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A Diffusion-Type Model of the Global Carbon Cycle for the Estimation of Dose to the World Population from Releases of Carbon-14 to the Atmosphere

G. G. Killough

Environmental Sciences Division
Publication No. 1036

OAK RIDGE NATIONAL LABORATORY

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**A DIFFUSION-TYPE MODEL OF THE GLOBAL CARBON CYCLE
FOR THE ESTIMATION OF DOSE TO THE WORLD POPULATION
FROM RELEASES OF CARBON-14 TO THE ATMOSPHERE**

G. G. Killough

**ENVIRONMENTAL SCIENCES DIVISION
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ABSTRACT

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A nonlinear dynamic model of the exchange of carbon among the atmosphere, terrestrial biosphere, and ocean is described and applied to estimating the radiation dose to the world's population from the release of ^{14}C to the atmosphere from the nuclear power industry. A computer implementation of the model, written in the IBM Continuous System Modeling Program III (CSMP III) simulation language, is presented. The model treats the ocean as a diffusive medium with respect to vertical transport of carbon, and the nonlinear variation of CO_2 partial pressure with the total inorganic carbon concentration in surface waters is taken into account in calculating the transfer rate from ocean to atmosphere. Transfers between the atmosphere and terrestrial biosphere are represented by nonlinear equations which consider CO_2 fertilization and impose a constraint on the ultimate total carbon mass in the biosphere. Sensitivity of the model to variation of its environmental parameters and scenarios (such as future production of nonradioactive carbon from the combustion of fossil fuels) has been studied with respect to its population dose estimates; the results of these studies indicate that the uncertainties in the environmental parameters of the model are sufficiently contained to provide adequate estimates of the population dose if the population growth model is stipulated. Comparisons of the model's predictions with a selection of population dose estimates given by other investigators show reasonable agreement, notwithstanding the differences among the models on which the estimates are based.

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1 INTRODUCTION

The identification of ^{14}C as a significant radiological effluent from nuclear power facilities has prompted a renewed interest in the assessment of its behavior in the environment and resultant radiation dose to man. Light-water-cooled reactors (LWRs) produce ^{14}C in their fuel and coolant; current estimates of the total production rate are in the range of 20 to 40 curies per GWe-yr (Davis 1977). At present it can be assumed that essentially all of that ^{14}C will be released to the atmosphere either at the reactor ($\sim 20\text{--}30\%$) or at the fuel reprocessing plant ($\sim 70\text{--}80\%$). This means that a fuel reprocessing plant having a capacity of 1500 metric tons of heavy metal (MTHM) per year could release up to 1000 Ci of ^{14}C per year of operation. Questions concerning the fraction of the total ^{14}C release that would be in the form of CO_2 as opposed to other chemical species (e.g., CH_4 and C_2H_6) remain to be answered satisfactorily.

Other nuclear fuel cycles likewise have a potential for significant environmental release of ^{14}C . A study of the potential impact of ^{14}C releases from a High Temperature Gas Cooled Reactor (HTGR) fuel reprocessing facility supporting generation of 50 GWe-yr has used an assumed annual release of 7500 Ci of $^{14}\text{CO}_2$ (Snider and Kaye 1976); this estimate excludes ^{14}C fixation, which is under development for HTGR fuel reprocessing plants. Oxide fuels in Liquid Metal Fast Breeder Reactors (LMFBRs) compare favorably with LWR fuels in their production of ^{14}C ; Davis (1977) gives a median estimate of 6.3 Ci/GWe-yr in the fuel as it enters the reprocessing plant (an additional 12.8 Ci is retained in the cladding and core structural materials). Some advanced fast breeder reactor fuels, however, have the potential for producing larger amounts of ^{14}C than LWR fuels; this increased production rate is particularly important for nitride fuel, and ^{15}N enrichment will likely be needed to minimize the production of ^{14}C in the fuel elements (Tennery et al. 1976).

When ^{14}C is released to the atmosphere as carbon dioxide, it is incorporated into the tissues of plants through photosynthesis and reaches man through his food chain; man also absorbs some of the airborne $^{14}\text{CO}_2$ by inhalation, but this intake pathway generally contributes negligibly to the radiation dose to members of the public. Near the source of a continuous release of ^{14}C to the atmosphere, the average dose rate is dependent upon local meteorology and location of sites of food production and will decrease abruptly when the emitting facility is finally taken out of service. But the release also increases the specific activity of carbon on a global scale, and because of the 5730-year half-life of ^{14}C , the result will be an increased dose rate to the world population for millennia. Estimation of the resultant population dose requires consideration of the dynamic distribution of nonradioactive carbon and ^{14}C among the several world reservoirs (atmosphere, biosphere, and ocean). It is to this aspect of the problem that the present report is addressed.

The interaction of cosmic neutrons with ^{14}N atoms in the upper atmosphere produces approximately 4×10^4 curies of ^{14}C per year (Light et al. 1973). This production rate and the radioactive decay rate of $1.21 \times 10^{-4} \text{ yr}^{-1}$ imply a steady-state inventory of about $3 \times 10^8 \text{ Ci}$, which sustains a specific activity of $\sim 6 \times 10^{-12} \text{ Ci/gC}$ in the biosphere, from which each individual receives 1.3 mrem/yr total-body radiation dose (Killough et al. 1976). Man has been exposed to this natural background cycle of ^{14}C throughout his history and has used it to advantage in estimating the age of dead organic material. This ^{14}C background is a convenient reference for a comparison with ^{14}C released from the nuclear fuel cycle and other activities of man. Moreover, measurements of the distribution of specific activity of natural radiocarbon among the world reservoirs furnish information for estimating the coefficients in compartment models of the world cycle.

From the 1940s through 1969, the atmospheric testing of nuclear weapons introduced approximately 9.7×10^6 curies of ^{14}C into the world cycle (Killough et al. 1976). In 1963, as a result of the production from weapons testing, the level of ^{14}C in the atmosphere reached a peak value about two times pre-1950 levels. This source of ^{14}C , essentially a sequence of pulse inputs into the world system, together with measurements of the activity remaining airborne, provides a second set of criteria to be met by a compartment model of the world carbon cycle.

During the industrial era, nonradioactive carbon dioxide from the combustion of fossil fuels has been released to the atmosphere at an annual rate which has grown exponentially, with only brief interruptions, at an average rate of just under three percent per year, and if the declines of the early 1930s and the two World Wars are left out of account, the average rate of growth has been in excess of four percent. While it serves to decrease the specific activity of carbon in man's exposure environment, and therefore his ^{14}C dose, the steadily increasing concentration of CO_2 in the atmosphere (331 parts per million by volume in 1974; see Baes et al. 1976) is being viewed with increasing concern regarding its potential climatological ramifications. Detailed estimates of CO_2 released from fossil-fuel combustion exist for the period from 1860 to the present (Keeling 1973b; Rotty 1975); with an input function based on this information, it is reasonable to require that the response of a model of the global carbon cycle approximate recent measurements of CO_2 concentration in the atmosphere.

In this report, we describe a model of the global carbon cycle, together with its computer implementation, which was assembled for the purpose of estimating the dose to the world population from any ^{14}C release into the cycle by way of the atmosphere. The model is defined by a nonlinear system of differential equations, which is integrated numerically by the computer.

Previous dynamic models of the carbon cycle have been pressed into service to perform similar tasks. A seven-reservoir finite-difference model of Machta (1971, 1973) was adapted by C. B. Nelson of the U. S. Environmental Protection Agency and used by Magno et al. (1974) and again by Fowler et al. (1976). Multiple-reservoir models also supported the work of Martin and ApSimon (1974) and Kelly et al. (1975). Moreover, a number of models of the global carbon cycle have been studied with objectives other than ^{14}C radiological assessment.

We offer three remarks in extenuation of our adding still another global carbon model to the list. The first is that the model is not new in the sense of pioneering an unexplored technique or incorporating any new basic science not yet considered in this context. But the second remark is that it does incorporate recent developments by two groups of investigators, namely the diffusive vertical transport of carbon in the deep ocean as implemented by Oeschger et al. (1975), and the nonlinear relationship between the concentration of inorganic carbon in ocean surface waters and the partial pressure of dissolved CO_2 , as analyzed by Keeling (1973a) and implemented by Bacastow and Keeling (1973). Both of these refinements are important to the dynamics of carbon transfer between the atmosphere and oceans on the time scales involved in our calculations, and we know of no available computer implementation of a model incorporating both. This leads to our third remark, concerning the availability of computer codes and detailed documentation associated with models to be used in applications of the sort we have in mind. Too often (perhaps inevitably) the models are labeled "developmental" and the undocumented computer implementations are given only private circulation, at best. We believe the interest in the carbon cycle, both from the standpoint of radiological assessment and the carbon dioxide problem, justifies publication of the detailed documentation of this working computer implementation.

It is certain that this model will not answer all questions that might be raised in the context of assessing the global impact of released ¹⁴C. It is our hope, however, that it will provide a starting point for investigators who have responsibility for such assessments and that it will encourage the prompt publication of more complete descriptions of the computer codes that are directly or indirectly used in environmental assessments. Users should realize, however, that this model is also "developmental" and carries no guarantees. We particularly urge that users familiarize themselves with the built-in assumptions and limitations before proceeding. The author will appreciate comments.

2 FORMULATION OF THE MODEL

The model described in this report has as its primary objective the prediction of the specific activity of ^{14}C in airborne carbon which results from the release of ^{14}C dioxide from the nuclear industry into the atmosphere. The ^{14}C radiation dose to the world population is proportional to the time integral of this quantity. The model consists of (1) a nonlinear system of differential equations which describes the transfer of nonradioactive carbon and ^{14}C among and within world reservoirs, (2) an input function which simulates the release to the atmosphere of carbon dioxide from the combustion of fossil fuels, using historical data for dates before 1974 and parametric trajectories for subsequent times, and (3) a model for world population levels in the recent past and the future. The world reservoirs considered in the model are the atmosphere, terrestrial biosphere (a slow and a rapid component), the surface waters of the ocean, the thermocline, and the deep ocean. The atmosphere, surface waters of the ocean, the two compartments of the biosphere are treated as well-mixed compartments, whereas the vertical transport of carbon in the thermocline and deep ocean is represented by a diffusive mechanism. Figure 2-1 is a schematic portrayal of the model.

The differential equations of the model are integrated from a preindustrial point in time, at which the nonradioactive carbon cycle is presumed to be in equilibrium, and prior to the introduction of the radiocarbon contaminant. Integration through 1974 permits the system to absorb the ~ 130 billion metric tons of carbon that had been released to the atmosphere through that date from the burning of fossil fuels and the production of cement. A preindustrial CO_2 concentration of 290 parts per million by volume (ppMV) in the atmosphere is assumed (this value can be varied), and one requirement imposed on the model is that it predict recently observed concentrations (e.g., 331 ppMV for the year 1974) in response to the simulated fossil-fuel input.

Figure 2-1. A schematic presentation of the box-diffusion model of global carbon transport described in this report.

A detailed description of the model and its computer implementation is presented in Secs. 2.1, 2.2, 2.2.1 through 2.2.4, and 2.3. Later sections of the report give details of the functions that define the historical and projected production of carbon from fossil-fuel combustion and the growth of the world population.

2.1 Dose to Man from ^{14}C in Global Reservoirs

In estimating the radiation dose to man from globally distributed ^{14}C , we have taken the atmosphere as man's exposure environment and have assumed that the specific activity of carbon in man's tissues maintains equilibrium with that of atmospheric carbon. The existence of a one- or two-year lag in response to changes in atmospheric specific activity is of no practical consequence in long-term dose estimates and has been ignored in our calculations.

Let $X(t)$ and $Y(t)$ denote the masses (g) of ^{14}C and nonradioactive carbon (^{12}C and ^{13}C), respectively, in the atmosphere at time t (yr). Then the specific activity of atmospheric carbon, $\text{SpA}(t)$, is

$$\text{SpA}(t) = \frac{4.46X(t)}{X(t) + Y(t)} \text{ Ci/gC}, \quad (2.1-1)$$

where 4.46 Ci/gC is the specific activity of pure ^{14}C . Since the carbon in man's tissues is assumed to be of the same specific activity, we may write the following equation for the dose rate, $\dot{D}(t)$, to man's total body:

$$\dot{D}(t) = (2.08 \times 10^8) \text{ SpA}(t) \text{ rem/yr}, \quad (2.1-2)$$

where 2.08×10^8 rem/yr per Ci/gC is a dose-rate factor for total body that is derived from dosimetric information tabulated by Killough et al. (1976). The population dose, $\mathcal{D}(t_0, t_1)$, delivered between times t_0 and t_1 (yr) is estimated as

$$\mathcal{D}(t_0, t_1) = \int_{t_0}^{t_1} N(t) \dot{D}(t) dt \text{ man-rem}, \quad (2.1-3)$$

where $N(t)$ denotes the number of individuals in the population at time t .

The population dose commitment is obtained from Eq. (2.1-3) by letting t_1 tend to infinity. To perform a numerical calculation, it is necessary to employ a dynamic model of the global carbon cycle, such as the one described in this report, to accept inputs of ^{14}C released from nuclear power facilities and nonradioactive carbon from combustion of fossil fuels, and to predict the time-varying distributions of radioactive and nonradioactive carbon among the world's reservoirs, and in particular, the atmospheric quantities $X(t)$ and $Y(t)$, which are used to compute $\dot{D}(t)$ [Eqs. (2.1-1) and (2.1-2)]. A computer implementation of such a model can only integrate the model's equations over a finite time interval (except in special cases where closed-form solutions are practical). Therefore, the integration is taken forward in time to a point t_1 at which a near-equilibrium state is achieved, subsequent to the cessation of fossil-fuel CO_2 production; the results are used to evaluate Eq. (2.1-3). To continue the integration analytically into the remote future, some scenario for the population $N(t)$ must be hypothesized; our assumption is that it remains at a constant level after time t_1 (details of the scenario are presented in Sec. 4). With this assumption, the population dose from time t_1 to infinity may be calculated as

$$D(t_1, +\infty) = (2.08 \times 10^8) \cdot N \cdot \text{SpA}(t_1) / \lambda^R \text{ man-rem}, \quad (2.1-4)$$

where

N = constant number of individuals in the world population after time t_1 , and

λ^R = radioactive decay rate for ^{14}C ($= 1.21 \times 10^{-4} \text{ yr}^{-1}$).

The total population dose commitment is the sum of Eqs. (2.1-3) and (2.1-4).

We need to point out that in application of the model $X(t)$ will be taken to mean the ^{14}C from a specific source, such as the nuclear industry or a particular facility. With such an interpretation, a conceptual error is introduced into Eq. (2.1-1); the denominator of the specific activity fraction should include all components of atmospheric carbon, including natural radiocarbon and the ^{14}C residue of nuclear weapons testing. But the mass, $Y(t)$, of nonradioactive carbon in the atmosphere exceeds 10^{17} g, whereas the steady-state level of naturally produced radiocarbon in all world reservoirs is about 7×10^7 g, and the testing of nuclear weapons through 1969 added approximately 2×10^6 g to the world inventory (Killough et al. 1976). Therefore, for any conceivable release of ^{14}C from man's activities, the mass of radiocarbon in the denominator of Eq. (2.1-1) is negligible in comparison with $Y(t)$.

In the light of the foregoing remarks, the population dose estimates computed with our model should be understood to represent only that component which may be ascribed to the ^{14}C input function, whatever the interpretation of the latter may be. In particular, components due to naturally produced radiocarbon (natural background) or the ^{14}C inventory from weapons testing are not estimated unless the ^{14}C input function has been defined in such a way as to include these sources.

An assumption that is implicit in the use of Eqs. (2.1-3) and (2.1-4) to estimate population dose is that the dose rate, $\dot{D}(t)$, as defined by Eq. (2.1-2) is appropriate for all age groups within the population. The dose-rate factor, 2.08×10^8 rem/yr per Ci/gC, is proportional to the fractional content of carbon in the body (grams of carbon per gram of tissue), which is unlikely to vary greatly with age. Therefore, the use of $\dot{D}(t)$ in Eq. (2.1-3) and the dose-rate factor in (2.1-4) is unlikely to introduce an appreciable error into the population dose estimates.

2.2 Differential Equations for Carbon Movement

Among World Reservoirs

Numerous investigators have based descriptions of the global carbon cycle on box-type models with rather gross aggregation of compartments which represent carbon in the ocean. When the coefficients of such models are adjusted to account for the distribution of natural ^{14}C , they differ substantially from values estimated from processes governed by different time constants, such as the ^{14}C produced by the testing of nuclear weapons and carbon dioxide from fossil-fuel combustion. Oeschger et al. (1975) have made a careful study of a model which accounts consistently for carbon transport over the range of time constants spanned by these components of the world cycle. Their model treats the ocean as a diffusive medium with respect to vertical transport of carbon, and they apply the term "box-diffusion" to models such as theirs which couple a diffusive thermocline and deep ocean to well-mixed "box" representations of other compartments, such as the ocean's mixed layer of surface water, the atmosphere, and components of the biosphere.

For computation, however, the diffusive processes are approximated by a substantial number of well-mixed ocean layers, with the result that the box-diffusion model is, in fact, implemented as a box model with more than the usual number of compartments. But Oeschger et al. have shown that the effect is quite different from that of the grosser aggregations.

In our model, we have adopted intact the layering of the ocean employed by Oeschger et al., although our choice of variables differs somewhat from theirs. Moreover, they have chosen to represent the carbon exchange between the atmosphere and terrestrial biosphere with time-delay equations; we have adopted different equations for this exchange.

A further refinement in our model concerns the variation of the so-called buffer factor with the total inorganic carbon

concentration in the surface water of the ocean. We have adapted a method of Keeling (1973a) and an algorithm and data of Bacastow and Keeling (1973) such that the model estimates the rate of evasion of carbon from ocean to atmosphere conformably with the evolving equilibria of inorganic carbon species as the ocean dissolves increasing quantities of carbon dioxide.

In the present version of the model, no attempt has been made to include marine organic carbon. This pool is $\sim 4\%$ of the total carbon in the ocean (Raes et al. 1976) and should have a negligible effect on the dynamics of carbon transfer between the ocean and atmosphere, from the point of view of estimating the specific activity of atmospheric carbon.

In Sec. 2.2.1 through 2.2.3 we present the differential equations and initial conditions that define the model.

2.2.1 Diffusive Model of the Ocean and Discrete Approximation

The idealized model of vertical transport of carbon in the thermocline and deep ocean which we have adapted from the work of Oeschger et al. (1975) may be formulated as follows. Let the depth coordinate z (m) increase downward, and let $z = 0$ correspond to the lower boundary of the mixed layer (assumed to be 75 m below the surface). Then the concentration $C(z,t)$ (g/m^3) of nonradioactive carbon at depth z and time t (yr) is assumed to satisfy the partial differential equation

$$C_t = KC_{zz} \quad (t \geq 0, z \geq 0) \quad (2.2.1-1)$$

with boundary conditions

$$C(0,t) = \text{concentration of nonradioactive carbon } (\text{g/m}^3) \\ \text{in the mixed layer at time } t, \quad (2.2.1-2)$$

$$C_z(h_d, t) = 0, \quad (2.2.1-3)$$

and initial condition

$$C(z,0) = \text{constant} = \text{initial concentration of nonradioactive} \\ \text{carbon in the mixed layer.} \quad (2.2.1-4)$$

In the foregoing equations, subscripts of C denote partial derivatives, and

K = diffusion coefficient (m^2/yr), and

h_d = depth of thermocline plus deep ocean (taken as 3725 m).

Let $c(z,t)$ denote the concentration of ^{14}C (g/m^3) at depth z and time t , and let λ^R stand for the radioactive decay constant for ^{14}C (yr^{-1}). Then the transport of radiocarbon is represented by the equation

$$c_t = Kc_{zz} - \lambda^R c \quad (t \geq 0, z \geq 0), \quad (2.2.1-5)$$

with boundary conditions

$$c(0,t) = \text{concentration of } {}^{14}\text{C (g/m}^3\text{) in the mixed layer} \\ \text{at time t, and} \quad (2.2.1-6)$$

$$c_z(h_d,t) = 0. \quad (2.2.1-7)$$

An appropriate initial depth distribution for radiocarbon depends on the interpretation. If the input function is regarded as a man-made source of recent origin, then in an integration which proceeds from a preindustrial point in time, the concentration at all depths is taken as zero:

$$c(z,0) = 0, \quad (0 \leq z \leq h_d). \quad (2.2.1-8a)$$

But if the input function contains naturally produced radiocarbon, the appropriate initial depth distribution is obtained by assuming preindustrial equilibrium ($c_t = 0$) and solving the ordinary differential equation

$$K \frac{d^2 c}{dz^2} - \lambda^R c = 0 \quad [\text{from Eq. (2.2.1-5)}]$$

with endpoint conditions

$$c|_{z=0} = c_0 = \text{steady-state preindustrial concentration} \\ \text{of natural } {}^{14}\text{C in the mixed layer, and}$$

$$\frac{dc}{dz}|_{z=h_d} = 0,$$

to obtain

$$c(z) = c_0 \left[\cosh \left(\sqrt{\frac{\lambda R}{K}} z \right) - \tanh \left(\sqrt{\frac{\lambda R}{K}} h_d \right) \cdot \sinh \left(\sqrt{\frac{\lambda R}{K}} z \right) \right],$$

$$(0 \leq z \leq h_c). \quad (2.2.1-8b)$$

We remark that a diffusion coefficient $K(z)$ which varies with depth might offer added realism to the ocean model. But it is unlikely that such a refinement would exert an important influence on the dynamics of transfer across the ocean surface, and we have followed Oeschger et al. in retaining the constant value. There is no conceptual or practical barrier, however, to incorporating a variable diffusion coefficient whenever data are assembled to permit a precise determination.

As we have mentioned before, a depth-discrete analog of the diffusion model is actually implemented. This discretization may be viewed as a partitioning of the thermocline and deep ocean into finite number of layers. Each layer is assumed to be well mixed, and transfers of carbon between contiguous layers occur by first-order kinetics.

Consider three consecutive layers for which z_{i-1} , z_i , and z_{i+1} are midpoint depth coordinates and Δz_{i-1} , Δz_i , and Δz_{i+1} are the respective layer thicknesses (m). Let C_i denote the nonradioactive carbon concentration $C(z_i, t)$ at a fixed time t , and let A_i (m^2) stand for the average sectional area of the ocean in the depth range from z_{i-1} to z_{i+1} . The carbon flow from layer $i-1$ to layer i is approximated by the central difference quotient

$$A_i K \left[\frac{C_i - C_{i-1}}{z_i - z_{i-1}} \right] = A_i K \left[\frac{Y_i / (A_i \Delta z_i) - Y_{i-1} / (A_i \Delta z_{i-1})}{(\Delta z_{i-1} + \Delta z_i) / 2} \right] \text{ g/yr},$$

$$(2.2.1-9)$$

where Y_{i-1} and Y_i denote the nonradioactive levels (g) in the respective layers. The expression for the flow from layer i to layer $i + 1$ is analogous. The difference of these flows is the rate of change, $\dot{Y}_i$, of carbon in the i -th layer:

$$\dot{Y}_i = K \left[C_{i-1} Y_{i-1} + U_{i+1} Y_{i+1} - (D_i + U_i) Y_i \right], \quad (2 \leq i < M), \quad (2.2.1-10a)$$

where

$$D_i = 2/[\Delta z_i (\Delta z_i + \Delta z_{i+1})], \quad (2 \leq i < M), \quad (2.2.1-11)$$

$$D_M = 0, \quad (2.2.1-12)$$

$$U_i = 2/[\Delta z_i (\Delta z_{i-1} + \Delta z_i)], \quad (2 < i \leq M) \quad (2.2.1-13a)$$

and

M = index of the deepest layer.

To make this description conformable with the computer implementation, we are defining $i = 2$ to correspond to the mixed layer, so that $i = 3$ is the uppermost layer of the thermocline. For this layer, the foregoing equations are still correct if Δz_2 is the depth of the mixed layer. The deepest layer requires a special equation, however:

$$\dot{Y}_M = K(D_{M-1} Y_{M-1} - U_M Y_M), \quad (2.2.1-10b)$$

which follows directly from Eq. (2.2.1-9).

For ^{14}C , Eqs. (2.2.1-10a) and (2.2.1-10b) are rewritten as follows:

$$\dot{\hat{x}}_i = K \left[D_{i-1} x_{i-1} + U_{i+1} x_{i+1} - (D_i + U_i + \lambda^R) x_i \right], \quad (3 \leq i < M), \quad (2.2.1-14a)$$

$$\dot{\hat{x}}_M = K \left[D_{M-1} x_{M-1} - (U_M + \lambda^R) x_M \right]. \quad (2.2.1-14b)$$

The equilibrium condition for nonradioactive carbon in the ocean compartments is that $Y_i/\Delta z_i$ be constant with time and independent of i :

$$Y_i(t)/\Delta z_i \equiv W = \text{constant} \quad (2.2.1-15)$$

When the integration proceeds from a preindustrial point in time at which the system is assumed to be in steady state, the condition (2.2.1-15) is used to set the initial values. When the radiocarbon input function includes naturally produced ^{14}C , the distribution described by Eq. (2.2.1-8b) needs to be taken into account.

The foregoing equations, together with the initial values described in Sec. 2.2.3, allocate preindustrial carbon to the deep ocean in the same concentration as in the mixed layer, but the concentration in the former should be somewhat higher. We introduce here a device for permitting the higher initial concentration beneath the mixed layer and preserving the initial equilibrium. We shall assume the higher concentration C_D throughout the thermocline and deep ocean, and define a parameter r by the equation

$$r = C_D/C_0, \quad (2.2.1-16)$$

where, as before, C_2 denotes the concentration in the mixed layer. We then redefine the coefficients U_i as follows:

$$U_i = \begin{cases} r \cdot 2 / [\Delta z_3 (\Delta z_2 + \Delta z_3)], & i = 3, \\ 2 / [\Delta z_i (\Delta z_{i-1} + \Delta z_i)], & i = 4, \dots, M. \end{cases} \quad (2.2.1-13b)$$

When the initial values are altered in such a way as to allocate carbon among the ocean's compartments consistently with Eq. (2.2.1-16), the reduced upward flow indicated by Eq. (2.2.1-13b) guarantees an initial state of equilibrium. This alternate arrangement will be utilized to compare the dynamics of exchange between the atmosphere and oceans, with regard to whether the difference is significant for the dose estimates computed with the model.

2.2.2 The Atmosphere, Terrestrial Biosphere, and Mixed Layer

In Sec. 2.2.1 the differential equations for nonradioactive carbon and ^{14}C in the thermocline and deep ocean were presented. These equations are coupled, through the respective isotopic levels Y_2 and X_2 in the mixed layer, to another set of equations for carbon levels in the mixed layer, atmosphere, and terrestrial biosphere. This latter set of equations will be presented in this section. We continue the previous notational conventions of using Y and X (with appropriate subscripts) to denote compartmental levels (in grams) of nonradioactive carbon and ^{14}C , respectively, and $Y^{(0)}$ and $X^{(0)}$ to stand for the corresponding initial values. Numeric subscripts have the following meanings: 1 = atmosphere, 2 = mixed layer, and 3 through M correspond to layers of the thermocline and deep ocean. Our model for the terrestrial biosphere makes use of two well mixed compartments: a slow-turnover component of long-lived biota (subscript B) and a rapid one (subscript S).

The equations follow:

Nonradioactive carbon

$$\begin{aligned} \dot{Y}_1 = & I^Y(t) - k_{12}Y_1 + (k_{12}/R_{12})(Y_2^{(0)}/P_{m0}) \cdot P_m(\Sigma C) \\ & + F_{B1}^Y + F_{S1}^Y - F_{1B}^Y - F_{1S}^Y, \end{aligned} \quad (2.2.2-1)$$

$$\begin{aligned} \dot{Y}_2 = & k_{12}Y_1 - (k_{12}/R_{12})(Y_2^{(0)}/P_{m0}) \cdot P_m(\Sigma C) \\ & + K(U_3Y_3 - D_2Y_2), \end{aligned} \quad (2.2.2-2)$$

$$\dot{Y}_B = F_{1B}^Y - F_{B1}^Y, \quad (2.2.2-3)$$

$$\dot{Y}_S = F_{1S}^Y - F_{S1}^Y, \quad (2.2.2-4)$$

where

$$F_{1B}^Y = (k_{1B}/R_{1B}) \left[1 + \beta_B \ln(Y_1/Y_1^{(0)}) \cdot \left(\frac{\eta_B - Y_B}{\eta_B - Y_B^{(0)}} \right) \right] Y_B, \quad (2.2.2-5)$$

$$F_{B1}^Y = (k_{1B}/R_{1B})Y_B, \quad (2.2.2-6)$$

$$F_{1S}^Y = (k_{1S}/R_{1S}) \left[1 + \beta_S \ln(Y_1/Y_1^{(0)}) \cdot \left(\frac{r_{1S} - Y_S}{\eta_S - Y_S^{(0)}} \right) \right] (Y_S^{(0)}/Y_B^{(0)}) Y_B, \quad (2.2.2-7)$$

$$F_{S1}^Y = (k_{1S}/R_{1S})Y_S. \quad (2.2.2-8)$$

Carbon-14

$$\begin{aligned} \dot{X}_1 = & I^X(t) - (\alpha_{1m} k_{12} + \lambda^R) X_1 + \alpha_{m1} (k_{12}/R_{12}) (Y_2^{(0)}/P_{m0}) \cdot P_m(\Sigma C) \cdot \\ & \cdot X_2/(X_2 + Y_2) + F_{B1}^X + F_{S1}^X - F_{1B}^X - F_{1S}^X, \end{aligned} \quad (2.2.2-9)$$

$$\begin{aligned} \dot{X}_2 = & \alpha_{1m} k_{12} X_1 - \alpha_{m1} (k_{12}/R_{12}) (Y_2^{(0)}/P_{m0}) \cdot P_m(\Sigma C) \cdot X_2/(X_2 + Y_2) \\ & - \lambda^R X_2 + K(U_3 X_3 - D_2 X_2), \end{aligned} \quad (2.2.2-10)$$

$$\dot{X}_B = F_{1B}^X - F_{B1}^X - \lambda^R X_B, \quad (2.2.2-11)$$

$$\dot{X}_S = F_{1S}^X - F_{S1}^X - \lambda^R X_S, \quad (2.2.2-12)$$

where

$$F_{1B}^X = \alpha_{1B} F_{1B}^Y X_1/(X_1 + Y_1), \quad (2.2.2-13)$$

$$F_{B1}^X = \alpha_{B1} F_{B1}^Y X_B/(X_B + Y_B), \quad (2.2.2-14)$$

$$F_{1S}^X = \alpha_{1S} F_{1S}^Y X_1/(X_1 + Y_1), \quad (2.2.2-15)$$

$$F_{S1}^X = \alpha_{S1} F_{S1}^Y X_S/(X_S + Y_S). \quad (2.2.2-16)$$

A number of quantities in Eqs. (2.2.2-1) through (2.2.2-16) require definition and comment.

$I^Y(t)$, $I^X(t)$ = input functions (gC/yr) of nonradioactive carbon and ^{14}C , respectively,

k_{ij} = transfer rate coefficient (yr^{-1}) of carbon from compartment i to compartment j ,

R_{ij} = initial mass ratio of total carbon in compartments j and $i = Y_j^{(0)} / Y_i^{(0)}$,

ΣC = total inorganic carbon concentration (mol/l) in the mixed layer,

$P_{\Sigma}(\Sigma C)$ = partial pressure (atmospheres) of dissolved CO_2 in the mixed layer, expressed as a function of the total inorganic carbon concentration,

$P_{\Sigma 0} = P_{\Sigma}(\Sigma C_0)$ = initial value of CO_2 partial pressure in the mixed layer,

n_B , n_S = limiting masses of carbon (g) in the long- and short-lived biota, respectively,

F_{ij}^Y , F_{ij}^X = flows (gC/yr) of nonradioactive carbon and ^{14}C , respectively, from compartment i to compartment j , and

α_{ij} = isotopic fractionation parameter for ^{14}C flow from compartment i to compartment j .

The partial pressure, $P_{\Sigma}(\Sigma C)$, of dissolved carbon dioxide in the mixed layer is a nonlinear function of the total dissolved inorganic carbon, ΣC , with equivalent mass essentially equal to Y_2 . This functional relationship has been derived and discussed by Keeling (1973a) and was utilized by Bacastow and Keeling (1973) with a six-reservoir box model to predict the atmospheric response to an increasing rate of production of fossil-fuel carbon dioxide. The increasing acidity of ocean water decreases its capacity to absorb additional carbon dioxide from the growing atmospheric

concentration. The numerical evaluation of $P_m(\Sigma C)$ has been adapted from the method of Bacastow and Keeling (1973) and is described in Appendix 3.

The coefficient of P_m in Eqs. (2.2.2-1) and (2.2.2-2) is derived as follows: At time $t = 0$, the system is assumed to be in equilibrium, so that the transfers between the atmosphere and mixed layer cancel each other, i. e.,

$$k_{12}Y_1 - \kappa_{21}P_m = 0,$$

when Y_1 and P_m have their initial values. Therefore the transfer coefficient κ_{21} (assumed constant with units appropriate to P_m) has the value

$$\kappa_{21} = \frac{k_{12}Y_1^{(0)}}{P_{m0}} = \frac{k_{12}}{R_{12}} \frac{Y_2^{(0)}}{P_{m0}}.$$

The corresponding flow of ^{14}C from the mixed layer to the atmosphere [Eqs. (2.2.2-9) and (2.2.2-10)] is scaled from its counterpart by the fractional content of ^{14}C in the inorganic carbon of the mixed layer, and it is corrected by the isotopic fractionation factor, α_{m1} , to account for the differential evasion characteristics of the heavier atom. The reverse flow of ^{14}C is corrected for isotopic fractionation by the factor α_{1m} . Analogous corrections have been made in the flows of ^{14}C between the atmosphere and compartments of the biosphere [Eqs. (2.2.2-13) through (2.2.2-16)].

The equations for the flows of nonradioactive carbon between the atmosphere and terrestrial biosphere [Eqs. (2.2.2-5) through (2.2.2-8)] are patterned after the formulation of Bacastow and Keeling (1973). The flows between the atmosphere and the long-lived biota are proportional to Y_B , reflecting the assumption that long-lived plants photosynthesize and respire in proportion to their mass. Bacastow and Keeling also made the flow from the atmosphere to the short-lived biosphere proportional to Y_B because of the association of a large fraction of short-lived plant growth with long-lived plants. The return flow, however, is proportional to Y_S .

Bacastow and Keeling included logarithmic terms of the form $B \ln(Y_1/Y_1^{(0)})$ to account for growth due to fertilization by the incremental CO_2 introduced into the atmosphere since the assumed preindustrial stasis. We have inserted the factors $(\eta_B - Y_B)/(\eta_B - Y_B^{(0)})$ and $(\eta_S - Y_S)/(\eta_S - Y_S^{(0)})$ to impose an ultimate limitation on such growth; otherwise, future equilibrium of the system would be impossible unless the atmospheric level, Y_1 , were to return to its preindustrial value, $Y_1^{(0)}$. The limiting levels, η_B and η_S (gC), are hypothesized by consideration of the availability of nutrients and land area and those activities of man that reduce biomass. A range of values will be explored in Sec. 5.3. It is convenient to prescribe the maximum values, η_B and η_S , in terms of the fractional increase of the biosphere over its preindustrial carbon mass. We define the parameter θ as this fractional increase, so that

$$\eta_b = (1 + \theta)Y_b^{(0)},$$

where the subscript $b = B$ or S . For example, $\theta = 2.0$ would give a three-fold increase over the preindustrial biosphere.

2.2.3 Initial Conditions

We have assumed a state of equilibrium for the carbon cycle at a preindustrial point in time, which we have taken as the year 1860. This assumption imposes a number of relationships among the initial compartmental levels and the transfer of coefficients. It has been our preference to specify initially (1) the total preindustrial nonradioactive mobile carbon in the system, $Y^{(0)}$, and (2) the ratios, R_{ij} , of initial compartment levels with corresponding transfer coefficients, k_{ij} . Specific equations for nonradioactive carbon are the following:

$$Y_1^{(0)} = Y^{(0)} \left[1 + R_{1B} + R_{1S} + R_{12} \left(1 + \frac{1}{\Delta z_2} \sum_{i=3}^M \Delta z_i \right) \right]^{-1}, \quad (2.2.3-1a)$$

$$Y_2^{(0)} = R_{12} Y_1^{(0)}, \quad (2.2.3-2)$$

$$Y_i^{(0)} = (\Delta z_i / \Delta z_2) Y_2^{(0)}, \quad (i = 3, \dots, M), \quad (2.2.3-3a)$$

$$Y_B^{(0)} = R_{1B} Y_1^{(0)}, \quad (2.2.3-4)$$

$$Y_S^{(0)} = R_{1S} Y_1^{(0)}, \quad (2.2.3-5)$$

as in Secs. 2.2.1 and 2.2.2, $R_{ij} = Y_j^{(0)} / Y_i^{(0)}$, and Δz_i is the thickness of the i -th layer of the ocean, where $i = 2$

designates the mixed layer. It may be verified directly that these expressions sum to $Y^{(0)}$. Equation (2.2.3-3) is based on the condition expressed by Eq. (2.2.1-15). Only the ratios R_{12} , R_{1B} , and R_{1S} are specified, together with k_{12} , k_{1B} , and k_{1S} . These quantities and the initial conditions (2.2.3-1) through (2.2.3-6) allocate the $Y^{(0)}$ grams of carbon among the system's reservoirs such that the initial state is one of equilibrium.

The corresponding assignments for radiocarbon are zero, unless naturally produced ^{14}C is to be included in the input function $I^X(t)$ [Eq. (2.2.2-9)]; a detailed treatment of such cases is beyond the scope of this report. For cases to be considered here,

$$x_B^{(0)} = x_S^{(0)} = x_i^{(0)} = 0 \quad (i = 1, \dots, M). \quad (2.2.3-6)$$

Equations (2.2.3-1a) through (2.2.3-5) assign the same initial concentrations of carbon to the mixed layer, thermocline, and deep ocean. Introducing the parameter r of Eq. (2.2.1-16) into two of these equations permits the latter two compartments to have a higher initial concentration than the mixed layer without sacrificing the initial equilibrium state. The revised equations are

$$y_1^{(0)} = y^{(0)} \left[1 + R_{1B} + R_{1S} + R_{12} \left(1 + \frac{1}{r \cdot \Delta z_2} \sum_{i=3}^M \Delta z_i \right) \right]^{-1}, \quad (2.2.3-1b)$$

$$y_i^{(0)} = \left(\frac{\Delta z_i}{r \cdot \Delta z_2} \right) y_2^{(0)}, \quad (i = 3, \dots, M). \quad (2.2.3-3b)$$

2.2.4 Parameterization of the Model

In this section, we indicate to the reader the sources of many of the parameters in our model and the methods used to calculate others.

We have followed Oeschger et al. (1975) in assigning a depth of 75 m to the mixed layer of the ocean and 3725 m as an average depth of the thermocline and deep ocean for a total average depth of 3800 m.

Keeling (1973a) calculated the mass of carbon in the preindustrial atmosphere as follows:

$$y_1^{(0)} = 2.90 \times 10^{-4} \times \frac{12.011}{28.964} \times 5.119 \times 10^{21} \text{ g} = 6.156 \times 10^{17} \text{ g},$$

where

$$\begin{aligned} 2.90 \times 10^{-4} &= \text{concentration (290 ppm by volume),} \\ 12.011, 28.964 &= \text{molecular weights for carbon and dry air,} \\ &\quad \text{respectively, and} \\ 5.119 \times 10^{21} &= \text{mass (g) of the atmosphere (dry air).} \end{aligned}$$

The assumed concentration of 290 ppm by volume in the preindustrial atmosphere would be in equilibrium with a corresponding partial pressure, P_{m0} , in the mixed layer:

$$P_{m0} = 2.90 \times 10^{-4} \text{ atmospheres}$$

which corresponds to an inorganic carbon concentration

$$IC_0 = 2.057 \times 10^{-3} \text{ mol/l}$$

in the mixed layer; this correspondence is derived from Eqs. (A3-1) and (A3-2) of Appendix 3. We can now calculate the initial mass of inorganic carbon in the mixed layer as

$$\begin{aligned} Y_2^{(0)} &= 2.057 \times 10^{-3} \frac{\text{mol}}{\text{T}} \times 12.01 \frac{\text{gC}}{\text{mol}} \times 3.53 \times 10^{14} \text{ m}^2 \times 75 \text{ m} \\ &\times 10^3 \frac{1}{\text{m}^3} = 6.540 \times 10^{17} \text{ g}, \end{aligned}$$

where

$$3.53 \times 10^{14} = \text{area of the ocean (Keeling 1973a).}$$

Baes et al. (1976) present the following levels and flows for the long- and short-lived biota for the year 1974:

	1974 level (gC)	1974 flow (gC/yr) *
long	1.60×10^{18}	3.9×10^{16}
short	1.62×10^{17}	7.4×10^{16}

There are used as target values to be aimed at by suitable adjustment of the uncertain preindustrial values. For $\beta_B = \beta_S = 0.1$, we might estimate, tentatively, an increase of $\sim 1\%$ in the biosphere between 1860 and 1974. Then

$$Y_B^{(0)} = 1.58 \times 10^{18} \quad \text{and} \quad Y_S^{(0)} = 1.60 \times 10^{17} \text{ g.}$$

*These flows are representative of gross primary production (GPP).

Also

$$k_{1B}/R_{1B} = 3.9 \times 10^{16} / 1.60 \times 10^{18} = 2.437 \times 10^{-2} \text{ yr}^{-1}, \text{ and}$$

$$k_{1S}/R_{1S} = 7.4 \times 10^{16} / 1.62 \times 10^{17} = 0.46 \text{ yr}^{-1},$$

so that

$$R_{1B} = 1.58 \times 10^{18} / 6.156 \times 10^{17} = 2.57,$$

$$k_{1B} = 2.57 \times 2.437 \times 10^{-2} = 6.26 \times 10^{-2} \text{ yr}^{-1},$$

$$R_{1S} = 1.60 \times 10^{17} / 6.156 \times 10^{17} = 0.260, \text{ and}$$

$$k_{1S} = 0.260 \times 0.46 = 0.120 \text{ yr}^{-1}.$$

These calculations provide a starting estimate for a parameterization and initial values for the biosphere that are consistent with the 1974 levels and flows estimated by Baes et al. They are refined by comparing the results of subsequent runs with the 1974 values, and the parameters shown in the listing of the CSMP III coding (Appendix 1) reflect such refinement.

We have generally assumed a growth ceiling on the biosphere of 50 percent in excess of its preindustrial carbon mass. If one considers only the limiting factors of available nutrients and land area, an increase in the range of two- to four-fold is conceivable with unrestricted CO₂. But the prospect of continued conversion of land to agricultural production to feed the growing human

population, the clearing of land for habitation and industrial use, and the destruction of biomass by fires render the 50 percent value more plausible than larger hypothetical increases that essentially leave man's activities out of account.

As we have noted previously (Secs. 2.2.1 and 2.2.3), the basic model assigns carbon to the mixed layer and deep ocean in uniform concentration, but a parameter, r , may be set to permit an initial differential [Eq. (2.2.1-16)]. The volume of water below the mixed layer is

$$3.53 \times 10^{14} \text{ m}^2 \times 3725 \text{ m} = 1.315 \times 10^{18} \text{ m}^3 = 1.315 \times 10^{21} \text{ l},$$

and an initial concentration of $2.057 \times 10^{-3} \text{ mol/l}$ in the mixed layer corresponds to

$$Y_D^{(0)} = 2.057 \times 10^{-3} \frac{\text{mol}}{\text{l}} \times 1.315 \times 10^{21} \text{ l} \times 12.01 \frac{\text{g}}{\text{mol}} = 3.249 \times 10^{19} \text{ gC}$$

initially in the thermocline and deep ocean if a uniform concentration is assumed. Keeling (1973a) cites an estimate of $2.33 \times 10^{-3} \text{ mol/l}$ for ocean water below a depth of 1000 m. If this concentration is extrapolated to all ocean water below the mixed layer, we have

$$Y_D^{(0)} = 2.33 \times 10^{-3} \frac{\text{mol}}{\text{l}} \times 1.315 \times 10^{21} \text{ l} \times 12.01 \frac{\text{g}}{\text{mol}} = 3.680 \times 10^{19} \text{ gC}$$

for a nonuniform distribution. For this case, the parameter r is given by

$$r = 2.057 \times 10^{-3} / 2.33 \times 10^{-3} = 0.883.$$

Table 2.2.4-1 summarizes the initial levels of carbon in the reservoirs of the model for the assumptions of uniform and nonuniform distribution between the mixed layer and deep ocean.

The parameters that are explicitly specified in the model are the total, $Y^{(0)}$; the ratios R_{12} , R_{1B} , and R_{1S} ; and their corresponding transfer coefficients k_{12} , k_{1B} , and k_{1S} . We have indicated an estimate for each of the latter except k_{12} , the relative rate from the atmosphere to the mixed layer. Oeschger et al. (1975) coupled their estimation of k_{12} with that of the diffusion coefficient, K (m^2/yr). They suggested

$$k_{12} = 1/7.3 \text{ yr}^{-1},$$

$$K = 3980 \text{ m}^2/\text{yr}$$

as midpoints of ranges representing most probable values derived from their analysis. We return to these ranges in Sec. 5.3 when we discuss sensitivity tests.

The particular scheme for partitioning the ocean into discrete layers is that of Oeschger et al. (1975), with

$$\Delta z_i = \begin{cases} 25 \text{ m} & \text{for } i = 3, \dots, 39, \\ 560 \text{ m} & \text{for } i = 40, \dots, M = 44. \end{cases} \quad (2.2.4-1)$$

Table 2.2.4-1 Assumed initial distributions of carbon among world reservoirs for uniform and nonuniform concentrations in the ocean

Reservoir	Initial level (gC)	
	Uniform ($r = 1.0$)	Nonuniform ($r = 0.883$)
Atmosphere [$Y_1^{(0)}$]	6.156×10^{17}	6.156×10^{17}
Slow biosphere [$Y_B^{(0)}$]	1.58×10^{18}	1.58×10^{18}
Rapid biosphere [$Y_S^{(0)}$]	1.60×10^{17}	1.60×10^{17}
Mixed layer [$Y_2^{(0)}$]	6.540×10^{17}	6.540×10^{17}
Thermocline and deep ocean [$Y_3^{(0)} \rightarrow Y_M^{(0)}$]	3.249×10^{19}	3.680×10^{19}
Total [$Y^{(0)}$]	3.550×10^{19}	3.981×10^{19}

Thus for the depth of the thermocline (925 m) the evolving concentration profile is represented in considerably greater detail than in the deep ocean.

The isotopic fractionation factors, α_{ij} , for the transfer of ^{14}C from reservoir i to reservoir j , were used by Bacastow and Keeling (1973). They are

α_{1B}	(atmosphere to biosphere)	=	0.964
α_{B1}	(biosphere to atmosphere)	=	1.000
α_{1m}	(atmosphere to mixed layer)	=	0.972
α_{m1}	(mixed layer to atmosphere)	=	0.955

Other parameters in the model are associated with the specification of future scenarios of fossil-fuel production of carbon dioxide. These will be defined in Sec. 3.2 and varied for sensitivity tests in Sec. 5.3.

2.3 Computer Implementation of the Model

The nonlinear system of differential equations which define our model of the global carbon cycle has been implemented in the IBM Continuous System Modeling program III (CSMP III) simulation language (IBM 1975). This language permits the specification of the dynamic equations by FORTRAN-like statements and the use of FORTRAN subprograms for auxiliary calculations, such as the evaluation of input functions. The relative ease in making structural changes in the model and the output tabulation and graphical capabilities of CSMP III are significant advantages of the use of the simulation language in preference to direct FORTRAN programming interfaced with a software package for numerical integration of the differential equations. These equations become stiff as the integration proceeds, and special methods are needed to deal with this property of the system; CSMP III offers an integration method that is suitable for stiff systems.

Computer runs have utilized the IBM/360 Models 75 and 91 at Oak Ridge National Laboratory. A typical execution time for translation, compilation, and integration over a 1940-year period is 3.46 CPU-minutes on the Model 91. All runs have executed in less than 180K bytes of core storage.

The source listing of the model's implementation in Appendix 1 consists of the CSMP III code and several FORTRAN IV subprograms. The latter are summarized as follows:

SUBROUTINE FERT - Computes the relative growth rates of the biospheric compartments due to CO_2 fertilization, while taking into account the assumed limiting levels of the compartments.

These are terms of the general form

$\beta \cdot \ln(Y_1/Y_1^{(0)}) \cdot (\eta - Y_b)/(\eta - Y_b^{(0)})$, where the subscript b refers to either compartment of the biosphere. See Eqs. (2.2.2-5) and (2.2.2-7).

C12PRO - Computes the rate (gC/yr) at which fossil-fuel carbon is being injected into the atmosphere at any given date T. See Secs. 3, 3.1, and 3.2.

C14PRO - Computes the rate at which ^{14}C is being released (g/yr) at any given date T. This subprogram would be rewritten for each specific application. The version shown injects 1 Ci (0.224 g) of ^{14}C into the atmosphere during the year 1985.

YLAG - Implements Lagrangian interpolation for other subprograms. Adapted from a routine given by Westley and Watts (1970).

POPULA - Returns the estimated world population at any date T (1925 or later). Based on linear interpolation in Table 4-1.

SUBROUTINE PARPRE - Computes the CO_2 partial pressure, P_{m} , in the mixed layer of the ocean as a function of the concentration of total inorganic carbon, ΣC . Also returns the pH of the water. The units of ΣC are mol/l, and P_{m} is in atmospheres. The calculation is described in Appendix 3.

The subprograms in the above list are of FUNCTION form, except where SUBROUTINE is explicitly noted.

3 PRODUCTION OF NONRADIOACTIVE CARBON FROM THE BURNING OF FOSSIL FUELS

The release of significant quantities of nonradioactive carbon into the global cycle has resulted from the combustion of fossil fuels. During the past century, until 1970, the rate of production of carbon as CO_2 from this source has tended to grow exponentially at an average annual rate just under three percent, and recent years have seen a higher rate of growth.

Detailed historical production rates have been tabulated by Keeling (1973b) and Rotty (1975), and this information has been used to construct a driving function for our model. These historical data are displayed in Sec. 3.1.

The estimation of dose to future populations from releases of ^{14}C now and in the future requires future scenarios for the production of fossil-fuel carbon. A functional formulation is presented in Sec. 3.2.

The historical data and future scenarios are combined in a FORTRAN subprogram, C12PRO, which returns to the calling program the annual rate of injection of fossil-fuel carbon (g/yr) at a specified date. This routine defines the input function, $I^Y(t)$ [Eq. (2.2.2-1)], for nonradioactive carbon.

3.1 Historical Data

From the papers of Keeling (1973b) and Rotty (1975) we have assembled estimates of the world's production of carbon dioxide due to the combustion of fossil fuels and the manufacture of cement during each year from 1860 to 1974. Table 3.1-1 displays this information, together with cumulative production. The exponential trend is evident in the graph shown in Fig. 3.1-1. The increasing inventory of nonradioactive carbon in man's exposure environment decreases the specific activity and hence the dose rate that would otherwise occur from a given release of ^{14}C .

The data in Table 3.1-1 are from an input file which is read and stored in an array by subprogram C12PRO on its first call of the simulation run. For dates between 1860 to 1974, the subprogram computes the corresponding production rates by linear interpolation in the stored data.

Table 3.1-1. Annual production rate and cumulative production of fossil-fuel carbon: 1960-1959 (Reeling, 1973b); 1960-1976 (Notcy, 1975)

Year	Annual rate (MMT/yr) ^a	Cumulative (MMT) ^a	Year	Annual rate (MMT/yr) ^a	Cumulative (MMT) ^a
1960	9.339E+01	5.330E+01	1917	9.453E+02	2.327E+04
1961	9.870E+01	1.920E+02	1918	9.320E+02	2.615E+04
1962	9.040E+01	2.904E+02	1919	8.209E+02	2.650E+04
1963	1.060E+02	3.964E+02	1920	9.589E+02	2.594E+04
1964	1.151E+02	5.115E+02	1921	8.280E+02	2.677E+04
1965	1.219E+02	6.334E+02	1922	8.504E+02	2.766E+04
1966	1.207E+02	7.621E+02	1923	1.003E+03	2.664E+04
1967	1.379E+02	9.000E+02	1924	9.985E+02	2.966E+04
1968	1.367E+02	1.037E+03	1925	1.004E+03	3.067E+04
1969	1.419E+02	1.179E+03	1926	1.004E+03	3.167E+04
1970	1.450E+02	1.324E+03	1927	1.098E+03	3.277E+04
1971	1.619E+02	1.485E+03	1928	1.091E+03	3.386E+04
1972	1.759E+02	1.661E+03	1929	1.177E+03	3.505E+04
1973	1.804E+02	1.850E+03	1930	1.078E+03	3.611E+04
1974	1.838E+02	2.034E+03	1931	9.682E+02	3.704E+04
1975	1.897E+02	2.223E+03	1932	8.738E+02	3.795E+04
1976	1.916E+02	2.414E+03	1933	9.188E+02	3.887E+04
1977	1.968E+02	2.610E+03	1934	9.965E+02	3.987E+04
1978	1.965E+02	2.807E+03	1935	1.032E+03	4.090E+04
1979	2.076E+02	3.014E+03	1936	1.144E+03	4.205E+04
1980	2.271E+02	3.262E+03	1937	1.224E+03	4.327E+04
1981	2.444E+02	3.464E+03	1938	1.161E+03	4.443E+04
1982	2.625E+02	3.726E+03	1939	1.233E+03	4.567E+04
1983	2.800E+02	4.028E+03	1940	1.300E+03	4.697E+04
1984	2.821E+02	4.311E+03	1941	1.337E+03	4.830E+04
1985	2.784E+02	4.686E+03	1942	1.334E+03	4.964E+04
1986	2.787E+02	5.163E+03	1943	1.364E+03	5.100E+04
1987	2.977E+02	5.163E+03	1944	1.352E+03	5.235E+04
1988	3.219E+02	5.484E+03	1945	1.204E+03	5.356E+04
1989	3.205E+02	5.814E+03	1946	1.271E+03	5.483E+04
1990	3.457E+02	6.163E+03	1947	1.422E+03	5.625E+04
1991	3.654E+02	6.529E+03	1948	1.518E+03	5.777E+04
1992	3.687E+02	6.898E+03	1949	1.470E+03	5.924E+04
1993	3.616E+02	7.259E+03	1950	1.613E+03	6.085E+04
1994	3.771E+02	7.636E+03	1951	1.744E+03	6.259E+04
1995	3.586E+02	8.039E+03	1952	1.767E+03	6.436E+04
1996	4.114E+02	8.466E+03	1953	1.808E+03	6.617E+04
1997	4.315E+02	8.879E+03	1954	1.862E+03	6.803E+04
1998	4.546E+02	9.333E+03	1955	2.027E+03	7.006E+04
1999	4.973E+02	9.830E+03	1956	2.151E+03	7.221E+04
1900	5.249E+02	1.035E+04	1957	2.248E+03	7.441E+04
1901	5.403E+02	1.090E+04	1958	2.322E+03	7.679E+04
1902	5.529E+02	1.145E+04	1959	2.466E+03	7.925E+04
1903	6.064E+02	1.205E+04	1960	2.670E+03	8.192E+04
1904	6.134E+02	1.267E+04	1961	2.630E+03	8.455E+04
1905	6.466E+02	1.331E+04	1962	2.768E+03	8.732E+04
1906	6.961E+02	1.401E+04	1963	2.930E+03	9.025E+04
1907	7.712E+02	1.478E+04	1964	3.102E+03	9.335E+04
1908	7.366E+02	1.552E+04	1965	3.229E+03	9.658E+04
1909	7.690E+02	1.625E+04	1966	3.399E+03	9.990E+04
1910	8.040E+02	1.705E+04	1967	3.459E+03	1.034E+05
1911	8.210E+02	1.791E+04	1968	3.683E+03	1.071E+05
1912	8.662E+02	1.878E+04	1969	3.900E+03	1.110E+05
1913	9.290E+02	1.971E+04	1970	4.188E+03	1.152E+05
1914	8.384E+02	2.055E+04	1971	4.303E+03	1.194E+05
1915	8.308E+02	2.138E+04	1972	4.503E+03	1.240E+05
1916	8.940E+02	2.227E+04	1973	4.756E+03	1.288E+05
			1974	4.894E+03	1.337E+05

^aMMT = million metric tons elemental carbon.

Figure 3.1-1. Annual release of carbon to the atmosphere from fossil fuel combustion and cement production. Estimates for 1860 - 1959 are from Keeling (1973b); those for 1960 - 1974 were compiled by Rotty (1975).

3.2 Future Scenarios

Future production of carbon dioxide from the combustion of fossil fuels is uncertain. Information about potential supplies of fossil fuels is incomplete, and future demand will depend upon the supply, on technological development, and on numerous interdependent social, economic, and political patterns that have yet to emerge. In the face of such uncertainty, it is natural to hypothesize a logistic function, such as that defined by the differential equation

$$\dot{P} = RP \left[1 - \frac{P}{P_{\infty}} \right], \quad P(t_0) = P_0,$$

where

- $\dot{P}(t)$ = fossil-fuel carbon production rate (g/yr) at time t ,
- P_{∞} = total fossil-fuel carbon ultimately produced (g),
- P_0 = cumulative production (g) prior to a specified time t_0 , and
- R = an adjustable parameter.

Keeling (1976) has suggested the introduction of a second fitting parameter, n , so that the differential equation becomes

$$\dot{P} = RP \left[1 - \left(\frac{P}{P_{\infty}} \right)^n \right], \quad P(t_0) = P_0. \quad (3.2-1)$$

Increasing the value of n hastens the approach of $P(t)$ to P_{∞} ; the effect is to increase the spiked appearance of a plot of $\dot{P}$ vs t (see Fig. 3.2-2).

The differential equation (3.2-1) can be solved to give

$$P(t) = \frac{P_{\infty}}{\left\{ 1 + \left[\left(\frac{P_{\infty}}{P_0} \right)^n - 1 \right] \cdot \exp[-nR(t - t_0)] \right\}^{1/n}}. \quad (3.2-2)$$

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Figure 3.2-2. Future scenarios of the production rate of atmospheric carbon from the combustion of fossil fuels. The curves shown correspond to a total release of $P_{\infty} = 3.08 \times 10^{18}$ gC.

For the application under discussion, we have taken t_0 to correspond to the year 1974. The value $P_0 = 1.36 \times 10^{17}$ g is the sum of the cumulative production from 1860 through 1974 (Table 3.1-1) and a backward exponential extrapolation. The value of R has been calculated from the differential equation (3.2-1) as

$$R = \frac{\dot{P}(1974)}{P_0 \left[1 - \left(\frac{P_0}{P_\infty} \right)^n \right]} \quad (3.2-3)$$

in order to provide a reasonably smooth fit to the historical data.

The FORTRAN subprogram C12PRO has been programmed to use Eq. (3.2-2) to calculate the rate of input of nonradioactive carbon into the global cycle for times subsequent to 1974. The parameters n and P_∞ (FORTRAN REAL variables N and $PINF$, respectively) may be varied to correspond to different assumptions about the remaining supply of fossil fuels and the rate at which they might be consumed. Keeling (1976) cites estimates of fossil fuel reserves by M. K. Hubbert (1969) which lead us to assign the range $3.08 \times 10^{18} \rightarrow 6.16 \times 10^{18}$ g to the parameter P_∞ for our dose estimates. The sensitivity of the dose estimates to variation of P_∞ within this range will be discussed in Sec. 5.3.

4 POPULATION GROWTH SCENARIO

Estimation of radiation dose to future populations of the world involves projected scenarios of growth which clearly involve considerable uncertainty. Estimates of the dose commitment following a release of ^{14}C require integration to infinity, which in practical terms requires the assumption of a population scenario spanning many thousands of years.

Our approach has been to construct a reference scenario consisting of the United Nations Population Bureau's "medium variant" projection (United Nations 1974), which forecasts a rise to 12.2 billion by the year 2075 with an apparently nonincreasing growth rate after 2000 (see Table 4-1 and Fig. 4-1). For the future beyond 2075, we have assumed a population which remains constant at the 12.2 billion level. We prefer not to speculate on whether such a scenario might prove conservative with respect to estimates of absolute numbers of radiation-induced health effects calculated from projected man-rem doses. It will be treated as a reference scenario, and population doses derived from it may reasonably be scaled upward or downward to accommodate alternative assumptions. Figure 4-1 shows the United Nations' "high" and "low" variants for the year 2075. Scenarios of the general shape of the one described here have been employed by other investigators of the hazard posed by the release of ^{14}C , and thus a degree of comparability of dose estimates might be expected. Some comparisons will be given in Sec. 5.2.

The population scenario is implemented in the model by means of the FORTRAN subprogram POPULA, which stores the information of Table 4-1 in two arrays and uses linear interpolation for nontabular points.

Table 4-1. World population scenario based on the United Nations Population Bureau's
"medium variant" projection^a

Date	1925	1950	1975	2000	2025	2050	2075
Population (billions)	1.960	2.505	3.988	6.406	9.605	11.163	12.210

^aUnited Nations 1974

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Figure 4-1. Reference scenario for world population growth (United Nations 1974). The population is assumed to be stationary at 12.21 billion after 2075.

5 ESTIMATES OF ^{14}C DOSE TO THE WORLD POPULATION

The basis for our estimates of radiation dose to the world population due to released ^{14}C is the model previously described, with computer listing in Appendix 1, together with the historical data and future scenarios presented in Secs. 3.1, 3.2, and 4. It is useful to think in terms of a reference case as a standard for comparisons and sensitivity tests. We suggest a reference case in Sec. 5.1.

Other investigators of ^{14}C hazard have published estimates of population doses. Some of these estimates have been based on models of the global carbon cycle for which published documentation is not available. In Sec. 5.2, we adjust a sampling of these estimates to our reference case, insofar as the published information permits, and compare the results with our own estimates.

Finally, in Sec. 5.3, we present the results of a number of sensitivity tests on our model.

5.1 Reference Case for ^{14}C Population Dose Estimates

The reference case that will be used for sensitivity tests and comparisons of our dose estimates with those offered by other investigators is defined by the computer code listing of Appendix 1. The principal parameter values are summarized in Table 5.1-1. The ^{14}C release is taken as one curie, injected into the atmosphere at a uniform rate during the year 1985. The computer integration is terminated at the year 3800, and the dose to infinity is computed from Eq. (2.1-4).

The joint choice of values for k_{12} , the transfer coefficient from the atmosphere to the ocean, and K , the diffusion coefficient for the ocean below the mixed layer, was suggested by the experience of Oeschger et al. (1975). From steady-state calculations with their model and empirical data on the $^{14}\text{C}/\text{C}$ distribution among the deep ocean, mixed layer, and atmosphere, they obtained the estimates from which the values in Table 5.1-1 were fine tuned. We quote some qualifying remarks from their paper: "K is an apparent global eddy diffusion coefficient which helps to reproduce an average transport phenomenon consisting of a series of overlapping mechanisms. To keep the model as simple as possible and the number of parameters small we assumed a constant preindustrial oceanic CO_2 concentration in contradiction to actually measured profiles. Similarly the ^{14}C minimum in the Pacific Ocean at a depth of approximately 2000 m has to be ignored.... The model is essentially intended to describe atmospheric responses to injections of CO_2 and ^{14}C into the atmosphere on a global scale. Severe approximation of the oceanographic data.... is therefore justified as long as the response of the model to atmospheric perturbations is satisfactory." Since the dose response is determined by the levels of ^{14}C and total carbon in the atmosphere, our objectives conform to those stated by Oeschger et al. (1975). Sensitivity of the dose response to the parameters k_{12} and K will be examined in Sec. 5.3.

Table 5.1-1. Parameters and scenarios for reference estimates of radiation dose to the world population from the release of 1 Ci ^{14}C to the atmosphere in 1985.

Environmental parameters

k_{12}	0.136 yr^{-1}	} k_{ij} = transfer coefficient from compartment i to compartment j. ^a
k_{1B}	0.0627 yr^{-1}	
k_{1S}	0.119 yr^{-1}	
R_{12}	1.063	} R_{ij} = ratio of preindustrial carbon in compartment j to that in compartment i. ^a
R_{1B}	2.574	
R_{1S}	0.258	
K	$3980 \text{ m}^2/\text{yr}$	Diffusion coefficient for ocean below mixed layer.
β_B	0.1	Growth factors for terrestrial biota.
β_S	0.1	
θ	0.5	Fractional growth limit for terrestrial biota.
r	1.0	Ratio of preindustrial carbon concentration in deep ocean to that in mixed layer.
$Y^{(0)}$	$3.55 \times 10^{19} \text{ g}$	Total preindustrial carbon available to model.
h_m	75 m	Depth of mixed layer.
h_d	3725 m	Average depth of thermocline plus deep ocean.
A	$3.53 \times 10^4 \text{ m}^2$	Area of ocean surface.
α_{1m}	0.972	} α_{ij} = isotopic fractionation factor for transfer of ^{14}C from compartment i to compartment j
α_{m1}	0.955	
α_{1B}	0.964	
α_{B1}	1.00	

^aSubscripts have the following meanings: 1 = atmosphere, 2 = m = mixed layer, B = long-lived biota, and S = short-lived biota.

Table 5.1-1. (Continued)

Fossil-fuel scenario

Historical: Table 3.1-1

Future^b:

n	1.0	Shape parameter for release scenario.
P_{∞}	3.08×10^{18} g	Ultimate cumulative release of fossil-fuel carbon.
P_0	1.34×10^{17} g	Cumulative release of fossil-fuel carbon through 1974.

World population scenario

Table 4.1

^bSection 3.2. Value of the fitting parameter, R, calculated by the computer program from Eq. (3.2-3) is

$$R = \frac{4.89 \times 10^{15}}{1.34 \times 10^{17} (1 - 0.134/3.08)} = 0.0382$$

for the reference case.

The parameter P_{∞} , representing the ultimate cumulative carbon release from fossil-fuel combustion, is chosen at the low end of a range suggested by Hubbert (1969) and equals five times the carbon content of the pre-industrial atmosphere. Keeling (1976) notes that higher published estimates have included consideration of shale oil and tar sands, but we retain Hubbert's lower estimate as a conservative one for dose estimation. The value of the shape parameter, n , is of less concern for dose estimates, as the sensitivity tests show, and we use $n = 1$ as a reference value, corresponding to a peak production of just under 3×10^{16} gC/yr late in the 21-st Century. The cumulative production through 1974, $P_0 = 1.34 \times 10^{17}$ g, is from Table 3.1-1.

It is difficult to argue for precise choices of parameter values that are associated with the terrestrial biosphere since many combinations of values give model predictions that fit observations within acknowledged uncertainties. We therefore have adopted the values of β_B , β_S , R_{1B} , R_{1S} , k_{1B} , k_{1S} , and θ shown in Table 5.1-1 as a working hypothesis. The method of arriving at R_{1B} , R_{1S} , k_{1B} , and k_{1S} was indicated in Sec. 2.2.4; at that point we also gave reasons for our choice of θ . The common value of β_B and β_S is the lower end of a range suggested by J. S. Olson (Olson et al. 1977) from his current work. We have taken the lower value simply because our model accurately predicts the 1974 atmospheric concentration of carbon when $\beta_B = \beta_S = 0.0$ and tends toward underprediction as the growth factors are increased. It is of course possible to readjust other parameters (k_{12} and K) to balance the biospheric uptake and maintain the accurate atmospheric response, but our sensitivity tests (Sec. 5.3) show such adjustment to be otiose in the context of the present application.

5.2 Comparisons of Dose Estimates

A number of world population dose estimates from ^{14}C have appeared in the literature. Table 5.2-1 summarizes these estimates, which we have adjusted (sometimes crudely) to be comparable with our reference case (Sec. 5.1), insofar as the published information has permitted. Some of the original estimates were also based on the assumption that 18 percent of the total body is carbon; we have adjusted the percentage upward to 22.8 to agree with data from Reference Man (ICRP 1975). In the following paragraphs, we give some additional information about the sources of these estimates.

The United Nations Scientific Committee on the Effects of Atomic Radiation (UNSCEAR) has used a simple steady-state model for obtaining an estimate of the dose commitment (United Nations 1972), namely

$$D = (\text{DRF})(\text{SpA}) \cdot N \cdot Q/B \quad \text{man-rem} \quad (5.2-1)$$

where

D = population dose commitment to infinite time (man-rem),

(DRF) = dose-rate factor = 2.08×10^8 rem/yr per Ci/gC for total body (sec. 2.1),

(SpA) = that component of the specific activity of carbon in man's exposure medium which is attributed to naturally produced ^{14}C (UNSCEAR has used 6.1×10^{-12} Ci/gC as being representative of the biosphere prior to the testing of nuclear weapons),

B = natural production rate ($\sim 3 \times 10^4$ Ci/yr is the value suggested by UNSCEAR; estimates by Light et al. (1973) give production rates closer to 4×10^4 Ci/yr, but we have used the former value to compute the entry in Table 5.2-1),

Table 5.2-1. World population dose estimates (man-rem) from the release of 1 Ci ^{14}C to the atmosphere^a

Source of estimate	30 yr	100 yr	Infinite time
United Nations Scientific Committee on the Effects of Atomic Radiation (UNSCEAR), (United Nations 1972)	—	—	5.2×10^2
United States Energy Research and Development Administration (ERDA 1975)	—	40	5.3×10^2
Magno et al. (1974), Fowler et al. (1976)	—	52	—
Martin and ApSimon (1974)	22	—	4.8×10^2
Kelly et al. (1975)	13	—	4×10^2
Brooks et al. (1974)	—	—	5.9×10^2
Present report (reference case, Sec. 5.1)	13	28	6.2×10^2

^aAll estimates have been adjusted to the population scenario of Table 4-1.

Q = one year's release (Ci) of ^{14}C from the nuclear fuel cycle, and

N = constant number of individuals in the world population (1.221×10^{10} is the stationary value from Table 4-1).

Equation (5.2-1) is based on the observation that the dose rate at steady state due to a constant production rate is equal to the infinite-time dose commitment due to one year's production at that rate, provided the differential equations which govern the dynamics are linear with constant coefficients. Therefore $\dot{D}$ may be replaced by $N \cdot \bar{D}$, where $\bar{D}$ is the average individual dose rate, and Q may be regarded as a constant production rate; with these interpretations the formula (5.2-1) becomes an obvious proportionality. This formula gives only an infinite-time dose estimate and is not applicable to shorter-term integrations. Moreover, the assumption of a stationary population level is necessary.

In the final Environmental Statement for the Liquid Metal Fast Breeder Reactor, the U. S. Energy Research and Developmental Administration (ERDA) utilized both the UNSCEAR model of Eq. (5.2-1) and an adaptation of a two-box dynamic model of the carbon cycle to estimate the dose to the world population 100 years after a release and for infinite time (ERDA 1975). The ERDA staff acknowledged that its analysis assumed a constant population and that it did not take into account the growing inventory of fossil-fuel carbon and other perturbations of the carbon cycle.

Magno et al. (1974) used a computer implementation of the seven-reservoir model developed by Machta (1971, 1973) to estimate the world population dose delivered in the first 100 years following a release in the year 1985. The production of fossil-fuel carbon was allowed to increase at an annual rate of 3.5 percent through the year 2080. The work of Fowler et al. (1976) is based on the same implementation of Machta's model.

Martin and ApSimon (1974) employed a global carbon transport model with four geographic regions by latitude and five environmental

reservoirs (stratosphere, troposphere, terrestrial biosphere, mixed ocean, and deep ocean) in each region. They reported estimates for 30 years and infinite time. Their paper does not indicate the nature of their assumptions about the production of carbon by the burning of fossil fuels.

The estimates in Table 5.2-1 derived from the information of Kelly et al. (1975) are based on their Tables 7 and 9 and our admittedly rough integration of graphical data from their Figure 25 with the population projection of Sec. 4 of this report. They employed a nine-reservoir dynamic model and neglected fossil-fuel carbon.

The UNSCEAR model described at the beginning of this section was the basis of an estimate by Brooks et al. (1974). Slightly different data on the production of natural ^{14}C account for the difference between their estimate and those of UNSCEAR and ERDA. Brooks et al. also used an unusually large dose-rate factor in their original calculation, but the entry in Table 5.2-1 has been corrected for this.

It should be emphasized that most of the papers cited above present very little detail concerning their models of the carbon cycle and associated scenarios of population growth and fossil-fuel carbon release. Therefore, it is possible that Table 5.2-1 might suggest more of a basis for comparison than the incomplete information permits. Even so, this tabulation indicates a fair insensitivity of the long-term dose estimates to the variation in model structure represented by this sampling, and this conclusion is consistent with our experience in evaluating models of simpler structure than the one finally assembled for our reference case. This remark should not be interpreted to mean that other variables are similarly insensitive, however, for such an inference would be seriously in error. The deeper questions of the climatologist, the oceanographer, or the terrestrial ecologist, for example, are not likely to be answered until additional refinements based on better data are incorporated into models of the global carbon cycle.

5.3 Sensitivity Tests

It is helpful to examine the effect of varying parameter values in the model over ranges suggested by the uncertainty with which they are believed to be known, and tabulating the corresponding variations in the response (i. e., the population dose). The result is an indication of the model's response sensitivity to the parameters in question. Table 5.3-1 shows the tabulation of 30-, 50-, and 100-year, and infinite-time population doses as predicted for our model corresponding to several deviations from the reference case defined in Sec. 5.1. One sees immediately that these dose estimates are fairly insensitive to the variations studied. We have rounded all numbers to two significant figures.

The first group of variations is based on the analysis of Oeschger et al. (1975) in their determination of "most probable" values of the parameters k_{12} and K . Their best joint estimate was based on natural radiocarbon specific activities in the mixed layer and deep ocean as percentages of that in the atmosphere. Uncertainties in these specific activities generated a rectangular region in the (k_{12}, K) plane, of which the pairs of values in Table 5.3-1 are the corners. The maximum deviation from the reference case is 13 percent.

In view of the uncertainty associated with the biosphere's long-term interaction with the atmospheric CO_2 levels, we have postulated two cases that are thought to be near extremes. In the first, the biota are assumed not to grow in response to increasing CO_2 concentrations but to maintain their preindustrial carbon level. The second case assigns a relatively large growth-rate factor ($\beta_B = \beta_S = 0.4$) and permits an ultimate three-fold increase ($\theta = 2.0$) in carbon mass, believed to be high if the activities of man are considered (Sec. 2.2.4). In both cases, the changes in the population doses are inconsequential.

Table 5.3-1. Sensitivity tests of reference case^a against several alternatives

	Population dose (man-rem)			
	30-yr	50-yr	100-yr	Infinite time
Reference case	13	18	28	6.2×10^2
Atmosphere-to-ocean transport: limits of "most probable" region ^b				
k_{12} (yr ⁻¹) K (m ² /yr)				
0.110 3560	14	19	30	6.4×10^2
0.115 5580	14	18	27	6.0×10^2
0.167 3060	13	18	28	6.5×10^2
Variation of biota growth-rate and growth-limitation factors ^c				
$\frac{R}{B} = \frac{\beta}{S}$ e				
0.0 —	13	18	28	6.2×10^2
0.4 2.0	13	18	28	6.3×10^2
Fossil-fuel scenarios				
n P_{∞} (gC)				
0.5 3.08×10^{18}	13	18	29	6.2×10^2
1.0 6.16×10^{18}	13	18	27	5.7×10^2
2.0 3.08×10^{18}	13	18	27	6.2×10^2
Ratio of initial carbon concentrations in surface waters and deep ocean				
$r = 2.057/2.33$	14	19	29	6.2×10^2

Table 5.3-1 (continued)

	Population dose (man-rem)			Infinite time
	30-yr	50-yr	100-yr	
Variation of year of release				
$t_0 = 2000$	16	21	31	6.3×10^2

^aThe reference case consists of the release of one Ci ^{14}C during the year 1985, subject to conditions and parameter values given in Section 5.1.

^bThe "most probable" region was indicated by the analysis of Oeschger et al. (1975).

^cThe parameter θ represents the fractional increase of the biosphere over its preindustrial mass: $n_b = (1 + \theta)Y_b^{(0)}$, where n_b = maximum size of biosphere (gC) and $Y_b^{(0)}$ = preindustrial size (gC).

The effect of doubling the cumulative production of fossil-fuel CO_2 is to decrease the infinite-time dose commitment, as one suspects. We alluded earlier to M. K. Hubbert's (1969) suggestion of a range between five and nine times the mass of carbon in the preindustrial atmosphere. We have substituted a range of five to ten times and used the lower value for our reference case as a conservative measure. Doubling the total fossil-fuel carbon, P_∞ , decreases the dose commitment eight percent but does not alter the short-term dose estimates appreciably. It is helpful to reexamine Fig. 3.2-1, which shows plots of the scenarios for three values of the shape parameter, n . The indicated values perturb the 100 year dose estimate less than four percent and leave the others unaffected.

The increased initial carbon in the thermocline and deep ocean and the attendant retardation of the upward flow to maintain the preindustrial equilibrium has the effect of increasing the 30-year population dose estimate by approximately eight percent and the other short-term doses by lesser fractions. The increase is due to the slower rate of return to the atmosphere of older carbon of lower specific activity. The infinite-time dose commitment is not significantly affected.

Since our reference case is based on the release of one curie in the year 1985, it is appropriate to inquire how the time of release affects the dose responses, and for a comparison we chose the year 2000, with the results shown in the table (maximum increase of 23 percent). This increase reflects the higher population level at the time of release but also attenuation due to the greater concentration of nonradioactive CO_2 in the atmosphere at that time. This is not an extreme comparison, however: the year 2075 would correspond to an even greater population at the time of release but probably is beyond the reasonable reach of ^{14}C release scenarios that might be formulated today.

In view of these results, it seems proper to conclude that uncertainties in environmental parameters in the model do little to disturb the dose estimates, which are also relatively insensitive to plausible variations in the fossil-fuel scenario. It is conceivable, of course, that structural variations and refinements in future models could have effects out of the range of those noted here, but our experience in examining models of simpler structure, as well as the comparisons discussed in Sec. 5.2, makes this contingency seem unlikely. More important by far is the population scenario which multiplies the dose rate.

6 CONCLUSIONS

We have defined a model of the global carbon cycle, described a computer implementation of the model, and discussed sensitivity tests conducted on the ^{14}C population doses estimated by the model. We have also compared the dose estimates computed with our model with those of other investigators of the ^{14}C hazard. The descriptions and computer code listing in this report should make the model available for use and further modification by others with access to IBM/360 computing hardware and the appropriate software (the CSMP III simulation language).

Comparison of our dose estimates with those in the literature, where the latter have been adjusted to a comparable basis to the extent permitted by the available information, has revealed no remarkable disparities. In fact, the degree of agreement among the various estimates, some of which were based on multiple-compartment dynamic models of the global carbon cycle, suggests that the time-integrated specific activity of the airborne carbon, which is proportional to the dose, is relatively insensitive to details of structure of the supporting model.

Sensitivity of the model to variation of its environmental parameters and scenarios has been studied with respect to its population dose estimates. The latter qualification is important, because as we have noted elsewhere, certain response variables may exhibit different characteristics in this regard, and in general, the appropriateness of a model for a given predictive application is dependent on the particular combination of response variables to be evaluated. Table 5.3-1 indicates trivial departures from the reference case except in conjunction with variation of atmospheric and oceanic parameters k_{12} and K ; when these quantities were set to their extreme values in a "most probable" region determined by the analysis of Oeschger et al. (1975), no dose estimate deviated from its reference counterpart by more than 13%. We accept these

results as indications that the environmental uncertainty of the model is sufficiently contained to provide adequate estimates of population dose.

We are aware of objections to the practice of integrating the population dose rate to infinity because of the uncertainties associated with projecting the population size into the remote future. When such estimates of dose commitment are multiplied by risk factors, such as those compiled by the Committee on the Biological Effects of Ionizing Radiation (BEIR 1972), it is argued that the resulting numbers of predicted health effects are subject to misinterpretation. We stipulate the difficulty and the possibility of distorted interpretations, but we cannot recommend omitting the integration to infinity on this excuse; if not to infinity, then where does one terminate?^{*} Periods of 30 and 100 years have been used, and one can make a case for examining these numbers along with the infinite-time commitment, for they stand as measures of impact of the release on those generations that will experience the highest individual dose rate. But to omit the infinite-time dose commitment from consideration is to lose sight of the persistence of the elevated ^{14}C specific activity in man's exposure environment and potential continuing exposure of many future generations. Some measure, even an imperfect one, of the radiological significance of this long-term environmental persistence is needed.

Estimates of radiation dose to the world population from a specified release of ^{14}C can be compared with the dose delivered by naturally produced radiocarbon. The appropriate comparison is with the dose from the natural ^{14}C produced within the same time interval as that spanned by the release. For example, for a release

^{*}We are not speaking here of the practical numerical problem of determining a point in time beyond which integration contributes negligibly to the total.

of 7.5×10^3 Ci [the annual source term for HTGR fuel reprocessing hypothesized by Snider and Kaye (1976)] during the year 2000, our model (last entry in Table 5.3-1) gives the population dose commitment

$$7.5 \times 10^3 \times 630 = 4.7 \times 10^6 \text{ man-rem.}$$

The natural radiocarbon produced during the year 2000 will amount to about 3.8×10^4 Ci (Light et al. 1973), and therefore the comparable natural background dose commitment is

$$3.8 \times 10^4 \times 630 = 2.4 \times 10^7 \text{ man-rem.}$$

To simulate the natural ^{14}C dose commitment for comparison with longer production periods, the computer code would need to be executed with the ^{14}C subroutine (C14PRO) written to inject at the desired constant rate ($3.8 \times 10^4 / 4.46 = 8.52 \times 10^3 \text{ g }^{14}\text{C/yr}$) during that period and zero otherwise.

The foregoing example is provided for purposes of illustration rather than to endorse piecemeal assessments of ^{14}C releases from individual facilities in the absence of a more comprehensive approach. We recommend rather, that scenarios of total ^{14}C production and release from the worldwide nuclear power industry be continuously revised and converted to estimates of dose commitment to the world population (scenarios for which should also be kept current). Only in such a global and long-term context, it appears to us, can an appropriate perception of the potential ^{14}C impact begin to emerge.

An added degree of refinement of the global carbon model should be investigated. Most of the ^{14}C released from the nuclear power industry in the next 50 years is likely to enter the atmosphere north of latitude 30°N . The increased ability of the colder

northern oceans to dissolve CO_2 , the greater industrialization of northern countries, and the different population levels and projected rates of growth north and south of this latitude should be examined in an elaborated global model. In particular, it seems appropriate to inquire whether a disproportionate share of the impact of the collective release of ^{14}C might be borne by populations that do not benefit (at least directly) from the power generated.

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APPENDIX 1
CSMP III SOURCE CODE LISTING
AND
FORTRAN IV SUBPROGRAMS

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-
- * A DIFFUSION-TYPE MODEL OF THE WORLD CARBON CYCLE FOR USE
 - * IN ESTIMATING RADIATION DOSE TO THE WORLD POPULATION FROM
 - * RELEASES OF C-14 TO THE ATMOSPHERE.

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* GENERAL NOTATION..

-
- * VARIABLE NAMES BEGINNING WITH Y REPRESENT GRAMS OF
 - * NONRADIOACTIVE CARBON. THOSE BEGINNING WITH X STAND
 - * FOR GRAMS OF C-14. THE LETTER L MEANS THE LONG-LIVED
 - * BIOSPHERE AND S THE SHORT-LIVED. FOR EXAMPLE, XL IS THE GRAMS
 - * OF C-14 IN THE LONG-LIVED BIOTA.

- * INDICES.. 1=ATMOSPHERE, 2=MIXED LAYER OF THE OCEAN, AND
- * 3 THROUGH 44=LAYERS OF THE SUBSURFACE OCEAN. SO Y(2)=
- * GRAMS OF NONRADIOACTIVE CARBON IN THE SURFACE WATERS, ETC.

- * THE CHARACTER ZERO IN A VARIABLE NAME INDICATES AN INITIAL
- * VALUE. FOR EXAMPLE, Y0(1)=GRAMS OF CARBON IN THE ATMOSPHERE
- * AT THE START OF THE RUN.

- * ALL RUNS ARE STARTED AT A PREINDUSTRIAL POINT IN TIME (WE
 - * GENERALLY HAVE USED 1860), WHEN THE SYSTEM IS ASSUMED TO BE
 - * IN EQUILIBRIUM. THIS INITIAL DATE IS SET BY THE TIME=
 - * PARAMETER IN THE TIMER STATEMENT.
-

INITIAL

FIXED I,M,N1
STORAGE U(44),D(44),DELZ(44),CONCY(44),CONCX(44),AD(44)

- * YTOTAL=TOTAL NONRADIOACTIVE CARBON (G) INITIALLY AVAILABLE TO
- * THE MODEL IN ALL COMPARTMENTS.

PARAMETER YTOTAL=3.54996E15

- * K12=TRANSFER-RATE PARAMETER (1/YR) FROM ATMOSPHERE TO MIXED
- * LAYER.

- * R12=RATIO OF INITIAL MASS OF CARBON IN MIXED LAYER TO THAT
- * IN ATMOSPHERE.

PARAMETER K12=C.136, R12=1.063

- * K18=TRANSFER RATE PARAMETER (1/YR) FROM ATMOSPHERE TO LONG-
- * LIVED BIOSPHERE.

- * R18=RATIO OF INITIAL MASS OF CARBON IN LONG-LIVED BIOSPHERE
- * TO THAT IN ATMOSPHERE.

PARAMETER K18=6.274E-2, R18=2.574

```

*
* KIS=TRANSFER-RATE PARAMETER (1/YR) FROM ATMOSPHERE TO SHORT-
*   LIVED BIOSPHERE.
* RIS=RATIO OF INITIAL MASS OF CARBON IN SHORT-LIVED BIOSPHERE
*   TO THAT IN ATMOSPHERE.
*
*   PARAMETER KIS=0.119, RIS=0.258
*
* HP=DEPTH (M) OF MIXED LAYER.
* HD=AVERAGE DEPTH (M) OF SUBSURFACE OCEAN.
* AREA=AREA (M**2) OF OCEAN SURFACE.
*
*   PARAMETER HM=75., HD=3725., AREA=3.53E14
*
* BETA=CO2-FERTILIZATION GROWTH FACTOR FOR TERRESTRIAL BIOTA.
*
*   PARAMETER BETA=0.1
*
* THETA IS THE LIMITING FRACTIONAL INCREASE OF THE TERRESTRIAL BIOSPHERE
* OVER ITS PREINDUSTRIAL SIZE.
*
*   PARAMETER THETA=0.5
*
* AL1B, ALB1, AL1M, AND ALM1 ARE ISOTOPIC FRACTIONATION FACTORS
* FOR C-14 RELATIVE TO INACTIVE CARBON FOR TRANSFERS BETWEEN
* ATMOSPHERE AND BIOSPHERE AND BETWEEN ATMOSPHERE AND MIXED LAYER.
*
*   PARAMETER AL1B=0.964, ALB1=1.000, AL1M=0.972, ALM1=0.955
*
* M-2 IS THE NUMBER OF LAYERS USED TO REPRESENT THE SUBSURFACE
* OCEAN.
*
*   PARAMETER M=44
NCSORT
*
* PARAMETER SMALLR PERMITS A DIFFERENTIAL INITIAL CONCENTRATION
* BETWEEN MIXED LAYER AND DEEP OCEAN. THE VALUE SMALLR=1.0 GIVES
* UNIFORM CONCENTRATION BETWEEN MIXED LAYER AND DEEPER OCEAN.
* IN GENERAL, SMALLR=CM/CD, WHERE CD=CONCENTRATION IN DEEP OCEAN
* AND THERMOCLINE AND CM=CONCENTRATION IN MIXED LAYER.
*
*   SMALLR=1.0
*
* BETAB AND BETAS ARE CO2 GROWTH FACTORS FOR THE LONG- AND SHORT-
* LIVED BIOTA, RESPECTIVELY. WE HAVE GENERALLY SET BOTH EQUAL
* TO BETA, HOWEVER.
*
*   BETAB=BETA
*   BETAS=BETA
*
* THE FOLLOWING STATEMENTS SET THE DEPTHS DELZ(I), I=2,M, OF THE
* OCEAN LAYERS (THE DELZ(I) ARE IN METERS). FROM THESE, COEFFICIENTS
* C(I) AND U(I) OF THE VERTICAL TRANSPORT EQUATIONS ARE COMPUTED.
*
*   DELZ(2)=HP
*   DO 10 I=3,39
* 10 DELZ(I)=25.0

```



```

DC 20 I=40,M
20 DELZ(1)=560.0
DC 30 I=3,M
  D(1-1)=2./(DELZ(1-1)+(DELZ(1-1)+DELZ(1)))
30 U(1)=2./(DELZ(1)+(DELZ(1-1)+DELZ(1)))
  U(3)=SMALLR*U(1)
  D(M)=0.0

```

- * THE FOLLOWING STATEMENTS COMPUTE INITIAL VALUES OF THE Y COMPARTMENTS
- * SO THAT THE INITIAL STATE IS ONE OF EQUILIBRIUM. INITIAL RADIOCARBON
- * LEVELS ARE SET TO ZERO (IF NATURAL C-14 IS INCLUDED IN THE SOURCE
- * TERM, THESE STATEMENTS MUST BE REWRITTEN).

```

SUMD=0.0
DC 40 I=3,39
40 SUMTH=SUMTH+DELZ(1)
DC 45 I=40,M
45 SUMC=SUMD+DELZ(1)
  SUM=SUMTH+SUMC
  YO(1)=YTOTAL/(1.+R12*(1.+SUM/(SMALLR*DELZ(2)))+R1R+R1S)
  YG(2)=R12*YO(1)
  XC(1)=0.0
  XO(2)=0.0
  YEO=R1R*YC(1)
  YSO=R1S*YO(1)
  XEO=0.0
  XSO=0.0
DC 50 I=3,M
  YO(1)=YO(2)+DELZ(1)/(SMALLR*DELZ(2))
50 XO(1)=0.0

```

- * ETAB AND ETAS ARE THE LIMITING LEVELS OF CARBON IN THE
- * LONG- AND SHORT-LIVED BIOSPHERES, RESPECTIVELY.

```

ETAB=(1.0+THETA)*YEO
ETAS=(1.0+THETA)*YSO

```

- * CCNV CONVERTS GRAMS OF CARBON IN THE MIXED LAYER TO MOLES/LITER.

```

CONV=1.0/(12.01*HM*AREA*1000.)
SIGCO=YO(2)*CCNV

```

- * SUBROUTINE PARPRE COMPUTES INITIAL CO2 PARTIAL PRESSURE PMO
- * (ATMOSPHERES) AND PH CORRESPONDING TO INORGANIC
- * CARBON CONCENTRATION SIGCO IN THE MIXED LAYER.

```

PMO,PH=PARPRE(SIGCO)

```

- * SPAWO=INITIAL CUMULATIVE SPECIFIC ACTIVITY OF ATMOSPHERIC CARBON.

```

SPAWO=0.0

```

SORT

- * KCAP=DIFFUSION COEFFICIENT (M**2/YR) FOR VERTICAL TRANSPORT OF
- * CARBON IN THE OCEAN.
- * LM=RADIOACTIVE DECAY COEFFICIENT (1/YR) FOR C-14.

PARAMETER KCAP=3980.
PARAMETER LM=1.21E-4

* FOSSTI-FUEL SCENARIC PARAMETERS

PARAMETER N=1.0, PINF=3.08E10, PO=1.34E17

DYNAMIC

- * THE PROCEDURE YDOT,XDOT=DIFF(Y,X) COMPUTES THE DERIVATIVES OF
- * THE ARRAYS Y AND X, WHICH REPRESENT NONRADIOACTIVE CARBON AND
- * C-14, RESPECTIVELY, IN THE ATMOSPHERE (Y(1),X(1)), MIXED LAYER
- * (Y(2),X(2)), AND SUBSURFACE OCEAN (Y(3) TO Y(M), X(3) TO X(M)).

PROCEDURE YDOT,XDOT=DIFF(Y,X)

YDOT(1)=Y(1)-K12*Y(1)+(K12/R12)*(Y(2)/PMO)*PM ...

+FYB1 + FYS1 - FY10 - FY15

YDOT(2)=K12*Y(1)-(K12/R12)*(Y(2)/PMO)*PM ...

+KCAP*(U(3)*Y(3)-D(2)*Y(2))

M1=M-1

DO 60 I=3,M1

60 YDOT(I)=KCAP*(D(I-1)*Y(I-1)+U(I+1)*Y(I+1) ...
-(D(I)+U(I))*Y(I))

YDOT(M)=KCAP*(C(M-1)*Y(M-1)-U(M)*Y(M))

XDOT(1)=X(1)-(ALIM*K12+LM)*X(1) ...

+ALM1*(X12/R12)*(Y(2)/PMO)*PM*(X(2)/(X(2)+Y(2))) ...

+FXB1 + FXS1 - FX10 - FX15

XDOT(2)=ALIM*K12*X(1)-ALM1*(K12/R12)*(Y(2)/PMO)*PM ...

*(X(2)/(X(2)+Y(2))) ...

-LM*X(2)+KCAP*(U(3)*X(3)-D(2)*X(2))

DO 70 I=3,M1

70 XDOT(I)=KCAP*(D(I-1)*X(I-1)+U(I+1)*X(I+1) ...
-(D(I)+U(I))*X(I)-LM*X(I))

XDOT(M)=KCAP*(C(M-1)*X(M-1)-U(M)*X(M))-LM*X(M)

ENDPROCEDURE

Y=INTGRL(YO,YECT,44)

X=INTGRL(XO,XECT,44)

- * SPA=CARBON SPECIFIC ACTIVITY (CURIES PER GRAM OF CARBON) IN
- * THE ATMOSPHERE.
- * SPAWGL=TIME INTEGRAL OF SPA.

SPA=4.46*X(1)/(X(1)+Y(1))

SPAWGL=INTGRL(SPAWO,SPA)

- * INVOKE SUBROUTINE FERT TO COMPUTE RELATIVE GROWTH RATES,
- * FRATEB AND FRATES, OF LONG- AND SHORT-LIVED BIOTA DUE TO
- * CO2 FERTILIZATION.

FRATEB,FRATES=FERT(ETAB,ETAS,YO,YBO,YS,YSO,BETAB,BETAS, ...
Y(1),YO(1))

```

*
* FY1B AND FYB1 ARE TRANSFER RATES (G/YR) FROM ATMOSPHERE TO
* LONG-LIVED BIOTA AND THE REVERSE, RESPECTIVELY, OF
* NONRADIOACTIVE CARBON.
*
  FY1B=K1B/R1B*(1.0+FPATEB)*YB
  FYB1=K1B/R1B*YB
*
* DIFFERENTIAL EQUATION AND INTEGRAL FOR INACTIVE CARBON IN
* THE LONG-LIVED BIOTA..
*
  YBDOT=FY1B-FYB1
  YB=INTGRL(YB0,YBDOT)
*
* FY1S AND FYS1 ARE TRANSFER RATES (G/YR) FROM ATMOSPHERE TO
* SHORT-LIVED BIOTA AND THE REVERSE, RESPECTIVELY, OF
* NONRADIOACTIVE CARBON.
*
  FY1S=K1S/P1S*(1.0+FRATES1)*YE*(YS0/Y90)
  FYS1=K1S/R1S*YS
*
* DIFFERENTIAL EQUATION AND INTEGRAL FOR INACTIVE CARBON IN THE
* SHORT-LIVED BIOTA.
*
  YSDOT=FY1S-FYS1
  YS=INTGRL(YS0,YSDOT)
*
* THE FOLLOWING EIGHT STATEMENTS ARE FOR C-14 IN THE
* BIOSPHERE AND ARE ANALOGOUS TO THE EIGHT PRECEDING
* STATEMENTS.
*
  FX1B=AL1B*FY1B*X(1)/(X(1)+Y(1))
  FXB1=ALB1*FYB1*XB/(XB+YB)
  XBDOT=FX1B-FXB1-LM*XB
  XB=INTGRL(XB0,XBDOT)
  FX1S=AL1B*FY1S*X(1)/(X(1)+Y(1))
  FXS1=ALB1*FYS1*XS/(XS+YS)
  XSDOT=FX1S-FXS1-LM*XS
  XS=INTGRL(XS0,XSDOT)
*
* PCP=WORLD POPULATION AT CURRENT TIME, EVALUATED BY SUBPROGRAM
* PCPULA.
*
  PCP=PCPULA(TIME)
*
* CRATE=DOSE RATE (REM/YR) TO AN INDIVIDUAL IN THE POPULATION.
*
  CRATE=2.08E8*SPA
*
* PCRATE=DOSE RATE (MAN-REM/YR) TO THE WORLD POPULATION.
*
  PCRATE=CRATE*PCP
*
* DCSE AND PDCE ARE THE DOSE TO AN INDIVIDUAL (REM) AND THE
* POPULATION (MAN-REM), RESPECTIVELY.
*
  DCSE=INTGRL(0.0,CRATE)
  PDCE=INTGRL(0.0,PCRATE)

```

```
*
* IXI=RELEASE RATE (G/YR) OF C-14 INTO THE ATMOSPHERE.
*
*      IXI=C14PRC(TIME)
*
*
* IYI=RATE CF RELEASE (G/YR) OF NONRADIOACTIVE CARBON FROM FOSSIL-
* FUEL COMBUSTION INTO THE ATMOSPHERE.
*
*      IYI=C12PRC(TIME)
NCSORT
*
* COMPUTE CONC. SIGC OF INORGANIC CARBON IN MIXED LAYER,
* AND FROM THIS, COMPUTE CO2 PARTIAL PPFESSURE AND PH.
*
*      SIGC=Y(2)*CONV
*      PM,PH=PARPRE(SIGC)
*
* THE FOLLOWING STATEMENTS CALCULATE GRAMS OF CARBON IN THE THERMO-
* CLINE (YTH,XTH) AND DEEP OCEAN (YDEEP,XDEEP), RESPECTIVELY,
* THE CONCENTRATION PROFILES (GRAMS/LITER) OF NONRADIOACTIVE
* CARBON (CONCY(3) TO CONCY(M)) AND C-14 (CONCX(3) TO CONCX(M)),
* AND THE SPECIFIC ACTIVITY PROFILE (AD(3) TO AD(M)) IN
* CURIES PER GRAM.
*
*      YTH=0.0
*      XTH=0.0
*      YDEEP=0.0
*      XDEEP=0.0
DC 80 I=3,39
    CONCY(I)=Y(I)/(DELZ(I)*AREA*1000.)
    CONCX(I)=X(I)/(DELZ(I)*AREA*1000.)
    AC(I)=4.46*X(I)/(X(I)+Y(I))
    YTH=YTH+Y(I)
80   XTH=XTH+X(I)
DC 85 I=40,M
    CONCY(I)=Y(I)/(DELZ(I)*AREA*1000.)
    CONCX(I)=X(I)/(DELZ(I)*AREA*1000.)
    AD(I)=4.46*X(I)/(X(I)+Y(I))
    YDEEP=YDEEP+Y(I)
85   XDEEP=XDEEP+X(I)
SCRT
-----
TERMINAL
-----
LABEL BCX-DIFFUSION ISOTOPIC GLOBAL CARBON CYCLING MODEL..
OUTPUT  Y(1),YB,YS,X(1),XB,XS,DCEP,F0 SE
OUTPUT  IYI
OUTPUT  Y(1),Y(2),YTH,YDEEP,YB,YS,SPA
OUTPUT  X(1),X(2),XTH,XDEEP,XB,XS
OUTPUT  Y(2),SIGC,PM,PH
OUTPUT  CONCY(3-39)
PAGE    CONTOR
OUTPUT  AD(3-39)
PAGE    CONTOR
METHOD  STIFF
TIMER   DELMIN=1.E-12, TIME=1860., FINTIM=2000., OUTOFL=5.
END
STCP
```

```

      SUBROUTINE FERT(ETAB,ETAS,YR,YBO,YS,YSO,
     1  BETAB,BETAS,YI,YIO,FRATER,FRATES)
C
C COMPUTES RELATIVE GROWTH RATES, FRATER AND FRATES, OF
C SLOW- AND FAST-TURNOVER BIOSPHERES, RESPECTIVELY. THE
C GROWTH IS DUE TO CO2 FERTILIZATION AND IS ASSUMED LIMITED
C BY MAXIMUM LEVELS ETAB AND ETAS (GRAMS OF CARBON).
      A = ALOG(YI/YIO)
      IF(ETAB .GT. YB .AND. ETAB .GT. YBO) GO TO 10
      FRATER=0.0
      GO TO 20
10  FRATER=BETAB*A*(ETAB-YB)/(ETAB-YBO)
20  IF(ETAS .GT. YS .AND. ETAS .GT. YSO) GO TO 30
      FRATES=0.0
      GO TO 40
30  FRATES=BETAS*A*(ETAS-YS)/(ETAS-YSO)
40  RETURN
      END

      FUNCTION C12PRC(T)
COMMON
C
C C12PRC RETURNS TO THE CALLING PROGRAM THE RATE (GRAMS PER YEAR)
C AT WHICH CARBON FROM THE COMBUSTION OF FOSSIL FUELS IS ENTERING
C THE ATMOSPHERE AS CO2. THE PARAMETER T IS THE DATE (YEAR A.D.).
C THE CSMP COMMON AREAS NEED TO BE VISIBLE IN ORDER FOR THE
C SUBPROGRAM TO HAVE ACCESS TO THE VARIABLES N, PO, AND PINF.
      REAL N,FOSDAT(115),T(115)
      LOGICAL FIRST
      DATA FIRST/.TRUE./
C
C ON FIRST CALL, READ HISTORICAL PRODUCTION RATES FOR FOSSIL-FUEL
C CARBON INTO THE ARRAY FOSDAT. GENERATE THE CORRESPONDING DATES
C AND STORE THEM IN THE ARRAY T1 (T1(1)=1860., ETC.). THE FOSDAT
C ENTRIES ARE IN UNITS OF MILLION METRIC TONS ELEMENTAL CARBON
C PER YEAR. ALSO USE DIFFERENTIAL EQUATION, EVALUATED AT THE YEAR
C 1974, TO CALCULATE THE RATE PARAMETER TO BE USED IN SUBSEQUENT
C CALCULATIONS WITH LOGISTIC PROJECTIONS.
      IF(.NOT. FIRST) GO TO 5
      FIRST=.FALSE.
      READ(12,100) (FOSDAT(I),I=1,115)
100  FORMAT(5E10.0)
      DO 2 I=1,115
2    T1(I)=1860.+FLOAT(I-1)
      RATE=1.E12*FOSCAT(115)/(PO*(1.-(PO/PINF)**N))
C
C CHOOSE APPROPRIATE METHOD OF CALCULATION ACCORDING TO VALUE OF T.
5  IF(T.LT.1860.) GO TO 10
   IF(T.LE.1974.) GO TO 20
   GO TO 30
C
C BACKWARD EXPONENTIAL EXTRAPOLATION FOR T.LT.1860.
10  C12PRC=9.2E13*EXP(0.0435*(T-1860.))
      GO TO 40

```

```

C
C INTERPOLATION IN TABLE OF HISTORICAL DATA FOR T BETWEEN 1960
C AND 1974, INCLUSIVE.
  20 C12PRJ=YLAG(T,T1,F0SDAT,0,2,115,1EX)*1.E12
  GC TO 40
C
C EVALUATION OF LOGISTIC PROJECTION FOR FUTURE RELEASE RATE
C (T.GT.1974).
  30 P=PINF/(1.+(PINF/PC)**N-1.)*EXP(-RATE*N*(T-1974.))**((1./N)
  C12PRC=RATE*P*(1.-(P/PINF)**N)
  40 RETURN
  END

```

FUNCTION C14PRJ(T)

```

C
C COMPUTES RATE (GRAMS PER YEAR) AT WHICH C-14 IS BEING INJECTED
C INTO THE ATMOSPHERE. THIS SAMPLE SUBPROGRAM PRODUCES
C ONE CURIE DURING THE YEAR 1985. THIS SUBPROGRAM WILL BE
C REWRITTEN FOR EACH SPECIFIC APPLICATION.
  C14PRJ=0.0
  IF(T.GE.1985. .AND. T.LE.1986.) C14PRJ=0.224
  RETURN
  END

```

FUNCTION YLAG(XI,X,Y,IND1,N1,IMAX,1EX)

```

C
C ADAPTATION OF DOUBLE PRECISION VERSION BY
C A.A. BROOKS AND E.C. LONG
C COMPUTING TECHNOLOGY CENTER
C UNION CARBIDE CORPORATION, NUCLEAR DIVISION
C OAK RIDGE, TENN.
C SEE WESTLEY AND WATTS (1970).
C
C LAGRANGIAN INTERPOLATION..
C XI IS INTERPOLATED ENTRY INTO X-ARRAY.
C N1 IS THE NUMBER OF TABULAR POINTS FOR INTERPOLATION.
C Y IS THE ARRAY FROM WHICH YLAG IS OBTAINED BY INTERPOLATION.
C IND1 IS THE MINIMUM I FOR WHICH X(I) .GT. XI..
C IF IND1=0, X-ARRAY WILL BE SEARCHED.
C IMAX IS MAXIMUM INDEX OF X- AND Y-ARRAYS.
C SUBPROGRAM RETURNS 1EX=-1 OR +1 ACCORDING AS EXTRAPOLATION
C BELOW OR ABOVE TABLE OCCURRED, =0 IF INTERPOLATION
C TOOK PLACE.
  DIMENSION X(1),Y(1)
  REAL P,0,5,XC,XI,X,Y,YLAG
  IND=IND1
  N=N1
  1EX=0
  IF(N.LE.IMAX) GO TO 10
  N=IMAX
  1EX=N
10  IF(IND.GT.0) GO TO 40
  DO 20 J=1,IMAX
    IF(XI-X(J)) 30,130,20
20  CONTINUE

```

```

      IEX=1
      GO TO 70
30    IND=J
40    IF(IND.GT.1) GO TO 50
      IEX=-1
50    INL=IND-(N+1)/2
      IF(INL.GT.0) GO TO 60
      INL=1
60    INU=INL+N-1
      IF(INU.LE.IMAX) GO TO 80
70    INL=IMAX-N+1
      INU=IMAX
80    S=0.
      P=1.
      DO 110 J=INL,INU
        P=P*(XI-X(J))
      D=1.
      DO 100 I=INL,INU
        IF(I.NE.J) GO TO 90
        XD=XI
      GO TO 100
        XD=X(J)
90    C=D*(XC-X(I))
100   S=S+Y(J)/D
110   YLAG=S*P
120   RETURN
130   YLAG=Y(J)
      GO TO 120
      END

```

FUNCTION POPULA(DATE)

```

C
C COMPUTES WORLD POPULATION FOR DATE (YEAR A.D. AFTER 1925)
C BY LINEAR INTERPOLATION IN TABULAR DATA. POPULATION IS
C IN BILLIONS. AFTER 2075, THE VALUE 12.210 IS RETURNED.
      DIMENSION PEOPLE(7),T1(7)
      DATA PEOPLE/1.960,2.505,3.928,6.406,9.605,11.163,12.210/
      DATA T1/1925.,1950.,1975.,2000.,2025.,2050.,2075./
      POPULA=0.0
      IF(DATE.LT.T1(1)) GO TO 30
      IF(DATE.LE.T1(7)) GO TO 20
      POPULA=PEOPLE(7)*1.0E9
      GO TO 30
20    POPULA=YLAG(DATE,T1,PEOPLE,C,2,7,IEX)*1.0E9
30    RETURN
      END

```

SUBROUTINE PARPRF(SIGC,PMH,PHH)

```

C
C INPUT..
C   SIGC = INORGANIC CARBON CONCENTRATION (MOLES/LITER) IN
C   SURFACE OCEAN WATER.
C

```

```

C CUTPUTS..
C   PPM = CARBON DIOXIDE PARTIAL PRESSURE (ATMOSPHERES) IN
C   SURFACE OCEAN WATER.
C   PHM = PH OF SURFACE OCEAN WATER.
C   LOGICAL FIRST
C   REAL KO,K1,K2,KB,KH,KW,K12
C   DATA FIRST/.TRUE./

C
C THE CONSTANTS KO, K1, K2, KB, KH, A, AND SIGA (=SIGMA-B)
C ARE DERIVED FROM BACASTON AND KEELING (1973).  IORDER IS
C THE NUMBER OF TABULAR POINTS FOR INTERPOLATION.
C   DATA KO/0.03347/,
C   K1/9.747E-7/,
C   K2/8.501E-10/,
C   KB/1.881E-9/,
C   KW/6.463E-15/
C   DATA A/0.002435/
C   DATA SIGA/4.05E-4/
C   DATA IORDER /4/
C   DIMENSION PHA(101), SIGCA(101),PMA(101)

C
C HPLUS(PH) IS THE HYDROGEN-ION CONCENTRATION (MOLES/LITER)
C FOR THE GIVEN PH VALUE.
C   HPLUS(PH)=EXP(-2.302585*PH)

C
C SIG(H) IS THE INORGANIC CARBON CONCENTRATION (MOLES/LITER)
C CORRESPONDING TO HYDROGEN-ION CONCENTRATION H.
C   SIG(H)=(K12+H*(K1+H))*(H+A-KH/H-KB*SIGB/(H+KB))/(K1*(H+2.0*K2))

C
C PM(H) IS THE CARBON DIOXIDE PARTIAL PRESSURE IN THE MIXED LAYER
C CORRESPONDING TO HYDROGEN-ION CONCENTRATION H.
C   PM(H)=H**2*SIG(H)/(KO*(K12+H*(K1+H)))
C   IF(.NOT. FIRST) GO TO 50

C
C ON FIRST CALL, RESET VARIABLE FIRST TO .FALSE., INITIALIZE,
C AND STORE FUNCTION TABLE IN THE ARRAYS PHA AND SIGCA.
C   FIRST=.FALSE.
C   K12=K1*K2
C   DPH=0.1
C   PH=0.9
C   DO 10 I=1,101
C     PH=PH+DPH
C     PHA(I)=PH
C     H=HPLUS(PH)
C     SIGCA(I)=-ALOG10(SIG(H))
C     PMA(I)=-ALOG10(PM(H))
C   10 CONTINUE
C   50 CONTINUE

C
C FOR GIVEN VALUE OF SIGC, USE CUBIC INTERPOLATION TO FIND
C PH AND PARTIAL PRESSURE.
C   PH=YLAG(-ALOG10(SIGC),SIGCA,PHA,0,IORDER,101,1EX)
C   PMM=YLAG(-ALOG10(SIGC),SIGCA,PMA,0,IORDER,101,1FX)
C   PMM=HPLUS(PMM)
C   PHM=PH
C   RETURN
C   END

```


Data cards read on unit 12 by subprogram C12PRD:

93.3	56.7	98.4	106.0	115.1
121.9	128.7	137.9	136.7	141.8
145.0	161.9	175.9	182.4	183.8
189.2	191.6	196.0	196.5	207.6
227.1	244.4	262.5	280.0	282.1
276.4	278.7	297.7	321.5	328.5
349.7	365.4	368.7	361.6	377.1
358.6	411.6	431.5	454.6	497.3
524.9	540.3	552.5	606.4	613.4
646.6	696.1	771.2	736.6	769.0
804.8	821.8	866.2	925.0	838.4
830.8	899.8	945.3	932.0	828.9
558.9	628.0	890.6	1005.3	998.5
1006.4	1006.0	1097.5	1050.5	1171.9
1077.5	568.2	873.8	918.8	996.5
1031.7	1146.4	1226.2	1161.4	1232.9
1300.4	1337.1	1334.4	1364.0	1352.2
1203.6	1270.5	1421.5	1517.5	1469.6
1613.0	1743.5	1766.5	1806.1	1861.9
2027.4	2150.7	2247.6	2331.7	2465.8
2670.	2630.	2767.7	2925.5	3101.5
3225.0	3395.3	3459.3	3682.6	3899.8
4186.0	4336.2	4502.5	4796.4	4694.4

APPENDIX 2
IBM/360-OS JOB CONTROL LANGUAGE

The following job control language (JCL) and indicated source code from Appendix 1 are appropriate to execute the model implementation on the IBM/360 Model 75 or 91 at Oak Ridge National Laboratory. The catalogued procedure CSMPIII is local, and the details of such a procedure would vary from one installation to another.

Job Card

```
//*CLASS CPU91=4M
// EXEC CSMPIII
//CSMP.SYSIN_DD_*
```

CSMP source deck, followed by the FORTRAN IV subprogram decks

```
ENDJOB
```

```
/*
```

```
//CSMP.FT12F001_DD_*
```

Data cards containing historical fossil-fuel production rates

```
/*
```

```
//
```

The //*CLASS card is local and overrides default limits for execution time, core storage allocation, and I/O calls. The ENDJOB statement is a terminal statement for the CSMP source module and not part of the JCL.

APPENDIX 3

**CALCULATION OF THE CO₂ PARTIAL PRESSURE IN THE
SURFACE WATERS OF THE OCEAN**

The differential equations which describe the transfer of carbon between the atmosphere and mixed layer of the ocean [Eqs. (2.2.2-1), (2.2.2-2), (2.2.2-9), and (2.2.2-10)] involve the quantity $P_m(\Sigma C)$, defined as the partial pressure of dissolved CO_2 in the mixed layer (in units of atmospheres), expressed as a function of ΣC , the concentration of total dissolved inorganic carbon (mol/l). The purpose of this appendix is to describe the computation of P_m , which is carried out by the FORTRAN subroutine PARPRE (listed in Appendix 1). The relationships and data for the calculation are taken from Bacastow and Keeling (1973).

From the equilibrium quotients

$$K_0 = \frac{[\text{CO}_2]}{P_m},$$

$$K_1 = \frac{[\text{H}^+][\text{HCO}_3^-]}{[\text{CO}_2]},$$

$$K_2 = \frac{[\text{H}^+][\text{CO}_3^{--}]}{[\text{HCO}_3^-]},$$

$$K_B = \frac{[\text{H}^+][\text{B(OH)}_4^-]}{[\text{B(OH)}_3]},$$

$$K_W = [\text{H}^+][\text{OH}^-],$$

and the equations for total boron and alkalinity

$$\Sigma B = [B(OH)_3] + [B(OH)_4^-],$$

$$A = [HCO_3^-] + 2[CO_3^{--}] + [B(OH)_4^-] + [OH^-] - [H^+],$$

it is possible to derive the relationships

$$P_m = [H^+]^2 \cdot (\Sigma C) \cdot [K_0(K_1K_2 + K_1[H^+] + [H^+]^2)]^{-1}, \quad (A3-1)$$

$$\Sigma C = \frac{([H^+]^2 + K_1[H^+] + K_1K_2) \left[[H^+] + A - K_W/[H^+] - K_B \Sigma B / ([H^+] + K_B) \right]}{K_1([H^+] + 2K_2)} \quad (A3-2)$$

For constant values of the alkalinity, A, and total boron concentration, ΣB , the last two equations effectively express P_m and ΣC as functions of the hydrogen ion concentration, $[H^+]$, and hence, implicitly, P_m as a function of ΣC . Numerical values, taken from Bacastow and Keeling, are as follows:

K_0	0.03347
K_1	9.747×10^{-7}
K_2	8.501×10^{-10}
K_B	1.881×10^{-9}
K_W	6.463×10^{-15}
A	2.435×10^{-3} eq/l
ΣB	4.09×10^{-4} mol/l

The values of the equilibrium constants correspond to an average surface temperature of 19.59°C and chlorinity 19.24 per mil. All concentrations are assumed to be expressed in moles per liter.

On first call, subroutine PARPRE uses Eqs. (A3-1) and (A3-2) to compute a table of the quantities

$$\text{pH} = -\log_{10}[\text{H}^+], -\log_{10}P_m, \text{ and } -\log_{10}(\Sigma\text{C}), \text{ pH} = 1(0.1)11.$$

Then on the first and subsequent calls, it interpolates in the table to find the value of pH and $-\log_{10}P_m$ corresponding to $-\log_{10}(\Sigma\text{C})$ and returns pH and P_m to the calling program. The logarithmic transformation proved necessary for satisfactory interpolation in the data. Numeric tests have indicated a computational accuracy of approximately three significant digits for P_m as a function of ΣC with this interpolation scheme. For greater accuracy, it would not be difficult to modify the subroutine to refine the interpolants by several iterations of the secant method.