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NUMERICAL INVESTIGATION OF HOSE INSTABILITY OF A BENNET BEAM

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NUMERICAL INVESTIGATION OF HOSE
INSTABILITY OF A BENNETT BEAM*

by

James M. Leary

ABSTRACT

The hose instability is examined numerically for an unmodulated beam with the Bennett current profile $J(r) \propto \left(1 + \frac{r^2}{a^2}\right)^{-2}$. Channel conductivity $\sigma(r)$ is also of this form. The dispersion relation of Lee⁽¹⁾ shows excellent agreement with numerical results. Transient behavior of beam is studied, and found consistent with Lee's work.

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I. INTRODUCTION

A pinched relativistic beam, when scattered by a gas evolves to a Bennett equilibrium.^(2,3) The current density profile is

$$J(r) = \frac{I}{ra^2} \left(1 + r^2/a^2\right)^{-2}, \quad (1)$$

where (I) is total beam current and (a) is the scale radius. A reasonable model of hose motion of a Bennett beam has been worked out by Ed Lee⁽¹⁾ with the assumption that the conductivity profile is the same shape as J(r). Lee also makes an educated guess as to the distribution of the response frequency.

Let

$$k_m^2 = \frac{2eI}{\beta\gamma mc^3 a^2} \quad (2)$$

$$\lambda = \frac{\pi\sigma\beta a^2}{2c}$$

$$s = \Omega^2/k_m^2$$

where the disturbance is of the form

$$y(x, z) \sim e^{-i(\Omega z + \omega x)} \quad (3)$$

Here y is the position of the center of mass and $x = \beta ct - z$.

Lee finds the dispersion formula

$$-i\omega\lambda = 6s \left[\frac{1}{2} - s + (s - s^2) \left(\pi i + \ln \frac{1-s}{s} \right) \right]. \quad (4)$$

This hose motion has also been done experimentally by E. J. Lauer.⁽⁴⁾ This note shows the results of the numerical simulation of this problem. An important finding is that the dispersion relation (4) is closely born out numerically.

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II. NUMERICAL SET-UP OF CODE BEAMSIM⁽⁵⁾

The beam is unmodulated with J given by (1) and with the conductivity profile having the same shape with the same scale radius (a) taken as 1.0 cm. The pinch field is fully built-up and the beam is in equilibrium with this field. The beam consists of 2196 particles per section. They move from five to eight betatron lengths in z depending upon the initial perturbation. Channel radius is 5.0 cm. Current is 600 amps with energy 6.0 MeV. Conductivity is 10"/sec. The imposed perturbation is as follows:

$$y(x, 0) = A \sin \omega x \quad (5)$$

$$\frac{\partial y}{\partial z}(x, 0) = 0$$

For our parameters we have

$$\begin{aligned} k_m^2 &= 0.0743 \\ \lambda &= 5.236 \end{aligned} \quad (6)$$

Four long computer runs were made with $\omega\lambda = 0.1507, 0.452, 1.178, 2.356$. These particular numbers were chosen in order to look at the important parts of Lee's dispersion relation.

III. RESULTS

Figure 1 shows the theoretical picture of the dispersion relation along with the results of the computer runs as indicated with the circled points. As usual Ω_r gives the frequency of oscillation while Ω_i gives the rate of growth of the solution. The results are summarized in Table 1. Ω_r is an average of 8% high and Ω_i is an average of 7% high for numerical runs.

Figure 2 shows the typical plots vs. z . The top two show motion of the center of mass in linear vs. linear and log vs. linear format. Notice the

growth is exponential. The pictures on the bottom show the beam radius vs. z . the beam is in good equilibrium.

Figure 3 shows y vs. x for $z = 700$ cm. Notice the transient behavior for approximately the first third of the graph and then the steady state sinusoid. The position in x and z where this transient dies out is of interest. It was measured in our runs for $z = 1/3L$, $2/3L$, and L where L is the total z length of the run. The mean of $z/x = 1.98$ with standard derivation = .19. Hence the transients die out with almost the same z/x in all cases. Lee⁽¹⁾ uses a saddle point evaluation to find the position of maximum growth of a wave packet. He finds that quantity also moves with constant z/x .

TABLE 1

$\omega\lambda$	ANALYTIC PREDICTION		NUMERICAL RESULTS	
	Ω_r/k_m	Ω_i/k_m	Ω_r/k_m	Ω_i/k_m
0.1507	.193	0.150	.211	.175
0.452	.365	0.1825	.423	.215
1.178	.588	0.120	.650	0.124
2.356	.708	0.00	.705	0.00

FIGURE CAPTIONS

Figure 1 Growth Rate for real ω .

Figure 2 Motion Down the Tank in z .

Figure 3 Motion Back in z .

FIGURE 1

DISTANCE BEHIND BEAMHEAD = 141 DELTAX

X = 8.490E + 02 CM.

FIGURE 2

FIGURE 3
CENTER OF MASS VS. DISTANCE BEHIND BEAMHEAD, STATION = 141

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