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AND ITS APPLICATION TO
PLASMA HEATING

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Parametric Decay of "Kinetic Alfvén Wave" and its Application to Plasma Heating

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ABSTRACT

Owing to its electrostatic properties, the rate of the parametric growth of the kinetic Alfvén wave [Alfvén wave with $\lambda \equiv k_{\perp}^2 c_s^2 / \omega_{ci}^2 \sim 0$ (1)] is shown to be larger than that of the classic result by a factor of $\lambda \omega_{ci} / \omega_A$ where ω_{ci} , ω_A are the ion cyclotron and the Alfvén frequencies, $k_{\perp}$ is the perpendicular wave number and c_s is the ion sound speed. This fact can be utilized to heat a plasma with a tokamak-type parameter.

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In the previous report,¹ we have shown that a plasma with a tokamak parameter can be heated by the kinetic Alfvén wave which can be excited in the plasma through the resonant mode conversion. The Alfvén wave which is excited by the resonant mode conversion has a perpendicular wave length, $k_{\perp}^{-1}$, comparable to the ion gyroradius, ρ_i , and accompanies an electrostatic component. Hence, it is called the kinetic Alfvén wave here. The dispersion relation of the kinetic Alfvén wave is given by¹

$$\begin{aligned}\omega^2 &= k_{\parallel}^2 v_A^2 \left[1 + \left(\frac{3}{4} + \frac{T_e}{T_i} \right) k_{\perp}^2 \rho_i^2 \right], \quad k_{\perp} \rho_i^2 \ll 1 \\ &= k_{\parallel}^2 v_A^2 \left(1 + \frac{T_e}{T_i} \right) k_{\perp}^2 \rho_i^2, \quad k_{\perp}^2 \rho_i^2 \gg 1.\end{aligned}\quad (1)$$

It was shown in the linear analysis that both electrons and ions are heated in a collisional regime, while only electrons are heated in a collisionless regime by the electron Landau damping.

In this report we show that, in a collisionless regime, ion heating is possible by utilizing either the parametric decay to the ion acoustic wave if $T_e \gg T_i$, or the ion nonlinear Landau damping if $T_e \lesssim 5T_i$.

Decay of a conventional Alfvén wave into another Alfvén wave and an ion acoustic wave has been studied by Sagdeev and Galeev² using the ideal MHD equations. Applications of this process for heating plasmas have been suggested by Vohala and Montgomery³ and Rashmore-Davies and Ong.⁴ However, because the growth rate is so small ($\sim \omega_0 \tilde{B}/B_0$; ω_0 is the pump frequency, $\tilde{B}$ the pump amplitude and B_0 the ambient magnetic field) that the actual application is considered difficult. Furthermore, if $T_e \lesssim 5T_i$, the ion acoustic wave is heavily ion Landau damped and such a decay becomes practically absent.

We show here that the parametric decay of the kinetic Alfvén wave to the ion acoustic wave has a growth rate larger than the Sagdeev-Galeev result by at least a factor of ω_{ci}/ω_A , where ω_A is the Alfvén wave frequency. This fact makes its application to the heating more realistic. We further show that even if $T_e \leq 5T_i$, where the decay is absent, the ion nonlinear Landau damping which has a large growth rate $(\omega_{ci}/\omega_A)^2 \beta^{-1}$ can be utilized to heat ions.

Such enhanced growth rates appear through the coupling due to the $\tilde{\mathbf{E}} \times \mathbf{B}_0$ and $v_{\parallel} \tilde{\mathbf{B}}/B_0$ drifts by the pump field $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$. These drifts can produce a charge separation which can not be cancelled owing to the effects of the finite ion gyroradius and finite electron inertia which are absent in the classic ideal MHD treatment.

We assume the plasma has a $\beta \sim (m_e/m_i)^{1/2}$.

In this case the compressional perturbation of the magnetic field B_z may be negligible. Hence, we use the classic two potential field,⁵ $E_z = -\partial\psi/\partial z$ and $\underline{E}_\perp = -\nabla_\perp\phi$ where z and $\perp$ are coordinates parallel and perpendicular to the ambient magnetic field. The field equation can then be written as¹

$$n_i^{(1)} + n_i^{(2)} = n_e^{(1)} + n_e^{(2)}, \quad (2)$$

$$\frac{\partial}{\partial z} \nabla_\perp^2 (\phi - \psi) = \mu_0 \frac{\partial}{\partial t} \left[J_{ze}^{(1)} + J_{zi}^{(1)} + J_{ze}^{(2)} + J_{zi}^{(2)} \right], \quad (3)$$

where subscripts i and e are for ions and electrons, n and J are number density and current density perturbations with the superscripts (1) and (2) showing the first and the second order perturbations in the wave amplitude.

For the kinetic equation, we use the drift kinetic equation,⁶

$$\frac{\partial f}{\partial t} + \underline{\nabla} \cdot (\underline{v}_d f) + \frac{q}{m} [E_z + (\underline{v}_d \times \underline{B}_\perp) \cdot \underline{e}_z] \frac{\partial f}{\partial v_z} = 0, \quad (4)$$

where

$$\underline{v}_d = v_z \underline{e}_z + \frac{\underline{E}_\perp \times \underline{B}_0}{B_0^2} + \frac{m}{qB_0} \frac{d\underline{E}_\perp}{dt} + \underline{v}_z \frac{\underline{B}_\perp}{B_0}, \quad (5)$$

$f(\underline{x}, v_z, t)$ is the drift distribution and the rest of the notations are standard.

The linear response for the electron and ion distribution functions, $f_e^{(1)}$ and $f_i^{(1)}$, are obtained from Eq. (4) in their Fourier amplitudes;

$$f_{ek}^{(1)} = f_e^{(0)} \bar{\psi}_k, \quad (6)$$

and

$$f_{ik}^{(1)} = -\lambda f_i^{(0)} \bar{\phi}_k + c_s^2 \frac{\partial f_i^{(0)} / \partial v_z}{v_z - \omega / k_z} \bar{\psi}_k, \quad (7)$$

where $\bar{\psi} = e\psi/T_e$, $\bar{\phi} = e\phi/T_e$, $c_s^2 = T_e/m_i$, and $\lambda = k_{\perp}^2 c_s^2 / \omega_{ci}^2$. Substitution of these distribution functions into the field equations (2) and (3) yields the linear dispersion relations which can be decoupled; for the kinetic Alfvén wave,

$$\epsilon_A(\omega, k) \equiv 1 - \frac{k_z^2 v_A^2}{\omega^2} (1 + \lambda_A) = 0, \quad (8)$$

$$\bar{\psi}_k = -\lambda_A \bar{\phi}_k, \quad (9)$$

and for the ion acoustic wave,

$$\epsilon_s(\omega, k) \equiv 1 + \lambda_s + \chi_1 = 0, \quad (10)$$

$$\bar{\psi}_k = \bar{\phi}_k, \quad (11)$$

and

$$\chi_i = - c_s^2 \int \frac{\partial f_i^{(0)} / \partial v_z}{v_z - \omega / k_z} dv_z . \quad (12)$$

We now introduce the parametrically coupled equations between these two modes through the pump potential $\bar{\phi}_0$, where $\bar{\phi}_0$ is related to the transverse component of the wave magnetic field $\tilde{B}_\perp$ of the pump through

$$\begin{aligned} \frac{\tilde{B}_\perp}{B_0} &= \frac{i(k_{\perp 0} \times k_{\parallel 0})}{\omega_0 \omega_{ci}} c_s^2 (\bar{\phi}_0 - \bar{\psi}_0) \\ &= \frac{i(k_{\perp 0} \times k_{\parallel 0})}{\omega_0 \omega_{ci}} c_s^2 (1 + \lambda_0) \bar{\phi}_0 , \end{aligned} \quad (13)$$

and the subscript 0 refers to the pump quantities.

There are many nonlinear terms that contribute to the coupling and the derivation of the coupling coefficient is nontrivial. However, we leave the derivation to the extended paper to be published elsewhere.⁷ Instead, we state the important terms which contribute to the coupling coefficient when λ is large (larger than ω / ω_{ci}). These are $\tilde{v}_E (= \tilde{E}_\perp \times B_0 / B_0^2)$ times the second term in Eq. (7) (the product of $\tilde{v}_E$ and the first term in Eq. (7) cancels with $\tilde{v}_E \cdot \nabla E_\perp$ term which originates from the ambipolar drift term) and $(\tilde{v}_E \times B_\perp) \cdot \partial f_i^{(0)} / \partial \tilde{v}_z$ for the ions. For the electrons,

in addition to the corresponding terms the term $v_z B_1 / B_0 f_e^{(1)}$ turns out to be crucial.

The coupled equations then read

$$\begin{aligned} \varepsilon_A(\omega, \underline{k}) \bar{\varphi}_{\underline{k}} &= \Lambda_A \bar{\varphi}_0 \bar{\varphi}_{\underline{k}-\underline{k}}^* \\ \varepsilon_s(\omega_0 - \omega, \underline{k}_0 - \underline{k}) \bar{\varphi}_{\underline{k}_0 - \underline{k}} &= \Lambda_s \bar{\varphi}_0 \bar{\varphi}_{\underline{k}}^* , \end{aligned} \quad (14)$$

where

$$\Lambda_A = \Lambda \frac{1}{\omega_A \lambda_A (1 + \lambda_A)} , \quad (15)$$

$$\Lambda_s = \Lambda \frac{1}{k_{zs}} \left(\frac{k_{z0}}{\omega_0} - \frac{k_{zA}}{\omega_A} \right) , \quad (16)$$

and

$$\Lambda = ic_s^2 \frac{(\underline{k}_A \times \underline{k}_s) \cdot \underline{e}_B}{2\omega_{ci}} [(1 + \lambda_0)(1 + \lambda_A) + \chi_i] . \quad (17)$$

In these expressions subscripts A and s refer to the kinetic Alfvén and the ion acoustic waves respectively, $\underline{e}_B$ is the unit vector in the direction of the ambient magnetic field B_0 .

Let us first look at the resonant decay process. This applies to a case with $T_e \gg T_i$ whereby the ion acoustic wave can exist. Then $\chi_i = -k_z^2 c_s^2 / \omega^2 = -(1 + \lambda_s)$. Unlike the MHD case in which only the backscattering is allowed, three different types of decay are possible here as shown in ω - k_z diagram in Fig. 1. The growth rate γ_D is obtained from the coupled equation (14),

$$\begin{aligned}
 \gamma_D &= [\omega_A \omega_s \Lambda_A \Lambda_s^* |\bar{\phi}_0|^2 / 4(1+\lambda_s)]^{1/2} \\
 &= \frac{\omega_{ci}}{4} \left[\frac{\omega_s \lambda_s}{\omega_A \lambda_0 (1+\lambda_0) (1+\lambda_s) (1+\lambda_A)} \right]^{1/2} \frac{v_A}{c_s} |\sin \theta| (1+\lambda_A) (1+\lambda_0) \\
 &\quad - (1+\lambda_s) \left| \frac{\tilde{B}_1}{B_0} \right|,
 \end{aligned}
 \tag{18}$$

where θ is the angle between k_{1A} and k_{1s} . The growth rate obtained above is larger than that derived by Sagdeev and Galeev² by a factor of $\lambda \omega_{ci} / \omega_A$ as is expected.

Let us now consider the nonlinear Landau damping process by the ions. This corresponds to a case with $T_e \lesssim 5T_i$. Note that the ion acoustic wave considered here has a phase velocity $c_s(1+\lambda_s)^{-1/2}$ which is smaller than c_s and hence is more likely subject to the ion Landau damping. To obtain the growth rate for a nonlinear Landau damping one normally must include a higher order response, $f^{(3)}$. However, because of the combined nonlinear contribution of electrons to the coupling, it turns out that this higher order response is unimportant. The growth rate of the kinetic Alfvén wave through the nonlinear Landau damping γ_N is then obtained again from Eq. (14), using $\omega_0 - \omega_A \sim (k_{z0} - k_{zA}) v_{Ti}$ (v_{Ti} is the ion thermal speed),

$$\begin{aligned}
 \gamma_N &= \omega_A \Lambda_A \Lambda_s^* |\bar{\phi}_0|^2 \frac{\text{Im} \chi_1}{2|\epsilon_s|^2} \\
 &= \frac{\omega_{ci} \lambda_s \sin^2 \theta}{8 \lambda_0 (1+\lambda_0) (1+\lambda_A)} \frac{\omega_{ci}}{\omega_0} \frac{v_A^2}{c_s^2} \frac{T_e/T_i [(1+\lambda_0)^2 (1+\lambda_A)^2 + T_e^2/T_i^2]}{(1+\lambda_s)^2 + T_e^2/T_i^2} \left| \frac{\tilde{B}_1}{B_0} \right|^2.
 \end{aligned}
 \tag{19}$$

Note here that the use of the drift kinetic equation did not allow the finite ion gyroradius effect which would reduce the coupling coefficient by a factor of $e^{-b} I_0(b)$, $b = k_1^2 \rho_1^2$. Hence, for a large value of λ in the above expression, γ_N is maximized roughly at $\lambda \sim T_e/T_i$.

For these growth processes to occur the pump field $\tilde{B}/B_0$ should exceed a threshold. For the case of the nonlinear Landau damping, the threshold value depends on the damping rate of the kinetic Alfvén wave δ_A (due to the linear electron Landau damping) which is on the order of 1%. Then for a reasonable choice of other parameters, the threshold amplitude of the magnetic field can be reduced to $\sim 10^{-2} B_0$, and the corresponding growth rate approaches the wave frequency. As was shown,¹ if the kinetic Alfvén wave is excited by the resonant mode conversion, its amplitude is enhanced by a factor $(\kappa\rho)^{-2/3}$ compared with the externally applied field, hence even with a field of several tens of gauss, the kinetic Alfvén wave will have an amplitude of several hundred gauss which can exceed the threshold of the nonlinear ion Landau damping.

The ion heating occurs due to the cascading of the nonlinear Landau damping. As in other cases of parametric heating,⁸ the heating rate of ions is roughly given by $\gamma(\tilde{B}^2/2\mu_0)(\omega_s/\omega_A) \text{ sec}^{-1}$ per unit volume.

In summary, we have shown that when $k_{\perp} c_s / \omega_{ci} \sim 0(1)$, the parametric excitation of the kinetic Alfvén wave has a growth rate larger by a factor of ω_{ci} / ω_A than the MHD result. The larger growth rate is due to the noncancelling charge separation created by the $\tilde{E} \times \tilde{B}$ drift of the pump field. Such enhanced growth rate and hence reduced threshold level can be utilized to heat ions in tokamak type plasmas even with $T_e \sim T_i$. If the pump field is applied through the resonant mode conversion the enhanced amplitude of the converted kinetic wave eases such a process.

ACKNOWLEDGMENTS

The research done by one of the authors (L. C.) was supported by U. S. Energy Research and Development Administration Contract E(11-1)-3073.

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Fig. 1. Three types of decay are possible from ω_0 to ω_A depending on the signs of k_{zs} and k_{zA} as well as the size of $\lambda_0 (= k_{i0}^2 c_s^2 / \omega_{ci}^2)$ with respect to $\lambda_A (= k_{iA}^2 c_s^2 / \omega_{ci}^2)$. In the MHD limit where $\lambda \rightarrow 0$, only case (c) is acceptable. In case (a), $|k_z|$ is decreased while in case (b) $|k_z|$ is increased in consequence of the decay.