RAPIDITY-DENSITY PATTERNS FOR EVENTS IN A STOCHASTIC-FIELD MULTIPARTICLE THEORY*

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Abstract

Typical-event rapidity distributions expected at energies of a few TeV are calculated in a stochastic-field multiparticle production theory. Short range rapidity correlations with characteristics of a Vander Waals fluid give rise to "domain" patterns in rapidity density, which have the appearance of clusters separated by rapidity gaps.

In a series of papers (I, ⁽¹⁾ II, ⁽²⁾ III⁽³⁾), we have reformulated a multi-Eikonal theory of multiparticle production (originally proposed⁽⁴⁾ by Arnold and Thomas) as a stochastic field theory, with a parameterization of long-range (Regge cut) pair correlations. Collective instabilities appearing at

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ultrahigh energies were discussed in I, the Regge intercept spectrum in II, and large- P_L cross sections in III.

In the present paper, we will discuss the characteristic features of event-structure in rapidity expected in such theories at "moderate" energies, $10^2 - 10^4$ GeV, where the long-range power-law decrease of rapidity correlations does not yet play a decisive role. In this energy regime, we will consider correlations which include negative short-range (kinematic) effects and positive moderate-range (1-2 units of rapidity) correlations.

The purpose of the stochastic-field theory^(1,2,3) is to calculate properties of multiparticle events when the correlations are given (experimentally or theoretically) for exclusive 2-particle production (in the central region). The exclusive amplitude in our papers (I-III) is parameterized as a product containing short-range ($\exp V_S$) and long-range ($\exp V_L$) correlations, with V_L non-negative. The short-range (V_S) correlations can be summed up with conventional MPM methods, while collective effects from V_L are computed as a functional integral over the short-range ensemble with a weight function determined by V_L .

Parameterizing V_S as a Vander Waals repulsion, as suggested by multiperipheral kinematics ($t_{\min}$ effect)⁽⁵⁾, and parameterizing V_L as a single Regge pole, we obtain exactly the KUH⁽⁶⁾ model of a one-dimensional fluid. The qualitative features of clustering in such a model have been discussed by Jalickee, Wiegel and Vezzetti.⁽⁷⁾ They showed it is sufficient to retain only 4th order in the expansion of the stochastic-field Lagrangian

in powers of the field. ⁽¹⁾

To obtain analytic results for clustering patterns, we note the functional saddle-points are determined by differential equations which have elliptic functions as their solutions. The specifics of this result are given by Krumhansl and Schrieffer, ⁽⁸⁾ who discuss a similar Lagrangian for describing structural phase transitions.

We determine characteristic parameters in the correlations by assuming a near-critical Vander Waals theory, as in Arnold's and Thomas' discussion ⁽⁹⁾ of multiplicity distributions. Exactly at criticality, the condition expected at asymptotic energy, the fluctuation theory is not valid. ⁽⁷⁾ However, as shown in Ref. 2, the behavior of cross sections at moderate energies is determined by an effective Pomeron singularity above unity, corresponding to a two-phase coexistence analog fluid just below the critical temperature. We leave $\epsilon^2 = \alpha_{\text{eff}} - 1$ as a small parameter to be determined by data (of order 1/20). Writing $\dot{\phi} = \frac{\partial \phi}{\partial y}$, we obtain the stochastic-field ^(1, 2, 3) Lagrangians appropriate to the problem posed here;

$$\mathcal{L}_0[\phi] = \frac{1}{2} [(\dot{\phi})^2 + \mu^2 \phi^2] \quad (1)$$

and

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1, \quad \mathcal{L}_1[\phi] = -\alpha_0 [e^{\phi}] \quad (2)$$

where $\alpha_0(z)$ satisfies the "hard-core" equation ^(5, 9):

$$zG = \alpha_0 e^{b\alpha_0} \quad (3)$$

We will determine G and b from the criticality assumptions of Ref. 9, and let μ^2 be a parameter slightly less than the critical value $(2a_c)^{-1}$, where a and b are the Vander Waals parameters. At Vander Waals criticality $b = 1/8$, $\mu^2 = 32/27$, according to Ref. 9.

Expanding f_1 around $\hat{\phi}$, the (constant) mean-field value of ϕ , we obtain

$$\mathcal{L}[\psi] = \mathcal{L}_0 + \mathcal{L}_1 = \frac{1}{2} (\dot{\psi})^2 + \frac{1}{2} (\mu^2 - C_2) \psi^2 - \frac{C_3}{3!} \psi^3 - \frac{C_4}{4!} \psi^4 + \dots \quad (4)$$

where $\psi = \phi - \hat{\phi}$; $\hat{\phi}$ satisfies

$$\mu^2 \hat{\phi} = \frac{z \partial a_0(z)}{\partial z} \Big|_{z=e^{\hat{\phi}}} ; \quad (5)$$

$$\mathcal{L} \equiv \frac{\mu^2}{2} \hat{\phi}^2 - a_0(e^{\hat{\phi}}) ; \quad (6)$$

and

$$C_p \equiv \left(z \frac{\partial}{\partial z} \right)^p a_0(z) \Big|_{z=e^{\hat{\phi}}} . \quad (7)$$

The event-distributions in rapidity are not dependent on $\hat{\phi}$; however, the C_p 's depend on the value of $\hat{\phi}$, and thus affect the y -distributions. All observables (functionals of ϕ) are obtained from the functional integral^(1,2) over ϕ , whose weight-function is proportional to $\exp \left\{ - \int_{-Y/2}^{+Y/2} \mathcal{L}[\phi(y)] dy \right\}$. We expect the functions which lead to stationary values of the integrand to give a reasonable idea of event-configurations in rapidity, as these will represent the saddle-points of the functional integral. These functions will satisfy:

$$\ddot{\psi} = (\mu^2 - C_2) \psi + \frac{C_3}{2} \psi^2 + \frac{C_4}{6} \psi^3 . \quad (8)$$

The solutions of (8) can be obtained in terms of elliptic functions. (10)

We will consider solutions near criticality; let

$$\epsilon^2 = C_2 - \mu^2 > 0, \text{ but } \epsilon^2 \ll 1 \quad ; \quad (9)$$

and we evaluate C_3 and C_4 at Vander Waals criticality, (9) whence

$$C_3 = \frac{3}{4} * \left(\frac{8}{9}\right)^5 \cong 0.3 \quad (10)$$

and

$$C_4 = \frac{3}{32} * \left(\frac{8}{9}\right)^7 \cong 0.03 \quad . \quad (11)$$

The amplitudes of oscillations in the solutions are determined by the roots of the right-hand side of (8);

$$\frac{C_4}{6} \psi_{\pm}^2 + \frac{C_3}{2} \psi_{\pm} - \epsilon^2 = 0 \quad . \quad (12)$$

Each solution $\psi_{(y)}$, periodic in y , will be determined by location of zeros and the slope at a representative zero. The former feature is uninteresting if we can shift the coordinates, appropriate at very high Y values. Thus, the essential parameter for each solution $\psi_{(y)}$ is an initial slope, $\dot{\psi}_{(0)}$, assuming $\psi_{(0)} = 0$.

An illuminating discussion of bubbles and droplets expected in a system described by (4) can be found in Ref. (7).

To illustrate the qualitative features expected as in Ref. (8), we consider the slightly simpler equation [obtained by dropping the quadratic term in (8)]:

$$\ddot{\psi} = \epsilon^2 \psi - \epsilon^2 \lambda^2 \psi^3 \quad (13)$$

[where $\epsilon\lambda \approx (.005)^{1/2}$ if we keep the C_4 coefficient (11.).] We can simplify if we scale the magnitude of ψ with λ , and scale the rapidity by ϵ ; thus writing

$$u = \lambda\psi \quad (14)$$

and

$$x = \epsilon y, \quad (15)$$

$$\ddot{u} - u + u^3 = 0 \quad (16)$$

(where $\dot{u} = \partial u / \partial x$, etc.). The solutions of (16) are given by

$$u(x) = \text{sn}(x, k), \quad (17)$$

where k , the modulus of the elliptic function sn , is related to the derivative of u with respect to x at $x = 0$; if

$$q = \dot{u}(0), \quad (18)$$

then

$$k^2 = \frac{1 - (1 - q^2)^{1/2}}{1 + (1 - q^2)^{1/2}} \quad (19)$$

and there are solutions only with $q^2 \leq 1$.

The probability of observing rapidity densities near the configuration $+Y/2$ $\psi_{(y)}$ is determined by the weight-function $\exp \left\{ - \int_{-Y/2}^{+Y/2} \mathcal{L}[\psi(y)] dy \right\}$, which is now a function of q .

For the following, we have borrowed from Langer's discussion of magnet theory, ⁽¹¹⁾ as well as the Krumhansl - Schrieffer ⁽⁸⁾ theory of structural phase transitions.

If q^2 is not close to unity, the function ψ_q will have many oscillations, and the integral of $(\dot{\psi})^2$ will be large, sharply reducing the contributions from such functions in the functional integral. Thus, only for q^2 near unity will the saddle-point functions have appreciable weight.

For $q^2 \approx 1$, we find $k^2 \approx 1$; the period $K(k)$ of the elliptic function is very long compared to unity (a short-range correlation length), and the transitions between $u \cong +1$ and $u \cong -1$ regions become very sharp. Thus, we find the phase-space in y is divided evenly into domains, between which the density changes suddenly between high and low values.

A sketch of a typical-event density, $\rho(y) \sim \exp[\psi(y)]$, with $\psi = \lambda^{-1}u$, is given in Fig. 1a. We see dense regions, which could be called "clusters," alternating with sparse regions, which could be called "gaps." We have chosen $\epsilon^2 = 0.05$, for illustration, since this is a typical value of $(a_{\text{eff}} - 1)$ seen in total cross section analyses⁽¹²⁾; this gives $\lambda \cong 0.3$ [with our assumptions on the proximity to Vander Waals criticality, Eqs. (10) and (11)].

The observable densities are obtained from functional averages over all solutions ψ . We may picture the true typical-event density as a superposition of solutions of (18), with various q^2 values near unity, when the fluctuations around the saddle-point functions are taken into account. The resulting density will be represented as ~~a~~^{an}aperiodic set of domains, whose wall-locations are distributed in a relatively random fashion. A sketch of such a typical-event density is given in Fig. 1b.

As pointed out by K. S., ⁽⁸⁾ and in a similar theory with N internal degrees of freedom discussed by Dashen, Ma, and Rajaraman, ⁽¹³⁾ the statistical distribution [following from (4)] of domain sizes and numbers can be estimated if one takes the domain walls or "kinks" to be a non-interacting gas of quasi-particles. (This is reasonable with finite-range correlations in one dimension, but not when long-range terms are included.) The distribution of kink-number n_k [for $Y \rightarrow \infty$] follows a Poisson probability distribution:

$$P(n_k) = \frac{(\bar{n}_k)^{n_k}}{n_k!} e^{-\bar{n}_k}, \quad (20)$$

where the average kink-density in our notation is given by ⁽⁸⁾:

$$\frac{\bar{n}_k}{Y} \approx \exp\left(-\frac{\epsilon^2}{2\lambda^2}\right). \quad (21)$$

If there are N internal degrees of freedom, with $O(N)$ symmetry, it can be shown [as in Ref. (13)] that:

$$\bar{n}_k/Y \approx \epsilon \exp(-N\epsilon^2/2\lambda^2). \quad (22)$$

As noticed by D. M. R., ⁽¹³⁾ this means that in the $N \rightarrow \infty$ limit, no kinks will be found in an asymptotic expansion in powers of N^{-1} .

The estimate (21) gives a fairly low average density of kinks; with our choice of parameters,

$$\frac{\bar{n}_k}{Y} = (.22) \exp(-.25) = \frac{1}{6}.$$

Note that fluctuations with several times this density will not be very rare, according to (20). We conclude that for $Y < 6$, only one kink is expected on the average (although realistic boundary conditions probably require an even number); but for $Y \gg 6$, we expect frequent occurrence of several kinks.

Since present accelerator experiments are limited to $Y < 6$ [where Y is the effective plateau length], it may not be reasonable to directly test these predictions by comparison with available data. There may be some evidence for dramatic density fluctuations in very high energy cosmic-ray experiments, but it does not seem possible to establish a systematic pattern with the limited number of events available.

At very high energies, if Eqs. (1) and (2) remain a reasonable description, we expect the effective ϵ^2 to be smaller than 0.05, and the value of λ^2 correspondingly greater. Then the amplitude of the fluctuations should increase, while the average kink density decreases; the "clusters" become more dense, but more widely separated on the average, and the "gaps" are populated more sparsely. However, the long-range rapidity correlations must play an increasingly important role, and we expect the ∇^σ terms omitted from (1) to influence the clustering structure in important ways. (1, 2, 3)

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Figure Caption

- Figure 1. (a) Typical-event rapidity density pattern from one elliptic function, equation (17).
- (b) Functional-averaged typical-event density with parameters defined in equation (21).

FIGURE 1a

FIGURE 1b