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PLASMA PRODUCTION AND FLOW IN ION BEAMS

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PLASMA PRODUCTION AND FLOW IN ION BEAMS

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ABSTRACT

Space charge neutralization of ion beams results from plasma generated by the beam. The two dimensional flow of plasma to walls surrounding a beam is analyzed to provide spatial distributions. Negative ion beams receive particular attention; the results are applied to electron control in beams subject to post-acceleration.

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PLASMA PRODUCTION AND FLOW IN ION BEAMS

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Introduction

The propagation of a non-magnetized intense charged beam requires neutralization by background plasma to prevent space charge blow-up. In the presence of plasma, electric fields are of the order of the electron temperature (in volts) divided by scale lengths, and induced changes in beam characteristics occur over propagation distances long compared with these scale lengths.

The intense neutral hydrogen and deuterium beams used in present magnetic fusion experiments originate as ion beams. These beams are neutralized by charge-exchange in gas. Collisions between the beam particles and the gas generate the background plasma. At the higher energies desired for future experiments and reactors, the charge-exchange efficiency decreases and it will be necessary to start with negative ion beams. Techniques for generating and handling these beams often require the beam to propagate some distance at low energy (1-10 keV) in order to separate the generation region from the high voltage accelerator (≥ 100 keV). This propagation permits control of gas and of electrons which could otherwise be accelerated.

In the following we analyze the flow of plasma generated by a negative ion beam. The gas ionization rate will be permitted to vary along and across the beam, so that the flow is two dimensional. Of particular interest is the flow of plasma along the beam from regions with high ionization rate.

The Model

The beam is taken to propagate in the z-direction and to have a fixed transverse profile, either independent of y (slab model) or of θ (cylindrical model). As the electric fields are generated by ambipolar forces in the plasma, the beam profile will be caused to vary over lengths of order $L \sim R_b v_b^2 / c_b^2$, where R_b is a transverse beam scale length, v_b the beam velocity, and $c_b = \sqrt{kT_e / M_b}$; thus $L \sim R_b E_b / T_e$. If, typically, $E_b = 10^3$ ev and $T_e = 1$ eV, $L \sim 1000 R_b$. It is interesting to note that the electron potential will generally confine electrons and thus focus the beam. Beam dynamics will not, however, be considered in the present analysis.

The background plasma is generated by collisions between the beam and background gas. Electron temperature will be assumed uniform and too low to cause ionization, c.f. Appendix 1 for discussion. The ionization rate is thus $S_i = \sigma v_b n_b n_g$, with σ the cross section, n_b and n_g the beam and gas densities. Of particular interest are negative deuterium beams at 1 keV, with $v_b = 3.56 \times 10^7$ cm/sec. If the background gas is deuterium², $\sigma \approx 2.5 \times 10^{-17}$ cm². In cesium, which is used as a charge-exchange medium for D^- production³, $\sigma \approx 2 \times 10^{-15}$ cm².

At low ionization rates the space charge of the beam becomes important. To estimate the "critical" gas density at which the plasma density becomes comparable to beam density, consider the continuity equation for a spatially uniform beam

$$\frac{1}{x^\beta} \frac{d}{dx} x^\beta n v = \sigma v_b n_b n_g.$$

with $\beta = 0$ for Cartesian coordinates and 1 for radial coordinates.

Integrating to the beam edge at $x = R_b$, setting $v = c = \sqrt{kT_e / M_i}$ (the speed of sound) and $n = n_b$, one identifies the critical density

$$n_c = \left(\frac{1+\beta}{R_b \sigma} \right) \left(\frac{c}{v_b} \right) \quad (1)$$

For gas densities $n_g \gg n_c$ the beam space charge will be negligible, but for $n_g \lesssim n_c$, $n \sim n_b$ and the electron density $\neq n$. It has been claimed that for $n_g < n_c$ negative ion beams will not propagate⁴, but both experimentally⁵ and theoretically (below) we find that beam propagation is possible. At low ionization rates ions must have a long residence time in the beam to provide space charge neutralizations. As will be seen quantitatively, this occurs when the electron density profile flattens inside the beam profile; the corresponding electric field, $E = -\nabla(T_e \ln n_e)$ is small as are the resulting ion velocities.

At walls around the beam a sheath will occur, as discussed in Appendix 2. For essentially all cases of interest the sheath will reflect electrons. As is well known, the "Bohm" sheath condition requires that the directed ion fluid velocity be $\geq c$, the speed of sound. In our calculations, which use a fluid model, this will be seen to be a naturally occurring result.

The electron energy balance is very sensitive to the coupling to the walls, to collisional effects, and to plasma stability. We will instead assume the electron temperature to be given and uniform in space. Furthermore, we will neglect ionization by the electrons, in agreement with experiment.⁵

The ion temperature will be taken to be zero. The consequences of this assumption will be considered later in Appendix 3.

Beam plasma instabilities have been considered elsewhere⁶ and shown to be unimportant when $v_b < \sqrt{kT_e/m_e}$. This condition is satisfied for the beams of interest here. Coupling to sound waves propagating across the beam and down the density gradient are of potential importance, but will not be considered here⁷.

Kino and Shaw have shown that the fluid model of a plasma discharge gives a good approximation to the kinetic theory results of Self⁹ and of Tonks and Langmuir¹⁰. It is consequently used here.

The time dependent equations for the plasma ions at zero temperature are thus taken as

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = \nu n_b \quad (2)$$

$$\frac{\partial n\mathbf{v}}{\partial t} + \nabla \cdot (n\mathbf{v}\mathbf{v}) = -c^2 n \nabla \ln n_e \quad (3)$$

$$n_e = n - n_b \quad (4)$$

$$\mathbf{v} = \sigma n_g \mathbf{v}_b \quad (5)$$

$$c^2 = kT_e/M_i. \quad (6)$$

These equations are solved below for the equilibrium in one and two dimensional flow.

Within the model described by Eqs. (2)-(6) the values of electron temperature and ion mass can be scaled away. Thus, define a dimensionless velocity as $\hat{\mathbf{v}} = \mathbf{v}/c$, distance as $\hat{r}/R_b$, and times as $\hat{t} = tc/R_b$. The dimensionless ionization rate is then $\hat{\nu} = \nu R_b/c$. Setting this parameter to $(1+\beta)$ leads to the critical density of Eq. (1).

One Dimensional Flow

In equilibrium one dimensional flow, Eqs. (2) and (3) become

$$\frac{1}{x^\beta} \frac{d}{dx} (x^\beta n v) = \nu n_b \quad (2a)$$

$$\frac{1}{x^\beta} \frac{d}{dx} (x^\beta n v^2) = -c^2 n \frac{d \ln n_e}{dx} \quad (3a)$$

The two cases $\beta = 0$ and 1 are considered separately.

(1) Cartesian coordinates:

Equation (2a) integrates to yield (with $v=0$ at $x=0$)

$$nv = \int_0^x v n_b dx \equiv F(x) \quad (7)$$

where F is the ion flux. This combines with Eqs. (2a) and (3a) to yield an equation for the density:

$$\left[\frac{n}{n - n_b} c^2 - \frac{F^2}{n^2} \right] \frac{dn}{dx} = \frac{n}{n - n_b} c^2 \frac{dn_b}{dx} - \frac{2v n_b F}{n} \quad (8)$$

For most cases $dn_b/dx \leq 0$, and for space-charge neutrality $n > n_b$ so that the rhs is ≤ 0 .

The boundary may be located either in a region with $n_b \neq 0$ or at large enough x that $n_b = 0$. Consider first the case $n_b \neq 0$ at the plasma boundary. The value of $dn/dx \rightarrow \infty$ when the plasma velocity $v = F/n = c\sqrt{n/(n-n_b)}$. This is the sheath edge; the divergence is typical of that encountered for quasineutral solutions [8]. The local speed of sound is $c\sqrt{n/(n-n_b)}$. This solution is thus similar to that obtained by Kino and Shaw; we get numbers by setting $v = c\sqrt{n/(n-n_b)}$ in Eq. (7) with $x = x$ (boundary). The result is solved for n at the boundary, and Eq. (8) is integrated backward to yield n (and thus v) for all x .

If the beam density goes to zero at $x = X_b < x$ (boundary), the analysis is slightly different. The density and velocity will be constant for $X_b < x < x(\text{boundary})$, so the sheath condition requires $v \geq c$ at $x = X_b$. On the other hand, for $x < X_b$ the density derivative will become singular

whenever $v = c\sqrt{n/(n - n_b)}$; this would be physical only at a sheath boundary, so $v < c\sqrt{n/(n - n_b)}$ for $x < X_b$. The only way to satisfy these two requirements is for $v = c$ at $x = X_b$. This speed is maintained to the boundary, so $v = c$ at $x = x(\text{boundary})$ defines the solution.

Equations (7) and (8) have been numerically integrated using a Gaussian beam profile, $n_b = 2 \times 10^9 \exp(-x^2)$. Results are shown in Figs. (1)-(3). The neutral density is scaled in terms of the critical density for slab geometry $n_{cx} = (1/X_b \sigma)(c/v_b)$; for our profile $X_b = 1$. At $n_g = 10 n_{cx}$ (Fig. (1)) the beam space charge is small. In the limit of negligible beam space charge, Eq. (3) integrates to $n = n_0 c^2 / (v^2 + c^2)$, and the density at the boundary is 0.5 that at the center. In the present case the boundary density is reduced to 0.37 the central density.

At the beam gas pressures illustrated in Figs. (2) and (3) beam space charge is important. Because of the reduced production rate the outward ion velocity is low within the beam, thus yielding an increased ion residence time. The electron density profile is flat, corresponding to a small electric field.

(2) Cylindrical coordinates

The equivalents to Eqs. (7) and (8) are

$$nv = \frac{1}{r} \int_0^r v n_b r dr \equiv G(r) \quad (9)$$

and

$$\left[\frac{n}{n - n_b} c^2 - \frac{G^2}{n^2} \right] \frac{dn}{dr} = \frac{1}{n} \left[\left(\frac{G^2}{r} - 2v n_b G \right) + \frac{n^2}{n - n_b} c^2 \frac{dn_b}{dr} \right]. \quad (10)$$

The term G^2/nr arises from the radial increase in the surface area of a cylinder. We see that the analysis is complicated by the fact that the right-hand-side of Eq. (10) can change sign inside the beam.

We require that the ion density, n , decay monotonically with r . (If it did not, our approximation would breakdown, and plasma would fill any minimum so that $dn/dr \leq 0$ everywhere.) A sufficient (but not necessary) condition for the r.h.s. of Eq. (10) to be negative everywhere, so that the coefficient of dn/dr is > 0 , is for $2vn_b > G/r$ for all r or

$$2vn_b r^2 > \int_0^r vn_b r dr . \quad (11)$$

Kino and Shaw had, in effect, $vn_b = \text{constant}$, so Eq. (11) was satisfied and the problem did not arise. Similarly, in our problem if the boundary radius is small enough r that Eq. (11) is satisfied everywhere, then the boundary is determined as in the cartesian case by the velocity equalling the speed of sound and dn/dr diverging when

$$\frac{n}{n - n_b} c^2 = \frac{G^2}{n^2} . \quad (12)$$

If Eq. (11) is not satisfied everywhere inside the boundary, any zeros of the r.h.s. of Eq. (10) must occur simultaneously with those of the coefficient of dn/dr . Thus consider Eq. (12) and

$$\frac{G^2}{r} - 2vn_b G + \frac{n^2}{n - n_b} c^2 \frac{dn_b}{dr} = 0. \quad (13)$$

Equations (12) and (13) are simultaneously satisfied when

$$n = N_0 = -r \frac{dn_b}{dr} G / [G - 2vn_b r] \quad (14)$$

The radius, r_0 , at which this occurs is found by substituting N_0 into Eq. (12):

$$\frac{N_0^3(r_0)}{N_0(r_0) - n_b(r_0)} = \frac{1}{c^2} G^2(r_0) . \quad (15)$$

The density $N_0(r)$ is sketched in Fig. 4a together with n . The solution goes supersonic at r_0 as indicated in Fig. 4b.

We emphasize: If r_0 from Eqs. (14) and (15) is $< r(\text{boundary})$, the flow goes supersonic at r_0 and enters the sheath with a velocity $>$ speed of sound. This still satisfies the Bohm sheath condition, of course. It is clear that in the cylindrical case the boundary condition cannot be $v = c$. It must, instead, be $v \geq c$.

Equation (10) for dn/dr is indeterminant at the supersonic transition. Expansion about r_0 yields the derivative

$$\begin{aligned} \frac{dn}{dr} &= - \left[B + \sqrt{B^2 + 4AC} \right] / 2A \\ A &= \frac{2G^2}{n^3} - \frac{c^2 n_b}{(n - n_b)^2} \\ B &= \frac{3G^2}{n^2 r} - \frac{4G}{n^2} v n_b + c^2 \frac{n_b}{(n - n_b)^2} \frac{dn_b}{dr} \\ C &= - \frac{3G^2}{nr^3} + \frac{4G}{nr} v n_b - 2 \frac{dv}{dr} \frac{n_b G}{n} - \frac{2v}{n} \frac{dn_b}{dr} G \\ &\quad - \frac{2v^2 n_b^2}{n} + \frac{n}{n - n_b} c^2 \frac{d^2 n_b}{dr^2} + \frac{n}{(n - n_b)^2} c^2 \left(\frac{dn_b}{dn} \right)^2 \end{aligned} \quad (16)$$

Solutions in the cylindrical geometry are found by integrating in both directions away from the transition point. Examples are shown in Figs. (5)-(7), with $n_{cr} = (2/R_b \sigma) (c/v_b)$ as a parameter. We again pick $n_b = 2 \times 10^9 \exp(-r^2)$ for illustration, so $R_b = 1$. As in the slab case the electron density gradient and thus electric field, are reduced for $n \lesssim n_{cr}$.

Two Dimensional Flow

We next consider problems in which the ionization rate is a function of z and r . These models represent, for example, a beam passing through a charge exchange cell in which the vapor is localized in space. The numerical solution to Eqs. (2) and (3) was obtained using a time dependent computer code "Plasflow" described in Appendix 4. The equilibrium was found by starting from an initial condition and calculating until steady conditions were reached. The code was based on the algorithm "Slide" which assumes cylindrical geometry, and slab geometry was not examined.

Before presenting the numerical results it is instructive to write Eqs. (2) and (3) in terms of s and τ the distance along and across velocity streamlines. Thus, define s and τ by

$$ds = (v_r dr + v_z dz)/v = \cos\theta dr + \sin\theta dz \quad (17)$$

$$d\tau = (-v_z dr + v_r dz)/v = -\sin\theta dr + \cos\theta dz \quad (18)$$

The equilibrium equations are

$$\frac{\partial}{\partial s} (nv) = vn_b - nv \left(\frac{\cos\theta}{r} + \frac{\partial\theta}{\partial\tau} \right) \quad (19)$$

$$\frac{\partial}{\partial s} \left(\frac{1}{2} v^2 + c^2 \phi \right) = - \frac{vn_b v}{n} \quad (20)$$

$$v^2 \frac{\partial\theta}{\partial s} = -c^2 \frac{\partial\phi}{\partial\tau} \quad (21)$$

where $\phi = \ln n_e$. These equations have simple interpretations. In Eq. (19) the $\cos\theta$ term results from the cylindrical expansion of the flow, whereas the $\partial\theta/\partial\tau$ term is >0 for expansion in the r - z plane. Eq. (20) has the same form as the one dimensional case: new particles are born at rest and hence reduce the average ion energy. Equation (21) is new: the r - z expansion

results from the electric field transverse to the streamlines.

Equations (19) and (20) are similar to the one dimensional form.

In particular,

$$\left(v^2 - \frac{nc^2}{n-n_b}\right) \frac{\partial n}{\partial s} = -nv^2 \left(\frac{\cos\theta}{r} + \frac{\partial\theta}{\partial\tau}\right) - \frac{nc^2}{n-n_b} \frac{\partial n_b}{\partial s} + 2vn_b v. \quad (22b)$$

As the ions flow away from the beam region, one expects $\partial n_b / \partial s < 0$ for most beam configurations. Consequently, the density decreases along the flow if:

(a) The beam space charge and ionization terms are larger than the expansion terms and the flow is subsonic, or

(b) The expansion terms are larger than the beam space charge and ionization terms and the flow is supersonic.

Furthermore, as in the one dimensional cases the right side of Eq. (22b) must be zero at any supersonic transition, and as in the one dimensional cylindrical case, the supersonic transition is within the beam.

For simplicity we consider a model which is symmetric about the plane $z=0$, so that $v_z(z=0) = 0$, $[\partial n / \partial z]_{z=0} = 0$ and $[\partial v_r / \partial z]_{z=0} = 0$. To represent a wide class of problems let vn_b be peaked at $r=0$, $z=0$, and fall-off monotonically toward the boundaries. $v_r = v \cos\theta \geq c$ at the radial boundary. We take $v_z = v \sin\theta = 0$ at the z wall, corresponding to the entrance to a negative ion accelerator. The flow pattern is sketched in Fig. 8.

Near the cylindrical axis, $r=0$, the equations may be expanded in r . There, $\partial/\partial s = \partial/\partial z$, $\partial/\partial\tau = -\partial/\partial r$, $n = n_0(z) + n_2(z)r^2 + \dots$, $n_b = n_{b0}(z) + n_{b2}(z)r^2 + \dots$, $v = v_{z0}(z) + v_{z2}(z)r^2 + \dots$, $v_r = v_{r1}(z)r + \dots$, and $\theta = \pi/2 - v_{r1}r/v_{z0}$. Equations (20)-(22) thus become

$$\frac{d}{dz}(n_0 v_{z0}) = vn_{b0} - 2n_0 v_{r1} \quad (23)$$

$$\frac{d}{dz} \left[\frac{1}{2} v_{z0}^2 + c^2 \ln(n_0 - n_{b0}) \right] = - \frac{v_{b0} v_{z0}}{n_0} \quad (24)$$

$$v_{z0}^2 \frac{d}{dz} \left(\frac{v_{r1}}{v_{z0}} \right) = 2c^2 \frac{n_2 - n_{b2}}{n_0 - n_{b0}} \quad (25)$$

The density variation along the axis is found from Eqs. (23) and (24):

$$\left(\frac{n_0 c^2}{n_0 - n_{b0}} \right) - v_{z0}^2 \frac{dn_0}{dz} = -2 \left[v_{b0} - v_{r1} n_0 \right] v_{z0}. \quad (26)$$

The reduction of average ion energy due to ionization plays a vital role in reducing the plasma density. In our problem $v_z = 0$ at $z=0$ and $z = z_{\max}$. If the ionization term were absent from Eq. (24), the density would be identical at the two ends and there would be no net reduction along the axis. Furthermore, from Eq. (23) we see that the average along the axis of $v_{b0} - 2n_0 v_{r1}$ will be zero; the extra factor of v_{b0} in Eq. (26) will reduce the density for subsonic flow.

Typical numerical results are shown in Figs. (9a) - (9d). The gas density is described by

$$n_g = n_{cr} [1 + 3 \exp(-z^2)].$$

The down stream solution differs from the one-dimensional result by $\leq 10\%$ due to numerical effects in the code. The region of high plasma density is well localized, extending only slightly farther along the axis than the ionization rate.

Discussion

Perhaps the most important conclusion to be drawn from these results is that regions of high plasma density are limited to regions of high ionization rates. As a result, plasma effects at the entrance to negative ion accelerators can be controlled by using a drift region between charge-exchange cells and the accelerator. Effects due to stripping cells can similarly be controlled. The required distances depend upon detailed beam and ionization profiles, but a rule of thumb would indicate several beam diameters.

As emphasised above, the flow is limited by the generation of new ions at zero energy. In the fluid model used here, the ion-ion collision frequency is assumed so high that the ion kinetic energy is instantly averaged over the entire population. This is, in fact, a poor approximation. An appropriate parameter is $\lambda_{ii}/R_b \approx 2 \times 10^{12} T_i^2 / n R_b$. At room temperature $T_i = 0.04 \text{ eV}$, and for $R_b = 1$, this is unity for $n = 2 \times 10^9 \text{ cm}^{-3}$. At 1 eV, however, $n = 1.3 \times 10^{12} \text{ cm}^{-3}$ is required. The ion distribution will thus be between the assumed mono-energetic one and more a free-flow distribution, with the result that the density will extend further away from regions of ionization than calculated here. Calculations with one dimensional plasmas suggest that this is a 25% - 50% effect.

Appendix 1 - Electron Temperature

The full ionization includes both electron and beam terms. In the presence of cesium, ionization by electrons is unimportant at electron temperatures below about 0.5 eV. Experiments indicate values of about this magnitude. For cases in which cesium vapor is low, the electron temp-

erature is found to be higher, but still low enough that ionization (e.g. of D_2) is unimportant.

If the beam velocity is greater than the average electron speed, beam plasma instabilities can occur. One nonlinear limiting mechanism for such instabilities is electron heating. Thus, problems might occur if ionization could be caused by an electron distribution with an average speed equal to the beam velocity.

The actual electron temperature and distribution require a detailed analysis of energy gain and loss. Thus, atomic excitation and ionization, thermal conductivity to walls, beam and plasma stability, and collisions must all be carefully considered. The problem is notoriously difficult and thus is not considered here.

Appendix 2 - Sheaths

The plasma model breaks down, of course, within a few Debye lengths of the wall. We have replaced a detailed analysis of the sheath and of the plasma-sheath transition by the well known Bohm sheath condition, $v \geq c$.

The present calculations assume that the sheath potential everywhere confines electrons. If this is true, the plasma potential at, say, the highest density point, can be determined relative to the walls by requiring that the ion and electron loss rates are equal either locally for non-conducting sections of walls or globally for conducting sections. The resulting sheath drops will generally vary along the walls, and the model will breakdown if this drop becomes less than a few T_e . In most cases, however, if the potential drop approaches zero, large local electron currents will flow which must be balanced by ion currents elsewhere. These currents

can usually not be sustained, however, except in small areas such as biased Langmuir probes.

The supersonic flow will also help to insulate the beam region from the walls. Because the transition point is essentially determined by volume ionization processes, the details at the wall will have little effect in the region of interest.

Appendix 3 - Ion Temperature Effects

In the present model the ion temperature has been set to zero, so that ion pressure effects are not included. As the ion temperature is usually much smaller than the electron temperature, this is not a serious problem when $n_e \approx n_i$. At low ionization rates, however, when the flattened electron density profile leads to a reduced electric field, the approximation breaks down. The electric field and pressure effects will be comparable when $T_i |\nabla n/n| \sim T_e |\nabla n_e/n_e|$. At $T_i = .04\text{eV}$ and $T_e = 1\text{eV}$, this will occur when the electron gradient length becomes an order of magnitude greater than the ion (and beam) gradient lengths. In this case the electron density profile will reverse to reduce ion loss and maintain a balance with ionization. The plasma flow characteristics will be much the same.

One result will be that the plasma ambipolar potential will defocus the negative ion beam rather than confine it. The defocussing fields will be small, however, of order of the ion temperature. As the temperature of the beam will probably be larger, this is not a major effect.

Appendix 4 - The Plasflow computer Code

The basic code used was developed by D.V. Anderson¹¹. The present version is written in two dimensional cylindrical coordinates, with the azimuthal angle assumed ignorable. Although the original version was written for

magnetized plasmas, the magnetic field has been set to zero for the present calculations. Problems involving axisymmetric fields could be analyzed. Details of the code will be presented elsewhere.

To keep the present model simple, the ion temperature has been set to zero. The code is capable, however, of handling temperatures and temperature gradients. Off-diagonal stress tensor terms are not included.

Wall boundary conditions have been generalized to include the Bohm sheath condition where appropriate.

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FIGURES

Figure 1. Plasma densities and ion velocity for slab geometry;

$$n_b = 2 \times 10^9 e^{-x^2}, n_g = 10 n_{cx}.$$

Figure 2. Plasma densities and ion velocity for slab geometry;

$$n_b = 2 \times 10^9 e^{-x^2}, n_g = n_{cx}.$$

Figure 3. Plasma densities and ion velocity for slab geometry;

$$n_b = 2 \times 10^9 e^{-x^2}, n_g = 0.1 n_{cx}.$$

Figure 4. Locating the supersonic transition in cylindrical geometry.

(a) Density. (b) velocity.

Figure 5. Plasma densities and ion velocity for cylindrical geometry;

$$n_b = 2 \times 10^9 e^{-r^2}, n_g = 10 n_{cr}.$$

Figure 6. Plasma densities and ion velocity for cylindrical geometry;

$$n_b = 2 \times 10^9 e^{-r^2}, n_g = n_{cr}.$$

Figure 7. Plasma densities and ion velocity for cylindrical geometry;

$$n_b = 2 \times 10^9 e^{-r^2}, n_g = 0.1 n_{cr}.$$

Figure 8. Plasma streamlines and coordinates in two dimensional flow.

Figure 9. Plasma densities and ion velocity in two dimensional ion flow;

$$n_b = 2 \times 10^9 e^{-r^2}, n_g = n_{cr} (1 + 3e^{-z^2}). \quad (a) \text{ Ion density, (b) Electron density, (c) Ion radial velocity, (d) Ion axial velocity.}$$

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9a.

Figure 9b.

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Figure 9c.

6/14/77 N3=3.89E+13 CXDENS=3. 2 DIMENSIONAL

Figure 9d.

NOTICE

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