BEAM SIZE BLOW-UP AND CURRENT LOSS IN THE FERMILAB
MAIN RING DURING STORAGE*G. Guignard
CERN, Geneva, Switzerland
M. Month

Brookhaven National Laboratory, Long Island, New York

MASTER1. Introduction

Observations at Fermilab during the storage mode of operation show characteristic forms of transverse beam size growth and current loss with time. There are three obvious mechanisms which can produce such blowup. The gas pressure is a source for immediate beam loss by direct nuclear scattering. Protons can also multiple coulomb scatter off the orbiting electrons of the gas atoms causing the transverse beam size to increase with time, t . This effect is therefore also proportional to the gas pressure. A third mechanism not related to the gas pressure is beam growth due to multiple crossing of betatron resonances arising from the synchrotron oscillations of the stored bunches. This simulates a random walk and causes the transverse beam size to grow with $\sqrt{t}$. Thus, we attempt to describe the observations with direct nuclear scattering, multiple coulomb scattering and multiple resonance crossing. Other effects, such as intrabeam scattering, are small and can be neglected. Also, we will not attempt to consider here the detailed influence of the changes in growth behavior with operating betatron tunes.

In addition to the loss rate from direct nuclear scattering, the presence of betatron resonances also contribute to particle loss. In fact this latter effect becomes dominant after the beam

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or for any errors or omissions, including those arising from copyright infringement, privately owned rights.

size reaches a critical value. This critical size is referred to as the resonance aperture. It is the size at which "fast" resonance crossing is no longer valid. The stopband width becomes so large (due both to emittance growth as well as the increase in magnetic field distortions) that particles are locked into the resonance and are extracted to the physical aperture.

In section 2, we describe our model in a phenomenological way, while in section 3 the coefficients involved are estimated. Theoretical curves for transverse beam growth and loss rate are plotted in section 4 and compared with some measured values. Finally, some general comments are given in section 5.

2. Phenomenology

The direct nuclear gas scattering gives a current loss rate linear in t ,

$$\frac{\Delta I}{I_0} = -C_{NS} t. \quad (1)$$

There are two contributions to the beam size growth, that due to multiple coulomb scattering (linear in t) and that due to resonance crossing (depending on $\sqrt{t}$):

$$\frac{\sigma}{\sigma_0} = 1 + C_{MS} t + C_{RES} \sqrt{t}. \quad (2)$$

I_0 and σ_0 are the initial values of the beam current and rms transverse beam size.

To see the effect of a resonance aperture, we take a parabolic density distribution for beam size:

$$\rho(x) = \frac{3}{4\ell^3} (\ell^2 - x^2),$$

where ℓ is the $\frac{1}{2}$ -width of the parabola, related to the rms size

by $\lambda = \sqrt{5} \sigma$, and ρ is normalized to unity. If we take a resonance aperture at $x = A$, we can estimate the fraction f of particles remaining by

$$f(t) = \int_0^A \rho(x) dx = \frac{3}{2} (\alpha - \frac{1}{3} \alpha^3), \quad (3)$$

where $\alpha = A/(\sqrt{5} \sigma)$, with σ given by (2).

This gives for the current as a function of time,

$$\frac{I(t)}{I_0} = f(t) - C_{NS} t \quad (4)$$

The parameters we need to estimate in the next section are C_{NS} , C_{MS} , C_{RES} and the resonance aperture, A/σ_0 , normalized to the initial rms beam size, σ_0 .

3. Estimation of Parameters

The current loss due to nuclear scattering in s^{-1} is given by¹⁾

$$\frac{dI}{I dt} = -\sigma_{n,N} \frac{ec^2}{\pi} P_{NS} = -C_{NS} \quad (5)$$

where $\sigma_{n,N}$ is the total nuclear cross-section for protons on nitrogen and P_{NS} is the pressure of nitrogen giving the same beam losses due to nuclear scattering as the actual residual gas. The solution of (5) is

$$\frac{I}{I_0} = e^{-C_{NS} t} \approx 1 - C_{NS} t \quad (6)$$

and the approximation holds for t small compared with $1/C_{NS}$.

Looking at the initial slope of the measured current losses in the main ring, we estimated C_{NS} to be $\sim 10^{-4} s^{-1}$. Using the explicit expression (5) for C_{NS} , we calculate the pressure of Nitrogen

$$P_{NS} = 1.36 \cdot 10^{-7} \text{ Torr}$$

which is not too far from the measured pressure, i.e. $\sim 10^{-7}$ Torr.

The rate of fractional rms beam dimension increase $[s^{-1}]$ due to multiple scattering in the residual gas is given by¹⁾

$$\sigma_y \frac{d\sigma_y}{dt} = k \frac{\bar{\beta}_y \beta_y}{p^2} P_{MS} = B \quad (7)$$

where $k = 5.557 \cdot 10^{-2}$, for β_y in m, P_{MN} in Torr and p in GeV/c. $\bar{\beta}_y$ is the average β -value (~ 50 in the main ring), p is the proton momentum (100 GeV/c), P_{MS} is the N_2 equivalent multiple scattering pressure and β_y is the β -value where σ_y is measured.

The solution of (7) is

$$\frac{\sigma}{\sigma_0} = \sqrt{1 + \frac{2B}{\sigma_0^2} t} \approx 1 + C_{MS} t \quad (8)$$

with $C_{MS} = \frac{B}{\sigma_0^2}$,

and the approximation holds for t small compared with $1/C_{MS}$.

Using (7), assuming an initial emittance of the order of $0.5 \cdot 10^{-6}$ at 100 GeV/c and taking an equivalent pressure P_{MS} of $5 \cdot 10^{-8}$ Torr ($\sim$ half of the pressure P_{NS} , as in the ISR), we can estimate C_{MS}

$$C_{MS} = 3.5 \cdot 10^{-4} s^{-1} \quad (9)$$

The transverse blow-up due to a single fast crossing of a monodimensional resonance is given by^{2,3)}

$$\frac{\Delta\sigma}{\sigma_0} = \sqrt{\frac{\pi\beta c}{2R}} \frac{\Delta e}{N^{3/2} \sqrt{\frac{dQ}{dt}}} \quad (10)$$

where N is the order of the resonance ($NQ=p$),

Δe is the bandwidth of the resonance, ($\Delta e = 2N\Delta Q$)

dQ/dt is the rate of change in the tune.

A similar formula exists for bidimensional resonances, but

it is much more complicated, so that we did not consider them in this first approach.

If we suppose that the particles are crossing a resonance because of the synchrotron oscillations in the bunch, the rate of change in the tune is given by³⁾

$$\frac{dQ}{dt} = 4 f_s Q' \frac{\Delta p}{p} \quad (11)$$

with f_s the synchrotron frequency and Q'/Q the chromaticity.

Because of these synchrotron oscillations, there are many crossings of the resonance and, since the phases of the crossings are random, the total increase of σ is roughly proportional to the square root of the number of crossings, i.e. $\sqrt{2f_s t}$. Putting it all together, we obtain

$$\frac{\Delta\sigma}{\sigma_0} = \frac{\Delta e}{N^{3/2}} \sqrt{\frac{\pi\beta c}{4RQ' \frac{\Delta p}{p}}} t = C_{RES} \sqrt{t} \quad (12)$$

which gives

$$\frac{\sigma}{\sigma_0} = 1 + C_{RES} \sqrt{t}$$

and

$$C_{RES} = \frac{\Delta e}{N^{3/2}} \sqrt{\frac{\pi\beta c}{4RQ' \frac{\Delta p}{p}}} \quad (13)$$

For the main ring, R is 1000 m, $\Delta p/p$ is $.53 \cdot 10^{-3}$ and Q' can vary between 3 and 20. A fit on the measurements gives C_{RES} and the formula (13) gives the corresponding bandwidth Δe , which is the unknown quantity.

The coefficients C_{NS} and C_{MS} have been estimated for the main ring, while C_{RES} comes from some measurements of the transverse

size low-up. Once these three parameters are known, a fit of the relation (4) on the measured current loss gives the resonance aperture A in units of σ_0 .

4. Comparison of the measurements with the theoretical predictions

Using two sets of measurements, we can try at first to fit the theoretical change in beam size (2), with $C_{MS} = 3.5 \cdot 10^{-4}$. The results of these fits are summarized in the two following tables, where C_{RES} is calculated from (2) for each measured point.

Table 1
Horizontal size variation at 100 GeV/c with $\sigma_0 = 1.95$

t [s]	σ_{measured}	C_{RES} [s ^{-1/2}]
10	2.15	.0324
20	2.3	.0401
60	2.5	.0364
180	2.8	.0325
300	3.0	.0311

The value of C_{RES} is fairly independent of t, which means that the expression (2) fits well with the measured curve. This is shown in Figure 1, where the average value $C_{RES} = .0345$ has been used for theoretical curve.

Table 2
Horizontal size variation at 100 GeV/c, with $Q_x = 19.375$ and $\sigma_0 = 2$

t [s]	σ_{measured}	C_{RES} [s ^{-1/2}]
100	3.26	.061
250	4.31	.068
300	4.25	.06
500	4.79	.055
750	5.33	.052
950	6.92	.07

The expression (2) fits again very well the measurements and the average value of C_{RES} is $.061 \text{ s}^{-1/2}$. The theoretical curve and the measurements are given in Figure 2.

For the second set of beam size measurements, the current loss has equally been measured. Using the parameter values of section 3 ($C_{NS} = 10^{-4}$, $C_{MS} = 3.5 \cdot 10^{-4}$) and of the preceding size fit ($C_{RES} = .061$), it is possible to fit the theoretical expression (4) on the measured current decrease. The parameter to be adjusted is now the resonance aperture. Doing this, we found

$$A = 4.25 \sigma_0 \quad (14)$$

The theoretical curve (4), with A given by (14), is plotted in Figure 3, as well as the measured point. The agreement is good as long as t is not too large, since the approximations (6) and (8) are valid and as long as $t \lesssim 1000 \text{ s}$.

It is worthwhile to deduce from the relation (13) what is the bandwidth associated with the value of C_{RES} , for both the cases which have been analyzed.

$$1) C_{RES} = .0345$$

$$\Delta e = 1.636 \cdot 10^{-6} N^{3/2} \sqrt{Q'}$$

Taking a fifth order monodimensional resonance ($N=5$) and some intermediate value for Q' ($=10$), the bandwidth is

$$\Delta e = 5.78 \cdot 10^{-5}$$

$$2) C_{RES} = .061$$

$$\Delta e = 2.893 \cdot 10^{-6} N^{3/2} \sqrt{Q'}$$

which gives for $N=5$ and $Q'=10$,

$$\Delta e = 1.02 \cdot 10^{-4}$$

These values can well be compared with the bandwidth observed in other machines for $N=5$. Nevertheless, the corresponding values of Δe for higher order ($N=8$) resonances seem somewhat large.

5. Comments

This simple analysis shows that the equivalent pressure for multiple scattering can only explain a small fraction of the blow-up in beam size. It is indeed necessary to introduce the mechanism of resonances in order to explain the observed blow-up and the initial rapid increase of the transverse size. The bandwidths which correspond to this blow-up look realistic.

In this paper, we supposed that the crossing of the resonances was due to the synchrotron motion. In actual fact, it can also be a consequence of the ripple in the power supplies and of some space charge effects. It is surely interesting to estimate these two last effects and to define the area which is covered by the tune of the beam. When this is known, it may be possible to find a tune region in order to avoid resonances of any order lower than six. In such a region, the size variation should be smaller. If it is not possible to avoid low order resonances, it becomes necessary to compensate the chromaticity and the ripple of the power supplies.

References

1. E. Fischer, CERN divisional report, CERN-ISR-VA/67-16 (1967).
 2. A. Schoch, CERN report, CERN 75-21 (1957).
 3. G. Guignard, CERN report, CERN 70-24 (1970).
- G. Guignard, CERN report, CERN 76-06 (1976).

* Research carried out under the auspices of the U.S. ERDA.

Fig.1 Horizontal size increase

Fig. 2 Horizontal size increase

Fig. 3 Beam current loss in V.R.