AN EXTENDED DIFFUSIVE MODEL FOR CALCULATING
THERMAL DIFFUSIVITY FROM SINGLE MONOPOLE
TOKAMAK HEAT PULSE PROPAGATION

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ABSTRACT

The problem of deducing χ_e from measurements of the propagation of a monopole heatpulse is considered. An extended diffusive model, which takes into account perturbed sources and sinks is extended to the case of a monopole heat input. χ_e is expressed as a function of two observables, the heat pulse velocity and the radial damping rate. Two simple expressions valid for two different ranges of the radius of the poloidal waist of the beam power profile are given. The expressions are valid in the heat pulse measurement region, extending radially $0.05a$ beyond the beam power waist to near $0.6a$. The inferred χ_e is a local value, not an average value of the radial χ_e profile.

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to a factor of two. Most significant is the effect of the perturbed source and sink terms, which are not accounted for by that model.

Lopes Cardozo et al. [2] treated the problem as a simple diffusion problem with the addition of a damping term to the heat equation to account for the perturbed source and sink terms. The damping term is characterized by a damping time constant T_d . They solved the equation numerically, allowing for realistic profiles of χ_e and n_e . Two observables descriptive of the heat pulse propagation were calculated:

The velocity of the heat pulse peak

$$v_{hp} = \left(\frac{dr}{dt} \right)^{-1}$$

and the rate at which the amplitude decreases with increasing radius, termed the radial damping rate

$$\alpha = 10 \frac{d}{dr} [\log_{10} T_e(r, t_p)]$$

A simple relation was found between these two quantities. They compared their new result with results of the solutions of the full set of transport equations, using the predictive 1.5D transport code JETTO[7]. They found that the result of the theory, when compared to results of simulations of the JET plasma, yielded χ_e accurate to within 10%. This is a significant improvement in accuracy over the formula derived by analytic calculations.

I have extended this approach to the case of a monopole heat pulse input with a Gaussian type of profile such as is produced by FEL heating in MTX. The overall form of the numerical results I obtained for the case of the monopole heat input was similar in nature to the results of Lopes Cardozo et al.[2]. There are some important differences. Because the minor radius of Alcator-C is considerably smaller than that of a tokamak such as JET, the expression relating χ_e to the two observables must be valid over a larger fraction of the minor radius than the formula in ref. [2]. Also, the propagation of the monopole heat pulse is affected by the edge boundary conditions at a lower value of the fractional minor radius than for the dipole. For this reason the formulas relating χ_e to the two observables characterizing the propagation of the monopole heat pulse are more complicated than the result of ref. [2]. When evaluated for a particular set of fixed measurement parameters, however, each formula yields a very compact equation, and can be used for a wider range of measurement parameters than the results of ref. [2].

Measurement of χ_e from monopole heat pulse has certain advantages compared to the measurement using sawtooth. The relative merits of the two are listed in Table 1.

In the following sections a detailed description of the model is given, along with an interpolation of the meaning of χ_e , n_p and an analysis of the influence of the shape of the χ_e profile on the value of $\chi_{e,Hp}$ determined.

Table 1

Issue	Comparison
Region of measurement of χ_e	For a narrow beam width, χ_e measurement possible for a monopole over a larger radial range and closer to the center of the plasma
Sensitivity to initial perturbation temperature profile	Detailed shape of monopole easier to quantify than for dipole; less uncertainty in analysis
Timing control	FEL input time is well known and virtually instantaneous. Sawtooth crash time is finite
Sensitivity to χ_e profile shape	Dipole gives a more localized measure of χ_e than monopole
Radial decay rate	Radial decay rate of the dipole is larger. This is offset by the small energy input by the FEL pulse

THE MODEL

The method proposed here utilizes two observables in a more accurate formula for χ_e . Simplification of the problem is obtained by assuming that between the $q=1$ and $q=2$ surfaces, the relaxation of the perturbed T_e profile is largely a diffusive process. Following the method of Lopes Cardozo, et al.[2], the radial spread of the T_e profile is treated as a simple diffusion problem in a radial geometry. A damping term is added, however, to account for the perturbed source and sink terms. The diffusion equation is solved numerically, allowing for realistic profiles for n_e and χ_e . Subtracting off the steady state transport equation from the perturbed equation we arrive at the fundamental equation governing the evolution of the T_e perturbation

$$(2) \quad \frac{3}{2} \frac{d}{dt} (n_e \tilde{T}_e) = -\nabla \cdot \tilde{Q} + \tilde{P}$$

where $\tilde{Q}$ is the heat flow per unit area and $\tilde{P}$ is the perturbed source power density. Quantities with a tilde superscript denote perturbation with respect to the equilibrium value $\tilde{Q} = Q - Q_{eq}$. The quantity $\tilde{Q}$ is related to $\nabla_r \tilde{T}_e$ via $\tilde{Q} = -n_e \chi_e \nabla_r \tilde{T}_e$. If we allow χ_e to be a function of T_e and ∇T_e , expanding to first order in perturbed quantities, neglecting possible perturbation in n_e , yields [5].

$$(3) \quad -\tilde{Q} = n_e \tilde{\chi}_e \nabla_r T_e + n_{e,eq} \chi_e \nabla_r \tilde{T}_e = n_e \chi_e^{HP} \nabla_r \tilde{T}_e + n_e \left(\frac{\partial \chi_e}{\partial T_e} \right) (\nabla_r T_e) \tilde{T}_e$$

with

$$\chi_e^{HP} = \chi_e^{eq} + \left(\frac{\partial \chi_e}{\partial \nabla_r T_e} \right) \nabla_r T_e$$

χ_e^{HP} is the quantity to be obtained through heat pulse analysis. The terms arising from a possible dependence of χ_e on T_e and ∇T_e play a role in the governing equations that is common for both the dipole and monopole cases[2]

- A $\nabla_r T_e$ dependence in χ_e replaces χ_e^{eq} by χ_e^{HP} but retains the diffusive nature of the heat pulse.
- A T_e dependence in χ_e adds a term $\tilde{Q}$ which has the form of a convective heat flow. This term mainly affects the radial behavior of the amplitude of the heat pulse. It can be shown that the effect on the determination of χ_e is small, except for very strong T_e dependences. A strong T_e dependence is incompatible with the results of power balance analysis. In tokamaks, no significant convective behavior has been associated with the propagation of heat pulses.

Taking that χ_e has no explicit dependence on T_e , substitution of eq. (3) in [2] yields a damped diffusion equation.

$$(4) \quad \frac{3}{2} n_e \tilde{T}_e = \nabla \cdot (n_e \chi_e^{HP} \nabla \tilde{T}_e) - \frac{n_e}{\tau_d} \tilde{T}_e$$

The source power density has been linearized in T_e with the help of a characteristic damping time constant. For $\tilde{T}_i \ll \tilde{T}_e$, τ_d can be expressed as [2]

$$(5) \quad \tau_d = \frac{3}{2} \left(\frac{3T_i}{T_e} - 1 \right) \frac{m_i}{m_e} \tau_{ei} + \frac{2}{3} \frac{n_e T_e}{\eta j^2} - n_e \left(\frac{\partial P_{rad}}{\partial T_e} \right)^{-1}$$

A figure of merit in assessing the effect of damping is the dimensionless parameter $K = \frac{a^2}{\tau_d \chi_e}$. This parameter can have values as large as 0(10) in the confinement region of the Alcator-C plasma for certain plasma parameters, indicating that damping can be significant. The radial profile of τ_d is complicated and depends on the equilibrium profiles which we do not want to involve in the analysis of the heat pulse. One improves on the assumption of $\tau_d \rightarrow \infty$ used in former theories by taking τ_d constant. Here τ_d is an effective damping time constant which represents a radially averaged effect of all contributions. Damping has the effect of reducing the time $t_p(r)$ at which $T_e(r)$ is maximum and reducing the amplitude of $\tilde{T}_e(r, t_p)$.

From ECE or other measurements $\tilde{T}_e(t)$ can be deduced at different radii, from which one can derive two quantities.

- the heat pulse velocity V defined locally by $V_{hp} = \left(\frac{dt_p}{dr} \right)^{-1}$
- the radial heat damping rate α defined by $\alpha = 10a \frac{d}{dr} (\log_{10} \tilde{T}_e(r, t_p))$ expressed in dB.

By including the damping rate in the analysis we can account for the effect of perturbed sources and sinks. It should be pointed out that the analysis is performed for a cylindrical geometry. In the course of data analysis V_{hp} and α should be corrected for the Shafranov shift. The corrected values are given by [2]

$$(6a) \quad V_{hp} = \sqrt{K} \frac{a}{a-s} V_{hp}^+ = \sqrt{K} \frac{a}{a-s} \left(\frac{d}{dr} t_p \right)^{-1}$$

$$(6b) \quad \alpha = \frac{a-s}{a} \alpha^+ = 10 (a-s) \frac{d}{dr} (\log_{10} A)$$

where S is the Shafranov shift and K would allow correction for an elongated plasma. The damped diffusion equation was solved for a variety of $\chi_e(r)$ profiles. The specific choice of a profile does not have a strong bearing on the results of this theory. The initial temperature perturbation profile was taken to be Gaussian in shape for a radius inside the thickness of the absorption layer (a slab in the vertical plane centered on axis) and was taken as the product of a Gaussian and a term accounting for redistribution of energy over flux surfaces (with a $1/r$ dependence) for radii greater than the half width of the absorption region.

Previously published models have considered the case of a perturbation profile due to an internal disruption which extends only to the mixing radius. The Gaussian pattern extends throughout the plasma. It was found, however, that in the region $0.05a$ beyond the beam power waist the

results of the theory are relatively insensitive to the shape of the initial temperature perturbation profile.

The numerical results yield that, to a very good approximation, α and $aV_{hp}/\chi_e(r)$ can be represented by linear functions of K .

$$(7a) \quad \frac{aV_{hp}}{\chi_e(r)} = p_v + q_v K$$

$$(7b) \quad \alpha = p_\alpha + q_\alpha K$$

Solving these for χ_e yields

$$(8) \quad \chi_{e, hp, ext} = C_1 \frac{aV_{hp}}{\alpha + C_2}$$

$$\text{where } C_1 = \frac{q_\alpha}{q_v} \text{ and } C_2 = C_1 p_v - p_\alpha$$

Figures 1 and 2 illustrate this, with each line representing the variation with K at a particular measurement radius. The linear relation between aV_{hp}/χ_e and α for the monopole at several radii is shown in Fig. (3). Here w is the radius of the beam power waist, 4.7 cm in these examples.

The range of r over which heat pulse measurements can be made is restricted by several constraints.

- - In the direct vicinity of the initial temperature perturbation, the solution to the diffusion equation is very sensitive to the shape of the initial perturbation. It is found that for the monopole, in analogy with the dipole, the far field region begins at $r = 0.05a + w$, five percent of the minor radius beyond the beam power waist. Note that in Figures 1 and 2 that at a radius of 4 cm, inside the beam power waist, the values of V_{hp} and α are quite different from the values in the measurement region. Inside the beam power radius the coefficients C_1, C_2 would be influenced strongly by changes in the heating profile.
- - Although the spatial damping of the heat pulse peak is much less for a monopole than for a dipole, the assumed 105 J energy input per pulse is relatively small and thus reliable measurement becomes very difficult at $r = 0.6a$. Even inside the measurement region the amplitude is small enough that averaging together results from several shots will be necessary.

It should be noted that the restriction on the region where heat pulse measurements can be made are not a particularity of the extended model, but of the radial range where $T_e(r)$ can be measured with sufficient accuracy.

One can subtract off the sawtooth temperature variation during a single sawtooth period from the measured temperature variation prior to the next sawtooth crash. The magnitudes of the sawtooth and monopole temperature perturbations will be small compared to the background temperature profile. Thus, one expects the perturbed source and sink terms

to exhibit mostly a linear dependence on the perturbed temperature. Thus, there should not be significant non-linear coupling between the two perturbations and the net profiles should contain information from the monopole heat pulse. For equivalent plasma parameters and beam widths the net monopole profiles from different shots may be simply added. Fluctuations in FEL output amplitude from shot to shot will not require renormalization for data analysis.

NUMERICAL RESULTS

It was found that in the measurement region, for a beam power width on axis of 4 cm or less the coefficients C_1 and C_2 are functions of the measurement position and nearly independent of the poloidal beam power width. To maximize the accuracy of the method numerical calculations were performed using a parabolic $\chi_e(r)$ profile inferred from previous heat pulse measurements in Alcator[3] and a density profile $n_e(r) = n(0) \left(1 - \left(\frac{r}{a+\delta} \right)^2 \right)^{1/2}$ similar to that measured on Alcator-C. δ is a small parameter to eliminate singularities in the domain. It will be shown that the results of the theory are still quite accurate for a variety of profiles. The nature of the density profile has very little influence on the results of the theory. C_1 and C_2 were determined by a least squares fit to numerically calculated data points in the measurement region. The formulas given are expressed in cm and are specifically for the case of $a = 16.5$ cm. Of course, when expressed in dimensionless coordinates they are applicable for any limiter dimension. For a beam power waist less than 4 cm (0.25a) in radius, the following formula was obtained.

$$(9) \quad x_{hp,ext} = \frac{(8.01 - 1.69r + 0.150r^2) a V_{hp}}{\alpha + 12.9 - 10.6r + 1.04r^2},$$

The parameter r is the radial distance from the axis of the machine to the center of the radial span between the two measurement positions, in centimeters.

For a monopole heat pulse the effect of the finite plasma radius alters the heat pulse propagation velocity and radial damping at a lower value of dimensionless minor radius than for the dipole. In addition, the FEL heating profile extends across the entire area of the plasma cross section, rather than being of finite radial extent as in the case of an internal disruption. Thus, as the beam power waist becomes larger than 4 cm, the coefficients exhibit a dependence on both the measurement radius and beam power width that is well modelled by a quadratic equation. For the larger beam waist ($w > 4$ cm) the result obtained is:

$$(10) \quad x_{hp,ext} = \frac{[(12.7 - 4.25 W + 0.270 W^2) + (-0.978 + 0.510 W - 0.346 w^2) r] a V_{hp}}{\alpha + (-5.12 - 2.03 r + 0.294 r^2) (1 + 0.227 W)}$$

The parameter w is the $1/e$ power radius of the FEL ray pattern in the vertical direction at the center of the resonant layer, assumed to be in the plane containing the plasma axis. Both r and w are to be given in centimeters. When evaluated at $w = 4$ cm and radii inside the measurement region the coefficients C_1 and C_2 in Equation (10) agree well with the coefficients in Equation (9).

Influence of the χ_e profile and heating profile on the determination of χ_e by means of heat pulse propagation analysis:

Because the heat equation is parabolic, the temperature at a point is affected by the shape of the χ_e profile. Consequently, χ_{hp} is not a measure of local χ_e , but a weighted radial average of the χ_e profile. Also, the quantity $\chi_e^{(inc)}$ measured by heat pulse propagation measurements is not the same $\chi_{e\ GLO}$ determined by power balance measurements, but rather an incremental χ_e describing an incremental change in the heat flow due to an incremental change in the temperature gradient.

$$\chi_e^{(inc)} = \frac{1}{ne} \frac{dQ}{dVr\ T_e} \Big|_{n_e = \text{constant}}$$

Thus, the χ_e value, determined by heat pulse measurements, should be compared with $\chi_e^{(inc)}$, rather than $\chi_{e\ GLO}$, determined from power balance measurements. There can be significant differences in the values of the two quantities. For example, if one considers the family of scaling laws $\tau_e = \text{const } P^{-\gamma}$, then τ_e^{inc} is related to τ_e by

$$(11) \quad \tau_e^{(inc)} = \frac{\partial W}{\partial p} = \frac{\partial}{\partial p} (\tau_e P) = (1-\gamma) \tau_e$$

Since χ_e is inversely proportional to τ_e

$$(12) \quad \chi_e^{(inc)} = \frac{1}{1-\gamma} \chi_{e, GLO}$$

Fredrickson et al. [4] investigated the influence of the shape of the χ_e profile on $\chi_{hp, dif}$ by numerically solving the diffusion equation for a number of different χ_e profiles. They found that $\chi_{hp, dif}$ was clearly affected by the shape of the χ_e profile. Lopes Cardozo, et al. [2] determined more quantitatively how different parts of the χ_e profile contribute to the measurement of χ_{hp} . Their approach was to write χ_{hp} as a weighted line average of χ_{hp}

$$(13) \quad \chi_{hp} = \int_0^a w(r) \chi_e(r) dr$$

and then to determine the weight function $w(r)$. One may consider $w(r)$ to be the instrument function of the heat pulse propagation method. A function $w(r)$ that peaks strongly in the measuring region would indicate that χ_{hp} is a local measure of $\chi_e^{(inc)}$, whereas a broader $w(r)$ would indicate χ_{hp} is a broader line average of $\chi_e^{(inc)}$.

This method is not generally applicable. In particular, the representation (13) breaks down for χ_e profiles that approach zero (or go to infinity) at any radius, because of the non-linearity of the problem. Lopes Cardozo, et al.[2] show that Eq. (13) is accurate for χ_e profiles with a dynamic range of a factor of 10. This means Equation. (13) is correct for all profiles interesting from the point of view of tokamak physics. The weight function was determined by using a test χ_e profile having a narrow step: a Heaviside

function stepping up at a particular radius, r_H . By determining the variation of $\chi_{e,dif}$ and $\chi_{e,ext}$ as a function of r_H for a particular measuring region and fixed initial temperature perturbation, the weight function could be determined. For the case of a dipole heat input Lopes Cardozo, et al[2] determined by the weight function for the extended diffusive model is sharply peaked in the measurement region. The purely diffusive model, on the other hand yielded a broad line averaged value of χ_e . Similar calculations for the monopole version of the extended diffusion theory also yield a weight function that is strongly peaked in the measurement region. Figure 4 illustrates the calculated weight function. The radius of the beam waist was taken to be 5.28 cm in this case and the quantities V_{hp} and α were averaged over the radial extent $0.43a < r < 0.53a$. As expected, the monopole version of the purely diffusive model (dashed line, Fig. 4) again yields a broad line averaged value of χ_e . The extended diffusive weight function peaks in the measurement region because a variation in χ_e outside of the measuring interval changes V_{hp} and α in much the same way, leaving the slope of $aV_{hp}/\chi_e(r)$ versus α , largely unaffected. Comparison of the numerical results for the cases of monopole and dipole heat inputs shows that the results for the monopole are considerably more sensitive to the shape of the χ_e profile than the results of the dipole case; the weight function for monopole version of the extended diffusive model has lobes extending further on either side of the peak than the weight function for the dipole version of the extended diffusion theory. The local values of χ_e were calculated with formulas (9,10) using numerical results for a variety of physically interesting χ_e profiles and were compared to the local analytic value. Representative profiles are shown in Figure 5. For all of the parabolic and linear profiles shown, both formulae gave χ_e values accurate to within 15% of the local analytic value for the stated

range of measurement radii and useful beam widths and K values. Excepting measurement radii at the very edge of the heat pulse measurement region the formulae gave results usually within a few percent of the local value. The χ_e profile in Figure 4 with a steep rise at $(r/a)=0.6$ is similar to the χ_e profile determined by power balance calculations for Alcator-C [3]. Throughout the measurement region extending radially from $0.05a$ beyond the beam power waist to $0.6a$, for all useful beam widths, the above formulae gave results for χ_e accurate within 25% of the local "power balance" χ_e . Except at the edge of the measurement region, the results for this case were typically within 15% of the local analytic value. For the same profiles the results of the purely diffusive model could deviate by up to 80% from the local value with no damping. When damping was included the diffusive model gave results for χ_e which were larger still. For the extended diffusive model the variation of χ_e in the measurement region is the most significant. Here the parabolic profiles most closely represent the $\chi_e(r)$ profiles observed in tokamaks. In all cases considered, compared to the purely diffusive model, the extended diffusive model yielded significant improvement in the accuracy of the local χ_e value obtained.

The results of numerical runs with a "hollow" monopole profile and a "square" FEL heating profile (consistent FEL amplitude inside a given radius) were examined. In the far field region the values of χ_e calculated from formulas (9,10) showed little change from the local χ_e value calculated with the Gaussian monopole. When the Gaussian heating peak was moved one centimeter off axis, calculated values changed by only a few percent.

It should be noted that although the fit for $w > 4$ cm (0.25a) may appear to be highly sensitive to the value of w , a moderate uncertainty in the value of w yields a modest error in the calculated value of χ_e . Typically a change in W of 1 cm (0.0625a) will cause a change of only a few percent in χ_e , as long as measurements are taken in the actual measurement region. Also, the heat pulse velocity and damping are functions of the radius which change rapidly over the small dimensions of Alcator. Both α and V_{hp} vary in a similar fashion, however, so that measurements of the heat pulse peak at points up to two centimeters from the center of the measurement region will introduce modest errors into the results. In this case the central $\tilde{T}_e$ in the denominator of the expression for α may be obtained through logarithmic interpolation.

The pulse peak velocity V_{hp} for the monopole is slower than that for the sawtooth peaks. An "averaged" sawtooth profile should be subtracted from the individual sawtooth profile containing the monopole. The remaining profile should consist of the monopole perturbation, other fluctuations and noise. By adding dozens of monopole peaks together one should be able to improve the signal to noise ratio. The monopole can only be extracted if the peak at a position can be measured before the first sawtooth crash after the FEL pulse. The reason for this is that the crash redistributes the monopole temperature distribution inside the mixing radius. Because the heat equation is parabolic this redistribution simultaneously effects the temperature evolution throughout the plasma. Figure (5) shows the effect on the evolution of the monopole heat pulse of flattening the perturbed temperature distribution inside a characteristic mixing radius (solid line). Shown is the evolution of the perturbed monopole temperature distribution

(with a beam power waist radius of 5.28 cm) at a measurement radius of 7.92 cm. The pulse is initialized at time = 0 and the profile is flattened inside a radius of 5.28 cm at a hypothetical time of 6.5 msec., simulating to first order the redistribution of profile energy by an internal disruption. This redistribution is equivalent to adding another dipole profile to the monopole; the sawtooth profile for this crash is different than for the other crashes. Because two indistinguishable perturbation profiles are then evolving simultaneously, the above analysis cannot be used for peaks measured after the first sawtooth crash that occurs after the FEL pulse. In Alcator there is a positive correlation between increased n_e , T_e and inversion radius and an increase in the sawtooth period^[3]. Note that at MIT it was observed that in the period immediately after pellet fueling the sawtooth period could become as long as 50 msec.

In order to check that the profile resulting after subtracting off the sawtooth is that of the monopole, the spatial attenuation and velocity of monopoles generated by FEL pulses at different times relative to the last sawtooth crash should be compared. These quantities should be independent of the relative time by which the last sawtooth crash preceeded the FEL pulse. If these quantities are dependent on this time span then this indicates that there is significant non-linear coupling between the sawtooth perturbation and the monopole perturbation and the above analysis cannot be used. Because the amplitudes of the monopole and dipole perturbations are small compared to the background temperature profile, one expects terms non-linear in the expansion of the perturbed sources and sinks to have a small effect on the evolution of the profiles. Both effects mentioned above would

also introduce large errors into the χ_e calculated from a single monopole input using the $\chi_{e,dif}$ model.

It is implicit that in order for this theory to yield improvements in the accuracy of the determination of χ_e , one must be able to make a localized measurement of the amplitude of the heat pulse peak. The magnitude of the two observables change rapidly across the minor radius of Alcator-C. Thus, a diagnostic with the highest radial resolution, such as the planned hetrodyne ECE, will likely prove the most useful for monopole heat pulse measurements generated by an isolated FEL pulse.

If it is found that the attenuation coefficient of the heat pulse cannot be accurately measured the diffusive model, with its larger systematic errors, can be applied. Numerical calculations with the radial diffusion code indicate that for the case of $\chi_e(r) \approx \chi_e(0)$ and an Alcator-like density profile $n_e(r) = n_0 (1 - (\frac{r}{a+\delta})^2)^{1/2}$ the dependence of the parameter r_{mix} in equation (1) on the beam waist radius w is well modelled by the relation.

$$(14) \quad r_{mix} \approx r_w = 2.25 W^{0.311}$$

for $4 < w < 8$ cm. Both r_{mix} and w are to be given in centimeters.

Another method of measuring χ_e from the monopole heat pulses from single FEL pulse shots would be to treat the diffusion of the heat pulse as a forced boundary value problem. The temperature measured at the channel near the beam power waist could be used as the source of the forced

boundary condition. The sawtooth profile could be subtracted off from the measured signal at each measurement position. Then, after adding the net profiles of several shots together, the evolution of the monopole heat pulse at positions in the confinement zone could be compared with the results from numerical calculations, and a χ_e profile fit to the data by a least squares criterion. In particular, the slopes of the temperature rise of the monopole pulse, before the first sawtooth crash, could be used. One minimizes the sum over the measurement radii of the weighted integrals of the variation of the measured temperature from the numerically calculated temperature

$$(15) \quad \min_{\vec{C}} \sum_k \int_{t_{fel}}^{1st \text{ crash}} |\tilde{T}_e(r_{k,t}) - \tilde{T}_{e,num}(r_k, t; \vec{C})| g(r_k, t) dt$$

and in theory calculates through the parameter $\vec{C}$, the best fit χ_e profile. Here $g(r_k, t)$ is a time dependent weighting function chosen to favor the initial rise phase of the pulse, when the temperature gradients are the largest. The cylindrical diffusion code written for the purpose of this analysis would be useful for such calculations. The principal disadvantage of this method is that it requires some assumption about the energy deposition profile of the FEL beam in the measurement region. Uncertainty about this profile limits the accuracy of this method.

When shots with multiple FEL pulses are eventually carried out, the Fourier transform method of analysis should prove the most useful^[6]. Here the periodic nature of the input perturbation profile is used. The measured periodic temperature variations at different radial positions are used to obtain the cross-power spectral density between the reference sensor and the other

sensors. The CPSD is the product of the Fourier transform of the cross-correlation function of the signal and reference. The CPSD consists of an amplitude-versus-frequency spectrum and a relative phase spectrum (between the signal and reference). Thus phase shifts for the various Fourier harmonics of the FEL monopole pulses are obtained as a function of position. In a slab geometry the χ_e is related to the phase shift between detectors by [6]

$$(16) \quad \chi_e = \frac{n\omega}{2} \left(\frac{\Delta x}{\Delta \phi} \right)^2$$

where ω is the input heat pulse frequency, n is the harmonic number, and Δx and $\Delta \phi$ are the relative differences between position and phase for the signal and reference. In cylindrical coordinates, numerical solutions of the diffusion equation are fit to the experimental results to determine χ_e . This method of analysis allows one to discriminate out the effect of the redistribution of the monopole profile by a sawtooth crash.

CONCLUSIONS

An extended diffusive model was applied to the case of a monopole heat pulse, such as created by FEL heating in MTX. The inclusion of a damping term accounted for the effect of perturbed sources and sinks and provided a far better localized measurement of χ_e than the original purely diffusive model. The effect of any sink in the perturbed equation is to increase both v_{hp} and α . A variation in χ_e outside of the measurement interval also affects v_{hp} and α in a similar manner, causing relatively little change in the linear relationship between the two quantities.

Unless the time constant τ_d associated with the sink is small compared to the time constant of the relaxation of the perturbed T_e profile, the effect of the sink is more global than local. This is typically the case for Alcator-C. In this case the radial distribution of sinks is not very important and the assumption τ_d is uniform is justified.

It was assumed that χ_e does not depend on T_e or $\nabla_r T_e$. The effect of such dependences on heat pulse studies has been summarized. A $\nabla_r T_e$ dependence does not change the linearized diffusion equation. It only replaces χ_e by [2]

$$(17) \quad \chi_e' = \chi_{e,eq} + \left(\frac{\partial \chi_e}{\partial \nabla T_e} \right) \tilde{\nabla T}_e$$

where the eq subscript refers to the unperturbed equilibrium situations. The analysis presented is still fully applicable. The dependence of χ_e on ∇T_e only affects the physical interpretation of χ_{hp} . A dependence of χ_e on T_e is formally equivalent to a convective term in the heat transport Equation [2]. Inspection of measured heat pulses so far has not revealed any significant non-diffusive behavior, which is why in heat pulse analysis χ_e is assumed to be independent of T_e .

Thus, it has been shown that the monopole version of the extended diffusive model has reduced the systematic error in heat pulse analysis to near the level of accuracy of measurement obtainable from the diagnostic,

and has greatly enhanced the localization of the χ_e value obtained from measurement.

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———— 4cm - - - - - 6cm $K = \sigma^2 / (X_e T_{damp})$

- . - . - 8cm 10cm

Fig. 1

— $4cm$ $6cm$ $K = \alpha^2 / (X_e T_{damp})$

--- $8cm$ -.-.- $10cm$ Fig. 2 27

fig. 3

WEIGHT FUNCTION

----- diffusive model

———— extended diffusive model

Fig. 4
29

Fig. 6 31

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