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A MICROSCOPIC ION-ION SPIN-ORBIT POTENTIAL

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A MICROSCOPIC ION-ION SPIN-ORBIT POTENTIAL*

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ABSTRACT

A microscopic, ion-ion spin-orbit potential is derived in a double folding model approach starting from a nucleon-nucleon interaction. Its geometry is determined by the structure of the target and projectile. An estimate of its strength is presented.

KEYWORD ABSTRACT

NUCLEAR REACTIONS: ion-ion spin-orbit potential, double folding model, heavy ions,

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I. INTRODUCTION

The standard Thomas form for the nucleon-nucleus spin-orbit potential can be derived, for example, in an impulse approximation¹ or a Glauber model approach.² The shape of the spin-orbit potential is given by $\frac{1}{r} \frac{d}{dr} \rho(r)$ where $\rho(r)$ is the target density, and at high energies the strength of the potential is proportional to the forward spin-flip nucleon-nucleon scattering amplitude.

Attempts have also been made to derive an ion-ion spin-orbit potential. Rawitscher³ derived an alpha-nucleus spin-orbit potential from a phenomenological alpha-nucleon spin-orbit potential. Love⁴ also used this approach, but he included nucleon-nucleon exchange effects also. Neither was able to obtain agreement with phenomenological alpha-nucleus spin-orbit potentials that were derived in fitting elastic alpha-nucleus scattering from non-zero spin targets.⁵⁻⁸

More recently, Thompson⁹ investigated the role of a spin-orbit potential in heavy ion scattering. In his approach, a phenomenological nucleon-nucleus spin-orbit potential is folded over the valence (non-spin saturated) nucleon's wave function. For example, for $^{13}\text{C} + ^{24}\text{Mg}$ a spin-orbit potential representing the nucleon- ^{24}Mg interaction is convoluted with the $1p_{1/2}$ neutron wave function in ^{13}C .

Finally, vector polarized ^6Li -nucleus elastic scattering and asymmetry measurements¹⁰ have been analyzed¹¹ by considering the ^6Li as an alpha-deuteron bound state and then folding the deuteron wave function with a phenomenological deuteron-nucleus spin-orbit potential. This analysis was also carried out for ^{14}N projectiles.

In this paper, we present a formalism for deriving an ion-ion spin-orbit potential from only the structure of the target and projectile nuclei and

from the nucleon-nucleon interaction. This will be done in the context of a double folding model that has proved quite successful in describing both elastic^{12,13} and inelastic^{14,15} heavy ion scattering. In the folding model approach of Refs. 12 and 14 it was found that the geometry of the ion-ion potential is well determined when accurate nuclear densities are used. Also, a smoothly energy-dependent complex strength emerges from the analysis of many different sets of elastic heavy ion data.^{12,14} This strength can, in principle, be related to a spin-isospin averaged, effective nucleon-nucleon forward laboratory scattering amplitude.¹² In a similar way, the geometry of the ion-ion spin-orbit potential can be fixed in the double folding model approach and the strength can be determined from a comparison to phenomenological potentials (unfortunately, there are very few) that have been derived.

In the energy range we will be considering (~ 5 MeV/nucleon) the effective nucleon-nucleon amplitude derived from fitting the data bears little resemblance to the free amplitude.¹² The real part of the free laboratory amplitude goes through zero at about a lab energy of 6 MeV and stays negative below that energy. The effective amplitude is fairly constant in that energy region at a value of about 1.8 fm.¹² At 5 MeV, the imaginary part of the free amplitude is large (~ 6 fm) and growing with decreasing energy, whereas for the effective amplitude it is small and decreasing.¹² Hence, it will not be surprising to find that the spin-flip amplitude needed to fit phenomenological spin-orbit potentials differs substantially from the free amplitude.

In the next section the formalism needed to derive the ion-ion spin-orbit potential is presented in some detail. The features of this poten-

tial are discussed in Section III. In Section IV the results of our calculations are shown and finally, a section of concluding remarks appears.

II. FORMALISM

The derivation of the central part of the ion-ion potential in the double folding model has been given elsewhere¹². In order to derive the spin-orbit term we need a nucleon-nucleon spin-orbit potential in coordinate space. We write

$$v_{so}(R+\vec{r}_2-\vec{r}_1) = g(|R+\vec{r}_2-\vec{r}_1|)(\vec{q}_1+\vec{q}_2) \cdot (R+\vec{r}_2-\vec{r}_1) \times \hat{k} \quad (1)$$

where g is a short ranged radial function, the $\vec{q}$'s are the Pauli spin operators, and the coordinates are defined in Figure 1. We write the ion-ion spin-orbit potential as the sum of all the nucleon-nucleon spin-orbit interactions averaged over the bound state nuclear wave functions, χ :

$$V_{so}(R) = \sum_{i,j} \int \int \chi_{1,i}^2(r_{1,i}) \chi_{2,j}^2(r_{2,j}) g_{ij}(|R+\vec{r}_{2,j}-\vec{r}_{1,i}|) \times (\vec{q}_{1,i}+\vec{q}_{2,j}) \cdot (R+\vec{r}_{2,j}-\vec{r}_{1,i}) \times \hat{k} d^3r_{1,i} d^3r_{2,j} \quad (2)$$

For simplicity, we will assume that nucleus 2 has total spin $I = 0$, and that nucleus 1 has total spin I given by the total spin of a single valence nucleon. Then,

$$\sum_j \langle \chi_{2,j} | \vec{q}_{2,j} | \chi_{2,j} \rangle = 0 \quad (3)$$

$$\sum_i \langle \chi_{1,i} | \vec{q}_{1,i} | \chi_{1,i} \rangle = \langle \tilde{\chi}_1 | \vec{q}_1 | \tilde{\chi}_1 \rangle$$

where χ_1 is the wave function of the valence nucleon. If we assume that the radial function g is independent of i and j , Equation 2 becomes:

$$V_{so}(R) = \iint \rho_2(r_2) \chi_1^2(r_1) g(|R+r_2-r_1|) \chi_1(R+r_2-r_1) \times k d^3 r_1 d^3 r_2. \quad (4)$$

We will now introduce a change of coordinates and proceed in a manner similar to Glauber's² approach in deriving the nucleon-nucleus spin-orbit potential. Let us define

$$\chi = R + r_2 - r_1 \quad (5)$$

$$2r = R + r_2 + r_1,$$

then Equation (4) becomes:

$$V_{so}(R) = \iint \rho_2(|R-r+\chi/2|) \chi_1^2(|R-\chi/2|) g(y) \chi \times k d^3 y d^3 z.$$

Since $g(y)$ is a short-ranged function (as compared to the size of the nuclei) we can attempt to expand the density and wave function squared about $y=0$. The zeroth order term leads to an integral of the form:

$$\int g(y) \chi d^3 y = 0,$$

so we must expand to first order in y . This leads to two terms involving derivatives, in one case of the density and in the other case of the wave function squared. Integrating by parts leads to the result

$$V_{so}(R) \approx \chi \cdot k \int d^3 y \chi g(y) \chi \cdot R \frac{d}{dR} \int \rho_2(|R-r|) \chi_1^2(z) d^3 z. \quad (6)$$

Now, using the fact that $\int d\Omega y_i y_j = \frac{4\pi}{3} y^2 \delta_{ij}$, Equation (6) becomes

$$V_{so}(R) = \chi \cdot k \times \frac{R}{R} \frac{d}{dR} \left[\int \rho_2(|R-r|) \chi_1^2(z) d^3 z \right] \frac{4\pi}{3} \int y^4 g(y) dy. \quad (7)$$

In an impulse approximation^{1,12} one could relate the radial factor $g(r)$ to the Fourier transform of the nucleon-nucleon isospin-averaged spin-flip laboratory amplitude $\bar{C}(q)$ ^{1,12}.

$$-\frac{2\pi\hbar^2}{m} \bar{C}(q) \hat{q} \cdot \hat{n} = \int e^{-i\mathbf{q} \cdot \mathbf{r}} g(r) \mathbf{r} \cdot \hat{q} \times \hat{k} d^3 r, \quad (8)$$

where m is the nucleon mass and $\hat{n} = \frac{k_0 \times k'_0}{|k_0 \times k'_0|}$, where k_0 and k'_0 are the initial and final nucleon-nucleon C.M. momentum. Using Eq. (8), one can show that the integral in Eq. (7) becomes

$$\frac{4\pi}{3} \int y^4 g(y) dy = -i \frac{2\pi\hbar^2}{m} k \bar{C}(0), \quad (9)$$

where $\bar{C}(q) \equiv \bar{C}(q)/(k k_0 \sin \theta_0)$, k is the ion-ion C.M. momentum, and θ_0 is the nucleon-nucleon scattering angle.

Finally, using $k = R \times k$, where k is the ion-ion C.M. angular momentum, Eq. (7) yields:

$$V_{so}(R) = i \chi \cdot k \frac{2\pi\hbar^2}{m} \bar{C}(0) \frac{1}{R} \frac{d}{dR} \int \rho_2(|R-r|) \chi_1^2(r) d^3 r. \quad (10)$$

Actually, the ion-ion spin-orbit potential should be written in terms of the total spin, I , of the nucleus. Using the Wigner-Eckhart Theorem, one can show that³:

$$\langle l \frac{1}{2} I^{\pm} | \sigma | l \frac{1}{2} I^{\pm} \rangle = \pm \frac{1}{l + \frac{1}{2}} \langle l \frac{1}{2} I^{\pm} | I | l \frac{1}{2} I^{\pm} \rangle, \text{ where we define } I^{\pm} \equiv l \pm \frac{1}{2}. \text{ Hence,}$$

the ion-ion spin-orbit potential becomes:

$$V_{sc}(R) = \pm i \frac{2\pi\hbar^2}{m} \tilde{C}(0) \frac{1}{k} \cdot \frac{1}{k} \frac{1}{R} \frac{d}{dR} \int \rho_2(|\vec{r}-\vec{R}|) \chi_1^2(r) d^3r \quad (11)$$

Since the nucleon-nucleon spin-flip amplitude is in general complex, the spin-orbit potential we have derived will also be complex. However, at 5 MeV lab energy, the real part of the factor $\frac{1}{\sin \theta_0} \left. \frac{d\tilde{C}(q)}{dq} \right|_{\theta_0=0}$ is about four times larger than the imaginary part. This would, at least partially, justify the use of real phenomenological spin-orbit potentials in this energy region.

III. DISCUSSION

It has been stated in the literature that the spin-orbit potential has a $1/A_1$ dependence (A_1 being the mass number of the non-zero spin nucleus) and hence is unimportant for heavy ions. For a fixed lab energy per nucleon the factor $\frac{C(q)}{k_0 \sin \theta_0} \Big|_{\theta_0=0}$ is a constant, and there are only two possible sources of an A_1 -dependence in Eq. (11). Firstly, there is the factor of k in the denominator:

$$k \approx k = \sqrt{2 \cdot mc^2 \cdot \frac{A_1 A_2}{(A_1 + A_2)} \cdot \frac{A_2}{(A_1 + A_2)} \cdot E_{Lab}}$$

Writing E_{Lab} as $A_1 \frac{E_{Lab}}{A_1} = \text{constant} \times A_1$, the A_1 -dependence of k becomes:

$$k \sim \frac{A_1 A_2}{A_1 + A_2}, \text{ which for } A_2 \gg A_1 \text{ goes like } A_1, \text{ and for } A_1 \gg A_2 \text{ goes like } A_2$$

(independent of A_1). However, this A_1 -dependence of the strength of the spin-orbit potential arising from the factor of k in the denominator is cancelled by the factor of k in the angular momentum, L , which appears in the numerator. Thus, although the strength of the spin-orbit potential may be smaller for heavier ions, this is partly compensated for by the

larger values of L that are important for larger A_1 .

Secondly, the nucleon-nucleon spin-orbit potential is usually written in terms of the derivative of the central potential. For a zero-range nucleon-nucleon potential, the central ion-ion potential is proportional to the convolution¹²:

$$\int \rho_2(\vec{r}-\vec{R}) \rho_1(\vec{r}) d^3r \quad (12)$$

Hence, if one were to use the derivative of the central potential to describe the spin-orbit term, the strength would have to be reduced by what might naively appear to be a factor of A_1 . This is because $\rho_1(r)$ is normalized to A_1 whereas χ^2 is normalized to 1. However, one must remember that heavy ion reactions are sensitive to relatively large separations of the ions¹⁶. (Satchler¹³ found that the radius $R_g = 1.5(A_1^{1/3} + A_2^{1/3})$ defined the important region for elastic scattering.) For these large R 's the density is dominated by the least-bound wave function which is just the valence wave function, χ . Thus, for large enough R , the ratio of $(\frac{1}{R} \frac{d}{dR})$ of the integral appearing in Eq. (12) and that appearing in Eq. (11) should approach 1. A plot of that ratio for $^{13}\text{C} + ^{40}\text{Ca}$ is presented in Figure 2. (At 14 fm the ratio is down to 1.5.)

In proton-nucleus scattering, where one is sensitive to smaller separations of the target and projectile, our results would give that the spin-orbit potential arising from any target spin would be approximately $1/A_1$ of the spin-orbit term due to the proton's spin. This is in agreement with a remark made by Feshbach¹⁷ and with experiment, where there has been no need to introduce a spin-orbit potential due to the target's spin in proton-nucleus scattering.

In contrast, for the strongly absorbed ${}^3\text{He}^{18}$ and ${}^4\text{He}^{3-8}$ projectiles, target spin dependence has been introduced. In the ${}^3\text{He}$ studies, the strength of the $\vec{L} \cdot \vec{S}$ term in the optical potential was compared for pairs of nuclei differing by one nucleon. It was found¹⁸ that the strength of the spin-orbit term increased by about 1 MeV for the non-zero spin member of the pair, independent of its spin. This is consistent with the result contained in Eq. (11), since the l -dependence of the potential is essentially cancelled by the $\frac{1}{2+1/2}$ factor. The differences between the spin-orbit potential for strongly and weakly absorbed particles, and its weak l -dependence in ${}^3\text{He}$ scattering have been discussed in Ref. (18).

IV. RESULTS

The phenomenological spin-orbit potential derived in the analysis⁵ of alpha- ${}^5\text{Be}$ scattering at $E_{\text{Lab}} = 20$ MeV is displayed in Figure 3. Also shown is the calculated potential of Eq. (11) normalized to the peak at $R = 3.6$ fm. This normalization gives a value of $\text{Re } \tilde{C}(0) = .39$ fm as opposed to the free space value of $.023$ fm¹⁹. As anticipated, these numbers differ dramatically.

The alpha density used is a three parameter Fermi distribution derived from electron scattering results²⁰. The ${}^9\text{Be}$ density and wave functions were obtained by using the Malaguti-Hodgson²¹ binding potentials with slight parameter adjustments to achieve better single particle binding energies.

Using the effective spin-flip amplitude found for the alpha- ${}^9\text{Be}$ scattering, an estimate of the ion-ion spin-orbit potential was calculated for ${}^{13}\text{C} + {}^{40}\text{Ca}$. It is shown in Figure 4 along with a Woods-Saxon derivative-type potential that was fixed to agree with the calculated potential

in the important peripheral region. This fit potential has the standard form

$$V_{\text{so}}(r) = \left(\frac{\mu}{m_{\pi}c}\right)^2 V_{\text{so}} \frac{1}{r} \frac{d}{dr} [1 + \exp(r-R)/a]^{-1} \vec{L} \cdot \vec{S}, \quad (13)$$

where m_{π} is the pion mass. The value of the parameters found are as follows: $V_{\text{so}} = -.73$ MeV, $R = 5.78$ fm, and $a = .79$ fm. Note that since the valence neutron in ${}^{13}\text{C}$ is in a $1p_{1/2}$ orbital, we have $I = 1/2$ and the potential of Eq. (11) has a sign opposite that of the normal nucleon-nucleus spin-orbit potential.

The geometrical parameters given above should be accurate, since the folding integrals were performed with carefully determined densities. However, the strength of the potential was determined by a comparison to only one phenomenological potential. It would be best to consider this number as only a very rough estimate for this reason. Above about 50 MeV/nucleon, there is hope that the impulse approximation, using the free nucleon-nucleon spin-flip amplitude, would give reasonably accurate results. Below that energy, the effective spin-flip amplitude should be taken as a free parameter.

The ${}^{13}\text{C}$ density and wave functions were also derived from modified Malaguti-Hodgson²¹ potentials. The ${}^{40}\text{Ca}$ density was produced from the results of a Hartree-Fock calculation²².

Although the spin-flip probability measured in the inelastic excitation by ${}^{13}\text{C}$ of the first $2+$ state in ${}^{58}\text{Ni}$ is quite small²³, there has been some recent evidence for the need of a spin-orbit potential in heavy ion transfer reactions. Fits to the transfer reaction ${}^{28}\text{Si}({}^{19}\text{F},$

$^{16}_0^{31}\text{P}$ have been greatly improved by the inclusion of a spin-orbit potential²⁴. The reaction $^{40}_{\text{Ca}}(^{13}_{\text{C}}, ^{14}_{\text{N}})^{39}_{\text{K}}(\text{g.s.})$ at 68 MeV cannot be fit in the DWBA using reasonable parameters²⁵. There are indications that a spin-orbit potential will improve the fit there also²⁶. The strength of the spin-orbit Woods-Saxon derivative potential estimated here was found to be too small to appreciably change the transfer cross section²⁷. Invoking an helicity-flip mechanism has been shown to provide a fit to the $^{40}_{\text{Ca}}$ transfer data²⁸.

V. CONCLUSIONS

A microscopic, complex, ion-ion spin-orbit potential has been derived in the framework of a double folding model. It is believed that it will provide a consistent geometry for the spin-orbit potential that can be used in semi-phenomenological analyses of heavy ion reactions. Of course, polarization and/or asymmetry measurements in heavy ion scattering would allow one to determine the spin-orbit potential in a more straightforward way.

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FIGURE CAPTIONS

Figure 1. The coordinate system used in deriving the spin-orbit potential.

Figure 2. The ratio $\frac{\frac{1}{R} \frac{d}{dR} \int \rho_2(|\vec{r}-\vec{R}|) \rho_1(r) d^3r}{\frac{1}{R} \frac{d}{dR} \int \rho_2(|\vec{r}-\vec{R}|) \chi_1^2(r) d^3r}$ for $^{13}\text{C} + ^{40}\text{Ca}$. The radius,

$$R_g = 1.5(A_1^{1/3} + A_2^{1/3}) \text{ is indicated by an arrow.}$$

Figure 3. The magnitude of the spin-orbit potential for $^4\text{He} + ^9\text{Be}$ at $E_{\text{Lab}} = 20 \text{ MeV}$. The solid line is the folding model result of Eq. (11). The dashed line is the Woods-Saxon derivative form of Eq. (13) using the parameters of Ref. 5.

Figure 4. The calculated spin-orbit potential for $^{13}\text{C} + ^{40}\text{Ca}$ at $E_{\text{Lab}} = 68 \text{ MeV}$ (solid line) and the Woods-Saxon derivative potential fit to it (dashed line). Again, R_g is indicated by an arrow.

Fig. 1

Fig. 2

Fig. 3

Fig. 4