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UCRL-52043

HELICAL-ROTOR EXPANDER APPLICATIONS FOR GEOTHERMAL ENERGY CONVERSION

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MS. date: April 1, 1976

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Abstract

The helical-rotor expander is one of numerous candidates for the direct "total flow" expansion of geothermal fluids. The potential exists for expansion efficiencies as high as 70%. Engine efficiencies will generally be below 70% because of an expansion-ratio limitation of 15 and a maximum pressure differential of 110 psi.

Single-stage expansion of self-pumped geothermal fluids from reservoirs at 350 to 752°F gives engine efficiencies in the range from 57.3 to 43.1%. Calculated rotor diameters range from 10.5 to 41.6 ft for a range of outputs of 5 to 40 MW.

Two-stage expansion of fluid from the 572°F reservoir gives an engine efficiency of 62.3% and a 21% reduction in the diameter of the larger rotor. Rotor diameters can be held to more reasonable sizes by using the expander in combination with a

vapor turbine. For the expansion of reservoir fluid at 572°F (360 psia at the wellhead with 18.9% quality) to helical-rotor-expander exhaust pressures ranging from 66 to 14.7 psia, calculations give rotor diameters of 4 to 8.5 ft for a 40-MW combined system power output. Corresponding system engine efficiencies vary from 58.7 to 55.3%.

The impulse contribution to the total work of the expander ranges from 7.7 to 8.5% when 572°F reservoir fluid is expanded to backpressures from 1.69 to 14.7 psia, respectively.

The better applications for the helical-rotor expander are for noncondensing, low-power-output uses or in a hybrid system with helical-rotor exhaust pressures above 1 atm.

Introduction

The helical-rotor (HR) expander is a leading contender in the positive-displacement approach to direct expansion of two-phase geothermal fluids. The expander consists of two side-by-side meshing rotors. Physical contact between the counterrotating helical rotors is prevented by a set of timing gears. Some HR air compressors operate without timing gears, however. The absence of internal wearing parts, inherent mechanical balance, and the

ability to operate at high rotary speeds are all attractive features. Since all internal surfaces (excluding inlet and outlet ports) are swept, scale buildup in the expander is self-limiting.

This paper is a compilation of undocumented work aimed at understanding and evaluating the expander as a potential geothermal prime mover. The approach taken is to illustrate by examples use of the HR expander in several configurations.

The Expander Model

VOLUMETRIC CAPACITY

The volumetric capacity is defined* by

$$V_c = \frac{A \cdot L \cdot N}{60} \cdot \frac{e}{e-1} \quad (1)$$

With a typical root diameter D_r equal to $0.6D_o$, the annular-area term becomes

$$A_e = \frac{\pi}{4} [D_o^2 - (0.6D_o)^2] = 0.5027D_o^2 \quad (2)$$

In practice the rotor length L is between 1 and 1.5 times the rotor outside diameter D_o . All examples studied in this paper assume a rotor length of $1.5D_o$. Combining Eqs. (1) and (2) and setting $L = 1.5D_o$ gives

$$V_c = 0.0126D_o^3 N \quad (3)$$

EXPANSION EFFICIENCY AND ALLOWABLE TIP SPEED

The experimental unit tested in our laboratory¹ has a rotor outside diameter of 6 in. and a tip speed of 131 ft/sec at the maximum speed of 5000 rpm. A unit with a rotor diameter of 16 in. is being built by Hydrothermal Power Company, Ltd., with a planned speed of 5000 rpm; this would result in a tip speed of 349 ft/sec. Some air-compressor data² show that adiabatic

compressor efficiencies as high as 69.5 to 73.4% have been obtained at compression ratios of 4.0 and tip speeds of 35% of sonic velocity. For many of the examples developed here, an efficiency of 70% was assumed at a tip speed of 300 ft/sec for the male rotor. These assumptions may be quite optimistic, especially for the lower-quality-exhaust conditions. Empirical data will be needed to determine whether an expansion efficiency of 70% can in fact be realized for operation at a tip speed as high as 300 ft/sec.

ROTOR DIAMETER

The rotor diameter is related to the tip speed by $U_t = \pi ND_o/60$. Solved for the rotor speed this gives

$$N = \frac{60U_t}{\pi D_o} \quad (4)$$

Combining Eqs. (3) and (4) gives

$$V_c = 0.2406D_o^2 U_t \quad (5)$$

However, V_c is also equal to the product of the mass flow rate and the exhaust specific volume. Thus

$$\dot{m} v_e = 0.2406D_o^2 U_t$$

This equation can be solved for the rotor diameter:

$$D_o = \frac{\dot{m} v_e}{0.2406U_t} \quad (6)$$

*The symbols and units for all calculations are listed in Appendix A.

EXPANSION RATIO

The expansion ratio is defined by the fully expanded lobe volume divided by the initial lobe volume existing when the inlet port has just closed:

$$R_v = \frac{V_e}{V_i} = \frac{mv_e}{mv_i} = \frac{v_e}{v_i}. \quad (7)$$

Helical-screw air compressors generally have compression ratios less than 8 (deduced from the maximum output at 125 psig). On the basis of the inlet pocket porting considerations, high steam pressures as the expansion ratio is increased, and associ-

ated leakage problems, an expansion ratio of about 15 seems to be a reasonable upper bound.

ALLOWABLE PRESSURE DIFFERENTIAL

Commercial Lysholm-type compressors are normally limited to a 5:1 pressure ratio³ (59 psid), though some large units are used up to 125 psig (110 psid). Large pressure differentials cause mechanical problems, such as deflections and high bearing loading.³ In light of the preceding information a maximum pressure differential of 110 psi between the inlet and exhaust ports is assumed to be the maximum practical limit for very large machines.

Single-Stage Expansion over a Range of Wellhead Characteristics

A number of reservoir temperatures and wellhead conditions for self-pumping wells are listed in Ref. 4. Reservoir temperatures are 350, 400, 500, and 572°F. Corresponding wellhead pressures, qualities, and enthalpies are listed in Table 1.

The object in this section is to calculate, at each of the four wellhead conditions, rotor diameters for single-stage expanders for outputs of 5, 10, 20, and 40 MW. Assumptions used in the calculations are an expansion ratio of about 15,

an expansion efficiency of 70%, a rotor-tip velocity of 35% of sonic, and a condenser temperature of 120°F. Throttling is done ahead of the expander in order to limit the expansion ratio required to 15. The process is shown in Fig. 1. Figure 2 shows sonic velocity at 120°F, plotted, from points calculated by Alger,⁵ as a function of quality.

A typical calculation is shown in Appendix B. Expansion takes place at constant enthalpy through the throttling device to

Table 1. Summary of steam conditions and performance.

Reservoir type	Wellhead			Ideal Δh_{1-4} (Btu/lb)	Expander						
	P_1 (psia)	h_1 (Btu/lb)	X_1		P_2 (psia)	X_3	v_3 (ft ³ /lb)	R_v	Δh_{2-3} (Btu/lb)	U_t (ft/sec)	η_e
I	50	318.2	0.074	33.86	19.0	0.2056	41.80	15.20	19.41	196	0.573
II	98	371.0	.083	50.02	21.5	.2505	50.94	15.14	26.09	220	.522
III	220	478.7	.137	86.83	25.0	.3418	69.48	14.97	40.18	264	.463
IV	360	562.5	0.189	119.6	27.0	0.4124	83.84	14.97	51.50	293	0.431

Fig. 1. Single-stage expansion with throttling. Point 1 is the wellhead state, point 2 is the expander inlet state, and point 3 is at the condenser temperature.

an intermediate pressure, determined by iteration, that will result in an expansion ratio of 15 for an expander of 70% efficiency. Results of the calculations are shown in Tables 1 and 2.

Effective engine efficiencies are higher for the lower energy reservoirs since less throttling is required. Rotor diameters, however, are larger because the mass flow rate per unit of power is higher and the tip velocities are lower.

The use of identical machines in parallel is not a very effective method of reducing rotor diameters. For example, if four 10-MW machines were used in parallel to replace one 40-MW unit, the rotor diameters would be reduced by only a factor of 2.

Table 2. Expander sizes.

Reservoir type	Shaft power (MW)	$\dot{m}$ (lb/sec)	D_o (ft)	L (ft)	N (rpm)
I	5	244.2	14.7	22.1	254
II	5	181.7	13.2	19.8	318
III	5	118.0	11.4	17.0	444
IV	5	92.05	10.5	15.7	535
I	10	488.4	20.8	31.2	180
II	10	363.5	18.7	28.0	225
III	10	236.0	16.1	24.1	314
IV	10	184.1	14.8	22.2	378
I	20	976.8	29.4	44.1	127
II	20	727.0	26.5	39.7	159
III	20	472.0	22.7	34.1	222
IV	20	368.2	20.9	31.4	267
I	40	1954	41.6	62.4	90
II	40	1454	37.4	56.1	112.3
III	40	944.0	32.1	48.2	157
IV	40	736.4	29.6	44.4	189

Fig. 2. Sonic velocity U_0 as a function of steam quality X at a temperature of 120°F .

Staging

This section considers the effect of using two stages on the effective engine efficiency and rotor diameter for one well-head condition (360 psia and $X_1 = 0.189$). An expander efficiency of 70% and a tip speed of 300 ft/sec are assumed. The maximum allowable pressure differential is 110 psi, with an expansion-ratio limit of 15.

Fig. 3. Two-stage expansion with throttling. Point 1 is the wellhead state, point 2 is the high-pressure expander inlet, point 3 is the high-pressure expander exhaust/low-pressure expander inlet, and point 4 is the low-pressure expander exhaust.

The process is shown in Fig. 3. The pressure is lowered from point 1 to point 2 by means of throttling. Points 2 and 3 are selected so that 110 psid is not exceeded in the high-pressure stage (stage 1) and the expansion ratio in the low-pressure stage (stage 2) is near 15. Point 4 is at the condenser temperature of 120°F (1.69 psia). No losses are assumed between the two stages. In practice, some pressure drop between stages would be expected. Details of the calculations are given in Appendix C.

Thermodynamic state properties are listed in Table 3 for each of the points.

Table 3. Thermodynamic state points for two-stage expansion example.

Parameter	Point			
	1	2	3	4
P (psia)	360	136	26	1.69
X	0.189	0.2756	0.3401	0.3886
h (Btu/lb)	562.24	562.24	533.99	486.58
S (Btu/lb-°F)	0.7763	0.7998	0.8171	0.8521
v (ft ³ /lb)	0.2593	0.9252	5.356	79.007

Table 4. Stage sizes.

Shaft power (MW)	$\dot{m}$ (lb/sec)	D_{o1} (ft)	L_1 (ft)	N_1 (rpm)	D_{o2} (ft)	L_2 (ft)	N_2 (rpm)
5	62.67	2.16	3.23	2657	8.28	12.42	692
10	125.34	3.05	4.58	1879	11.71	17.57	489
20	250.67	4.31	6.47	1328	16.56	24.85	346
40	501.35	6.10	9.15	939	23.43	35.14	245

An effective engine efficiency of 63.3% is obtained, with 37.3% of the work accomplished in stage 1 and 62.7% in stage 2. A comparison between the type IV reservoir (see Table 1) results in the preceding section shows a 47% improvement in effi-

ciency (62.3 versus 43.1) as well as a reduction of 21% in the diameter of the rotor. As indicated in Table 4, the expanders turn at different speeds. This would require gearing to match them to a common alternator.

A Hybrid System

A hybrid system utilizing a helical-rotor expander, a steam separator, and a conventional vapor turbine is shown in Fig. 4. Throttling is done ahead of the HR expander in order to limit the pressure differential across the machine to 110 psi. Expansion efficiencies of both the HR expander and the turbine are assumed to be 70%.

The process is shown on a temperature-entropy diagram (Fig. 5). Calculations were done for HR expander exhaust pressures from 14.7 to 66 psia. A loss in availability of about 3 Btu/lb was used to establish the pressure drop through the separator. Results of the calculations are presented in Table 5 and Figs. 6 and 7. The maximum overall engine efficiency of 58.7% occurs

Fig. 4. Hybrid system.

Fig. 5. Hybrid-system state points.

Parameters: $P_1 = 360$ psia,
 $X_1 = 0.189$, $P_3 - P_2 = 110$ psi,
 $T_7 = 120^\circ\text{F}$, and $\Delta h_{1-8} = 119.6$
 Btu/lb.

Fig. 6. Rotor diameters for the HR expander vs exhaust pressure for a combined-system shaft power of 5, 10, 20, and 40 MW.

Fig. 7. Performance of hybrid system.
 The maximum overall engine
 efficiency occurs at $P_3 = 14.7$
 psia.

Table 5. Results of hybrid-system calculations.

P_2 (psia)	x_2	v_2 (ft ³ /lb)	P_3 (psia)	x_3	v_3 (ft ³ /lb)	Δh_{2-3} (Btu/lb)	P_5 (psia)	Δh_{5-7} /lb mixture (Btu/lb)	x_7
125	0.2816	1.023	14.696	0.3557	9.543	37.01	13	33.18	0.9438
135	.2761	0.9333	25	.3413	5.575	28.86	22	40.31	.9300
145	.2709	0.8568	35	.3304	3.942	23.99	31	44.38	.9211
156	.2654	0.7840	46	.3203	2.961	20.33	41	47.28	.9138
166	.2606	0.7265	56	.3122	2.402	17.85	50	49.06	.9087
176	0.2559	0.6756	66	0.3050	2.013	15.92	58	50.24	0.9048

at $P_3 = 14.7$ psia (Fig. 7). Operation with $P_3 > 14.7$ psia (1 atm) is desirable since rotor diameters and expansion ratios increase rapidly with exhaust pressure reduc-

tion (see Figs. 6 and 7). Significant reductions in rotor diameter (Fig. 6) can be made with only a small penalty in efficiency (Fig. 7).

Impulse Contribution

In preceding examples a throttling process has been used ahead of the expander in order to limit either the pressure differential or the expansion ratio across the machine. Since throttling represents dissipation of available energy, it is desirable to minimize its use. A variable-area convergent nozzle placed at the rotor inlet pocket (Hydrothermal Power Company, patent pending) allows converting a portion of the available energy into work. High-velocity fluid at the nozzle exit is deflected by rotor surfaces, and the resulting impulse adds torque to the rotor. This section appraises the contribution from impulse to the total work for one wellhead state (360 psia, $x = 0.189$) and three exhaust pressures (14.7, 4.0, and 1.69 psia). A maximum tip velocity of 300 ft/sec (male rotor), a positive-displacement expansion ratio of 15, and an associated expansion efficiency of 70% were assumed for the calculations.

The thermodynamic model is shown in Fig. 8. Partial expansion of the steam

occurs in the nozzle from point 1 to point 2_n at essentially constant entropy. Sonic velocity is attained at 2_n . The remaining isenthalpic expansion occurs from 2_n (the nozzle exit plane) to 2_a (the first impulse stage, i.e., the mesh point). The steam quality at point 2_a is used as a factor to establish relative amounts of work from liquid and vapor phases. An increase in entropy occurs between points 2_a and 2

Fig. 8. Thermodynamic state points for HR expansion with an impulse contribution.

since only a portion of the kinetic energy is converted into work. Expansion, after the inlet port is closed, then takes place at 70% efficiency between points 2 and 3. Impulse work is accomplished between points 1 and 2, and work from positive-displacement expansion is accomplished between points 2 and 3.

Evaluation of the impulse work was based on interaction between the nozzle effluent and the retreating male and female lobe surfaces (Fig. 9). Nozzle placement is assumed to be such that the stream is di-

Fig. 9. Impulse model ("top" view).

vided equally near the mesh intersection. Because of the abrupt change in stream direction at the first turn (stage 1), it is assumed that liquid droplets strike the surfaces and lose their relative velocity. The vapor goes on to the second stage formed by the intersection of the lobe surfaces and the end wall. A 15% loss in relative vapor velocity is assumed to occur at each stage because of the adverse geometry (corners instead of smooth curves). Turning angles, $\theta_1, \theta_2, \theta_3$, and θ_4 were calculated from a flat pattern layout of the mesh intersection lines (Fig. 10). Average lobe velocities (U_{rm} and U_{rf}) are shown in Fig. 11. On the basis of the analysis outlined in Appendix E, "blade" efficiencies for the liquid and vapor phases were calculated to be 40 and 81%, respectively.

Thermodynamic results are listed in Table 6. The impulse contribution is rather small, ranging from 7.7 to 8.5% for

Fig. 10. Flat pattern layout.

exhaust pressures from 1.69 to 14.7 psia. Overall engine efficiency varies from 46.2% (1.69-psia exhaust) to 64.7% (14.7-psia exhaust). The 171-psi pressure differential across the expander with atmospheric exhaust pressure (required to attain a 15:1 expansion ratio) may be too high a value for large units. A maximum value of 110 psi was used in other sections. Calculated flow rates and expander sizes are given in Table 7.

Fig. 11. Lobe velocities.

Table 6. Summary of steam conditions and performance with an impulse contribution.

$$P_1 = 360 \text{ psia}, X_1 = 0.189, \\ h_1 = 562.24 \text{ Btu/lb}, S_1 = 0.7763 \text{ Btu/lb-}^\circ\text{F}$$

Parameter	P_3 (psia)		
	14.7	4.0	1.69
P_2 (psia)	186.0	60.0	27.0
h_{2a} (Btu/lb)	554.49	554.49	554.49
X_{2a}	0.2423	0.3194	0.3600
h_2 (Btu/lb)	558.37	558.13	558.00
X_2	0.2469	0.3234	0.3637
v_2 (ft ³ /lb)	0.6197	2.332	5.528
S_2 (Btu/lb- ^o F)	0.7866	0.8205	0.8498
h_3 (Btu/lb)	516.71	509.64	507.05
X_3	0.3469	0.3863	0.4086
v_3 (ft ³ /lb)	9.309	35.02	83.06
W_i (Btu/lb)	3.87	4.11	4.24
W_e (Btu/lb)	41.66	48.49	50.95
W (Btu/lb)	45.53	52.60	55.19
η_e (%)	64.7	52.3	46.2
v_3/v_2	15.02	15.02	15.02
v_3/v_1	35.91	135.1	320.3
W_i/W (%)	8.5	7.8	7.7

Table 7. Single-stage expander sizes with an impulse contribution.

Shaft power (MW)	P_3 (psia)	$\dot{m}$ (lb/sec)	D_o (ft)	L (ft)	N (rpm)
5	14.7	104.14	3.66	5.49	1566
5	4.0	90.14	6.61	9.92	867
5	1.69	85.91	9.94	14.91	576
10	14.7	208.28	5.18	7.77	1106
10	4.0	180.29	9.35	14.03	613
10	1.69	171.82	14.06	21.09	408
20	14.7	416.56	7.33	11.00	782
20	4.0	360.57	13.22	19.83	433
20	1.69	343.65	19.88	29.82	288
40	14.7	833.12	10.36	15.54	553
40	4.0	721.14	18.70	28.05	306
40	1.69	687.30	28.12	42.18	204

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Appendix A

Symbol Names and Units

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
A_e	Annular area between tip and root diameters	$\text{ft}^2/\text{rev.}$
D_o	Rotor outside diameter	ft
D_r	Rotor root diameter	ft
h	Enthalpy	Btu/lb
L	Rotor length	ft
m	Fully charged lobe mass	lb
$\dot{m}$	Mass flow rate	lb/sec
N	Rotor speed	rpm
P	Pressure	psia
P	Expander shaft power	MW
R_o	Rotor outside radius	ft
R_v	Expansion ratio	--
S	Entropy	Btu/lb-°F
T	Temperature	°F
U_M	Mesh point velocity	ft/sec
U_0	Sonic velocity	ft/sec
U_{rf}	Female-rotor lobe velocity at $0.8R_o$	ft/sec
U_{rm}	Male-rotor lobe velocity at $0.8R_o$	ft/sec
U_t	Rotor tip velocity	ft/sec
U_{tf}	Female-rotor tip velocity	ft/sec
U_{tm}	Male-rotor tip velocity	ft/sec
V_c	Volumetric capacity of fully expanded mixture	ft^3/sec
V_e	Fully expanded lobe volume	ft^3
V_i	Initial lobe volume	ft^3
v	Specific volume	ft^3/lb
v_e	Exhaust specific volume	ft^3/lb
v_i	Inlet specific volume	ft^3/lb

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
W	Expander work	Btu/lb
W_e	Expansion work	Btu/lb
W_i	Impulse work	Btu/lb
X	Steam quality	--
Δh	Enthalpy change	Btu/lb
η	Expander efficiency	--
η_e	Effective engine efficiency	--
η_g	Vapor-blade efficiency	--
η_L	Liquid-droplet blade efficiency	--

Subscripts

f	Liquid	--
fg	Vapor value - liquid value	--
g	Vapor	--
s	Process at constant entropy	--

Appendix B

Sample Calculation for Single-Stage Expansion

The example selected is a 10-MW expander using wellhead fluid from a type I reservoir (see Table 1). Available energy is represented by an isentropic expansion from point 1 to point 4 (see Fig. 1). Initial entropy is calculated from the stated wellhead pressure and quality:

$$S_1 = S_{1f} + X_1 S_{1fg} = 0.4110 + 0.074(1.2474) \\ = 0.50331 \text{ Btu/lb-}^\circ\text{F.}$$

By definition, $S_4 = S_1$, and therefore

$$X_4 = \frac{S_4 - S_{4f}}{S_{4fg}} = 0.19148,$$

$$h_4 = h_{4f} + X_4 h_{4fg} = 284.34 \text{ Btu/lb.}$$

The available energy between the wellhead and the condenser is $h_1 - h_4$:

$$h_1 - h_4 = 33.86 \text{ Btu/lb.}$$

Expansion between points 1 and 2 is accomplished by throttling. Therefore

$$h_2 = h_1 = 318.2 \text{ Btu/lb.}$$

Pressure at point 2 was determined by trial and error to be 19 psia. Furthermore,

$$X_2 = \frac{h_2 - h_{2f}}{h_{2fg}} = 0.1297,$$

$$v_2 = X_2 v_{2g} + (1 - X_2) v_{2f} = 2.75 \text{ ft}^3/\text{lb},$$

$$S_2 = S_{2f} + X_2 S_{2fg} = 0.5139,$$

$$S_2 = S_{3s} = 0.5139,$$

$$X_{3s} = \frac{S_{3s} - S_{3sf}}{S_{3sfg}} = 0.1975,$$

$$h_{3s} = h_{3sf} + X_{3s} h_{3sfg} = 290.46 \text{ Btu/lb},$$

$$\Delta h_{2-3} = \eta \Delta h_{2-3s} = 0.7(318.20 - 290.46)$$

$$= 19.41 \text{ Btu/lb},$$

$$h_3 = h_2 - \Delta h_{2-3} = 298.79 \text{ Btu/lb},$$

$$X_3 = \frac{h_3 - h_{3f}}{h_{3fg}} = 0.2056,$$

$$v_3 = v_{3f} + X_3 v_{3fg} = 41.80 \text{ ft}^3/\text{lb},$$

$$R_v = \frac{v_3}{v_2} = \frac{41.80}{2.75} = 15.20,$$

$$\eta_e = \frac{\Delta h_{2-3}}{\Delta h_{1-4}} = \frac{19.41}{33.86} = 0.573, \text{ or } 57.3\%.$$

From Fig. 2, with $X = X_3 = 0.2056$, we obtain a sonic velocity of 560 ft/sec, and hence

$$U_t = 0.35 U_0 = 0.35(560) = 196 \text{ ft/sec.}$$

The machine can now be sized for a 10-MW output:

$$\dot{m} = (10,000) \text{ kW} (0.948) \frac{\text{Btu}}{\text{sec-kW}} \left(\frac{1}{19.41} \right) \frac{\text{lb}}{\text{Btu}}$$

$$= 488.4 \frac{\text{lb}}{\text{sec}}$$

From Eq. (6) the outside diameter of the rotor is found to be

$$D_o = \left[\frac{488.4(41.8)}{0.2406(196)} \right]^{1/2} = 20.8 \text{ ft},$$

and the length of the rotor is found to be

$$L = 1.5D_o = 1.5(20.8) = 31.2 \text{ ft}.$$

From Eq. (4) the rotor speed is calculated as

$$N = \frac{60(196)}{\pi(20.8)} = 180 \text{ rpm}.$$

Appendix C

Two-Stage Expansion Calculations

THROTTLING FROM POINT 1 TO POINT 2

The enthalpy at point 1 is

$$h_1 = h_{1f} + X_{1fg} h_{1fg} = 412.67 + 0.189(791.4) \\ = 562.24 \text{ Btu/lb.}$$

Since the process is throttling, $h_2 = h_1$.
The pressure at point 2 is 136 psia.
Furthermore,

$$X_2 = \frac{h_2 - h_{2f}}{h_{2fg}} = 0.2756,$$

$$S_2 = S_{2f} + X_2 S_{2fg} = 0.7998 \text{ Btu/lb-}^\circ\text{F,}$$

and

$$v_2 = X_2 v_{2g} + (1 - X_2) v_{2f} = 0.9252 \text{ ft}^3/\text{lb.}$$

EXPANSION FROM POINT 2 TO POINT 3 (STAGE 1)

The assumed expander efficiency η is 0.7.
The pressures at points 2 and 3 (P_2 and P_3) were determined by trial and error to be 136 and 26 psia, respectively. The other parameters are calculated as follows:

$$S_{3s} = S_2 = 0.7998,$$

$$X_{3s} = \frac{S_{3s} - S_{3f}}{S_{3fg}} = 0.3274,$$

$$h_{3s} = h_{3f} + X_{3s} h_{3fg} = 521.89 \text{ Btu/lb,}$$

$$\Delta h_{2-3} = \eta \Delta h_{2-3s} = 28.246 \text{ Btu/lb,}$$

$$h_3 = h_2 - \Delta h_{2-3} = 533.99 \text{ Btu/lb,}$$

$$X_3 = \frac{h_3 - h_{3f}}{h_{3fg}} = 0.3401,$$

$$v_3 = (1 - X_3) v_{3f} + X_3 v_{3g} = 5.356 \text{ ft}^3/\text{lb,}$$

$$\frac{v_3}{v_2} = \frac{5.356}{0.9252} = 5.79.$$

EXPANSION FROM POINT 3 TO POINT 4 (STAGE 2)

The assumed expander efficiency η is 0.7.
The method of calculation is identical with that used for stage 1, and the expansion ratio from point 3 to point 4 is found to be

$$\frac{v_4}{v_3} = \frac{79.007}{5.356} = 14.75.$$

EXPANDER SIZES

The mass flow rate is

$$\dot{m} = \frac{P \text{ (MW)} (948.3) \text{ (Btu/sec-MW)}}{75.66 \text{ Btu/lb}} = 12.534 P.$$

From Eq. (6) the outside diameter of the rotor is found to be

$$D_o = \left[\frac{\dot{m} v_e}{0.2406(300)} \right]^{1/2}.$$

For stage 1,

$$L = 1.5D_o,$$

$$D_o = 0.2724m^{\frac{1}{2}},$$

and from Eq. (4) the speed of the rotor is calculated to be

and for stage 2,

$$D_o = 1.0462m^{\frac{1}{2}}.$$

$$N = \frac{60(300)}{\pi D_o} = \frac{5730}{D_o}$$

The length of the rotor is

for both stages.

Appendix D

Sample Calculation for Hybrid System

The case of an HR expander exhaust pressure of 14.7 psia (1 atm) is considered for this example.

EXPANSION FROM POINT 1 TO POINT 2

The wellhead enthalpy h is 562.24 Btu/lb. Because of the throttling process, $h_2 = 562.24$ Btu/lb. Furthermore,

$$P_2 = 125 \text{ psia,}$$

$$X_2 = \frac{h_2 - h_{2f}}{h_{2fg}} = 0.2816,$$

$$S_2 = S_{2f} + X_2 S_{2fg} = 0.8023 = S_{3s},$$

$$v_2 = (1 - X_2) v_{2f} + X_2 v_{2g} = 1.023 \text{ ft}^3/\text{lb.}$$

EXPANSION FROM POINT 2 TO POINT 3

$$P_3 = 14.696 \text{ psia,}$$

$$X_{3s} = \frac{S_{3s} - S_{3f}}{S_{3fg}} = 0.3394,$$

$$h_{3s} = h_{3f} + X_{3s} h_{3fg} = 509.37 \text{ Btu/lb,}$$

$$\Delta h_{2-3} = \eta \Delta h_{2-3s} = 0.7 (h_2 - h_{3s})$$

$$= 37.01 \text{ Btu/lb,}$$

$$h_3 = h_2 - \Delta h_{2-3} = 525.23 \text{ Btu/lb,}$$

$$X_3 = \frac{h_3 - h_{3f}}{h_{3fg}} = 0.3557,$$

$$S_3 = S_{3f} + X_3 S_{3fg} = 0.8259 \text{ Btu/lb-}^\circ\text{F}$$

$$= S_{4s},$$

$$v_3 = (1 - X_3) v_{3f} + X_3 v_{3g} = 9.543 \text{ ft}^3/\text{lb,}$$

$$\frac{v_3}{v_2} = 9.329.$$

LOSS OF AVAILABLE ENERGY IN THE SEPARATOR

$$P_4 = P_5 = P_6 = 13.0 \text{ psia,}$$

$$X_{4s} = \frac{S_{4s} - S_{4f}}{S_{4fg}} = 0.3574,$$

$$h_{4s} = h_{4f} + X_{4s} h_{4fg} = 522.10 \text{ Btu/lb,}$$

$$h_4 = h_3 \text{ (throttling).}$$

The loss of availability is defined as

$$h_4 - h_{4s} = 3.13 \text{ Btu/lb,}$$

and

$$X_4 = \frac{h_4 - h_{4f}}{h_{4fg}} = 0.3606 \frac{\text{lb vapor}}{\text{lb mixture}}.$$

EXPANSION FROM POINT 5 TO POINT 7

$$T_7 = 120^\circ\text{F,}$$

$$h_5 = h_{4g} = 1148.1 \text{ Btu/lb,}$$

$$s_5 = s_{5g} = 1.7665 \text{ Btu/lb-}^\circ\text{F} = s_{7s},$$

$$x_{7s} = \frac{s_{7s} - s_{7f}}{s_{7fg}} = 0.9054,$$

$$h_{7s} = h_{7f} + x_{7s} h_{7fg} = 1016.67 \text{ Btu/lb},$$

$$\Delta h_{5-7} = \eta \Delta h_{5-7s} = 0.7(h_5 - h_{7s})$$

$$= 92.00 \text{ Btu/lb},$$

$$h_7 = h_5 - \Delta h_{5-7} = 1056.10 \text{ Btu/lb},$$

$$x_7 = \frac{h_7 - h_{7f}}{h_{7fg}} = 0.9438,$$

$$\Delta h_{5-7}/\text{lb mixture} = x_4 \Delta h_{5-7} = 33.18 \text{ Btu/lb}.$$

WORK RATIOS

The total work consists of two components: 52.7% (37.01 Btu/lb) from the HR expander and 47.3% (33.18 Btu/lb) from the vapor turbine. Thus

$$\text{total work} = 37.01 \text{ (HR expander)}$$

$$+ 33.18 \text{ (vapor turbine)}$$

$$\Sigma \Delta h \quad \underline{\quad} \quad 70.19 \text{ Btu/lb}.$$

The effective engine efficiency is calculated as follows:

$$\eta_e = \frac{\Sigma \Delta h}{\Delta h_{1-8}} = \frac{70.19}{119.6}$$

$$= 58.7\%.$$

Appendix E

Calculations for Estimating the Impulse Contribution to the Total Work

NOZZLE EXIT VELOCITY

The wellhead pressure is 360 psia with a quality of 0.189. The other parameters are as follows:

$$s_1 = 0.7763,$$

$$v_1 = 0.2593,$$

$$h_1 = 562.24.$$

By a process of iteration it was determined that a nozzle exit pressure of 235 psia (P_{2n}) results in a velocity that is close to the sonic velocity of 623 ft/sec at that pressure and entropy. For an ideal nozzle with negligible approach velocity,

$$U_2 = U_n = 223.8 (h_1 - h_{2n})^{1/2}.$$

Thus

$$h_1 - h_{2n} = \left(\frac{U_n}{223.8} \right)^2 = \left(\frac{623}{223.8} \right)^2 = 7.75 \frac{\text{Btu}}{\text{lb}},$$

$$x_{2n} = 0.2220,$$

and

$$h_{2n} = h_1 - \Delta h_{1-2n} = 554.49 \text{ Btu/lb.}$$

IMPULSE-ENERGY CONVERSION

Geometry

The rotor geometry is based on a rotor with an L/D_o ratio of 1.5. The male rotor

has four lobes with a 300° wrap angle; the female rotor has six lobes with a 200° wrap angle (Figs. 10 and 11). From Fig. 10 the following angles are calculated:

$$\theta_1 = 49.3^\circ,$$

$$\theta_2 = 40.7^\circ,$$

$$\theta_3 = 60.2^\circ,$$

$$\theta_4 = 29.8^\circ.$$

The tip velocity for the male rotor, U_{tm} , is assumed to be 300 ft/sec; the tip

velocity for the female rotor is

$U_{tf} = \frac{4}{6} (300) = 200 \text{ ft/sec}$. The lobe velocities for the male and female rotors are

$$U_{rm} = 0.8 (300) = 240 \text{ ft/sec},$$

and

$$U_{rf} = 0.8 (200) = 160 \text{ ft/sec}.$$

The mesh-point velocity (U_M) is calculated to be 171.9 ft/sec.

Liquid Droplets

With an approach velocity of 623 ft/sec (no phase slip), complete loss of relative velocity (droplets stick), and a final velocity equal to the mesh velocity of 171.9 ft/sec, the blade efficiency $\eta_L = 0.40$.

Vapor

Based on the angles, velocities, and assumed 15% loss in relative velocity at

each turn (details omitted for brevity), the "blade" efficiencies for the female and male rotors were calculated to be 0.84 and 0.78, respectively. With the assumption of equal mass flow rate, the combined vapor blade efficiency is the average of the two; that is, $\eta_g = 0.81$.

THERMODYNAMIC AND PERFORMANCE CALCULATIONS

Expansion to Exhaust Pressure

The expansion to $P_3 = 4.0$ psia is used as an example. Here

$$h_{2a} = h_{2n} = 554.49 \text{ Btu/lb,}$$

$$\Delta h_{1-2a} = 7.75 \text{ Btu/lb (available to do work).}$$

By trial and error, a value of 60 psia is arrived at for P_2 , in order to yield an expansion ratio near 15. With values for P_2 and h_{2a} , the steam quality is found to be $X_{2a} = 0.3194$.

The impulse work is

$$W_i = (1 - X_{2a})\eta_L \Delta h_{1-2a} + X_{2a}\eta_g \Delta h_{1-2a}$$

(work from droplets) (work from vapor)

$$= 4.11 \text{ Btu/lb.}$$

The enthalpy is

$$h_2 = h_1 - \Delta h_{1-2} = 562.24 - 4.11$$

$$= 558.13 \text{ Btu/lb,}$$

and the steam quality, specific volume, and entropy are

$$X_2 = 0.3234,$$

$$v_2 = 2.332 \text{ ft}^3/\text{lb,}$$

$$S_2 = 0.8205 \text{ Btu/lb-}^\circ\text{F.}$$

For an isentropic expansion, $S_{3s} = S_2$. Proceeding, we calculate the following parameters:

$$P_3 = 4.0 \text{ psia (assigned),}$$

$$X_{3s} = 0.3657,$$

$$h_{3s} = 488.86 \text{ Btu/lb,}$$

$$\Delta h_{2-3s} = 69.27 \text{ Btu/lb,}$$

$$\Delta h_{2-3} = \eta \Delta h_{2-3s} = 48.49 \text{ Btu/lb} = W_e,$$

$$h_3 = h_2 - \Delta h_{2-3} = 509.64 \text{ Btu/lb,}$$

$$X_3 = 0.3863,$$

$$v_3 = 35.02 \text{ ft}^3/\text{lb,}$$

$$\frac{v_3}{v_2} = \frac{35.02}{2.33} = 15.02.$$

This value corresponds to the positive-displacement expansion ratio.

$$\frac{v_3}{v_1} = \frac{35.02}{0.2593} = 135.06$$

is the apparent (wellhead to exhaust) expansion ratio.

The total work = $W_i + W_e = 52.60$ Btu/lb, W_i contributing 7.8% of this total. An isentropic expansion from point 1 to temperature T_3 yields 100.52 Btu/lb of work.

The effective engine efficiency is found to be

$$\eta_e = \frac{W_1 + W_e}{\Delta h_{1-T_3}} = \frac{52.60}{100.52} = 0.523,$$

or 52.3%.

Mass Flow Rate and Expander Size

The mass flow rate is

$$\dot{m} = \frac{P(948.3)}{W}$$

For a 10-MW expander,

$$\dot{m} = \frac{9483}{52.60} = 180.29 \frac{\text{lb}}{\text{sec}}$$

From Eq. (6) the outside rotor diameter is found to be

$$D_o = \left[\frac{180.29(35.02)}{0.2406(300)} \right]^{1/2} = 9.35 \text{ ft},$$

and the rotor length is

$$L = 1.5D_o = 14.03 \text{ ft}.$$

Finally, from Eq. (4), the rotor speed is found to be

$$N = \frac{60(300)}{\pi(9.35)} = 613 \text{ rpm}.$$