.96}{\sqrt{n_{jT}}} \hat{\sigma}_{tj},$$

$$\gamma_{1j} = \hat{\mu}_{1j} + \frac{1.96}{\sqrt{n_{jT}}} \hat{\sigma}_{1j}, \text{ and}$$

$$\gamma_{mj} = \hat{\mu}_{mj} + \frac{1.96}{\sqrt{n_{jT}}} \hat{\sigma}_{mj}.$$

For the population:

$$\gamma_t = \hat{\mu}_t + \frac{1.96}{\sqrt{n_T}} \hat{\sigma}_t,$$

$$\gamma_1 = \hat{\mu}_1 + \frac{1.96}{\sqrt{n_T}} \hat{\sigma}_1, \text{ and}$$

$$\gamma_m = \hat{\mu}_m + \frac{1.96}{\sqrt{n_T}} \hat{\sigma}_m.$$

The computed values of γ are printed as Items S, U, and W of Figure 1.

For the 95 percent confidence limit values, Γ , of the extended dollars, the following are computed and printed as Items T, V, and X of Figure 1.

For each stratum j:

$$\Gamma_{tj} = N_j \gamma_{tj}$$

$$\Gamma_{tj} = N_j \gamma_{tj}, \text{ and}$$

$$\Gamma_{mj} = N_j \gamma_{mj}.$$

For the population:

$$\Gamma_t = N \gamma_t,$$

$$\Gamma_1 = N \gamma_1, \text{ and}$$

$$\Gamma_m = N \gamma_m.$$

Decision Criteria

Management should be alerted if there has been a significant change in the average dollar difference in the current month. Comparisons are made utilizing the computed 95 percent confidence limits to determine if the latest sample differs significantly.

The following comparison tests are made on unit (per job) dollar differences. If the test fails, an appropriate comment is printed as Item Y in Figure 1.

For each stratum j:

$$\text{If } m_{tj} > \gamma_{tj},$$

$$m_{1j} > \gamma_{1j}, \text{ or}$$

$$m_{mj} > \gamma_{mj}.$$

For the population:

$$\text{If } m_t > \gamma_t,$$

$$m_1 > \gamma_1, \text{ or}$$

$$m_m > \gamma_m.$$

Smoothing Constant Selection

The α and β factors used in the exponential smoothing formulae will determine the response time to process changes. Response time is the number of months required for the population estimate to approach the true value within 10 percent, given that a step function change has occurred. Large factors improve the response time while increasing the variance of the estimate. A tradeoff between response time and estimate variance must be made.

The variance of the sample variance, $\text{Var}(s^2)$, is also dependent upon the sample size taken each month. A rule of thumb of four months was set for response time to step function change in stratum variance. Since each stratum will have a different sample size in addition to a different tendency toward variance change, a simulation program was written to test various values of β .

Appendix B is the program listing of the simulation program which was written in the BASIC language.

A number of simulation runs were performed using the various strata sizes and initial sample sizes to determine an acceptable value for β . A sample of 3 percent of the stratum was used as a basis for evaluation. As a result, a β value of 0.2 will be used.

ACCOMPLISHMENTS

A theoretical basis for in-process product sampling has been established. A sound foundation is provided for implementation of a system which results in considerable cost savings. The practicalities of sampling in a constantly changing product mix have been considered and included in the plan. Continuous updating of estimates of dollar differences between record and physical inventory is possible with the outlined approach. Management control is provided by decision criteria which alert management to deviations.

FUTURE WORK

The sampling process will be monitored for possible changes in the smoothing constants used for estimation of strata means and variances. The definition of deviations may be adjusted as a result of experience and the degree of management control desired.

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Appendix A

POPULATION VARIANCE PROGRAM

Statistical data collected from the 100 percent physical inventory is shown in Tables 2, 3 and 4 of the text. A computer program, written in the BASIC language for remote terminal operation, was necessary to compute the total population mean, variance and average dollar value per item. In addition, the program calculates the sample size for each stratum in the current month. The sample output from the program and program listing is shown below. Data shown is from Table 2.

COMPUTE POPULATION VARIANCE, STD DEV, MEAN, AND
AVERAGE DOLLAR VALUE. COMPUTES GROUP SAMPLE SIZES.

INPUT PERCENT CLOSENESS DESIRED = ? .5
INPUT NUMBER OF GROUPS = ? 10

POPULATION STATISTICS				
N	MEAN	STD DEV	VARIANCE	AVERAGE \$
6680	12.0782	113.543	12891.9	1919.67
GROUP	POPULATION #	SAMPLE #	PERCENT	
1	78	3	3.8	
2	24	3	12.5	
3	125	3	2.4	
4	212	3	1.4	
5	572	5	.8	
6	739	4	.5	
7	321	3	.9	
8	1043	12	1.1	
9	1181	5	.4	
10	2385	15	.6	
TOTAL	6680	56	.8	

INPUT PERCENT CLOSENESS DESIRED = ? .3

GROUP	POPULATION #	SAMPLE #	PERCENT
1	78	3	3.8
2	24	3	12.5
3	125	4	3.2
4	212	5	2.3
5	572	12	2
6	739	11	1.4
7	321	8	2.4
8	1043	34	3.2
9	1181	12	1
10	2385	41	1.7
TOTAL	6680	133	1.9

INPUT PERCENT CLOSENESS DESIRED = ? 0

```

100=100 DATA 78,24,125,212,572,739,321,1043,1181,2385
101=101 DATA 24938.6,39.4675,66854.9
102=102 DATA 5.65551,.439713,8853.05
103=103 DATA 22388.5,13.9959,14988.4
104=104 DATA 16731.1,16.3852,5546.08
105=105 DATA 12656.8,12.2399,2158.52
106=106 DATA 6816.76,6.29612,1844.67
107=107 DATA 18735.2,-10.859,611.559
108=108 DATA 32325.7,17.1284,678.599
109=109 DATA 3079.71,9.96668,441.971
110=110 DATA 9229.16,14.4932,135.418
200=200 DIM V(20),N(20),M(20),D(20),M1(20),D1(20),S(20),R(20)
210=210 PRINT "COMPUTE POPULATION VARIANCE, STD DEV, MEAN, AND "
220=220 PRINT "AVERAGE DOLLAR VALUE. COMPUTES GROUP SAMPLE SIZES."
230=230 PRINT
240=240 PRINT
250=250 RESTORE
260=260 PRINT "INPUT PERCENT CLOSENESS DESIRED = ";
270=270 INPUT C
280=280 IF C= 0 THEN 999
290=290 LET C=C/100
300=300 IF Z9= 1 THEN 670
310=310 PRINT "INPUT NUMBER OF GROUPS = ";
320=320 INPUT J
330=330 PRINT
340=340 PRINT
350=350 PRINT
360=360 FOR T= 1 TO J
370=370 READ N(T)
380=380 NEXT T
390=390 FOR T= 1 TO J

```

```

400=400 READ V(T),M(T),D(T)
410=410 NEXT T
420=420 LET N1= 0
430=430 LET M2= 0
440=440 LET D2= 0
450=450 LET S3= 0
460=460 LET Q= 0
470=470 FOR I= 1 TO J
480=480 LET S(I)= SQR(V(I))
490=490 LET M1(I)= N(I)*M(I)
500=500 LET D1(I)= N(I)*D(I)
510=510 LET N1=N(I)
520=520 LET M2= M2+M1(I)
530=530 LET D2= D2+D1(I)
540=540 LET S2= ((N(I)*(N(I)-1)*V(I))+(M1(I)**2))/N(I)
550=550 LET S3= S3+S2
560=560 LET Q= Q+(N(I)*S(I))
570=570 NEXT I
580=580 LET V1= ((N1*S3)-(M2*M2))/(N1*(N1-1))
590=590 LET D3= D2/N1
600=600 LET M3= M2/N1
610=610 LET S4= SQR(V1)
620=620 IF Z9= 1 THEN 670
630=630 PRINT "          POPULATION STATISTICS"
640=640 PRINT "N","MEAN","STD DEV","VARIANCE","AVERAGE $"
650=650 PRINT
660=660 PRINT N1,M3,S4,V1,D3
670=670 PRINT
680=680 PRINT
690=690 LET N2= ((1.96*S4)/(C*D3))**2
700=700 LET N2= INT(1+N2/12)
710=710 LET N3= 0
720=720 FOR I= 1 TO J
730=730 LET R(I)= (N2*N(I)*S(I))/Q
740=740 LET R(I)= INT(1+R(I))
750=750 IF R(I) 2 THEN 770
760=760 LET R(I)= 3
770=770 LET N3= N3+R(I)
780=780 NEXT I
790=790 PRINT "GROUP","POPULATION #","SAMPLE #","PERCENT"
800=800 PRINT
810=810 FOR I= 1 TO J
820=820 PRINT I,N(I),R(I),INT(1000*R(I)/N(I))/10
840=840 NEXT I
850=850 PRINT
860=860 PRINT "TOTAL",N1,N3,INT(1000*N3/N1)/10
870=870 PTINY
880=880 LET Z9=1
890=890 PRINT
900=900 PRINT
910=910 GO TO 240
999=999 END

```

Appendix B

SMOOTHING CONSTANTS PROGRAM

The following computer program in the BASIC language was written to simulate the use of different smoothing constants. A sample output is provided where the standard deviation of the changed population is equal to two (the standard deviation of the population is always equal to one for starting purposes). The example is for a population of 100 items and sample equal to 3 percent of the population. Five months of random samples from a population whose mean is zero and standard deviation is two is shown.

The value S2 is the sample variance and S3 is the sample standard deviation. Ten values follow each random sample. These are the exponentially smoothed values (or best estimates of population standard deviation) using progressively larger values for the weight given the sample standard deviation.

It is seen that more rapid response to the change is possible using larger values. However, the last figures (standard deviation of the variances) show that a greater variability in the estimate is the price one pays for this rapid response.

RUN B

PROGRAM COMPUTES THE STANDARD DEVIATION OF THE SAMPLE AND THE WEIGHTED STANDARD DEVIATION OF THE SAMPLE AND THE POPULATION.

THE WEIGHT GIVEN TO THE SAMPLE IS FROM .1 TO .28 IN STEPS OF .02 I.E. EXPONENTIAL SMOOTHING OF THE VARIANCES.

THE STANDARD DEVIATION OF THE SMOOTHED VALUES IS GIVEN AT THE END.

INPUT STANDARD DEVIATION OF CHANGED POPULATION = ? 2

INPUT N, PERCENT SAMPLE = ? 100,3

INPUT FIRST RANDOM NUMBER = ? 0

INPUT NUMBER OF RUNS =, NUMBER OF MONTHS = ? 1,5

S2= 6.25

S3= 2.5

1.23

1.28

1.32

1.36

1.39

1.43

1.47

1.5

1.54

1.57

S2= 19.19

S3= 4.38

1.81

1.93

2.04

2.15

2.25

2.34

2.43

2.51

2.6

2.67

S2= .19	S3= .44			
1.73	1.82	1.9	1.98	2.04
2.1	2.16	2.2	2.24	2.28

S2= 8.52	S3= 2.92			
1.88	1.98	2.08	2.16	2.23
2.29	2.35	2.39	2.44	2.48

S2= 3.15	S3= 1.77			
1.87	1.96	2.04	2.1	2.15
2.2	2.23	2.26	2.28	2.3

STD DEV OF VARIANCES

.84	.96	1.07	1.18	1.28
1.38	1.47	1.56	1.65	1.74

```

10=10 PRINT "PROGRAM COMPUTES THE STANDARD DEVIATION OF THE ";
20=20 PRINT "SAMPLE AND THE WEIGHTED"
30=30 PRINT "STANDARD DEVIATION OF THE SAMPLE AND THE POPULATION."
40=40 PRINT " THE WEIGHT GIVEN TO THE SAMPLE IS FROM .1 TO .28";
50=50 PRINT "IN STEPS OF .02"
60=60 PRINT " I.E. EXPONENTIAL SMOOTHING OF THE VARIANCES."
70=70 PRINT " THE STANDARD DEVIATION OF THE SMOOTHED VALUES IS ";
80=80 PRINT "GIVEN AT THE END."
90=90 PRINT
100=100 PRINT "INPUT STANDARD DEVIATION OF CHANGED POPULATION =";
110=110 INPUT B2
120=120 DIM U2(1,10)
130=130 DIM V3(1,10)
140=140 LET A= .02
150=150 PRINT "INPUT N, PERCENT SAMPLE =";
160=160 INPUT N,P
170=170 LET P= P/100
180=180 PRINT "INPUT FIRST RANDOM NUMBER =";
190=190 INPUT X
200=200 PRINT "INPUT NUMBER OF RUNS =, NUMBER OF MONTHS =";
210=210 INPUT B,B1
220=220 PRINT
230=230 LET M= INT(.99+N*P)
240=240 FOR W= 1 TO B
250=250 FOR K= 1 TO 10
260=260 LET V(K)= 1
270=270 LET H(K)= 0

```

```

280=280 LET L(K)= 0
290=290 NEXT K
300=300 FOR J= 1 TO B1
310=310 LET S= 0
320=320 LET S1= 0
330=330 FOR K= 1 TO M
340=340 LET Z= 0
350=350 FOR I=1 TO 12
360=360 LET Y= RND(X)
370=370 LET Z= Y+Z-.5
380=380 NEXT I
390=390 LET Z= B2*Z
400=400 LET S= S+Z
410=410 LET S1= S1+Z*Z
420=420 NEXT K
430=430 LET S2= (M*S1-S*S)/(M*(M-1))
440=440 FOR K= 1 TO 10
450=450 LET V(K)= ((.92-A*K)*V(K)+(.08+A*K)*S2)
460=460 LET V2(K)= SQR(V(K))
470=470 LET V2(K)= INT(100*V2(K)+.5)/100
480=480 LET H(K)= H(K)+V(K)
490=490 LET L(K)=V(K)+V(K)
500=500 NEXT K
510=510 LET S2= INT(100*S2+.5)/100
520=520 LET S3= INT(100*SQR(S2)+.5)/100
530=530 PRINT "S2= ";S2,"S3= ";S3
540=540 MAT V3= TRN(V2)
550=550 MAT PRINT V3
560=560 NEXT J
570=570 FOR K= 1 TO 10
580=580 LET U(K)= (B1*L(K)-H(K)*H(K))/(B1*(B1-1))
590=590 LET U(K)= SQR(U(K))
600=600 LET U1(K)= INT(100*U(K)+.5)/100
610=610 NEXT K
620=620 PRINT
630=630 PRINT "STD DEV OF VARIANCES"
640=640 MAT U2= TRN(U1)
650=650 MAT PRINT U2
660=660 NEXT W
999=999 END

```


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