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ONE-DIMENSIONAL MODEL CALCULATIONS

OF EBT-P PERFORMANCE

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I. INTRODUCTION

This appendix describes model calculations for possible EBT-P designs using a one-dimensional, fluid-transport code. We begin (Section II) by giving a detailed description of our most updated and complete version of the code itself, so that the appendix is, as far as practical, self-contained. For the reader who is not too much interested in details of the code makeup, Section II may be skimmed. The relevant equations of Section II for the EBT-P model calculations are Equations (1)-(6), (10)-(19), and (23)-(27). In Section III, we discuss calculations performed with the code for the EBT-1 experiment to compare with previous ORNL calculations, and then we present 1-D calculations for possible EBT-P experimental designs in Section IV.

II. THE EBT CODE

In this section we describe a one-dimensional, time-dependent model of plasma particle and energy transport coupled with magnetic field evolution in a geometry appropriate to EBT. The transport equations used are derived from exact moments of the Boltzmann equation¹, and the magnetic field is calculated from Faraday's and Ampere's laws. The set of transport equations is closed by incorporating into them transport coefficients derived from the appropriate kinetic equation.

The unique features of the present model are the inclusion of geometrical effects in the code and the effect of plasma pressure on the magnetic field. Other attempts at modeling EBT in one dimension consist of treating the plasma transport in cylindrical geometry while maintaining a constant magnetic field profile.^{2,3}

Geometrical effects are accounted for in the model by employing the magnetic field lines as the basis of the coordinate system. Transport of mass and heat in a plasma is much faster along a field line than across it. Thus, on the diffusion time scale, in which we are interested, the plasma density and temperature can be considered constant along a field line. Surfaces of constant pressure, or equivalently surfaces of constant magnetic flux, can then be mapped out, and variation of the plasma quantities is considered only normal to these surfaces.

A multi-dimensional equilibrium calculation is required to determine the shapes and positions of the flux (pressure) surfaces, and thus the geometry, at each instant of time. Once this family of surfaces is determined, the one-dimensional calculation of plasma and field diffusion across these surfaces can be performed. Thus, the fully self-consistent treatment requires a calculation of flux surface shapes changing in response to plasma diffusion which is influenced by the instantaneous shapes. This technique has been successfully employed in the modeling of tokamaks (Ref. 4 and references therein).

The equilibrium calculation determines the shapes of the surfaces and not the value of the flux on each surface. If a knowledge of the shapes of the family of surfaces is assumed a priori and these shapes are taken to remain

constant in time, the description is reduced to a one-dimensional one. This is the basis of the one-dimensional version of the code that we will use for the modeling studies presented further on.

A. BASIC EQUATIONS

1. Transport Equations

The general transport equations describing the evolution of particle density and energy density may be obtained from either moments of the bounce averaged drift kinetic equation² or from moments of the Boltzmann equation.¹

We assume quasi-neutrality for the electrons and ions, i.e., $n_e \approx n_i$. This implies that the electron flux, Γ_e , is equal to the ion flux, Γ_i . The particle conservation equation becomes

$$\frac{\partial n}{\partial t} + \nabla \cdot \Gamma = S \quad (1)$$

where S is the source provided by ionization of neutrals and impurities.

The electron and ion energy conservation equations are

$$\frac{3}{2} \frac{\partial p_e}{\partial t} + \nabla \cdot Q_e + e \Gamma E_r + Q_\Delta + R = E \cdot J + P_{\mu e} \quad (2)$$

$$\frac{3}{2} \frac{\partial p_i}{\partial t} + \nabla \cdot Q_i + \frac{3}{2} n n_0 \langle \sigma v \rangle_x k (T_i - T_0) =$$

$$P_i + Q_\Delta + e \Gamma E_r + \frac{3}{2} n n_0 \langle \sigma v \rangle_i k T_0 \quad (3)$$

Here $p = \frac{3}{2} n k T$ is the electron or ion energy density, and Q_e and Q_i are the total electron and ion energy fluxes which include both convective and conductive losses; $E \cdot J$ is the Joule heating from any plasma currents and inductive electric fields; $P_{\mu e} = (\text{const}) \times n^\alpha$ is the electron heating rate due to microwave absorption, which we take to be proportional to density raised to some arbitrary power α . The electron-ion energy exchange rate, Q_Δ , is given by

$$Q_\Delta = \frac{3n}{\tau_{ei}} \frac{m_e}{m_i} k (T_e - T_i)$$

where τ_{ei} is the electron-ion collision time

$$\tau_{ei} = \frac{3}{4} \sqrt{\frac{m_e}{2\pi}} (kT_e)^{3/2} / (e^4 n \ln \Lambda)$$

and $\ln \Lambda$ is the Coulomb logarithm. E_r is the radial ambipolar electric field determined self-consistently by equating $\Gamma_e = \Gamma_i$. Energy losses from line and recombination radiation arising from collisions between electrons and impurities is represented by R . Neutral ionization and charge exchange rates are given by the terms, $n_0 \langle \sigma v \rangle$, where n_0 is the neutral density, and T_0 is the neutral temperature. P_i is any additional ion heating, e.g., from RF sources or neutral beams.

2. Magnetic Field Equations

As we have discussed, the magnetic field in the plasma depends on the plasma pressure, so that a self-consistent calculation is required for determining the field. Also, since the plasma pressure changes on a diffusion time scale, the change in magnetic field must also be computed on this time scale.

The magnetic field and the plasma are related by the equation of magnetohydrodynamic equilibrium and Ampere's law. The equilibrium equation

$$\mathbf{J} \times \mathbf{B} = c \nabla p \quad (4)$$

relates the currents, field, and total pressure, $p_e + p_i$. Ampere's law relates the field to the currents

$$\nabla \times \mathbf{B} = 4\pi \frac{\mathbf{J}}{c} \quad (5)$$

Finally, conservation of magnetic flux is expressed by Faraday's law

$$\frac{\partial \mathbf{B}}{\partial t} = -c \nabla \times \mathbf{E} \quad (6)$$

where $\mathbf{E}$ is the inductive electric field.

B. ONE-DIMENSIONAL EQUATIONS

As discussed in the introduction, a reduction in complexity in the description of EBT can be accomplished if the magnetic field geometry is used as the basis for the coordinate system. The diffusion of plasma quantities can be modeled with one-dimensional equations instead of three-dimensional ones.

In this section we derive a set of one-dimensional equations suitable for modeling EBT.

The magnetic field can be written in terms of the scalar variables α and ψ , the toroidal flux, such that

$$\mathbf{B} = \nabla \alpha \times \nabla \psi \quad (7)$$

where $\mathbf{B}$, $\nabla \alpha$, and $\nabla \psi$ are mutually orthogonal. We adopt a third coordinate variable, χ , which lies along $\mathbf{B}$. This system is illustrated in Figure 1. Assuming that no current flows in the "radial" or ψ direction, we see from Ampere's law, Equation (5), that the current lies in the direction of $\nabla \alpha$

$$\frac{4\pi}{c} \mathbf{J} = \frac{4\pi}{c} J_\alpha = \frac{1}{h_\chi h_\psi} \frac{\partial}{\partial \psi} \frac{h_\chi}{h_\alpha h_\psi} \quad (8)$$

where the h are metric coefficients, i.e., $\nabla_i = \frac{1}{h_i} \frac{\partial}{\partial x_i}$. It then follows from Equation (4) that the pressure is constant on a surface of constant ψ ,

$$(\nabla p)_\psi = J_\alpha B_\chi \quad (9)$$

Substituting from Equation (8),

$$4\pi \frac{\partial p}{\partial \psi} + \frac{1}{h_\psi h_\chi h_\alpha} \frac{\partial}{\partial \psi} \frac{h_\chi}{h_\alpha h_\psi} = 0 \quad (10)$$

we arrive at the Grad-Shafranov equation^{5,6} expressed in the general geometry coordinate system.

The time rate of change of the flux, from Equation (6), is given by

$$\dot{\psi} = h_\alpha c E_\alpha \quad (11)$$

We make the assumption that heat flow is infinitely fast along a field line, and thus the temperatures are constant on the constant pressure surfaces. This, then, implies that the density is also a constant since $n = \frac{p}{kT}$. Therefore by averaging Equations (1)-(3) over a small volume enclosing a flux surface, all reference to χ and α can be eliminated. We introduce the additional surface label, V , where V is the volume contained within a ψ surface. The flux surface average of a quantity A is defined by

$$\langle A \rangle = \frac{\partial}{\partial V} \int_V d^3x A \quad (12)$$

where the integral is over the volume contained within the surface. From Gauss' theorem

$$\nabla \cdot A = \frac{\partial}{\partial V} \langle A \cdot \nabla V \rangle \quad (13)$$

for any vector A .

With the notation $\langle n \rangle = n$, $\langle p \rangle = p$ and $\frac{\partial}{\partial t} = \frac{\partial}{\partial t} \Big|_V$, the conservation equations, Equations (1)-(3), become

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial V} \langle \Gamma \cdot \nabla V \rangle = \langle S \rangle \quad (14)$$

$$\frac{3}{2} \frac{\partial p_e}{\partial t} + \frac{\partial}{\partial V} \langle Q_e \cdot \nabla V \rangle = \langle S_e \rangle \quad (15)$$

$$\frac{3}{2} \frac{\partial p_i}{\partial t} + \frac{\partial}{\partial V} \langle Q_i \cdot \nabla V \rangle = \langle S_i \rangle \quad (16)$$

where the S 's are the remaining terms in Equations (1)-(3). The equations determining the evolution of the magnetic field, Equations (11) and (10), are

$$\langle \dot{\psi} \rangle = \langle h_\alpha c E_\alpha \rangle \quad (17)$$

$$4\pi \frac{\partial p}{\partial V} + \frac{\partial \psi}{\partial V} \frac{\partial}{\partial V} K \frac{\partial \psi}{\partial V} = 0 \quad (18)$$

where K is a purely geometrical function and depends only on the shapes of the flux surfaces,^{7,8}

$$K = \left(\oint B \, dl \right) \left(\oint dl/B \right) \quad (19)$$

The one dimensional equations (14)-(17) are the conservation equations of mass, energy, and flux. The equilibrium equation, Equation (18), is a constraint equation on the field and pressure. These equations are exact in the sense that no approximations have been made in reducing their dimensionality.

A two or three-dimensional equilibrium calculation is required to determine the shapes and positions of the flux surfaces and then the geometry coefficients for insertion into the one dimensional set. The flux (or B) is determined from Faraday's law, Equation (17), with an electric field consistent with the transport.

The difference equations representing Equations (14)-(17) are written such that they preserve the conservation properties of these equations, so that only physical processes create or destroy particles, energy and magnetic flux. The method of obtaining the simultaneous solution of the nonlinear set of one-dimensional equations, Equations (14)-(18), is discussed in Reference 9.

A self-consistent solution requires alternating between the 1-D set and the 2 or 3-D equilibrium equation.^{4,7,8} If the geometry changes are expected to be relatively minor, then the geometry can be specified a priori and the code run in a 1-D mode. The 1-D mode is used for the model calculations presented in this appendix.

C. EQUILIBRIUM AND GEOMETRY

The role of the equilibrium calculation is to determine the geometrical coefficients for insertion into the 1-D equation set. For computational ease in calculating equilibria, we approximate the EBT device by its infinite aspect ratio limit, the axisymmetric bumpy cylinder, Figure 2. Effects of toroidal curvature are included through the transport coefficients. With this approximation the geometry function, K , given by Equation (19) reduces to $K = \langle |\nabla V|^2 / r^2 \rangle$.

A background magnetic field from given external currents plus a current distribution in the annulus region is required to complete the description.

Given profiles of $p(V)$ from the 1-D set of equations, the grid shape is altered while preserving the volume inside each flux surface until the solution for ψ from Equation (10) agrees with that obtained from the 1-D set. Standard techniques for solving elliptic partial differential equations are employed for the solution of Equation (10). The iteration technique for solving for the grid shape is described in more detail in Reference 4. Since the shape changes occur on a much slower time scale than the plasma diffusion, very few iterations are required for convergence.

D. TRANSPORT

In order to close the system of equations, Equations (14)-(18), transport coefficients in the particle and energy fluxes, must be specified. Transport in EBT is the subject of intense investigation, and a review of its status can be found in Reference 10.

The fluxes can be expressed in terms of integrals over the solution of a suitable bounce averaged drift kinetic equation

$$\Omega \frac{\partial f}{\partial \alpha} + V_D \frac{\partial f}{\partial \psi} = Cf \quad (20)$$

where α is again the poloidal angle variable and ψ designates constant pressure surfaces, Ω is poloidal drift frequency, $\Omega = -\frac{\partial J}{\partial \psi}$, V_D is the vertical drift, $V_D = \frac{\partial J}{\partial \alpha}$, J is the adiabatic invariant, $J = \oint ds v_{||}$, and Cf is the bounce averaged collision operator. Bounce averaging is justified in forming Equation (20) since the bounce frequency is much greater than the poloidal drift and collision frequencies.

The particle and energy fluxes normal to a pressure surface are determined from the solutions of Equation (20),

$$\Gamma_{\psi} = \int \frac{d\alpha}{2\pi} \int d^3v V_{D\psi} f \quad (21)$$

$$Q_{\psi} = \int \frac{d\alpha}{2\pi} \int d^3v \quad v_{D\psi} \quad \frac{mv^2}{2} f \quad (22)$$

Results of these calculations give the fluxes in terms of diffusion coefficients and gradients

$$\Gamma_j = -D_{nj} \frac{\partial n}{\partial r} - D_{Tj} \frac{\partial T_j}{\partial r} + \frac{ne_j}{kT_j} D_{nj} E_r \quad (23)$$

$$Q_j = -K_{nj} \frac{\partial n}{\partial r} - K_{Tj} \frac{\partial T_j}{\partial r} + \frac{ne_j}{kT_j} K_{nj} E_r \quad (24)$$

where the subscript j refers to particle specie, and the "radial" derivative refers to the derivative across the pressure surface and is given, for example, by $\frac{\partial n}{\partial r} = \frac{\partial n}{\partial V} \langle \nabla V \rangle$, where V is the volume enclosed by the surface. The quantity E_r in these equations is the radial ambipolar electric field. This field is determined self consistently in the code by solving the nonlinear equation $\Gamma_e = \Gamma_i$ for E_r .

There are three regimes of neoclassical transport in EBT, and the diffusion coefficients appropriate to each regime have been calculated at least in some approximation.

The collisional regime is characterized by the parameter $\nu/\Omega_0 > 1$, where ν is the collision frequency and Ω_0 is the poloidal precession frequency of a thermal particle in a mirror field with radius of curvature, R_c , $\Omega_0 = \frac{ckT}{eBrR_c}$. This regime was first treated by Kovrizhnykh¹¹ in the large electric field^c limit, $\frac{e\phi}{kT} \gg 1$, where ϕ is the potential associated with the ambipolar field. Even for smaller fields, the results give a good approximation to the transport.¹² The coefficients for this regime are

$$D_{nj} = \frac{v_{oj}^2}{24} \frac{v_j}{v_j^2 + \Omega_j^2}$$

$$D_{Tj} = \frac{2n_j}{T_j} D_{nj}$$

$$K_{nj} = \frac{7}{2} kT_j D_{nj}$$

$$K_{Tj} = \frac{101}{2} kn_j D_{nj} \quad (25)$$

where

$$V_{oj} = 2kT_j / (m \omega_{cj} R_T)$$

is the average vertical drift speed due to the toroidal curvature, R_T is the major radius of the torus, ω_c is the cyclotron frequency, ν is the collision frequency, and Ω is the total poloidal bounce averaged precession frequency,

$$\Omega = \frac{ckT}{eBrR_c} + \frac{cE_r}{Br} \quad (26)$$

where R_c represents the effect of the mirror radius of curvature and the magnetic field gradient. In these equations all quantities refer to their mid-plane values.

In the more collisionless regimes of operation where $\nu/\Omega_0 < 1$, there are classes of particles for which the electric and magnetic drifts cancel (ions for radially inward pointing electric fields), in which case the flux can be enhanced.

Hazeltine, et al.,¹² have given expressions in integral form for the fluxes in the regime $\delta^{3/2} < \nu/\Omega_0 < 1$, where $\delta = a/R_T$ is the inverse aspect ratio. They find that the fluxes are independent of collision frequency, hence the name "plateau." Jaeger, et al.,¹³ have evaluated the integrals using a model magnetic field and an ad hoc value of the poloidal component of the ambipolar field so that equipotential surfaces and contours of constant B are aligned in the mid-plane. These results are convenient for use in modeling and can be written as follows:

$$D_n = \frac{3\pi}{8/a} \left(1 + \frac{1}{2a}\right)^2 \frac{r}{R_T^2} \frac{ckTR_c}{eB} e^{-\hat{E}}$$

$$D_T = \frac{n}{T} D_n (1 + \hat{E})$$

$$K_n = kT D_n \left(\frac{5}{2} + \hat{E}\right)$$

$$K_T = kn D_n \left(5 + \frac{7}{2} \hat{E} + \hat{E}^2\right) \quad (27)$$

where $a = 2.1$, and $\hat{E} \equiv -e R_c E / kT > 0$. For radially inward electric fields, the Kovrizhnykh transport coefficients of Equation (25) give a good approximation to the electron transport even for $\nu/\Omega_0 < 1$ in this regime, since the poloidal magnetic and electric field drifts are in the same direction.

In the plateau regime the particles with $\Omega \approx 0$ suffer a collision before they complete a closed orbit. In the very collisionless operating regime such that $v/\Omega_0 < \delta^{3/2}$, the orbits are closed and are shaped like crescents or bananas. Here the spatial drifts of particles with $\Omega \approx 0$ become particularly important in the calculation of the diffusion coefficients. Calculations are in progress using Fokker-Planck collision integrals and preliminary results have been reported.¹³⁻¹⁵ However, due to the small value of the inverse aspect ratio, δ , the banana regime may be difficult to access. In fact, recent point model calculations of the EBT reactor¹⁶ indicate favorable steady state burn points in the more collisional regimes of operation.

In the modeling calculations discussed in succeeding sections, the neo-classical transport package that we use in the code encompasses the collisional and plateau regimes. Specifically, we use the transport coefficients given in Equations (25) and (27).

E. NEUTRAL MODEL

Since the neutral particles are not constrained by the magnetic field, the neutral profiles adjust instantaneously (on the plasma diffusion time scale) to a steady-state. The neutral profile thus changes only parametrically in response to plasma diffusion. In the code there are two models for determining the steady state neutral density.

The first method¹⁷ calculates the solution, in cylindrical geometry, to the time independent neutral transport equation for each generation of charge exchange neutrals. The transport equations are integrated to give an integral equation for each generation of charge exchange neutrals in terms of the previous generation. By summing over all generations, one can obtain a single integral equation involving the charge exchange rate at each point in terms of the rate at every other point. The total rate is calculated numerically by a matrix inversion. Once this rate is determined, all other quantities of interest such as ionization rates and energy loss rates can be calculated.

The alternate method of modeling the neutrals is based on a diffusion approximation and treats two separate neutral fluids: a cold fluid from the wall and a hot fluid generated by charge exchange collisions between cold neutrals and hot ions. The equations determining the steady state neutral profiles are

$$\nabla \cdot \Gamma_c = n_i n_c \langle \sigma v \rangle_{xc} - n_e n_c \langle \sigma v \rangle_{ic} \quad (28)$$

$$\nabla \cdot \Gamma_h = n_i n_c \langle \sigma v \rangle_{xc} - n_e n_h \langle \sigma v \rangle_{ih}$$

where subscripts h and c refer to hot and cold species, Γ is the neutral particle flux, n_e and n_i are the electron and ion densities, and n_h and n_c the hot and cold neutral densities. The ionization and charge exchange rates, $\langle \sigma v \rangle_i$ and $\langle \sigma v \rangle_x$, are obtained from Reference 18.

The diffusion approximation assumes that the mean free path between collisions is small compared to the gradient scale lengths in the system. The flux is then given by $\Gamma = -D \nabla n$, where the diffusion coefficient is

$$D = \frac{v_o^2}{3(n_i \langle \sigma v \rangle_x + n_e \langle \sigma v \rangle_i)} \quad (29)$$

and v_o is the mean neutral speed. The hot neutral temperatures are determined in this model from the solution of an energy transport equation analogous to Equation (28) using a heat conductivity analogous to Equation (29).

The boundary condition on the hot neutrals in the diffusion model is that the density goes to zero at 0.71 of an absorption length outside of the plasma. A boundary condition for the hot neutrals is not required for the transport model. For both codes an isotropic source of cold neutrals at the edge of the plasma with an energy, E_o , is assumed. Both codes require the flux of cold neutrals into the plasma to be specified.

The advantage of the transport model is its accuracy in calculating the neutral density at low plasma densities ($\approx 10^{12} \text{ cm}^{-3}$) where the mean free paths are long. Its disadvantages are that the bumpiness of the geometry is not accounted for in the calculation and the relatively long running times to calculate a neutral density profile.

On the other hand, at low plasma densities while assumptions in deriving the diffusion equations break down, the neutral densities calculated with this model are in good agreement with the transport calculation.³ Since the geometry effects of the bumps can be included through the divergences in Equation (28), and the running time of the diffusion code is negligible compared to a plasma

transport step, the diffusion code is employed in the 1-D calculations even for EBT-1, while the transport code is used periodically to confirm the diffusion calculations. At the densities of EBT-P operation, the use of the diffusion approximation is quite well justified.

F. BOUNDARY CONDITIONS

The one-dimensional equations, (14)-(18), and the two-dimensional equation, (10), require boundary conditions for their solution. The code requires either the specification of the value of a variable or its flux at the boundary; e.g., either n or Γ , T_e or Q_e , T_i or Q_i . In the case of the equilibrium equation, (18), $K \frac{\partial \psi}{\partial V}$ is $\frac{4\pi}{c}$ times the current enclosed within a flux surface. Thus Equation (18) requires as a boundary condition either the specification of the total current in the system or the amount of magnetic flux being introduced into the system. (The flux in the system would change if the currents in the external coils were to change. The normal boundary condition is $\dot{\psi} = 0$.) The outermost surface is assumed to be fixed in these calculations and is not a free boundary. Thus the total volume of the system remains constant.

III. COMPARISON WITH EBT-1 RESULTS

Previous attempts at modeling the EBT-1 device^{8,19} have led to solutions which became thermally unstable in the collisionless regime in disagreement with the experimental observation of stable operation. These calculations have assumed a reflux of cold neutrals which was sufficient to maintain a constant number of plasma particles. This flux necessarily varied in time in response to the plasma outflux which varied with changing density, temperature, and gradients.

In the higher temperature collisionless regime the particle diffusion and electron heat conduction coefficients increase with density and decrease with temperature. Thus, if the number of plasma particles is kept constant, the electron heat flow decreases if the temperature increases. If the heat input is maintained constant, the result is a net overall addition of heat and thus an increase in temperature. A thermal runaway ensues.

Recently Jaeger, et al.²⁰ have reported 1-D calculations of stable operation in the collisionless regime by applying an alternate boundary condition which maintains a constant flux of cold neutrals at the plasma edge irrespective of the plasma outflow. With this boundary condition a feedback mechanism is instituted which suppresses any thermal excursion. The mechanism is as follows:

With the decrease in plasma outflux resulting from the increase in electron temperature, the plasma density increases since the neutral influx is unchanged. The increase in density is sufficient to offset the increase in temperature in the transport coefficients and limits any further thermal excursion. In fact the temperature and density damp back to their steady state values.

Jaeger, et al.²⁰ find that the steady state values of density and temperature are functions of the edge flux of neutrals. Plots of central temperature and density at steady state versus the ambient neutral pressure for fixed microwave power show the temperature and density increasing with decreasing edge neutral pressure in the collisionless regime. This behavior is similar to that observed in the experiment,²¹ and indicates that the neutrals do in fact provide the damping mechanism on the thermal excursions.

We have performed a number of one-dimensional calculations with the present code to compare with the ORNL calculations. The parameters that were employed were: plasma radius $a = 10$ cm, major radius $R_T = 150$ cm, 24 mirror sectors, magnetic field on axis $B_0 = 6.4$ kG, cold neutral energy $E_0 = 0.5$ eV, edge plasma density $n = 3 \times 10^{10}$ cm $^{-3}$, and edge temperatures T_e and $T_i = 13.5$ eV. We employed the same magnetic field dependence on radius as did the Jaeger calculations.^{2,20} The field is given by

$$\frac{B(r)}{B_0} = \exp \left\{ - \frac{r^2}{2rR_{co}} \left[1 - \frac{rR_{co}}{a^2} \left(\frac{r}{a} \right)^6 \ln(1 - \beta_A) \right] \right\} \quad (30)$$

where $rR_{co} = 2 (R_T/N_S)^2$, and N_S is the number of mirror sectors. Here β_A is the beta of the hot, non-interacting electron ring, and was taken to be 0.37. For these calculations the geometry was taken to be that of concentric cylinders to compare with Jaeger, et al.,²⁰ who use a concentric cylinder geometry. The electron diffusion coefficients were the Kovrizhnykh ones, Equation (25), while the plateau coefficients, Equation (27), were used for ions. The power per unit volume deposited in the electrons by the ECH was chosen proportional to the density. Lastly, the diffusion model was used to obtain the neutral profiles.

In Figure 3 we plot the results from a number of runs. We plot the central density and temperature versus edge neutral density for various input powers. Each point on a curve represents a steady state operating point for that particular neutral edge density and input power. In the collisionless regime, the density, temperature and potential (not plotted) all increase as the edge neutral pressure increases. This behavior is consistent with that observed in the experiment.²¹ At higher pressures the density is seen to increase with increasing pressure, which is a demonstration of collisional scaling. Figure 3 is similar to the plots displayed in Reference 20. Since the temperatures, density, and ambipolar field calculated here are consistent with their experimental values and with previous ORNL calculations, we feel confident in applying the model to EBT proof of principle designs. This is discussed in the following section.

IV. MODELING THE EBT PROOF OF PRINCIPLE EXPERIMENT

The EBT-1 device has demonstrated favorable scaling in which the energy confinement time increases with temperature. This scaling is consistent with neoclassical scaling in the collisionless regime ($v/\Omega_0 < 1$). Furthermore, the magnitudes of the confinement time, the temperatures and densities, and the shape of the ambipolar potential (inward pointing electric field) are all consistent with neoclassical predictions of the 1-D code. We therefore apply the 1-D code in this section to determine EBT-P operating points.

The EBT-P experiment is intended to form a bridge between the present device and reactor size devices. It must be able to test the collisionless neoclassical scaling in a device with the dimensionless parameters of the reactor (except possibly the plasma β) and with larger more reactor-like absolute parameters (n , T , τ , etc.) than EBT-1.

We now describe the results of a large number of 1-D model calculations for an EBT-P baseline design, as well as calculations for a possible upgrade to the baseline magnetic field. The results of these calculations are summarized in Figures 4-18.

Just as in the EBT-1 calculations discussed above, a constant neutral influx is assumed at the plasma boundary, with the neutral energy taken as 0.5 eV and the numerical value of the neutral flux scaled from the EBT-1 results.²² Again we took $P_{\mu e}$ proportional to density and allowed for direct ion heating such that $P_i/P_{\mu e} = \text{constant}$; i.e., power per unit volume deposited directly into the ions was chosen proportional to density. Also the midplane magnetic field radial variation is the same as for the EBT-1 calculations, Equation (30), as is the neoclassical transport package; viz Kovrizhnykh electrons and plateau ions for the collisionless ion regimes that we find with negative ambipolar electric field. In contrast to the concentric cylinder geometry used in the EBT-1 calculations, we use a bumpy cylinder geometry to model EBT-P.

Figures 4-14 display the code results for the McDonnell Douglas baseline EBT-P design. The machine parameters used for these calculations are

$$B_0 = 14 \text{ kG}$$

$$R_T = 550 \text{ cm (major radius)}$$

$$a = 32 \text{ cm (at the midplane)}$$

$$\begin{aligned} a &= 20 \text{ cm (at the throat)} \\ N &= 36 \text{ sectors} \\ \beta_A &= 0.4 \text{ (annular beta)} \end{aligned}$$

Figures 15-18 display the results for an upgraded EBT-P experiment in which the midplane magnetic field on axis is doubled to 28 kG and the other machine parameters are unchanged.

Examining the calculations for the baseline case as a function of the assumed power input to the plasma, a number of interesting results and trends are evident. From Figures 4-7 we see that the plasma quality improves in all respects by increasing either the electron or ion power deposition; in particular we note that the plasma density increases with ion power as well as the temperature. Thus the use of ion heating power to supplement the microwave ECH power, seems very beneficial to achieving an overall more reactor grade plasma. Since RF ion power is considerably cheaper than ECH power, these results indicate that significant economic advantage may be achieved in reaching a desired EBT-P operating state by way of direct ion heating.

From Figure 8 we see that varying the neutral influx at the plasma edge provides a way of improving (optimizing) plasma performance for EBT-P as it does in EBT-1, and the scaling is similar to that in EBT-1 calculations. Figures 9-12 show the steady state plasma and potential profiles for several of the operating states of Figures 4-8. The density and temperature profiles are quite flat except in the outer plasma region, and the ambipolar field is negative as in EBT-1; indeed the electric field was found to be negative for all of the steady states calculated in this section. Although the ambipolar potential is shown rising to only several hundred electron volts in Figures 9-12, it actually rises to many kilovolts at the plasma edge. Thus the electric fields are very large in the outer several centimeters of the plasma, where the plasma gradients are large, e.g., in Figure 9 the wall potential is $\phi = 2.6 \text{ keV}$, while in Figure 11 it is $\phi = 5.5 \text{ keV}$ and in Figure 12 it rises to $\phi = 8.3 \text{ keV}$ at the wall.

Examining Figures 5 and 12, we see that raising the ion power well in excess of the electron power does not produce $T_i > T_e$ nor does it radically alter the potential profile. Rather the major effect of substantially increasing the ion power beyond the electron power is to significantly increase the operating plasma density, as well as both the electron and ion temperatures. The results

at the highest values of ion power should be treated with caution however, since the low values of ion collisionality actually push them into the banana regime. Thus a more accurate treatment of these cases must await further development of the neoclassical transport theory in this regime.

In Figures 13 and 14 we plot sample curves of power deposited into and lost from a volume with midplane radius r . These plots correspond to Figures 9 and 12. In both plots the total power into the electrons is 1.5 MW. In Figure 13 the total power deposited to the ions is 1 MW and in Figure 14 it is 6 MW. The electron power deposition and the conduction heat loss are nearly the same for both cases, so they are displayed in Figure 14 only. From the plots it can be seen that for the case with the larger ion heating, the ion conduction loss term rises accordingly to maintain a steady state. In order to maintain a roughly constant electron-ion coupling, the density and electron temperature increase in response to the larger ion temperature. The drop in ion heat conduction loss at the boundary in both figures is compensated by a large charge exchange loss (not shown) in this region. This loss drops to zero in a few centimeters from the wall as does the neutral density.

In summarizing, the calculations presented for the baseline EBT-P design (Figures 4-14) demonstrate (1) EBT-P is capable of containing interesting plasmas in steady state if powers in the megawatt range are coupled into the plasma, i.e., $n \sim 10^{13} \text{ cm}^{-3}$, T_e and $T_i \sim 1\text{-}2 \text{ keV}$, $n\tau \sim 10^{12} \text{ cm}^{-3} \text{- sec}$. In general the more power the better over the range studied. (2) Very interesting plasmas can be contained if large ion power is coupled to the plasma, e.g., $n\tau \sim 3 \times 10^{12} \text{ cm}^{-3} \text{- sec}$ at $n \sim 3.6 \times 10^{13} \text{ cm}^{-3}$ and $T_e \sim T_i \sim 4 \text{ keV}$ for $P_e = 1.5 \text{ MW}$ and $P_i = 6.0 \text{ MW}$.

Scaling trends are similar for an upgraded EBT-P in which the magnetic field is increased to 28 kG. From Figures 15 and 16 we see that all plasma quantities improve as either the electron or ion power is increased, including the density. A significantly improved plasma relative to the baseline case can be contained if power in the five megawatt range is delivered to the plasma, i.e., $n \sim 5 \times 10^{13} \text{ cm}^{-3}$, $T_e \sim 4 \text{ keV}$, $T_i \sim 3 \text{ keV}$ and $n\tau \sim 5 \times 10^{12} \text{ cm}^{-3} \text{- sec}$. Figure 17 demonstrates that varying the neutral influx provides a mechanism for optimizing plasma parameters at fixed power. For this MDAC upgraded design, PLT type parameters are achievable if total power deposited in the plasma is in

the 10 MW range; e.g., $n\tau$ values approaching $10^{13} \text{ cm}^{-3} \cdot \text{sec}$ at $n \sim 5 \times 10^{13}$ and $T_i \sim 4 \text{ keV}$. Like other magnetic fusion schemes, and tokamak in particular, these calculations indicate that reactor-like EBT-plasmas require provisions for large amounts of power.

We now briefly discuss some preliminary additional 1-D calculations for the EBT-P baseline case which indicate that more favorable operating states may exist than those summarized in Figures 4-14, for the same range of powers. Figure 19 displays these apparently more favorable operating states, which are to be compared directly with those shown in Figure 5 for the same electron power input and neutral influx. These new operating points were discovered by initializing the calculations at larger central densities than for Figure 5, and were approached from the high density side. These operating states indicate the possible existence of multivalued equilibria for a given power absorption. We reiterate that these results should be regarded as preliminary, since they were discovered rather late in our study and constraints on available computer time prohibited us from exploring them in the detail that we would have preferred. However, since this new branch of operating equilibria implies the potential for truly exciting EBT-P operation [$n\tau > 10^{13} \text{ cm}^{-3} \cdot \text{sec}$, $n \sim 4 \times 10^{13} \text{ cm}^{-3}$ (approaching microwave cutoff), $T_e \sim T_i \sim 4 \text{ keV}$] in a reasonably sized device at reasonable power, $P_{\mu e} \sim P_i \sim 1 \text{ MW}$, these new operating states should be vigorously studied in the future.

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22. The values of the neutral influx as well as the machine dimensions for EBT-P can be estimated from the following crude 0-D scaling arguments:

The power into the device is

$$P = \frac{nTV}{\tau_E} \quad (1)$$

where V is the volume and τ_E is the energy confinement time. In the collisionless regime

$$\frac{1}{\tau_E} = C_0 \frac{n}{T^{3/2}} \frac{(aR_c)^2}{R_T^2} \frac{1}{a^2} \quad (2)$$

where a is the minor radius, R_T is the major radius, R_c is the mirror radius of curvature and C_0 is a constant. From the previous section $aR_c = 2 \frac{R_T^2}{N^2}$ where N is the number of mirror sectors. The power balance becomes

$$P = C_1 \frac{n^2}{T^{3/2}} \frac{R_T^3}{N^4} \quad (3)$$

The particle balance is likewise

$$\begin{aligned} \Gamma_0 A &= \frac{n}{\tau_p} V \\ \Gamma_0 A &= C_2 \frac{n^2}{T^{3/2}} \frac{R_T^3}{N^4} \end{aligned} \quad (4)$$

where Γ_0 is the neutral influx, A is the surface area, and we assume that the particle and energy fluxes differ only by a constant factor. Thus $C_2/C_1 = \Gamma_0 A T/P$. Using the EBT-1 values of Γ_0 , T , and P , the required neutral flux to sustain EBT-P at a desired T and a given P is then readily determined. Equation (3) can be used to estimate the major radius required to contain a desired density for EBT-P, and the minor radius can be estimated from $aR_c = 2R_T^2/N^2$ for a given number of sectors and curvature. In order to size a baseline EBT-P design, we require that at a minimum, it be able to achieve $n \sim 10^{13} \text{ cm}^{-3}$, $T \sim 1 \text{ keV}$ for $P \sim 1 \text{ MW}$.

Figure Captions

- Figure 1. Coordinate system for EBT model equations.
- Figure 2. Bumpy cylinder model of EBT.
- Figure 3. EBT-1 model calculations for comparison with experiment²¹ and with recent ORNL calculations²⁰. Central temperatures and densities versus neutral pressure for 6 kW and 8 kW of ECH power deposited in the electrons.
- Figure 4. Plots of density ($\bullet$), electron temperature ($\times$), ion temperature ($\circ$) and energy confinement time (Δ) on axis vs. electron power absorption for fixed ion power absorption, $P_i = 1.0$ MW, and neutral wall flux $\Gamma_0 = 1.2 \times 10^{14}$ particles/cm²/sec/sector.
- Figure 5. Same as Figure 4 but with $\Gamma_0 = 1.0 \times 10^{14}$ particles/cm²/sec/sector and fixed electron power, $P_e = 0.75$ MW and variable ion power.
- Figure 6. Same as Figure 4 but with fixed electron power $P_e = 1.5$ MW and variable ion power.
- Figure 7. Same as Figure 6 but with $P_e = 3.0$ MW.
- Figure 8. Central plasma parameters vs. neutral wall flux for fixed electron power, $P_e = 1.5$ MW and fixed ion power $P_i = 1.0$ MW.
- Figure 9. Plasma and potential profiles for $P_e = 1.5$ MW, $P_i = 1.0$ MW, $\Gamma_0 = 1.2 \times 10^{14}$ particles/cm²/sec/sector.
- Figure 10. Same as Figure 9 for $P_e = 3.0$ MW, $P_i = 0.5$ MW.
- Figure 11. Same as Figure 10 for $P_e = 3.0$ MW, $P_i = 2.0$ MW.
- Figure 12. Same as Figure 9 but with $P_e = 1.5$ MW and $P_i = 6.0$ MW.
- Figure 13. Plots of the power into and out of the EBT-P plasma out to radius r corresponding to the parameters of Figure 9, $P_e = 1.5$ MW and $P_i = 1.0$ MW.
- Figure 14. Same as Figure 13 for the parameters of Figure 12, $P_e = 1.5$ MW and $P_i = 6.0$ MW.

Figure Captions (continued)

Figure 15. Calculations for upgraded EBT-P with $B = 28$ kG. Central plasma parameters and energy confinement times vs. electron power for fixed ion power and neutral wall flux, $\Gamma_0 = 2.5 \times 10^{14}$ particles/cm²/sec/sector.

Figure 16. Same as Figure 15 but for fixed electron power $P_e = 4$ MW and variable ion power.

Figure 17. Central plasma density, temperatures and energy confinement times versus neutral influx for fixed electron and ion power.

Figure 18. Same as Figure 16 but with $P_e = 6$ MW and $\Gamma_0 = 2.0 \times 10^{14}$ particles/cm²/sec/sector.

Figure 19. More favorable operating states indicating possible existence of multivalued equilibria for a given power input. Figure 19 is to be compared directly with Figure 5.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18

Figure 19