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WATER RESOURCES RESEARCH PROGRAM

REVIEW AND APPRAISAL OF NUMERICAL CIRCULATION MODELS FOR LAKE MICHIGAN

by

J. H. Allender

Great Lakes Project

ARGONNE NATIONAL LABORATORY

ENERGY AND ENVIRONMENTAL SYSTEMS DIVISION

Prepared for the U. S. ENERGY RESEARCH
AND DEVELOPMENT ADMINISTRATION
under Contract W-31-109-Eng-38

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Printed in the United States of America
Available from
National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22161
Price: Printed Copy \$5.50; Microfiche \$3.00

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

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September 1976

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REVIEW AND APPRAISAL OF NUMERICAL CIRCULATION
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J. H. Allender

ABSTRACT

Output from four numerical-hydrodynamical models for the lake is compared to data from the field program of the Federal Water Pollution Control Agency (FWPCA) for September 1963 and/or to other limited data from which lake currents could be inferred. The comparisons between model-generated and observed currents are made by using real-time graphs for fixed locations, power spectra, lake-wide plots of average motion, progressive vector diagrams, cumulative scalar- and vector-averaged currents at fixed locations; horizontal scalar averages at various depths; and Fourier norms. The major merit of the models is their apparent ability to simulate circulation patterns. This ability can be used to expose important avenues of exchange between the northern and southern basins of the lake and to locate dominant features in the motion of the lake. The major failing of the models is their inability to simulate time-series currents at a fixed location. This failing is shown to be primarily the result of inadequate knowledge of wind and temperature fields. Choice of an appropriate means for data and model comparison evolves as a difficult and important task. Suggestions are made for model improvement and synthesis with future observations in order to focus on average motion in the deep lake.

"The purpose of computing is insight, not numbers," according to R. W. Hamming.

1. INTRODUCTION

Hamming's statement is very likely true from a theoretical viewpoint, but in a practical situation the numbers can be as, or more, important than the insight. Simulating lake circulations illustrates a situation of that kind, since legal and economic considerations contribute significantly to the impetus for such work.

*Portions of the results described below also have appeared in Allender, Berger, and Saunders (1975, 1976).

An operationally useful lake-process model must simulate the modeled quantity reasonably accurately, and must be transportable. That is, it must be relatively easy for persons trained in programming to implement the model for their specific needs. The circulation models chosen for the present study satisfy the transportability criterion. They are the models by Katz and Kizlauskas (1973), Allender (1975), Simons (1973), and Bennett (1975). The last three models had not been applied heretofore to Lake Michigan. A testing of the Bennett Model is continuing, while experimentation with the other models has been concluded for the present time.

The primary aim of this report is to compare the response of each of the four models with current-meter data taken from Lake Michigan in 1963 under the auspices of the Federal Water Pollution Control Administration (FWPCA). Lesser aims are: (1) to compare simulated currents with other limited data from which currents could be inferred, (2) to compare and contrast the numerical techniques used in the circulation models, and (3) to discuss the ways in which each of the models represents the physics of the lake circulation.

The next section of this report is a brief outline of previous numerical circulation modeling in the Great Lakes. Section 3 gives the equations of motion and the salient features of each of the finite-difference models that were tested in this study. Sections 4 and 5 describe the observational data used and the methods for comparison of observed and computed currents, respectively. Section 6 presents the methods used to determine time-space-dependent surface stresses. Sections 7 and 8 summarize the results of a variety of model tests for an idealized basin. Numerous comparisons of simulated circulation with observed currents are given in Section 9. Most of these comparisons are based on FWPCA current-meter data. Section 10 offers a recapitulation of the behavior of the circulation models, with specific reference to their ability to simulate the observed motions.

2. HISTORICAL BACKGROUND

In early lake circulation modeling, the equations of motion were integrated over cross sections perpendicular to a major axis of the body. The resulting one-dimensional equations were then used to study free oscillations (seiches) in the lakes. This technique was used by Defant (1953), Mortimer (1965), and Rockwell (1966). The first two-dimensional, storm-surge calculations for Lake Michigan were made by Platzman (1958). In that study, currents and water heights were calculated for the southern end of Lake Michigan for a surge that occurred on June 26, 1954. The linearized, inviscid, vertically-integrated, barotropic equations were used. Fischer (1959) devised a simplified finite difference scheme for solution of the storm-surge equations that has been widely used with various modifications. Welander (1961) wrote an extensive summary of progress in numerical surge prediction.

Steady-state, two-dimensional, finite difference models have been presented by Gedney and Lick (1970, 1971) for Lake Erie and by Murty and Rao (1970) for all of the Great Lakes. Cheng (1970) has applied a finite element method to steady, wind-driven motion in Lake Ontario. Birchfield (1974) has modeled two-dimensional circulation in Lakes Michigan and Huron by a technique that maps the irregular shoreline onto a series of connected rectangles, prior to solution of the equations.

Katz and Kizlauskas (1973) used a time-dependent, two-layer model for summer flows in Lake Michigan. Output from their work was used to describe episodes of pollutant dispersal in the lake (see also Katz and Schwab, 1975). The largest modeling effort for any of the Great Lakes was made by Simons (1973, 1974, 1975). Simons used ideas from the work of Platzman and Fischer to design multilayer circulation models for Lake Ontario. In his latest work Simons used data taken during the International Field Year on the Great Lakes (IFYGL) to calibrate and improve his models. Finally, Bennett (1975) developed a seven-layer model for Lake Ontario in which surface gravity waves are ignored and three-dimensional, finite difference methods are used.

The preceding synopsis of Great Lakes circulation models is not meant to be exhaustive. Additional information may be found in the reference lists of the articles cited.

3. MODEL PRINCIPLES

All of the models tested solve the vertically-integrated, linear, shallow-water equations on an f plane.* Three of the models allow two or more superimposed layers of fluid. The models are treated below in order of increasing complexity. The analytical equations used in each model are given, and the inherent assumptions in the equations are discussed. The accompanying finite difference methods are described, and their properties are enumerated. (Details of the methods are given in Appendix A.)

a. ADI Model

The alternating-direction-implicit** (ADI) model was applied to the following equations:

$$\frac{\partial \vec{U}}{\partial t} + f \hat{k} \times \vec{U} = -gh \nabla_H \zeta + (\vec{\tau}_w - \vec{\tau}_B) \quad (3.1)$$

and

$$\frac{\partial \zeta}{\partial t} + \nabla_H \cdot \vec{U} = 0, \quad (3.2)$$

where

$$\vec{U} = \int_{-h}^{\zeta} \vec{u} \, dz = \text{horizontal transport vector,}$$

$\vec{u} = (u, v)$ = components of flow in the eastward (+x) and northward (+y) directions, respectively,

$$\nabla_H = (\partial/\partial x, \partial/\partial y),$$

h = depth of the fluid,

ζ = height of the free surface,

$$\vec{\tau}_w = \text{surface stress,}$$

$$\vec{\tau}_B = \text{bottom stress,}$$

*See Glossary for definition of this and other terms pertaining to geophysical fluid dynamics.

**See Glossary for definition of this and other terms pertaining to computational fluid dynamics.

f = Coriolis parameter,

$\hat{k}$ = unit vector in the $+z$ direction (upward and perpendicular to the undisturbed free surface),

ρ = density, which is taken to be unity,

and

g = acceleration of gravity.

The major assumptions used in deducing Eqs. 3.1 and 3.2 were: (1) that the nonlinear field acceleration and horizontal diffusion terms are small compared to other terms in the equations; (2) a hydrostatic equilibrium exists; and (3) the motion is barotropic.

The wind stress at the free surface was written as:

$$\vec{\tau}_w = \rho_{\text{air}} C_D u_A^2 \hat{u}_A, \quad (3.3)$$

where

ρ_{air} = density of air,

C_D = drag coefficient,

u_A = mean wind at anemometer height,

and

$\hat{u}_A$ = unit vector in the direction of the mean wind.

The bottom stress was formulated as:

$$\vec{\tau}_B = \rho C'_D |\vec{u}| \vec{u} / h^2, \quad (3.4)$$

where C'_D is the bottom drag coefficient. In this study C_D is dependent upon wind speed and anemometer height (see Sec. 6). For initial tests, $C_D = 2.6 (10^{-3})$, however. $C'_D = 2.5 (10^{-3})$, unless otherwise noted. Boundary conditions are that $\vec{U} \cdot \hat{n} \equiv 0$, where $\hat{n}$ is a unit normal to the shoreline.

The ADI scheme was introduced by Peaceman and Rachford (1955) and by Douglas (1955). The technique has been applied to estuarine research by Leendertse (1967, 1970). As applied to Eqs. 3.1 and 3.2, the ADI method

conserves mass and momentum ignoring friction and the rotation of the earth. Truncation errors are $O(\Delta t, \Delta x^2, \Delta y^2)$ for u and v , and $O(\Delta t^2, \Delta x^2, \Delta y^2)$ for ζ , where Δt represents the time step, and Δx and Δy represent the grid spacings in the x and y directions, respectively. Unconditional stability has been proven (Allender, 1975), as applied to Eqs. 3.1-3.3, neglecting the Coriolis term and using a linear bottom friction. The method is dispersive.

A space-staggered grid system was used. The shoreline was approximated by using the usual stepping pattern, which maps the boundary onto tangent lines perpendicular to the flow component at the nearest grid point.

b. Katz-Kizlauskas Model

The two-layer, time-dependent circulation model by Katz and Kizlauskas (1973), hereafter referred to as the K Model, was developed to simulate summer flows in Lake Michigan. The model was applied to the following equations:

$$\frac{\partial \vec{U}}{\partial t} + f \hat{k} \times \vec{U} = -g D \nabla_H \zeta + \frac{1}{\rho} (\vec{\tau}_w - \vec{\tau}_z), \quad (3.5)$$

$$\frac{\partial \zeta}{\partial t} + \nabla_H \cdot \vec{U} = - \frac{\partial \zeta_\ell}{\partial t}, \quad (3.6)$$

$$\begin{aligned} \frac{\partial \vec{U}_\ell}{\partial t} + f \hat{k} \times \vec{U}_\ell = & -g D_\ell \left(\frac{\rho}{\rho_\ell} \nabla_H (\zeta - \zeta_\ell) + \nabla_H \zeta_\ell \right) \\ & + \frac{1}{\rho_\ell} (\vec{\tau}_z - \vec{\tau}_B), \end{aligned} \quad (3.7)$$

and

$$\frac{\partial \zeta_\ell}{\partial t} + \nabla_H \cdot \vec{U}_\ell = 0, \quad (3.8)$$

where

ℓ = lower-layer quantities,

$\vec{\tau}_z$ = stress between the layers,

$D = \zeta + h_\ell - \zeta_\ell$,

and

$$D_\ell = h - h_\ell + \zeta_\ell.$$

The two major assumptions made in deducing Eqs. 3.5-3.8 were: (1) that nonlinear field acceleration and horizontal diffusion terms are small compared to other terms in the equations; and (2) that a hydrostatic equilibrium exists and the vertical density structure is well-approximated by two homogeneous, horizontal layers with densities ρ and ρ_ℓ , separated by a movable, impermeable interface. The formulation of $\vec{\tau}_w$ and $\vec{\tau}_B$ was the same as for the ADI Model. Boundary conditions were that $\vec{U} \cdot \hat{n} = \vec{U}_\ell \cdot \hat{n} \equiv 0$, where $\hat{n}$ is a unit normal to the horizontal boundaries. A Newtonian interface stress law was used:

$$\vec{\tau}_z = 2 A_z \left(\frac{\vec{U}}{h} - \frac{\vec{U}_\ell}{h_\ell} \right) (h + h_\ell)^{-1}, \quad (3.9)$$

where A_z is the effective vertical eddy viscosity. Initial interface depth was chosen to be 16 m for this work.

The finite difference method used in the K-Model was also used by Reid and Bodine (1968). A time-and-space-staggered, explicit scheme was used. The method, as applied here, conserves mass and momentum, ignoring friction and the rotation of the earth. Truncation errors are $O(\Delta t^2, \Delta x^2, \Delta y^2)$. An estimate for the stability criterion can be found from the usual Courant condition. The scheme is dispersive, but not dissipative (ignoring the friction and Coriolis terms). The shoreline is fitted, as in the ADI Model, with the help of a system of markers that are used to identify the various possible cases for land-water geometry.

c. Simons Model

The four-layer, time-dependent circulation model by SIMONS (1973), hereafter referred to as the S Model, has been applied to Lake Ontario. Data from the IFYGL were used to improve and calibrate the model as applied to that lake.

A continually changing density field is found from the time-space-dependent temperature distribution. The computed circulation is used to

prescribe advection in the temperature equation. Pertinent equations for the S Model may be written:

$$\left(\frac{\partial \vec{U}}{\partial t} + f \hat{k} \times \vec{U} \right)_{k-1/2} = -g \left[D(\nabla_H \zeta + \nabla_H \xi) + R \nabla_H Z \right]_{k-1/2} \quad (3.10)$$

$$+ \nabla_H \left[A_H D \nabla_H \cdot \left(\frac{\vec{U}}{D} \right) \right]_{k-1/2} + \chi_{k-1} - \chi_k,$$

$$\left(\frac{\partial D}{\partial t} + \nabla_H \cdot \vec{U} \right)_{k-1/2} + w_{k-1} - w_k = 0, \quad (3.11)$$

$$\left[\frac{\partial T}{\partial t} + \nabla_H \cdot \left(\frac{\vec{U} T}{D} \right) \right]_{k-1/2} - \nabla_H \cdot \left[A_\theta D \nabla_H \left(\frac{T}{D} \right) \right]_{k-1/2} \quad (3.12)$$

$$- \gamma_{k-1} + \gamma_k = 0,$$

and

$$\sigma = -6.8(10^{-6}) \theta^2 \quad (3.13)$$

where

D = layer depth,

ξ = baroclinic pressure function,

$R \nabla_H Z$ = baroclinic pressure correction that accounts for variable layer depth,

A_H = horizontal eddy viscosity coefficient,

χ = vertical momentum flux,

χ_o = surface stress,

$\chi_{\max}$ = bottom stress,

w = vertical velocity,

T = layer heat content,

A_θ = horizontal eddy heat conductivity coefficient,

γ = vertical heat flux,

$\sigma \equiv (\rho - \rho_0)$ = a measure of the density anomaly, where ρ_0 is the value of maximum density,

and

θ = deviation of the temperature from the temperature of maximum density.

The following definitions have been introduced:

$D_{k-1/2} \equiv Z_{k-1} - Z_k$, where Z_k = the depth of the k th interface, k increasing downward,

$\Phi \equiv \int_{Z_k}^{Z_{k-1}} \phi \, dz$, where ϕ = an arbitrary dependent variable,

$Z_{k-1/2} \equiv 1/2(Z_{k-1} + Z_k)$,

$\xi_{k-1/2} \equiv \int_{Z_{k-1/2}}^{\zeta} \sigma \, dz$,

and

$R \equiv \int_{Z_k}^{Z_{k-1}} \sigma \, dz$.

The major assumptions made to deduce Eqs. 3.10-3.13 are that:

- (1) nonlinear field accelerations are small compared to other terms in the momentum equations; (2) a hydrostatic equilibrium exists; (3) the vertical density structure can be approximated by four vertically homogenous, horizontal layers with densities calculated on the basis of Eqs. 3.12 and 3.13; (4) horizontal diffusion of heat and momentum can be modeled based upon the familiar eddy diffusion concept; (5) the density varies slowly in each layer, which leads to the two baroclinic pressure terms in Eq. 3.10; and finally, (6) Eq. 3.13 is a satisfactory equation of state. Boundary conditions are that $\vec{u} \cdot \hat{n} \equiv 0$ and $\vec{u} \cdot \hat{r} \equiv 0$, where $\hat{r}$ is a unit vector tangent to the lateral boundary. Interface stresses were formulated as in the K Model by using the transports calculated from adjacent layers. Interface depths were chosen to be 16, 20, and 50 m for this work.

The finite difference method used in the S Model employs leapfrog time-differencing and centered space-differencing. A conjugate lattice system is used that interchanges the locations of u and v on alternate time steps. Diffusion and dissipation are evaluated at the old time step, while the pressure gradient, divergence, Coriolis, and nonlinear terms are evaluated at a time step centered between the new and old values. The four layer equations are transformed to a vertical mean-flow equation and three shear-layer equations, by summing all layer equations and subtracting adjacent layer equations. This procedure (used by Simons) effectively decouples the barotropic (external) and baroclinic (internal) modes. Thus, a much longer time step may be used for the internal mode than for the external mode. The advection terms in Eq. 3.12 were treated by using a Lax-Wendroff scheme in Simon's original work (1973). The advection terms in the version of the model that was tested in this study were treated by a forward-differencing technique, which progresses along zigzag lines in a rotated coordinate frame.

The basic coordinates are rotated 45° and space differences are evaluated along the rotated axes. Horizontal viscosity couples the space lattices more effectively in the rotated system, thus reducing grid dispersion. Both flow components are specified as zero at lateral boundaries. Special procedures are required where boundaries pass through elevation points (Simons, 1973). Forward space differences for the continuity equations are applied at such points. This technique insures conservation of mass. Tangential velocity components near the shore in the rotated system are found as follows: The equations of motion in the transformed coordinates are applied at the boundary streampoint by introducing a fictitious surface elevation at this point and by evaluating the pressure gradients by forward differences.

The S Model conserves mass, temperature, temperature squared, and momentum, ignoring dissipative effects and the rotation of the earth. Truncation errors are $O(\Delta t^2, \Delta x^2, \Delta y^2)$, except where forward-difference procedures are used, which reduces the method to first order. The Courant condition provides rough stability estimates. The scheme for the flow is essentially nondissipative, but it is dispersive.

d. The Bennett model

The three-dimensional, time-dependent model by Bennett (1975), hereafter referred to as the B Model, was developed to simulate circulation in Lake Ontario. A three-dimensional, finite-difference grid, having seven levels in the vertical was used. A newer, presently unpublished, eight-level version of the model was tested on Lake Michigan in this study, and may be summarized as follows:

$$\frac{\partial \vec{u}}{\partial t} + f \hat{k} \times \vec{u} = -\frac{1}{\rho_0} \nabla_H (p + p_s) + \frac{\partial}{\partial z} (A_z \vec{u}) + B.V. \quad (3.14)$$

$$\frac{\partial}{\partial t} L\psi = -f \nabla_H \psi \times \nabla_H \left(\frac{1}{h} \right) - \nabla \times \left[-\nabla_H p + \frac{\partial}{\partial z} \left(A_z \frac{\partial \vec{u}}{\partial z} \right) \right]^Z \quad (3.15)$$

$$\nabla_H \cdot \vec{u} = -\frac{\partial w}{\partial z} \quad (3.16)$$

$$\frac{\partial T}{\partial t} + \nabla \cdot (\vec{u} T) + \frac{\partial}{\partial z} \left(wT - A_\gamma \frac{\partial T}{\partial z} \right) = 0 \quad (3.17)$$

$$\sigma = -6.8(10^{-6}) \theta^2 \quad (3.18)$$

$$p = g \int_0^Z \sigma dz \quad (3.19)$$

In Eq. 3.14, B.V. represents a bulk viscosity term. This smoothing term was introduced by Bennett in lieu of horizontal diffusion, and takes the form of the truncation errors arising from a four-point, spatial average for the Coriolis term. The smoothing acts only on the vertical motion field, not on the vorticity.

In Eq. 3.15, $L\psi$ is the vorticity of the vertically-averaged flow, $L\psi = \nabla_H \cdot (1/h \nabla_H \psi)$; p is the baroclinic pressure, as defined by Eq. 3.19;

p_s is the surface pressure; w is the vertical component of the velocity; A_y is the vertical eddy heat conductivity coefficient; and the remaining variables were as defined above. Midpoints for vertical levels were chosen to be approximately 3, 10, 20, 35, 55, 84, 125, and 183 m below the lake surface.

The major assumptions made to deduce Eqs. 3.14-3.19 were: (1) surface gravity waves may be ignored (the rigid-lid approximation); (2) nonlinear field accelerations are small compared to other terms in the momentum equations; (3) a hydrostatic equilibrium exists; (4) diffusion of temperature is small compared to advection of temperature; and (5) Eq. 3.18 is a satisfactory equation of state.

The finite difference method for solving Eqs. 3.14-3.19 uses centered-space differences on a single Richardson lattice. The time-differencing method is similar to that used by Platzman (1972). An initial estimate of the new velocity field is found by ignoring the barotropic part of the pressure field, p_s . A new value of the stream function then is calculated by over-relaxation, using a Dufort-Frankel method, and it is used to correct the velocity field for neglect of p_s . Vertical motion is found next from the continuity equation. The new temperature field is found from Eq. 3.17 by using an Adams-Bashforth scheme for the advection terms and forward differencing for the diffusion terms. Finally, the baroclinic pressure field is computed from Eqs. 3.18 and 3.19. The method would appear to be $O(\Delta t^2, \Delta x^2, \Delta y^2)$. The overall method is both dissipative and dispersive.

4. OBSERVATIONAL DATA

a. Bathymetry

The complex bathymetry of Lake Michigan is shown in Fig. 4.1. This contour plot is based on a 2-minute interpolation of all available soundings (Hughes, Birchfield, and Matthies, 1972). The dominant features of the topography are the relatively deep, northern and southern basins of the lake that are separated by a broad sill.

Additional physical characteristics of the lake are (Rockwell, 1966): volume = $464(10^{10})\text{m}^3$; surface area = $525(10^8)\text{m}^2$; mean depth = 89 m; length = 558 km; and mean width = 93 km.

Figure 4.2 gives the depth distribution used as input to all of the numerical model runs. A uniform grid, with 10.8-km spacing, was used to obtain input depth values.

b. Winds

Wind data for this study consisted of hourly speed and direction values from four first-order synoptic stations around Lake Michigan and twenty-minute observations from eight of the FWPCA meteorological buoys. Figure 4.3 shows the locations of the 12 stations. Each moored, floating buoy was equipped with a three-cup, Geodyne anemometer at 3-m height and a wind vane to measure speed and direction. Each revolution and tenth revolution of the anemometer cups was recorded digitally on 16-mm film for a 6-second sampling period during each 20-minute interval. Two direction readings were obtained during each observational interval, and the modulo 180 mean of the two was taken to be the true direction.

Wind data obtained before the summer of 1964 were subject to some speed errors, since the anemometers were not necessarily oriented in a favorable direction with respect to the wind. A vane was later added to the buoy stations to maintain a favorable orientation. See FWPCA, 1967 for a more comprehensive description of these wind data.

The effects of surface pressure variations were not considered in this study. No emphasis is placed upon motions that could arise from a pressure-induced setup.

Fig. 4.1. Contour Plot of Lake Michigan Bathymetry

40 METER DEPTH CONTOURS

PLOT 0 10.49.03 MON 12 JAN, 1976 JOB-ELEVCON. 1530, 5:55PM VER 4.11

Fig. 4.2. Contour Plot of Model Bathymetry

c. Lake Currents

The current-meter observations used in this study were collected by the Great Lakes Illinois River Basin Project under the FWPCA in 1962-64. Previous analysis of portions of those data have been made by Verber (1964, 1965, 1966), Malone (1968, 1972), Birchfield and Davidson (1967), and Mortimer (1968). Strings of Richardson current meters were deployed at 34 stations in Lake Michigan by using subsurface moorings (see Fig. 4.3). Nominal station depths were 10, 15, 60, and 90 m or greater. The sampling period was 50 sec, once every 20 min. The speeds were averaged over this 50-sec interval. Current directions were found as follows: two readings of the vane were taken at the trisection points of the read interval; a single compass reading was taken at the center of the interval; the vane readings were averaged vectorially; and the actual direction was calculated. A description of the rest of the initial data processing may be found in Mehr (1965). The effects of aliasing of the records due to mooring line noise were not studied in this work. Gould, Schmitz and Wunsch (1974) concluded that subsurface moorings were a stable platform from which to make current observations in deep water. Gould and Sambuco (1975) have shown that aliasing effects are still poorly understood, however.

Fig. 4.3. Locations of FWPCA Meteorological Buoys and Current Meters

5. METHODS FOR DATA AND MODEL COMPARISONS

In the present study, vector fields such as simulated, quasi-steady-state circulations and surface stresses, were compared pairwise by calculating the root-mean-square (rms) speed deviation per grid point in relative measure $\Delta\hat{u}$, and the rms direction deviation per grid point in radians, $\Delta\beta$. Let $\vec{u}_{jk} = r_{jk} e^{i\beta_{jk}}$ represent a given vector at any grid point j,k . The rms relations then become:

$$\Delta\hat{u} = \left[\frac{\sum_{jk} (M_{r_{jk}} - \text{ref}_{r_{jk}})^2}{\sum_{jk} \text{ref}_{r_{jk}}^2} \right]^{1/2} \quad (5.1)$$

$$\Delta\beta = \left[\sum_{jk} \left(\left[M_{\beta_{jk}} - \text{ref}_{\beta_{jk}} \right] \bmod 180 \right)^2 / N \right]^{1/2}, \quad (5.2)$$

where the pre-superscript identifies the model or wind field, ref indicates usage of one vector field as a reference, and the summations extend over all the points, N , common to the given fields.

Time-dependent, computed and observed currents were compared by using real-time graphs and by the following qualitative and quantitative means. Some or all of these methods were applied to various data segments. Results from the desired station(s) and the corresponding model grid points were recovered from tape storage. Low-pass filtering of the data was performed in many cases. The Fourier norm and the norm of absolute differences were calculated (a) between the calculated and observed data, (b) between the computed data and zero, and (c) between the observed data and zero. These relations for arbitrary vectors $\vec{P}$ and $\vec{Q}$ may be written:

$$\|\vec{P}, \vec{Q}\|_n \equiv \left[\frac{M\Delta t}{\sum_{t=\Delta t} |\vec{P}(t) - \vec{Q}(t)|^n / M} \right]^{1/n} \quad (5.3)$$

The Fourier norm is found when $n=2$, and the norm of absolute differences requires $n=1$. The calculation of these norms was a useful way of quantifying flow comparisons.

The following additional calculations and comparisons were made. The average velocities over a given time period were computed; detrended spectra were found by using the Blackman-Tukey method and a Hanning window; progressive vector diagrams for daily-average velocities were prepared; and three-, eight-, and thirty-day-average velocities were compared in geographical plots of the lake. Cumulative scalar- and vector-averaged velocities were graphed for model and observed data, and flow patterns arising from climatologically-derived wind stresses were compared with currents inferred from limited ERTS satellite data (Strong and Stumpf, 1975).

6. DETERMINATION OF TIME-AND-SPACE-DEPENDENT SURFACE STRESSES

The interpolation of irregularly-spaced data onto a regular grid (objective analysis) continues to be a difficult problem. For some cases, polynomial surfaces have been fitted to the data by using a generalized least-squares procedure (Panofsky, 1949; Dixon, 1969; Dixon and Spackman, 1970). Weighted averages based on an inverse power of the distance to data stations has been used, also (Simons, 1974). Cressman (1959) developed a method, now widely used by synoptic meteorologists, which essentially consists of applying corrections to a first-guess field (McDonnell, 1973; Kruger and Asselin, 1967). Cole (1968) used Cressman's method to interpolate wind data over the Great Lakes.

A second problem with the determination of surface stresses is the choice of an empirical relation for the surfacedrag coefficient. The nature of the dependence of this parameter on atmospheric stability, wind speed, and fetch is not well known. Deacon and Webb (1962) have summarized the methods for estimating surface stress, and additional research on airflow over water and above changes in surface heat flux, temperature, and roughness has been conducted by Taylor (1970, 1971, 1972), among others. Estimates of the average drag coefficient for Lake Ontario were inferred from wind-induced setup by Donelan, Elder, and Hamblin (1974). Boundary-layer profiles of wind speed, temperature, and humidity were studied by Donelan, Birch, and Beesley (1974). More recently, Donelan (1975) has investigated the dependence of the drag coefficient on wave conditions. Finally, Hicks (1975) has developed a procedure for formulating these bulk transfer coefficients at any reasonable height in the boundary layer, incorporating the effects of wind speed and atmospheric stability.

In the succeeding paragraphs, the empirical relations for the drag coefficient and the interpolation procedures used in this study are described.

a. Formulation of Drag Coefficients

The bulk transfer coefficient appearing in Eq. 3.3 was formulated from the work by Hicks (1975). For the land stations, the relation is:

$$C_D = [1 + 0.07(u_{10} - 5)] (10^{-3}) \quad (6.1)$$

and for the overwater stations the result is:

$$C_D = [.79 + 0.13 u_3] (10^{-3}), \quad (6.2)$$

where u_{10} and u_3 are the wind speeds in m/sec at 10 m and 3 m above the surface, respectively.

These formulas are based on neutral atmospheric stability. The time-dependent simulations described below use data primarily from September, 1963. Frequency-of-occurrence histograms for various atmospheric stabilities over Lake Michigan (Hicks, 1975) indicate that neutral conditions best describe the situation during a typical September.

b. Spatial-interpolation Procedures

Four techniques for spatial interpolation of wind data were tried in this study. These methods are: polynomial fitting by least squares, a weighted-average based on all stations simultaneously, a weighted-average based on the three stations closest to a given point, and an analysis similar to that used by Cressman (1959) and Cole (1968). The raw wind data were converted to north-south and east-west stresses by using the formulas above [Eq. 6.1 or 6.2 and Eq. 3.3], prior to application of an interpolation procedure. The given technique was then applied to each stress component independently.

Polynomial fitting was found to be unsuitable for this study. Unrealistic spatial variations of wind stresses were obtained, and radically different results were found for polynomials of order two and three. Too few data stations were available for polynomial fitting for the area covered by the lake, even when the available data were supplemented with as many as 8 fictitious stations, each of which were averages of the real station data.

Interpolation based on weighted-averages of the data, τ_m^{obs} , may be summarized by the formula:

$$\tau(x,y) = \sum_{\ell=1}^N \prod_{m=1}^{N-1} d(x,y)_{\ell m} \tau_{mc}^{obs} / \sum_{\ell=1}^N \prod_{m=1}^{N-1} d(x,y)_{\ell m} \quad (6.3)$$

For a given ℓ the $d_{\ell m}$ represent a member of the subset of $\binom{N}{N-1}$ combinations of distances from x, y to the N stations. The distance to station m is always chosen to be the complement of this subset. Stress values are found at each spatial location $(i\Delta x, j\Delta y)$ required by the numerical schema by using Eq. 6.3.

Weighting all twelve stations ($N=12$) simultaneously produces reasonable results, although they may be oversmoothed, and the method is relatively time-consuming from a computational viewpoint. Weighting the three stations nearest to a given location $(i\Delta x, j\Delta y)$ produces reasonable results, too, but they may contain some unrealistic spatial gradients.

The modified Cressman analysis used in this study begins with an initial guess, τ^* , for the wind field, which is taken to be the arithmetic average of all stress values at a given time. A new field is computed, point-by-point, by correcting the initial guess:

$$\tau(x, y)^{**} = \tau^* + \frac{1}{N'} \sum_{m=1}^{N'} \left[\frac{L^2 - d(x, y)_m^2}{L^2 + d(x, y)_m^2} \left(\tau_m^{\text{obs}} - \tau^* \right) \right], \quad (6.4)$$

where

$d(x, y)_m$ = the distance to m th station,

L = the scan radius,

and

N' = the number of stations within radius L of (x, y) .

Thus the correction is a mean value of weighted errors. τ^{**} then replaces τ^* in Eq. 6.4, the scan radius is reduced, and a further correction is made. This procedure is repeated 3 or 4 times, as a rule. In this work, L values were chosen to be 10, 5, and 2.5 times the mesh size (10.8 km).

The Cressman technique appears to give the best results of all the techniques, since it preserves important spatial gradients in a relatively smooth manner. Figure 6.1 shows a typical, interpolated stress field at one moment in time. The stresses over the northern end of the lake are

CRESS. AVG.

Fig. 6.1. Wind Field from Cressman Interpolation

dominated by the initial guess, τ^* , due to the lack of wind stations in that area. The stress field over the southern end of the lake is undoubtedly more realistic, because most of the data come from that region.

The interpolation methods were compared quantitatively by calculating rms pairwise deviations (Sec. 5). Figures 6.2 and 6.3 show a time history of the rms stress magnitude deviation (in percent), and the rms direction deviation in radians between pairs of differently interpolated results, respectively. Typical deviations between the twelve-station average and the nearest three-station average (solid lines in figures) are larger than the deviations between the twelve-station average and the modified Cressman analysis (dashed lines in figures). It is clear that the three methods produce significantly different results. The circulation models were shown to be sensitive to these differences (Sec. 8).

It was instructive to perform tests with the numerical models by applying them to an idealized basin and by subjecting them to simplified external influences. Such simple experiments served as a means of familiarization with the various models, and helped to insure that no absurd errors had been made in adaptation and application of the models for this research. Further, these tests showed important differences among model responses and afforded insights regarding the dynamical response of the actual lake.

The results of the ideal basin experiments are described in Sec. 7, and part a of Sec. 8.

Fig. 6.2.. Root-mean-square Deviations between Differently Interpolated Wind Fields (Magnitude of Stress)

Fig. 6.3. Root-mean-square Deviations between Differently Interpolated Wind Fields (Direction of Stress)

7. MODEL COMPARISONS FOR AN IDEALIZED BASIN

The comparability of the S, K, and ADI Models was first tested by studying the vertically-integrated, barotropic response of an idealized basin that possessed some of the attributes of Lake Michigan. (The B Model was not available when these comparisons were made.) The model lake is depicted in Fig. 7.1. Grid size was 10.8 km. Time steps were 100 sec, 120 sec, and 6 min for the S, K, and ADI Models, respectively. The east-west width was 97.2 km, the north-south length was 475.2 km, and the depth increased from 0 to 90 m over two grid lengths, inward from the boundary. Each of the models, using one layer, was run to quasi-steady-state (200 hr) under the action of a south wind at 0.5 dyne/cm^2 . Graphical results showed that each model develops a qualitatively similar circulation pattern like the one illustrated in Fig. 7.2. The flow is downwind in the shallow regions near the meridional boundaries, and returns upwind in the deeper, central portion of the basin. This behavior may be explained by forming a vorticity balance from Eq. 3.1. Defining $\omega_z = \nabla_H \times \vec{u}$, and applying the curl operator, the result is

$$\left(\frac{\partial \omega_z}{\partial t} + f \nabla_H \cdot \vec{u} \right) \hat{k} = -g \nabla_H h \times \nabla_H \zeta + \nabla_H \times (\vec{\tau}_w - \vec{\tau}_B) \quad (7.1)$$

The two terms on the left side of Eq. 7.1 are zero when steady state is reached. The two terms on the right are the topographic curl effect (vortex stretching) and the combined effect of wind-stress curl and bottom friction.

The gyral pattern in Fig. 7.2 is a direct result of the topographic curl effect. The effects of the Coriolis force are most apparent near the latitudinal boundaries where the flow is westward and the return flow slightly farther inward is skewed toward the northwest and southeast corners of the basin. Most of the vorticity (in this inviscid solution) is generated near the latitudinal boundaries where the flow is least well-aligned with the depth contours. Vorticity dynamics in large lakes have been discussed by Csanady (1973) and Bennett (1974). An additional graphical demonstration of these effects on Lake Michigan is presented in Sec. 9.a.

The model solutions are compared quantitatively in Figs. 7.3 and 7.4. Total squared transport vs. time, an aggregate property of the solution, is

Fig. 7.1. Sketch of Idealized Basin

Fig. 7.2. ADI-Model Flow Pattern for Ideal Basin

Fig. 7.3. Sum of Squared Transport vs. Time for Model Responses for an Idealized Basin

Fig. 7.4. Root-mean-square Deviations between Model Responses for an Idealized Basin

plotted for each model in Fig. 7.3. The K Model shows the highest "kinetic energy." A stability analysis suggests that the K Model allows a growing solution, the amplitude of which is proportional to $(1 + f^2 \Delta^2)^{n/2}$, where n is the number of time steps. The ADI solution does not appear to reach steady-state, and apparently also allows a weak instability. The S solution appears to be more damped than the other models, but does reach a more or less steady value. It is important to note that the total "kinetic energies" given by the model solutions are much more similar during the first 72 hr than they are after that period. Weather systems also have a time of passage of approximately three days.

Figure 7.4 shows that the normalized rms differences in transport magnitude (per grid point), $\hat{\Delta u}$, are smallest for a given model pair between approximately 40 and 70 hr. Comparisons are more divergent outside of this time interval. The $\hat{\Delta u}$ values at 20 hr show that the models "spin-up" quite differently, a fact that was not apparent from the "kinetic energy" calculations. The direction deviations for model pairs usually are slightly greater than one radian. These deviations also show that the various solutions are more different pointwise than might have been anticipated from the aggregate property comparisons.

These tests showed that the models give significantly different results when applied to a simple geometry and subjected to time-independent winds. These variant results are clearly due to the different amplification and dispersion properties possessed by the numerical schemes. The implication, perhaps obvious, is that it would be due to luck alone if a given model correctly simulated a real geophysical system without careful "tuning" based on observational data.

The Bennett Model was first tested for a rectangular basin 100 x 500 km. For the depth distribution, an elliptic paraboloid, 200 m at the center of the basin and shoaling to 12 m at the boundary, was used. A variable, horizontal mesh spacing was used, having a minimum resolution of approximately 2 km near the boundary, and increasing to about 40 km in the interior. The midpoints of vertical levels were fixed at approximately 3, 10, 20, 35, 55, 84, 125, and 183 m below the surface. The time step was 1 hr.

Numerous experiments were performed with and without thermal stratification. The results, obtained from two particularly simple experiments, are noteworthy. For these experiments, the wind stress was set to zero and a horizontal temperature gradient was introduced in the upper four levels. These gradients were chosen to imitate the tendency for a lake to be warmer near the boundaries than in the center during the summer (Ayers et al., 1958; FWPCA, 1967). Each simulation lasted for ten days. The temperature distribution prescribed initially, and at the beginning of each day thereafter, was given by

$$T(x,y,t)_k = T(0,0,0)_k - T_o(1-x^2)(1-y^2), \quad (7.2)$$

where k ranges from 1 to 4 and T_o (the temperature variation from the center to the boundary) was taken first as 2°C and then as 4°C . The initial temperatures at the center of the basin, $T(0,0,0)_k$, were chosen to be 20.0, 19.4, 14.2, 8.0, 4.5, 4.4, 4.3, and 4.2°C for levels 1-8, respectively.

A symmetric, counterclockwise pattern is produced as a result of a simple, geostrophic balance. Doubling the horizontal temperature variation doubles the strength of flow, as expected. A 4°C variation causes currents of several cm/sec after a few days of simulation. These model currents are the same order of magnitude as the daily-average currents in Lake Michigan (Sec. 9). It is possible that thermally-induced flow accounts for a significant fraction of the average observed currents. Inadequate knowledge of the spatial temperature distribution thus poses a difficulty for simulation of the motions of the real lake.

8. RESPONSES OF THE ADI MODEL TO CHANGES IN THE WIND FIELD

a. Experiments Using an Idealized Basin

The ADI Model was chosen to test the effects of spatially-varying wind stress on the circulation of the idealized, barotropic basin shown in Fig.

7.1. In these tests, a north-south or east-west variation in the wind stress was introduced. A time history of the total square transport and total transport vorticity, $\int(\partial V/\partial x - \partial U/\partial y)$, was calculated from the model results for each test.

The wind conditions for the cases that were studied are summarized in Table 8.1. Magnitudes of the spatial gradients of the stress were based on climatological wind data for Lake Michigan (Phillips and McCulloch, 1972). The significant features of the results from cases 1, 2, 5, 6, 8, and 9 are discussed below.

The total squared transport for cases 1, 2, 5, and 6 is graphed in Fig. 8.1. The integral of the wind stress over the area of the basin was the same for all cases. (That is, the total momentum fluxes through the free surface were equal.) It is immediately clear that the south wind (Case 1) produces a much greater "kinetic energy" than does the west wind (Case 5). This effect can be understood as follows. Csanady (1973) showed that the transport across a section near the center of such a basin should be of the form

$$U \propto 1 - h/\bar{h}, \quad (8.1)$$

where $\bar{h}$ is the average depth along the section. The ratio of flows for north-south (N-S) and east-west (E-W) sections is then

$$V/U \propto \left(1 - h/\bar{h}_{E-W}\right) / \left(1 - h/\bar{h}_{N-S}\right). \quad (8.2)$$

Since $\bar{h}_{N-S} > \bar{h}_{E-W}$, $V/U > 1$ when $h > \bar{h}$. In other words, the interior flow is stronger across an E-W section than a N-S section. This behavior may be contrasted to Lake Ontario, the semimajor axis of which is oriented more or less east-west.

Figure 8.1 also shows that typical east-west changes in τ_y (Case 2) produce larger relative changes in "energy" than do typical north-south

Table 8.1. Wind Conditions for Ideal Basin Tests

Case	τ_y	τ_x
1	0.5 dyne/cm ² , everywhere.	0.0
2	0.3 dyne/cm ² , along western boundary; $\partial\tau_y/\partial x = 0.37(10^{-7})$ dyne/cm ³ .	0.0
3	As in 2, except $\partial\tau_y/\partial x = 0.46(10^{-7})$ dyne/cm ³ .	0.0
4	As in 2, except $\partial\tau_y/\partial x = 0.56(10^{-7})$ dyne/cm ³ .	0.0
5	0.0	0.5 dyne/cm ² , everywhere.
6	0.0	0.3 dyne/cm ² , along southern boundary; $\partial\tau_x/\partial x = 0.8(10^{-9})$ dyne/cm ³ .
7	0.0	As in 6, except $\partial\tau_x/\partial y = 1.0(10^{-9})$ dyne/cm ³ .
8	$\tau_y = (0.3 + \pi/10 \sin x)$ dyne/cm ² ; ($\tau_y = 0.3$ along eastern and western boundaries.)	0.0
9	$\tau_y = (0.4 + \pi/20 \sin x)$ dyne/cm ² ; ($\tau_y = 0.4$ along eastern and western boundaries.)	0.0

Fig. 8.1. Sum of Squared Transport for ADI-Model Responses to Different Wind Fields

changes in τ_x (Case 6). This suggests that the east-west dependence of the wind field may be more important, generally, than the north-south dependence when long-term trends are being studied for Lake Michigan.

The last two experiments on the idealized basin were intended to simulate the differing responses that could be produced by interpolation errors in the wind field. A symmetric error was introduced in the wind field for each case (see Table 8.1). The integral of the wind stress over the area of the basin was the same for these two cases as it was for Case 1. The total transport vorticity for these two cases shows little change compared to Case 1, since the wind field was symmetric. A 25% change in the total squared transport was produced by a relative increase of $\pi/8$ in wind stress (Case 9), after 72 hr, however. From these tests, it appears that interpolation errors could have a significant effect on simulated circulations. The effects of such errors will be discussed further in the next subsection.

b. Simulated Lake Michigan Circulations Using Differently Interpolated Stress Fields

In Sec. 6, three different methods for spatial interpolation of wind data were presented. The sensitivity of the numerical models to these differently interpolated stress fields was studied by using the one-layer ADI Model. In Fig. 8.2 the total transport vorticity vs. time is plotted for three different simulations of Lake Michigan currents based on identical, observed wind data. Differences as large as 100% in this aggregate property can be seen after 130 hr of simulation. A similarly large disparity was found for total squared transport, although this property showed smaller deviations among the simulations in general than those found for the transport vorticity.

Significant differences between simulations also were found when the rms deviations per grid point between pairs of solutions were calculated. Typical values for the rms direction deviation per grid point ranged from 0.4 to 0.7 radians between the simulations that used the 12-station weighted average and the Cressman stress fields. The range of direction deviations was from 0.7 to 1.0 radians between the simulations that used the 12-station weighted-average stress fields and those of the three-nearest-stations. The normalized speed deviations per grid point were typically ± 30 to $\pm 50\%$.

Fig. 8.2. Vorticity Comparisons for ADI-Model Responses to Differently Interpolated Stresses

These rms comparisons give a measure of the inherent uncertainty at a selected grid point in the various numerical solutions due to possible errors in the interpolation of the wind field. These comparisons show one reason for caution when comparing model-generated currents with observed currents at a fixed location in the lake.

9. COMPARISON OF SIMULATED LAKE MICHIGAN CIRCULATION WITH OBSERVATIONAL DATA

a. Model Responses to Climatological Wind Data and Comparison with Currents Inferred from Satellite Imagery

Quasi-steady-state model responses were generated by using wind fields derived from climatological data. Data for the months of April and July (Phillips and McCulloch, 1972) from Chicago, Illinois; Green Bay, Wisconsin; and Grand Rapids, Michigan, were fitted to a planar surface. (The average wind data are summarized in Table 9.1.) The S, K, and ADI Models were then run for 100 hours using these stress fields. The rms comparisons between pairs of models showed deviations similar to those found during the idealized basin tests (see Sec. 8.a). The differences among aggregate properties of the various model solutions also were similar to those calculated for the idealized basin.

Table 9.1. Averages^a for Climatological Wind Data

	<u>Chicago, Illinois</u> speed, direction ^b	<u>Green Bay, Wisconsin</u> speed, direction	<u>Grand Rapids, Michigan</u> speed, direction
April	5.7, 92	6.2, 57	6.7, 120
July	4.1, 38	4.3, 57	4.7, 75
October	5.1, 75	4.8, 43	5.7, 75
January	5.7, 92	5.7, 75	5.7, 98

^a Inferred from Phillips and McCulloch, 1972.

^b All speeds in m/sec; all directions in degrees toward.

Figure 9.1 shows the ADI-Model transport (vertically-integrated velocity) field produced by the April wind data. This figure represents a typical computer drawing of output from the models. The topographic-curl effect (see Sec. 7, Eq. 7.1) for the ADI April response was calculated and is graphed in conjunction with the lake bathymetry in Fig. 9.2. The + signs indicate CCW rotation and the - signs indicate CW rotation. The April winds, which were essentially WNW, are seen to produce three major "vortex pairs." For each pair downwind flow is favored near the boundary and a return flow

Fig. 9.1. ADI-Model Transport Field for April Climatological Wind Data

Fig. 9.2. Topographic-curl Effect for ADI-Model Response to April Wind

overlies the deeper waters. This behavior has been described for idealized basins (see Sec. 7, Fig. 7.2). It is instructive, however, to see the effects of topographic control in a realistic basin.

The thickness of the Waukegan member of the sediments in southern Lake Michigan is well-correlated with the barotropic transport under the prevailing westerlies. (The Waukegan member is the uppermost stratigraphic unit as defined by seismic profiling.) Figure 9.3 shows a contour map of the thickness of these sediments. The areas of greatest thickness correspond well with the two major gyral flow features along the eastern shore, which can be seen in Fig. 9.1. This correspondence suggests that these gyres may be persistent features during non-stratified times of the year, and reinforces the role of the barotropic component of circulation for sedimentation processes.

Recent technological developments have made satellite imagery a potential tool for studying the circulation of the upper 10-20 m of the Great Lakes. Figures 9.4 and 9.5 are a synthesis of surface currents inferred from ERTS satellite imagery (Strong and Stumpf, 1974) and currents resulting from model responses to climatologically-derived wind data. Arrow lengths in these two figures have only relative meaning.

Figure 9.4 shows the satellite-inferred, surface currents under W winds during non-stratified conditions (November) and selected features of the ADI-Model response to WNW winds (April climatological data). The locations of cities mentioned below are given in Fig. 4.1. The model currents compare remarkably well with the inferred currents. The major features for comparison are: (1) a large, clockwise gyre along the eastern shore (between Benton Harbor and Muskegon, Michigan), (2) a large counterclockwise gyre in the southeastern part of the lake (between Gary and Michigan City, Indiana), (3) a convergence along the western shore (between Milwaukee and Racine, Wisconsin), and (4) a convergence along the western shore (north of Chicago in the Highland Park area).

Figure 9.5 shows the satellite-inferred, surface currents under SW winds during stratified conditions (June) and the selected features of the S-Model response to SW winds (July climatological data). (Model currents are from the top layer of a four-layer simulation.) The agreement between

Fig. 9.3. Contour Map of Thickness of Waukegan Member of Lake Sediments (from Edgington and Robbins, 1973)

Fig. 9.4. Synthesis of Surface Currents Inferred from ERTS Imagery (W Wind) and ADI-Model Response (April Wind)

Fig. 9.5. Synthesis of Surface Currents Inferred from ERTS Imagery (SW Wind) and S-Model Response (July Wind)

model and inferred currents is not as good as for the non-stratified period represented in Fig. 9.4. Agreement is quite good north of a line between Milwaukee, Wisconsin, and Holland, Michigan. The clockwise gyre exposed by the ERTS data along the eastern shore is very weak in model results. The general offshore motion in the Chicago vicinity is shown as a clockwise northward motion in the numerical model.

b. Comparison of Multilevel Model Currents with FWPCA Current Data

A multiplicity of numerical experiments (Table 9.2) were performed. Pertinent model parameters were varied, different initial temperature fields were prescribed, the duration of simulations was changed, and so on. Several general remarks about the experiments should be made before the results are given.

Model temperatures at required depths (see Table 9.2) were based on average, observed temperatures for September, 1963, at FWPCA station 8 (see FWPCA, 1967). No data were available from other stations and, therefore, no spatial variability could be included in the initial temperature fields. A grid spacing of 10.8 km was chosen for all the experiments. This choice allowed depth data to be selected without interpolation; resulted in easily manageable storage requirements for the computer algorithms; and very likely produced currents with the same essential features as would appear in a simulation that used one-half this grid spacing. (Some experiments were conducted with the ADI Model while using a 5.4-km grid spacing. The resulting patterns were very similar to corresponding patterns found by using a 10.8-km spacing. Currents in the lake certainly undergo significant variations at space scales of less than 5 km. It is not clear that the apparently more detailed patterns arising at a 5.4-km spacing should compare to observed data at a fixed location any better than the patterns found for the 10.8-km spacing.) No attempt was made to formulate vertical diffusion and/or diffusivity in terms of Richardson numbers. Such formulations did not significantly improve comparability of model-generated and observed currents during the studies of Lake Ontario circulation (Bennett, 1975). This deficiency, however, does not necessarily preclude improving comparability in this manner.

Table 9.2. Summary of Conditions for Major, Model Simulations of Lake Michigan

Model	Time step(s)	Grid size	No. of layers	Duration	Wind data	Vert. diffusion	Horiz. diffusion	Temp. structure	Comments
K	2 min	10.8 km	2	1188 hr	4 shore stations	50 cm ² /sec	n/a	20°/4°C (fixed)	
K	2 min	"	"	"	4 shore & 8 over-	"	n/a	"	computed speeds unreasonably large
K	1 min	"	"	864 hr	"	"	n/a	"	"
K	1-1/3 min	"	"	"	"	100 cm ² /sec	n/a	"	"
S	2 min/15 min	"	"	"	4 shore stations	50 cm ² /sec	10 ⁶ cm ² /sec		
S	1 min/12 min	"	4	"	4 shore stations	"	"	20°/16°/7.5°/4.5°C (initially)	
S	"	"	"	"	4 shore + 8 over-water stations	"	"	20°/16°/7.5°/4.5°C (initially)	additional wind data improves results
S	"	"	"	"	"	"	"	19°/16°/3.5°/4.5°C (initially)	modified temp. structure caused little change
S	"	"	"	"	"	100 cm ² /sec	"	"	
S	"	"	"	"	"	75 cm ² /sec	10 ⁴ cm ² /sec	"	small differences in comparability of average results with obs. currents
S	"	"	"	192 hr	"	25 cm ² /sec	10 cm ² /sec	"	
B	60 min	"	8	576 hr	"	40 cm ² /sec	n/a	20°/19.4°/14.2°/8.0°/4.5°/4.3°/4.2°C (initially)	
B	60 min	"	"	216 hr	"	f(wind stress) ≥ 5 cm ² /sec; ≤ 49 cm ² /sec	"	"	poor results
B	40 min	"	"	200 hr	"	30	"	"	

Very little emphasis was placed on nearshore currents in this study. This was due to the lack of suitable temperature data for model testing and to the aims of the research as funded.

In the results given below, all K-Model simulations were based on overland wind data only. Unrealistic currents resulted in the K Model during the period when a heavy storm (September 10-18, 1963) was active. These strong currents persisted after the storm passed and represent results of little practical value. Several unsuccessful attempts were made to remedy the difficulty. The cause of the problem may have been the instability of the interface position between two fluids having a ratio of densities close to unity. Katz and Kizlauskas (1973) offer this interpretation in their original paper, extrapolating from theoretical work by Rao and Simons (1970, 1972). The computer algorithm also may generate an instability when the interface between layers intersects the surface in a complex manner. In any case, the available observations often show very different currents at 30, 60, and 90 m, and strongly suggest that a two-layer model would be inadequate for simulating the motion of the deep lake even given complete wind data.

Finally, Simons (1974, 1975) obtained better agreement between model and observations during his Lake Ontario studies than was obtained between models and observations in this Lake Michigan study. Simons more conclusive results seem to indicate that the IFYGL data base provided significantly better wind, temperature, and current meter data than were available for this study.

Observed currents (FWPCA) at various stations in the lake were available for a period during late summer through early fall, 1963. The emphasis on comparison of models and observations was for an 8-day period, September 10-18, 1963, during which strong northeasterly winds prevailed. The period, though relatively short, was chosen because earlier work (Simons 1974, 1975) suggests that circulation models are best able to imitate observed currents during periods when surface stresses are high. In addition, average lake behavior over periods as long as a month can be dominated by one or, perhaps, several storms. A few figures pertaining to average behavior for September 1-30 also will be presented.

Three-day averages of the available, over-the-water wind data for the period 12:00 pm, Sept. 10 to 12:00 pm, Sept. 19, 1963, are shown in Fig. 9.6. The first three-day average is dominated by a strong northeasterly flow that prevailed during the last 12 hours. Note that there seems to have been an important curl in the stress field. A less strong easterly flow was in effect for the next three-day period and, finally, the average winds became southeasterly and much weaker during the last three-day period.

Figure 9.7 allows a comparison of overwater and overland winds during the storm period. The overwater stresses are presented as 6-hr vector averages for the eight FWPCA meteorological stations used in this study (see Figs. 4.3 and 9.6). The overland stresses are presented as 6-hr averages of the synoptic data from Chicago and Milwaukee. (The records from Green Bay and Muskegon showed only minor indications of the storm.) Note that the magnitude of overwater stresses was consistently larger than the magnitude of overland stresses. At the height of the storm, the ratio of these magnitudes was almost 4. Directional agreement between overland and overwater winds existed for only about 24 hours, during the height of the storm. These results suggest that it may not be possible to generate realistic, time-dependent numerical simulations by using shore-based wind data alone.

Real-time plots of observed and model-generated currents show the nature of the motion at the surface and at depth, and give an indication of the range of speeds that can exist. In Fig. 9.8, observed and model-generated currents at 10-m depth at FWPCA station 8 have been plotted vs. time for the period Sept. 10-18, 1963 (Julian dates 252-260). Total depth at this station is approximately 100 m, and it is on a parallel with Waukegan, Illinois. Computed results were obtained from the top layer of the four-layer S Model. Speeds are in mm/sec and direction is that toward which the current flows; solid lines are the observed dates; dashed lines are the computed data. (All data have been filtered at 12.5 hr, since periods less than or equal to the fundamental seiche period of the lake (9.0 hr) have little effect on average motion and would have been strongly affected by surface pressure variations, which were not included in this work.) The strong "nor'easter" cited in Figs. 9.6 and 9.7 causes the current speed to rise sharply to about 25 cm/sec at Julian 254.7. The numerical model imitates this major event, including interial motions, rather

AVERAGE OBSERVED WIND STRESS

→ - 0.3 DYNES / cm²

Fig. 9.6. Three-day Averages of Overwater Winds for 12:00 pm, Sept. 10 to 12:00 pm, Sept. 19, 1963

Fig. 9.7. Comparisons of Overwater and Overland Winds for Sept. 11-13, 1963

Station No : 8 depth : 10m
 Numerical Model - sim.h2
 Observed data - FWPCA 1963

Fig. 9.8. Observed and S-Model-generated Currents at FWPCA Station 8, 10-m Depth for Sept. 10-18, 1963

well. The direction record is dominated by the inertial motions, displaying a pattern that is typical for midlake stations during stratified conditions. The S Model imitates the midlake inertial activity better than the other models tested because of the method used for approximating the Coriolis term (see Sec. 3.c). In Fig. 9.9, a power-spectral density estimate is given based on the data in Fig. 9.8. The only peak that can be identified is near the inertial frequency. This is a typical spectrum for a record of several days duration from above the thermocline.

In Fig. 9.10, observed and S-Model-generated currents at 60-m depth have been plotted vs. time for the period Sept. 10-18, 1963. Inertial activity is evident at this depth, but is much less dominant than at the 15-m depth (see Fig. 9.8). Model currents show a compensatory flow toward the SE during Julian 255, which is in accord with classical Ekman dynamics. The observed currents show a similar tendency, but clearly also have a rotational motion during this period. The character of motions at 60 m tends to vary more among the stations than at 10 m. Different parts of the lake play different roles in large-scale mass adjustment. These roles are exposed more clearly in the lakewide plots of currents given later in this subsection, however. Power-spectral estimates for data at this depth show no identifiable peaks for a record of 8 days' duration.

In Fig. 9.11, model-generated and observed currents are compared at 10-m depth at FWPCA station 12. Total depth at this station is approximately 18 m, and it is located about 6 km offshore from Wind Point, near Racine, Wisconsin. Computed results were obtained from level 2 of the 8-level B Model. The model imitates observed directions reasonably well for the first 3-4 days. Speeds are less well simulated than directions, but the correct trends are produced. The model predicts a relatively strong southward flow during the latter half of this simulation. The observed data show little or no current during this period. Currents are highly shore-parallel, as expected. The other two multilayer models (S and K) behave similarly, although the B Model seems to give a slightly better comparison. The computed data were taken from the westernmost grid square in the model at this latitude. These results illustrate typical comparability for nearshore stations based on a model with a spatial step size that is probably as large or larger than the width of the coastal-boundary layer itself.

Station No : 8 depth : 10m
 Numerical Model - sim.h2
 Observed Data FWPCA 1963

Fig. 9.9. Power Spectral Density Estimate for Data in Fig. 9.8

Fig. 9.10. Observed and S-Model-generated Currents at FWPCA Station 8, 60-m Depth for Sept. 10-18, 1963

Fig. 9.11. Observed and B-Model-generated Currents at FWPCA Station 12, 10-m Depth for Sept. 10-18, 1963

Figures 9.8-9.11 have shown that there are large discrepancies between computed and observed motions when they are compared pointwise in space and time. The difficulty arising from spatial interpolation of wind data was shown to be a contribution to these discrepancies. The choice of certain model parameters also can have significant effects. Data from the S Model shown in Figs. 9.8-9.10 were obtained by using a vertical diffusion coefficient of momentum (see Sec. 3.c) of $50 \text{ cm}^2/\text{sec}$. Figures 9.12 and 9.13 present rms deviations per grid point between these data and data from an S-Model simulation in which the vertical diffusion was prescribed as $100 \text{ cm}^2/\text{sec}$; September 10-18 corresponds to 348 to 540 hours. For the top layer, the normalized speed deviations range from a few percent to nearly 175%. The direction deviations range from a fraction of a radian to greater than one radian. For the bottom layer, these quantities range over similar values, although the deviations are in general smaller than for the top layer. It is clear from these figures that varying the vertical diffusion in such a model can have a drastic effect on pointwise comparisons in space.

The comparisons presented thus far suggested that more fruitful results could be obtained if various sorts of averages of the data were calculated. In Fig. 9.14, progressive-vector diagrams for daily-average currents for September 10-18, for station 8, depth 10 m, are presented for (1) the observed currents, (2) S-Model currents, (3) B-Model currents, and (4) K-Model currents. All diagrams are based on a square grid and are, therefore, shape-preserving. The scale, however, is different from diagram to diagram. The observed data show a relatively strong westerly flow during the third day (the height of the storm), a very small N-S particle displacement, and an E-S displacement of $O(10^4)$ m. The S and B Models give approximately correct displacements for the first three days. The K Model underestimates this E-W motion, but gives only a small N-S displacement, as do the observations. All of the models show an E or SE motion, not seen in the actual currents, during days four through eight.

In Fig. 9.15, progressive-vector diagrams for daily-average currents for September 10-18, for station 8, depth 60 m, are presented for (1) the observed currents, (2) S-Model currents, (3) B-Model currents, and (4) K-Model currents. The net displacement for the observed motions is $O(10^4)$ m. The observed data also show a relatively strong southeasterly flow during the

Fig. 9.12. Effect of Changing Vertical Diffusion from 50 to 100 cm^2/sec in S-Model/rms Comparison between Top Layer Transports

Fig. 9.13. Effect of Changing Vertical Diffusion from 50 to 100 cm^2/sec in S-Model/rms Comparison between Bottom Layer Transports

Fig. 9.14. Progressive-vector Diagrams for Sept. 10-18, 1963, at FWPCA Station 8, Depth 10 m (Distances in Meters)

Fig. 9.15. Progressive-vector Diagrams for Sept. 10-18, 1963, at FWPCA Station 8, Depth 60 m (Distances in Meters)

third day, compensating for Ekman drift in the upper layers during the height of the storm. This compensatory flow is strongest for the S Model during the fourth day; it is about the same in relation to later days for the B Model, and it is underestimated and somewhat northward of the correct direction in the K Model. The S and B Models give nearly correct directions for the eight-day-resultant displacement.

The daily-average observed currents at this station do not exceed 10 cm/sec; a more representative value would be 5 or 6 cm/sec. Typically observed horizontal temperature gradients in the lake can produce a quasi-geostrophic, thermally-driven flow (see Sec. 7), which could account for 50% or more of the daily-average currents. It is not surprising that model-generated and observed currents agree best when surface stress effects are dominant.

Cumulative vector and scalar averages of daily-average currents at FWPCA station 10 were calculated for September 10-18 to further investigate the comparability of the models and the observations. The total depth at this station is about 70 m. It was on a parallel with station 8, but was on the eastern side of the southern basin rather than the western side. Fig. 9.16 shows normalized, cumulative scalar and vector averages at the 15-m depth for the observed data and for the S and B Models. The scalar average for the observed data shows that the daily-average current magnitudes do not change markedly in spite of the storm (which began on the third day). The S and B Models underestimate daily-average magnitudes during the first three days even though they were started well before the zero(th) day. The vector averages for the observed data imply that there was a significant change in current direction during the third day, but that the direction changed slowly thereafter. The S and B Models clearly behave more like each other than like the observed data.

Figure 9.17 shows normalized, cumulative scalar and vector averages at the 60-m depth for the observed data and for the S and B Models. The rather similar shapes of the scalar and vector averages for the observed data indicate that the average current direction was fairly persistent during this period. The S and B Models are evidently able to simulate the trends in average current magnitude much better at 60 m than at 15 m. The similarity of the responses of the S and B Models is again notable. Finally, the

Fig. 9.16. Cumulative Scalar- and Vector-averaged Currents at FWPCA Station 10, Depth 15 m for the Observed Data and for the S and B Models

Fig. 9.17. Cumulative Scalar- and Vector-averaged Currents at FWPCA Station 10, Depth 60 m for the Observed Data and for the S and B Models

greatest contributions are seen to occur on the fifth day and thereafter. Thus, the deep water at this station in the lake begins to make a major adjustment more than a day after the onset of the storm. The models also exhibit this adjustment; however, the raw data show that the actual flow is toward the northeast during this period, whereas model currents are essentially toward the northwest.

The comparative calculations discussed above have concentrated on only one station at a time. Further insight regarding comparability of actual and model-generated currents may be gained by examining the synoptic currents at all available stations. Figures 9.18-9.25 summarize three- and eight-day average currents for September 10-18 for the observed data and for the S, B, and K Models. The observed data averages are reproduced in each drawing of the lake. Model currents from several neighboring model grid points for each observed data station have been included. The inclusion of currents from several neighboring points was motivated by results presented in previous sections, which suggested that comparability at a fixed point in space must be treated cautiously.

Figure 9.18 shows average, observed and model-generated currents at the 10-m depth for the period September 10-13, 1963. The K Model underestimates the currents, suffering from a lack of wind data as mentioned previously. Current directions compare poorly at stations 9, 10, 27, and 29. Station 8 is the only deep-water location where the models simulate direction correctly. The predominantly shore-parallel currents at stations 12, 16, and 31 are best imitated by the B Model. Station 41 shows a relatively strong northward drift, which also is seen in the B Model results. Note that the models exhibit similar deviations from the actual currents (stations 9, 10, 27, 29).

Figure 9.19 shows comparative results for the 60-m depth for September 10-13, 1963. (Observed flows at 90 m at stations 9, 20, and 41 also are presented. The B Model, which can differentiate between 60 and 90 m, is compared to the 90-m stations in Fig. 9.24.) Motion at station 9 is poorly imitated by all the models. Current directions at station 8 compare reasonably well, although average speeds there are best given by the B Model. The B Model also gives the most correct currents at station 10. It is curious to note that the S Model response near station 20 seems to be a

Fig. 9.18. Average, Observed and Model-generated Currents at 10-m Depth for Sept. 10-13, 1963

Fig. 9.19. Average, Observed and Model-generated Currents at 60- and 90-m Depths for Sept. 10-13, 1963

Fig. 9.20. Average, Observed and Model-generated Currents at 10-m Depth for Sept. 13-16, 1963

Fig. 9.21. Average, Observed and Model-generated Currents at 60- and 90-m Depths for Sept. 13-16, 1963

Fig. 9.22. Average, Observed and Model-generated Currents at 10-m Depth for Sept. 10-18, 1963

Fig. 9.23. Average, Observed and Model-generated Currents at 60- and 90-m Depths for Sept. 10-18, 1963

Fig. 9.24. Average, Observed and E-Model Currents at 90-m Depth for Sept. 10-13, 13-16, and 10-18, 1963

Fig. 9.25. Average, Observed and S-Model Currents at 10-, 60-, and 90 m for Sept. 1-30, 1963

combination of the observations at 60 and 90 m. The B Model shows a shoreward flow at station 20, not seen in the data. Current directions are in good agreement at station 29, although the B Model gives slightly more realistic magnitudes. The deep water at station 41 is seen to move in opposition to the S-Model currents. K-Model currents are disorganized at this location.

Figure 9.20 shows actual and model results for the 10-m depth for September 13-16. Currents are evidently stronger at most locations than they were during the previous three-day period. The storm that occurred on September 13 caused this increase. The observed flows at stations 8, 9, and 10 suggest a sense of rotation opposite to that seen in Fig. 9.18. The motions at stations 9 and 10 suggest a strong flow out of the southern basin. Model results show a similar tendency, the B-Model results being most correct. It is clear that motions in the northern part of the lake are poorly simulated. This is due in part to the lack of wind data for that portion of the lake. The complicated bathymetry may generate confusion in the models, which also contributes to the discrepancies.

Figure 9.21 shows comparative results at the 60-m depth for September 13-16. The observed currents are seen to be much stronger than during the preceding three-day interval (see Fig. 9.19). The data suggest a counterclockwise rotation that is imitated by the models. The agreement between models and observations has improved for this period (during which surface stresses were high). Note that there are strong northerly flows (out of the southern basin) at station 9 (90 m) and station 10 (60 m). The flow at 60 m at station 9 appears to be westward, however.

The deepest currents (90 m) tend to be stronger than the currents at 60 m at station 9. The deep waters of the southern basin thus have their own response to storms even under stratified conditions. Observed currents at 60 and 90 m at station 20 appear to be more related than at station 9. The models have a tendency to overestimate the northward motion at this location. The ENE observed currents evidently feed a persistent upwelling during this period. The B Model suggests a convergence at this location. Comparability with the models could have been very different if station 20 were displaced by several km to the north. Motion at stations 29 and 41 is

poorly imitated, which suggests that winds over the northern portion of the lake may exert a significant influence on the deep waters therein.

Figure 9.22 shows the average currents at 10 m for the entire period September 10-18. The influence of the storm is clearly dominant (compare Fig. 9.20). The models suggest a more organized flow in the southern basin than is apparent from the observed data. The trend for CCW rotation is clear in all the models. The B Model imitates flow at stations 9 and 10 rather well. Model currents are not as shore-parallel at stations 12 and 16 as are the actual motions. The average motion at station 27 remains northward, while stations 29 and 31 show a flow to the southwest. The models are similarly wrong with respect to these latter three stations. Model flow at station 27 is southward, and model flow at 29 and 31 apparently overestimates the Ekman drift.

The average motion at 60 m for September 10-18 is depicted in Fig. 9.23. The influence of the storm is again dominant (compare Fig. 9.21).

The results from the K Model again show the deficiencies arising from lack of wind data. The currents at station 8, 9, and 10 are best imitated by the B Model, although the S Model also gives reasonable current directions at these locations. The B Model flows at station 20 strongly overestimate the actual amount of northward flow, while the S Model results again seem to be a combination of the observed flows at 60 and 90 m. All the model results suggest the presence of an eastward flow at moderate depth, toward station 20, which then joins with a northward flow that extends from the area of stations 9 and 10 to about the latitude of station 29. This northward flow meanders leftward into the deep northern basin (obeying conservation of potential vorticity) and thereafter becomes disorganized. The S Model imitates the actual directions at station 29, but is unable to resolve the northward flow at 90 m at station 41. The B and K Models respond incorrectly at station 29. The K Model disorganization is probably due largely to the lack of wind data, and the B Model apparently overestimates the surface drift resulting in a wrongly flowing countercurrent at depth.

The B Model, which was used with eight vertical levels, encompasses the 90 m depth in its sixth level. Figure 9.24 shows actual and model-generated currents at 90 m at stations 9, 20, and 41 for September 10-13,

13-16, and 10-18, respectively. The observed flows at stations 9 and 41 are essentially northward, while station 20 recorded an eastward flow, as noted previously. Simulated currents compare better with data at stations 9 and 41 than with the corresponding results from the S Model shown in previous figures. Comparisons at station 20 are problematic due to the complexity of the local flow regime. On the other hand, this complexity may be partially a model artifact and the persistent eastward flow at this station may feed a strong coastal current that is not resolved by the models.

Scalar, spatial averages for September 10-18 for the observed currents, and the S- and B-Model currents, are summarized in Table 9.3. The average, observed currents at moderate depths are seen to be about 40-60% greater during the storm period. Both models tend to overestimate this increase. Average current magnitudes are well imitated during the storm period by the S Model, and are overestimated significantly by the B-Model. Eight-day averages compare better for the surface layer than for the deeper waters.

The lakewide presentations of average currents shown thus far have not spanned more than eight days. Figure 9.25 shows average, observed and S-model-generated currents for September 1-30. There is a remarkable similarity between these monthly-average data at 10 m (Fig. 9.25a) and the eight-day average data at this level (compare Fig. 9.22). The tendency for CCW rotation in the southern basin is still present in the model. The average flow at stations 12 and 16 is again southward. The comparability of the actual and model results does not seem particularly better or worse than for the eight-day case.

The monthly-average flows at 60 and 90 m are shown in Fig. 9.25b. These flows also show a striking similarity to the averages for September 10-18 (compare Fig. 9.23). Model results clearly compare less well with the actual data over this longer time period. Model currents at station 10 are essentially zero, and they are flowing in the wrong direction at stations 20, 29, and 41. (Reasonable results were found for stations 20 and 29 for Sept. 10-18.) Average currents from the B Model for Sept. 1-18 are shown for 60 and 90 m in Fig. 9.26(a) and (b), respectively. (Results at 10 and 15 m show similar comparability to the observations, as do the S-Model results.) The lakewide plots in Fig. 9.26 (machine drawn) show current vectors wherever

Table 9.3. Lakewide Scalar Average of Observed and Model-generated Currents (cm/sec) for Sept. 10-18, 1963

Date	Depth(m)	Observed	S Model	B Model
Sept. 10-13	10 & 15	5.4	4.8	3.9
	60	2.5	1.2	1.5
	90	1.8	-	1.7
Sept. 13-16	10 & 15	5.6	6.6	9.5
	60	3.5	3.1	6.1
	90	2.9	-	3.8
Sept. 10-18	10 & 15	3.9	4.3	4.3
	60	2.5	1.7	7.8
	90	2.4	-	3.8

Fig. 9.26. Average B-Model Currents at 60 m, 90 m for Sept. 1-18, 1963

the magnitude of flow exceeds 0.4 cm/sec. These current vectors may be compared with the observed vectors shown in Fig. 9.25, which are for Sept. 1-30. Observed currents are reasonably well imitated by the B-Model average currents for Sept. 1-18 at stations 9, 10, 20, and 41, although B-Model currents are much too small at station 29. The increased vertical resolution offered by the B Model may be an important factor for long-term simulation of deep flows under stratified conditions.

The last two figures suggest that processes affecting the relatively long-term lake dynamics at 60 and 90 m are apparently not well represented in the models. These processes might include horizontal temperature gradients somewhat above these depths, incorrect model time constants for post-storm recovery of the thermocline in various parts of the lake, and also the familiar statement about incorrect vertical diffusion of momentum.

The monthly-average results also suggest a CCW motion in the southern basin. Figure 9.27 offers a reason for this behavior of the long-term average. In this figure, the cumulative wind impulse for the daily, vector-averaged N-S and E-W components of all the overwater wind data is plotted vs. the day of the month. Three storm events can be noted at about September 13, 20, and 28. These storms contribute relatively large southward impulses, which are not counteracted by the prevailing westerlies (positive slope in the graph of the N-S component). Note also that a westward impulse accompanies each southward impulse, as expected. Thus, the average winds for September, 1963 added a cumulative impulse to the lake that clearly favored the south and west. The CCW motion is then as expected very likely assuming linearity, based on previous work that claims a CW sense of rotation under the prevailing westerlies (Ayers et al., 1958; Bellaire, 1964).

The final figure for comparison of the characteristics of observed and model-generated currents summarizes all of the calculations of Fourier norms (see Sec. 5) for September 1-30. In Fig. 9.28, the norms of observed and S-Model data for all available stations and depths are plotted vs. station latitude. Careful study of this information reveals the following facts: (1) the ratio of the norms of the observations with respect to zero at 10 and 15 m compared to 60 and 90 m is typically about 2; (2) the norms vary over a smaller range at 60 and 90 m than at 10 and 15 m; (3) currents in the deep northern basin are somewhat less variable than in the deep

Fig. 9.27. Cumulative Wind Impulse for September, 1963

Fig. 9.28. Fourier Norms for Observed and S-Model Currents for September, 1963

southern basin; (4) the fact that the norm of the observed and computed data is greater than the norm of the observed data and zero, given that the latter norm is greater than the norm of zero and the computed data, implies that the computed currents are in the wrong direction a certain percentage of the time; thus, stations 7, 8, 27, and 29 have model-current directions that are more correct than for other stations; and (5) model-current directions are therefore somewhat better at moderate depths than near the surface.

10. CONCLUSIONS

a. Relevance of the Circulation Models to Large-scale Transport Over Days and Weeks

Diverse facts pertaining to the behavior of the circulation models under a variety of circumstances were described in the previous section. These facts can be fruitfully gathered in order to make statements about the specific utility of present models for the study of transport phenomena in the Great Lakes. Some of these statements are quite general, others are weather-specific or are clearly applicable for Lake Michigan only. The following statements are subdivided accordingly.

1. Selection of an appropriate mean(s) for comparison of observed and model-generated currents is a difficult task. Future model improvement may be dependent on making a judicious selection.

2. Barotropic circulation patterns produced by the models were found to correlate reasonably well with currents inferred from satellite imagery and with certain features of the sediment pattern in the lake. An organized lakewide circulation pattern is more likely to persist under homogeneous conditions, which supports this correlation with the sediments. In fact, redistribution of sediments may be most dependent on storm events during non-stratified conditions. Strong currents (> 50 cm/sec) have been observed in the deep waters of the lakes during winter storms (Sloss, 1976). Currents inferred from satellite imagery were helpful in this work.

3. A variety of stratified circulation patterns were generated by adjusting model parameters, such as the coefficients for vertical and horizontal diffusion of momentum and heat, initial temperature fields, layer depths, time steps, and bulk transfer coefficients in surface stress formulae. Realistic adjustment of these parameters can have significant effects on the pointwise comparability of models with observations. Comparison with observed currents at fixed locations also was shown to be strongly influenced by our limited knowledge of the wind and temperature fields. That is, (1) the model results were shown to be highly dependent on the method chosen for spatial interpolation of wind data; and (2) thermally-induced currents could account for a significant fraction of the daily-average currents.

4. Various sorts of averaging of the computed data are necessary in order to obtain meaningful comparisons. One- to several-day averages of the data, for example, are able to suppress some of the influences cited in item 3. Progressive-vector diagrams based on such averages show that the S and B Models can simulate several-day particle displacements that are often of the correct order of magnitude and often within $\pm 45^\circ$ of the correct resultant direction. The models are better able to imitate such displacements at moderate depth than near the surface.

5. Comparability with observations was found to be best during periods of high surface stress. In fact, the utility of one- to several-day averages appears to be largely confined to such periods, and very likely is limited to approximately a week during stratified conditions. Thereafter, thermal effects (such as unknown gradients in the temperature field and incorrect recovery of model thermoclines) cause comparisons to go awry.

6. Nearshore comparability was understandably limited in this study by the coarse grid spacing of the models as applied. Previous work using smaller grid spacing (Simons, 1974, 1975; Bennett, 1975) showed, however, that nearshore lake physics are not well represented in these models.

7. The utility of the barotropic patterns cited in item 2 seems to be a forte of the multilevel models, also. Circulation patterns at various depths arising from several-day averages of the model output were found to imitate similar averages of the observed data. Comparisons were best understood when results from several model grid points in the neighborhood of a given data station were presented. Model results suggest that some data stations were located in regions of complex flow. Model currents are more comparable with observed flows when consideration is given to effects such as strong local divergence. For ecological purposes, the clear implication is that the models are current-pattern predictors, not specific-current predictors.

The following numbered statements refer to specific features of the circulation of Lake Michigan.

8. There are two important counterclockwise gyres in the barotropic circulation pattern arising under prevailing westerlies; these are well correlated with the thickest parts of the Waukegan sediment member (see

Figs. 9.1 and 9.3). The larger of these two gyres is located in the east-central part of the southern basin and stretches approximately from South Haven to Muskegon, Michigan. The smaller gyre is somewhat nearer to the eastern shore than the above, and stretches approximately from Michigan City, Indiana, to South Haven, Michigan. A smaller, clockwise gyre, nearshore between Holland and Muskegon, Michigan, was found to correlate with currents inferred from satellite imagery (see Fig. 9.4).

9. The general sense of rotation for non-stratified conditions under prevailing westerlies appears to be counterclockwise (see Fig. 9.1). The largest transports occur along the eastern shore. Integrated motion in the northern basin is essentially to the south. There are apparently at least two convergence zones along the western shore. These are located near Highland Park, Illinois, and in the Milwaukee-Racine, Wisconsin, area. Model results also suggest a rather strong convergence on the eastern shore near Big Sable Point, north of Ludington, Michigan. Current speeds cannot be inferred reliably from one-layer, vertically-integrated flow. The average speed of all particles that comprise a vertical, unit column can be calculated, however. Average northward speeds along the eastern side of the southern basin are thus found to be about 5 cm/sec. A demarcation seems to exist between the flow regimes of the northern and southern basins along a line connecting Sheboygan, Wisconsin, and Little Sable Point, Michigan. It appears that at least some of the northward-flowing water mass from the southern basin returns southward without reaching the deep waters of the northern basin. Travel time for a water parcel moving at 5 cm/sec from the southern end of the lake along the eastern shore to this proposed demarcation would be about one-and-a-half months (other things being equal).

10. Data and model comparisons for stratified flow were made primarily during a storm period and/or for a month during which the cumulative wind impulse was dominated by northeasterly winds. The remarks to be made subsequently apply for the above conditions, and extension to behavior under other wind regimes must be made cautiously, if at all. Several-day averages of data at the 10 and 15 m depths show rather changeable, disorganized patterns (see Figs. 9.18, 9.20, and 9.22). The probable sense of rotation of the surface waters of the southern basin is counterclockwise under northeasterly winds. Net motion of the surface waters across the broad, midlake

sill appears to be north-northwest over several-day periods including a storm(s). Several-day particle displacements are $O(10^4)$ m in the southern basin. Daily average speeds range from about 5 to 10 cm/sec.

Several-day averages of data at 60 and 90 m depths show a somewhat less-changeable pattern than was found for the surface waters (see Figs. 9.19, 9.21, 9.23, and 9.24). Synthesis of observed and model-generated currents yields the following qualitative description of flow for the deep waters averaged over a storm period: the sense of rotation of the southern basin is essentially counterclockwise; flow at 60 m in the western and central parts of the basin is southerly; flow at 90 m in the central part and at 60 m in the eastern part is northerly; this northerly flow is joined by a relatively strong easterly flow in the deep lake west of Muskegon, Michigan; this convergence of flows appears to feed a large upwelling zone in this area; northerly flow persists along the eastern shore as far north as Frankfort, Michigan, in the eastern part of the northern basin; this flow is supplemented by a northeasterly flow in the central part of the northern basin; a broader, less intense, southward flow prevails in the western part of this basin and seems to feed the eastward flow in the central part of the lake and the counterclockwise motion in the southern basin. Northward, particle displacements are $O(10^4)$ m over eight days in the eastern part of the southern basin, and somewhat smaller in the western half of the basin.

b. Future Research

State-of-the-art numerical circulation models for the Great Lakes have limited utility for current prediction and for simulating average lake behavior over several weeks during stratified periods. The first of these weaknesses is clearly dependent on the type of predictions required. Comparability with observations at fixed locations, even for variations having periods as large as twelve hours, is likely to remain as a significant problem. This problem exists for the reasons stressed in previous sections, and probably because of the fact that nature does not operate according to the deterministic principles built into these models.

Simulation of average behavior over days and weeks probably can be improved. This calls for utilization of observed temperature data to correct/update model temperature fields periodically. (Simons (1976) has

recently reported on such a procedure.) This improved simulative capability also requires use of a model that can operate with very little friction, such as the B Model, so as not to erode the vertical thermal structure. Vertical diffusion of heat and momentum also can be (and has been) formulated in terms of Richardson numbers. The original version of the B Model included these formulations, but apparently did not give significantly better results with regard to massive transport of warm water around the Lake Ontario basin (Bennett, 1975). Results from the present study showed that kinetic energy accumulation in the deep waters is reasonably well imitated by the models during storm periods. A more sophisticated means for representing vertical diffusion and diffusivity may be required, however, during periods of minimal surface stress.

The utility of the S, B, and K Models for non-stratified conditions could not be adequately tested in this study. The necessary observational data were not available for these lake conditions. The cooperative effort amongst NOAA, the University of Wisconsin, and Argonne, presently being conducted, should provide new current and temperature data at several depths during homogeneous and stratified lake conditions.

These new data can be used, as described above, for further experiments and improvement of the models. Synthesis of such additional model-generated and observed currents can provide further insights into lake dynamics under various conditions. In addition, time-series correlations between current-meter observations and between current and wind observations are being made presently from the FWPCA data. These correlations are being used to test the validity of inferring circulation patterns from a few, rather widely-scattered data stations. For example, these calculations also may provide estimates for the influence of winds in the northern part of the lake on the motion in the southern part. Further, the correlations may establish time constants for typical motions at various levels. Results from the circulation models can be subjected to similar analysis. Comparison of model and lake dynamics, as exposed by the correlation calculations, may prove to be a viable means for useful adjustment of model parameters.

Present lake-circulation models are useful research tools. However, the quote offered at the outset of this report seems apropos, and is given again without counterpoint: "The purpose of computing is insight, not numbers," according to R. W. Hamming.

APPENDIX A

FINITE DIFFERENCE FORMULATIONS FOR THE NUMERICAL MODELS

APPENDIX A. FINITE DIFFERENCE FORMULATIONS FOR THE NUMERICAL MODELS

The following notation is introduced to facilitate presentation of the numerical methods (ϕ represents an arbitrary dependent variable):

$$\phi_{jk}^n \equiv \phi(x_j, y_k, t_n) \quad , \quad x_j = j\Delta x, \quad y_k = k\Delta y, \quad t_n = n\Delta t$$

$$\begin{aligned} \bar{\phi}_{jk} \equiv & 1/4(\phi_{j-1/2, k-1/2} + \phi_{j-1/2, k+1/2} \\ & + \phi_{j+1/2, k+1/2} + \phi_{j+1/2, k-1/2}) \end{aligned}$$

$$\bar{\phi}_{j,k} \equiv 1/2(\phi_{j-1/2, k} + \phi_{j+1/2, k}) \quad , \quad \delta_x \phi \equiv 1/\Delta x(\phi_{j+1/2, k} - \phi_{j-1/2, k})$$

$$\delta_{2x} \phi \equiv 1/2\Delta x(\phi_{j+1, k} - \phi_{j-1, k}) \quad , \quad \delta_{2t} \phi \equiv 1/2\Delta t(\phi^{n+1} - \phi^{n-1})$$

$$\delta_{1/2}^t \phi \equiv 2/\Delta t(\phi^{n+1/2} - \phi^n) \quad , \quad \phi_{at n}^+ \equiv \phi^{n+1/2} \quad , \quad \phi_{at n}^- \equiv \phi^{n-1}$$

Definitions for all dependent variables in the equations below are given in sections 3a.-3d.

The ADI method used in this study may be written:

$$\delta_{1/2}^t U - f\bar{V} + gh_U \delta_x \zeta^+ - \tau_w^+(x) + C_D' U^2/h_U = 0, \quad \text{at } n, j+1/2, k \quad (A.1)$$

$$\delta_{1/2}^t \zeta + \delta_x U^+ + \delta_y V = 0, \quad \text{at } n, j, k \quad (A.2)$$

$$\delta_{1/2}^t V + f\bar{U} + gh_V \delta_y \zeta^+ - \tau_w^+(y) + C_D' V^2/h_V = 0, \quad \text{at } n+1/2, j, k+1/2 \quad (A.3)$$

$$\delta_{1/2}^t \zeta + \delta_x U + \delta_y V^+ = 0, \quad \text{at } n+1/2, j, k \quad (A.4)$$

A space-staggered grid was used. The U transport and the depths for U points, h_U , were described at integer and one-half values of j and integer

values of k . The V transport and the depths for V points, h_V , were described at integer and one-half values of k and integer values of j . The surface height, ζ , is specified at integer values of j and k .

The time-and-space staggered, explicit method used in the K Model may be summarized:

$$\delta_t U - f\bar{V} + g^{\overline{x_D}^+} \delta_x \zeta^+ - 1/\rho(\tau_w(x) - \tau_z(u)) = 0, \text{ at } n, j+1/2, k \quad (\text{A.5})$$

$$\delta_t V + f\bar{U} + g^{\overline{y_D}^+} \delta_y \zeta^+ - 1/\rho(\tau_w(y) - \tau_z(V)) = 0, \text{ at } n, j, k+1/2 \quad (\text{A.6})$$

$$\delta_t \zeta + \delta_x U + \delta_y V - \delta_t \zeta_\ell = 0, \text{ at } n, j, k \quad (\text{A.7})$$

$$\delta_t U_\ell - f\bar{V}_\ell + g^{\overline{x_D}^+} \delta_x \zeta_\ell^+ + \frac{(\rho_\ell - \rho)}{\rho_\ell} \delta_x \zeta_\ell^+ \quad (\text{A.8})$$

$$- 1/\rho_\ell (\tau_z(U) - \tau_B(U)) = 0, \text{ at } n, j+1/2, k$$

$$\delta_t V_\ell + f\bar{U}_\ell + g^{\overline{y_D}^+} \delta_y \zeta_\ell^+ + \frac{(\rho_\ell - \rho)}{\rho_\ell} \delta_y \zeta_\ell^+ \quad (\text{A.9})$$

$$- 1/\rho_\ell (\tau_z(V) - \tau_B(V)) = 0, \text{ at } n, j, k+1/2$$

$$\delta_t \zeta_\ell + \delta_x U_\ell + \delta_y V_\ell = 0, \text{ at } n, j, k \quad (\text{A.10})$$

The interface stress, τ_z , is given by:

$$\tau_z(U) = 2A_z \left(\frac{U}{x_{\overline{D}}} - \frac{U_\ell}{x_{\overline{D}_\ell}} \right) / \left(x_{\overline{D}} + x_{\overline{D}_\ell} \right) \quad (\text{A.11})$$

and similarly for $\tau_z(V)$. The formulation of $\vec{\tau}_w$ and $\vec{\tau}_B$ was the same as in the ADI model. The spatial locations of the variables was also the same as in the ADI model, except that $D(D_\ell)$ was given at $\zeta(\zeta_\ell)$ points.

The four layer, S Model is described in essence by the difference equations:

$$\delta_{2t} U - f V^* + g [D(\delta_{2x} \zeta + \delta_{2x} \xi) + S \delta_{2x} Z] \quad (A.12)$$

$$+ \delta_{2x} [A_H D \delta_{2x} (U/D)]^- - \chi_{\ell-1}^-(U) + \chi_{\ell}^-(U) = 0 \text{ at } n, j, k$$

$$\delta_{2t} V + f U^* + g [D(\delta_{2y} \zeta + \delta_{2y} \xi) + S \delta_{2y} Z] \quad (A.13)$$

$$+ \delta_{2y} [A_H D \delta_{2y} (V/D)]^- - \chi_{\ell-1}^-(V) + \chi_{\ell}^-(V) = 0 \text{ at } n, j, k$$

$$\delta_{2t} D^+ + \delta_{2x} U + \delta_{2y} V + w_{\ell-1} - w_{\ell} = 0 \text{ at } n, j, k \quad (A.14)$$

$$\delta_{2t} T + \delta_{2x} \left[U^+ \overline{x(T/D)} \right] + \delta_{2y} \left[V^+ \overline{y(T/D)} \right] \quad (A.15)$$

$$+ 1/2 \left[w_{\ell-1}^+ \overline{z(T/D)}_{\ell-1/2} - w_{\ell}^+ \overline{z(T/D)}_{\ell+1/2} \right] - \delta_x [A_{\theta} D \delta_x (T/D)]^-$$

$$- \delta_y [A_{\theta} D \delta_y (T/D)]^- - \gamma_{\ell-1} + \gamma_{\ell} = 0 \text{ at } n, j, k$$

where level subscripts have been omitted unless they differ from $\ell-1/2$. The Coriolis terms are Euler-implicit, i.e., $V^* \equiv 1/2[V^{n+1} + V^{n-1}]$, and similarly for U^* . The spatial locations of the variables was the same as in the K Model, with these exceptions (additions): (1) a conjugate lattice was used which interchanges the locations of U and V on alternate time steps, and (2) T , D , and w were defined at ζ points. The first term in (3.25) is zero, except when the barotropic motion is considered, since rigid, permeable levels were chosen for this work. Expressions of the following form were used for interfaces stresses:

$$\chi_{k-1}(U) = A_Z [(U/D)_{k-1/2} - (U/D)_{k+1/2}] / Z_{\bar{D}_{k-1/2}} \quad (\text{A.16})$$

and similarly for $\chi(V)$. The surface and bottom stresses (χ_0 and $\chi_{\max}$, respectively) were formulated as in the K Model. The vertical heat flux γ is formulated in the same manner as vertical momentum flux (Eq. A.16).

The eight level, B Model can be summarized by the following relations:

$$\delta_t u - f\bar{v} = -\delta_x p + \chi(u)_{\ell+1} - \chi(u)_{\ell} \quad \text{at } j+1/2, k, n \quad (\text{A.17})$$

$$\delta_t v + f\bar{u} = -\delta_y p + \chi(v)_{\ell} - \chi(v)_{\ell+1} \quad \text{at } j, k+1/2, n \quad (\text{A.18})$$

$$\begin{aligned} \delta_t (L\psi) = & f/2 [\delta_x (1/h_u) \delta_y \psi - \delta_y (1/h_v) \delta_x \psi] \\ & + \overline{\delta_x [-\delta_y p + \chi(v)_{\ell+1} - \chi(v)_{\ell}]}^Z \\ & - \overline{\delta_y [-\delta_x p + \chi(u)_{\ell+1} - \chi(u)_{\ell}]}^Z \quad \text{at } j+1/2, k+1/2, n \end{aligned} \quad (\text{A.19})$$

$$u^{n+1} = u - \bar{u}^Z - 1/h_u \delta_y \psi^{n+1} \quad \text{at } j+1/2, k \quad (\text{A.20})$$

$$v^{n+1} = v - \bar{v}^Z + 1/h_v \delta_x \psi^{n+1} \quad \text{at } j, k+1/2 \quad (\text{A.21})$$

$$\delta_Z w = -\delta_x u - \delta_y v \quad \text{at } j, k, n+1 \quad (\text{A.22})$$

$$\begin{aligned} \delta_t T + \delta_x (u T^{AB}) + \delta_y (v T^{AB}) \\ - \delta_Z (w T^{AB} - \gamma) = 0 \quad \text{at } j, k, n \end{aligned} \quad (\text{A.23})$$

where level subscripts have been omitted unless they differ from $\ell + 1/2$.

The first estimates for the horizontal velocity, u^* and v^* , are forced to be consistent with the transport stream function in Eqs. A20-21. The vertical average operator is $\overline{(\quad)}^Z \equiv 1/h \sum_{\ell=1}^8 (\quad) h_{\ell}$. T^{AB} signifies the Adams-Bashforth temperature approximation: $3/4(T_{\ell} + T_{\ell+1})^n - 1/4(T_{\ell} + T_{\ell+1})^{n-1}$. Formulation of vertical diffusion, χ , and diffusivity, γ , was the same as for the S Model. Spatial locations of variables was the same as in the ADI model with these exceptions (additions): (1) ψ was defined at integer and one-half values of j and k ; (2) T and p were defined at integer values of j and k ; and (3) w was given at level interfaces. Eq. A.19 is solved for ψ by relaxation.

The vertical velocity field, i.e., the divergent part of the motion is smoothed by adding a bulk viscosity term to the pressure. This term has the form:

$$-AQQ \, f \, \Delta x \, \Delta y (\delta_x u + \delta_y v), \quad (A.24)$$

where AQQ controls the amount of smoothing which is applied.

GLOSSARY

The following, short glossary is included for the reader who is unfamiliar with terms used in computational fluid dynamics and/or in geophysical fluid dynamics. The definitions given are conceptual in nature, and are formulated as applicable to their use in this report. Precise definitions may be found, e.g., in Richtmeyer and Morton (1967) or Roache (1972), Fofonoff (1962) or Neumann and Pierson (1966).

ADAMS-BASHFORTH METHOD. An explicit, one-step, 3-time-level, forward time scheme with second-order accuracy as applied to the advection equation (see Eq. 3.17).

ALTERNATING-DIRECTION-IMPLICIT METHOD. An implicit, time-splitting scheme with second-order accuracy as applied to the linear equations (see Eq. 3.1-3.2) operating on each coordinate direction independently and requiring only the inversion of a tridiagonal matrix for solution.

BAROCLINIC. A field of mass in which isobaric and isopycnal surfaces intersect.

BAROTROPIC. A field of mass in which isobaric and isopycnal surfaces coincide.

CENTERED-SPACE DIFFERENCING. Any finite difference approximation for a spatial derivative found by using values of the given dependent variable located symmetrically around the given location.

COURANT CONDITION. An upper bound for the computational time step, expressing the fact that waves in the numerical grid cannot propagate more rapidly than allowed by the continuum equations.

DISPERSIVE METHOD. A numerical method in which the speed of propagation of a given wave depends on its wavelength.

DISSIPATIVE METHOD. A numerical method in which the amplitude of the solution decays without specific inclusion of frictional terms.

DUFORT-FRANKEL LEAPFROG METHOD. An explicit, one-step scheme with first-order time and second-order space accuracy, which combines leapfrog differences for the advection terms with forward time, centered-space differences for diffusion terms.

EXPLICIT METHOD. A numerical method in which the dependent variable(s) at the advanced time level is(are) given directly in terms of values of the dependent variable(s) at present and/or previous time level(s).

IMPLICIT METHOD. A numerical method in which the dependent variable(s) at the advanced time level is(are) given in terms of values of the dependent variable(s) at the advanced and present and/or previous time level(s).

f PLANE. A domain of definitions for the shallow-water wave equations (e.g., Eqs. 3.1-3.2), consisting of a plane tangent to the surface of the earth at the point where the vertical component of the rotation vector of the earth is calculated (and defined as f).

FORWARD TIME DIFFERENCE. A finite difference approximation for a first time derivative which uses the values of the given dependent variable at the advanced and present time levels.

ISOBARIC SURFACE. A surface on which pressure is constant.

TSOPYCNAL SURFACE. A surface on which density is constant.

TWO-STEP, LAX-WENDROFF METHOD. A two-step, second-order accurate scheme which can be interpreted as a second-order Taylor series expansion in time, and which uses spatial averaging for the given dependent variable at the previous time level in the first step and midpoint leapfrog calculation for the second step.

LEAPFROG METHOD. A second-order accurate scheme for the first time derivative which expresses the new value of the given dependent variable in terms of the value of that dependent variable at the previous time level and other dependent variables at the present time level.

MIDPOINT LEAPFROG METHOD. A second-order accurate scheme which uses leapfrog time differencing and centered-space differencing (for first space derivatives).

ORDER OF ACCURACY. A concept which refers to the magnitude of truncation errors as the spatial step (Δs) and the temporal step (Δt) approach zero; estimates for these errors are found by substituting Taylor series expansions of all dependent variables into a given numerical method, and canceling common terms.

OVER-RELAXATION. An iterative method for solving the Poisson equation, which uses a multiplicative constant called the relaxation factor to optimize the convergence of the dependent variable based on the value of the prognostic variable at the given time level.

RICHARDSON LATTICE. A spatial disposition of dependent variables in which one velocity component is given on opposite sides of the basic grid element and the other velocity component is given on the remaining two sides; other dependent variables are prescribed at the center and/or corners of the basic grid element depending upon the positions required by the numerical method being used.

SPACE-STAGGERED SCHEME. A method which uses one or more Richardson lattices to locate its variables.

TIME-STAGGERED METHOD. Any numerical scheme in which all dependent variables are never present at the same time level simultaneously; a scheme of this type is sometimes useful for solving coupled equations (see e.g., Eqs. 3.5-3.6).

TOPOGRAPHIC CURL EFFECT. The manner in which the pressure gradient and the depth gradient interact to change the vorticity of the flow.

ACKNOWLEDGMENTS

The author is especially grateful to Ms. M. J. Berger, scientific programmer, whose insight and enthusiasm contributed significantly to this work. Dr. K. D. Saunders played a key role in helping to shape the scope and outline of this research, and also helped to develop many of the methods for data and model comparisons.

Dr. J. R. Bennett (Massachusetts Institute of Technology) and Dr. A. W. Green (The University of Michigan) are thanked for reviewing this manuscript and for offering constructive criticism.

Dr. P. W. Harrison, Mr. D. L. McCown, and Dr. R. A. Paddock are thanked for their numerous suggestions and daily interaction.

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