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GEOMETRICAL THEORY OF NONLINEAR PHASE DISTORTION OF
INTENSE LASER BEAMS*

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ABSTRACT

Phase distortion arising from whole beam self-focusing of intense laser pulses with arbitrary spatial profiles is treated in the limit of geometrical optics. The constant shape approximation is used to obtain the phase and angular distribution of the geometrical rays in the near field. Conditions for the validity of this approximation are discussed. Geometrical focusing of the aberrated beam is treated for the special case of a beam with axial symmetry. Equations are derived that show both the shift of the focus and the distortion of the intensity distribution that are caused by the nonlinear index of refraction of the optical medium. An illustrative example treats the case of beam distortion in a Nd:Glass amplifier.

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129

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Introduction

Self-focusing of intense laser beams was first discussed over a decade ago.¹ Since that time it has been the subject of numerous experimental and theoretical investigations.² It has become customary to discuss this phenomenon in the context of the whole-beam and small-scale regimes; the two being distinguished largely by the spatial scale over which the focusing occurs. Intensity induced distortions resulting from whole beam self-focusing become important when a laser amplifier is operated well above the critical power^{3,4} but below threshold for precipitous filamentary beam breakup.⁵⁻⁸ Such distortions can give rise to complicated temporal and spatial variations of phase and intensity when the beam is brought to a focus.^{9,10} Interest in understanding and controlling whole-beam phase front aberrations has increased in recent time with the development of terawatt Nd:Glass laser systems for use in fusion research. Such systems are being designed for high irradiance of targets as small as 100 microns in diameter.

A full diffraction treatment of whole beam effects becomes necessary when near field intensity modulations arising from diffraction become appreciable or where regions near a caustic surface are being considered. This problem requires extensive numerical computation.^{11,12,13,14} In treating practical amplifier systems, however, a geometrical optics treatment can be used to obtain the aberrated phase front and its trajectory within the near field. Free space propagation codes which include diffraction can subsequently be used to examine far field behavior. The justification for a ray optics approach is two fold: (1) amplifiers are generally placed in the near field of limiting apertures, and (2) the magnitude of distortions of interest are of the order

of waves across apertures whose diameters are of the order of centimeters. The treatment below is, within stated limits of approximation, quite general and may be applied to beams with arbitrary spatial intensity profiles. This is an important feature since high energy systems often employ spatially non-Gaussian beams.

Ray Optics Approximation

In the ray approximation, the orthogonal trajectory and phase of the wavefront can be determined from the following equations:

$$\frac{d}{ds} \left(n \frac{dr}{ds} \right) = \nabla n(r) \quad (1)$$

$$\phi = \frac{2\pi}{\lambda} \int_c n(r) ds + \text{const.} \quad (2)$$

where ds is the differential path length along the ray, r the position vector of a point on the ray, dr/ds the unit vector tangent to the ray, c the optical path, and $n(r)$ the index of refraction at the vector position r . In the following analysis it is assumed that all rays are paraxial with respect to the z direction. Figure 1 is a schematic of the geometry employed for calculations that follow.

We now obtain an expression for the angular deviation of a ray from the $\hat{z}$ axis. This angle will be denoted as $\delta\theta$ and for small displacements of the ray can be approximated by

$$\delta\theta \approx \left| \frac{dr}{ds} \times \hat{z} \right|. \quad (3)$$

Using Eq. (1), this expression becomes

$$\delta\theta = \frac{1}{n(\underline{r})} \left| \hat{z} \times \int_c \nabla n(\underline{r}) ds \right| \quad (4)$$

which, after carrying out the vector operation, becomes

$$\delta\theta = \frac{1}{n(\underline{r})} \left| \int_c \nabla n(\underline{r}) ds \right|. \quad (5)$$

It is now convenient to write the index of refraction in the presence of an intense electromagnetic field as

$$n(\underline{r}) = n_0 + \gamma I(r_{\perp} z) \quad (6)$$

where $I(r_{\perp} z)$ is the field intensity for a wave propagating paraxially with respect to the z direction and $\gamma I(r_{\perp} z)$ the nonlinear contribution to the index. (It is implicitly assumed that $I(r_{\perp} z)$ and $n(\underline{r})$ vary with time). For field intensities that are of interest,

$$\frac{\gamma I(r_{\perp} z)}{n_0} \ll 1. \quad (7)$$

Substituting Eq. (6) into Eqs. (2) and (5) gives

$$\delta\theta_{NL} = \frac{\gamma}{n_0} \int_c \nabla I(r_{\perp} z) ds \quad (8)$$

and

$$\phi = \frac{2\pi n_0}{\lambda} \int_C ds + \Delta\phi_{NL} + \text{const.} \quad (9)$$

where

$$\Delta\phi_{NL} = \frac{2\pi \gamma}{\lambda} \int_C I(r_{\perp} z) ds \quad (10)$$

is a small intensity dependent contribution to the phase. In general, $\Delta\phi_{NL}$ will represent an aberration of the unperturbed low intensity wavefront.

Constant Shape Approximation

The integrals in Eq. (8) and (10) can be simplified if it is assumed that the spatial intensity profile of the beam retains its initial shape over the entire propagation path. Several authors have used this so-called "constant shape approximation" in their treatment of Gaussian beam propagation.^{4,15} The approximation is quite general, however, and only requires that the bending of the rays resulting from the nonlinearity be small compared to characteristic spatial dimension of the beam. This is equivalent to requiring

$$\left(\frac{dr_{\perp}}{dz} \right)^2 \ll 1, \quad (11)$$

at all points $(r_{\perp} z)$ along the propagation path.

Equation (11) allows the differential path length function, ds , to be simplified. Writing ds as

$$ds = dz \left(1 + \frac{dr_{\perp}}{dz} \cdot \frac{dr_{\perp}}{dz} \right)^{1/2}, \quad (12)$$

and noting that the lowest order term is quadratic in $|dr_{\perp}/dz|$ allows us to write

$$ds \sim dz \text{ (first order, const. shape).} \quad (13)$$

Further simplification of the integrals appearing in Eqs. (8) and (10) can be made by writing the intensity, $I(r_{\perp}z)$, for paraxial rays as

$$I(r_{\perp}z) = F(r_{\perp}/a(z)) I(z) \quad (14)$$

where $I(z)$ is the intensity on-axis and $F(r_{\perp}/a(z))$ is a radial intensity distribution. The justification for this approximation is found in the treatment of aberration free self-focusing¹⁶ where it is shown that an exact solution of the nonlinear propagation equations can be written in the form

$$I = I(z) F(r_{\perp}/a(z)) = \left(\frac{a(0)}{a(z)} \right) I_0 F(r_{\perp}/a(z)) \quad (15)$$

where

$$\frac{1}{a} \frac{da}{dz} = \frac{1}{2R(z)} \quad (16)$$

and the phase is

$$\phi = \frac{r_{\perp}^2}{2R(z)} + \rho(z). \quad (17)$$

The quantity $a(z)$ appearing in the above equations plays the same role as the waist size parameter in the self similar Laguerre-Gaussian modes encountered in linear diffraction theory; i.e., the transverse shape of the beam remains unchanged over the propagation path whereas its overall size at any point along the propagation path is determined by $a(z)$ and $R(z)$ is the dynamic radius of curvature. For paraxial rays it is easy to show that $r_{\perp}/a(z)$ is independent of z .

We now assume that the length of the propagation path is short enough that no appreciable change in the intensity profile takes place due to the accumulated nonlinear phase distortion. This is the essence of the constant shape approximation when treating the aberrations arising from self-focusing. This approximation allows us to substitute Eqs. (13) and (14) into Eqs. (8) and (10) to obtain

$$\delta\theta_{NL} = \frac{\gamma}{n_0} \left| \nabla_{\perp} F(r_{\perp}/a(z)) \right| \int_c I(z) dz \quad (18)$$

and

$$\Delta\phi_{NL} = \frac{2\pi\gamma}{\lambda} F(r_{\perp}/a(z)) \int_c I(z) dz. \quad (19)$$

Equations (18) and (19) provide a simple means of estimating intensity induced phase changes and the concomitant angular deviation of the rays in the near field. These equations can be used to evaluate paraxial beams with arbitrary radial profiles (not restricted to axial symmetry) and provide a basis for choosing profiles and/or corrective elements that will minimize whole beam effects.

Equation 18 is similar to that used by Dabby et.al. in their analysis of the steady state thermal distortion of a low power Argon laser beam in lead glass.¹⁷

Limitations of the Constant Shape Approximation

Although Eqs. (18) and (19) are applicable for a wide range of beam and system parameters, two aspects concerning their limitations should be addressed. The first concerns the range of values for which the linear approximation to the differential path length function, ds , is valid. This can be investigated by extending the calculation of $\Delta_{NL}\phi$ to second order.

Expanding Eq. (12) and retaining the quadratic term yields

$$ds = dz \left[1 + \frac{1}{2} \left(\frac{dr}{dz} \right)^2 \right]. \quad (20)$$

Noting that $dr_L/dz \approx \delta_{NL}\theta$ from (3) we write

$$ds \approx dz \left[1 + \frac{\delta_{NL}^2 \theta^2}{2} \right]. \quad (21)$$

Substituting Eq. (21) into Eq. (10) and using Eq. (18) for $\delta_{NL}\theta$ yields

$$\Delta_{NL}\phi = BF(u) \left[1 + \frac{\lambda^2 B^2}{8\pi^2 a^2 n_o^2} |v_u F(u)|^2 \right] \quad (22)$$

where

$$B \equiv \frac{2\pi\gamma}{\lambda} \int I(z) dz \quad (23)$$

and

$$u \equiv r_L/a \quad (24)$$

The first term in Eq. (22) is simply the constant shape solution as expressed in Eq. (19). The condition that must be satisfied for the constant shape approximation to retain its validity is now seen to be

$$\frac{\lambda^2 B^2}{8\pi^2 a^2 n_0^2} |\nabla_u F(u)|^2 \ll 1 \quad (25)$$

We can investigate (25) for the Nd:Glass amplifier ($\lambda = 1.06 \mu$, $\gamma = 4.4 \times 10^{-16} \text{ cm}^2/\text{W}$). For this system small scale beam instabilities limit the value of B that can be tolerated. Practical designs generally constrain this parameter to values of $B \lesssim 15$. The gradient of the spatial profile, $\nabla_u F(u)$, is a measure of how effectively the beam fills the clear aperture of an amplifier. The constraint imposed on filling the aperture is the need to avoid the growth of near field diffraction rings which in turn lead to small scale beam instabilities. It has been found that this gradient is less than 3 for beams which successfully meet this constraint.¹⁸ Limitations on the aperture size, a, generally follow from practical amplifier design considerations. It can range from about 0.1 cm in preamplifier stages to over 20 cm in disk power amplifiers. The considerations place an upper limit of about 10^{-5} on the correction factor of Eq. (25); hence, in treating whole-beam intensity induced aberrations it can generally be neglected.

The second limitation to the use of the constant shape approximation concerns the free space propagation of the beam. If the distance between nonlinear elements for which the approximation is being applied is sufficiently

long, adjacent rays, whose angular trajectories are given by Eq. (18) will intersect. As points of intersection develop forming a caustic surface the intensity and phase profiles will distort; fundamentally, geometrical optics no longer becomes applicable in this limit.

The distance, Z_c , at which adjacent rays intersect is easily determined. With the aid of Fig. 2, we have, for paraxial rays.

$$h + \Delta r_{\perp} = Z_c \delta_{NL} \theta(r_{\perp} + \Delta r_{\perp}) \simeq Z_c [\delta_{NL} \theta(r_{\perp}) + \nabla_{\perp} \delta \theta(r_{\perp}) \Delta r_{\perp}] \quad (26)$$

and

$$h = Z_c \delta_{NL} \theta(r_{\perp}). \quad (27)$$

Solving these equations for Z_c and using Eq. (18) for $\delta_{NL} \theta$ along with the substitutions of Eqs. (23) and (24) the crossing distance becomes

$$Z_c = \frac{a^2}{B \nabla_{\perp}^2 F(u)} \quad (28)$$

In Eq. (28), $\nabla_{\perp}^2 F(u)$ is a fundamental property of the undistorted intensity profile. The first ray crossings occur where this quantity is a maximum. The quantity a^2/B , however, depends on the details of amplifier staging. (Generally a increases as B increases). It is difficult to give a meaningful estimate of Z_c without considering a specific staging. For the

CYCLOPS system at Livermore (see example below), Z_c attains a minimum value of about 60 meters in early stages of amplification. It is easy to construct configurations, however, where the rays first cross at distances less than a meter. Such staging configurations are seldom practical.

Geometrical Focusing of the Distorted Beam

Although the complete mathematical description of how an aberrated light beam propagates after it passes through a focusing lens requires diffraction theory, three quantities of interest for a circular beam can be easily calculated with reasonable accuracy using only geometrical optics. These three quantities are:

- 1) The distance from the focal point of the lens that a ray crosses the optical axis of the system (this is measure of the dynamic motion of the focus).
- 2) The envelope of points of intersection in the focal volume of two adjacent rays in the exit pupil; i.e., the first caustic surface.
- 3) The distortion of the intensity distribution that results from the first caustic surface.

(In a recent publication Suydam has treated the far-field spot size for a gaussian beam with whole-beam nonlinear distortion.¹⁹⁾)

Figure 3 is a ray diagram that will be used to calculate the focal point shift ΔZ for the aberrated beam. (It should be recalled that the beam intensity $I(z)$ is time dependent and so ΔZ represents a dynamic shift of the focus). In the paraxial approximation, the phase ϕ associated the aberrated beam is given by

$$\phi = Z - \frac{r^2}{2f} - 8 F \left(\frac{r}{a} \right) \quad (29)$$

where f is the focal length of the lens and the origin of the coordinate system is chosen in the plane of the exit pupil.

The rays directions, $\underline{s}$, described by this phase front are given by

$$\underline{s} = \frac{\nabla\phi}{|\nabla\phi|}. \quad (30)$$

Invoking the paraxial ray or small angle approximation, (see Fig. 1), requires that

$$|\frac{\partial\phi}{\partial r}|^2 \ll 1 \quad (31)$$

or

$$\tan(\theta + \Delta\theta) \approx \theta + \Delta\theta \quad (32)$$

Observing that the transverse coordinate of the perturbed and unperturbed ray in the exit pupil are the same, we find that the focal point shift ΔZ is given by

$$\Delta Z = f \frac{\Delta\theta}{\theta}. \quad (33)$$

where second order quantities in Equation 30 have been set equal to zero.

Evaluating the above expression in focal volume units we obtain

$$\frac{k\Delta Z}{(f\#)^2} = 4 \, k_B \, \frac{1}{\mu} \, \frac{dF}{d\mu} \quad (34)$$

where $\mu = \frac{r}{a}$ and the F number, $f\#$, of the lens is defined as $f\# = f/2a$.

The envelope of intersection points in the focal volume of two rays which are adjacent in the exit pupil defines a caustic surface. These adjacent ray pairs and their intersection point define a ray tube. Since the energy in a ray tube is conserved, once the intersection point is obtained it is a simple matter to obtain the intensity in the ray tube.

Figure 4 shows such a ray tube along with the pertinent coordinates needed to obtain the point of intersection. At the point of intersection, the transverse position of the two rays is equal, hence

$$(Z' - \Delta Z) \tan \{ \theta(\mu + \Delta\mu) + \Delta\theta(\mu + \Delta\mu) \} = \{ Z' - \Delta Z - \Delta Z(\mu) + \Delta Z(\mu + \Delta\mu) \} \tan \{ \theta + \Delta\theta \} \quad (35)$$

where Z' is the longitudinal coordinate of the ray crossing measured from the focal point of the unaberrated beam. In the limit that $\Delta\mu \rightarrow 0$ Equation 35 becomes

$$(Z' - \Delta Z) = a^2 \mu \frac{\partial(\Delta Z)}{\partial \mu} \left(\frac{1 + \Delta Z/f}{1 + f/\phi \frac{\partial \Delta \theta}{\partial \mu}} \right) \quad (36)$$

Equation 35 can be further simplified by observing that the focal plane shift ΔZ is small compared to the focal length f of the lens and that the aberration B is small compared to the aperture size a . In this limit Equation 36 becomes

$$(Z' - \Delta Z) = a^2 \mu \frac{\partial(\Delta Z)}{\partial \mu} \quad (37)$$

Inserting the expression for ΔZ from Equation 34 into Equation 37 and converting to focal volume units we obtain

$$\frac{kZ'}{(f\#)^2} = 4 \text{ kB} \frac{d^2 F}{d\mu^2} \quad (38)$$

which is the equation for the caustic surface.

The intensity $F(\mu, Z)$ associated with the ray μ is now obtained by invoking conservation of energy in the ray tube with the result

$$(f\#)^2 \frac{F(\mu, Z)}{4(ka)^2} = \frac{F(\mu, Z_{\text{exit}})}{\left[\frac{kZ}{(f\#)^2} - 4 \text{ kB} \frac{d^2 F}{d\mu^2} \right]^2} \quad (39)$$

The transverse position of this ray in the plane $kZ/(f\#)^2 = \text{constant}$ is given by

$$\frac{kr}{f\#} = \frac{u}{2} \left[\frac{kZ}{(f\#)^2} - 4 \text{ kB} \frac{1}{u} \frac{dF}{du} \right] \quad (40)$$

Hence Equations 36 and 37 can be used to obtain the transverse intensity profile in the plane $kZ/(f\#)^2 = \text{constant}$.

Example

We apply the preceding formalism to the CYCLOPS Nd:Glass laser which is currently in operation at Livermore.²⁰ For this system, the gaussian beam from the oscillator and preamplifier stages is apodized to form a beam with a "super-gaussian" spatial profile with axial symmetry for injection into the power amplifiers. This beam shape is described by the following equation;

$$F(r/a) = \exp(-6.5(r/a)^5). \quad (41)$$

At full power (~ 1.5 terawatts) the value of B from equation (23) is about 2.2 waves ($\gamma = 4.4 \times 10^{-16} \text{ cm}^2/\text{watt}$) at the peak of the time pulse and at the exit aperture of the final 20 cm diameter power amplifier. Using these values, equations (18) and (19) yield the nonlinear phase retardation and angular distribution of the rays²¹ at the exit aperture of the system. These quantities are plotted in figure 5. From this figure it is noted that the peak angular deflection of the rays attains the rather substantial value of about 100 microradians. Moreover it is observed that appreciable deflections occur in spatial regions of the beam which are far removed from the axial rays but which contribute the preponderance of the output power.

Figure 6 shows the normalized axial shift of the geometrical foci as calculated using equation (34). Even for fast optical systems it is observed that the displacement of the rays is appreciable. An $f\#2$ system, for example,

displaces the rays by as much as 150 microns. This behavior clearly complicates the problem of determining the plane of best focus especially when uniform illumination of targets as small as 100 microns is desired.

The intensity distribution of the beam at various receiving planes following the focusing lens can be calculated using equation (39). Figure 7 depicts the evolution of the relative intensity pattern as the beam passes from a plane whose origin is at $kz/(f\#)^2 = -3000$ to a plane at -750 . The dominant feature displayed by these figures is the rapid transformation of the initially "super-gaussian" profile to an annular distribution as the geometrical focus is approached. Diffraction calculations show that this annular behavior is preserved in the intermediate field.²² However, at the "best" focus the intensity has changed to an "Airy-like" pattern. Similar calculations for an aberrated gaussian beam show that the gradient of the intensity increases near the axis resulting in an axial maxima in the near field but changes to an "Airy-like" pattern in the intermediate field up to the "best" focus. Beyond the "best" focus it assumes a doughnut-like structure. Structure of this type has been observed experimentally by Campillo et. al.⁹

The time dependence of the intensity induced aberration complicates the problem of locating the optimum focal plane for target irradiation as well as of specifying the surface figure of a corrective phase plate. A correction of the aberration at time t' can be made with a lens whose surface figure varies as $B(t') F(r/a)$. It seems reasonable that the best average correction however can be obtained by minimizing the energy weighted mean square phase variation over the temporal period of the pulse. For a gaussian pulse it is easy to show that this correction is made with a surface variation of $\phi_p/\sqrt{2} F(r/a)$ where ϕ_p is the axial phase retardation at the temporal peak of the pulse ($\phi_p = 2.2$ waves for the preceding example).

Concluding Remarks

In this paper it has been shown that aberrations arising from whole-beam self-focusing and their effect on the focusing properties of a high power laser beam can be treated analytically with geometrical optics providing:

- 1) conditions for the applicability of the constant shape approximation are upheld as the beam propagates through both the laser and the intervening medium between the laser and the final focusing lens (Equations 11, 25 and 28) and
- 2) focal regions of interest are forward of the first caustic surface.

These conditions are often met in the design of a fusion laser where amplifiers are placed within the near field, and targets in the intermediate field. For this case Equation 39 can be used to investigate transverse intensity profiles and to determine the best combination of beam shape, $F(r/a)$, and target position for a given irradiation geometry.

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FIGURE CAPTIONS

1. Geometry used to specify the trajectories of the geometrical rays as they pass through a medium exhibiting a nonlinear index of refraction. All rays are paraxial with respect to the central ray which propagates along the Z direction.
2. Geometry used to calculate the axial distance, Z_c , at which two adjacent rays intersect in free space propagation. The plane of the focusing lens is at $z = 0$.
3. Geometry used to calculate the focal point shift, ΔZ , for the aberrated beam. The plane of the focusing lens is at $z = 0$. The undeviated ray is assumed to cross the axis at a distance f from the lens.
4. Geometry used to calculate the point of intersection of two rays in the focal volume. The adjacent ray pairs and their intersection point define a ray tube. The plane of the focusing lens is at $z = 0$.
5. Calculated phase retardation and angular distribution of the geometrical rays at the exit aperture of the 20 cm diameter power amplifier on the CYCLOPS system. These quantities were calculated for the peak of the gaussian time pulse. The intensity distribution is that of the super-gaussian given by Eq. 41.
6. Normalized axial shift of the geometrical foci as calculated from Eq. 34 for the focused output of the CYCLOPS system. The same conditions as in Figure 5 are assumed.
7. Calculated intensity distribution of the beam at various receiving planes following the focusing lens. These figures depict the evolution of the relative intensity pattern as the beam passes from a plane whose origin is at $kZ/(f\#)^2 = -3000$ to a plane at -750 . The initial aberration is that shown in Figure 5.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7