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Collisional Effects on Trapped Electron Instabilities

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ABSTRACT

The effects of collisions on dissipative trapped electron instabilities are evaluated by distinguishing between the perpendicular and parallel electron velocity components when calculating trapped and untrapped electron contributions. The growth rate is obtained for all regimes of collisionality and reduces to previous results in appropriate limits. We show that the dominant effect of finite collisionality is to determine the number of trapped and untrapped electrons and that collisional broadening removes the resonant electron response only for collision frequencies greater than or equal to ten times the wave frequency, $\nu_e \gtrsim 10 \omega$. For such large ν_e/ω , ion-ion collisions are found to exert a far stronger stabilizing influence than broadening.

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1. INTRODUCTION

Dissipative trapped electron modes¹ are among the instabilities most threatening to confinement in large and reactor-size tokamaks. As in the case of ordinary drift waves, magnetic shear has a strong stabilizing influence² on these instabilities. Usually, however, the shear strength required is impractically large, especially when the resonances associated with the ∇B -drift³ of the trapped electrons are included. Prompted by the observation that resonant electrons can result in stabilizing contributions for collisionless drift waves, the effect of resonant electrons on trapped electron modes was first investigated by Tang et al.,⁴ and later by Hinton and Ross,⁵ Horton,⁶ and Catto et al.⁷ Reference 7 introduces a model useful for all collisionalities, ν_* , but neglects resonance broadening. (The collisionality ν_* is the ratio of effective collision frequency to the trapped electron bounce frequency for a thermal electron.) The observation⁷ that trapped electrons cannot effectively contribute to the collisionless resonance reduces the argument of the logarithm of the usual slab result from $(ML_n/m_e L_s)$ to $1/\epsilon$ for $\nu_* \rightarrow 0$,⁴ where m_e and M are the electron and ion masses, L_n and L_s are the density gradient and shear scale lengths, and ϵ is the inverse aspect ratio associated with the rational surface under consideration. Because the trapped electrons are permitted in effect to resonate with the wave in the analysis in Ref. 5 in the absence of collisional broadening, the resulting resonant response is the same as for a collisionless drift wave in a slab geometry. The analysis of Ref. 6 does not properly treat magnetic shear in resonant electron contributions.

It is our purpose to derive a resonant electron term applicable in all collisionality regimes for a sheared magnetic field. In order to do so, we have to distinguish carefully between resonant and nonresonant electron responses.

We employ a $v_{||}$ (parallel electron velocity) and $v \equiv |\vec{v}|$ (the total speed) plot as shown in Fig. 1. Independent variables $v_{||}$ and v are used because they are more convenient in solving the kinetic equation with an appropriate collision operator. The line $v_{||} = \epsilon^{1/2} v$ roughly represents the poloidal average location of the boundary between trapped and untrapped electrons in the collisionless limit. When finite collisionality is taken into account, particles with $v_{||} < \epsilon^{1/2} v$ can be detrapped by collisions if their velocity is below v_* , where $v_* = v_e^{1/4} v_e$, $v_e = (2T_e/m_e)^{1/2}$, $v_* = 2^{1/2} v_e Rq/\epsilon^{3/2} v_e$, v_e is the average electron collision frequency, and R and q are the major radius and safety factors of the tokamak. The argument is as follows. For the trapped electrons with $v_{||} < \epsilon^{1/2} v$, the effective collision and bounce frequencies are ν/ϵ and $(\epsilon/2)^{1/2} \nu/Rq$ respectively, where $\nu = \nu_e (v_e/v)^3$. Hence for $v < v_*$, the effective collision frequency is larger than the bounce frequency and the electron is detrapped by collisions. Because $v_{||}$ is never larger than v , there are no particles in Fig. 1 in the region with $v_{||} > v$. Between the forbidden region and the trapped region, there is a region where particles follow the field line without mirroring, just as particles in a slab geometry. The collisional effect on these particles in a sheared field is the principal topic of this paper.

The procedure of splitting up the velocity space according to the local collisionality and treating each region separately has been found

to give good agreement with exact calculations in cases where pitch angle scattering is dominant.⁸ We neglect the treatment of the boundary layer between trapped and untrapped electrons here, because its effect is apparently negligible since its contribution to the dispersion relation is proportional to the volume of the boundary layer in velocity space.

2. SOLUTION FOR COLLISIONAL AND UNTRAPPED ELECTRONS

For a perturbed potential of the form $\phi = \hat{\phi}(r) \exp(-i\omega t + im\theta - il\zeta)$, the perturbed resonant electron distribution function may be written as

$$f_e = \frac{e}{T_e} \hat{\phi} F_e + \hat{g} \exp(-i\omega t + im\theta - il\zeta),$$

where F_e is the equilibrium electron distribution function.

$$F_e = N(\pi v_e^2)^{-3/2} \exp(-v^2/v_e^2).$$

The departure from an adiabatic electron response, g , satisfies the following equation:

$$(\omega - k_{||}v_{||} - iC) \hat{g} = -(\omega - \omega_*^T) \frac{e}{T_e} F_e \hat{\phi},$$

where

$$k_{||} = \frac{1}{Rq} (m - lq), \quad \omega_*^T = \omega_* \{1 + \eta_e [(v^2/v_e^2) - \frac{3}{2}]\}, \quad \omega_* = kcT_e/eBL_n, \quad k = m/r,$$

and C is a collision operator. For electrons in the collisional portion of velocity space, it is convenient to employ $v_{||}$ and $v = (v_{||}^2 + v_{\perp}^2)^{1/2}$ as the independent variables. Neglecting the mirror force on these electrons and using a pitch angle scattering model collision operator⁹

$$Cg = v \frac{\partial}{\partial v_{||}} \left[(v^2 - v_{||}^2) \frac{\partial g}{\partial v_{||}} \right],$$

we obtain

$$\left[v_{||} - \frac{\omega}{k_{||}} - \frac{iv}{k_{||}} \frac{\partial}{\partial v_{||}} (v^2 - v_{||}^2) \frac{\partial}{\partial v_{||}} \right] G = \frac{\omega}{k_{||}}, \quad (1)$$

where

$$G \equiv \hat{g} \left(\frac{\omega - \omega_*^T}{\omega} \frac{e}{T_e} F_e \hat{\phi} \right)^{-1}, \quad v = v_e (v_e/v)^3, \quad v_e = \frac{2\pi e^4 Z N_e \ln \Lambda}{m_e^2 v_e^3}.$$

Equation (1) can be solved by Fourier transforming in $v_{||}$,

$$G(p) = \int_{-v}^v dv \quad G(v_{||}) \exp(i p \cdot v_{||}).$$

In the transform of Eq. (1), the $\partial^2 G / \partial p^2$ and $v p \partial G / \partial p$ terms may be neglected because they only contribute for $v v^2 p^3 \gg k_{||}$, and $\sin(pv)/p$ may be approximated by $\pi \delta(p)$. The approximate solution is then

$$G = i \frac{\omega}{|k_{||}|} \int_0^\infty dp \exp(-isv_{||} p + i \frac{\omega}{|k_{||}|} p - \frac{v v^2}{3|k_{||}|} p^3), \quad (2)$$

where $s = \text{sign}(k_{||})$.

Within the accuracy of our model collision operator and the preceding approximations, Eq. (2) should describe the collisional electrons correctly until v is so small that trapping occurs in the toroidal field. Consequently, Eq. (2) is applicable everywhere in the velocity space except in the trapped region.

The perturbed electron density due to G is

$$\tilde{N}_e = i \int_{UT} d\vec{v} \frac{\omega - \omega_*^T}{|k_{||}|} \frac{e\hat{\phi}}{T_e} F_e \int_0^\infty dp \exp(-isv_{||} p + i \frac{\omega}{|k_{||}|} p - \frac{v v^2}{3|k_{||}|} p^3),$$

where $\int_{UT} d\vec{v}$ indicates that the velocity integration is over all the untrapped region in Fig. 1. As a result, we have

$$\int_{UT} d\vec{v} = 2\pi \left[\int_0^\infty v dv \int_{-v}^{+v} dv_{||} - \int_{v_*}^\infty v dv \int_{-v_e^{1/2}}^{v_e^{1/2}} dv_{||} \right],$$

and

$$\tilde{N}_e = 4\pi i \frac{e\hat{\phi}}{T_e} \int_0^\infty dp \left[\int_0^\infty v dv \frac{\sin(vp)}{p} - \int_{v_*}^\infty v dv \frac{\sin(vp\varepsilon^{1/2})}{p} \right] x$$

$$F_e \frac{\omega - \omega_*^T}{|k_{||}|} \exp \left(i \frac{\omega p}{|k_{||}|} - \frac{v v^2}{3|k_{||}|} p^3 \right).$$

3. DISPERSION RELATION

Taking into account the response of the ions and the trapped and untrapped electrons, quasi-neutrality gives

$$\frac{\partial^2 \hat{\phi}}{\partial x^2} - \left[\Lambda - \mu^2 x^2 + i\sigma(|x|) \right] \hat{\phi} = 0, \quad (5)$$

where $\mu = (\Omega/\omega) |k/L_s| S^{1/2}$, $\Omega = eB/Mc$, $L_s^{-1} = (\epsilon/q) \partial \ln q / \partial r$,

$$\Lambda = \frac{2}{\rho_i^2 D} \{ \omega [1 + \tau(1 - \Gamma_0)] - \omega_* [\Gamma_0 - \eta_i b (\Gamma_0 - \Gamma_1)] + (\omega - \omega_*) A_2 \\ - \eta_e \omega_* A_1 + \frac{i v_i}{\omega} [(\omega\tau + \omega_*) \Gamma_0 - \eta_i \omega_* b (\Gamma_0 - \Gamma_1)] \},$$

$$A_1 = - \sum_{p=1}^{\infty} \frac{8}{\pi^{1/2} \alpha_p^2} \int_{v_*^{1/2}}^{\infty} \frac{dt t^{1/2} (t - \frac{3}{2}) \exp(-t)}{1 - \frac{\omega_{De}}{\omega} t + \frac{i v_e \alpha_p^2}{4 \epsilon \omega t^{3/2}}},$$

$$A_2 = - \sum_{p=1}^{\infty} \frac{8}{\pi^{1/2} \alpha_p^2} \int_{v_*^{1/2}}^{\infty} \frac{dt t^{1/2} \exp(-t)}{1 - \frac{\omega_{De}}{\omega} t + \frac{i v_e \alpha_p^2}{4 \epsilon \omega t^{3/2}}},$$

$$i\sigma = \frac{2 \omega T_e \tilde{N}_e}{e N_e \rho_i^2 \hat{\phi} D}, \quad D = (\omega\tau + \omega_*) (\Gamma_0 - \Gamma_1) + \eta_i \omega_* [\Gamma_0 - 2b(\Gamma_0 - \Gamma_1)],$$

and $S = D^{-1} \{ (\omega\tau + \omega_*) \Gamma_0 + \eta_i \omega_* [\Gamma_0 - b(\Gamma_0 - \Gamma_1)] \}$.

In the preceding, $v_i^2 = 2T_i/M$, $\rho_i = v_i/\Omega$, $b = k^2 \rho_i^2 / 2$, $\Gamma_j = I_j(b) \exp(-b)$,

$\tau = T_e/T_i$, $\eta_j = d \ln N / d \ln T_j$, $\omega_{De} = \omega_* L_n / R$, $L_n^{-1} = - d \ln N / dr$, $x = r - r_{m\ell}$,

$m = \ell q(r_{m\ell})$, α_p is the p^{th} zero of $J_0(x)$ and ¹⁰

$$v_i = \begin{cases} 0 & b \leq 1 \\ \frac{2(2\pi)^{1/2} Z^4 e^4 N_i \ln \Lambda}{M^{1/2} T_i^{3/2}} & b > 1 \end{cases}$$

The A_1 and A_2 are found by using an approximate collision operator valid for deeply trapped electrons and demanding $g = 0$ at the collisionless trapping boundary.¹¹

Solving Eq. (5) for the lowest even unstable $\hat{\phi} = \hat{\phi}_0 \exp(-i\mu x^2/2)$ by including the resonant electron term iteratively yields the eigenvalue equation

$$\Lambda + i\mu = -2 \left(\frac{i\mu}{\pi}\right)^{1/2} i \int_0^\infty dx \sigma(|x|) \hat{\phi}^2.$$

The x , p , and v integrations of $\int_0^\infty dx \sigma \hat{\phi}^2$ can be performed as indicated in the Appendix to obtain the final eigenvalue equation

$$\begin{aligned} \omega [1 + \tau(1 - \Gamma_0)] - \omega_* [\Gamma_0 - \eta_i b(\Gamma_0 - \Gamma_1)] + (2\varepsilon)^{1/2} (\omega - \omega_*) A_2 \\ - (2\varepsilon)^{1/2} \eta_e \omega_* A_1 + \frac{i v_i}{\omega} [(\omega\tau + \omega_*) \Gamma_0 - \eta_i \omega_* b (\Gamma_0 - \Gamma_1)] \end{aligned} \quad (6)$$

$$+ i s^{1/2} \frac{|\omega_* L_n|}{\omega \tau |L_s|} \{ (\omega\tau + \omega_*) (\Gamma_0 - \Gamma_1) + \eta_i \omega_* [\Gamma_0 - 2b(\Gamma_0 - \Gamma_1)] \}$$

$$+ \frac{(i-1)}{2^{1/2}} \frac{\chi_e}{\chi_T} [(\omega - \omega_*) C_2 + \eta_e \omega_* C_1] = 0,$$

where $C_2 = \bar{C}_2 - \tilde{C}_2$, $C_1 = \bar{C}_1 - \tilde{C}_1$,

$$\frac{\chi_T}{\chi_e} = \left[\frac{|\omega_*|}{\omega s^{1/2}} \left(\frac{2M|L_n|}{m_e |L_s|} \right) \right]^{1/2},$$

$$\bar{C}_2 = \ln \left(\frac{x_T^2}{x_e} \right) - 2\gamma - i \frac{\pi}{2} - \frac{4L_2(0, \nu_e/3\omega)}{S \sqrt{\frac{\nu_e}{3\omega}}} \quad (7)$$

$$\bar{C}_2 = \exp(-\nu_e^{1/2}) \left[\ln \left(\frac{x_T^2}{x_e} \right) - 2\gamma - i \frac{\pi}{2} - \frac{4L_2(0, \nu_e/3\omega)}{S \sqrt{\frac{\nu_e}{3\omega}}} \right] \quad (8)$$

for trapped electrons and assuming $\nu_e = 0$ at the collisionless

$$\bar{C}_1 = \ln \left(\frac{x_T^2}{x_e} \right) - 2\gamma - i \frac{\pi}{2} - \frac{8L_1(0, \nu_e/3\omega)}{S \sqrt{\frac{\nu_e}{3\omega}}} \quad (9)$$

by including the resonant electron term iteratively yields the eigen-

$$\bar{C}_1 = \frac{1}{2} \exp(-\nu_e^{1/2}) \left\{ (1 - 2\nu_e^{1/2}) \left[\ln \left(\frac{x_T^2}{x_e} \right) - \gamma - i \frac{\pi}{2} \right] + \frac{8L_1(0, \nu_e/3\omega)}{S \sqrt{\frac{\nu_e}{3\omega}}} \right\} \quad (10)$$

as determined by the L_1 and L_2 functions (see Appendix A) and can be performed as

indicated in the Appendix to obtain the final eigenvalue equation

$$L_1(a, \omega) = \int_a^\infty dt \left(\frac{3}{2} - [t] \exp(-t) \right) \operatorname{Re} E_1 \left[\frac{t^{3/4}}{\alpha^{1/2}} \exp(-i \frac{\pi}{4}) \right] + i \gamma \quad (11)$$

and

$$L_2(a, \omega) = \int_a^\infty dt t \exp(-t) \operatorname{Re} E_1 \left[\frac{t^{3/4}}{\alpha^{1/2}} \exp(-i \frac{\pi}{4}) \right] + i \gamma \quad (12)$$

In the preceding, $E_1(x) \equiv \int_x^\infty dt t^{-1} \exp(-t)$ is the exponential integral

and $\gamma = 0.577\dots$. For $\nu_e/\omega \rightarrow 0$, L_1 and L_2 go to zero. As a

result, the influence of collisions on the resonant electron inter-

action is contained in the L 's, and for $L_1 = 0 = L_2$ the forms of C_1

and C_2 are the generalizations of the results of Ref. 7, where $C = C_0$

In the strong collisional broadening limit, $\nu_e/\omega \rightarrow \infty$, the resonance broadening corrections are only valid until the broadening has removed the resonance. That is, each $\bar{C}$ and/or $\tilde{C}$ expression is to be replaced

by zero for v_e/ω larger than the critical value that changes the sign of the real part as v_e/ω is increased from very small to very large values. If there is a value of v_e/ω for which $\text{Re } \tilde{C} = \text{Re } \bar{C}$, then both $\tilde{C}$ and $\bar{C}$ are to be set equal to zero above this v_e/ω . (Recall that the $\tilde{C}$'s are just the modifications to the resonant electron response caused by the inability of trapped electrons to resonate with the wave.) The sign change in $\tilde{C}_1$ as a function of $v_*^{1/2}$ for fixed $v_e/\epsilon\omega$ occurs for reasons similar to those that cause the sign change in A_1 with v_* . The expressions for the C 's fail for large enough v_e/ω because in evaluating the x integrations in the Appendix $x_T/x_0 \gg 1$ is assumed. When $v_e/\omega \rightarrow \infty$, however, $x_T/x_0 \rightarrow 1$ and the Landau resonance must be removed.

Before the discussion of the electron resonance terms in various limits, we want to point out a limitation of Eq. (6), which is derived from the flutelike mode assumption of the perturbed electrostatic potential. When this assumption is abolished, the eigenmode equation becomes two-dimensional. In the two-dimensional eigenfunction problem, modes centered at adjacent mode rational surfaces are coupled together by the trapped electron terms. Our radial eigenmode equation has been solved by assuming that the free energy which drives the instability originates very close to the mode rational surface and is convected outward to the region where ion Landau damping occurs, $x \geq x_i$, where $k_{||}(x_i)v_i \equiv \omega$. As a result, it has been implicitly assumed that $x_i < \delta$, where $\delta = (\epsilon \partial q / \partial r)^{-1}$, is the distance to the next rational surface having the same toroidal mode number but a poloidal mode number of $m \pm 1$. The condition $x_i < \delta$ requires that the mode be strongly ion Landau damped before encountering an adjacent mode rational surface. For $b < 1$, $x_i < \delta$ results in the

inequality $b < L_n / \tau R q$. This inequality provides us with an upper bound for b beyond which Eq. (6) should be modified because of the radial overlapping of the mode over many rational surfaces. This inequality is not exact, but it does represent the most conservative estimate. A more careful treatment¹² of the multiple rational surface problem will hopefully relax this inequality somewhat.

The nonadiabatic untrapped electron contributions contained in Eq. (6) differ from the results of Hinton and Ross⁵ in their dependence on v_* and v_e/ω . Hinton and Ross argue that due to collisional coupling between trapped and untrapped electrons at the trapping boundary, the untrapped electron resonance is carried over to the trapped electrons so that the resonant electron response is not sensitive to the trapped-untrapped boundary (that is, v_*). This effect enters because of the addition of a homogeneous solution to their trapped electron response, Eq. (35), which when matched to their untrapped electron response, Eq. (44), does not vanish provided one neglects the difference between the $\exp(im\theta)$ of the untrapped and the $\exp(i l q \theta)$ of the trapped electron responses. It is not clear why they do not also add a homogeneous solution to the untrapped electrons. We do not believe that these θ dependences can be matched and so we do not obtain any additional homogeneous solutions. Even if such a coupling is possible, we expect on physical grounds that the effect should increase monotonically with increasing v_* . Eq. (45) of Hinton and Ross obviously violates this belief. In addition, Eq. (45) is not consistent with the $m = lq$ assumption.

As for the v_e/ω dependence, we note that Hinton and Ross assume $\epsilon\omega \ll v_e < \omega$, while we consider arbitrarily v_e/ω . To recover the results

of Hinton and Ross from Eqs. (7)-(10), we note that $\varepsilon\omega \ll v_e$ results in $\mathcal{C}_1 \approx 0 \approx \mathcal{C}_2$, while $v_e < \omega$ permits the L's in Eqs. (7) and (9) to be neglected. The resulting C_1 and C_2 correspond to the collisionless drift wave limit of a slab geometry.

4. DISCUSSION

From Eqs. (7)-(10) and (A1)-(A2) the influence of resonance broadening can be evaluated. For $\nu_* \gg 1$ and ν_e/ω , arbitrary $\tilde{C}_1 \approx 0 \approx \tilde{C}_2$ so that C_1 and C_2 simplify to the resonance broadened slab result $C_1 = \bar{C}_1$ and $C_2 = \bar{C}_2$. In $\bar{C}_1$ and $\bar{C}_2$ the forms of the $L_j(0, \alpha)$ useful for small and large α are obtained from Eqs. (A1) and (A2), respectively, by letting $\nu_*^{1/2} \rightarrow 0$ while keeping $\alpha = \nu_e/3\omega$ finite. From the plots of $L_j(0, \alpha)$, $L_2(0, \alpha)$, $\text{Re } \bar{C}_1$, and $\text{Re } \bar{C}_2$ in Figs. 2-4, resonance broadening of this $\nu_* \gg 1$ limit is seen to be important when $\nu_e/3\omega \gtrsim 10^{-1}$ for C_1 and when $\nu_e/3\omega \gtrsim 1$ for C_2 .

For $\nu_*^{3/8} > (\nu_e/3\epsilon\omega)^{1/2}$ and ν_* arbitrarily, the $L_j(\nu_*^{1/2}, \nu_e/3\epsilon\omega)$ corrections in C_1 and C_2 are exponentially small and so may be neglected. As a result, the resonance broadening in C_1 and C_2 is again due only to the $L_j(0, \nu_e/3\omega)$ so that the $\nu_* \gg 1$ limit is the same as the preceding case; while for $\nu_* \ll 1$, $(\nu_e/3\omega) < \epsilon < 1$ and resonance broadening is not important. Consequently, the resonant electron contribution obtained in Ref. 4 is valid for $(\nu_e/3\epsilon\omega)^{4/3} < \nu_* \ll 1$ and not $\nu_* \ll \epsilon^2 \omega^2 / \omega_0^2$ as claimed in Ref. 5, where $\omega_b = (\epsilon/2)^{1/2} \nu_e/Rq$ is the thermal electron bounce frequency.

For both $\nu_* < 1$ and $\nu_*^{3/8} < (\nu_e/3\epsilon\omega)^{1/2}$ life is somewhat more complicated because of the cancellation caused by subtracting out the trapped electrons ($\nu_* < 1$) and because the $L_j(\nu_*^{1/2}, \nu_e/3\epsilon\omega)$'s are not exponentially small. For $\nu_* < 1$, Eqs. (7)-(10) may be combined to give

$$C_1 = -(1/2)\ln\epsilon/z + (3/2)v_*^{1/2} \left[\ln(v_*^{1/2}\epsilon\chi_T^2/\chi_e^2) - \gamma - 1 - i\pi/2 \right] \\ + 4 \left[L_1(v_*^{1/2}, v_e/3\epsilon\omega) - L_1(0, v_e/3\omega) \right] \\ \text{and} \quad (11)$$

$$C_2 = -\ln\epsilon + v_*^{1/2} \left[\ln(v_*^{1/2}\epsilon\chi_T^2/\chi_e^2) - \gamma - 1 - i\pi/2 \right] \\ + 4 \left[L_2(v_*^{1/2}, v_e/3\epsilon\omega) - L_2(0, v_e/3\omega) \right],$$

where various limits of the L's depend on $v_*^{1/2}$, $v_*^{3/8}/(v_e/3\epsilon\omega)^{1/2}$, and $v_*^{3/8}/(v_e/3\omega)^{1/2}$ versus unity. In considering the various limits it is useful to note that for $v_*^{1/2} < \alpha^{2/3} < 1$, Eq. (A1) may be employed to obtain

$$L_1(v_*^{1/2}, \alpha) = 3\alpha^{2/3} \Gamma(4/3)/4 + 5\alpha^{4/3} \Gamma(8/3)/8 \\ - (3/2)v_*^{1/2} \left[3/4 - \gamma + \ln(\alpha^{1/2}/v_*^{3/8}) \right] + \dots, \quad (12)$$

$$L_2(v_*^{1/2}, \alpha) = \alpha^{2/3} \Gamma(4/3)/2 + \alpha^{4/3} \Gamma(8/3)/4 \\ - v_*^{1/2} \left[3/4 - \gamma + \ln(\alpha^{1/2}/v_*^{3/8}) \right] + \dots,$$

while for $v_*^{1/2} < 1 < \alpha^{2/3}$ Eq. (A2) gives

$$L_1(v_*^{1/2}, \alpha) = (1/2)(\ln \alpha^{1/2} - \gamma/4) + 3/4 - \Gamma(7/4)(2\alpha)^{-1/2}/4 \\ - (3/2)v_*^{1/2} \left[\ln(\alpha^{1/2}/v_*^{3/8}) - \gamma + 3/4 \right] + \dots \quad (13)$$

$$\text{and} \\ L_2(v_*^{1/2}, \alpha) = \ln \alpha^{1/2} - \gamma/4 + \Gamma(7/4)(2\alpha)^{-1/2} - v_*^{1/2} \left[\ln(\alpha^{1/2}/v_*^{3/8}) \right. \\ \left. - \gamma + 3/4 \right] + \dots$$

Considering the $\nu_*^{1/2} < (\nu_e/3\epsilon\omega)^{2/3} < 1$ limit first, employing Eq. (12) in Eq. (11), shows that collisional broadening is not important in this limit because $\nu_e/3\omega$ must also be much less than one.

For $\nu_*^{1/2} < 1 < (\nu_e/3\epsilon\omega)^{2/3}$, Eq. (13) may be inserted in Eq. (11).

For $\nu_e/3\omega \lesssim 1$, broadening is significant if $\ln(\nu_e/3\epsilon\omega)^2 \gtrsim \ln(1/\epsilon) + 3\nu_*^{1/2} \ln(\nu_*^{1/2} \epsilon \chi_T^2 / \chi_e^2) > 1$. For $\nu_e/3\epsilon\omega > 1$, Eqs. (11) and (13) yield

$$C_1 = -3(\ln\epsilon)/2 + (3/2)\nu_*^{1/2} \left[\ln \nu_*^2 \epsilon \chi_T^2 \chi_e^{-2} \right] (\nu_e/3\epsilon\omega)^{-2} + \dots \quad (14)$$

and

$$C_2 = -3 \ln\epsilon + \nu_*^{1/2} \ln \left[\nu_*^2 \epsilon \chi_T^2 \chi_e^{-2} (\nu_e/3\epsilon\omega)^{-2} \right] + \dots$$

Consequently, collisional broadening is always significant for $\nu_* < 1$ and $\nu_e/3\epsilon\omega > 1$. Expression (14) remains valid as long as the real parts of $\bar{C}_1$, $\bar{C}_1$, $\bar{C}_2$, and/or $\bar{C}_2$ do not change sign as ν_e/ω is increased or until $\text{Re}(\bar{C} - \bar{C}) = 0$.

The behavior of the L's and C's as a function of α for various ν_* and $x_T^2/x_e^2 = 368$ is summarized in Figs. 2-4. These numerical results, obtained by using the integral representation of the L's, the $\text{Re}\bar{C}_1$ and $\text{Re}\bar{C}_2$ for $\nu_* \gtrsim 1$ and $\nu_* \gtrsim 10$ respectively, are not shown in Figs. 3 and 4 because they are small enough to neglect.

5. SUMMARY AND CONCLUSIONS

The preceding limits can be summarized qualitatively by writing

$$\bar{C}_1 = \frac{1}{2} \left\{ \ln \left[\frac{x_T^2/x_e^2}{1 + \bar{\sigma}_1 (v_e/3\omega)^2} \right] - 2\gamma - 2 - i\pi/2 \right\}, \quad (15)$$

$$\bar{C}_2 = \ln \left[\frac{x_T^2/x_e^2}{1 + \bar{\sigma}_2 (v_e/3\omega)^2} \right] - 2\gamma - i\pi/2, \quad (16)$$

$$\begin{aligned} \mathcal{C}_1 = \frac{1}{2} \exp(-v_*^{1/2}) & \left((1-2v_*^{1/2}) \left\{ \ln \left[\frac{\epsilon x_T^2/x_e^2}{1 + \tilde{\sigma}_1 (v_e/3\epsilon\omega)^2} \right] - \gamma - i\pi/2 \right\} \right. \\ & \left. - \frac{3}{2} v_*^{1/2} \ln v_* + v_*^{1/2} - 2 - \gamma \right), \end{aligned} \quad (17)$$

and

$$\mathcal{C}_2 = \exp(-v_*^{1/2}) \left\{ \ln \left[\frac{\epsilon x_T^2/x_e^2}{1 + \tilde{\sigma}_2 (v_e/3\epsilon\omega)^2} \right] - \frac{1}{2} v_*^{1/2} \ln v_* + v_*^{1/2} - 2\gamma - i\pi/2 \right\}, \quad (18)$$

where $\bar{\sigma}_1 \approx 130$, $\bar{\sigma}_2 \approx 1.3$, $\gamma = .577 \dots$, and $\tilde{\sigma}_1$ and $\tilde{\sigma}_2$ start out at $\bar{\sigma}_1$ and $\bar{\sigma}_2$ (for $v_* = 0$) but decrease with increasing v_* . Table 1 gives $\tilde{\sigma}_1$ and $\tilde{\sigma}_2$ for $10^{-2} \leq v_* \leq 10$. For $v_* \gtrsim 1$ we may set $\tilde{\sigma}_1 = 0$ because broadening does not occur before $\text{Re}(\bar{C}_1 - \mathcal{C}_1) = 0$. Similarly, we may set $\tilde{\sigma}_2 = 0$ for $v_* \gtrsim 10$ because broadening does not occur before $\text{Re}(\bar{C}_2 - \mathcal{C}_2) = 0$. The approximated forms for $\bar{C}_1$ and $\bar{C}_2$ in Eqs. (15) and (16) are plotted as the dashed lines in Figs. 3 and 4 to indicate the validity of approximate forms. The approximate forms for $\mathcal{C}_1$ and $\mathcal{C}_2$ exhibit even better agreement with the exact forms.

From the discussion leading up to Eqs. (15)-(18) the following conclusions can be reached:

(i) For $v_* \lesssim 1$ the depletion in the number of resonant electrons caused by the presence of a substantial trapped electron population is an important modification to the nonadiabatic resonant electron response. Similarly, for $v_* \gtrsim 1$ the reduction in the number of trapped electrons will strongly reduce the nonadiabatic trapped electron response.

(ii) Collisional broadening of the resonant electron interaction plays a significant role for $v_e/3\omega \gtrsim 10^{-1}$ when $v_* \gg 1$, and for $v_e/3\omega \gtrsim \epsilon/10$ for $v_* \ll 1$. In particular, for $v_e/3\omega \gtrsim 10$ the broadening completely removes the resonant electron response.

The eigenvalue equation, Eq. (6), can be solved approximately by assuming $\omega[1 + \tau(1 - \Gamma_0)] = \omega_*[\Gamma_0 - \eta_i b(\Gamma_0 - \Gamma_i)]$ to the lowest order and then iterating to find $\delta\omega$ with $\text{Im } \omega = \gamma$, the growth rate. Plotting $\gamma L_n (T_e/M)^{-1/2}$ versus b for various v_* and $L_n/L_s = 0.01, 0.03$, and 0.06 results in Figs. (5), (6), and (7), respectively. The remaining parameters are taken to be $q = 2$, $\eta_e = 1 = \eta_i$, $\tau = 1$, $\epsilon = 0.2 = L_n/R$, and $M/m_e = 1840$. An exact solution of Eq. (6) may result in quantitative changes in Figs. 5-7 but the qualitative structures will be as indicated.

From these plots the following observations can be obtained:

(i) The peak in γ at lower b is due to the trapped electron contributions while that at higher b is due to resonant electron effects. The stabilizing influence of shear can be seen by comparing Figs. 5-7 for a fixed value of v_* .

(ii) A strong stabilizing influence on the trapped electron contributions occurs as v_* increases beyond unity, as would be expected because of the lower limit in A_1 and A_2 .

(iii) Starting with the smallest ν_* and increasing ν_* destabilizes the resonant electron contribution until $\nu_* \sim 1$. This destabilizing influence occurs because more electrons are able to resonate with the wave; that is, the number of trapped electrons is reduced as ν_* is increased. For $\nu_* \gtrsim 1$ effectively all electrons resonate with the wave.

iv) For $\nu_* \gtrsim 1$ and $\nu_e/3\omega \lesssim 1$ the resonant electron contributions reach a maximum, and increasing ν_* further causes $\nu_e/3\omega > 1$ so that resonant broadening has a stabilizing influence. For $\nu_e/3\omega > 1$, however, ion-ion collisions begin exerting a strong stabilizing force which can cancel out the destabilizing resonant electron contributions. In fact, in order for ion-ion collisions to be negligible, $\omega > \nu_i \sim 2b(m_e/M)^{1/2} \nu_e$, and resonant broadening to be important, $\nu_e/3\omega \gtrsim 1$, when $b > 1$ requires $3 \lesssim \nu_e/\omega < (2b)^{-1} (M/m_e)^{1/2}$ or $1 < b < (M/m_e)^{1/2}/b$. Consequently, resonant broadening enters only for a very limited range of b and $\nu_* \gtrsim 1$.

The preceding can be summarized by saying that the dominant collisional effects enter via ν_* and ion-ion collisions, and not because of collisional broadening of the resonant electron response. The ν_* sensitivity occurs because ν_* is a measure of the relative number of trapped and untrapped electrons.

ACKNOWLEDGMENTS

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APPENDIX

The purpose of this appendix is to evaluate the x , p , and v integrations of Eq. (6).

To evaluate the x integration and retain order unity corrections to the logarithm, it is necessary to note that the solution of Eq. (2) fails as $k_{||} \rightarrow 0$. The failure occurs because the $vpv/k_{||}$ correction to $\partial G/\partial p$ can no longer be neglected and because it is no longer sufficient to approximate $\sin(pv)/p$ by $\pi\delta(p)$. Because $\omega/k_{||} \rightarrow \infty$ corresponds to $v_{||} \rightarrow \infty$ and therefore $v \rightarrow \infty$, the solution of the equation for $G(p)$ in the small $k_{||}$ limit is actually independent of x . As a result in

$\int_0^\infty \frac{dx}{x} \exp(-i\mu x^2) G(p)$, the path in the complex x plane which keeps the integrand pure real starts out from $x = 0$ along the real $x > 0$ axis and ends up on a path $-\pi/4$ from the real $x > 0$ axis as $|x| \rightarrow \infty$. Consequently,

$$\int_0^\infty \frac{dx}{x} \exp[-i\mu x^2 + (ia - b)/x] \rightarrow -i \frac{\pi}{4} + \int_0^\infty \frac{dx}{x} \exp(-x^2/x_T^2 - x_0/x)$$

where $a = \omega p |L_S/k|$, $b = v\omega^2 p^3 |L_S/3k|$, $x_0 = |b - ia| = (a^2 + b^2)^{1/2}$, and $x_T = |\mu^{-1/2}|$ so that the x integral that remains is now pure real.

Integrating for $x_T/x_0 \gg 1$

$$\int_0^\infty \frac{dx}{x} \exp[-i\mu x^2 + (ia - b)/x] \rightarrow -i \frac{\pi}{4} + \left[\ln\left(\frac{x_T}{x_0}\right) - \frac{3}{2} \gamma + \dots \right],$$

where $\gamma = .577$ is Euler's constant.

Next we evaluate the p integrals, again being careful to retain order unity corrections. Letting pv or $pve^{1/2}$ equal z we can obtain these order unity corrections to the p integrations by noting that

$$\begin{aligned}
\int_0^\infty \frac{dz}{z} \sin z \ln z &= \int_0^{\delta \rightarrow 0} dz \ln z + \operatorname{Im} \int_{\delta \rightarrow 0}^\infty \frac{dz}{z} (\ln z) \exp(iz) \\
&= \frac{\pi}{2} [\ln \delta + E_1(\delta)] + \operatorname{Im} \int_\delta^\infty \frac{dy}{y} (\ln y) \exp(-y) \\
&= -\frac{\pi}{2} \gamma.
\end{aligned}$$

Using the preceding gives

$$\begin{aligned}
\int_0^\infty \frac{dp}{p} (\sin pu) \ln \left\{ \frac{\sigma}{pu[1+\alpha(pu)^4]^{1/2}} \right\} \\
&= \frac{\pi}{2} \ln \sigma - \int_0^\infty \frac{dz}{z} (\ln z) \left\{ \ln z + \frac{1}{2} \ln [1 + \alpha z^4] \right\} \\
&= \frac{\pi}{2} [\ln \sigma + \gamma] - \frac{1}{2} \int_0^\infty \frac{dz}{z} (\sin z) \ln (1 + \alpha z^4) \\
&= \frac{\pi}{2} \left\{ \ln \sigma + \gamma - 2 \operatorname{Re} E_1 [(1-i)/2^{1/2} \alpha^{1/4}] \right\}.
\end{aligned}$$

Using the preceding results for the x and p integrations results in

$$\begin{aligned}
&\frac{T_e}{i4\pi e \hat{\phi}} \left| \frac{k}{L_s} \right| \int_0^\infty dx \exp(-i\mu x^2) \tilde{N}_e \\
&\approx \left[\int_0^\infty dv v F_e(\omega - \omega_\star^T) \int_0^\infty \frac{dp}{p} \sin(pv) - \int_{v_\star}^\infty dv v F_e(\omega - \omega_\star^T) \int_0^\infty \frac{dp}{p} \sin(\varepsilon^{1/2} pv) \right] \\
&\quad \times \left\{ \ln \left[\frac{x_T}{x_0(p, v)} \right] - \frac{3}{2} \gamma - i \frac{\pi}{4} \right\} \\
&= \frac{\pi}{2} \int_0^\infty dv v F_e(\omega - \omega_\star^T) \left\{ \ln \left[\frac{x_T}{a(p=1/v)} \right] - \frac{\gamma}{2} - i \frac{\pi}{4} - 2 \operatorname{Re} E_1 \left[\frac{(1-i)t^{3/4}}{(2v/3\omega)^{1/2}} \right] \right\} \\
&\quad - \frac{\pi}{2} \int_{v_\star}^\infty dv v F_e(\omega - \omega_\star^T) \left\{ \ln \left[\frac{x_T}{a(p=1/\varepsilon^{1/2}v)} \right] - \frac{\gamma}{2} - i \frac{\pi}{4} - 2 \operatorname{Re} E_1 \left[\frac{(1-i)t^{3/4}}{(2v/3\varepsilon\omega)^{1/2}} \right] \right\}
\end{aligned}$$

$$\begin{aligned}
&= \frac{N}{8\pi^{1/2}v_e} \left(\int_0^\infty dt \exp(-t) \{ \omega - \omega_* [1 + \eta_e (t - \frac{3}{2})] \} \{ \ln(\frac{t\chi_T^2}{\chi_e}) - \gamma - \frac{i\pi}{2} \right. \\
&\quad \left. - 4\operatorname{Re} E_1 \left[\frac{(1-i)t^{3/4}}{(2v_e/3\omega)^{1/2}} \right] \right\} \\
&- \int_{v_*}^\infty dt \exp(-t) \{ \omega - \omega_* [1 + \eta_e (t - \frac{3}{2})] \} \{ \ln(\frac{t\epsilon\chi_T^2}{\chi_e}) - \gamma - \frac{i\pi}{2} \\
&\quad \left. - 4\operatorname{Re} E_1 \left[\frac{(1-i)t^{3/4}}{(2v_e/3\epsilon\omega)^{1/2}} \right] \right\} \Bigg) \\
&\equiv \frac{N}{8\pi^{1/2}v_e} \left[(\omega - \omega_*) C_2 + \eta_e \omega_* C_1 \right].
\end{aligned}$$

Employing

$$\begin{aligned}
\int_{v_*}^\infty dt (\ln t) \exp(-t) &= E_1(v_*^{1/2}) + \left[\exp(-v_*^{1/2}) \right] \ln v_*^{1/2}, \\
\int_{v_*}^\infty dt t (\ln t) \exp(-t) &= E_1(v_*^{1/2}) + \left[\exp(-v_*^{1/2}) \right] [1 + (1 + v_*^{1/2}) \ln v_*^{1/2}],
\end{aligned}$$

and the definitions of L_1 and L_2 , the expressions for C_1 and C_2 of Eqs. (7) to (10) are obtained.

In order to estimate the influence of resonance broadening the following alternate forms of L_1 and L_2 are useful:

$$L_1(v_*^{1/2}, \alpha) = \left[\left(\frac{1}{2} - v_*^{1/2} \right) \exp(-v_*^{1/2}) - \frac{1}{2} \right] \operatorname{Re} E_1[v_*^{3/8} \exp(-i\pi/4) \alpha^{-1/2}] \\ - \operatorname{Re} \sum_{n=1}^{\infty} (n+1/2) [-\alpha^{2/3} \exp(i\pi/3)]^n \Gamma\left(\frac{4n}{3}, \frac{v_*^{3/8} \exp(-i\pi/4)}{\alpha^{1/2}}\right) / n! ,$$

$$L_2(v_*^{1/2}, \alpha) = [\exp(-v_*^{1/2}) - 1] \operatorname{Re} E_1[v_*^{3/8} \exp(-i\pi/4) \alpha^{-1/2}] \\ - \operatorname{Re} \sum_{n=1}^{\infty} [-\alpha^{2/3} \exp(i\pi/3)]^n \Gamma\left(\frac{4n}{3}, \frac{v_*^{3/8} \exp(-i\pi/4)}{\alpha^{1/2}}\right) / n! ,$$

and

$$L_1(v_*^{1/2}, \alpha) = (1/2 - v_*^{1/2}) \exp(-v_*^{1/2}) \operatorname{Re} E_1[v_*^{3/8} \exp(-i\pi/4) / \alpha^{1/2}] \\ - 3E_1(v_*^{1/2})/8 + 3/4 \operatorname{Re} \exp\{-v_*^{1/2} - [v_*^{3/8} \exp(-i\pi/4) / \alpha^{1/2}]\} \\ - \frac{3}{4} \sum_{n=1}^{\infty} \left(\frac{1}{2} - \frac{3n}{4} \right) \frac{(-1)^n \cos n\pi/4}{n! \alpha^{n/2}} \Gamma\left(\frac{3n}{4}, v_*^{1/2}\right) , \quad (A2)$$

$$L_2(v_*^{1/2}, \alpha) = \exp(-v_*^{1/2}) \operatorname{Re} E_1[v_*^{3/8} \exp(-i\pi/4) / \alpha^{1/2}] - 3E_1(v_*^{1/2})/4 \\ - \frac{3}{4} \sum_{n=1}^{\infty} \frac{(-1)^n \cos n\pi/4}{n! \alpha^{n/2}} \Gamma\left(\frac{3n}{4}, v_*^{1/2}\right) ,$$

where $\Gamma(a, x) = \int_x^{\infty} dt t^{a-1} \exp(-t)$ is an incomplete gamma function for which $\Gamma(a+1, x) = a\Gamma(a, x) + x^a \exp(-x)$. Equations (A1) and (A2) are obtained by employing the series representations of $\exp(-t)$ and $E_1[t^{3/4} \exp(-i\pi/4) / \alpha^{1/2}]$, respectively.

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Table 1. Empirical values of $\tilde{\sigma}$'s in the analytic fits for $\tilde{C}$'s

ν_*	$\tilde{\sigma}_1$	$\tilde{\sigma}_2$
0	130	1.3
10^{-2}	33	.71
10^{-1}	12	.22
1	0	2.7×10^{-2}
10	0	0

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FIGURE CAPTIONS

- Fig. 1. Plot of electron velocity space indicating the trapped (cross-hatched) and untrapped regions. The untrapped electrons resonating with the wave are indicated schematically by the shaded region.
- Fig. 2. Plots of L_1 and L_2 versus α for various values of v_* . The dash-dot curves are L_1 , while the solid waves are for L_2 .
- Fig. 3. Plots of the real parts of $\bar{C}_1$ versus $v_e/3\omega$ (upper abscissa) and $\tilde{C}_1$ versus $v_e/3\epsilon\omega$ (lower abscissa) for various v_* . The $\tilde{C}_1$ for $v_* \gtrsim 1$ are small enough to neglect. The dashed curve is the $\bar{C}_1$ approximation of Eq. (15).
- Fig. 4. Plots of the real parts of $\bar{C}_2$ versus $v_e/3\omega$ (upper abscissa) and $\tilde{C}_2$ versus $v_e/3\epsilon\omega$ (lower abscissa) for various v_* . The $\tilde{C}_2$ for $v_* \gtrsim 100$ are small enough to neglect. The dashed curve is the $\bar{C}_2$ approximation of Eq. (16).
- Fig. 5. Plots of the normalized growth rate versus $b = k^2 T_i / M \Omega^2$ for $L_n/L_s = 0.01$ and various v_* . The solid, dashed, dotted, and dash-dot curves correspond to v_* equal 10^{-3} , 10^{-2} , 10^{-1} , and 1. The parameters for Figs. 5-7 are $q = 2$, $\eta_e = 1 = \eta_i$, $\tau = 1$, $\epsilon = 0.2 = L_n/R$, and $M/m_e = 1840$.
- Fig. 6. Plot of the normalized growth rate versus b for $L_n/L_s = 0.03$ and v_* equal 10^{-3} , 10^{-2} , 10^{-1} , and 1.
- Fig. 7. Plot of the normalized growth rate versus b for $L_n/L_s = 0.06$ and v_* equal 10^{-3} , 10^{-2} , 10^{-1} , and 1.

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Fig. 1. Plot of electron velocity space indicating the trapped (cross-hatched) and untrapped regions. The untrapped electrons resonating with the wave are indicated schematically by the shaded region.

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Fig. 2. Plots of L_1 and L_2 versus α for various values of ν_* . The dash-dot curves are L_1 , while the solid waves are for L_2 .

Fig. 3. Plots of the real parts of $\bar{C}_1$ versus $\nu_e/3\omega$ (upper abscissa) and $\tilde{C}_1$ versus $\nu_e/3\epsilon\omega$ (lower abscissa) for various ν_* . The $\tilde{C}_1$ for $\nu_* \gtrsim 1$ are small enough to neglect. The dashed curve is the $\bar{C}_1$ approximation of Eq. 15.

Fig. 4. Plots of the real parts of $\bar{C}_2$ versus $\nu_e/3\omega$ (upper abscissa) and $\bar{C}_2$ versus $\nu_e/3\epsilon\omega$ (lower abscissa) for various ν_* . The $\bar{C}_2$ for $\nu_* \gtrsim 100$ are small enough to neglect. The dashed curve is the $\bar{C}_2$ approximation of Eq. 16.

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Fig. 5. Plots of the normalized growth rate versus $b = k^2 T_i / M \Omega^2$ for $L_n / L_s = 0.01$ and various v_* . The solid, dashed, dotted, and dash-dot curves correspond to v_* equal to 10^{-3} , 10^{-2} , 10^{-1} , and 1. The parameters for Figs. 5-7 are $q = 2$, $\eta_e = 1 = \eta_i$, $\tau = 1$, $\epsilon = 0.2 = L_n / R$, and $M / m_e = 1840$.

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Fig. 6. Plot of the normalized growth rate versus b for $L_n/L_s = 0.03$ and v_* equal to 10^{-3} , 10^{-2} , 10^{-1} , and 1.

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Fig. 7. Plot of the normalized growth rate versus b for $L_n/L_s = 0.06$ and v_* equal to 10^{-3} , 10^{-2} , 10^{-1} , and 1.

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74. CTR Reading Room, c/o Prof. R. Gross, Plasma Research Lab., Columbia Univ., New York, NY 10027
75. CTR Reading Room, c/o Prof. Roy Gould, California Inst. of Tech., M.S. 116-81, Pasadena, CA 91125
76. Dr. Nicholas A. Krall, Science Applications, Inc., P.O. Box 2354, 1200 Prospect St., La Jolla, CA 92037
77. CTR Reading Room, c/o Dr. Jay P. Boris, Plasma Physics, Naval Research Laboratory, Washington, DC 20390
78. Professor A. Simon, Dept. of Mechanical & Aerospace Sciences, University of Rochester, Rochester, NY 14627
79. CTR Library, c/o Dr. Alan F. Haught, United Technologies Research Labs, East Hartford, CT 06108
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