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Reversed Field Decay Times

The rate at which a reversed field mirror plasma will decay due to electron drag is not known. Estimates¹ using a slowing-down model of the electron-ion drag and momentum balance indicate that the ion current in the plasma would be cancelled by an electron component to the current on very short time scale (the electron-ion momentum exchange time). This time scale is so short that it would probably rule out the possibility of a thermonuclear reactor using a field-reversed mirror configuration held together by an ion current. The present paper is an attempt to justify why this time scale ordering is inappropriate here because of the closed nature of the system and the high beta necessarily present. (Beta is a measure of the pressure in the plasma and equals $\frac{8\pi P}{B^2}$ in cgs units.)

The problem can be set up as a two-fluid equilibrium plasma model with electron-ion drag terms taken from the Spitzer coefficients.² An equilibrium cold electron background is assumed to exist which balances the electrical charge of the rotating ion ring. The ions can be assumed to have a directed velocity $\langle \vec{v}_\theta^i \rangle$ which is on the order of their mean thermal velocity v_{th}^i . The entire system is viewed as being only a few gyroradii in extent, so that a characteristic length $L \equiv (\frac{\nabla B}{B})^{-1}$ is on the order of the ion gyroradius.

The problem is tractable where the dynamics of the pressure tensor are ignorable. Since the only nonzero mean velocity is in the $\hat{\theta}$ direction, interaction with the scalar pressure gradient is considered unimportant. The pressure has a tensorial nature when the velocity distribution is dependent strongly on angle. If the velocity distribution is isotropic about a shifted frame of reference, then the dynamics of the pressure is

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ignorable. We make an assumption, then, that this holds for the problem at hand, so that we work only with scalar pressure.

The two-fluid equations then become:

$$\frac{d\vec{v}_i}{dt} = \frac{e}{m_i c} \vec{v}_i \times \vec{B} + \frac{e}{m_i} \vec{E} - \frac{\vec{\nabla} p_i}{n_i m_i} - \frac{(\vec{v}_i - \vec{v}_e)}{\tau_{ie}} \quad (1a)$$

$$\frac{d\vec{v}_e}{dt} = \frac{-e}{m_e c} \vec{v}_e \times \vec{B} - \frac{-e}{m_e} \vec{E} - \frac{\vec{\nabla} p_e}{n_e m_e} - \frac{(\vec{v}_e - \vec{v}_i)}{\tau_{ei}} \quad (1b)$$

conservation of momentum gives the side equation

$$\frac{m_i}{\tau_{ie}} = \frac{m_e}{\tau_{ei}} \quad (2)$$

The set of equations must possess self-consistency through the Maxwell equations

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{J} \quad (3)$$

$$-\frac{1}{c} \frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \vec{E} \quad (4)$$

This is a very important part of the problem. If the system was not closed, and if the gyroradius of the ion was zero, then the two equations (1a and 1b) could be uncoupled and the slowing-down rate could be seen from the momentum balance

$$m_i \vec{v}_i + m_e \vec{v}_e = \text{constant} = \langle p_\theta^i \rangle + \langle p_\theta^e \rangle \quad (5)$$

But here the magnetic field is high and hence the field configuration is strongly coupled to the current. The current is the ion velocity, and the

slowing down force it experiences will be counteracted by an electric field or "back-emf" as the magnetic field changes. This follows from Lenz's law.

The perturbed equations are (letting $\langle \vec{v}_i \rangle|_{t=0} \equiv v_0 \hat{\theta}$ and taking the θ components of the perturbed form of 1a and 1b):

$$\frac{dv_i}{dt} = \frac{e}{m_i} E_\theta - \frac{(v_0 + v_i - v_e)}{\tau_{ie}} \quad (6a)$$

$$\frac{dv_e}{dt} = \frac{-e}{m_e} E_\theta - \frac{(v_e - v_0 - v_i)}{\tau_{ei}} \quad (6b)$$

there v_i and v_e are the perturbed velocities. Hence momentum balance reads

$$m_i v_i + m_e v_e = 0 \quad (7)$$

coupling this to the perturbed Maxwell equation

$$(\vec{\nabla} \times \vec{\nabla} \times \vec{E})_\theta = \frac{-4\pi ne}{c^2} \frac{d}{dt} (v_i - v_e) \quad (8)$$

or

$$(\nabla^2 - \frac{1}{R^2}) E_\theta = \frac{\omega_{pi}^2}{ec^2} m_i \frac{d}{dt} (v_i - v_e) \quad (8a)$$

using (8a) and (6a) we get, ignoring $\frac{m_e}{m_i}$ compared to one,

$$(\nabla^2 - \frac{1}{R^2}) \frac{dv_e}{dt} = \frac{\omega_{pi}^2}{c^2} \left(\frac{m_i}{m_e} \right) \frac{dv_e}{dt} + \left(\frac{m_i}{m_e} \right) (\nabla^2 - \frac{1}{R^2}) \frac{(v_0 - v_e)}{\tau_{ie}} \quad (9)$$

$(\frac{\omega_{pi}}{c})$ is the reciprocal of the ion collisionless skin depth and is of the same order as an inverse length or gradient operator for a high beta, large gyroradius plasma. Hence we can ignore the left-hand term in (9) and write

$$\frac{\omega_{pi}^2}{c^2} \frac{dv_e}{dt} = \frac{1}{\tau_{ie}} (\nabla^2 - \frac{1}{R^2}) (v_0 - v_e)$$

This has the form of a diffusion equation with characteristic time τ_{ie} for a plasma with $\frac{c}{\omega_{pi}} \sim L$. The total current is cancelled roughly in this time. The ions thus slow down much more slowly, since the equilibrium, zero-current ion velocity is still $(1 - \frac{m_e}{m_i})v_0$ for a linear model of the decay. Sketches of the time-dependent electron and ion velocities follow. It is important to note that

$$\tau_{ie} \sim \frac{1.1 \times 10^{12} \tau_{keV}^{3/2}}{n_e}$$

which for a thermonuclear plasma is on the order of hundreds of milliseconds. Hence this time scale is on the order of classical diffusion, and is a factor of $\frac{m_i}{m_e}$ larger than the electron-ion momentum exchange time. This contradicts the results of Hartman¹, since he did not couple the fluid-drag equations to the Maxwell equations. That the results parallel an estimate based on classical diffusion arguments is only to be expected, since classical diffusion is equivalent to magnetic relaxation in the plasma.

References

1. C. W. Hartman, LLL Report (unpublished), "Field Reversal with Axis Encircling Ions," 6 May 1976.
2. Lyman Spitzer, Jr., Physics of Fully Ionized Gases, Interscience, 1962, New York, p. 123.

Fig. 1. Sketch of the time dependent behavior of the fluid velocities and total current for the reversed field plasma.