VELOCITY-SPACE STABILITY OF
COUNTERSTREAMING-ION TOKAMAK
PLASMAS

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TOKAMAK PLASMAS

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ABSTRACT

The steady-state CBT/CIT ion velocity distributions are stable to all electrostatic and electromagnetic infinite-medium modes. This stability is due to the large thermal spread of the distributions ($T_{\text{hot}} \gtrsim 1.5 T_e$, $v_{ti}/u > 1/3$), and to the fact that tokamak equilibrium allows the mean streaming velocity, u , to be only a small fraction of the Alfvén velocity.

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VELOCITY-SPACE STABILITY OF COUNTERSTREAMING-ION TOKAMAK PLASMAS

In counterstreaming-ion tokamak plasmas, the ion population makes up two distinct, quasi-thermal velocity distributions, oppositely displaced in velocity along the magnetic axis, as shown in Fig. 1. These distributions are maintained by introducing all plasma ions by means of energetic neutral-atom beams, injected parallel and antiparallel to the magnetic axis, and by minimizing recycling of decelerated ions that leave the hot plasma region. Counterstreaming distributions of deuterons and tritons form the basis of the CBT fusion reactor concept¹⁻³ (also referred to as the CIT, for counterstreaming-ion torus). The optimal performance of this fusion reactor depends, inter alia, on the absence of microinstabilities that would enhance the slowing-down rate of the energetic ions.¹⁻⁴ In this note we summarize stability treatments that show that for all practical conditions, the steady-state CBT/CIT velocity distributions are stable to electrostatic and electromagnetic modes. This stability is due, first of all, to the large thermal spread of the distributions, and second, to the fact that tokamak equilibrium allows the streaming velocity to be only a small fraction of the Alfvén velocity.

Fokker-Planck calculations¹ have shown that the ion velocity distributions $f_i(v)$ can be approximated by shifted Maxwellians (see Fig. 1):

$$f_i(v) = \frac{M_i^{3/2} n_b}{(2\pi)^{3/2} T_{\perp} T_{\parallel}} \exp[-M_i (v_{\parallel} \pm u_b)^2 / 2T_{\parallel} - M_i v_{\perp}^2 / 2T_{\perp}] \quad (1)$$

Here, $v_{\parallel}$ and $v_{\perp}$ are velocities parallel and perpendicular to the magnetic axis, while $T_{\parallel}$ and $T_{\perp}$ are the parallel and perpendicular temperatures of the ion distributions. In the following, the electrons are taken as Maxwellian, and the plasma is assumed to be homogeneous. For simplicity, we assume that the counterstreaming distributions have the same amplitude, temperature, and displaced velocity, and that all ions are identical.

We define $\Delta W = 2u_b v_{ti} M_i$, where $v_{ti} = (2T_{\parallel}/M_i)^{1/2}$. Then $\Delta W/W_b = 4(T_{\parallel}/W_b)^{1/2}$, where $W_b = 1/2 M_i u_b^2$. The Fokker-Planck calculations for steady-state operation reveal the following relations: $T_{\parallel} \geq 1.5 T_e$, $T_{\perp} > T_{\parallel}$, $v_{ti}/u_b > 1/3$, $\Delta W/W_b > 4/3$.

The sources of free energy for velocity-space instability are $\partial f_i / \partial v_{\parallel}^2 > 0$, for $|v_{\parallel}| < u_b$, and the anisotropic ion pressure ($p_{\parallel} > p_{\perp}$). First we consider electrostatic modes, with ω not near ω_{ci} , the ion cyclotron frequency. The stability boundaries for such modes in counterstreaming systems have been discussed by Stringer⁵ and by Forslund and Shonk.⁶ They show that ion-ion modes (including the "two-stream" instability) are stable if $T_{\parallel} \geq 1/2 T_e$. When $T_{\parallel} > T_e$ (as in the present situation), electron-ion modes are stable if $u_b \lesssim 10(T_e/M_i)^{1/2}$; for injection energy $\lesssim 100$ keV, this condition is satisfied if $T_e \gtrsim 0.7$ keV. These criteria hold for waves propagating at arbitrary angles.

The stability of electromagnetic modes with ω not near ω_{ci} depends in part on the accessible range of u_b . In calculating the poloidal beta of a CBT plasma, the streaming energy density can be treated formally as if it were a thermal pressure (A. H. Boozer, unpublished). That is, even when the thermal energy of the plasma is negligible, for equilibrium we require that

$$\frac{8\pi M_i n_i u_b^2}{B_p^2} \approx \frac{1}{\epsilon} \quad (2)$$

so that

$$\frac{u_b}{v_{\text{Alf}}} \approx \frac{1}{q} \left(\frac{\epsilon}{2} \right)^{1/2} \quad (3)$$

where v_{Alf} is the Alfvén velocity, ϵ is the inverse aspect ratio, and $q = \epsilon B_t / B_p$ is the safety factor at the limiter. For practical conditions, $q > 2$ and $\epsilon \lesssim 1/3$, so that $u_b / v_{\text{Alf}} \approx 0.2$.

Gaffey⁷ shows that a minimum condition for the excitation of ordinary electromagnetic waves propagating perpendicular to $\vec{B}$ is that $(u_b / v_{\text{Alf}})^2 \gtrsim 1/2$. Montgomery *et al.*⁸ obtain numerically a similar requirement for exciting magnetosonic and Alfvén modes, propagating at arbitrary angles to the magnetic field. This instability threshold cannot be reached by the CBT velocity distributions.

Electrostatic ion cyclotron waves ($k_\perp \gg k_\parallel$) have been treated by Weibel⁹ and by Perkins.¹⁰ For $T_\parallel > T_e$, their treatments give essentially the same criterion, namely that these modes are stable when $\Delta W / W_b \gtrsim 0.8 T_e / T_i$; this condition is always satisfied by the steady-state CBT distributions. A recent experiment in a linear device¹¹ has demonstrated the validity of this stability criterion.

Finally, the excitation of electromagnetic ion cyclotron waves by counterstreaming ions has been investigated by Perkins.¹⁰ While interaction between the counterstreaming distributions tends to make the waves grow, an important stabilizing mechanism is cyclotron damping by each distribution itself.

Excitation of these modes requires that $u_b \gtrsim 1/3 v_{Alf}$, and in any event there is stability when $\Delta W/W_b \gtrsim 1$.

Table 1 summarizes the stability criteria, and indicates the range of parameters of "ideal" CBT/CIT distributions,^{1,2} which are bound to be the least stable. That is, a substantial population of colder ions enhances the slowing-down rate of energetic ions, resulting in a smaller u_b and increased ΔW and $T_{||}$. (Of course, the cold-ion population also degrades reactor performance.) Although we have assumed in our survey that the counterstreaming distributions are identical, we expect essentially the same results if this condition is relaxed.

(The above analysis applies only to steady-state conditions. At the start of injection, the ion distributions are more peaked, with a smaller ΔW and a larger u_b , so that instabilities can be excited. This transient phase, however, lasts only one or two slowing-down periods, typically less than 0.5 s.)

Turning briefly to MHD modes that could result from the anisotropic pressure, it is clear that the "firehose" instability cannot arise in tokamak operation, where $p_{||}/B^2 \ll 1$. Connor and Hastie¹² have shown that parallel beam injection leading to $p_{||} > p_{\perp}$ actually has a stabilizing influence on the localized interchange modes, provided that $dp_{||}/dr < 0$.

Concerning the other two potential problems that have been mentioned by Cordey⁴ in his discussion of the CBT concept, we note that tokamak operation with comparable ion and electron energy densities has already been achieved in recent beam-injection experiments on the ATC, ORMAK, and TFR devices.

[unpublished results]. It seems likely that with further increases in injection power, large ratios of ion energy to electron energy can be obtained. Then the fundamental plasma physics problem facing implementation of CBT/CIT operation is the need to minimize the population of nonstreaming ions. Further theoretical investigations are required also concerning stability against finite-medium modes, such as universal instabilities and the trapped-electron mode.

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Table 1.
Stability Criteria for Counterstreaming-Ion
Velocity Distributions

Mode	Sufficient Condition for Stability	Practical CBT Parameters
Electrostatic, ω not near ω_{ci}	$T_{\parallel} > 1/2 T_e$, $u_b \lesssim 10 (T_e/M_i)^{1/2}$	$T_{\parallel} \gtrsim 1.5 T_e$ $u_b < 5 (T_e/M_i)^{1/2}$
Electromagnetic, ω not near ω_{ci}	$u_b < 1/2 v_{Alf}$	$u_b \gtrsim 1/5 v_{Alf}$
Electrostatic ion cyclotron waves	$\Delta W/W_b > 0.8 T_e/T_i$	$\Delta W/W_b > 2 T_e/T_i$
Electromagnetic ion cyclotron waves	$u_b < 1/3 v_{Alf}$ or $\Delta W/W_b \gtrsim 1$	$\Delta W/W_b > 4/3$

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Fig. 1. Counterstreaming-ion velocity distributions parallel to the toroidal magnetic field. Ions are injected at $v_{||} = v_{inj}$, and removed at $v_{||} < 2$. u_b is the mean streaming velocity, and v_{ti} is the ion parallel "thermal" velocity.