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**MODELING THE TEMPERATURE-DEPENDENT SCALE
ACCUMULATION FROM GEOTHERMAL BRINE**

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MODELING THE TEMPERATURE-DEPENDENT SCALE ACCUMULATION FROM GEOTHERMAL BRINE

Abstract

A simple model for calculating the temperature-dependent deposition of silica scale from geothermal brine is outlined. A finite-difference technique for solving the moving boundary problem is described. The implicit formulation is solved directly, using matrix inverse methods appropriate for banded matrices. The multimaterial problem is defined in a way that maintains the compact matrix bandwidth of a single material. Sample problems are presented in

dimensionless form, including a comparison between the finite-difference results and an analytical solution for a single-material slab problem. These initial studies show that the temperature at the brine-scale interface is not significantly perturbed by the growth of a thin scale layer, although pipe thickness and different experimental scale-deposition curves significantly affect the total amount of scale forming over a given period of time.

Introduction

This report covers the initial phase of a study of the thermally dependent, silica-based scale precipitation from geothermal brines.¹

The remainder of the report is in three sections. The first describes the simple heat-transfer model and the procedure that was used to solve the model equations. The second section describes some sample calculations that were made using the model described below. Not included are a discussion of the nature of silica-based scale, the reaction kinetics, or the experimental scale-deposition

studies that have been carried out, which are beyond the scope of this report. The last section summarizes the report.

Experimental curves of geothermal scale deposition, as a function of temperature in converging-diverging nozzles, are given in Ref. 2. The tests utilized the hypersaline brine from three different geothermal wells at the Salton Sea Geothermal Field (SSGF). The nozzles were instrumented with thermocouples at the outer (insulated) radius along the diverging flow section. They were

subsequently sectioned, and the scale thickness along the diverging portion of the nozzle gave the deposition rate as a function of temperature. The nature of the boundary layer in the nozzle, and the dependence of scaling on other hydrodynamic effects, were not investigated, since the dependence of scaling on temperature was believed to be the dominant phenomenon.

The modeling of temperature-dependent scale accumulation on the walls of pipes and associated hardware falls into the class of mathematical problems referred to as moving boundary problems. These include melting/condensation, phase changes, and chemical or combustion processes. In our case of scale deposition, three factors were of particular interest in this initial study:

1. Temperature as a function of time at the interfaces between brine and pipe, pipe and air, and brine and scale (in the cases where scale was forming)
2. Indication of the total growth of scale for various assumed parameters
3. Sensitivity of scale deposition to variations in the deposition curves.

The third factor is important, since the high-temperature portion of the deposition curve is poorly defined,

and substantial variation in rate dependence occurs from brine samples that flowed from different wells in the SSGF.

The model for scale deposition described in detail below includes the following simplifying assumptions:

- Brine is in kinetic equilibrium at all times (i.e., the chemical reaction rates are short compared to our times of interest).
- Rate of deposition as a function of temperature is known (e.g., a curve of scale thickness in $\mu\text{m/s}$ versus temperature is available).
- Hydrodynamic effects, such as velocity variations, turbulence, etc., are not included in the model at this time. This means that the actual phenomena are reduced to a simple static heat diffusion problem, with one moving material boundary and two stationary material boundaries.

Since future studies may incorporate more sophisticated and complicated models, analytical solutions for general applicability are not feasible. Instead, we derived a finite-difference solution in one dimension. The computer program that we used for numerical studies is written with initial and boundary conditions that are sufficiently general to accommodate modifications at some later time.

In the case of scale accumulation, the moving boundary "grows" slowly ($\approx 0.01 \mu\text{m/s}$); hence, the scale thickness at our times of interest — small fractions of a year — will be a small fraction of a metre. Although the scale grows slowly, its initial buildup may eventually be studied

in greater detail, since the initial scale formation may have a catalytic effect on subsequent deposition. Finally, the scale may act as a thermal insulator, and the effect of this growing layer on the heat flow from the geothermal brine must be determined.

Model Development

A qualitative view of the physical problem is shown in Fig. 1. The governing, partial differential equation for this one-dimensional, heat conduction problem is the diffusion equation

$$\rho C \frac{\partial T}{\partial t} = \frac{1}{r^\alpha} \frac{\partial}{\partial r} \left(r^\alpha K \frac{\partial T}{\partial r} \right), \quad (1)$$

where

ρ = density (kg/m^3)

C = specific heat capacity ($\text{J/kg}\cdot\text{K}$)

T = temperature (K)

t = time (s)

r = radius (m)

K = thermal conductivity ($\text{W/m}\cdot\text{K}$)

and

$\alpha = 1$ for cylindrical geometry
 $= 0$ for Cartesian geometry.

Equation (1) can be rewritten in dimensionless form by defining

$$T^* = \frac{T - T_{\text{air}}^0}{T_{\text{brine}}^0 - T_{\text{air}}^0}, \quad (2)$$

$$t^* = \frac{k_{\text{pipe}} t}{(r_p - r_0)^2}, \quad (3)$$

Fig. 1. A qualitative view of the scale-deposition problem.

and

$$r^* = \frac{r - r_0}{r_p - r_0}, \quad (4)$$

where the zero superscript refers to the initial value. Value r_p is a reference value, which we take to be the pipe's outer radius, and r_0 is some inner reference radius (for Cartesian problems, r_0 can be negative). The thermal diffusivity is defined by

$$k = \frac{K}{\rho C}, \quad (5)$$

and when we define

$$\phi = \frac{K}{K_{\text{pipe}}}, \quad (6)$$

the dimensionless form of (1), after dropping the $*$, can be written as

$$\frac{\partial T}{\partial t} = \frac{1}{\phi r^\alpha} \frac{\partial}{\partial r} \left(\phi r^\alpha \frac{\partial T}{\partial r} \right). \quad (7)$$

The pipe's conductivity K_p is chosen as the reference value, because the pipe conductivity is usually greater than the conductivities of the other materials in the problem. Thus, the dimensionless conductivity ϕ is in the interval $0 < \phi \leq 1$.

When no scale is growing, the solution dependence in the problem is reduced to a single parameter ϕ for each material and for a specified geometry. In our formulation, the parameter ϕ may be a function of position in any, or all, of the

materials. This may prove useful in modeling the flowing brine in more detail in future studies. The boundary conditions are of the Dirichlet type with time dependence, i.e.,

$$\begin{aligned} T(r_0, t) &= f(t) \\ T(r_N, t) &= g(t), \end{aligned} \quad (8)$$

and this gives the added constraints

$$\begin{aligned} \left. \frac{\partial T}{\partial t} \right|_{r=r_0} &= \frac{df}{dt} \\ \left. \frac{\partial T}{\partial t} \right|_{r=r_N} &= \frac{dg}{dt}, \end{aligned} \quad (9)$$

where r_N is the outer boundary radius of the problem.

This problem is approximated by a finite-difference grid, as shown in Fig. 2. The numbers (and symbols NB, NA, and N) in Fig. 2 refer to "grid points," and the regions between adjacent grid points are referred to as "zones". As the scale grows, the dotted line in Fig. 2 moves to the left in discrete steps at each time step. The reason for including brine zones in the problem configuration, instead of applying a boundary condition at the labeled grid point NB, is to allow for future inclusion of boundary effects in these zones with a minimum of effort. The pipe/air boundary is treated symmetrically, and it could also be handled by

Fig. 2. Finite-difference grid for the multimaterial problem with a moving boundary.

defining a boundary condition at the labeled grid point NA.

The finite-difference analogue of (7) can be written in implicit form for all interior grid points (grid points not adjacent to material boundaries) as

$$T_i - \tilde{T}_i = \frac{\Delta t}{\phi_i r_i^\alpha (r_{i+1/2} - r_{i-1/2})} (Q_{i+1/2} - Q_{i-1/2}), \quad (10)$$

where i is the grid point index. The dimensionless heat flow for interior points is defined by

$$Q_j = r_j^\alpha \phi_j \left(\frac{T_{j+1/2} - T_{j-1/2}}{r_{j+1/2} - r_{j-1/2}} \right), \quad (11)$$

where j is a dummy, grid point index. All of the variables are for arbitrary, m th time-step values and $\tilde{T}$ is the value from the $(m-1)$ th time step. The equations for the interior grid points can be rewritten in the form:

$$B_{i,1} T_{i-1} + B_{i,2} T_i + B_{i,3} T_{i+1} = \tilde{T}_i \quad (12)$$

for

$$i = 2, 3, \dots, N-1,$$

where

$$i \neq NB-1, NB, NA-1, NA.$$

The coefficients $B_{i,1}$ for the interior points are given by

$$B_{i,1} = \frac{r_{i-1/2}^\alpha \phi_{i-1/2} \phi_i}{(r_i - r_{i-1})},$$

$$B_{i,3} = \frac{r_{i+1/2}^\alpha \phi_{i+1/2} \phi_i}{(r_{i+1} - r_i)},$$

$$B_{i,2} = 1 - B_{i,1} - B_{i,3}, \quad (13)$$

where

$$\phi_i = - \frac{\Delta t}{r_i^\alpha \phi_i (r_{i+1/2} - r_{i-1/2})} \quad (14)$$

The form of the difference equations presented above allows unequal zone sizes.

The family of equations given by (12), plus the equations for the problem boundaries and the material boundaries, form a matrix equation. The solution vector gives the temperatures for each grid point at the m th time step, and the right-hand side is the column vector of temperatures at time step $m-1$ (a known quantity). The coefficient matrix is a banded $N \times N$ matrix, with the bandwidth defined by the three $B_{i,j}$ values. This banded matrix of B coefficients involves only the time step, material constants, and grid information. When scale is not being deposited (the grid is stationary), the coefficient matrix is constant. But when scale is growing, some of the banded matrix elements change at each time step. When the coefficient matrix is constant, the matrix inverse must be computed only once, but when the scale is growing and the matrix is changing with time, the coefficient matrix-inverse must be computed at each time step.

To complete the problem definition, the boundaries must be modeled. The approach presented here is not unique, but it has the desirable characteristic of maintaining the bandwidth of the coefficient matrix

at its minimum value of three. This could be important if a large number of grid points are used, or if the method is extended to more detailed modeling, where computation time becomes important.

The zone in which scale will begin to grow is isolated in Fig. 3. Let the stationary, inner boundary of the pipe be midway between grid points $NB-1$ and NB . This choice allows an equation form that maintains the compact bandwidth of the coefficient matrix. However, this is not the case when the boundary is situated on a grid point. The pipe's inner radius is then

$$r_B = \frac{1}{2} (r_{NB-1} + r_{NB}) , \quad (15)$$

at which point the temperature is defined as T_B . The inner scale boundary is given by

Fig. 3. Configuration of moving boundary zone.

$$r_S = r_B - \int_0^t \delta(T_p) dt, \quad (16)$$

at which point the temperature is defined as T_S . The deposition rate $\delta(T_p)$ is the dimensionless, temperature-dependent parameter defined by

$$\delta(T_p) = \frac{L \Delta(T_p)}{k_{\text{brine}}}, \quad (17)$$

where L is a characteristic length, k_{brine} is the thermal diffusivity of the brine, and $\Delta(T_p)$ is the experimentally determined, scale-deposition rate curve. The experimental quantity $\Delta(T_p)$ is a measured function of the temperature (T_p) at the pipe's outer radius. The quantity δ is the temperature-dependent parameter that represents the ratio of the scale "diffusion" into the brine to the thermal diffusion in the brine — in general it is a small number. The characteristic length L is taken to be the reference value in the problem — in our case we define L as $L = r_p - r_0$. The temperature at r_p is T_p , and it is this temperature that the deposition rate depends upon. It would be easy to change the computer program to allow a temperature from some other radius to determine the deposition rate. But in the sample calculations, we have used T_p

to determine the amount of scale deposition from (16).

Equation (10) can be applied to the material boundary between grid points NB-1 and NB. However, there are three unknowns in the resulting equations for grid points NB-1 and NB: T_S , T_B , and r_S . The linear heat-flow equations at the following points in the zone between NB-1 and NB are

$$\begin{aligned} \text{pipe: } Q &= R_p^\alpha \psi_p \left(\frac{T_{\text{NB}} - T_B}{r_{\text{NB}} - r_B} \right) \\ \text{scale: } Q &= R_S^\alpha \phi_S \left(\frac{T_B - T_S}{r_B - r_S} \right) \\ \text{brine: } Q &= R_B^\alpha \psi_B \left(\frac{T_S - T_{\text{NB-1}}}{r_S - r_{\text{NB-1}}} \right). \end{aligned} \quad (18)$$

The quantity ψ is a dimensionless, heat-transfer coefficient given by

$$\psi = \mu h. \quad (19)$$

Here, h is the dimensionless, heat-transfer coefficient for ideal heat transfer across a material boundary, and μ is a function that specifies nonideal contact (transfer) at the boundary. In the scale layer, we have $(\mu h)_{\text{scale}} = \phi_S$ (the dimensionless conductivity), and at the scale-pipe interface, $\psi_p = \mu$ (since $\phi_p = 1$). The radii between the grid points and interfaces (R_B and R_p), and the radius at the midpoint of the scale (R_S), are given by

$$\begin{aligned}
R_B &= \frac{1}{2} (r_{NB-1} + r_S) \\
R_S &= \frac{1}{2} (r_S + r_B) \\
R_p &= \frac{1}{2} (r_B + r_{NB}) . \quad (20)
\end{aligned}$$

Elimination of the unknown values of T_B and T_S in (18) gives the required expression for $Q_{NB-1/2}$ in terms of known grid values (the value of r_S at the $(m-1)th$ time step is used to find the new values at time step m).

Expressed in the form of (11), we have

$$Q_{NB-1/2} = r_{NB-1/2}^\alpha h_B \left(\frac{T_{NB} - T_{NB-1}}{r_{NB} - r_{NB-1}} \right), \quad (21)$$

where h_B is the time-dependent, dimensionless, heat-transfer coefficient obtained from (18) and (20), i.e.,

$$\begin{aligned}
h_B &= \left[\left(\frac{r_B}{R_p} \right)^\alpha \frac{1}{2\mu} + \left(\frac{r_B}{R_S} \right)^\alpha \left(\frac{r_B - r_S}{r_{NB} - r_{NB-1}} \right) \frac{1}{\phi_S} \right. \\
&\quad \left. + \left(\frac{r_B}{R_B} \right)^\alpha \left(\frac{r_S - r_{NB-1}}{r_{NB} - r_{NB-1}} \right) \frac{1}{\psi_B} \right]^{-1}. \quad (22)
\end{aligned}$$

The factor 2π from radial integration was canceled throughout the derivation. Thus, the B coefficients for the two grid points bounding the scale zone are

$$\begin{aligned}
B_{(NB-1),1} &= \frac{r_{NB-3/2}^\alpha \phi_{NB-3/2} \phi_{NB-1}}{(r_{NB-1} - r_{NB-2})}, \\
B_{(NB-1),3} &= \frac{r_{NB-1/2}^\alpha h_B \phi_{NB-1}}{(r_{NB} - r_{NB-1})}, \quad (23)
\end{aligned}$$

and

$$\begin{aligned}
B_{NB,1} &= \frac{r_{NB-1/2}^\alpha h_B \phi_{NB}}{(r_{NB} - r_{NB-1})}, \\
B_{NB,3} &= \frac{r_{NB+1/2}^\alpha \phi_{NB+1/2} \phi_{NB}}{(r_{NB+1} - r_{NB})}. \quad (24)
\end{aligned}$$

The values of $B_{(NB-1),2}$ and $B_{NB,2}$ have the form of (13).

For the pipe-air interface the derivation is identical, except for the scale layer, and we have

$$h_p = \frac{2\mu r_{NA-1/2}^\alpha}{\left(\frac{1}{R_a^\alpha \phi_a} + \frac{1}{R_p^\alpha \phi_p} \right)}, \quad (25)$$

where

$$\begin{aligned}
R_p &= \frac{1}{2} (r_{NA-1} + r_p), \\
R_a &= \frac{1}{2} (r_p + r_{NA}). \quad (26)
\end{aligned}$$

The matrix elements are given by equations of the form (23) and (24), with NA replacing NB and h_p replacing h_B . For the inner boundary of the

finite-difference approximation at $r = r_i$, we have

$$\frac{\partial T}{\partial t} = \frac{df(t)}{dt}, \quad (27)$$

or in finite-difference form,

$$T_1 = \tilde{T}_1 + f - \tilde{f}, \quad (28)$$

and the matrix elements are

$$\begin{aligned} B_{1,1} &= B_{1,3} = 0, \\ B_{1,2} &= 1. \end{aligned} \quad (29)$$

Similarly, for the problem outer boundary at $r = r_N$,

$$\begin{aligned} B_{N,1} &= B_{N,3} = 0 \\ B_{N,2} &= 1, \end{aligned} \quad (30)$$

and

$$T_N = \tilde{T}_N + g - \tilde{g}. \quad (31)$$

The temperatures at the material boundaries can be computed from (18). For the pipe's inner radius $r = r_B$ (brine-pipe interface for no scale), the temperature is found to be,

$$T_B = \frac{T_{NB-1} + C T_{NB}}{1 + C}, \quad (32)$$

where

$$\begin{aligned} C &= \left(\frac{R_P}{R_S} \right)^\alpha \left(\frac{\mu}{\phi_S} \right) \left(\frac{r_B - r_S}{r_{NB} - r_S} \right) \\ &+ \left(\frac{R_P}{R_B} \right)^\alpha \left(\frac{\mu}{\phi_B} \right) \left(\frac{r_S - r_{NB-1}}{r_{NB} - r_B} \right). \end{aligned} \quad (33)$$

When scale is present, the temperature at the brine-scale interface ($r = r_S$) is given by

$$T_S = \frac{T_{NB-1} + AT_B}{1 + A}, \quad (34)$$

where

$$A = \left(\frac{R_S}{R_B} \right)^\alpha \left(\frac{\phi_S}{\psi_B} \right) \left(\frac{R_S - R_{NB-1}}{r_B - r_S} \right), \quad (35)$$

and of course T_S is undefined when $r_S = r_B$. At the pipe-air interface $r = r_{NA-1/2}$, the temperature that will control scaling is given by

$$T_p = \frac{T_{NA-1} + BT_{NA}}{1 + B}, \quad (36)$$

where

$$B = \left(\frac{R_a}{R_p} \right)^\alpha \left(\frac{\psi_a}{\mu} \right) \left(\frac{r_p - r_{NA-1}}{r_{NA} - r_p} \right). \quad (37)$$

This completes the definition of the problem. A computer program has been written to solve the matrix equation using LU decomposition.³ This is then followed by simple back substitution. In our case, this direct solution technique (rather than iterative approach) was desirable, since our problems required relatively few grid points (< 500) and the model was one-dimensional. When the scaling is "turned off," the program performs the decomposition once, at the start

of the problem, and succeeding time steps required only back-solving. When the scale is growing, the program performs decomposition at each time step, followed by the solution.

Note that the manner in which the model is outlined here does not limit its applicability to a moving boundary that is less than a distance $r_B - r_{NB-1}$ from the inner zone radius r_{NB-1} , because the computer program can easily check to see when the scale thickness exceeds the zone half-width. When such a situation occurs, the problem is redefined by the program, as shown in Fig. 4.

The problem redefinition is a "bookkeeping" problem. After

redefinition, the scale is growing in the outer half of the NB-2 zone instead of in the inner half of zone NB-1.

Fig. 4. Grid configuration for the case in which scale thickness exceeds the zone half-width.

Calculational Results

The main purpose of the computations presented here is to demonstrate that a computer program exists for use in future numerical studies of scale deposition. This program also provides the basis for future computational development, using more sophisticated models.

The first consideration is to compare the results from the finite-difference calculation with those of a problem that has been solved analytically. Reference 4 gives the solution for a simple slab problem,

which can be defined by the initial and boundary conditions

$$\begin{aligned} T(x, 0) &= 1, \\ T(0, t) &= T(1, t) = 0, \end{aligned} \quad (38)$$

where

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \quad \begin{aligned} 0 &\leq x \leq 1 \\ t &\geq 0 \end{aligned} \quad (39)$$

The solution is

$$T(x, t) = \frac{4}{\pi} \sum_{m=1,3,5,\dots}^{\infty} \left[\frac{1}{m} e^{-m^2 \pi^2 t} \sin(m\pi x) \right]. \quad (40)$$

Our finite-difference model definition allows this problem to be computed, when $\alpha = 0$ and $\phi = 1$ in the pipe zones, $\phi \gg 1$ outside the pipe zones, the scale deposition is "turned off," and the initial and boundary conditions are used from (38). In Fig. 5, the computed results are compared to those of Ref. 4 for three different dimensionless times. The calculation shown here is for 100 grid points and 100 time steps. One second of CDC-7600 computer time was required for the solution, including the complete input and output for the problem. We did not make a detailed error analysis, but the figure shows

that for this particular problem satisfactory agreement is apparent.

The computer program allows problems to be run with scale deposition either turned on or turned off. Sample calculations in radial geometry that simulate some typical pipe problems are shown in Fig. 6, where the temperatures at the inner and outer radii of the pipe (T_B and T_p) are given for three different, dimensionless, pipe inner radii r_B when no scale was forming. The problem parameters are given in Table 1.

The calculations presented in Fig. 6 result in a parametric family of curves that typify this type of problem. That is, as pipe thickness

Fig. 5. The slab problem.⁴ Solid curves represent the analytical solution. The crosses represent the points computed from the model described in the text.

Fig. 6. Typical pipe problem without scale deposition, showing dimensionless temperatures at the pipe's inner and outer radii (T_B and T_p) for three values of the pipe's inner radius.

$r_p - r_B$ decreases ($r_B \rightarrow 1.0$), temperatures in the pipe approach equilibrium values more rapidly. Also, the temperature difference between the inner and outer radii decreases as $r_B \rightarrow 1$. Although the plot gives the appearance of $T_B \rightarrow T_p$ at large values of time, there remains a very small difference in temperature between the radii r_B and r_p for $t \gg 1$. Transient, radial, layered problems do not have simple analytical solutions; hence, we make no comparison between the finite-difference results presented in Fig. 6 and analytical calculations.

Table 1. Problem parameters for the calculations without scale deposition (total 7600 computer, problem time = 1.5s)

$\phi_B = 0.014$	$\mu = 1$
$\phi_p = 1$	$r_B = 0.25, 0.5, 0.75$
$\phi_a = 0.0005$	$r_p = 1.0$

A calculation for a dimensionless, pipe inner radius of 0.995 is presented in Fig. 7. The approximate equilibration takes place at much earlier dimensionless times than those that occurred in the problems with relatively thick walls, shown in Fig. 6. The

Fig. 7. Thin-wall pipe simulation problem with no scale and $r_B = 0.995$.

same parameters were used in the calculations for Figs. 6 and 7 (see Table 1).

Two dimensionless deposition curves are shown in Fig. 8. Recall that $\delta(T_p)$ is the ratio of the scale "diffusivity" in brine to the corresponding thermal diffusivity in brine. The curve labeled MM2 is from measurements made using Magmamax #2 and #3* brine from the geothermal wells of the Magma Imperial Power Company. The curve labeled S4 is from flow experiments at the Sinclair #4 well.⁵ Data at dimensionless temperatures less than 0.48 are not available from the Sinclair well, and it is not known how high the scaling rate goes as the temperature decreases. The scaling rate at $T_p = 0$ was arbitrarily set at 10^{-2} , and a straight line was used to connect that value to the first data point at $T = 0.48$.

Fig. 8. Dimensionless scale-deposition rate as a function of dimensionless temperature at the pipe's outer radius. MM2 refers to the MagmaMax wells, and S4 refers to the data from the Sinclair #4 geothermal well. The crosses represent the experimental data and the solid lines represent the approximations used in our calculations.

In the deposition calculation, the scale had a parameter value, $\phi_s = 0.005$, with all other values from Table 1. As might be expected from the values of δ in Fig. 8, the growth of scale at these rates does not perturb the heat flow significantly. The values of dimensionless temperature at the brine-scale boundary, using the curves of Fig. 8, differed from the temperature at the pipe's inner radius by less than 0.1%. Although the temperature is not perturbed significantly for the deposition rates referred to, the total scale deposited when the problem terminated for the two different deposition curves in Fig. 8 was significant. For the lower curve (MagmaMax #2), the total dimensionless deposition is 4.14. For the higher deposition curve (Sinclair #4), the total scale deposition is 43.6.

In terms of dimensioned quantities, we use the relationships in (2) to (4), (6), and (17). The values of the dimensioned parameters for a typical problem are given in Table 2. From these relationships we have

$$t = 833.33t^*,$$

$$r = 0.1r^*.$$

Table 2. Conversion values for steel pipe problems with $\sim 10\%$ brine salinity.

$T_0 = 50^\circ\text{C}$
$T_{\text{brine}}^0 = 300^\circ\text{C}$
$r_p = 0.1 \text{ m (3.936 in.)}$
$k_{\text{brine}} = 2 \times 10^{-7} \text{ m}^2/\text{s (water with } \sim 10\% \text{ NaCl)}$
$k_{\text{pipe}} = 1.2 \times 10^{-5} \text{ m}^2/\text{s (steel)}$
$K_{\text{pipe}} = 50 \text{ W/m}\cdot\text{C (steel)}$
$r_B = 0.0936 \text{ m (3.916 in.)}$

From Fig. 6 for the case of $r_B = 0.25$, the equilibrium time for $T_p^* \approx 0.94$ is approximately 4.6 hr. For $r_B = 0.5$, the equilibrium time to $T_p^* \approx 0.94$ is about 1.4 hr, and for $r_B = 0.75$, the equilibrium time to $T_p^* \approx 0.9$ is about 55 min. For the thin-wall pipe problem ($r_B = 0.995$), the equilibrium time is about 33 s (see Fig. 7).

Summary

In the previous sections, we outlined a simple model for calculating the thermally dependent deposition of scale from geothermal brine. We described a finite-difference technique that uses an implicit approximation to determine the temperature-dependent, moving boundary radius, and we presented the temperature as a function of time for several configurations.

We used the finite-difference technique to first calculate the solution to a simple slab problem, with one material and no moving boundaries, for comparison with an analytical solution. Then, we calculated the solution to four different radial problems to show

how the temperature, as a function of time, depends on pipe thickness. Finally, we ran a thin-wall pipe problem on the computer, using scale deposition for two different deposition curves. The rate curves were based on experimental data from three wells in the Salton Sea Geothermal Field.

The deposition curves in dimensionless form represent the ratio of the scale "diffusivity" to the thermal diffusivity in the brine. The values are small fractions and they suggest that the deposition will not significantly perturb the inner wall temperature of the pipe, even for small values of thermal

conductivity for the scale. The computed values support this suggestion, and the pipe wall temperatures (with and without scale) differ less than 1/10 of one percent. However, we found that the total deposition depends strongly on the deposition curve, since the two experimental curves that we used in the calculations differed by more than an order of magnitude at some temperatures. Apparently, the brine's temperature and salinity strongly affect the deposition rate curves, although more experimental data are required to understand this phenomenon more fully.

The model described above is rudimentary and can be made more sophisticated by incorporating elementary boundary layer theory,⁶ or a convective boundary condition at the brine interface, or both. The computer program that produced the calculations described above can be used for a variety of calculations

other than those for geothermal scale deposition. Problems involving melting or condensation in slabs or cylinders can be computed, if the rate of melting or condensation vs. temperature is given. The computer program can also be utilized for calculating fluid flow in porous media for both slab and cylinder geometries, by using appropriate choices of the storativity and transmissivity.⁷

Future studies should include a more detailed numerical analysis of the early time deposition, and the effect of the high-temperature portion of the deposition curve on the total deposited scale layer. An obvious area of weakness in this study is the lack of any solution chemistry or chemical kinetic effects. The incorporation of the chemistry of siliceous scale precipitation is a much more difficult problem than the isolated, thermally dependent phenomena associated with the hydrodynamic aspects of flow.

References

1. L.B. Owen, *Precipitation of Amorphous Silica from High-Temperature Hypersaline Geothermal Brines*, Lawrence Livermore Laboratory, Rept. UCRL-51866 (1975).
2. L.B. Owen, Lawrence Livermore Laboratory, private communication (1976).
3. A.C. Hindmarsh, *Solution of Banded Linear Systems*, Lawrence Livermore Laboratory, Rept. UCID-30045 (1972).
4. G.G. O'Brien, M.A. Hyman, and S. Kaplan, "A Study of the Numerical Solution of Partial Differential Equations," *J. Math. Phys.* 29, p. 223 (1951).
5. R.C. Schroeder, *Reservoir Engineering Report for the Magma-SDG&E Geothermal Experimental Site Near the Salton Sea, California*, Lawrence Livermore Laboratory, Rept. UCRL-52094 (1976).
6. R.S. Brodkey, *The Phenomena of Fluid Motions* (Addison-Wesley Publishing Co., 1967), p. 117.
7. A.E. Scheidegger, *The Physics of Flow Through Porous Media* (University of Toronto Press, Toronto, Canada, 1974), 3rd ed.