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Energy Flux Distributions for Direct Converters

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ABSTRACT

Expressions are derived for the relative ion flux in the fans of a mirror fusion reactor. Numerical values are presented for typical reactor parameters.

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INTRODUCTION

An essential part of a mirror fusion reactor is the direct converter used to recycle the energy of the escaping ions. Such converters are limited in the maximum power per unit area they can handle, and, when placed in the fan of a minimum-B mirror, must be sufficiently removed from the mirror region to reduce the energy flux to a tolerable level. In general, the escaping ions will not be distributed uniformly on the field lines which intersect the direct convert, but will reflect the plasma density distribution and, the shape of the magnetic well in the reactor.

At present, it is not possible to predict the absolute ion flux and energy distribution for a mirror reactor. We can, however, make use of certain general scaling relations to predict the relative flux from certain parameters characterizing the magnetic field and the variation of number density within the reactor. Then, from a prediction of the total power escaping and the shape of the vacuum magnetic field we can compute the distribution of energy flux in the fan regions.

The basic equations for this problem are developed in the next section. Calculations for typical reactor conditions are reported in the third section.

THEORY

The rate of particles lost from an element of flux tube of area dA and length ds is

$$d^2 I = \frac{n dA ds}{\tau} \quad (1)$$

where n is the number density and τ is the time it takes a particle to scatter out of the system. The area dA can be related to the area at the midplane dA_0 by $dA = B_0 dA_0 / B = dA_0 / \psi$. If we assume that the density distribution can be factored into a term involving the radius r at the midplane and a term dependant on the local value of ψ , then we get

$$d^2 I = \frac{n_0^2 (r) f^2(\psi) dA_0 ds}{n \tau \psi} \quad (2)$$

A reasonable approximation for $n \tau$ is

$$n \tau = \gamma \bar{E}^{3/2} \log_{10} R \quad (3)$$

where R is the mirror ratio in the flux tube, $\bar{E}$ is the mean plasma energy and γ is a constant. We can integrate Eq. (2) over the length of flux tube to get the power lost per unit midplane area

$$p_0(r) = \frac{\bar{E}_L L n_0^2(r) C^{(2)}(R)}{\gamma \bar{E}^{3/2} \log_{10} R} \quad (4)$$

where $\bar{E}_L$ is the mean energy of the escaping ion, L is the total length between mirrors and

$$C^{(2)}(R) = \frac{2}{L} \int_1^R f^2(\psi) \frac{1}{\psi} \frac{ds}{d\psi} d\psi \quad (5)$$

The quantity $C^{(2)}$ has been evaluated numerically by Moir and Taylor¹ for a Holdren distribution² and found to be nearly constant for a wide range of mirror ratios. It will be taken as constant for present purposes.

As a model of the radial density profile we can take¹

$$n = n_0 \left[1 - (r/r_p)^j \right] \quad (6)$$

where n_0 is the central density, r_p is the plasma radius, and j is an integer characterizing the profile. For the fusion-fission (F-F)³ design, $j = 3$ seems appropriate, while a much higher j , 20-30, is indicated for a large pure fusion reactor.⁴

In the long-thin approximation, the local mirror ratio R is related to the vacuum mirror ratio R_v by

$$R = R_v / (1 - \beta)^{1/2} \quad (7)$$

β will depend on the local amount of plasma present, which depends on radius. We thus take ($x = r/r_p$)

$$\beta = \beta_0 (1 - x^j) \quad (8)$$

where β_0 is the value at the center of the plasma and is usually taken as slightly less than the mirror-mode stability-limited value.

For radial stability, the vacuum B-field will have a radial well. In the present case we can approximate the radial variation by

$$R_v = R_{ov} \left[1 + (R_r - 1) x^2 \right] \quad (9)$$

where R_{ov} is the vacuum axial mirror ratio at the center of the device and R_r is the radial mirror ratio. It has been tacitly assumed that the plasma will fill the radial well out to the last closed $|B|$ contour at the center of the machine and that the field at the mirror is constant.

The total power leaking out a mirror reactor could be found by integrating

Eq. (4) over radius after substituting Eqs. (5)-(9). In practice, however, the total power will be known from other considerations, for example, from the injected power and the Q value

$$P_{TOT} \approx P_{inj} (1 + 0.2Q)$$

We can therefore eliminate the unknown factors in Eq. (4) in favor of P_{TOT} to get

$$p_o(x) = \frac{P_{TOT}}{\pi r_p^2 I} \frac{(\log_{10} R_o)(1 - x^j)^2}{\log_{10} R(x)} \quad (10)$$

with $R_o \equiv R_{ov}/(1 - \beta_o)^{1/2}$, the numerical factor I defined by

$$I \equiv (\log_{10} R_o) \int_0^1 \frac{(1 - y^{j/2})^2 dy}{\log_{10} R(y)} \quad (11)$$

and R found as a function of $y \equiv (r/r_p)^2 = x^2$ from Eqs. (7)-(9). In a numerical calculation, it is most convenient to plot the dimensionless flux at the mirror since the field there is assumed constant

$$p_m(x) \equiv \frac{p_o(x)}{(P_{TOT}/\pi r_p^2)} \frac{R(x)}{R_o} \quad (12)$$

The energy flux at the location of a direct converter where the ratio of the local to the mirrorfield is B/B_m is then

$$p(x,B) = \frac{P_{TOT}}{\pi r_p^2} \frac{R_o B}{B_m} p_m(x) \quad (13)$$

For some cases, it is only necessary to know the peak to average flux at the mirror. It is easily seen that this is given by

$$\frac{p_{\max}}{p_{\text{ave}}} = \frac{1}{I}$$

The equations of the previous section give the power flux as a function of the radial position of the flux surface at the midplane with plasma present. In order to compute the energy flux incident on a particular location of the direct converter, it is necessary to follow the field lines from the midplane to the direct converter location. It is not possible to do so at present with plasma present, but we can do so with the MAFCO code⁵ if the radius of the equivalent vacuum field line is known. We can find its location from flux conservation.

In terms of variables already introduced, flux conservation can be expressed by

$$\begin{aligned} & \pi r_p^2 B_{0v} \int_0^{y_v} \left[1 + (R_r - 1) y \right] dy \\ &= \pi r_p^2 B_{0v} \int_0^{y_p} \left[1 + (R_r - 1) y \right] \left[1 - \beta_0 (1 - y^{j/2}) \right]^{1/2} dy \end{aligned} \quad (14)$$

where $y = (r/r_p)^2$. The left-hand side can be integrated analytically, but the right-hand side must be evaluated numerically. We find

$$y_v^2 + \frac{2}{R_r - 1} y_v - \frac{R_r + 1}{R_r - 1} (1 - \beta_0)^{1/2} J(y_p) = 0 \quad (15)$$

which must be solved for y_v given y_p . The integral is

$$J(y_p) \equiv \frac{2}{R_r + 1} \frac{1}{(1 - \beta_0)^{1/2}} \int_0^{y_p} \left[1 + (R_r - 1) y \right] \left[1 - \beta_0 (1 - y^{j/2}) \right]^{1/2} dy \quad (16)$$

Two special cases can be noted. If $R_r = 1$, then

$$y_v = \int_0^{y_p} \left[1 - \beta_0 (1 - y^{j/2}) \right]^{1/2} dy \quad (15a)$$

and the integral may be evaluated exactly for various values of j , for example, $j = 2$:

$$y_v = \frac{2}{3\beta_0} \left[(\beta_0 y_p + 1 - \beta_0)^{3/2} - (1 - \beta_0)^{3/2} \right] \quad (15b)$$

$j = 4$:

$$y_v = \frac{y_p}{2} (\beta_0 y_p^2 + 1 - \beta_0)^{1/2} + \frac{1 - \beta_0}{2\sqrt{\beta_0}} \left\{ \ln \left[y_p \beta_0^{1/2} + (\beta_0 y_p^2 + 1 - \beta_0)^{1/2} \right] - \frac{1}{2} \ln \beta_0 \right\} \quad (15c)$$

The other special case arises for $j = \infty$,

$$y_v = \begin{cases} (1 - \beta_0)^{1/4} y_p & (R_r = 1) \\ (1 - \beta_0)^{1/4} \left[y_p^2 + \frac{2}{R_r - 1} y_p + \frac{(1 - \beta_0)^{1/2}}{(R_r - 1)^2} \right]^{1/2} - \frac{1}{R_r - 1} & (R_r > 1) \end{cases} \quad (15d)$$

Equations (15b) and (15c) are useful approximations for low density machines such as F - F, while Eq. (15d) could be employed for a high density fusion reactor.

NUMERICAL RESULTS

There are evidently a large number of combinations of R_{ov} , R_r , β_0 and j for which Eqs. (10) - (12) and (15) - (16) could be evaluated. However, other considerations dictate that only certain ranges of the first three of these variables are desirable, typically $R_{ov} \approx 3$, $R_r \approx 1.15$, and $\beta_0 \approx 0.80$. The j parameter depends on a number of factors, possible values ranging from 2 to 50. It turns out that $p_m(x)$ is fairly insensitive to variations in R_{ov} , R_r and β_0 . In addition, as can be seen from Eq. (14), the radii of the vacuum field lines are completely independent of R_{ov} and also depend only weakly on R_r and β_0 .

Table I gives the ratio p_{max}/p_{ave} at the mirror for $R_r = 1.15$ various values of j and $R_{ov} = 3$, $\beta_0 = .7, .8$, $R_{ov} = 3.5$, $\beta_0 = .8$. We see the expected high flux on the axis for low j values reflecting the highly peaked density distributions. The results show that, for low j , direct converters placed on axis would have to be located much further from the mirror, i.e., higher B_m/B , than would be expected from considerations of the average flux. Those near the edge can accordingly be placed much closer to the mirror. This can be seen from the function p_m which is plotted in Fig. 1 for $R_{ov} = 3$, $R_r = 1.15$, $\beta_0 = .80$ and the same j values.

The ratio of the outer field-line radius in vacuum to the plasma radius is listed in Table II for typical conditions, while a plot of y_v vs. y_p is given in Fig. 2. From the latter we note that $y_v(y_p)$ is nearly linear for high j , reflecting the fact that the nearly constant diamagnetism in the central region yields $y_v \approx y_p (1 - \beta_0)^{1/4}$.

The results given here should be adequate for locating a direct converter for most likely plasma parameters. Should parameters vary considerably from

those considered, then it may be desirable to repeat these calculations. The program used is available on the MFE computer network as outlined in Appendix A.

Input cards are also explained there.

ACKNOWLEDGEMENTS

Thanks to Bill Barr for helpful discussion and to Jim Hanson for much of the coding.

RSD:tb

APPENDIX A

The source is available from FILEM under

317 .PD PDS

the controllee from

317 .PD PDRUN

and sample input in

317 .PD PDIN

Input is via NAMELIST, for which the variables are:

JSCRIPT - j

RR - R_r

BETA0 - β_0

RP - r_p (default = 1.0)

INTSTEP - number of intervals in Simpson integration (default = 32)

RV - R_{ov}

PTOT - P_{TOT} (default = 1.0)

IQUIT - set = 1 to terminate input.

A typical execution line is

PDRUN PDIN+ KK. TEST↑U52# /t v

where PDIN is the data file name, KK will keep both print and plot files on disk, TEST is the name you wish to give to this run, and U52 is the box number.

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3. R. W. Moir et al., Progress on the Conceptual Design of a Mirror Hybrid Fusion-Fission Reactor, UCRL-51797.
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5. W. A. Perkins and J. C. Brown, MAFCO - A Magnetic Field Code for Handling General Current Elements in Three Dimensions, UCRL-7744, Rev. II.

TABLE I.

RATIO OF MAXIMUM TO AVERAGE ENERGY FLUX
AT THE MIRROR FOR $R_r = 1.15$

		$R_{ov} = 3.0$	$R_{ov} = 3.5$
$\beta_0 =$	.70	.80	.80
$j = 2$	2.53	2.40	2.45
3	1.94	1.86	1.89
4	1.67	1.61	1.64
5	1.51	1.47	1.49
6	1.41	1.38	1.40
10	1.22	1.21	1.22
20	1.09	1.08	1.09
50	1.01	1.01	1.01

TABLE II

RATIO OF VACUUM FIELD LINE RADIUS TO PLASMA

RADIUS FOR $R_r = 1.15$, $R_{ov} = 3.0$

$\beta_0 =$	0.70	0.80
$j = 2$	0.90	0.88
3	0.88	0.85
4	0.86	0.83
5	0.84	0.81
6	0.83	0.80
10	0.81	0.76
20	0.78	0.73
50	0.77	0.70

FIGURE CAPTIONS

Fig. 1. Energy flux in mirror plane vs. r/r_p for $R_{ov} = 3$,
 $R_r = 1.15$, $\beta_0 = 0.8$ and various j values.

Fig. 2 y_v vs. y_p for $R_{ov} = 3$, $R_r = 1.15$, $\beta_0 = 0.8$ and
various j values.

Fig. 1

Fig. 2

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