MASTER

THEORETICAL HIGH ENERGY PHYSICS

Progress Report

for the Period

May 1, 1976 - April 30, 1977

*

Principal Investigators:

T. D. Lee

R. Serber

Department of Physics
Columbia University
New York, N. Y. 10027

May 1977

Prepared for

The U. S. Energy Research and Development Administration
under Contract. EY-76-C-02-2271.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

EB

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Notice

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product or process disclosed or represents that its use would not infringe privately owned rights.

PUBLICATION LIST

EY-76-C-02-2271

1. C. Bernard and E. J. Weinberg, "The Interpretation of Pseudoparticles in Physical Gauges" (to be published in Physical Review D).
2. C. Bernard and A. Duncan, "Regularization and Renormalization of Quantum Field Theory in Curved Space-Time" (submitted to Annals of Physics (N.Y.)).
3. N. H. Christ, A. H. Guth and E. J. Weinberg, "Canonical Formalism for Gauge Theories with Applications to Monopoles", Nuclear Physics B114, 61 (1976).
4. G. Feinberg and T. D. Lee, "Unconfined, Heavy Fermion Model of J/ψ Particles", Physical Review D13, 3071 (1976).
5. G. Feinberg, "Parity-Violating Electromagnetic Interactions of Nuclei" (to be published in the Annals of the New York Academy of Sciences).
6. J. Finkelstein and J. Koplik, "f-Dominance and Final-State Counting in Multi-peripheral Cluster Models", Physical Review D14, 1437 (1976).
7. J. Finkelstein and J. Koplik, "Energy Dependence of the ρ Regge Trajectory", Physical Review D14, 1467 (1976).
8. J. Finkelstein and S. Pinsky, "Regge Spectroscopy of Charmed Mesons", Physical Review D15, 360 (1977).
9. J. Finkelstein and S. F. Tuan, "Regge Spectroscopy of Charmed Baryons", Physical Review D15, 902 (1977).
10. J. Finkelstein and Y. Eylon, "J-Dependence of the Twist Parameter in Dual Unitarity", Physics Letters 66B, 154 (1977).
11. R. Friedberg, T. D. Lee and A. Sirlin, "Class of Scalar-Field Soliton Solutions in Three Space Dimensions", Physical Review D13, 2739 (1976).
12. R. Friedberg, T. D. Lee and A. Sirlin, "Gauge-Field Nontopological Solitons in Three Space Dimensions (I)", Nuclear Physics B115, 1 (1976).
13. R. Friedberg, T. D. Lee and A. Sirlin, "Gauge-Field Nontopological Solitons in Three Space Dimensions (II)", Nuclear Physics B115, 32 (1976).
14. R. Friedberg and T. D. Lee, "Fermion-Field Nontopological Solitons I", Physical Review D15, 1694 (1977).

15. R. Friedberg and T. D. Lee, "Fermion-Field Nontopological Solitons II. Models for Hadrons" (to be published in *Physical Review D*).
16. R. Friedberg, "Nontopological Solitons", to be published in the proceedings of the *Orbis Scientiae*, Coral Gables, 1977.
17. R. Gonsalves, "Quantizing Solitons without Using a Box", *Physical Review D*14, 2610 (1976).
18. P. Gujrali, "High-Temperature Series Expansions for Isotropic Heisenberg Model in One and Two Dimensions I" (to be published in *Physical Review D*).
19. A. Guth, M. Goldberger and D. Soper, "Large Q^2 Behavior of the Pion Electromagnetic Form Factor", *Physical Review D*14, 1117 (1976).
20. A. Guth, M. Goldberger and D. Soper, "Light-Cone Behavior of the Pion Bethe-Salpeter Wave Function in the Ladder Model", *Physical Review D*14, 2633 (1976).
21. F. Krausz, "Mass Singularities in Yang-Mills Theories", *Physics Letters* 66B, 251 (1977).
22. F. Krausz, "Cancellations of Mass Singularities in Yang-Mills Theories" (submitted to *Nuclear Physics*).
23. T. D. Lee, "Nontopological Solitons", to be published in the proceedings of the *Symposium on Frontier Problems in High Energy Physics*, Pisa, 1976.
24. T. D. Lee and G. C. Wick, "Abnormal Nuclear States", to be published in *Mesons in Nuclei*, edited by M. Rho and D. Wilkinson (North-Holland).
25. T. D. Lee, "Nontopological Solitons and Applications to Hadrons" (to be published in *Comments on Nuclear and Particle Physics*).
26. A. H. Mueller, "A Geometrical Interpretation of the Reggeon and Pomeron Intercepts in the String Model", *Nuclear Physics B*118, 253 (1977).
27. A. H. Mueller, "Lectures on Hadron-Nucleus Collisions at High Energies", to be published in the proceedings of the *Summer Institute for Theoretical Physics*, Bielefeld, 1976.
28. A. Sen, "On a Simplified Polyakov Model", *Physical Review D*13, 2335 (1976).
29. R. Serber, "A Simple Nuclear Model", *C*14, 718 (1976).
30. A. Sinha, "SU(3) Magnetic Monopoles", *Physical Review D*14, 2016 (1976).

31. A. Sinha, "Lorentz Transformations for Gauge Theory Monopole Solutions", (submitted to Physical Review D).
32. E. J. Weinberg and A. H. Guth, "Nonexistence of Spherically Symmetric Monopoles with Multiple Magnetic Charge", Physical Review D 14, 1660 (1976).
33. G. C. Wick, "Three Lectures on Solitons", to be published in the proceedings of the Erice Summer School, 1976.

Dissertations

- A. Gujrati, "High-Temperature Series Expansions for Isotropic Heisenberg Model in One and Two Dimensions", 1977.
- F. Krausz, "Mass Singularities in Yang-Mills Theories", 1977.

Percentages of Time Devoted to Project by Investigators

November 1, 1976 - October 31, 1977

<u>Name</u>	<u>% of Time</u>	<u>Period</u>
Professors N. H. Christ	100	3 summer months
	~ 12	9 academic months
G. Feinberg	100	3 summer months
H. M. Foley	100	1 summer month
R. Friedberg	100	2 summer months
T. D. Lee	100	3 summer months
	33	2½ academic months
	13	2 academic months
A. H. Mueller	100	3 summer months
	15	9 academic months
R. Serber	100	3 summer months
G. C. Wick	100	3 summer months
Assistant Professors J. Finkelstein	100	3 summer months
	50	9 academic months
E. J. Weinberg	100	3 summer months
	50	9 academic months
Research Associates C. Bernard	100	12 months
A. Duncan	100	2 months
A. H. Guth	100	10 months

Note: During the academic year, the average teaching load for our faculty members is one course. A considerable amount of research (in addition to that specified above) is performed by the several investigators during those nine months.

During the past year, Dr. Bernard has worked on two distinct problems of current interest. The first of these was the study of classical solutions to the Yang-Mills field equations in Euclidean space, the so-called "instantons", or pseudoparticles. This work has led to a paper with Erick Weinberg, "The Interpretation of Pseudoparticles in Physical Gauges", to be published in Physical Review D. The paper shows how the physical effects of instantons can be understood in gauges like axial gauge without the necessity of introducing multiple vacua. For the past few months Dr. Bernard has worked with Erick Weinberg, Norman Christ and Alan Guth on understanding and characterizing pseudoparticle solutions with topological number greater than one.

The second problem under investigation is the study of field theories in the presence of a background metric. Together with Anthony Duncan, of the Institute for Advanced Study, Dr. Bernard has written a paper, "The Regularization and Renormalization of Quantum Field Theory in Curved Space-time", which they have submitted to Annals of Physics (N.Y.). The paper describes how Pauli-Villars regulators and counterterms can be used to render the energy-momentum tensor of a two-dimensional scalar theory finite in perturbation theory. The trace anomaly of the tensor is then calculated and found to be non-vanishing. Finally, an exactly soluble model is presented and the results are shown to agree with the perturbation theory answers. This work helps to clear up a controversy about the existence of the trace anomaly, and may also have implications for the study of particle radiation from black holes.

During the past year Professor Christ's research has been directed at the general goal of obtaining a fundamental theory of hadrons with particular emphasis on the possibilities of the Yang-Mills theory. The topics studied include: i) The interaction of classical Yang-Mills charges with classical and quantum Yang-Mills fields, in particular the instability of large classical charges pointed out by Mandula, ii) Some general properties of a quantum mechanics based on quaternions or octonians as a possible way of introducing "quark" variables, iii) The properties and quantum-mechanical implications of classical, Euclidean, Yang-Mills solutions. This last topic is being actively pursued at present, in collaboration with Claude Bernard, Alan Guth and Erick Weinberg—some progress having been made on the possible application of mathematical tools developed by Atiyah and Ward to the explicit construction of certain "multi-instanton" Yang-Mills solutions.

In addition to the above investigation, Professor Christ has directed the work of two students and one CNRS Fellow from Paris. The most senior student, Mr. Amarendra Sinha, has completed his study of SU_3 Yang-Mills soliton solutions (published in Physical Review D), and turned his attention to an extension of the classical mechanics of Yang-Mills solitons recently developed by Christ, Guth and Weinberg (Nuclear Physics). He has now succeeded in generalizing the original discussion of angular momentum to all the generators of the Poincaré group—showing that with proper care, these generators can be realized by Poisson brackets obeying the required commutation

relations. This work has been written up and submitted for publication in the Physical Review.

A second student, Mr. Emil Mottola, has been investigating by direct calculation the axial Ward identity for Euclidean amplitudes in the one-instanton sector of an SU_2 Yang-Mills theory with fermions. At zero momentum transfer this Ward identity is readily seen to be valid, but away from $q^2 = 0$ the situation appears to be quite interesting—requiring both a careful treatment of normal ordered products and proper accounting of the zero-frequency fermion modes.

Finally, Dr. Olivier Babelon, a CNRS Fellow from Paris, has studied the contribution of the zero-frequency modes to transition amplitudes involving one-dimensional solitons. Developing an idea of Professor Lee, he has been able to show that such zero-frequency modes do not contribute to Feynman amplitudes calculated in the presence of a soliton provided there is no net momentum transfer to the soliton. This work is currently being prepared for publication.

Professor Feinberg has worked principally in two areas:

a. He has written a paper on the parity non-conserving electromagnetic interactions of nuclei. These interactions are classified according to their oddness or evenness under time reversal, which determines whether they can originate from ordinary weak interactions or from time reversal violating weak interactions. A discussion of the prospects for measuring such nuclear properties is included in the paper, which is to be published in the Annals of the New York Academy of Sciences.

b. He has continued to work on the spectrum and radiative decays of charmonium. An analysis was carried out of a suggestion by H. Harari that one of the observed charmonium states should be identified as the 1D_2 state. He has concluded that the pattern of decays to and from this state that is observed is in strong disagreement with this suggestion. This work will soon be published as part of a more detailed study of charmonium radiative decays.

(1) Professor Finkelstein has investigated several questions involving the masses of the newly-discovered charmed particles. Even though the $SU(4)$ symmetry that presumably relates these particles to more ordinary ones is quite badly broken, it may still be possible to use some of the concepts familiar from studies of the more ordinary particles (such as duality, Regge Behavior, and exchange degeneracy) to analyse the spectra of the new ones. In particular, the use of the Ademollo-Veneziano-Weinberg relation together with the measured values of the masses of the lightest charmed mesons permits the prediction of the masses and spin assignments of more highly excited charmed mesons. This work has been reported in two papers in Physical Review: one an analysis of the charmed meson spectrum, done in collaboration with Professor S. Pinsky of Ohio State University, the other an analysis of the charmed baryon spectrum, done in collaboration with Professor S. F. Tuan of the University of Hawaii.

(2) Together with Dr. Y. Eylon of the Lawrence Berkeley Laboratory, Professor Finkelstein has been studying the effects of terms in the dual topological expansion beyond those of the planar approximation. They have realized that the twist parameter which governs the breaking of exchange degeneracy, Zweig-rule violation, exotic exchange, and the relative production of vector and tensor mesons, is J -dependent, and have constructed a simple model for this dependence which reconciles the smallness of exotic exchange with the large ratio of vector to tensor production, and which successfully predicts the sign and magnitude of the splitting of the ρ and A_2 trajectory intercepts. This work was reported in a paper in Physics Letters.

(3) Also in collaboration with Dr. Eylon, Professor Finkelstein has been able to formulate the rules for calculating quark diagrams for processes involving baryons. This involves an understanding of the relation between the six possible twists of a baryon line, and a knowledge of how to sum quark diagrams to obtain a physical amplitude. A report of this work is in preparation.

Progress Report

Professor Henry M. Foley

The formulation of the general theory of short-range atomic perturbations, e.g., fine structure, hyperfine structure, other nuclear interactions, in terms of the asymptotic behavior of the electron wave function, i.e., "quantum defect" theory, is proceeding. The point is to include many electron effects into what appears finally as a single electron formula. Thus the many "closed channels" of configurations with which the given perturbation interacts are to be included as an "effective potential" or as an effective one particle operator. The project should permit a scaling out of most of the energy and Z-dependence of these perturbations. The single particle work of Fano, Seaton, et al. serves as models. Professor Foley's graduate student, Y. Lee, is engaged on this problem.

The theory of "cascade" in exotic H atoms (with muons, π^- , etc.), including energy transfers to the nucleons, is virtually completed. Scaling relations for the dependence on mass and input energy of the cascade rates at various excited levels are being worked out.

Professor Friedberg and Professor Lee have continued their work on solitons. They have studied the quasiclassical approximation for solutions of quantum field theories containing a Fermion interacting with a scalar field, and they have concluded that this approximation is probably fairly good for strongly bound relativistic states even with only two or three Fermions. They have then applied this approximation to an extended field model of hadrons, in the manner of the SLAC-bag work but with different parameters which can be varied over a wide range. They have discovered a variety of solutions, ranging in form from the MIT bag to a SLAC-like bag. As is already known, the agreement with empirical static properties is reasonably good.

Professor Friedberg and his student, Brian Coffey, have been studying the spontaneous decay from complete inversion of a small number of atoms. They are now completing a paper which treats the interplay of the Coulomb interaction at distances $\ll \lambda$ with intrinsic detuning among the atoms. At the same time they have conducted preliminary investigations concerning a small number of atoms (> 2) separated by several wavelengths.

During the past year Dr. Guth has, in collaboration with Erick J. Weinberg, investigated the possibility of generalizing the magnetic monopole solutions discovered by G. 't Hooft. In particular, they have examined the possibility of monopoles of multiple magnetic charge, within the same $SU(2)$ gauge theory as the original monopoles. This research led to a proof of the non-existence of multiply charged monopoles which are spherically symmetric. ("Spherical symmetry" was defined in the following sense: a configuration is considered to be spherically symmetric if the effects of a rotation amount to no more than a gauge transformation.)

With Norman H. Christ and E. J. Weinberg, Dr. Guth has also completed the development of a canonical formalism for gauge theories. The formalism was developed primarily for the purpose of treating configurations such as the 't Hooft monopole which are not amenable to the standard canonical treatments in either the unitarity or Coulomb gauges. The formalism is based on axial gauge, within which the monopole can be expressed without introducing singularities. It is found that the boundary conditions imposed on the gauge fields are rather delicate, and naive boundary conditions are found to lead to a failure of the Jacobi identity for the generators of the rotation group. With a more complicated choice of boundary conditions, it can be shown that this problem is remedied.

Dr. Guth has also continued to work with Davison E. Soper of Princeton University. They have been working on an extension of their previous derivation of the conformal

invariant operator product expansion of two spinor fields. By generalizing the method developed by Bonora, Sartori and Tonin, they have produced a relatively simple procedure for constructing the most general possible conformal invariant operator product expansion for operators of arbitrary spin. They have also settled several technical questions about the validity of this method which were not addressed by the original authors. This work is essentially completed, but has not yet been written up for publication.

Currently, Dr. Guth is working with N. H. Christ, E. J. Weinberg and Claude Bernard in a study of the "instanton" (four-dimensional pseudoparticle) solutions to gauge-invariant field theories. They are trying to learn more about the nature and the implications of the configurations involving many "instantons".

During this past year Professor Lee has worked in the following areas:

1. Solitons

The study of nontopological soliton solutions that began with the Physics Reports paper [T. D. Lee, Physics Reports 23C, 254 (1976)] has been extended to all renormalizable local field theories in four space-time dimensions [R. Friedberg, T. D. Lee and A. Sirlin, Physical Review D13, 2739 (1976), Nuclear Physics B115, 1, 32 (1976); T. D. Lee, to be published in the proceedings of the Symposium on Frontier Problems in High Energy Physics, Pisa, 1976; R. Friedberg and T. D. Lee, Physical Review D15, 1694 (1977), and Columbia preprint CO-2271-89 (to be published in Physical Review D); T. D. Lee, Columbia preprint CU-TP-102 (to be published in Comments on Nuclear and Particle Physics)]. It is found that except for quantum electrodynamics and the related theory of massive photon quantum electrodynamics, all other renormalizable theories have soliton solutions. Under suitable conditions, these solitons are the lowest energy solutions, not the usual plane wave solutions. This fact changes quite drastically our understanding of bound states. The soliton description of a bound state is quite different from the usual Bethe-Salpeter description.

2. Application to Hadrons

In a recent paper, Professor Lee (together with Professor Friedberg) has examined the possibility and the consequences that in a relativistic local field theory, consisting of color quarks q , scalar gluon σ , color gauge field V_μ

and color Higgs field ϕ , the mass of the soliton solution may be much lower than any mass of the plane wave solutions; i.e., m_q the quark mass, m_g the gluon mass, etc. There appears a rather clean separation between the physics of these low mass solitons and that of the high energy excitations, in the range of m_q and m_g , provided that the parameters $\xi \equiv (\mu/m_q)^2$ and $\eta \equiv \mu/m_g$ are both $\ll 1$, where μ is an overall low energy scale appropriate for the solitons (but the ratio η/ξ is assumed to be $O(1)$, though otherwise arbitrary).

Under very general assumptions, they showed that independently of how many parameters in the original Lagrangian, the mathematical problem of finding the quasi-classical soliton solutions reduces, through scaling, to that of a simple set of two coupled first order differential equations, neither of which contains any explicit free parameters. The resulting solitons exhibit physical characteristics very similar to those of a "gas bubble" (i.e., the soliton) immersed in a "medium" (i.e., the vacuum): there is a constant surface tension and a constant pressure exerted by the medium on the gas; in addition, there are the "thermodynamical" energy of the gas and the related gas pressure, which are determined by the solutions of the reduced equations. Both a SLAC-like bag and the Creutz-Soh version of the MIT bag may appear, but only as special limiting cases.

These soliton solutions are applied to the physical hadrons; their static properties are calculated and, within a 10-15% accuracy, agree with observations. What emerges is then a new formulation of the relativistic quark model of hadrons, based on the soliton solution.

3. Abnormal Nuclear States

Work connected with the possible existence of a new form of matter at high density suggested by Lee and Wick [Phys.Rev. D9, 2291 (1974)] is being continued. Their recent paper [Columbia preprint CO-2271-99, to be published in Mesons in Nuclei, edited by M. Rho and D. Wilkinson] gives a thorough survey of the field; it also examines the best experimental direction in which to investigate such problems. In this connection, it is hoped that the proposed construction at the Lawrence Berkeley Laboratory will extend the capability of the Bevalac to produce relativistic ions up to uranium, and increase its versatility to produce ions in the intermediate 35 to 200 MeV/amu range; thereby it may open the door to studying possible new coherent nuclear states at twice the normal nuclear density.

4. Besides the above-mentioned topics, Professor Lee is continuing his work on CP nonconservation and spontaneous symmetry breaking, questions related to vacuum excitation and vacuum stability, the possible existence of the intermediate boson, high energy neutrino reactions, scaling properties, and, above all, a possible unification of the electromagnetic, weak and strong interactions.

During the summer of 1976 Professor Mueller completed work on a geometrical interpretation of the string model. A paper has been published in Nuclear Physics B. The conclusion of this work was that the intercepts of the trajectories in both the open and closed string models are intrinsic properties of a reparametrization-invariant one-dimensional system existing in more than one space dimension. This indicates that truly severe modifications of the string Lagrangian will be needed before an acceptable physical theory can emerge.

Work was also completed on some extension of a previous work on hadron-nucleus collisions. This is to be published by Springer as a set of lectures given in Bielefeld, Germany last summer.

Since late in the fall, Professor Mueller has been collaborating with A. Duncan on an investigation into the properties of a realistic vacuum in Yang-Mills type theories. Their present, preliminary conclusions are that the exact many-instanton solutions will not survive quantum corrections when an infinite number of instantons is present. They have constructed several model vacua, of finite energy density, and have verified that instantons cannot confine quarks or other objects with color charge. A general formalism for superimposing classical solutions, and almost-classical solutions, has been developed.

Progress Report

Professor Robert Serber

Professor Serber has carried out calculations of nuclear binding energies and charge distributions for light nuclei, where a treatment in terms of volumes and surface energies is inadequate. The results were sufficiently unsurprising that it did not seem worthwhile to pursue the investigation, since the chief virtue of the model being examined, its simplicity, was rapidly being lost. A model with mesons of two masses was also examined. The charge distribution was found to be insensitive to such a generalization.

Professor Serber also did some work on classical solutions of soliton equations, and on representations of SU_n groups.

During the past year Professor Weinberg's interest has been primarily in studying the properties of solutions of classical field theories and the relevance of such solutions for the corresponding quantum theories. In particular, he has been interested in the magnetic monopole and pseudoparticle (or instanton) solutions of non-Abelian gauge theories.

- Several years ago 't Hooft and Polyakov showed that one can obtain from an $SU(2)$ gauge theory a monopole of unit magnetic charge. In collaboration with Alan Guth, Professor Weinberg investigated the properties of monopole solutions with multiple magnetic charge. Since the 't Hooft-Polyakov monopole is spherically symmetric, it seemed natural to look for solutions with spherical symmetry. They were able to show that there are no such solutions; in fact, they proved the stronger result that there are no finite energy spherically symmetric configurations with multiple magnetic charge. A paper describing this work has appeared in Physical Review D.

It has been shown that Yang-Mills theories in four-dimensional Euclidean space have stable solutions, known as pseudoparticles or instantons, which have remarkable consequences for the Minkowski space field theory. Among these are a mechanism for the spontaneous breaking of chiral symmetry without introducing a "ninth" Goldstone boson, and a possibility of introducing CP violation in a new manner. Previous authors had explained these effects by working in a particular gauge (one with $A_0 = 0$), and had concluded that the theory possessed multiple vacua. Working with Claude Bernard,

Professor Weinberg was able to show that the effects of the pseudoparticles can be explained in other gauges. However, they showed that the multiple vacua do not appear if one works in what they define as "physical" gauges; rather, the multiple vacua are artifacts of a particular method of treating the $A_0 = 0$ gauge. A paper describing this work will be published in Physical Review D.

At present, Professor Weinberg is still investigating the properties of instanton and multi-instanton solutions, in collaboration with Claude Bernard, Norman Christ and Alan Guth.

In the period between June 1976 and now, Professor Wick's interest has turned more and more towards the study of the connection between topology and the stability of certain classes of solitons.

Part of his effort was aimed at reaching a simple understanding of the mathematical facts underlying the said connection. The result of this effort was a short series of lectures he gave in July and August at the Erice Summer School. Among the subjects discussed in the lectures was the topological interpretation of the charge of the 't Hooft monopole, and of the analogous quantity for the Polyakov instanton. In particular, a very simple technique was exhibited to prove the integral nature of topological charges by means of singular differential forms.

Professor Wick's work on this subject has gone on through the fall and winter, but progress has been slow. He decided, in fact, that he had to spend some time acquiring some of the basic mathematical tools of differential geometry that may become necessary in the future, especially in the study of gauge fields. Some justification for this conclusion seems to be afforded by the fact that recently some very significant progress in finding interesting solutions of gauge-field theories, such as multi-instanton solutions, has been made by mathematicians or by physicists using methods involving at least some knowledge of the mathematical techniques referred to.

In addition to this work, Professor Wick has prepared, in collaboration with Professor Lee, a review of the question about the possible existence of an anomalous state of

nuclear matter. In this review, the original discussion of the problem, presented in various papers by Professor Lee, by Lee and Wick, and by Lee and Margulies, was reworked and presented in a simpler form, taking also into account objections and remarks by other authors. In addition, an attempt was made to put this subject in a proper perspective in relation to the so-called "pion condensation" phenomenon which represents an alternative phase transition, considered by other physicists.