AIR BLAST EFFECTS ON CONCRETE WALLS

by

C. A. Kot and P. Turula

Components Technology Division

U of C-AUA-USERDA

for

Nuclear Regulatory Commission
Office of Standards Development

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ABSTRACT

The effects of airblast due to explosive detonation in close proximity of a concrete wall are investigated analytically. Estimates are obtained both for the spalling of the back-face of the concrete wall and for the overall wall response produced by the total impulsive load of the air blast.

Assuming elastic wave propagation in the concrete wall, it is found that as spall thickness increases the spall velocity decreases. This holds for normal as well as oblique wave incidence on the back-face of the wall. Therefore, for debris which has significant mass, the ejection velocity produced by spalling action alone is quite moderate.

Plastic yield-line analysis of the wall segment subjected to the impulsive loading of the air blast indicates that for sufficiently large explosions substantial displacements and peak velocities can occur in typical shield walls. Thus for close-in explosions severe wall damage and/or breaching should be expected. Also coupling between spallation and the gross wall motion is possible. This occurs when debris produced by spalling action is ejected at high velocities by the rapid wall motion.

1. INTRODUCTION AND PROBLEM DESCRIPTION

The subject of this report is an investigation of the effects of explosive detonations on concrete walls and structures. The phenomena associated with such explosions in the vicinity of a structure are quite complex and in general involve a multiplicity of loadings, i.e., air blast, ground shock and direct shock in the case of contact placement of the explosive. In the current effort, attention is limited to the effects of air blast and the interaction of the explosion with the ground surface is not considered. This reduces a general three-dimensional problem to two dimensions, as shown in Figure 1, and simplifies the effort sufficiently so as to permit some analytical estimates.

Past efforts concerned with the effects of air blast on structures focus their attention primarily on the low and moderate overpressure range. It is usually assumed that the dimensions of the blast wave are large relative to those of the structure and that the shock wave is plane [1]. While such treatments are appropriate for very large explosions, such as those resulting from nuclear events, they are not applicable when moderate amounts of explosives (100 lb to 40,000 lb) are detonated in close vicinity of large structures (wall dimensions of the order of 100 feet). The blast loading in the latter case is often very local and the variation of blast wave strength in its interaction with the structure must be accounted for. Also, blast wave durations are shorter than the structure's clearing times, thus limiting the loads to a single wall. Past work for this case has been restricted to developing design procedures for explosive storage and manufacturing facilities [2]. The major emphasis in these procedures is to determine the overall response of heavily reinforced concrete walls to the impulse load generated by the explosion. Loading on the wall face

Figure 1. Schematic of Explosive Charge and Concrete Wall

is assumed to be uniform. Again while these procedures are applicable to the problem under consideration, modifications must be introduced to account for the nonuniformity of loading which is important for the case of large walls.

For explosions close to the structure, a direct strong shock wave is also transmitted into the wall due to the reflection of the air blast wave at the surface. Wall damage may result from this shock wave in the form of spallation on the inner wall surface after the wave reflects as a strong tension wave. Thus two aspects of air blast loading of concrete structures must be considered for the case of close-in explosive detonation. The first of these is the above described direct spallation which is an early-time effect produced by the shock reflection and is strongly dependent on the blast wave pressure-time history. It may lead to the formation of high velocity debris. The second effect is the overall response of a portion of the wall to the impulse of the blast wave. The wall motion produced by this impulse may continue long after the blast load ceases and thus is a late-time effect which is independent of the details of the pressure-time history and depends only on the impulse distribution over the wall. The wall deflection due to the impulse loading may result in secondary spallation called scabbing [2] which normally separates a layer of concrete down to the back face reinforcement. If the motions are sufficiently large, complete wall destruction may result by means of plug shear separation or complete fragmentation of the loaded area. Coupling may occur between the two damage mechanisms (direct spall and wall deflection by impulse) under certain conditions. This happens when substantial direct spallation of the back-face of the wall has taken place but the velocity of the formed debris is low in comparison to the velocity induced in the affected portion of the

wall by the impulse loading. It will then be the latter motion which leads to the high speed ejection of spallation debris formed by the action of the direct shock wave reflection.

The air blast loading of a typical wall segment is shown schematically in Figure 2 at two instances of time. In the first instant, blast wave reflection is still regular having a small angle of incidence while at the later time Mach reflection is indicated forming the familiar three shock configuration. This occurs at large angles of incidence (approximately angles larger than 40°). Also indicated in the figure are the shock waves induced in the wall by blast wave reflection and the stress waves resulting from their reflection at the back face of the wall.

Due to the complexity of the blast wave interaction with the wall, no analytical description of the blast loading is possible. Therefore, loading definition is obtained from experimental data systematized in the form of graphs and charts and collected over many years, primarily for defense applications. In the current work, Army Technical Manual TM5-1300 [2] is primarily used. Some inconsistencies exist in the data of this document; however, the information presented is reasonably complete. Where necessary, the data has been supplemented with information from other sources.

While a large amount of experimental data exists on concrete wall response to explosive detonation effects, the available information [3,4,5] is not systematized and cannot be readily employed for general applications. Test results were usually obtained with specific applications in mind and lack consistency. Therefore, in the current study primary reliance is placed on analytical predictions of wall response when subjected to air blast loading. Basic engineering methods are used to predict both direct spall and overall wall motion due to impulse action. While this approach

DEFINITIONS

p_s - Incident Air Shock
 p_r - Reflected Air Shock
 p_m - Mach Stem Shock
 p_α - Compression Wave

σ_R - Reflected Dilatation Wave
 τ_R - Reflected Shear Wave
 r - Radial Distance
 α - Angle of Incidence

(a) Regular Reflection Time t_1

(b) Mach Reflection Time t_2

Figure 2. Air Blast Interaction with a Wall

requires considerable simplification of the complex response phenomenon and the results may only be qualitatively correct, it has the merit of consistency and enables one to discern trends as well as obtain more generalized conclusions.

In limiting this study to the case of air blast loading, contact placement of the explosive on the wall is not considered. While such placement may result in the severest local damage, it does not lend itself to simple analytical investigation. Also, the multiple interactions of extremely strong (hundreds of kilobars) shock and stress waves requires material behavior data for the concrete and explosive which is not readily available. On the other hand it is believed that more reliable data exist, e.g., Refs. 3 and 4, on the action of contact explosives than is the case for air blast loading. Thus this data may be used to arrive at engineering estimates of damage. An obvious difficulty arising in this respect is the definition of contact placement. In the current effort the closest charge placement is arbitrarily limited to a scaled distance of $0.2 \text{ ft/lbm}^{1/3}$ or approximately 1.5 charge radii. While this distance is well within the fireball (6 charge radii) of the explosion, the peak reflected pressure at a wall ($\sim 68,000$ psi) is still sufficiently moderate so that direct interaction and impedance matching of explosive to wall material may be neglected. It should also be pointed out that attention is restricted in this study to the effects of bulk explosives. The effects of special charge geometries and in particular shaped charges are not considered. To further simplify the analysis it is also assumed that the explosive charge is of spherical shape. The pressure loading of the wall is assumed to be that of a rigid surface. Thus wall motion or deformation does not alter the loading. In light of the impedance mismatch, between air and wall material, and the shortness of a typical pressure loading (a couple of msec) this is a reasonable assumption.

Section 2 of this report outlines the procedures for predicting the gross motion of a portion of a reinforced concrete wall subjected to impulse loading resulting from explosive detonations in its close vicinity. Also presented are a number of specific and general calculational results, the latter in graphical form. Section 3 deals in a similar manner with some aspects of direct spallation induced by the initial shock wave reflection from the back-face of the wall. Finally, Section 4 presents general conclusions and recommendations based on the results of this study.

2. STRUCTURE RESPONSE

The present analysis is a "first cut" derivation of the curve defining W , the explosive charge weight needed to inflict serious structural damage to a shield wall, as a function of y , the distance from the outside face of wall to the center of the explosive charge (see Figure 1). Four major assumptions made for this analysis are: first, the shield wall is flat and uniform, and extends sufficiently far in all directions from the point nearest the explosion so that its boundaries can be neglected; second, the elastic behavior of the material is not significant and there is enough ductility to sustain large displacements under a constant ultimate load; third, the explosive charge is spherical, uncased and not in contact with the shield wall; and, fourth, the wall is prestressed or conventionally reinforced but without a liner and is of one of two designs as specifically described below. The analytical procedures used are outlined in sections 2.1 and 2.2; computational details are given in Section 2.3.

The structural analysis of the effects of the explosion consists of two phases: first, computation of the blast loading; and, second, evaluation of the wall resistance. In the first phase it is assumed that the wall provides no material strength to resist the impulse associated with the applied blast load. The only resistance available is provided by the inertia of the mass of the wall. In the present approach, given the total impulse and the total mass, a single velocity imparted to the entire wall segment under consideration is computed. During the resistance phase the ultimate load carrying capacity of the reinforced concrete structure is developed to provide a decelerating force which brings the wall back to rest. The total distance traveled is compared to a criterion based on the maximum deformation at which serious damage will be inflicted.

2.1 LOADING PHASE

The basic data for the characteristics of the blast loading, as well as the general approach to the evaluation of the blast effects, are taken from the Army Technical Manual TM5-1300 [2], which is based on weapons test data. Figure 4-5 of TM5-1300 provides the peak incident blast pressure in the direction of a blast wave moving radially from the center of an explosion of a spherical charge in free air. The peak incident pressure, p_{so} , is given as a function of the scaled distance, z , where $z = r/W^{1/3}$, r is the distance to the center of the explosion and W is the TNT equivalent charge weight. It should be noted that the curves referred to here are not non-dimensionalized although they are scaled with respect to $W^{1/3}$. Hence specific units: lb for W , ft for r , ft-lb^{-1/3} for z and psi for p_{so} , must be used. Figure 4-5 of TM5-1300 also provides the blast wave time of arrival, the positive pressure phase duration time and the negative pressure phase duration time.

The maximum pressure experienced by a flat surface that is in the path of the blast wave is the peak reflected pressure denoted by p_{ra} . This is dependent on the angle of incidence, α , which is the angle between the direction toward the center of the explosion and the direction of the normal to the surface of the wall, and on the value of p_{so} at the point under consideration. The pressure p_{ra} is given by $p_{ra} = C_{ra} p_{so}$ where the coefficient $C_{ra}(\alpha, p_{so})$ is experimentally determined. Figure 4-6 of TM5-1300 was used for obtaining C_{ra} values for α less than 40°; more recent data obtained by Carpenter and Brode [6] was used for α over 40°. Variations of the peak reflected pressure p_{ra} across the face of a wall, for a number of charge stand-off distances, are shown in Figure 3. With p_{ra} determined, Figure 4-5 of TM5-1300 is reentered to find a corresponding value of i_{ra} , the impulse

Figure 3. Variation of Peak Reflected Pressure Along Wall (z_0 - Scaled Stand-off Distance, $\text{ft}/\text{lbm}^{1/3}$)

associated with the positive reflected pressure. The one to one correspondence between p_{ra} and i_{ra} (as well as the negative impulse) postulated by TM5-1300 is accepted for the purposes of this study even though it is obvious that such a direct correspondence is not possible. The assumption is sufficiently good for scaled distances below $z = 1$. The total positive reflected impulse, I_r , provided by the blast to any section of wall can be computed by integrating i_{ra} values as computed above over the area of interest. Similarly, the total force, F , is obtained by integrating the reflected pressure, p_{ra} . Finally, under the assumption that only inertia is available in the loading phase to resist the blast impulse, the maximum unrestrained wall velocity, u , imparted to the entire wall segment under consideration is given by $u = I_r/M$ where M is the mass of the wall segment. A velocity of any section of wall accelerating without restraint can be computed as $u = i_{ra}/m$ where m is the mass per unit area of the wall. Velocities for various charge weights and distances are shown in Figure 4.

2.2 WALL RESPONSE AND RESISTANCE

The separation of the blast analysis into two phases as is done here is not necessarily intended to imply a separation in time, but rather it is a computational convenience. An implied assumption is that the motion of the wall does not affect the magnitude of the blast pressures. This assumption is valid for the present analysis because the duration of the impulse is short relative to the response time of the structure; it would not be valid for blast loadings where the impulse duration is much longer, as for example due to nuclear weapons. For the most extreme detonations considered here the wall will move only about 4 inches, typically less than 1 inch, during the application of the major part of the positive blast pressure and thus remains effectively rigid while the load is applied. In this time

Figure 4. Variation of Maximum Velocity with Charge Weight and Standoff Distance, Wall Type a

period the wall can be accelerated to a velocity of up to 500 ft/sec, far in excess of any velocity that it is able to restrain in the long run. Since the wall remains subjected to the full reflected pressure during the positive phase of the blast, it will absorb the full reflected impulse of the blast.

For the blast levels being considered the instantaneous forces resulting from the application of the positive reflected pressure are high enough to make any resistance provided by the strength of the wall negligible. That is, the loaded segment of the wall is free to move in response to the applied force. This is true for large/close-in explosions but becomes less valid as the charge weight decreases or the distance increases. For example, the peak reflected pressures from a 4 ton charge at 4 feet will produce a shear stress of 40,000 psi in a conventionally reinforced wall, which is about 100 times as high as the shear strength of the concrete. At the other extreme a 27 lb charge at 3 feet from a prestressed wall will produce shear stresses of about 1800 psi which is about the same as the shear strength of typical prestressed concrete. Hence, for the present range of interest, the positive phase forces generated are large enough to cause shear cracking at the periphery of the loaded segment of the wall; but of short enough duration that the segment will not move far enough to experience a significant geometry change.

Several mechanisms can be postulated to compute the maximum deflection of a wall segment, of mass M , which has an initial velocity, u , due to the impulse of the blast. Ultimate strength analysis, which is commonly used for the design of reinforced concrete structures, assumes plastic yielding under a constant applied force, R . Hence the deflection can be computed from the kinetic energy of the wall segment as $\Delta = \frac{1}{2} Mu^2/R$. A second mechanism considers the energy absorbed by the ductile deformation of the

reinforcing steel acting as a net to restrain the motion of the concrete wall. As a third mechanism, applicable to prestressed containments with contact liners, one can consider the ductile capacity of the full length of the unbonded post-tensioned prestressing tendons to absorb the energy of the blast initially absorbed by the wall segment. The first two mechanisms will be considered in more detail. The third will not be covered in this report since it will require a separate investigation into the behavior of concrete shells with steel liner plates anchored to the inside wall.

In order to present numeric results, two specific shield wall configurations are considered. Wall type a is a relatively thin conventionally reinforced wall; wall type b is a prestressed wall. More details on the assumed wall geometries and material properties is given in section 2.3. R, the capacity of the wall to resist blast loading induced motion, computed by ultimate strength analysis is between $2 \cdot 10^6$ lb, for wall type a, and $9 \cdot 10^6$ lb, for wall type b. The resulting displacement computed for wall type a is 2.7 feet for a 1000 lb charge of TNT detonated at a distance of 2 feet from the wall. Displacement for wall type b is about 1/6 of that for wall type a. The displacement decreases to about 1/3 if the detonation distance is increased to 10 ft. If the scaled detonation distance (distance divided by cube root of the charge weight) is fixed then the displacement is proportional to the charge weight raised to the 4/3 power. Figure 5 shows displacements of wall type a for various charge weights and distances.

Two criteria can be used to evaluate these displacements: limiting total displacement and limiting total rotation. If displacement is to be limited to one foot, for example, the maximum charge weights that can be tolerated at specific distances are shown in Figures 5 and 6 for wall types a and b respectively. Other displacement limits are also shown. The limit

Figure 5. Deflections and Deformation Criteria as a Function of Charge Parameters, for Wall Type a

Figure 6. Deflections and Deformation Criteria as a Function of Charge Parameters, for Wall Type b

on displacement can be considered a "functional" limit. Limits on rotation, on the other hand, are based on maximum reinforced concrete ductility.

TM5-1300 (Chapter 6) recommends that rotation at a yield line, as defined in conventional ultimate load analysis, be limited to 5° where structural integrity is to be maintained. The corresponding curves for maximum charge weight are shown in the figures noted above. If large deformations and scabbing are to be allowed then TM5-1300 (Chapter 5) recommends a 12° limit on yield line rotation and this is also shown on the figures. It can be seen that the critical explosive charge weight at any particular charge distance can change by an order of magnitude depending on the assumptions made. For example, for a two foot distance the computed charge weight can be between 170 and 1700 lb.

The second mechanism for computing the wall deflection noted above considers the energy associated with the deformation of the steel during the process of forming a blast crater shaped displacement. If, for simplicity, we assume a parabolic shaped crater (see section 2.3 for details), then the total energy absorbed by deformation, computed on the assumption that the displacements are large enough so that the elastic portion can be neglected, is dependent on the material and geometry of the wall and is proportional to the square of the deflection at the center. The energy is independent of the extent of the deformed area. For a conventionally reinforced wall (wall type a) under these assumptions, allowing a one foot maximum displacement, the curve for the acceptable charge weight as a function of distance, is essentially the same as that obtained for a three inch allowed displacement using the ultimate strength method (Figures 5 and 6). Allowing a two foot displacement would give the same curve as allowing a one foot displacement under the ultimate strength method.

2.3 BLAST COMPUTATIONS

Table I shows a portion of the computational data for the determination of timing, blast pressure, blast impulse values, etc. The full tabulations were carried out for four z values (0.2, 0.3, 0.5 and 1.0) and for incidence angles up to 80° with 5° increments. Much of this data is derived by interpolation of various graphs, hence the work was done by hand rather than by computer. The basic curves will have to be digitized for computer use so that analyses over a wider range of the parameters can be carried out.

The table gives values for various points on the face of the wall; i.e., at the point nearest the blast ($\alpha = 0$) and for concentric circles about this point. Most of the items in the table are scaled with respect to some power of the cube root of the charge weight. This scaling parameter will be referred to as q ; $q = w^{1/3}$. Its unit, $\text{lb}^{1/3}$, will be referred to as b . Parameters z_0 and z are then defined as y/q and r/q . Dimensions x , y , r and α are shown in Figure 1. Figure 4-5 of TM5-1300 was used to get values of peak incident pressure, p_{so} , time of arrival, t_a , positive pressure phase duration, t_+ , and negative pressure phase duration, t_- , corresponding to various values of the scaled distance, z . Note that the time parameters are scaled with respect to q , whereas the pressure is given directly. Factors $C_{r\alpha}$ to convert peak incident pressure, p_{so} , to a reflected pressure applied against any point on the wall are provided by experimental data. $C_{r\alpha}$ is a function of both α and p_{so} . The reflected pressure, $p_{r\alpha}$, which is the pressure loading the wall, is computed by $p_{r\alpha} = C_{r\alpha} p_{so}$. Figure 4-6 of TM5-1300 provides curves for $C_{r\alpha}$, however for incidence angles over 40° these do not agree with more recent data provided by Carpenter and Brode [6]. The curves used in this analysis were a composite of these two sources with the

TABLE I. SAMPLE BLAST COMPUTATIONS ($b = 1b^{1/3}$, $q = w^{1/3}$)

$z_o = y/q$	0.2				0.5				ft/b
α	0	20	40	60	0	20	40	60	deg.
$z = r/q$	.200	.213	.261	.400	.500	.532	.653	1.000	ft/b
x/q	.000	.073	.168	.347	.000	.182	.420	.867	ft/b
t_a/q	3	3	7	16	23	26	37	73	10^{-6} sec/b
$(t_a + t_+)/q$	44	44	48	59	69	73	87	138	"
$(t_a + t_+ + t_-)/q$	6300	6300	6300	6350	6450	6450	6600	7050	"
P_{so}	5500	5300	4400	2950	2250	2100	1650	850	lb/in ²
C_{ra}	12.3	10.9	8.2	2.0	10.5	9.3	7.1	2.0	-
P_{ra}	67.7	57.8	36.1	5.9	23.0	19.5	11.7	1.7	10^3 lb/in ²
i_{ra}/q	3.25	2.65	1.55	0.30	1.00	0.85	0.53	0.125	lb/in ² ·sec/b
x/q	.0	.4	.8	1.2	.0	.4	.8	1.2	ft/b
P_{ra}	68	5	2	1	23	12	2	1	10^3 lb/in ²
i_{ra}/q	3.25	.25	.16	.10	1.00	.55	.17	.10	lb/in ² ·sec/b
z_a	.025	.450	.900	1.300	.025	.450	.900	1.300	ft/b
P_r/q^2	19	1680	2400	2890	7	1420	2750	3240	$10^3 \cdot lb/b^2$
I_r/W	0.9	76	126	171	0.3	63	135	180	lb·sec/b ³
T/q	10.2	49.5	35.4	29.5	3.5	41.7	40.5	33.1	10^3 lb/in/b
i_a/q	3.25	.83	.34	.22	1.00	.69	.37	.24	lb·sec/in ² /b
Δ/q^4	9	189	131	116	1	133	150	128	10^{-6} ft/b ⁴
θ/W	362	420	145	73	34	296	167	99	$10^{-6}/b^3$
K/q^4	222	4622	3195	2833	21	3257	3669	3139	in·lb/b ⁴

former used without change in the range $\alpha < 40^\circ$ but the latter modifying the basic TM5-1300 curves in the range $\alpha > 40^\circ$.

Finally the positive pressure phase impulse density, i_{ra} , must be determined. This is defined by the integral of the reflected pressure over time from t_a to $t_a + t_+$. The method for obtaining i_{ra} used here is that specified in TM5-1300; namely, a one to one correspondence is assumed between p_{ra} and i_{ra} . It should be noted, however, that such a correspondence does not appear justified since it ignores the influence of the phase duration and of the incidence angle. For example, given $p_{ra} = 1000$ psi, TM5-1300 prescribes $i_{ra} = 0.093$ psi-sec/b. However, this p_{ra} can be the result of a normal incidence with $\alpha = 0^\circ$, $z = 2.45$ ft/b and $t_+ = 280$ μ -sec/b; or, it can be the result of $\alpha = 60^\circ$, $z = 1.55$ and $t_+ = 95$. The pressure wave shape would have to be drastically different in these two cases to give the same impulse for the same peak pressure with a 3:1 ratio in durations, t_+ . The impulse is reasonable for the $\alpha = 0$ case but is practically impossible for the other case.

Although this procedure is physically suspect, in the area of primary interest here, which is the low z range ($z < 1$), the positive phase duration curve is rather flat, ranging from 40 μ -sec/b at $z = 0.2$ to 63 at 1.0. Hence, the discrepancy is less severe.

Another difficulty with the TM5-1300 curves should be pointed out, namely t_+ and i_r curves in the low z range do not correspond well with more recent data [6]. Moreover, they are not internally consistent as can be seen by looking at $z = 0.2$ where $p_r = 68,000$ psi, t_+ is 40 μ -sec/b and $i_r = 2.1$ psi-sec/b. With the extreme assumption of a linear pressure wave shape i_r can be at most 1.36 psi-sec/b. Nevertheless the TM5-1300 curves are used in this study because no other authoritative source of comprehensive

data is available. Moreover, the inaccuracies are on the conservative side; that is the blast loading is overestimated.

For further computation the p_{ra} and i_{ra} values as shown in Table I are plotted against x/q , the scaled distance along the wall, and are then retabulated with respect to this parameter. The plot for p_{ra} is shown in Figure 3. A small part of this new tabulation is shown in Table I. The p_{ra} and i_{ra} values are then integrated over the area of the wall segment in question. The integration is carried out over circles with increasing radii in order to arrive at the critical choice of circle size. The integration computation is done by a computer routine using an averaging technique. Table I shows the integration results listing z_a , radius of the circle to which the actual integration extended; P_r , the peak reflected force (an approximate value since it ignores the variation in t_a with respect to z); and, I_r the total impulse absorbed by the wall segment. The shear force per unit length around the circumference, T , associated with the peak force and the average impulse per unit area of wall segment, i_a , are computed.

Further computations are dependent on the details of the wall under consideration. Two specific configurations are considered here: first, wall type a, a conventionally reinforced concrete wall of 30 inch thickness, concrete strength $f'_c = 4500$ psi, reinforced with 60 ksi steel, 1.5 sq. in./foot in each face in each direction; and, second, wall type b, a prestressed concrete wall of 42 inch thickness, prestressed to 1800 psi horizontally and 1000 psi vertically.

The average shear stress can be computed as $T/30$ in. and $T/42$ in. for wall a and wall b respectively. For any of the z values considered, the wall segment is subjected to shear forces, T/q , of at least $25 \cdot 10^3$ lb/in./b

which gives a shear stress of 600 psi/b for wall b. With a charge as low as 27 lb ($q = 3b$) the computed shear stress would be 1800 psi which is considerably above the shear strength of the prestressed concrete of wall type b. For wall type a the corresponding stress is 2500 psi, an order of magnitude more than the shear strength for conventionally reinforced concrete. The maximum T/q value is about $60 \cdot 10^3$ lb/in./b giving 2000 and 1400 psi/b respectively, far in excess of any available concrete shear strength even with a very small charge.

The initial velocity due to the impulse of the blast computed as i_a/m . The mass per unit area of wall, m , is 0.0809 and 0.1132 (lb-sec/in.²)(ft/sec) for walls a and b respectively. Using $i_a/q = 3.25$ (lb/in.²)(sec/b) this gives a maximum velocity $u/q = 40$ (ft/sec)/b to be developed in about 40μ -sec/b giving an approximate maximum displacement during the time of application of the blast load, $\Delta/q^2 = 0.5 u t_+ / q^2$, of about 0.01 in./b². For a 4 ton blast ($q = 20$ b) this gives a displacement of 4 inches. More typically i_a/q is about 1 with about the same time, hence the deflections at the end of the positive phase are generally about 1 inch or below. Deflections computed on the basis of peak reflected blast force, P_r , are much smaller, reflecting the internal inconsistency of Figure 4-5 of TM5-1300 noted previously. For example, assuming a linear positive blast pressure variation and double integrating up to t_+ gives a displacement of 0.005 in./b² at a velocity of only 17 (ft/sec)/b under the above conditions.

The maximum velocities as a function of charge weight and distance, attained by a type a wall are shown in Figure 4. For wall type b, the velocities will be about 70% of these.

Yield line analysis [7] of a flat circular isotropic plate loaded at the center shows that the ultimate load capacity of $4\pi M_u$ if the plate is clamped around the circumference and $2\pi M_u$ if it is supported but free to rotate. If the load is uniformly distributed over the entire plate then the ultimate capacity is 3 times as high. Here, M_u is the ultimate moment capacity assuming the plate is equally reinforced on both faces, or the average ultimate moment if the reinforcement percentages are different. The wall loading under consideration here is expected to form a circular yield line pattern similar to that for a clamped circular plate due to the restraining effect of the inertia of the entire wall surrounding the loaded wall segment. This would give an ultimate load capacity of $4\pi M_u$, which should be reduced somewhat due to the fact that the circumferential support condition is somewhat less than clamped and increased somewhat because the load is distributed and not concentrated. In the following analysis, $10 M_u$ is used.

For wall type a the ultimate moment capacity is computed by conventional reinforced concrete principles as $M_u = A_s f_s (d - \frac{1}{2} a)$. Taking the steel cross-sectional area $A_s = 1.5 \text{ in.}^2/\text{ft}$, the steel yield strength $f_s = 60,000 \text{ lb/in.}^2$, the effective depth $d = 27 \text{ in.}$ and the concrete compression block depth $a = 2 \text{ in.}$, $M_u = 0.195 \cdot 10^6 \text{ ft-lb/ft}$ and the ultimate load is $1.95 \cdot 10^6 \text{ lb}$. Wall b is considerably more complicated. The ultimate moment capacity must be computed on the basis of the full plastic strength of the prestressing tendons which is taken as $2 \cdot 10^6 \text{ lb}$ (170 - 1/4" diameter strands at 240,000 psi). Vertical tendons are assumed to be at 48 inch spacing located on the wall centerline. Assumed horizontal steel consists of tendons at 27 inch spacing located 6 inches away from the outside face, and #10 bars of 60,000 psi steel at 12 inch spacing near the inside face. The

resulting ultimate moments are $0.7 \cdot 10^6$ ft-lb/ft vertically; and, $2.0 \cdot 10^6$ and $0.24 \cdot 10^6$ horizontally for the two directions of bending. An average value of $0.9 \cdot 10^6$ will be used giving an ultimate load capacity of $9 \cdot 10^6$ lb. The above calculations for a prestressed wall follow current practice; however, they are merely a rough idealization of the actual behavior of a post-tensioned wall since the ductility of the entire length of the post-tensioned unbonded prestressing tendon can be counted on to resist local blast effects, particularly if a substantial and sufficiently anchored liner is available to provide some local restraint.

The displacement of the wall segment can now be computed. Recall that the mass, m , was previously computed as 0.0809 and $0.1132 \cdot (\text{lb-sec/in.}^2) / (\text{ft/sec})$ for wall type a and b respectively. The displacement is $\Delta = \frac{1}{2} M u^2 / R$ where M is the mass of the segment in question $M = \pi x^2 m$, the velocity is given by $u = i_a / m$ and R is the resisting force computed above. So, $\Delta = x^2 i_a^2 / (2mR/\pi)$. The factor $2mR/\pi$ is dependent on the wall type and is $(27 \text{ lb-sec/in.}^2)^2 \text{ ft}$ and $(67 \text{ lb-sec/in.}^2)^2 \text{ ft}$ for wall type a and b respectively. The displacement for wall a is given in Table I and in Figure 5; that for wall b is approximately 6 times smaller.

If displacement is limited to a fixed value, then the computed displacement can be used to compute the maximum charge weight for any given scaled distance. For example, for wall type a with $z_o = 0.2$ the maximum computed scaled deflection (which occurs at $z_a = 0.25$) is $270 \cdot 10^{-6} \text{ ft/b}^4$, which will be less than 1 foot if $q^4 < 3700 \text{ b}^4$; i.e., if the charge weight is less than 475 lb.

Similarly, if we limit the rotation to, say, 5° , then the θ value, computed as Δ/x , as shown in Table I is used. Again for wall a with $z_o = 0.2$, maximum θ occurs at $z_a = 0.125$ and has a scaled value of $1,168 \cdot 10^{-6} / \text{b}^3$

which gives 5° (0.08727 radians) at $w = 75$ lb. Graphs for these and other limits derived as indicated above are shown in Figures 5 and 6.

In computing the energy capacity associated with the ductility of the reinforcing steel it is assumed that the wall displacement in ρ, θ polar coordinates is of the form

$$\Delta = (1 - \rho^2/r^2)\Delta_{\max} \quad (2.1)$$

where $\Delta_{\max}$ is the deflection at the center and r is the extent of the deformed segment of the wall. Using x, y for rectangular wall coordinates this gives

$$\Delta = [(r^2 - x^2 - y^2)/r^2]\Delta_{\max} \quad (2.2)$$

The strain of a fully bonded horizontal bar is approximated by

$$\epsilon = \frac{1}{2} (d\Delta/dx)^2 = 2x^2\Delta_{\max}^2/r^2 \quad (2.3)$$

Assuming rigid-plastic steel deformation at a yield stress of σ_y , the total strain energy for the horizontal steel is given by

$$\int_{-\pi/2}^{\pi/2} \int_{-r \cos \theta}^{r \cos \theta} \sigma_y 2x^2\Delta_{\max}^2/r^4 s \cos \theta dx d\theta \quad (2.4)$$

where s is the steel area per unit length of wall cross-section. Assuming s is the same for both horizontal and vertical steel and carrying out the integration, the total strain energy is found to be $\pi s \sigma_y \Delta_{\max}^2$. For wall type a this gives $6.8 \cdot 10^6$ in.-lb using $s = 2$ 1.5 in.²/ft; $\sigma_y = 60,000$ psi and $\Delta_{\max} = 1$ foot. Note that this analysis is not carried out for wall type b since the prestressing tendons are assumed not to be bonded. The kinetic energy of the wall segment is computed as $K = I_r^2/2M$, where M is the wall segment mass defined previously. This is tabulated in Table I. For example, at $z_0 = 0.2$ ft/b, the maximum value of K/q^4 is 6617 in.-lb/b⁴ which occurs at $z_a = 0.25$. Setting K to $6.8 \cdot 10^6$ in.-lb gives $q = 5.66$ b or

$w = 182 \text{ lb}$ as the maximum charge weight. Alternatively, one can note that the $6.8 \cdot 10^6 \text{ in.-lb}$ capacity is about the same as that available using the ultimate strength mechanism ($2 \cdot 10^6 \text{ lb}$) described previously with about 3 inches displacement; thus, the criterion considered here is equivalent to the 3 in. deflection criterion shown in Figure 5.

2.4 DISCUSSION OF WALL RESPONSE RESULTS

The results of this part of the study are summarized in Figures 5 and 6. While these results are strongly dependent on various assumptions noted throughout the preceding sections, they indicate that severe damage can result from fairly moderate amounts of explosives detonated in the vicinity of reinforced concrete walls. Moreover, the possibility of an explosive detonated in contact with the wall, or of a shaped charge, is not considered. The latter may cause unacceptable damage with a weight that is an order of magnitude smaller than indicated by the figures noted above. For example, test results from the National Defense Research Committee [3] indicate that a conventionally reinforced wall (type a) can be penetrated with a cone shaped charge of as little as 9 lb of TNT. For a thicker wall (type b) the corresponding weight would be 19 lb.

3. DIRECT SPALLATION

As indicated earlier spallation may occur on the back face of a concrete wall subjected to air blast on its front face. The phenomenon of spall or separation of layers of material occurs when strong rarefaction or tension waves are reflected into the wall from the free surface at the back face and interact with the decaying shock or compression wave traversing the wall. The mechanisms underlying spall and crack formation are exceedingly complex and depend on many material parameters as well as loading variables [8 and 9]. Thus, in general, the spalling process is time dependent. However, for brittle materials, such as concrete, whose tensile strength is but a small fraction of the compressive strength (5% - 7%), spalling can be assumed to occur instantaneously whenever the dynamic tensile limit is exceeded [10]. Assuming further that the material is restricted to linear elastic response it is possible, under certain loading conditions, to estimate analytically the spallation response of a concrete wall. In particular, it is possible to obtain analytical solutions for the reflections of plane compression waves from a free surface, both for normal and oblique incidence [10], at least for wave forms that can be described by simple analytical expressions.

The shock waves induced in a wall by air blast reflection are in general curved and spread in a circular fashion from the first contact point (see Figures 1 and 2). However, the reflection of the center portion of these waves approximate the normal reflection of a plane wave. Similarly, the reflection of the far portions of the curved wave approaches that of a plane wave reflecting obliquely. These approximations become better as the radius of curvature of the compression wave increases, i.e., for large charges where the standoff distance from the wall is large. In the current application we use the plane wave approximation to estimate the size (thickness)

and velocities of the air blast induced spalls. To make this estimate more realistic, the strength of the incident compression wave arriving at the backface of the wall is, however, correctly attenuated taking into account its spherical divergence.

3.1 BLAST AND SHOCK WAVE FORMS

In contrast to the late-time wall response (discussed in Section 2) which depends solely on the total impulse imparted to the wall segment, spallation is strongly dependent on the wave form (pressure-time history) of the shock wave induced in the wall by blast wave reflection. Considering that the static tensile rupture strength of concrete is around 400 psi [11] and the dynamic strength may be considerably higher our interest lies mainly in the high pressure range of the blast wave. Little information is available concerning wave reflection in this range. In fact very recent studies [6] indicate that wave forms are rather complex particularly in the Mach stem reflection region where two pressure peaks have been observed. To carry out analytical estimates of spallation, simple mathematical wave form descriptions must be provided. The blast wave decay is in general of exponential form and the only analytical descriptions for the high pressure region have been derived for the case of nuclear explosions in air [12]. These have been found to be in reasonably good agreement with wave forms of a TNT explosion obtained by detailed numerical calculations [13]. Therefore, in the absence of readily usable better information, the wave forms of Brode [12] are employed for the current application. The analytical approximation of these wave forms is given as

$$p_1(\tau) = p_{so}(1 - \tau)(ae^{-\alpha\tau} + be^{-\beta\tau} + ce^{-\gamma\tau}) \quad (3.1)$$

Here p_1 is the time dependent air blast pressure, p_{so} is the peak pressure, $\tau = t/t_+$, t is time measured from the instant of shock arrival,

and t_+ is the positive phase duration of the wave. The parameters a , b , c , α , β , and γ depend on peak pressure p_{so} and are available in graphical form (Figure 24) in Reference 12. Consistent with the assumptions for impulse loading [2] it is assumed that the duration of the reflected wave is equal to that of the incident wave and that the decay of both waves is similar. Thus, the parameters corresponding to the incident peak overpressure are used to describe the reflected wave. The peak reflected overpressure, both for normal and oblique reflection $p_{r\alpha}$, is obtained from experimental data presented in Reference 2 (Figure 4-6). This data is supplemented for the Mach reflection region by information from Reference 6 (Figure 8). Some of these results are summarized in Figure 3. Similarly all other blast wave data, such as positive phase durations t_+ , are taken from Reference 2 (Figure 4-5) where they are presented in terms of scaled variables with $W^{1/3}$ (charge weight-lb $_m^{1/3}$) as the scaling factor. Based on the assumptions outlined above, Eq. 3.1 can be directly employed to give the wave form for the reflected pressure decay p_r as a function of the peak reflected pressure $p_{r\alpha}$.

To obtain the time history of the compression wave incident on the back face of the wall, it is assumed that the wave form remains similar to that of the air blast and the peak pressure in the wall decays linearly with distance through the wall. This is consistent with a linear elastic material behavior and constant wave speed as well as with the spherical divergence of the pressure wave. Hence for any angular position α (angle of incidence) the value of the peak compression arriving at the back face of the wall p_α is given in terms of the reflected pressure at the front face as:

$$p_\alpha = p_{r\alpha} \left(\frac{r}{r + h/\cos \alpha} \right) \quad (3.2)$$

Here r is the radial distance from the point of burst to the wall at angle α and h is the wall thickness. Since $r = y/\cos \alpha$, where y is the normal distance from the explosion point to the wall, Eq. (3.2) can be written irrespective of the angle of incidence α as follows:

$$p_{\alpha} = p_{r\alpha} \left(\frac{y}{y+h} \right) = p_{r\alpha} \left(\frac{z_o}{z_o + z_w} \right) \quad (3.3)$$

where the second equality is simply written in terms of scaled variables. Once p_{α} is determined the decay of the compression wave at angle α is obtained by the assumption of wave form similarity, i.e., an expression similar to Eq. (3.1) applies.

$$p(\tau) = p_{\alpha} (1 - \tau) (a'e^{-\alpha'\tau} + be^{-\beta'\tau} + c'e^{-\gamma'\tau}) \quad (3.4)$$

The positive phase duration t_+ is consistent with the load duration and thus the peak incident pressure p_{so} . However, to account approximately for the wave decay through the wall the wave shape parameters ($a', b', c', \alpha', \beta', \gamma'$) are based on an incident peak pressure p'_{so} , which upon reflection, at angle α , would result in a peak reflected pressure p_{α} , equal to the strength of the compression wave at the backface of the wall. In deriving (3.4) it has been tacitly assumed that the wave speed in the concrete wall is equal to the blast wave speed in air and that precursor or wave lag phenomena are negligible. For the pressure range of interest this is approximately valid both speeds being of the order of 10,000 fps.

While equation (3.4) appears simple it is not possible to obtain analytical spall solutions with this wave form. Therefore, the decaying pressure wave is locally approximated by still simpler forms. Specifically two forms are used; an exponential and a power law, i.e.,

$$p(\tau) = p_{\alpha} e^{-\gamma\tau} \quad (3.5)$$

and
$$p(\tau) = p_{\alpha} (1 - \tau^{\gamma}) \quad (3.6)$$

Here the exponent γ varies along the decaying compression wave and is evaluated by matching for each spallation the values of pressure obtained from (3.5) or (3.6) to those obtained by Eq. (3.4).

3.2 SPALL FOR NORMAL INCIDENCE

The reflection of a plane compression wave from a free surface at normal incidence is a one-dimensional problem as shown in Figure 7. The instant of wave arrival at the free surface is illustrated in Figure 7a. Assuming that the material is linearly elastic with constant wave speed, the condition of a stress free boundary implies that a tension or rarefaction wave, equal in strength and wave shape to that of the compression wave, is reflected back into the wall material as illustrated in the figure. The actual state of stress of any location is then the algebraic sum of the two stress waves (compression and tension). Whenever this net stress σ_N at the head of the reflected tensile wave equals the dynamic tensile rupture strength σ_T of the material, spallation will occur as illustrated in Figure 7. Designating the stress values of the compression wave by p (positive quantities) one can then write for the instant of the first spall

$$\sigma_T = \sigma_{N_0} = \sigma_0 - p_1(t_1) = p_0 - p_1(t_1) \quad (3.7)$$

Here the subscripts 0 refer to the peak initial value of stress in the wave and p_1 is the stress in the compression wave at the location of the first spall. The values of σ_0 and p_0 are obviously equal. The time t_1 is not the physical time (after wave reflection at the free surface) at which spall occurs, but rather measures the time along the wave form of the compression wave when p_1 is just equal to the value satisfying Eq. (3.7).

Figure 7. Spall Configuration for Normal Incidence

Considering the symmetry of the waves and the constant wave speed it can be easily demonstrated that time t_1 is given by

$$t_1 = \frac{2\delta_1}{c} \quad (3.8)$$

where δ_1 is the thickness of the first spall and c is the wave speed.

It should be evident that if the initial peak compression is much larger than the tensile rupture strength σ_T and the wave form is decaying with time then it is possible to obtain multiple spalls. In fact, the theoretical number of spalls n is given as

$$n \leq \frac{p_0}{\sigma_T} \quad (3.9)$$

Since a new free surface is generated every time a spall appears, the reasoning applied to the first spall can be extended to all spalls. Thus, Eq. (3.7) can be generalized for an arbitrary spall k as follows:

$$\sigma_T = \sigma_{N_k} = p_{k-1}(t_{k-1}) - p_k(t_k) \quad (3.10)$$

Here p_k is the value of the compression wave when the k 's spall occurs and p_{k-1} is the value corresponding to that for the preceeding spall. Again considering wave symmetry and constant wave speed the time t_k along the wave form corresponding to stress p_k can be simply obtained from

$$t_k = \frac{2(\delta_1 + \delta_2 + \dots + \delta_{k-1} + \delta_k)}{c} \quad (3.11)$$

From the above it can be seen that the thicknesses of the spalls depend strongly on the wave form of the compression wave.

The velocity of the spall layers for this simple one-dimensional case is obtained by equating the momentum of the spall layer to the portion of

the impulse imparted to it by the compression wave and still trapped in it at the time of spall. Expressing all quantities per unit area one can write for the first spall

$$i_1 = \int_0^{t_1} p(t) dt = \rho_w \delta_1 u_1 \quad (3.12)$$

where i_1 is the trapped impulse, ρ_w the density of the wall material and u_1 the velocity of the first spall layer. By simple analogy, the velocity u_k for any arbitrary spall k can be obtained from:

$$i_k = \int_{t_{k-1}}^{t_k} p(t) dt = \rho_w \delta_k u_k \quad (3.13)$$

Again, the strong dependence of spall velocity on the form of the compression wave can be readily discerned. To derive specific expressions for spall thickness and velocity the wave form approximations discussed in Section 3.1 must now be employed.

3.2.1 Calculation of Spall for Exponential Wave Forms

Considering first the exponential wave form given by Eq. (3.5) one obtains from Eq. (3.7) for the first spall

$$p_0 - \sigma_T = p_1(\tau_1) = p_0 e^{-\gamma \tau_1} = p_0 e^{-\gamma \frac{2\delta_1}{ct_+}}$$

The last expression is obtained from equation (3.8) and the relationship

$\tau_1 = t_1/t_+$. Solving for δ_1 yields

$$\delta_1 = \frac{ct_+}{2\gamma} \ln\left(\frac{p_0}{p_0 - \sigma_T}\right) \quad (3.14)$$

Proceeding similarly for the second spall using Eqs. (3.10) and (3.11) one obtains

$$\begin{aligned}\sigma_T &= p_1 - p_2 = p_o e^{-\gamma \tau_1} - p_o e^{-\gamma \tau_2} \\ &= p_o e^{-\frac{2\gamma \delta_1}{ct_+}} \left(1 - e^{-\frac{2\gamma \delta_2}{ct_+}}\right) = (p_o - \sigma_T) \left(1 - e^{-\frac{2\gamma \delta_2}{ct_+}}\right)\end{aligned}$$

Solving for δ_2 yields

$$\delta_2 = \frac{ct_+}{2\gamma} \ln \left(\frac{p_o - \sigma_T}{p_o - 2\sigma_T} \right) \quad (3.15)$$

In the same manner the spall thickness for an arbitrary spall k is calculated to be:

$$\delta_k = \frac{ct_+}{2\gamma} \ln \left[\frac{p_o - (k-1)\sigma_T}{p_o - k\sigma_T} \right] \quad (3.16)$$

The velocity of the first spall is obtained from the impulse integral (3.12) which can be written as

$$\rho_w \delta_1 u_1 = t_+ p_o \int_0^{\tau_1} e^{-\gamma \tau} d\tau = \frac{p_o t_+}{\gamma} (1 - e^{-\gamma \tau_1})$$

Remembering that $\tau_1 = \frac{2\delta_1}{ct_+}$ one obtains for the velocity

$$u_1 = \frac{p_o t_+}{\gamma \rho_w \delta_1} \left(1 - e^{-\frac{2\gamma \delta_1}{ct_+}} \right) \quad (3.17)$$

A similar operation can be carried out for arbitrary spall k employing Eq. (3.13) and using expression (3.11); this yields

$$u_k = \frac{t_+ [p_o - (k-1)\sigma_T]}{\gamma \rho_w \delta_k} \left(1 - e^{-\frac{2\gamma \delta_k}{ct_+}} \right) \quad (3.18)$$

3.2.2 Calculation of Spall for Power Law Wave Forms

Substituting the compression wave approximation (3.6) into Eq. (3.7) and using Eq. (3.8) yields the following relationship for the first spall thickness

$$\delta_1 = \frac{ct_+}{2} \left(\frac{\sigma_T}{p_o} \right)^{1/\gamma} \quad (3.19)$$

For the second spall using Eq. (3.10) and (3.11) together with expression (3.6) we can write

$$\begin{aligned} \sigma_T &= p_1 - p_2 = p_o [-\tau_1^\gamma + \tau_2^\gamma] \\ \text{or} \quad \frac{\sigma_T}{p_o} &= - \left(\frac{2\delta_1}{ct_+} \right)^\gamma + \left[\frac{2\delta_1}{ct_+} + \frac{2\delta_2}{ct_+} \right]^\gamma = - \frac{\sigma_T}{p_o} + \left[\left(\frac{\sigma_T}{p_o} \right)^{1/\gamma} + \frac{2\delta_2}{ct_+} \right]^\gamma \end{aligned}$$

where Eq. (3.19) was used to obtain the last expression. Solving for δ_2 yields

$$\delta_2 = \frac{ct_+}{2} \left[\left(\frac{2\sigma_T}{p_o} \right)^{1/\gamma} - \left(\frac{\sigma_T}{p_o} \right)^{1/\gamma} \right] \quad (3.20)$$

A similar relationship exists for all subsequent spalls, leading to the following general equation for the k's spall thickness

$$\begin{aligned} \delta_k &= \frac{ct_+}{2} \left\{ \left(\frac{k\sigma_T}{p_o} \right)^{1/\gamma} - \left[\frac{(k-1)\sigma_T}{p_o} \right]^{1/\gamma} \right\} \\ &= \frac{ct_+}{2} \left(\frac{\sigma_T}{p_o} \right)^{1/\gamma} [k^{1/\gamma} - (k-1)^{1/\gamma}] \end{aligned} \quad (3.21)$$

The spallation velocities again obtain from the impulse Eqs. (3.12) or (3.13).

For the first spall we find after integration and substitution of (3.8)

$$\rho_w \delta_1 u_1 = t_+ p_o \tau_1 \left(1 - \frac{\tau_1^\gamma}{\gamma + 1} \right) = t_+ p_o \frac{2\delta_1}{ct_+} \left[1 - \frac{1}{\gamma + 1} \left(\frac{2\delta_1}{ct_+} \right)^\gamma \right]$$

This simplifies finally by using (3.19) to

$$u_1 = \frac{2p_o}{c\rho_w} \left[1 - \frac{1}{\gamma + 1} \frac{\sigma_T}{p_o} \right] \quad (3.22)$$

The same procedure can be applied to any spall and after much simplification one obtains for the velocity of the k's spall

$$u_k = \frac{2p_o}{c\rho_w} \left\{ 1 - \frac{1}{\gamma + 1} \frac{\sigma_T}{p_o} \left[k + \frac{1}{\left(\frac{k}{k-1}\right)^{1/\gamma} - 1} \right] \right\} \quad (3.23)$$

It is interesting to note that the spall velocities explicitly do not depend on the blast wave duration t_+ and thus the explosive weight. The velocity depends, however, on p_o the peak compression stress which in turn depends somewhat on explosion size by virtue of Eq. (3.3). The same holds also for the exponential wave form discussed in Section 3.2.1 as can be easily verified.

3.3 SPALL FOR OBLIQUE INCIDENCE

When a plane compression wave in an elastic medium strikes a free surface obliquely the reflection process becomes considerably more complex than in the case of normal incidence. To maintain a stress free boundary condition at the surface two waves must be reflected back into the material, namely a dilatational or longitudinal wave and a transverse or shear wave [14]. The transverse wave differs from a dilatational wave in that the particle motion behind the wave is normal rather than parallel to the direction of wave propagation. A typical reflection process of this type is illustrated in Figure 8. For linear elastic media the reflection angle for the longitudinal wave must be equal to α the angle of incidence because its wave speed is equal to that of the incident wave. The transverse wave having a different wave speed c' reflects, however at a different angle β in order to stay in contact with the point of wave incidence as it moves across the surface. The relationship between these angles, wave speeds, and the Poisson's ratio ν

p_α - Incident Compression Wave
 σ_R - Reflected Dilatational Wave
 τ_R - Reflected Shear Wave

Figure 8. Free Surface Reflection of a Compression Wave at Oblique Incidence

of the material can be obtained from Snell's law [14] and is written as

$$\frac{\sin \alpha}{\sin \beta} = \frac{c}{c'} = \left[\frac{2(1 - \nu)}{1 - 2\nu} \right]^{1/2} \quad (3.24)$$

Concerning the strength of the reflected waves, these are obtained from the condition that the sum of the resultant stresses normal to the surface must be zero. Using a reflection coefficient η_R it can be shown [10] that the following relationships hold.

$$\sigma_R = \eta_R \sigma_I \quad (3.25)$$

$$\tau_R = [(\eta_R + 1) \cot 2\beta] \frac{c}{c'} \sigma_I \quad (3.26)$$

$$\eta_R = \frac{\tan \beta}{\tan \beta} \frac{\tan^2 2\beta - \tan \alpha}{\tan^2 2\beta + \tan \alpha} \quad (3.27)$$

Here σ_I is the amplitude of the incident dilatation wave and σ_R and τ_R are the amplitudes of the reflected dilation and shear wave respectively.

Equation (3.24) implies that for physically acceptable values of ν the dilatational wave speed is always larger than the transverse wave speed and thus angle β is smaller than angle α . The above equations also indicate that the reflection process is independent of the amplitude of the incident wave and is only a function of material properties and the angle of incidence α . Using typical material properties for concrete [11,15] values of the angle β , and the ratios of the amplitudes of reflected to incident waves are obtained. The following properties are assumed for concrete:

Compressive Strength $S_c = 4,000$ psi

Static Tensile Strength $S_T = 400$ psi

Poisson's Ratio $\nu = 0.15$

Dilatational Wave Speed $c = 10,000$ fps

The results are shown in Figure 9. The most interesting feature is that the reflection factor η_R (Fig. 9b) changes sign twice as the angle of incidence varies from zero to 90 degrees. This implies that the phase of the reflected dilatation wave changes, ie. for angles up to approximately 45° a compression wave reflects as a tension wave, while for angles larger than this value (except for angles close to 90°) it will reflect as a compression wave. Since reflections of blast waves are very weak for angles approaching 90° (glancing incidence) the above results indicate that spallation in concrete as produced by the reflected tension wave is limited to incidence angles no larger than 45°. Similar results have been obtained for other materials having Poisson's ratios less than 0.25 [16].

Assuming that spallation occurs before arrival of the shear wave one can proceed similarly as for the case of normal incidence, taken however into account geometric modifications. The geometry of a typical oblique spall is shown in Figure 10. Also shown are the profiles of the compression and reflected tension wave. Introducing again the pressure notation p (positive quantities) for the stress in the incident compression wave and letting η be the absolute value of the reflection coefficient η_R one can write for the first spall

$$\sigma_T = \sigma_R - p_1(t_1) = \eta p_\alpha - p_1(t_1) \quad (3.28)$$

where the second expression obtains from (3.25) together with the identity $\eta_R \sigma_I = \eta p_\alpha$. The thickness of the spall δ_1 is now related to the time t_1 (when the pressure is p_1) by the following modified equation:

$$t_1 = \frac{2\delta_1 \cos\alpha}{c} \quad (3.29)$$

Figure 9. Parameters for Free Surface Reflection of Longitudinal Waves (Poisson's Ratio - 0.15)

Figure 10. Free Surface Spallation at Oblique Incidence

For strong compression waves multiple spalls are again possible. Their theoretical total number n can be estimated from the following expression

$$n \leq - \frac{\ln \left[(1-\eta) \frac{p_\alpha}{\sigma_T} + 1 \right]}{\ln \eta} \quad (3.30)$$

For an arbitrary spall k the relations giving spall thickness are:

$$\sigma_T = \eta p_{k-1}(t_{k-1}) - p_k(t_k) = \eta^k p_\alpha - \sigma_T \eta \left(\frac{1-\eta^k}{1-\eta} \right) - p_k(t_k) \quad (3.31)$$

and

$$t_k = \frac{2(\delta_1 + \delta_2 + \dots + \delta_{k-1} + \delta_k) \cos \alpha}{c} \quad (3.32)$$

Spall velocities are again obtainable from the impulse integral. However, since that impulse is acting in the direction of the incident wave, the velocity will also be in this direction and the impulse is trapped in a length given by $\delta/\cos \alpha$. Thus for the first spall the relationship is

$$\rho_w u_1 \frac{\delta_1}{\cos \alpha} = \int_0^{t_1} p(t) dt \quad (3.33)$$

For an arbitrary spall k one obtains

$$\rho_w u_k \frac{\delta_k}{\cos \alpha} = \int_{t_{k-1}}^{t_k} p(t) dt \quad (3.34)$$

Finally the velocity normal to the surface u_{Nk} for any spall is given by

$$u_{Nk} = u_k \cos \alpha \quad (3.35)$$

Applying the approximate wave forms, given by equations (3.5) and (3.6), to the above relationships one obtains specific expressions for spall thicknesses

and velocities. Without presenting the details of the derivation and using the same notation as in the preceeding sections the following formulas are obtained:

(a) Exponential wave form $p(\tau) = p_\alpha e^{-\gamma\tau}$; $\tau = t/t_+$.

Thickness of 1st spall

$$\delta_1 = \frac{ct_+}{2\gamma\cos\alpha} \ln \left(\frac{p_\alpha}{\eta p_\alpha - \sigma_T} \right) \quad (3.36)$$

Thickness of k^{th} spall

$$\delta_k = \frac{ct_+}{2\gamma\cos\alpha} \ln \left[\frac{\eta^{k-1}(1-\eta) - (1-\eta)^{k-1} \frac{\sigma_T}{p_\alpha}}{\eta^k(1-\eta) - (1-\eta)^k \frac{\sigma_T}{p_\alpha}} \right] \quad (3.37)$$

Velocity of 1st spall

$$u_1 = \frac{p_\alpha t_+}{\gamma \rho_w \delta_1} \cos\alpha \left(1 - e^{-\frac{2\gamma\delta_1\cos\alpha}{ct_+}} \right) \quad (3.38)$$

Velocity of k^{th} spall

$$u_k = \frac{t_+\cos\alpha}{\gamma \rho_w \delta_k} \left[\eta^{k-1} p_\alpha - \left(\frac{1-\eta^{k-1}}{1-\eta} \right) \sigma_T \right] \left[1 - e^{-\frac{2\gamma\delta_k\cos\alpha}{ct_+}} \right] \quad (3.39)$$

(b) Power Law wave form $p(\tau) = p_\alpha (1-\tau^\gamma)$; $\tau = t/t_+$.

Thickness of 1st spall

$$\delta_1 = \frac{ct_+}{2\cos\alpha} \left(1-\eta + \frac{\sigma_T}{p_\alpha} \right)^{1/\gamma} \quad (3.40)$$

Thickness of k^{th} spall

$$\delta_k = \frac{ct_+}{2\cos\alpha} \left\{ \left[(1-\eta^k)^{1/\gamma} - (1-\eta^{k-1})^{1/\gamma} \right] \left[1 + \frac{1}{1-\eta} \frac{\sigma_T}{p_\alpha} \right]^{1/\gamma} \right\} \quad (3.41)$$

Velocity of 1st spall

$$u_1 = \frac{2p_\alpha}{(\gamma+1)c\rho_w} \cos^2 \alpha \left[(\gamma+\eta) - \frac{\sigma_T}{p_\alpha} \right] \quad (3.42)$$

Velocity of k^{th} spall

$$u_k = \frac{2p_\alpha \cos^2 \alpha}{(\gamma+1)c\rho_w} \left\{ \left[(\gamma+\eta^k) - \left(\frac{1-\eta^k}{1-\eta} \right) \frac{\sigma_T}{p_\alpha} \right] - \frac{\left(\frac{1-\eta}{1-\eta^{k-1}} + \frac{\sigma_T}{p_\alpha} \right) \eta^{k-1}}{\left(\frac{1-\eta^k}{1-\eta^{k-1}} \right)^{1/\gamma} - 1} \right\} \quad (3.43)$$

Since in general multiple spalls may occur, it is of interest to estimate the total depth of spalled material, i.e., the sum of all spall thicknesses. Because the exponent γ is variable no exact algebraic expression can be derived for computing this total depth. However, assuming an average (constant) value for γ over the entire pressure decay one can obtain an approximation of the total spall depth, for n spalls, by performing a summation on equation (3.41).

$$D_{n\alpha} = \sum_{k=1}^n \delta_k = \frac{ct_+}{2 \cos \alpha} \left[(1 - \eta^n) \left(1 + \frac{1}{1 - \eta} \frac{\sigma_T}{p_\alpha} \right) \right]^{1/\gamma} \quad (3.44)$$

This reduces for the case of normal incidence ($\alpha = 0$) to the following simpler expression, as can be verified by summing equation (3.21) directly.

$$D_{n0} = \frac{ct_+}{2} \left(n \frac{\sigma_T}{p_0} \right)^{1/\gamma} \quad (3.45)$$

In the latter case when the number of spalls is large ($p_o \gg \sigma_T$) we see from equation (3.9) that $n \approx p_o / \sigma_T$ and the above expression can be further approximated to yield

$$D_{n_o} \approx \frac{ct}{2} \quad (3.46)$$

This implies that the total depth of spall is one half the wave length of the compression wave. No such simplification is possible for the more general case of oblique spall and equation (3.44) must be used to estimate the total depth of spall.

3.4 COMPUTATIONAL PROCEDURE

In applying the methods outlined in the preceding sections for the calculation of spall, it was found to be preferable to use the power law wave form approximation (equation (3.6)) rather than the exponential approximation (equation (3.5)). This results in more consistent values of the spall variables, particularly for the tail end of the compression wave where the simple exponential is a poor approximation. Thus all results presented in the following are based on equation (3.6). The variation of the exponent γ as a function of incident blast wave pressure is given in Figure 11. The pressure ratio (current pressure to peak blast wave pressure) is used as a parameter and the values of γ are computed by equating pressures calculated from equation (3.6) to those obtainable from equation (3.4). The material properties for concrete used in computing spallation are those given in Section 3.3. However, the dynamic tensile rupture strength was assumed to be 800 psi or twice the static value which is the more appropriate stress level for the formation of fine cracks. Indications exist that to produce cohesive spall, i.e., separation of material layers, considerably higher stress levels are required [8], hence the value 800 psi was selected.

Figure 11. Variation of Pressure Wave Form Exponent

The procedure for obtaining the spall effects, for a given set of input parameters, involves a multiplicity of steps which define the air blast loading, the compressional wave in the wall, the stress wave reflection process and finally the spallation thickness and velocity. While this procedure is relatively straightforward it is not readily reversible, i.e., given a specific spall effect it is not possible to define the conditions which produced it.

Starting with a given amount W (lbm) of explosive (TNT), a wall thickness h (ft) and a standoff distance y (ft), the spall parameters can be computed for any arbitrary angle of incidence α as follows:

- (i) Calculate scaled distances in $\text{ft}/\text{lbm}^{1/3}$

$$Z_o = y/W^{1/3}, \quad Z_w = h/W^{1/3}, \quad Z = Z_o/\cos\alpha$$

- (ii) From TM 5-1300 [2] Figures 4-5 and 4-6 obtain the blast wave parameters: positive phase duration - $t_+/W^{1/3}$ (msec/lbm^{1/3}), peak incident blast wave pressure - $p_{s\alpha}$ (psi), and peak reflected pressure $p_{r\alpha}$ (psi).
- (iii) Compute the peak pressure of the compression wave p_α (psi) incident on the backface of the wall from equation (3.3).
- (iv) Find the incident shock pressure $p'_{s\alpha}$ corresponding to a reflected pressure of p_α (Figure 4-6, TM 5-1300). Using the value of $p'_{s\alpha}$ obtain parameters a' , b' , c' , α' , β' and γ' (Figure 24, Reference 12) to define the compression wave decay as a function of time (equation 3.4).
- (v) Obtain the reflection coefficient η from equation (3.27) or Figure 9.
- (vi) For the first spall compute the strength of the reflected tension wave $\sigma_{R\alpha} = \eta p_\alpha$ (psi).

- (vii) Since the first spall occurs when $\sigma_T = \sigma_{R\alpha} - p_1(\tau_1)$, from Fig. 11 find a value of the exponent γ , for equation (3.6), corresponding to an incident pressure $p'_{s\alpha}$ and a pressure ratio

$$\frac{p}{p_\alpha} = \left(\sigma_{R\alpha} - \frac{\sigma_T}{2} \right) / p_\alpha \quad (3.47)$$

Alternately the value of γ may be found by matching equation (3.4) and (3.6) at the above specified pressure ratio. This requires iteration. The pressure ratio at which γ is defined is arbitrarily chosen to be at the midpoint between $\sigma_{R\alpha}$ and p_1 . This results in a reasonably accurate approximation of the pressure wave form over the first spall.

- (viii) Compute the first spall thickness and velocity using equations (3.40) and (3.42) respectively.
- (ix) For any subsequent spall k the procedure is repeated starting with step (vi). The strength of the reflected tension wave is now

$$\sigma_{\alpha_k} = \eta p_{k-1} = \eta^k p_\alpha - \eta \left(\frac{1 - \eta^k}{1 - \eta} \right) \sigma_T \quad (3.48)$$

The exponent γ is again defined at the midpoint between σ_{α_k} and p_k and the thickness and velocity of spall are obtained from equations (3.41) and (3.43).

- (x) If $\alpha = 0$, i.e., normal incidence, the procedure is simplified in that $\eta = 1$ and the strength of the tension wave for arbitrary spall k is simply

$$\sigma_k = p_0 - (k-1)\sigma_T \quad (3.49)$$

To obtain spall thickness and velocity the appropriate equations

are now (3.19) and (3.22) for the first spall, and (3.21) and (3.23) for any arbitrary spall.

- (xi) If the total depth of spall is of interest then a value can be estimated using equation (3.44) for oblique spall and equation (3.45) and/or (3.46) for normal spall. Average values of γ at the midpoint of the compression wave form can be used or the values may be bracketed by using γ for the first and last spall. If only a few spalls occur it is more reliable to compute the individual spalls and add their thicknesses.

3.5 SPALL RESULTS

Using the procedure outlined in the preceding section representative spall results were computed. To limit the amount of computation to a reasonable level only the most important features of spallation are investigated and the results are restricted to indicate trends rather than provide specific results for all possible ranges of variables. Such a complete parametric representation of results would require the computerization of the calculational procedure and is considered beyond the scope of the current effort.

Figure 12 presents spall thickness and velocity at normal shock incidence ($\alpha = 0$), for various charge standoff distances, as a function of residual stress ratio. The latter quantity is the ratio of the reflected tensile stress σ_k to the peak compression stress p_0 . As can be seen from equation (3.49) this ratio is not a continuous variable but takes on discrete values with each new spall. Since the number of spalls varies widely depending on the charge standoff from the wall this stress ratio is a more convenient variable for result presentation. The computational data are given in scaled form and to obtain results for specific charge weights the

Figure 12. Variation of Spall Parameters with Residual Stress Ratio, Normal Wave Incidence

distances (thickness of spall and charge standoff) must be multiplied by the cube root of the charge weight $W^{1/3}$ ($\text{lbm}^{1/3}$). The data in Figure 12 are for zero wall thickness, i.e., no attenuation of the shock wave in the wall is assumed to take place. This yields the highest possible spall velocities. As expected the spall thickness increases as the stress ratio decreases while the spall velocity shows an opposite trend. Thus the first spall has the highest velocity but its thickness is a minimum. Also spall thickness increases with charge standoff but the spall velocity decreases. These trends reflect the dependence of spall thickness on the compression wave form which is steepest at the front and becomes flatter as it decays. Similarly the wave form becomes less steep as the peak pressure decays, i.e., the standoff distance increases. The same data is replotted in Figure 13 as a function of standoff distance with the stress ratio now being a parameter. While this may appear to be a better way of representing the data it is somewhat unsatisfactory because the stress ratio is a discrete variable while the data presentation seems to imply continuity.

In general one finds that for explosions close in to the wall most spalls are very thin. Considering a charge weight of 1000 lb at a standoff of 2 ft ($Z_o = 0.2 \text{ ft/lbm}^{1/3}$) the thickness of the first spall is only 0.002 in. while the velocity is about 430 fps. By the time the stress wave has decayed to ten percent of its peak value the spall thickness increases to about 0.10 in. but the velocity is only 19 fps. Increasing the charge to 40,000 lb (at the same scaled standoff $Z_o = 0.2$) will not alter the velocities and increases thickness only by a factor of approximately 3.42. At large standoff distances ($Z_o = 1.8 \text{ ft/lbm}^{1/3}$) the 1000 lb charge produces a first spall 4.2 in. thick, however, its velocity is only 9 fps. In fact it is found that the kinetic energy per unit area of spall varies only by a factor of three over the entire range of parameters considered in Figure 12.

Figure 13. Variation of Spall Parameters with Standoff Distance, Normal Wave Incidence

The specific values are $4 - 12 \text{ ft} \cdot \text{lb}/\text{ft}^2 \cdot \text{lbm}^{1/3}$ or for a charge weight of 1000 lb, $40 - 120 \text{ ft} \cdot \text{lb}/\text{ft}^2$. An even narrower range of values results from the estimate of total depth of spall as computed by equation (3.45) or by direct addition of spall thicknesses. The values vary from a low of $2.4 \text{ in.}/\text{lbm}^{1/3}$ for $Z_0 = 0.2$ to $3.5 \text{ in.}/\text{lbm}^{1/3}$ for $Z_0 = 2.0$. Again for the 1000 lb charge this gives 24 to 30 inches. Spalls to such depth will probably not occur in real walls which usually have a layer of reinforcing bars just a few inches below the backface surface and which should prevent the actual separation of concrete layers beyond this depth.

The effect of wall thickness on spall velocity and thickness is illustrated in Figure 14, where results are given for the first spall variables at normal incidence as a function of charge standoff distance. The data is again presented in scaled form; this also applies to the wall thickness. Thus for a fixed charge weight and wall thickness results lie on curves parallel to those indicated. However if only the wall thickness is fixed and the charge weight varies then the results of interest lie on different curves even for a fixed value of scaled standoff distance. For fixed charge weight and standoff distance the spall thickness increases with increasing wall thickness while the spall velocity exhibits just the opposite behavior. Similar results are obtained for subsequent spalls. This trend is again a consequence of the flattening of the compression wave form as it decays by propagating through the wall. For a 1000 lb charge at a standoff of 2 ft from the wall we see that the first spall thickness is 0.0020 in. for zero wall thickness and becomes 0.0145 in. at 4 ft thickness. At the same time the velocity is reduced from 430 fps to 142 fps. Again the kinetic energy per unit area over the full range of parameters is found to lie in the band indicated earlier for zero wall thickness.

Figure 14. Effect of Wall Thickness on Spall Parameters, Normal Wave Incidence

Similarly the range of total depth of spall is found to be in agreement with that obtained for $Z_w = 0$.

A very strong effect on spall thickness and velocity is produced by variations in angle of shock incidence α . This is illustrated in Figure 15 which gives the spall variables for the first spall as a function of standoff distance for various values of the angle of incidence α . Again the case of zero wall thickness or no shock attenuation by the wall is presented. As indicated earlier no spalls occur beyond $\alpha = 45^\circ$ because the reflection factor changes phase at this point (see Fig. 9). For a given standoff distance the spall thickness increases rapidly with angle of incidence while the velocity decreases equally fast. Relative to normal incidence the spall thickness for the oblique case is increased by the factor $1/\cos\alpha$ as well as by the effect of the reflection coefficient. The velocity is decreased by $\cos^2\alpha$ and again a reflection coefficient effect. However, in computing the kinetic energy per unit area for oblique incidence, values substantially higher than those for normal incidence are encountered. Also the range of values is much broader than shown earlier. Values as high as $55.0 \text{ ft}\cdot\text{lb}/\text{ft}^2\cdot\text{lbm}^{1/3}$ and as low as $2.0 \text{ ft}\cdot\text{lb}/\text{ft}^2\cdot\text{lbm}^{1/3}$ are computed. The maximum kinetic energy occurs at the closest charge placement $Z_o = 0.2 \text{ ft}/\text{lbm}^{1/3}$ for an angle of incidence $\alpha \approx 20^\circ$. In fact the maximum for most charge standoff distances appears to occur around 20° . In terms of a 1000 lb charge this maximum represents $550 \text{ ft}\cdot\text{lb}/\text{ft}^2$ and is produced by a spall 0.078 in. thick having a velocity of 193 fps.

A comparison of the maximum spall velocities (first spall, normal incidence) with the maximum velocities induced by the impulse of the blast is shown in Figure 16. The latter velocities are obtained by considering the gross motion of the entire loaded portion of the wall and are calculated by the procedure outlined in Section 2. The results are given in scaled

Figure 15. Effect of Wave Incidence Angle on Spall Parameters

Figure 16. Comparison of Maximum Spall and Wall Displacement Velocities

form as a function of charge standoff distance for a number of wall thicknesses. It is seen that except for the thickest wall $Z_w = 0.6 \text{ ft/lbm}^{1/3}$ the velocities induced by impulse loading are substantially higher than those produced by direct spalling. A similar behavior is observed at other angular positions. Hence coupling between the two motions can be expected. The impulsive motion is a late time effect and a number of stress wave passages will occur before this motion commences. Therefore the small high velocity spall debris is expected to be ejected from the wall before the gross motion takes effect. However, the thicker spall debris which has but little velocity is expected to stay in contact (or near contact) with the wall and will be ejected by the wall motion at quite high velocities. Considering a standoff of $Z_o = 0.4 \text{ ft/lbm}^{1/3}$ and a wall thickness also of $Z_w = 0.4 \text{ ft/lbm}^{1/3}$ one finds that wall velocities of the order of 100 fps may be encountered. Coupling this velocity to some of the larger spall debris which is approximately $0.6 \text{ in./lbm}^{1/3}$ thick one finds kinetic energies per unit area exceeding $1000 \text{ ft}\cdot\text{lb/ft}^2\cdot\text{lbm}^{1/3}$. These values increase for smaller standoff distances and are more than an order of magnitude larger than those produced by direct spall. Thus it appears that the severest effects of spallation may be produced by the coupling to the gross motion induced by the impulsive loading.

The spall analysis given here provides no information on the lateral dimensions of the spall debris. However experimental evidence indicates [17], that due to the perturbations of the shock surfaces the thin spall layers have a tendency to break up into small particles. Thus lateral dimensions of the order of five to ten times the thickness may be expected. At the other end of the thickness spectrum where spall layers are of the same magnitude as the depth of the steel reinforcement cover, lateral spall debris dimensions are limited by the size of the reinforcement pattern.

A comparison of the analytical results with experimental data [17,18] indicates the computed spall velocities are of the proper magnitude. Reported experimental spall velocities range from 20 to 300 fps and vary with charge weight, wall thickness and standoff distance. It is not possible to carry out a direct one to one comparison between reported experimental data and computations. The reason for this is the lack of specificity in the experimental data. Only gross overall velocities are reported and no information on spall debris thickness is given. For smaller charges (below 1000 lb) experiments [17] seem to indicate some dependence of spall velocity on charge weight contrary to what the analysis predicts. The reason for this is not clear and the experimental data is not sufficiently specific to ascertain which velocity was measured. For charge weights equal or larger than 1000 lb the experimental data indicates velocities which are independent of the charge weight (for fixed wall thickness and standoff distance). The predicted threshold for spallation, with typical wall thicknesses of two to four feet, is approximately at a scaled distance of $Z_0 = 2.0 \text{ ft/lbm}^{1/3}$. While this threshold is somewhat dependent on the assumed dynamic tensile rupture strength, it is found to be in good agreement with experimental results, at least for the larger charge weights [17].

4. CONCLUSIONS AND RECOMMENDATIONS

The analysis and results given in the preceding section represent a "first cut" approach to determining the effects of air blast, resulting from explosive detonation, on concrete walls. The approach can be used to predict expected trends both for the direct spall and the gross motions or failures produced by the impulse loading of a portion of the wall. The results appear to be qualitatively correct and yield values which are of the same magnitude as those obtained by experiments. However, better experimental verification is required. As indicated earlier, most of the available experimental data seems to lack consistency and the quality of measurements is often marginal. The search for more reliable data should therefore continue and comparisons with the predictions should be carried out in order to establish their validity.

Due to the limitations of the current effort many assumptions are made in the analysis. Where possible these assumptions should be further investigated and the effect of neglecting or simplifying certain aspects should be evaluated. For the problem of direct spall some of the restrictions which merit further evaluation are: the effect of the shear wave in oblique spall, the effect of sphericity of the compression wave which was neglected, and the influence of plastic and time dependent material response. For both the gross wall motion and spallation the effect of time phasing of the loading, which was neglected, should be investigated. The substantial computational effort that this would require suggests that the calculational procedures be computerized. At the same time the inconsistencies existing in the currently used air blast loading information [2] should be eliminated. Improved air blast data may now be available [6]. Computerization of the procedures would also permit a better evaluation of

the coupling effects between spallation and gross wall motion. In particular the timing and interdependence of the two phenomena could be investigated.

To complete the study of explosion effects on concrete walls the effect of steel liners and the problem of contact placement of explosives must be studied. While these aspects are completely neglected in the current effort they are expected to have strong influences on both spallation and gross wall motions. The liner problem may be amenable to approximate analytical treatments. However, the study of the effects of contact explosives will require either elaborate computer calculations or the empirical evaluation of the problem from available experimental data together with simple one dimensional shock impedance calculations. For the current application the latter approach appears at this point to be adequate.

The results obtained to date and also future efforts in this area depend strongly on the material properties assumed in the analysis. Since air blast loadings are at times extreme and occur very rapidly high strain rates are encountered. Data for concrete at such loadings are currently not available. Of particular importance is the dynamic tensile rupture strength which directly influences the thickness of debris formed by spalling. Information of this type can only be generated by careful experimentation. However, the data so obtained would not be restricted in its application to the problem of current interest.

Should the effects of explosives on shield walls and containment structures become of major importance, then it is believed that a much more rigorous analytical and computational approach will be required. Lagrangian finite difference computer codes can be used to study the early time effects including spallation. For the late time gross wall motions finite element formulations and codes would appear to be more appropriate.

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