RESEARCH IN THEORETICAL PHYSICS

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Abstract Gauge theories on coset spaces and dynamical mechanisms for generating internal symmetries have been investigated. Gauge theories on cosets with respect to maximal subgroups of local internal symmetry groups contain reduced number of dynamically independent gauge fields. The transformation law of the gauge fields is non-linear. Such theories can be interpreted in terms of a spontaneously broken internal symmetry. Internal symmetries may be generated in multidimensional geometric theory with vacuum solutions of the cylindrical type. Spinor fields on such spaces support intrinsically broken, approximate internal symmetries.

1. Introduction

This Report covers research performed during the contract period between September 1, 1976 and March 31, 1977.

The major problem left unresolved in the investigations reported previously^{1,2)} has been the local structure of the algebra of observables satisfying the triality rule as proposed by Gürsey and his collaborators³⁾. This problem and our attempts made at its resolution constitute the main body of the present report.

In particular, the basic physical aspects of the problem are outlined in Sec. 2. The subsequent Sections (Secs 3, 4) are devoted to a brief description of the progress made towards the resolution of these problems. The last part of this Report (Sec. 5) summarizes the results obtained so far and gives an outlook towards further research to be performed along these lines.

2. Problems of the algebra of local observables

Gürsey and his collaborators³⁾ proposed that the two fundamental principles governing particle physics should be:

- a) A symmetric treatment of all "fundamental constituents of matter".
- b) An algebraic (i.e. basically kinematic) treatment of quark selection rules, otherwise known as color selection rules.

In particular, assumptions a) and b) imply that leptons (possibly including "heavy leptons") are to be treated on the same footing as quarks are. While one may be justified in questioning the "fundamental" nature of these assumptions, there is little doubt concerning the phenomenological usefulness of the latter.

Further, the symmetric treatment of quarks and leptons as fundamental constituents of matter provides a natural framework for a GIM mechanism to operate, which is known to be useful from the point of view of weak interaction phenomenology. It was also suggested by Gürsey, in particular that an algebraic treatment of the color selection rules may provide a natural explanation of the unobservability of quarks⁴⁾.

Soon thereafter, we pointed out that the algebraic framework proposed by Gürsey and his collaborators cannot possibly explain the unobservability of quarks unless it is implemented locally¹⁾.

We gave two, admittedly oversimplified, models for the "algebraic confinement" of quarks.

The first model (ref. 1) is based on a minimal, local generalization of the algebras proposed by Gürsey et. al. Quark confinement can indeed be achieved together with maintaining a quark-lepton symmetry.

However, the model contains some undesirable features; specifically,

there exist no separate conservation laws for baryon and lepton numbers. As a consequence, the stability of the proton can be achieved through a symmetry breaking mechanism, which, however, is somewhat artificial. (More elaborate unified theories based on any of the exceptional Lie groups face the same problem.)

The second model considered²⁾ has been based on a superspace approach and it means a more radical departure from the formalism - although not from the spirit - of Gürsey's approach. We succeeded in constructing a supergauge-algebra which respects all the necessary selection rules. Hence, it proved to be a good candidate for a supergauge algebra describing known (and some predicted) states in terms of quarks and leptons as fundamental building blocks of matter. There were two fundamental difficulties with the "superspace" approach as described in ref. 2.

--First, the not-associative nature of the algebra (a necessary consequence of the color selection rules, as we have repeatedly emphasized^{1,2)}) made it difficult to define the concept of parallel translation on the superspace. As a consequence, we have been unable to construct physically sensible Lagrangian models - containing derivatives - in ref. 2 for fields on the colored superspace. Given the fact that the exponential of a Lagrangian also serves as a local weight factor in a Feynman functional integral, the difficulty in constructing appropriate "classical" Lagrangians also means that one is unable to construct a physically meaningful quantum theory based on the colored superspace.

--Second, it became clear that the role of internal symmetries remained somewhat obscure in the construction described in ref. 2, for at least two reasons.

- i) From a physical point of view, a unified description of leptons and colored quarks seems to require the introduction of very large internal symmetry

groups. At present the exceptional group $E(7)$ appears to be a good candidate, according to the analysis of Gürsey and Sikivie⁵⁾. However, the fundamental representation of $E(7)$ has dimension = 56, which, according to the decomposition into $SU(6) \times SU(3)_{\text{color}}$:

$$(56) = (20, 1) + (6, 3) + (\bar{6}, \bar{3}) \quad (2.1)$$

contains 20 leptons and 6 quarks (together with 6 antiquarks). The adjoint representation has a dimension of the order of 10^2 . Thus, if any of these internal symmetry groups is considered a local symmetry, one has to introduce as many "fundamental" gauge fields. To be specific, $\dim \text{Ad } E(7) = 133$, with the $SU(6) \times SU(3)_{\text{color}}$ decomposition:

$$(133) = (35, 1) + (1, 8) + (15, \bar{3}) + (\bar{15}, 3) \quad (2.2)$$

This implies the existence of 35 color singlet vector bosons alone; in principle, all of them should be observable! In addition, the color triplet bosons tend to destabilize the proton¹⁾, hence, they must be "superheavy" ($m \sim m_{\text{Planck}}$) in order to make the life-time of the proton comparable with the age of the Universe. (The color octet boson is relatively "harmless": it is unobservable if color is "confined" and, presumably, it mediates strong interactions; hence, it is "light".)

This is a very unsatisfactory situation. One needs $133 + 56 = 189$ "elementary building blocks" of matter. Even if color is "confined", the $20 + 35 = 55$ color singlet fields should give rise to particles, which, in principle, are observable.

At this point, we do not wish to enter a detailed discussion of the problems of color confinement or of the symmetry breaking mechanisms which could make some gauge fields "superheavy", others "moderately heavy"-

$m \sim 40$ GeV - and still others "light". Nevertheless, it may be worth pointing out that relatively simple mechanisms can be devised which account for the superheavy vector bosons⁶⁾. However, apparently, no one has succeeded in finding even a moderately convincing argument to explain mass ratios which are of the order of $10^{20} : 10 : 1 : 0$)

ii) From a mathematical point of view, the use of the exceptional Lie groups for the description of quarks and leptons seems to be somewhat less compelling than it was believed previously. Indeed, in ref. 2 we gave a general constructive procedure by means of which one can arrive at "superspaces" satisfying the well-known color selection rules. Although the details of that constructive procedure have not been fully explored yet, it seems to us that superspaces may well exist which imply the existence of fewer local observables than hitherto suspected.

The research performed during the current contract period has been entirely devoted to a search for a solution to the two main problems outlined above. Our main results can be summarized as follows.

1) We achieved a better understanding of the basic space-time structure of local observables. In particular, we found that theories with local symmetries (gauge-type theories) can be constructed in such a way that the number of dynamically independent gauge fields is smaller than the dimension of the adjoint representation of the local symmetry group.

2) We have tentative results indicating that models can be constructed with the following properties.

2a) The division of the exact symmetries into "space-time" and "internal" symmetries arises as a dynamical phenomenon, as a consequence of a spontaneous symmetry breaking mechanism.

2b) The fundamental spinor fields entering the theory support the representations of additional, "internal" symmetries (typically, groups of the series $SU(n)$). There exist asymptotically conserved currents corresponding to these groups.

2c) The order, n , of the "ghost symmetry" groups depends on the energy region under consideration; generally, n is an increasing function of the highest available energy.

2d) None of the "symmetries" $SU(n)$ is exact. Indeed, none of these groups is a true symmetry at all. (Hence the name, "ghost symmetry".) However, the groups $SU(n)$ - or possibly another series of groups - serve to classify the fundamental fields contained in the theory. The symmetry breaking terms (all of them are "soft", typically quadratic in the spinor fields) are exactly determined by the geometrical structure of the theory.

The following two Sections contain a more detailed description of these results. For the sake of clarity, the main emphasis is on the underlying physical ideas, while formal arguments are largely suppressed or merely sketched.

3. Gauge theories on coset spaces

This problem arose in connection with certain models constructed for the purpose of understanding a possible dynamical origin of internal symmetries. Models of this type are discussed in Sec. 4 of this Report. It was found, however, that the problem of constructing "gauge"-theories (i.e. theories possessing local symmetries) with a "small" number of gauge fields is of considerable physical interest, independently of the context in which this problem arose. It was decided, therefore, to publish the results of this investigation in a general form⁷⁾, without reference to specific models.

The structure of gauge theories has been the subject of investigations ever since the fruitfulness of the idea of constructing theoretical models with local symmetries has been recognized. Remarkably, the underlying geometrical structure of such theories has not been understood until the mid-sixties, even though H. Weyl's original investigations were clearly motivated by geometrical considerations. The full clarification of the geometrical structure of gauge theories as one understands them at present, is due to the works of DeWitt, Trautman, Kerner and more recently of Cho and Freund, Chang, Macrae and Mansouri⁸⁾.

In the light of these works, the geometry of gauge theories appears to be quite a simple one. The easiest way of describing that structure is to give a set of constructive prescriptions. (For operational purposes, the set of prescriptions may be looked at as a "machine" with a primitive geometry serving as an input, the output being a consistent gauge theory.)

The constructive prescriptions are the following:

- 1) Assume the existence of a model of space-time, M . (Technically, M should be at least a differentiable manifold.)
- 2) Erect at each $p \in M$ a space, S , which supports a realization of a local symmetry group, G . (From the physical point of view, G is a local internal symmetry, hence G may be assumed to be a simple Lie group.)
- 3) Locally, therefore, the total space, B , has a direct product structure, $B \sim U \times S$, with $p \in U \subset M$. In this sense, M is called the base manifold. (Technically, it is a locally trivial fibre bundle).

One induces a connection on B as follows. One considers a curve in B ; one may imagine that the curve is given in terms of its local coordinates, say, $x^\mu(t)$ and $s^a(t)$, where x^μ and s^a are coordinates of a point in M and S , respectively. The geometrical significance of a connection (in the

sense of E. Cartan) is that it determines a curve in B entirely in terms of its projection onto M. This means that we must have

$$\frac{ds^a}{dt} = C_{\mu}^a \frac{dx^{\mu}}{dt} ;$$

C_{μ}^a is called (up to a factor) a connection coefficient. (The dependence of C_{μ}^a on s^a is restricted by certain invariance requirements.)

4) Finally, one gives B a metric structure and uses the usual machinery of Riemann geometry to compute the relevant geometrical quantities (curvature tensor, etc.).

The remarkable insight gained in the works quoted in ref. 8 is that this geometrical language gives a complete characterization of gauge theories, with the correspondence rules:

$S \equiv G$ (self-realization of the symmetry)

Connection coefficient $C_{\mu}^a \simeq$ gauge potential

Ricci scalar of B = Einstein-Yang-Mills Lagrangian density

Geometrically, this structure appears to be quite inflexible and it seems to offer no escape from the usual conclusion, namely, that one needs as many gauge fields as the dimension of the adjoint representation of G.

Remarkably enough, such as "escape" nevertheless does exist⁷⁾.

We can, in particular, choose S to be a coset of G, instead of G itself. The dimension of the coset is lower than the dimension of the Lie algebra of G, therefore, one expects to need fewer gauge fields than in the case of a self-realization of G.

This turns out to be the case. At least in a classical theory, the number of dynamically independent gauge fields is equal to the dimension of the coset. The rest of the construction (in particular, step 4) goes through as it does in standard gauge theories.

There is a price to be paid, however, for diminishing the number of independent gauge fields: the transformation laws of the gauge potential (and of the field tensors) becomes nonlinear. We have explicitly analyzed the transformation laws of the gauge fields in the case of a large and physically important class of coset spaces. (The coset spaces considered - classical, off-diagonal cosets with respect to a maximal subgroup - comprise all the spaces considered in gauge models with a broken internal symmetry, with the exception of Gürsey's works quoted earlier.) We found that the nonlinear transformation law of the gauge fields can be understood physically, if it is interpreted in terms of a "spontaneously" broken symmetry.

We recall in this respect the transformation law of fields in a "usual" theory with spontaneously broken symmetry, e.g. a Higgs model with U(1) symmetry. One has the Lagrangian:

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu + ieA_\mu)\varphi^*\varphi(\partial_\mu - ieA_\mu)\varphi + V(\varphi^*\varphi), -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

where $V(\varphi^*\varphi)$ stands for the non-derivative part of the Lagrangian. Upon making the polar decomposition, $\varphi = \chi \exp i\theta$ one gets

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu + ieB_\mu)\chi(\partial^\mu - ieB^\mu)\chi + V(\chi^2) - \frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}$$

where $B_\mu = A_\mu - \partial_\mu\theta$, $\tilde{F}_{\mu\nu} = B_{[\mu,\nu]}$. One notices that B_μ is now invariant under gauge transformations, due to the fact that a longitudinal field $(\partial_\mu\theta)$ has been absorbed into the original gauge potential. This phenomenon (the well-known Higgs phenomenon, in fact) is easily generalized to more complicated gauge groups. The polar decomposition has to be replaced by an appropriate Cartan decomposition of the gauge group and one finds that the "Higgs-transformed" gauge fields (generalizations of B_μ above) transform trivially under the maximal subgroup $K \subset G$, whereas they transform nonlinearly under

the coset G/K .

A similar phenomenon occurs in the models investigated in ref. 7. One finds that the dynamically independent gauge fields transform trivially under K , while their transformation law is nonlinear under G/K . The basic difference between conventional Higgs-type models and the ones considered here is that in the latter, certain gauge-fields can be eliminated, whereas Higgs-type models allow for the elimination of spinless fields (the pseudo-Goldstone bosons) only. It seems therefore that models based on associated bundles, as described in ref. 7, allow for the existence of theories possessing local symmetries but of a relatively small number ($\dim G/K = \dim G - \dim K$) of independent gauge fields.

There is no serious problem in extending the above considerations to coset spaces of other types than the ones considered in ref. 7. (In particular, coset spaces with respect to maximal subgroups of exceptional groups could be handled as well as classical Cartan cosets were in our work.) The problem is purely a technical (although a non-trivial) one: one needs to construct an appropriate parametrization of the respective cosets in order to be able to carry out the calculations explicitly.

However, the quantization of such gauge theories raises more serious problems. Indeed, since the transformation law of the dynamically independent gauge fields is both nonlinear and non-local, standard quantization methods are inapplicable. One way out would be to use redundant fields, thereby linearizing the transformation laws. In that case, quantization can be carried out in a standard fashion, although at the cost of losing some of the elegance and compactness of the classical theory. This problem is presently under investigation.

4. Dynamical origin of internal symmetries

Detailed calculations regarding this subject are still in progress. Hence, in this Section, we restrict ourselves to the presentation of the main ideas and preliminary results obtained so far. It is expected that the first report on this subject will be completed before 3.31.77. The purpose of the investigation is to examine the question whether approximate internal symmetries and the associated symmetry breaking mechanism can be generated dynamically. Preliminary investigations of similar questions go as far back as Kaluza and Klein; they have been reexamined recently in connection with dual models⁹⁾ and supergauge theories¹⁰⁾. The model we examine is based on a "superspace" approach and in this respect it is similar to the attitude adopted by the authors of ref. 10.

We consider the possibility that physical space-time may contain more than $3 + 1$ dimensions; however, some of the spacelike dimensions are "spontaneously compactified". From a geometrical point of view this means that one considers an appropriate variational principle which is a functional of the components of the metric and asks whether there exist classical (vacuum) solutions with a "cylindrical" topology. The best-known example of such spaces is an Einstein static universe in $(3 + 1)$ dimensions, with a metric

$$- ds^2 = dt^2 - \left(\frac{dr^2}{1-r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right),$$

having the topology $R \times S^3$. Models in more than four dimensions can be easily constructed which yield topologies of the type $M^4 \times S^k$, M^4 being Minkowski space and S^k standing for a k -dimensional sphere. None of the models found so far is an entirely satisfactory one; however, for the moment one may proceed by assuming that a model has been found which generates a vacuum metric with

a cylindrical topology. In order to fix the ideas, we assume that k is small, typically $k = 2, 3$. (The value $k = 1$ turns out to be rather uninteresting from the physical point of view.)

Next, one examines fluctuations around the vacuum metric. To this end, one introduces the (multidimensional analogs of) tetrad fields as dynamical variables.

We may write for the general metric in matrix form:

$$G = V^T \eta V,$$

where η stands for $\text{diag} (-1, +1, \dots, +1)$.

In general,

$$V = O T$$

where O is pseudoorthogonal, $O^T \eta O = \eta$, and T is triangular. The assumed cylindrical structure of the vacuum metric, $G^{(0)}$, means that the corresponding vacuum tetrad field ($T^{(0)}$) is actually block-diagonal:

$$T^{(0)} = \begin{matrix} & 3+1 & & \\ & & p^{(0)} & 0 \\ & & \vdots & \vdots \\ & k & 0 & q^{(0)} \end{matrix} \quad ,$$

where $p^{(0)}$ (the tetrad corresponding to $3 + 1$ dimensional flat space-time) can be chosen to be the unit matrix.

Fluctuations around $T^{(0)}$ can be written again in a triangular form, viz.

$$T = T^{(0)} T^{(1)},$$

where $T^{(1)}$ is a general triangular matrix; hence, it represents the most general fluctuation term around the "vacuum-tetrad", $T^{(0)}$. The diagonal

elements of $T^{(1)}$ describe fluctuations of the metric around its vacuum expectation value, while the off-diagonal elements may be interpreted as gauge fields. (Depending on the nature of the subspace characterized by the tetrad matrix $q^{(0)}$, the off-diagonal elements of $T^{(1)}$ are either ordinary gauge fields, or gauge fields on a coset, see previous Section.)

From the physical point of view, the crucial question is the existence and representation of spinor fields on the space with a geometrical structure described above. Obviously, the properties of these spinor fields (quarks, leptons), depend on the representations of the Clifford algebra on $M^4 \times S^k$. In any case, a spinor field on this space has $2^{2+k/2}$ components for $k = \text{even}$ and $2^{2+(k-1)/2}$ components for $k = \text{odd}$, while each component depends on $4 + k$ local coordinates. The spinor fields may then be expanded in terms of the orthogonal harmonics on the k -dimensional compact space (say S^k). In this way, one gets a set of spinor fields on M^4 .

This set of spinor fields can be shown to support the representations of various higher "internal symmetry" groups, for instance, groups of the type $SU(n)$. None of these "ghost symmetries" is exact, however. The symmetry is always broken by "mass-terms" in a way dictated by the underlying geometry, although there exist "asymptotically conserved currents". In a quark language, this means that the spinor fields on the higher dimensional space are equivalent to a (possibly infinite) set of quark fields in ordinary Minkowski space, with an internal symmetry. The latter is broken by the quark mass terms. According to the parity of k , the theory is either parity conserving ($k = \text{even}$) or parity violating ($k = \text{odd}$). In the latter case, parity is violated "softly", similarly to vector-like theories. (This follows from the representation theory of Clifford algebras.)

It is to be pointed out that this situation is quite similar to the one encountered in atomic physics. There, the "true" symmetry of atomic wave functions is just $SO(3) \times S_n$, where S_n stands for the permutation group of n electrons. Nevertheless, multielectron wave functions support the representations of various "ghost symmetries": groups which are never true symmetries of the system. However, they play an important role in the classification of states and often they may be considered as good approximate symmetries, see e. g. Judd¹¹⁾.

5. Summary and outlook

The research described in this Report has been motivated by both theoretical and phenomenological considerations.

i) From the phenomenological point of view, recent experimental results on inelastic ν -scattering and electron-pair annihilation seem to require the existence of an alarmingly high number of "fundamental constituents of matter".

ii) Our previous investigations (refs. 1, 2) seem to have loosened the seemingly very tight connection between "color" and "internal" degrees of freedom.

We have put forward the suggestion that what is presently considered the "internal symmetry" of hadrons and leptons, may, in fact, have a closer connection with space-time geometry than hitherto suspected. The model outlined in Sec. 4 would explain the existence of the seemingly high number of fundamental constituents in very simple geometrical terms. Such a theory has a relatively high predictive power: for instance, the number of quarks accessible in any given energy range can be relatively easily calculated,

hence the Richter ratio, inelastic lepton cross sections, etc. can be simply predicted and confronted with data in the near future.

Clearly, the theory as outlined in the preceding two Sections contains a drastic simplification: color degrees of freedom are either neglected or put in by hand.

However, our experience with colored supersymmetry algebras²⁾ and recent results obtained by MacDowell and Mansouri concerning geometrical theories based on Z_2 -graded algebras encourage one to believe that an algebraic realization of color selection rules together with a dynamical description of internal symmetries will become possible in the not-so-distant future. Models incorporating such properties may account for a substantial number of presently available experimental data.

6. Scientific Personnell

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Footnotes and References

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