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Plasma Physics Laboratory

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Studies of Tokamak Fusion Reactor Dynamics
Covering the Period
June 1, 1975 to January 31, 1977

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I. EXECUTIVE SUMMARY

A. Scope of Activities

We report here research conducted under Special Research Source Agreement #EY-76-S-02-2735, Studies of Tokamak Reactor Dynamics, during the period June 10, 1975 to Jan. 31, 1977. The purpose of this research has been to investigate the time dependent phenomena associated with the inherently pulsed nature of the tokamak fusion reactor. Although the significant difficulties associated with the cyclic mode of operation were pointed out in early conceptual design studies,⁽¹⁾ detailed understanding of dynamic plasma behavior was not available at that time and the purpose of this research was to develop an understanding of this aspect of tokamak reactor plasma physics.

Several sub-tasks were delineated and carried forward: A multi-node point kinetics computer code has been developed in which the plasma is viewed as a set of conductors inductively coupled to one another and to a system of poloidal field coils. This code incorporates aspects of both the plasma equilibrium problem (force balance) as well as the plasma diffusion problem (particle and energy balance). Efficiency of operation is obtained by using simple model equations. (e.g., Rather than solve the MHD equilibrium equation, an approximate force balance equation is used.) A more detailed description of this code will be found in Section III of this report and in the attached appendix.

The importance of the MHD equilibrium configuration of proposed reactor designs was recognized (different configurations^(1,2) proved to differ by an order of magnitude in stored poloidal field energy that must be transferred during each cycle of operation); consequently studies of the equilibrium properties of various reactor designs were undertaken. (This work has been reported in the literature⁽³⁾ and in a previous progress report under this contract and will not be discussed further here.⁽⁴⁾)

Several types of operating cycles may be envisioned for tokamaks. Some involve energy transfers from auxiliary power systems on time scales that are short compared to the resistive time scales of the plasma and consequently will induce adiabatic transformations (compressions, expansions and more general heating or cooling of the plasma. A preliminary investigation of such adiabatic transformations has been carried out, but conclusions are incomplete at this time.

Finally we have initiated a study of plasma heating by alpha particle deposition. Initial work was done by a graduate student in the Department of Chemical Engineering at Princeton University and is being continued under different sponsorship.

The next section of this report is a more detailed introduction to the subject of Tokamak Reactor Dynamics, and subsequent sections provide detailed information on the activities carried out under this contract as indicated above.

II. TOKAMAK REACTOR DYNAMICS

Tokamaks, as opposed to stellarators, are inherently non-steady-state devices. At best, with the provision of divertors to provide an adequate exhaust system as well as impurity control and a scheme successful in providing fuel at a rate that is sufficient to balance the particle loss and consumption rates, the burn phase of the tokamak operating cycle is limited by the ability to provide sufficient transformer capacity to compensate the resistive IR drop of the plasma. Previous conceptual cycle designs⁽¹⁾ based on the premise that it was worthwhile to extend this phase of the cycle for as long as possible have indicated that pulses of ~100 minutes might be possible. These estimates are, of course, design dependent as the available volt-seconds or transformer drive will depend on the available core size and the maximum design field strength. The general form of the tokamak operating cycle has, however, been established. It consists of an ignition phase, an extended, or in some designs abbreviated, burn phase and a shutdown or quench phase. The investigation of those phenomena that are peculiar to these cycles of operation comprises the subject matter of studies of tokamak reactor dynamics.

It is important, before proceeding further, to note that during the ignition and quench phases of the cycle, large fractions of the magnetic energy associated with the plasma current and transverse magnetic field required for equilibrium

will be transferred in and out of the system. The large plasma currents (tens of MA) and large sizes (≥ 10 m) imply that the stored energies involved may well exceed 10 GJ). Consequently, if reasonable assumptions are made for the power supplies required to execute this energy transfer, it is apparent that time periods measured in tens of seconds will be necessary. As these time periods are commensurate with the time required for pumping down the vacuum system (i.e., exhausting helium and resupplying deuterium and tritium after each cycle), one can reasonably establish that the interruption time between pulses will be of the order of one minute. These times are, of course, longer by orders of magnitude than the pulse lengths of operating tokamak experiments.

In the course of each cycle plasma conditions vary greatly from a cold post-ionization state to the high temperature fully ignited state. In many startup scenarios low density is prescribed to reduce radiation losses during some or all of the startup cycle, and most recently proposals have been made to use strong heating on adiabatic time scales.⁽⁵⁾ We have developed a multi-node point kinetic code designed to be used in analyzing these problems. (It is described in detail in Section III of this report and in Appendix A.) This code treats each element of the plasma, which can be discretized in both the radial or flux normal direction as well as in the poloidal direction, as a circuit element that is inductively coupled to its neighboring

elements as well as to a system of external conductors. The effect of the distribution of electric fields over the plasma volume is treated in this manner. This strategy has been adopted because the buildup by the plasma current and the proper programming of the various coil systems are important. In addition provision is made for including the effects of particle diffusion and energy transport by conduction, convection, and radiation. These phenomena are modeled on a coarse grid that is consistent with the level of detail desired.

One of the important phenomena of interest in startup calculations is the level of line radiation produced by partially ionized impurity ions. As was noted in reference (1), the resulting lamp effect is accentuated in reactor scale devices that are usually operated at low current density in comparison to today's experiments. Consequently the reduced ohmic heating power, ηj^2 , available can more easily be dominated by line radiation.⁽⁶⁾ Overcoming such transient effects will require specially designed ignition programs that optimize heating of the plasma by both ohmic and auxiliary heating processes. To date heavy emphasis has been placed on the use of neutral beams to provide the necessary supplemental heating. Provision has also been made for including these effects in the code that we have developed.

A proven technique for increasing the internal energy of a plasma is adiabatic compression. More generally we may speak of an adiabatic transformation of the plasma as one

which maps one equilibrium state into another under the constraints that: i) the poloidal and toroidal flux within any flux surface is conserved and ii) the plasma obeys an adiabatic equation of state. Such a transformation provides a means of transferring energy from magnetic fields to the internal energy of the plasma itself; it must be carried out in a time span that is short compared to that on which plasma diffusion occurs. A variant of this form of plasma heating is accomplished by rapid injection of neutral beam energy which raises the temperature of the plasma on this time scale. In this case a stationary adiabatic-transformation of the plasma occurs in that the equilibrium magnetic fields must be adjusted to preserve the location of the plasma equilibrium. We have performed a preliminary analysis aimed at improving our understanding of adiabatic-transformations as part of this research effort.

III. A MULTI-NODE POINT KINETICS PLASMA MODEL

A computer code has been developed for the analysis of time-dependent Reactor Dynamics phenomena. We shall summarize the modeling that has been chosen and incorporated in this code. The basic premise is that it is possible to analyze the plasma as a set of N inductively coupled circuits. This discretization of the discharge is accomplished by dividing the cross section into n flux shells (concentric cylinders in work done to date) and then further subdividing each of these flux shells into m segments such that $N=mn$. The system is

driven by M poloidal field coils and the inductance matrix for these N + M circuits is calculated. Solution of the circuit equations for specified voltages applied to the driving coils provides the first approximation to the behavior of the plasma during the operating cycle. The plasma, however, is not a nested set of rigid wires. Its equilibrium position and configuration are maintained by the action of the magnetic fields produced by the M poloidal field coils and will change as the currents in these coils vary; consequently the variation in the inductance matrix elements produced by this changing geometry must also be calculated. The geometry changes or "strains" must be compatible with a set of compatibility conditions that state the two elements nearby to each other in the initial configuration are proportionately nearby to each other in the new configuration. In the simplest case considered the major radius of the discharge is found by using⁽⁷⁾

$$B_v = \frac{I}{R} \left[\ln \frac{8R}{a} - 1.25 + \beta_\theta \right] ,$$

and the compatibility conditions are provided by requiring the conservation of flux within the plasma.

More complicated cases can be analyzed. Non-circular geometries can be discretized as easily as circular ones and longer time scale phenomena (resistive effects, etc.) can be included. In the later case, however, new compatibility

conditions not based on flux conservation must be developed. Provision has been made in the code for including the effects of ohmic heating, alpha particle and neutral beam heating, radiation losses and species equilibration.

The results obtained from the analysis of several test problems considered in initiating the implementation of this code and a detailed description of the physical and numerical models used may be found in the report included as appendix A.

References

¹R. G. Mills, ed., A Fusion Power Plant, Princeton Plasma Physics Laboratory Report MATT-1050 (1974).

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³A. M. M. Todd, S. L. Gralnick and H. E. Dalhed, MHD Properties of Tokamak Fusion Reactor Designs, Nuclear Fusion 16, 687 (1976).

⁴R. G. Mills and S. L. Gralnick, Progress Report on Studies of Tokamak Reactor Dynamics, COO-2735-1 (1976).

⁵J. F. Clarke and D. G. McAlees, High Beta Predemonstration Fusion Devices, Oak Ridge National Laboratory Report TM-5692 (1976).

⁶R. J. Hawryluk and J. A. Schmidt, Effects of low-Z Impurities during the Startup Phases of a Large Tokamak, Nuclear Fusion 16, 775 (1976).

⁷J. M. Greene, J. L. Johnson and K. Weimar, Tokamak Equilibrium, Physical Fluids 14, 671 (1971).

APPENDIX A

TOROIDAL PLASMA TIME EVOLUTION

PART I: MULTIPLE-NODE MODEL

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Progress Report

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<u>TABLE OF CONTENTS</u>	ii
Summary	iii
Introduction	iv
List of Symbols	v
1. Formulation of a Single-Node-Toroidal-Plasma Heating	1
2. Formulation of a Multi-Node-Toroidal-Plasma Heating	9
3. Computational Flow Chart	13
4. Comments on Model	18
5. Results	22
6. Conclusions	26
Appendix A	28
Appendix B	29
Acknowledgements	32
List of Tables	33
List of Terms in Kirchhoff Equations.....	34
List of Terms in Transport Equations	35
Tables	36
List of Figures	53
Figures	54

SUMMARY

Toroidal plasmas time evolution is being analyzed based on a multiple circuit plasma model. The inductive coupling between plasma and external coils and between various plasma subsections is taken into account. The plasma is considered in quasistatic equilibrium in the magnetic field generated by external coils which control the compression. Numerical results are provided for the Reference Design described in MATT-1050 (Princeton University, Plasma Physics Laboratory).

INTRODUCTION

The present analysis represents an extension of the study done by Dr. S. L. Gralnick on the toroidal plasma time evolution published in MATT-1050 Report, Princeton University. It attempts to add information on the plasma-coils inductive coupling and on the quasistatic compression of a finite cross section plasma.

The adequacy of multiple-circuit plasma modeling of tokamaks is being contemplated due to the high flexibility and computational efficiency of such modeling.

Further parts of the present Report, cover:

Part II: Tokamak plasma heating based on multiple-circuit plasma model.

Part III: Correction to previous results when realistic magnetic surfaces are being considered.

Part IV: Comparative analysis of multiple mode model and space variable model results.

LIST OF SYMBOLS

I_p	plasma current, single plasma
I_c	control coil current
I_ξ	vertical and divertor coil currents
R_e	electric resistance
R_p	major radius
R	radius in cylindrical coordinates referred to the torus axis
a	minor radius
$M_{\alpha\beta}$	mutual inductance coefficient
L	self-inductance coefficient
n	set of number densities
T	set of temperatures
nT	sum of the number density x temperature of all species
r, θ	polar coordinates in the poloidal plane
B_{VDZ}	Z component of the vertical and divertor magnetic field
S_i	ion source intensity
$L_{c,s}$	conductive losses of species
W_i	energy per injection ion
$U_{\alpha i}, U_{\alpha e}$	portion of the fusion energy transferred on an average from the α particles to the ions/electrons
W_{line}	line radiation power loss density (absolute value)
W_{compr}	compressional power gain (absolute value)
W_{Brem}	Bremsstrahlung radiation power loss density
W_{cycl}	cyclotron radiation power loss density

I_{pj}	plasma current of j^{th} plasma shell
a_j	outer minor radius of the j^{th} shell
L_{jj}	self induction coefficient of the j^{th} plasma shell
$L_{\xi\xi}$	self induction coefficient of the ξ coil (vertical or divertor)
I_{pj_k}	plasma current of the k^{th} cell in the j^{th} shell
$I_{pj'_s}$	plasma current of the s^{th} cell in the j'^{th} shell

1. FORMULATION OF SINGLE-NODE-TOROIDAL-PLASMA HEATING

During the initial heating step of a toroidal plasma, electric current is generated as a result of the plasma inductive coupling with the control, vertical and divertor coils. The electric current is determined at any moment by the time rate of change of the coil current magnetic fluxes through the plasma circuit area and by the time rate of change of the plasma self flux:

$$R_e I_p = - \dot{\phi}_{\text{coils}} - \dot{\phi}_{\text{self}} = - M \dot{I}_{\text{coils}} - \dot{M} I_{\text{coils}} - L \dot{I}_p - \dot{L} I_p \quad (1)$$

where the symbols are defined in the List of Symbols, or:

$$\begin{aligned} R_e I_p = & - M_{cp} \dot{I}_c - \left(\frac{\partial M_{cp}}{\partial a} \dot{a} + \frac{\partial M_{cp}}{\partial R_p} \dot{R}_p \right) I_c + \\ & - \sum_{\xi} M_{\xi p} \dot{I}_{\xi} - \sum_{\xi} \left(\frac{\partial M_{\xi p}}{\partial a} \dot{a} + \frac{\partial M_{\xi p}}{\partial R_p} \dot{R}_p \right) I_{\xi} + \\ & - L \dot{I}_p - \left(\frac{\partial L}{\partial a} \dot{a} + \frac{\partial L}{\partial R_p} \dot{R}_p \right) I_p \end{aligned} \quad (2)$$

which can be summarized as:

$$I_p \equiv I_p (\dot{I}_p, \dot{I}_{\xi}, \dot{a}, \dot{R}_p, a, R_p, n, T) \quad (2a)$$

While heating, the plasma is compressed quasi-statically under the action of the vertical and divertor magnetic fields. Conservation of the toroidal flux of a circular cross section plasma during compression, for a toroidal magnetic field varying as $1/R$, namely:

$$\begin{aligned}\phi_T &\equiv B_{T,in} R_{in} \int_0^{2\pi} \int_0^a \frac{r dr d\theta}{R} \\ &= B_{T,in} R_{in} \int_0^{2\pi} \int_0^a \frac{r dr d\theta}{R_p + r \cos\theta}\end{aligned}\quad (3)$$

where $B_{T,in}$, R_{in} are the values of the toroidal magnetic field and distance from the torus axis at the inner radius of the torus, leads to a relation between the minor and major radius of the plasma:

$$f(a, R_p, B_T(R)) = 0 \quad (4)$$

as expressed in Appendix A. Differentiating with respect to time Equation (4), one obtains a relation between the two plasma radii compression rates:

$$\frac{\dot{a}}{a} = \frac{\dot{R}_p}{R_p} Y(a, R_p) \quad (5)$$

For a first approximation in which the magnetic field is considered constant over the plasma cross section, but varying as $1/R_p$ (while the plasma is compressed) Equations (4) and (5) become respectively:

$$\frac{a^2}{R_p} = \text{const} \quad (4')$$

and

$$\frac{\dot{a}}{a} = \frac{1}{2} \frac{\dot{R}_p}{R_p} \quad (5')$$

Substituting (5') in Equation (2) leads to the following equation:

$$\begin{aligned} R_e I_p = & - M_{cp} \dot{I}_c - \frac{\dot{R}_p}{R_p} \left(\frac{\partial M_{cp}}{\partial a} \cdot \frac{a}{2} + \frac{\partial M_{ep}}{\partial R_p} R_p \right) I_c + \\ & - \sum_{\xi} M_{\xi p} \dot{I}_{\xi} - \frac{\dot{R}_p}{R_p} \sum_{\xi} \left(\frac{\partial M_{\xi p}}{\partial a} \frac{a}{2} + \frac{\partial M_{\xi p}}{\partial R_p} R_p \right) I_{\xi} + \\ & - L \dot{I}_p - \frac{\dot{R}_p}{R_p} \left(\frac{\partial L}{\partial a} \cdot \frac{a}{2} + \frac{\partial L}{\partial R_p} R_p \right) I_p \end{aligned} \quad (6)$$

or

$$I_p \equiv I_p (\dot{I}_p, \dot{I}_{\xi}, \dot{R}_p, R_p, n, T) \quad (6')$$

Equation (6) may be written in a contracted form as:

$$\frac{\dot{I}_p}{I_p} = - \frac{\dot{M}}{L} - \frac{\dot{R}_p}{R_p} \frac{S}{L} \quad (7)$$

where:

$$M \equiv R_e + M_{cp} \frac{\dot{I}_c}{I_p} + \sum_{\xi} M_{\xi p} \frac{\dot{I}_{\xi}}{I_p} \quad (7a)$$

$$\begin{aligned}
 S \equiv & \left(\frac{\partial M_{cp}}{\partial a} \cdot \frac{a}{2} + \frac{\partial M_{cp}}{\partial R_p} R_p \right) \frac{I_c}{I_p} \\
 & + \sum_{\xi} \left(\frac{\partial M_{\xi p}}{\partial a} \cdot \frac{a}{2} + \frac{\partial M_{\xi p}}{\partial R_p} R_p \right) \frac{I_{\xi}}{I_p} \\
 & + \frac{\partial L}{\partial a} \cdot \frac{a}{2} + \frac{\partial L}{\partial R_p} R_p
 \end{aligned} \tag{7b}$$

The plasma tends at every moment to adjust itself to that major radius R_p for which it is in a stable equilibrium position under the inward Lorentz forces of the vertical magnetic field. The equilibrium condition formulated by J. Greene, J. Johnson and K. Weimer, (Phys. Fluids 14, 3 (1971) 671).

$$B_{eq} = - 10^{-7} \frac{I_p}{R_p} \left(\ln \frac{8R_p}{a} - \frac{5}{4} + \frac{2\mu n T}{K^2 I_p^2} \right) \tag{8}$$

or

$$B_{eq} \equiv B_{eq} (I_p, R_p, n, T) \tag{8'}$$

where B_{eq} is the necessary magnetic field which would keep the plasma in equilibrium. The available magnetic field is the vertical component (z component) of the vertical and divertor magnetic field:

$$B_{VDz} \equiv \sum_{\xi} P_{\xi z} I_{\xi} \tag{9}$$

The plasma equilibrium location results therefore from the following equation:

$$B_{VDZ}(I_{\xi}) = B_{eq}(I_p, R_p, n, T) \quad (10)$$

As B_{VDZ} is an independent function of time controllable through the coil generators, and as the concentration and temperature at every moment depend on the time history of the plasma radii and current, Equation (10) represents at every moment, for given coil current time evolution, a relation between the plasma current and the major radius.

By differentiating Equation (10) with respect to time, the following equation results:

$$\begin{aligned} \int_{\xi} P_{\xi z} \dot{I}_{\xi} + \int_{\xi} \left(\frac{\partial P_{\xi z}}{\partial R} \right)_{R=R_p} I_{\xi} \dot{R}_p \\ = -10^{-7} \frac{I_p}{R_p} \left[\frac{\dot{I}_p}{I_p} \left(\ln \frac{8R_p}{a} - \frac{5}{4} - \frac{2\mu n T}{I_p^2 k^2} \right) \right. \\ + \frac{\dot{R}_p}{R_p} \left(-\ln \frac{8R_p}{a} + \frac{7}{4} - \frac{2\mu n T}{I_p^2 k^2} \right) \\ + \frac{\dot{R}_p}{R_p} \left(\frac{R_p}{k} \frac{\partial k}{\partial R} + \frac{a}{2k} \frac{\partial k}{\partial a} \right) \left(-\frac{4\mu n T}{k^2 I_p^2} \right) \\ \left. + \frac{\dot{n}}{nT} \cdot \frac{2\mu}{k^2 I_p^2} \right] \quad (11) \end{aligned}$$

Eliminating $\frac{\dot{I}_p}{I_p}$ between Equations (7) and (11) leads to:

$$\frac{\dot{R}_p}{R_p} = \frac{A+B+C}{D+E+F+G} \quad (12)$$

where:

$$A \equiv - \frac{1}{L} \left(R_e + M_{cp} \frac{\dot{I}_c}{I_p} + \sum_{\xi} M_{\xi p} \frac{\dot{I}_{\xi}}{I_p} \right) \left(\ln \frac{8R_p}{a} - \frac{5}{4} - \frac{2\mu n T}{I_p^2 k^2} \right)$$

$$\equiv - \frac{1}{L} M N \quad (12a)$$

$$N \equiv \ln \frac{8R_p}{a} - \frac{5}{4} - \frac{2\mu n T}{I_p^2 k^2} \quad (12b)$$

$$B \equiv \frac{\dot{nT}}{k^2 I_p^2} \equiv \frac{\dot{nT}}{nT} \beta_{\theta} \quad (12c)$$

$$C \equiv 10^7 R_p \sum_{\xi} P_{\xi z} \frac{\dot{I}_{\xi}}{I_p} \quad (12d)$$

$$D \equiv \ln \frac{8R_p}{a} - \frac{7}{4} + \frac{2\mu n T}{I_p^2 k^2} \quad (12e)$$

$$E \equiv \left(\frac{R_p}{k} \frac{\partial k}{\partial R_p} + \frac{a}{2k} \frac{\partial k}{\partial a} \right) \left(\frac{4\mu n T}{k^2 I_p^2} \right) \quad (12f)$$

$$F \equiv \frac{1}{L} \left[\sum_{\xi} \left(\frac{\partial M_{\xi p}}{\partial a} \frac{a}{2} + \frac{\partial M_{\xi p}}{\partial R_p} R_p \right) \frac{I_{\xi}}{I_p} + \frac{a}{2} \frac{\partial L}{\partial a} + R_p \frac{\partial L}{\partial R_p} \right. \\ \left. + \left(\frac{\partial M_{cp}}{\partial a} \cdot \frac{a}{2} + \frac{\partial M_{cp}}{\partial R_p} R_p \right) \frac{I_c}{I_p} \right] \left(\ln \frac{8R_p}{a} - \frac{5}{4} - \frac{2\mu n T}{I_p^2 k^2} \right)$$

$$\equiv \frac{1}{L} S N \quad (12g)$$

$$G \equiv - 10^7 R_p^2 \sum_{\xi} \left(\frac{\partial P_{\xi z}}{\partial R} \right)_{R=R_p} \frac{I_{\xi}}{I_p} \quad (12h)$$

Equations (7) and (12) constitute a system of equations which provide the relative time rate of change of the plasma current $\dot{I}_p/I_p$ and of the major radius $\dot{R}_p/R_p$ for given coil current time rate of change.

Introducing the ratios of various vertical and divertor coil currents with respect to one of them:

$$a_{\xi} \equiv \frac{I_{\xi}}{I_1} \quad (13)$$

and the ratio of the single turn control coil current with respect to the same reference coil current

$$b_c \equiv \frac{I_c}{I_1} \quad (14)$$

one may solve Equation (12) for the reference coil current time rate of change:

$$\begin{aligned} \frac{\dot{I}_1}{I_1} = & \frac{\frac{R_p}{R_p} (D+E+F+G) - B + \frac{R_e N}{L} + \frac{N}{L} \left(M_{cp} \dot{b}_c \frac{I_1}{I_p} + \sum_{\xi} M_{\xi p} \dot{a}_{\xi} \frac{I_1}{I_p} \right) - 10^7 R_p \sum_{\xi} P_{\xi z} \dot{a}_{\xi} \frac{I_1}{I_p}}{-\frac{N}{L} \left(M_{cp} b_c + \sum_{\xi} M_{\xi p} a_{\xi} \right) + 10^7 R_p \sum_{\xi} P_{\xi z} a_{\xi}} \end{aligned} \quad (15)$$

The number density and average kinetic energy density result from the respective conservation equations:

Nr density conservation

$$\frac{d}{dt} n_i = S_i - \frac{1}{2} (\overline{\sigma v}) n_i^2 - L_{c,i} \quad (16)$$

$$\frac{d}{dt} n_{\alpha} = \frac{1}{4} (\overline{\sigma v}) n_i^2 - L_{c,\alpha} \quad (17)$$

Energy Conservation

$$\begin{aligned} \frac{d}{dt} \left(\frac{3}{2} n_i T_i \right) = S_i W_i + \frac{1}{4} (\overline{\sigma v}) U_{\alpha i} n_i^2 - \frac{1}{2} (\overline{\sigma v}) \left(\frac{3}{2} n_i T_i \right) n_i - \\ - C_1 \frac{\left(\frac{3}{2} n_i T_i \right) n_e}{T_e^{3/2}} \left(1 - \frac{T_e}{T_i} \right) - \left(\frac{3}{2} T_i \right) L_{c,i} - W_{\text{line}} + W_{\text{compr}} \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{d}{dt} \left(\frac{3}{2} n_e T_e \right) = \frac{1}{4} (\overline{\sigma v}) U_{\alpha c} n_i^2 + C_1 \frac{\left(\frac{3}{2} n_i T_i \right) n_e}{T_e^{3/2}} \left(1 - \frac{T_e}{T_i} \right) + n j^2 - \\ - \left(\frac{3}{2} T_e \right) L_{c,e} - W_{\text{Brem}} - W_{\text{cycl}} + W_{\text{compr}} \end{aligned} \quad (19)$$

$$W_{\text{cycl}} = 3.5 \times 10^3 \left(1 + 2 \frac{n_{\alpha}}{n_i} + z_I \frac{n_I}{n_e} \right)$$

$$\times n_i^{1/2} B_T^{5/2} (T_e^{11/4} + 0.0049 T_e^{15/4})$$

$$W_{\text{Brem}} = 3.0 \times 10^{-21} n_e T_e^{1/2} (n_i + 4n_{\alpha} + z_I^2 n_I)$$

2. FORMULATION OF A MULTI-NODE-TOROIDAL PLASMA HEATING

The plasma current time evolution is described independently for each sub plasma j according to the equivalent equations of Equation (2):

$$\begin{aligned}
 L_{jj} \dot{i}_{pj} = & - M_{cj} \dot{i}_c - I_c \left(\frac{\partial M_{cj}}{\partial a_j} \dot{a}_j + \frac{\partial M_{cj}}{\partial a_{j-1}} \dot{a}_{j-1} + \frac{\partial M_{cj}}{\partial R_p} \dot{R}_p \right) + \\
 & - \sum_{\xi} M_{\xi j} \dot{i}_{\xi} - \sum_{\xi} I_{\xi} \left(\frac{\partial M_{\xi j}}{\partial a_j} \dot{a}_j + \frac{\partial M_{\xi j}}{\partial a_{j-1}} \dot{a}_{j-1} + \frac{\partial M_{\xi j}}{\partial R_p} \dot{R}_p \right) + \\
 & - \sum_{j'} M_{j'j} \dot{i}_{pj'} - \sum_{j'} I_{pj'} \left(\frac{\partial M_{j'j}}{\partial a_{j'}} \dot{a}_{j'} + \frac{\partial M_{j'j}}{\partial a_{j'-1}} \dot{a}_{j'-1} + \frac{\partial M_{j'j}}{\partial a_j} \dot{a}_j + \right. \\
 & \quad \left. + \frac{\partial M_{j'j}}{\partial a_{j-1}} \dot{a}_{j-1} + \frac{\partial M_{j'j}}{\partial R_p} \dot{R}_p \right) + \\
 & - R_{ej} I_{pj} - I_{pj} \left(\frac{\partial L_{jj}}{\partial a_j} \dot{a}_j + \frac{\partial L_{jj}}{\partial a_{j-1}} \dot{a}_{j-1} + \frac{\partial L_{jj}}{\partial R_p} \dot{R}_p \right) \quad (20)
 \end{aligned}$$

where a_{j-1} and a_j are the inner and outer radii of the j th sub-plasma annulus.

The constancy during compression, of the toroidal magnetic flux through each plasma subsection, therefore, in the present case the constancy of the toroidal flux expressed in Equation (3) through each circular cross section of radius a_j leads to a set of equations determining the minor radii a_j as functions of the plasma major radius R_p :

$$2\pi B_T (R_p) R_p^2 \left(1 - \sqrt{1 - \frac{a^2}{R_p^2}} \right) = \text{const}$$

or

$$f(a_j, R_p, a_{j0}, R_{p0}) = 0 \quad (21')$$

where the subindex zero stands for the respective initial values.

Differentiating versus time Equation (21) one obtains:

$$\dot{a}_j a_j = \dot{R}_p R_p \left(1 - \sqrt{1 - \frac{a^2}{R_p^2}} \right) \quad (22)$$

or

$$\frac{\dot{a}_j}{a_j} = \frac{\dot{R}_p}{R_p} \cdot y_j(a_j, R_p) \quad (22')$$

which substituted in Equation (20) result in:

$$\begin{aligned}
 L_{jj} \frac{\dot{I}_{pj}}{I_{pj}} = & - M_{cj} \frac{\dot{I}_c}{I_{pj}} - \frac{\dot{R}_p}{R_p} \left(\frac{\partial M_{cj}}{\partial a_j} a_j Y_j + \left(\frac{\partial M_{cj}}{\partial a_j} a_{j-1} Y_{j-1} + \frac{\partial M_{cj}}{\partial R_p} R_p \right) \frac{I_c}{I_{pj}} + \right. \\
 & - \sum_{\xi} M_{\xi j} \frac{\dot{I}_{\xi}}{I_{pj}} - \frac{\dot{R}_p}{R_p} \sum_{\xi} \left(\frac{\partial M_{\xi j}}{\partial a_j} a_j Y_j + \frac{\partial M_{\xi j}}{\partial a_{j-1}} a_{j-1} Y_{j-1} + \frac{\partial M_{\xi j}}{\partial R_p} R_p \right) \frac{I_{\xi}}{I_{pj}} + \\
 & - \sum_{j'} M_{j'j} \frac{\dot{I}_{pj'}}{I_{pj}} - \frac{\dot{R}_p}{R_p} \sum_{j'} \left(\frac{\partial M_{j'j}}{\partial a_{j'}} a_{j'} Y_{j'} + \frac{\partial M_{j'j}}{\partial a_{j'-1}} a_{j'-1} Y_{j'-1} \right. \\
 & \quad \left. + \frac{\partial M_{j'j}}{\partial a_j} a_j Y_j + \frac{\partial M_{j'j}}{\partial a_{j-1}} a_{j-1} Y_{j-1} + \frac{\partial M_{j'j}}{\partial R_p} R_p \right) \frac{I_{pj'}}{I_{pj}} - \\
 & - R_{ej} - \frac{\dot{R}_p}{R_p} \left(\frac{\partial L_{jj}}{\partial a_j} a_j Y_j + \frac{\partial L_{jj}}{\partial a_{j-1}} a_{j-1} Y_{j-1} + \frac{\partial L_{jj}}{\partial R_p} R_p \right) \quad (23)
 \end{aligned}$$

or:

$$\begin{aligned}
 \frac{\dot{I}_{PJ}}{I_{PJ}} + \sum_{j'} \frac{M_{j'j}}{L_{jj}} \frac{\dot{I}_{pj'}}{I_{pj}} = \\
 = - \frac{M_i}{L_{jj}} - \frac{\dot{R}_p}{R_p} \frac{S_j + S_{jj'}}{L_{jj}} \quad (23')
 \end{aligned}$$

where:

$$M_j \equiv R_{ej} + M_{ej} \frac{\dot{I}_c}{I_{pj}} + \sum_{\xi} M_{\xi j} \frac{\dot{I}_{\xi}}{I_{pj}} \quad (23a)$$

$$S_j \equiv \frac{R_p}{\dot{R}_p} \left(\dot{M}_{cj} \frac{I_c}{I_{pj}} + \sum_{\xi} \dot{M}_{\xi j} \frac{I_{\xi}}{I_{pj}} + \dot{L}_{jj} \right) \quad (23b)$$

$$S_{jj'} \equiv \frac{R_p}{\dot{R}_p} \sum_{j'} \dot{M}_{j'j} \frac{I_{pj'}}{I_{pj}} \quad (23c)$$

The plasma major radius R_p time evolution results from the equilibrium Equation (10) applied to the central plasma subsection ($j=1$), or from its time differential expression, namely Equation (11).

Eliminating $\frac{\dot{R}_p}{R_p}$ between Equations (23) and Equation (11) applied to the central plasma, leads to a system of equations which provide the relative time rate of change of the plasma currents, and time rate of the major radius for given coil current time rate of change.

The coil currents time evolution are controlled by the inductive coupling with the plasma, as described below:

$$L_{\xi\xi} \dot{I}_{\xi} = \zeta_{\xi} - M_{p\xi} \dot{I}_p - \dot{M}_{p\xi} I_p - \sum_{\xi'} M_{\xi'\xi} \dot{I}_{\xi'} - R_{\xi} I_{\xi} \quad (24)$$

$$L_{cc} \dot{I}_c = \zeta_c - M_{pc} \dot{I}_p - \dot{M}_{pc} I_p - \sum_{\xi} M_{\xi c} \dot{I}_{\xi} - R_c I_c \quad (25)$$

3. COMPUTATIONAL METHODS

Following methods have been pursued in the plasma heating evaluation:

First Method (Single-node plasma).

- 1). Input:
 $n(0), T(0), a(0), I_p(0), R_p(t), I_\xi(t), \Delta t$
- 2). Guess $\dot{\phi}_c$
- 3). Calculate: $\dot{I}_p$ from Eq. (7),
 $\dot{n}, \dot{T}$ from Eqs. (16)-(19).
- 4). Propagate along one time step:
 I_p, n, T, ϕ_c
- 5). Calculate B_{eq} from Eq. (8)
- 6). Calculate B_{VDZ} from Eq. (9)
- 7). If Eq. (10) is satisfied repeat from 2)
 If Eq. (10) is not satisfied, improve guess of $\dot{\phi}_c$
 and repeat from 3)
- 8). Results: $n(t), T(t), I_p(t), \phi_c(t), a(t)$

Second Method (single-node plasma).

- 1). Input:
 $n(0), T(0), R_p(0), a(0), I_p(0), I_\xi(0), \dot{I}_\xi(t), \Delta t$
- 2). Calculate:
 $\frac{\dot{R}_p}{R_p}$ from Eq. (12)
 $\frac{\dot{I}_p}{I_p}$ from Eq. (7)
 $\dot{n}$ from Eqs. (16), (17)
 $\frac{\dot{n}}{nT}$ from Eqs. (18), (19)

3). Propagate along one time step:

$$R_p, I_p, n, T, a$$

4). Repeat from 2)

$$\text{Results: } n(t), T(t), I_p(t)$$

Third Method (single-node plasma)

1). Input:

$$n(0), T(0), R_p(0), a(0), I_p(0), I_\xi(0), a_\xi(t), b_c(t), \\ \dot{R}_p(t), \Delta t$$

2). Calculate:

$$\frac{\dot{I}_1}{I_p} \text{ from Eq. (15)}$$

$$\frac{\dot{I}_p}{I_p} \text{ from Eqs. (7), (12)}$$

$$\dot{n} \text{ from Eqs. (16), (17)}$$

$$\frac{\dot{n}}{nT} \text{ from Eqs. (18), (19)}$$

3). Propagate along one time step:

$$I_1, I_p, n, T, R_p, a$$

4). Repeat from 2

Fourth Method (multi-node plasma)

1). Input:

$$n_j(0), T_j(0), R_p(0), a_j(0), I_{pj}(0), I_\xi(0), a_\xi(t), \\ b_c(t), I_1(t), \Delta t$$

2). Calculate:

$$\frac{\dot{R}_p}{R_p} \text{ from Eq. (12)}$$

$$\frac{\dot{I}_{pj}}{I_p} \text{ from Eqs. (23)}$$

$$\dot{n}_j \text{ from Eqs. (16), (17)}$$

$$\frac{\dot{n}}{nT} \text{ from Eqs. (18), (19)}$$

3). Propagate along one time step:

$$R_p, I_{pj}, n_j, T_j, a_j$$

4). Repeat from 2)

Fifth Method (multi-node plasma)

1) Input:

$$n_j(0), T_j(0), R_p(0), a_j(0), I_{pj}(0), I_\xi(0), \zeta_c(t), \\ \zeta_\xi(t), \Delta t$$

2) Calculate:

$$\begin{aligned} \dot{I}_{pj}, \dot{I}_c, \dot{I}_\xi & \text{ from Eqs. (23), (24), (25)} \\ \dot{R}_p & \text{ from Eq. (10)} \\ \dot{a}_j & \text{ from Eqs. (22)} \\ \dot{n}_j & \text{ from Eqs. (16), (17)} \\ \dot{nT} & \text{ from Eqs. (18), (19)} \end{aligned}$$

3) Propagate along one time step:

$$I_{pj}, I_c, I_\xi, R_p, A_j, n, T$$

4) Repeat from 2)

In principle the first four methods describe the plasma time evolution regarding heating and contraction for prescribed coil currents.

The first method, iterative on the control coil flux through the plasma circuit, is devised to provide information on the degree of control of the plasma heating through the control coil current, based on the fact that the inductive coupling between the control coil and plasma affects the plasma current time rate, but not the magnetic field in which the plasma has to equilibrate dynamically.

The plasma current reactivity towards control coil rate variation is represented by the following parameter:

$$\eta_{Ip} \equiv \frac{\delta(\dot{I}_p/I_p)}{\delta(\phi_c/\phi_c)}$$

and the plasma major radius and temperature reactivity towards plasma current rate variation is represented by:

$$\eta_{RI} \equiv \frac{\delta(\dot{R}_p/R_p)}{\delta(\dot{I}_p/I_p)}$$

and

$$\eta_{TI} \equiv \frac{\delta(\dot{T}/T)}{\delta(\dot{I}_p/I_p)}$$

Once $\dot{R}_p$ and $\dot{I}_\xi$ are fixed in the input data, $\dot{\phi}_c$ results.

The second and third methods are noniterative, the contraction and plasma heating resulting directly as function of the coil current time variation (second method) or the coils and plasma time evolution resulting directly from an imposed contraction evolution (third method). The following parameters are relevant:

$$\eta_{R\xi} \equiv \frac{\delta(\dot{R}_p/R_p)}{\delta(\dot{I}_\xi/I_\xi)}$$

$$\eta_{I\xi} \equiv \frac{\delta(\dot{I}_p/I_p)}{\delta(\dot{I}_\xi/I_\xi)}$$

$$\eta_{IR} \equiv \frac{\delta(\dot{I}_p/I_p)}{\delta(\dot{R}_p/R_p)}$$

The fourth method represents a generalization of the second one to a multi-node plasma, in which the subplasmas cross-sections are approximated by concentric annuli. The

central subplasma dynamic quasistatic equilibration determines the major radius time evolution, and the constancy of the toroidal flux determines the minor radii time evolution.

Finally the fifth method solves simultaneously the subplasmas and coil currents as well as the plasma contraction, for applied coil electromotive forces.

COMMENTS ON MODEL

The description of a plasma behaviour via electric circuits is realistic as long as one can identify current paths, whose knowledge is necessary for mutual inductance coefficient calculations.

The concept of magnetic surfaces on which the current is flowing favors the modeling of independent circuits, the plasma being subdivided into shells separated by magnetic surfaces as in Fig. 1. As the current and current time rate of change are not necessarily uniform within a shell, a further subdivision is made through azimuthal toroidal planes, each plasma current I_{pj_k} being described in Equation (B 1).

The shape of the plasmas whose cross sections are represented in Fig. 1 is arbitrary, the azimuth of a given (jk) cross section in the respective j shell being cyclicly variable along the toroidal direction. Noncircular subplasma shapes affect the values of the inductance coefficients and ohmic resistances in Equation (B 1).

A substantial reduction in the number of plasma circuits takes place if uniform conditions over each shell are assumed to exist, as in Equation (B 4), or if an a priori known distribution can be stated, as in Equation (B 9).

A first approximation of the above plasma model consists of a subdivision of the plasma into concentric circles. The integration of the induction Equation (B 4), which is identical with Equation (20), for the case of concentric circle cross section plasmas, is independent of the shapes of the subplasma toroidal axes.

The plasma compression is being described by taking into account the magnetic invariants during the compression, namely the toroidal flux for the approximation of circular cross section, and the toroidal and poloidal flux for cross sections delimited by magnetic surfaces.

The plasma multiple circuit modeling serves primarily to analyze the possibility of control of the plasma heating through external coil currents and injected power sources. It is relevant therefore to identify quantitative criteria necessary to be met in order to achieve the desired plasma heating conditions, as follows:

The plasma currents are generated by the inductive fluxes of the variable external coil current rates of change and by the energy density accumulation in the constituent particles, as described by the quasiequilibrium Equation (11):

$$\dot{R}_p \equiv \dot{R}_p (\dot{i}_\xi, \dot{i}_p, \dot{nT}, \dots) \quad (27)$$

Therefore the external coil current variation and the energy density accumulation are the determining factors in the plasma current variation, as results by eliminating $\dot{R}_p$ between Equation (6) and (11):

$$\dot{i}_p \equiv \dot{i}_p (\dot{i}_\xi, \dot{i}_c, \dot{nT}, \dots) \quad (28)$$

The ion energy density accumulation depends on the ion fusion and compressional sources on one side, and on the energy

sinks constituted by convective, fusion, radiative and collisional events on the other side. A main contribution is constituted by the convective loss, which depends strongly on the current intensity:

$$\dot{nT} \quad \dot{nT} (I_p, \dots) \quad (29)$$

Thus in order to achieve a plasma heating, following conditions are necessary and sufficient:

$$\dot{nT} > 0 \quad (\text{over most heating period}) \quad (30)$$

$$\dot{n}_i > 0 \quad " \quad (31)$$

$$\dot{n}_\alpha > 0 \quad " \quad (32)$$

$$\dot{I}_p > 0 \quad " \quad (33)$$

The inequality (33) can be written explicitly as:

$$\begin{aligned} \frac{\dot{I}_p}{I_p} &\equiv \frac{A}{N} - \frac{F}{N} \cdot \frac{A+B+C}{D+E+F+G} \equiv \\ &\equiv \frac{A}{N} \left(1 - \frac{F}{D+E+F+G} \right) - \frac{F}{N} \cdot \frac{C}{D+E+F+G} - \\ &\quad - \frac{F}{N} \cdot \frac{B}{D+E+F+G} > 0 \end{aligned}$$

According to the expressions of A, ... G, the first two terms in the second expression of $\dot{I}_p/I_p$ represent the control of the external coil currents on the plasma current increase, and the third term represents the limitation in the plasma current increase due to energy losses.

The above conditions have to be fulfilled simultaneously at each level of number density, energy density and plasma current intensity.

Among the various terms that control the inequalities (30) - (33), some are independent of the intrinsic state of the plasma characterized by the following ensemble of parameters:

$$\{ \equiv \{n, nT, I_p\}$$

and are controllable independently at each level of the plasma heating. For a given impurity content which affects the radiated power, these terms are the power injected through a high energy beam and the electromotive force induced by the external coil variable currents.

It results therefore that plasma heating can be achieved by determining the injected power and external coil currents as function of time such that inequalities (30) - (33) should be satisfied. Obviously the solution is not unique. The acceptable range of solutions are described in Part II of the present report.

5. RESULTS

Numerical calculations were performed on a single mode plasma following the First Method described in chapter 3 and on multi-mode plasmas (6 modes), following the Fifth Method.

Three sets of results are presented, one for a single mode plasma (plasma A) and two for a six mode plasma (plasma B and plasma C).

The initial conditions for plasmas B and C are listed on Table 1. All initial conditions are similar except the minor radii values.

Both plasmas B and C undergo a slow compression of $\frac{1}{13.2}$ m/s.

The plasma current time rates for multi mode plasmas and the time rate of one of the vertical field coils ($\xi=1$) are calculated by matrix inversion technique from equations (11) and (23).

Figure 1 represents a plasma poloidal cross section for general shape magnetic surfaces.

The location of the vertical, divertor and control coils is indicated on Figure 2. The five pairs of vertical coils and the two pairs of divertor coils are connected in parallel, the ratio of the voltages applied to the two groups of coils being $V_D/V_V = -0.421$. The current distribution among coils and therefore the current ratios $a_\xi \equiv I_\xi/I_\xi = 1$ result from the coils cross sections listed in MATT-1050 Report, page 144.

Figures 3A - 3C (see list of Figures) and Tables 2, 3,

describe the A, B, C, plasmas time evolution.

Figure 3 presents the coil currents that are necessary to equilibrate the plasma quasistatically in time, and the plasma current time variation.

The electromotive forces which determine the plasmas B and C current evolution are listed on Tables 2 and 3 for typical moments, under the subtitle Kirchhoff voltage equation terms (equation 23). The uncircled numbers leading the columns of Kirchhoff terms on Tables 2 and 3 are used to identify the nature of the terms as indicated on the list of terms in Kirchhoff's equation, and the numbers at the left of each line identify the plasma section, number 1 representing the central plasma.

At time $t=0$ the predominant EMF's which determine the time variation of plasma B are indicated in columns 2, 3, and 4 and are respectively the control coil applied voltage, the equilibrium coils applied voltage and the plasma sections mutually induced voltage generated by the electric current variation, the latter having a negative effect on the plasma current increase. While the terms 2 and 3 do not vary with the minor radius, term number 4 does, and therefore the plasma sections mutual interaction constitutes the cause of the uneven distribution of the plasma current along the minor radius. For the conditions chosen at $t=0$ the ohmic voltage drop (term 5) is small, therefore the radial distribution of the plasma currents time rates is controlled primarily by the mutual inductance.

At a later moment in time $t \approx 20$ sec, the significant terms in Kirchhoff voltage equation for plasma B (Table 2) are the

mutual inductive terms due to plasma motion (terms 6 and 7) followed by the mutual inductive terms due to plasma current variation (term 4). No term is negligible in this case. The radial variation of the overall effect expressed by term 1 is produced by the radial variation of terms 7 and 4 .

Similar qualitative behavior is shown on Table 3 for plasma C where at time $t \approx 0.3$ seconds the control coil and equilibrium coils induced EMF's (terms 2 and 3) and the plasma mutual inductive voltage (term 4) are the determinant terms in the Kirchhoff equation and at $t \approx 4.7$ seconds all terms excluding the ohmic voltage drop contribute equally to the plasma current time variation, the radial distribution of term 1 being determined by the radial distribution of terms 4 and 7 .

If compression does not exist, the only significant terms in Kirchhoff equation are terms 2 - 5, and as terms 2 and 3 do not vary with the minor radius, the radial distribution of the electric current time variation is controlled by the plasma sections mutual inductance (term 4) if the ohmic voltage drop (term 5) is negligible.

Figure 4 represents the particle density for plasmas A, B and C.

The number density time evolution is determined in the present model by the net effect of injected particle deposition and convective loss. Charge neutrality is assumed in order to calculate the electron density. Impurity density is considered a fixed fraction of the ion density.

The ion source is constant along the minor radius for

plasma B and linearly variable for plasma C as shown by the terms under the heading "ion particle continuity equation terms" in Tables 2 and 3, the order of the columns being indicated on the "List of Terms in the Transport Equations".

Clearly the time evolution of the radial distribution of the ion density as shown on Figure 4 is determined by the source function, the particle fluxes used being too low as compared to the sources to level the density gradients towards the center of the plasma.

The temperature time evolution is represented on Figure 5 and contributions to the ion and electron energy conservation are listed on Tables 2 and 3 under the heading "Ion/Electron energy continuity equation terms", for two specific moments.

In the initial phase the ohmic heating is responsible for the electron temperature increase and the electron-ion collisions are responsible for the ion heating.

At a later moment the injected neutral particles heat the ions, but the electrons are still primarily heated ohmically.

Figure 6 represents various magnetic properties of the plasma.

6 CONCLUSIONS

A two dimensional transport problem is substituted with a system of zero dimensional transport problems by subdividing a toroidal plasma in a set of toruses represented in the poloidal cross section on Figure 1.

The present work assumes a θ symmetry in the poloidal plane and is therefore equivalent to a one-dimensional transport solution.

The main aspect of the results at the step presently described is the realistic correlation between the plasma and the magnetic coils in a fusion reactor, and therefore the estimate of the inductive electric fields which are built up in the plasma.

The plasma evolution in the start-up and shut-down phase is dominated by induction phenomena as described by the induction equation (equation 23).

However, the temporal behavior of the results presented on Figure 3 on plasma current distribution are unrealistic, as far as an electric current density increasing with the minor radius is not observable experimentally except for short times of skin current formation.

This aspect suggests that the radial particle and energy transport are incompletely treated, assumption confirmed by the unrealistic density and temperature profiles of the plasma, peaked towards the outer shells. (Figures 5 and 7).

Anticipating on results presented in Part II of the Toroidal Plasma Time Evolution, one may state that a more elaborate treat-

ment of the diffusion phenomena leads to density and temperature profiles which have their maxima in the center of the plasma (looking at a poloidal cross-section) but also leads to electric current density radial profiles which have their maxima on the outer shell, consistent with the results presented on Figure 3.

Also considering the present causes of the assumed radially inverted electric current density profiles, namely the terms of the Kirchhoff voltage equations on Tables 2 and 3, it is obvious that density and temperature distribution affect the electric current time rate distribution only through the electrical resistance in the ohmic voltage drop, and as this term is negligible at high temperatures, a change in temperature profiles would not affect the electric current distribution.

It therefore results that more consideration needs to be given to the simplified Ohm's law which is imbedded in the induction equation regarding the tensorial effects of particle motion in electric and magnetic fields. These effects are presented in Part III of the Toroidal Plasma Time Evolution.

Besides, a correct priority of approximations built in a physical model can only result from computational ↔ experimental comparison, and an analysis of the existing experiments with the present computational model is felt necessary.

APPENDIX A

Circular Cross Section Contraction

If the magnetic surfaces had circular cross sections, the constancy during compression of the toroidal flux which is:

$$\begin{aligned}\phi_T &\equiv B_{T,in} R_{in} \int_0^{2\pi} \int_0^a \frac{r dr d\theta}{R_p + r \cos\theta} = \\ &= 2\pi B_{T,in} R_{in} R_p \left(1 - \sqrt{1 - \frac{a^2}{R_p^2}} \right)\end{aligned}$$

leads to the following compression relation:

$$\dot{a} a = \dot{R}_p R_p \left(1 - \sqrt{1 - \frac{a^2}{R_p^2}} \right)$$

APPENDIX B.

Plasma Circuitry

Subdividing the plasma into circuits delimited by magnetic surfaces and by azimuthal toroidal planes convergent along a curve of maximum pressure, as in Figure 1, one may describe the currents independently according to Faraday's law:

$$\begin{aligned}
 R_{ej_k} I_{pj_k} = & -\dot{\phi}_{ext,j_k} - \frac{\partial}{\partial t} \left[\sum_{j'} \sum_s I_{pj'_s} \int_0^{R_{j_k}} \int_0^{2\pi} \int_{j'_s} \frac{d\vec{l}_{j'_s} \times \vec{\delta}_{j'_s} j_k}{\delta_{j'_s j_k}^3} \cdot \vec{e}_z R dR d\phi \right. \\
 & \left. - L_{j_k j_k} \dot{I}_{pj_k} \right] = \\
 = & -\dot{\phi}_{ext,j_k} - \sum_{j'} \sum_s \dot{I}_{pj'_s} M_{j'_s j_k} - \sum_{j'} \sum_s I_{pj'_s} \dot{M}_{j'_s j_k} + \\
 & - L_{j_k j_k} \dot{I}_{pj_k} - \dot{L}_{j_k j_k} I_{pj_k}
 \end{aligned} \tag{B1}$$

where k, s represent indexes within single shells and j, j' indexes over shells.

By summing over the j^{th} shell Equation (B 1) becomes:

$$\begin{aligned}
 \sum_k R_{ej_k} I_{pj_k} = & -\dot{\phi}_{ext,j} - \sum_{j'} \sum_s \dot{I}_{pj'_s} M_{j'_s j} - \sum_{j'} \sum_s I_{pj'_s} \dot{M}_{j'_s j} + \\
 & - \sum_k L_{j_k j_k} \dot{I}_{pj_k} - \sum_k \dot{L}_{j_k j_k} I_{pj_k}
 \end{aligned} \tag{B2}$$

and if neither the current nor the current time rate of change do vary considerably within a shell:

$$\frac{\partial I_{pj_k}}{\partial k} = \frac{\partial I_{pj'_s}}{\partial s} = 0, \quad \frac{\partial \dot{I}_{pj_k}}{\partial k} = \frac{\partial \dot{I}_{pj'_s}}{\partial s} = 0 \quad (B3)$$

summations over the j_s^{th} shell may be performed and (B1) becomes:

$$\begin{aligned} I_{pj} \sum_k R_{ej_k} = & - \dot{\phi}_{\text{ext},j} - \sum_{j'} \dot{I}_{pj'} M_{j'j} - \sum_{j'} I_{pj'} \dot{M}_{j'j} + \\ & - \dot{I}_{pj} \sum_k L_{jkj_k} - I_{pj} \sum_k \dot{L}_{jkj_k} \end{aligned} \quad (B4)$$

where

$$M_{j_s j} = \sum_k M_{j_s j_k} \quad (B5)$$

and

$$M_{j'j} = \sum_s M_{j'_s j} \quad (B6)$$

In general the current time rates of change are not independent of the indexes k or s over given shells and a reduction in the number of plasma circuits may be realized in this case by introducing in (B2) distribution functions of current and current rates over given shells:

$$I_{pj'_s} \equiv I_{pj'} \phi_{j'_s}(s) \quad (B7)$$

$$\dot{i}_{pj_s} \equiv \dot{i}_{pj} \phi_{j_s} + I_{pj} \dot{\phi}_{j_s} \quad (B8)$$

Equation (B2) becomes:

$$\begin{aligned} I_{pj} \sum_k R_{ej_k} \phi_{j_k} = & -\phi_{ext,j} - \sum_{j'} \dot{i}_{pj'} \sum_s M_{j_s'j} \phi_{j_s'} - \sum_{j'} I_{pj'} \sum_s M_{j_s'j} \dot{\phi}_{j_s'} + \\ & - \sum_{j'} I_{pj'} \sum_s \phi_{j_s'} \dot{M}_{j_s'j} - \dot{i}_{pj} \sum_k L_{j_k j_k} \phi_{j_k} + \\ & - I_{pj} \sum_k L_{j_k j_k} \dot{\phi}_{j_k} - I_{pj} \sum_k \dot{L}_{j_k j_k} \phi_{j_k} \end{aligned} \quad (B9)$$

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LIST OF TABLES

1. Input parameters for plasmas A, B, C.
2. Plasma B parameters.
3. Plasma C parameters.

List of terms in Kirchhoff equation, on Tables 2 and 3.

- ① $L_{ii} \dot{I}_{pi}$
- ② $- M_{ci} \dot{I}_c$
- ③ $-\sum_{\xi} M_{\xi i} a_{\xi} \dot{I}_i$
- ④ $-\sum_{j \neq i} M_{ij} \dot{I}_{pj}$
- ⑤ $- R_{ei} I_{pi}$
- ⑥ $-\sum_{\xi} a_{\xi} I' \dot{M}_{\xi i}$
- ⑦ $-\sum_{j \neq i} I_{pj} \dot{M}_{ij}$
- ⑧ $- I_{pi} \dot{L}_{ii}$
- ⑨ $-\sum_{\xi} M_{\xi i} a_{\xi} \dot{I}'$
- ⑩ $- I_c \dot{M}_{ci}$

Note: Term ① = $\sum$ (Terms ② - ⑩)

List of Terms in the Transport (continuity) Equations on
Tables 2 and 3

Ion particle continuity equation terms:

1. Neutral injection
2. Fusion loss
3. Boundary loss
19. Compression effect

Alpha particle continuity equation terms:

4. Fusion source
5. Boundary loss
20. Compression effect

Ion energy continuity equation terms.

6. Neutral injection
7. Alpha particle-ion collision
8. Fusion loss
9. Electron-ion collisions
10. Boundary loss
17. Compression effect

Electron energy continuity equation terms:

11. Alpha particle-electron collisions
12. Ion-electron collisions
13. Boundary loss
14. Ohmic heating
15. Bremsstrahlung radiation
16. Cyclotron radiation
18. Compression effect

TABLE 1

INITIAL CONDITIONS

FOR PLASMAS B AND C

Ion Temperature	10 eV
Electron Temperature	10 eV
Ion Density	$4.6 \text{ E } 17 \text{ m}^{-3}$
Plasma Current	6.0 E 5 A
Alpha Particle Density	0.0

Initial conditions are spatially uniform.

TABLE 2

PLASMA B

TIME = 0

KIRCHHOFF VOLTAGE EQUATION TERMS

	1	2	3	4	5	6	7	8
1	1.7 E 2	7.0 E 2	2.6 E 3	-3.1 E 3	-1.8 E 1	-3.0 E- 2	5.0 E- 2	1.2 E- 2
2	3.6 E 2	7.0 E 2	2.6 E 3	-3.0 E 3	-2.2 E 1	-3.0 E- 2	4.5 E- 2	9.8 E- 3
3	5.9 E 2	7.0 E 2	2.6 E 3	-2.7 E 3	-2.7 E 1	-3.0 E- 2	3.9 E- 2	8.8 E- 3
4	8.6 E 2	7.0 E 2	2.6 E 3	-2.4 E 3	-3.7 E 1	-3.0 E- 2	3.3 E- 2	8.3 E- 3
5	1.1 E 3	7.0 E 2	2.6 E 3	-2.2 E 3	-5.4 E 1	-3.0 E- 2	2.7 E- 2	8.1 E- 3
6	1.1 E 3	7.0 E 2	2.6 E 3	-2.1 E 3	-1.2 E 2	-3.0 E- 2	2.3 E- 2	7.8 E- 3

ION PARTICLE CONTINUITY EQUATION TERMS

	1	2	3
1	0.0	0.0	-1.6 E 1
2	0.0	0.0	-1.3 E 1
3	0.0	0.0	-9.8 E 0
4	0.0	0.0	-6.7 E 0
5	0.0	0.0	-4.2 E 0
6	0.0	0.0	-1.6 E 0

ALPHA PARTICLE CONTINUITY EQUATION TERMS

	4	5
1	0.0	0.0
2	0.0	0.0
3	0.0	0.0
4	0.0	0.0
5	0.0	0.0
6	0.0	0.0

PLASMA B

TIME = 0

ION ENERGY CONTINUITY EQUATION TERMS

	6	7	8	9	10	17
1	0.0	0.0	0.0	0.0	-2.4 E- 1	3.0 E 13
2	0.0	0.0	0.0	0.0	-1.9 E- 1	3.0 E 13
3	0.0	0.0	0.0	0.0	-1.5 E- 1	3.1 E 13
4	0.0	0.0	0.0	0.0	-1.0 E- 1	3.1 E 13
5	0.0	0.0	0.0	0.0	-6.3 E- 2	3.1 E 13
6	0.0	0.0	0.0	0.0	-2.4 E- 2	3.1 E 13

ELECTRON ENERGY CONTINUITY EQUATION TERMS

	11	12	13	14	15	16	18
1	0.0	0.0	-2.5 E- 1	1.3 E 19	-8.7 E 13	-7.3 E 8	3.2 E 13
2	0.0	0.0	-2.0 E- 1	1.8 E 19	-8.7 E 13	-7.3 E 8	3.3 E 13
3	0.0	0.0	-1.5 E- 1	2.7 E 19	-8.7 E 13	-7.3 E 8	3.3 E 13
4	0.0	0.0	-1.0 E- 1	5.1 E 19	-8.7 E 13	-7.3 E 8	3.4 E 13
5	0.0	0.0	-6.6 E- 2	1.1 E 20	-8.7 E 13	-7.3 E 8	3.7 E 13
6	0.0	0.0	-2.5 E- 2	5.0 E 20	-8.7 E 13	-7.3 E 8	5.5 E 13

PLASMA B

TIME = 0

MINOR RADII

1.9 2.5 3.0 3.2 3.4 3.5

RESISTANCES

1.8 E- 4 2.2 E- 4 2.7 E- 4 3.7 E- 4 5.3 E- 4 1.2 E- 3

PLASMA - FIELD COIL INDUCTANCES

	1	2	3	4	5	6	7
1	7.2 E- 6	1.1 E- 5	1.5 E- 5	1.9 E- 5	2.1 E- 5	9.1 E- 6	4.4 E- 6
2	7.2 E- 6	1.1 E- 5	1.5 E- 5	1.9 E- 5	2.1 E- 5	9.1 E- 6	4.4 E- 6
3	7.2 E- 6	1.1 E- 5	1.5 E- 5	1.9 E- 5	2.1 E- 5	9.1 E- 6	4.4 E- 6
4	7.2 E- 6	1.1 E- 5	1.5 E- 5	1.9 E- 5	2.1 E- 5	9.1 E- 6	4.4 E- 6
5	7.2 E- 6	1.1 E- 5	1.5 E- 5	1.9 E- 5	2.1 E- 5	9.1 E- 6	4.4 E- 6
6	7.2 E- 6	1.1 E- 5	1.5 E- 5	1.9 E- 5	2.1 E- 5	9.1 E- 6	4.4 E- 6

PLASMA B

TIME = 0

MATRIX A

9.7 E- 8	1.8 E- 8	1.8 E- 8	1.8 E- 8	1.8 E- 8	1.8 E- 8	1.8 E- 8
7.0 E- 5	3.1 E- 5	2.1 E- 5	1.8 E- 5	1.6 E- 5	1.5 E- 5	1.4 E- 5
7.0 E- 5	2.1 E- 5	2.5 E- 5	1.8 E- 5	1.6 E- 5	1.5 E- 5	1.4 E- 5
7.0 E- 5	1.8 E- 5	1.8 E- 5	2.2 E- 5	1.6 E- 5	1.5 E- 5	1.4 E- 5
7.0 E- 5	1.6 E- 5	1.6 E- 5	1.6 E- 5	2.0 E- 5	1.5 E- 5	1.4 E- 5
7.0 E- 5	1.5 E- 5	1.5 E- 5	1.5 E- 5	1.5 E- 5	2.0 E- 5	1.4 E- 5
7.0 E- 5	1.4 E- 5	1.4 E- 5	1.4 E- 5	1.4 E- 5	1.4 E- 5	1.9 E- 5

VECTOR B

-2.7 E- 2	6.8 E 2	6.7 E 2	6.7 E 2	6.6 E 2	6.4 E 2	5.8 E 2
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VECTOR X

-3.8 E 7	5.3 E 6	1.4 E 7	2.7 E 7	4.2 E 7	5.7 E 7	6.1 E 7
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TABLE 2

PLASMA B

TIME = 19

KIRCHHOFF VOLTAGE EQUATION TERMS

	1	2	3	4	5	6	7	8
1	1.4 E 0	4.8 E- 2	3.8 E- 1	-1.2 E 0	-1.0 E- 2	-1.9 E 0	3.8 E 0	2.1 E- 1
2	1.5 E 0	4.8 E- 2	3.8 E- 1	-8.3 E- 1	-2.2 E- 2	-1.9 E 0	3.5 E 0	3.5 E- 1
3	1.2 E 0	4.8 E- 2	3.8 E- 1	-7.6 E- 1	-3.2 E- 2	-1.9 E 0	3.0 E 0	5.0 E- 1
4	3.6 E- 1	4.8 E- 2	3.8 E- 1	-1.1 E 0	-4.2 E- 2	-1.9 E 0	2.4 E 0	6.4 E- 1
5	-4.8 E- 1	4.8 E- 2	3.8 E- 1	-1.6 E 0	-5.6 E- 2	-1.9 E 0	1.8 E 0	8.0 E- 1
6	-1.4 E 0	4.8 E- 2	3.8 E- 1	-2.1 E 0	-8.0 E- 2	-1.9 E 0	1.4 E 0	9.4 E- 1

ION PARTICLE CONTINUITY EQUATION TERMS

	1	2	3
1	5.0 E 17	-1.3 E 16	-2.1 E 14
2	5.0 E 17	-1.5 E 16	-1.6 E 13
3	5.0 E 17	-2.1 E 16	-5.1 E 12
4	5.0 E 17	-2.8 E 16	-3.5 E 12
5	5.0 E 17	-3.5 E 16	-2.7 E 12
6	5.0 E 17	-4.2 E 16	-2.4 E 12

ALPHA PARTICLE CONTINUITY EQUATION TERMS

	4	5
1	6.4 E 15	-7.1 E 11
2	7.7 E 15	-6.3 E 10
3	1.1 E 16	-2.7 E 10
4	1.4 E 16	-2.6 E 10
5	1.8 E 16	-2.5 E 10
6	2.1 E 16	-2.6 E 10

PLASMA B

TIME = 19

ION ENERGY CONTINUITY EQUATION TERMS

	6	7	8	9	10	17
1	1.5 E 19	5.8 E 18	-2.7 E 17	-4.8 E 18	-4.5 E 15	1.1 E 18
2	1.5 E 19	7.4 E 18	-3.6 E 17	-4.4 E 18	-3.6 E 14	1.2 E 18
3	1.5 E 19	1.1 E 19	-5.7 E 17	-3.7 E 18	-1.4 E 14	1.4 E 18
4	1.5 E 19	1.8 E 19	-9.8 E 17	-3.1 E 18	-1.2 E 14	1.8 E 18
5	1.5 E 19	2.7 E 19	-1.5 E 18	-1.8 E 18	-1.2 E 14	2.2 E 18
6	1.5 E 19	4.1 E 19	-2.3 E 18	1.8 E 18	-1.3 E 14	2.8 E 18

ELECTRON ENERGY CONTINUITY EQUATION TERMS

	11	12	13	14	15	16	18
1	1.7 E 19	4.8 E 18	-4.2 E 15	1.3 E 17	-1.6 E 18	-1.5 E 18	1.0 E 18
2	1.9 E 19	4.4 E 18	-3.4 E 14	7.1 E 17	-1.7 E 18	-1.9 E 18	1.1 E 18
3	2.5 E 19	3.7 E 18	-1.3 E 14	2.1 E 18	-1.9 E 18	-3.2 E 18	1.3 E 18
4	3.1 E 19	3.1 E 18	-1.2 E 14	5.1 E 18	-2.1 E 18	-6.3 E 18	1.7 E 18
5	3.5 E 19	1.8 E 18	-1.2 E 14	1.3 E 19	-2.4 E 18	-1.3 E 19	2.2 E 18
6	3.3 E 19	-1.8 E 18	-1.5 E 14	4.6 E 19	-3.0 E 18	-4.0 E 19	3.2 E 18

PLASMA B

TIME = 19

MINOR RADII

1.8	2.4	2.9	3.1	3.3	3.4
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RESISTANCES

5.8 E- 9	6.1 E- 9	5.7 E- 9	5.5 E- 9	5.6 E- 9	6.8 E- 9
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PLASMA - FIELD COIL INDUCTANCES

	1	2	3	4	5	6	7
1	7.5 E- 6	1.0 E- 5	1.4 E- 5	1.6 E- 5	1.8 E- 5	8.4 E- 6	4.6 E- 6
2	7.5 E- 6	1.0 E- 5	1.4 E- 5	1.6 E- 5	1.8 E- 5	8.4 E- 6	4.6 E- 6
3	7.5 E- 6	1.0 E- 5	1.4 E- 5	1.6 E- 5	1.8 E- 5	8.4 E- 6	4.6 E- 6
4	7.5 E- 6	1.0 E- 5	1.4 E- 5	1.6 E- 5	1.8 E- 5	8.4 E- 6	4.6 E- 6
5	7.5 E- 6	1.0 E- 5	1.4 E- 5	1.6 E- 5	1.8 E- 5	8.4 E- 6	4.6 E- 6
6	7.5 E- 6	1.0 E- 5	1.4 E- 5	1.6 E- 5	1.8 E- 5	8.4 E- 6	4.6 E- 6

PLASMA B

TIME = 19

MATRIX A

1.1 E- 7	1.8 E- 8	1.8 E- 8	1.8 E- 8	1.8 E- 8	1.8 E- 8	1.8 E- 8
6.5 E- 5	2.9 E- 5	1.9 E- 5	1.6 E- 5	1.8 E- 5	1.3 E- 5	1.3 E- 5
6.5 E- 5	1.9 E- 5	2.3 E- 5	1.6 E- 5	1.4 E- 5	1.3 E- 5	1.3 E- 5
6.5 E- 5	1.6 E- 5	1.6 E- 5	2.0 E- 5	1.4 E- 5	1.3 E- 5	1.3 E- 5
6.5 E- 5	1.4 E- 5	1.4 E- 5	1.4 E- 5	1.4 E- 5	1.3 E- 5	1.3 E- 5
6.5 E- 5	1.3 E- 5	1.3 E- 5	1.3 E- 5	1.9 E- 5	1.8 E- 5	1.3 E- 5
6.5 E- 5	1.3 E- 5	1.3 E- 5	1.3 E- 5	1.3 E- 5	1.3 E- 5	1.7 E- 5

VECTOR B

9.8 E-4	2.2 E 0	1.9 E 0	1.6 E 0	1.1 E 0	7.1 E- 1	3.7 E- 1
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VECTOR X

-6.2 E 3	4.9 E 4	6.6 E 4	6.0 E 4	1.9 E 4	-2.6 E 4	-7.8 E 4
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TABLE 3

PLASMA C

TIME = 3.0 E- 1

KIRCHHOFF VOLTAGE EQUATION TERMS

	1	2	3	4	5	6	7	8
1	1.6 E 1	1.9 E 2	4.5 E 2	-6.3 E 2	-4.1 E- 1	-5.0 E- 1	1.0 E 0	4.3 E- 2
2	4.6 E 1	1.9 E 2	4.5 E 2	-6.0 E 2	-4.2 E- 1	-5.0 E- 1	9.7 E- 1	6.8 E- 2
3	8.8 E 1	1.9 E 2	4.5 E 2	-5.6 E 2	-3.9 E- 1	-5.0 E- 1	8.6 E- 1	1.1 E- 1
4	1.4 E 2	1.9 E 2	4.5 E 2	-5.1 E 2	-3.5 E- 1	-5.0 E- 1	7.0 E- 1	1.7 E- 1
5	2.1 E 2	1.9 E 2	4.5 E 2	-4.4 E 2	-3.0 E- 1	-5.0 E- 1	5.4 E- 1	2.3 E- 1
6	2.7 E 2	1.9 E 2	4.5 E 2	-3.8 E 2	-2.5 E- 1	-5.0 E- 1	4.0 E- 1	3.0 E- 1

ION PARTICLE CONTINUITY EQUATION TERMS

	1	2	3
1	0.0	0.0	-1.9 E 10
2	0.0	0.0	-5.7 E 9
3	0.0	0.0	-1.9 E 9
4	0.0	0.0	-5.3 E 8
5	0.0	0.0	-1.4 E 8
6	0.0	0.0	-3.0 E 7

ALPHA PARTICLE CONTINUITY EQUATION TERMS

	4	5
1	0.0	0.0
2	0.0	0.0
3	0.0	0.0
4	0.0	0.0
5	0.0	0.0
6	0.0	0.0

PLASMA C

TIME = 3.0 E- 1

ION ENERGY CONTINUITY EQUATION TERMS

	6	7	8	9	10	17
1	0.0	0.0	0.0	2.4 E 17	-4.3 E 8	9.3 E 14
2	0.0	0.0	0.0	2.6 E 17	-1.3 E 8	9.7 E 14
3	0.0	0.0	0.0	2.3 E 17	-4.4 E 7	9.4 E 14
4	0.0	0.0	0.0	1.9 E 17	-1.1 E 7	8.6 E 14
5	0.0	0.0	0.0	1.5 E 17	-2.6 E 6	7.5 E 14
6	0.0	0.0	0.0	1.0 E 17	-4.2 E 5	5.7 E 14

ELECTRON ENERGY CONTINUITY EQUATION TERMS

	11	12	13	14	15	16	18
1	0.0	-2.4 E 17	-9.2 E 8	8.7 E 17	-4.9 E 14	-9.0 E 12	2.0 E 15
2	0.0	-2.6 E 17	-4.7 E 8	2.1 E 18	-6.4 E 14	-4.1 E 13	3.7 E 15
3	0.0	-2.3 E 17	-2.9 E 8	4.3 E 18	-8.5 E 14	-2.0 E 14	6.7 E 15
4	0.0	-1.9 E 17	-1.4 E 8	8.1 E 18	-1.1 E 15	-9.8 E 14	1.2 E 16
5	0.0	-1.5 E 17	-6.6 E 7	1.5 E 19	-1.5 E 15	-4.7 E 15	2.1 E 16
6	0.0	-1.0 E 17	-3.0 E 7	3.3 E 19	-2.2 E 15	-4.1 E 16	4.7 E 16

PLASMA C

TIME = 3.0 E- 1

MINOR RADII

1.3 1.8 2.1 2.3 2.4 2.5

RESISTANCES

2.4 E- 6 1.2 E- 6 6.1 E- 7 3.4 E- 7 2.0 E- 7 1.3 E- 7

PLASMA - FIELD COIL INDUCTANCES

	1	2	3	4	5	6	7
1	7.2 E- 6	1.0 E- 5	1.5 E- 5	1.9 E- 5	2.1 E- 5	9.1 E- 6	4.4 E- 6
2	7.2 E- 6	1.0 E- 5	1.5 E- 5	1.9 E- 5	2.1 E- 5	9.1 E- 6	4.4 E- 6
3	7.2 E- 6	1.0 E- 5	1.5 E- 5	1.9 E- 5	2.1 E- 5	9.1 E- 6	4.4 E- 6
4	7.2 E- 6	1.0 E- 5	1.5 E- 5	1.9 E- 5	2.1 E- 5	9.1 E- 6	4.4 E- 6
5	7.2 E- 6	1.0 E- 5	1.5 E- 5	1.9 E- 5	2.1 E- 5	9.1 E- 6	4.4 E- 6
6	7.2 E- 6	1.0 E- 5	1.5 E- 5	1.9 E- 5	2.1 E- 5	9.1 E- 6	4.4 E- 6

PLASMA C

TIME = 3.0 E- 1

MATRIX A

9.7 E- 8	2.1 E- 8	2.1 E- 8	2.1 E- 8	2.1 E- 8	2.1 E- 8	2.1 E- 8
7.0 E- 5	3.6 E- 5	2.7 E- 5	2.3 E- 5	2.1 E- 5	2.0 E- 5	2.0 E- 5
7.0 E- 5	2.7 E- 5	2.9 E- 5	2.3 E- 5	2.1 E- 5	2.0 E- 5	2.0 E- 5
7.0 E- 5	2.3 E- 5	2.3 E- 5	2.7 E- 5	2.1 E- 5	2.0 E- 5	2.0 E- 5
7.0 E- 5	2.1 E- 5	2.1 E- 5	2.1 E- 5	2.5 E- 5	2.0 E- 5	2.0 E- 5
7.0 E- 5	2.0 E- 5	2.0 E- 5	2.0 E- 5	2.0 E- 5	2.4 E- 5	2.0 E- 5
7.0 E- 5	2.0 E- 5	2.0 E- 5	2.0 E- 5	2.0 E- 5	2.0 E- 5	2.4 E- 5

VECTOR B

2.6 E- 4	2.1 E 2	2.1 E 2	2.1 E 2	2.1 E 2	2.1 E 2	2.1 E 2
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VECTOR X

-7.2 E 6	5.0 E 5	1.7 E 6	3.6 E 6	6.3 E 6	9.5 E 6	1.2 E 7
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TABLE 3

PLASMA C

TIME = 4.7

KIRCHHOFF VOLTAGE EQUATION TERMS

	1	2	3	4	5	6	7	8
1	3.0 E 0	1.4 E 0	5.0 E 0	-8.8 E 0	-5.2 E- 2	-4.7 E 0	1.0 E 1	2.4 E- 1
2	4.5 E 0	1.4 E 0	5.0 E 0	-7.0 E 0	-5.0 E- 2	-4.7 E 0	9.2 E 0	5.8 E- 1
3	4.5 E 0	1.4 E 0	5.0 E 0	-6.3 E 0	-5.8 E- 2	-4.7 E 0	8.1 E 0	1.0 E 0
4	2.5 E 0	1.4 E 0	5.0 E 0	-7.3 E 0	-6.7 E- 2	-4.7 E 0	6.6 E 0	1.6 E 0
5	4.3 E- 1	1.4 E 0	5.0 E 0	-8.4 E 0	-8.0 E- 2	-4.7 E 0	5.1 E 0	2.2 E 0
6	-2.1 E 0	1.4 E 0	5.0 E 0	-1.0 E 1	-1.0 E- 1	-4.7 E 0	3.7 E 0	2.9 E 0

ION PARTICLE CONTINUITY EQUATION TERMS

	1	2	3
1	5.0 E 16	0.0	-1.3 E 16
2	1.0 E 17	-4.4 E 13	-9.9 E 15
3	2.0 E 17	-1.8 E 14	-1.6 E 16
4	3.0 E 17	-4.1 E 14	-1.3 E 16
5	4.0 E 17	-7.3 E 14	-9.3 E 15
6	5.0 E 17	-1.1 E 15	-6.2 E 15

ALPHA PARTICLE CONTINUITY EQUATION TERMS

	4	5
1	0.0	0.0
2	2.2 E 13	0.0
3	9.2 E 13	-1.3 E 12
4	2.1 E 14	-2.0 E 12
5	3.7 E 14	-2.0 E 12
6	5.7 E 14	-1.8 E 12

PLASMA C

TIME = 4.7

ION ENERGY CONTINUITY EQUATION TERMS

	6	7	8	9	10	17
1	1.5 E 18	0.0	0.0	-1.1 E 16	-1.3 E 16	3.3 E 16
2	3.0 E 18	3.6 E 14	-6.6 E 14	-8.3 E 16	-1.5 E 16	6.7 E 16
3	6.0 E 18	4.0 E 15	-3.8 E 15	1.9 E 17	-3.3 E 16	1.3 E 17
4	9.0 E 18	2.9 E 16	-9.6 E 15	3.6 E 17	-3.1 E 16	2.0 E 17
5	1.2 E 19	1.6 E 17	-1.8 E 16	5.5 E 17	-2.3 E 16	2.7 E 17
6	1.5 E 19	7.6 E 17	-3.0 E 16	7.2 E 17	-1.6 E 16	3.4 E 17

ELECTRON ENERGY CONTINUITY EQUATION TERMS

	11	12	13	14	15	16	18
1	0.0	1.1 E 16	-1.3 E 16	1.1 E 18	-4.7 E 15	-4.7 E 16	3.2 E 16
2	4.2 E 15	-8.3 E 16	-2.3 E 16	4.0 E 18	-1.3 E 16	-6.1 E 17	1.0 E 17
3	2.2 E 16	-1.9 E 17	-6.1 E 16	1.1 E 19	-3.5 E 16	-3.0 E 18	2.4 E 17
4	8.1 E 16	-3.6 E 17	-7.7 E 16	2.8 E 19	-7.5 E 16	-1.1 E 19	4.8 E 17
5	2.3 E 17	-5.5 E 17	-7.7 E 16	6.8 E 19	-1.4 E 17	-3.8 E 19	8.9 E 17
6	3.9 E 17	-7.2 E 17	-8.4 E 16	2.4 E 20	-2.6 E 17	-1.8 E 20	1.7 E 18

PLASMA C

TIME = 4.7

MINOR RADII

1.3	1.8	2.1	2.3	2.4	2.5
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RESISTANCES

3.2 E- 8	1.1 E- 8	6.5 E- 9	4.6 E- 9	3.7 E- 9	3.8 E- 9
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PLASMA - FIELD COIL INDUCTANCES

	1	2	3	4	5	6	7
1	7.3 E- 6	1.0 E- 5	1.5 E- 5	1.8 E- 5	2.0 E- 5	8.9 E- 6	4.5 E- 6
2	7.3 E- 6	1.0 E- 5	1.5 E- 5	1.8 E- 5	2.0 E- 5	8.9 E- 6	4.5 E- 6
3	7.3 E- 6	1.0 E- 5	1.5 E- 5	1.8 E- 5	2.0 E- 5	8.9 E- 6	4.5 E- 6
4	7.3 E- 6	1.0 E- 5	1.5 E- 5	1.8 E- 5	2.0 E- 5	8.9 E- 6	4.5 E- 6
5	7.3 E- 6	1.0 E- 5	1.5 E- 5	1.8 E- 5	2.0 E- 5	8.9 E- 6	4.5 E- 6
6	7.3 E- 6	1.0 E- 5	1.5 E- 5	1.8 E- 5	2.0 E- 5	8.9 E- 6	4.5 E- 6

PLASMA C

TIME = 4.7

MATRIX A

1.0 E- 7	2.1 E- 8	2.1 E- 8	2.1 E- 8	2.1 E- 8	2.1 E- 8	2.1 E- 8
6.8 E- 5	3.5 E- 5	2.6 E- 5	2.3 E- 5	2.1 E- 5	2.0 E- 5	1.9 E- 5
6.8 E- 5	2.6 E- 5	2.8 E- 5	2.3 E- 5	2.1 E- 5	2.0 E- 5	1.9 E- 5
6.8 E- 5	2.3 E- 5	2.3 E- 5	2.6 E- 5	2.1 E- 5	2.0 E- 5	1.9 E- 5
6.8 E- 5	2.1 E- 5	2.1 E- 5	2.1 E- 5	2.4 E- 5	2.0 E- 5	1.9 E- 5
6.8 E- 5	2.0 E- 5	2.0 E- 5	2.0 E- 5	2.0 E- 5	2.3 E- 5	1.9 E- 5
6.8 E- 5	1.9 E- 5	1.9 E- 5	1.9 E- 5	1.9 E- 5	1.9 E- 5	2.3 E- 5

VECTOR B

2.0 E- 3	6.5 E 0	6.1 E 0	5.5 E 0	4.6 E 0	3.7 E 0	2.9 E 0
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VECTOR X

-6.7 E 4	8.4 E 4	1.5 E 5	1.7 E 5	9.8 E 4	1.5 E 4	-9.7 E 4
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LIST OF FIGURES

1. Poloidal cross section of the model plasma
2. Reactor poloidal cross section
- 3(A,B,C) Coils and plasma currents versus time for
respectively plasmas A, B, and C.
- 4(A,B,C) Density (number density) versus time for plasmas
A, B, C.
- 5(A,B,C) Temperature versus time for plasmas A, B, and C.
- 6(A,B,C) Magnetic properties for plasmas A, B, C.

Figure 1

Figure 2

(from MATT 1050, Princeton)

Figures:

3A

3B

3C

Figures:

3A

3B

3C

Figures:

3A

3B

3C

Figures:

4A

4B

4C

Figures:

4A

4B

4C

Figures:

4A

4B

4C

Figures:

5A

5B

5C

Figures:

5a

5b

5c

Figures:

6A

6B

6C