MASTER

EXTRINSIC FACTORS AFFECTING
ACCURACY OF ULTRASONIC FLOWMETERS
FOR LMFBRs

by

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NOMENCLATURE

A	area
c	speed of sound in a medium
CRBR	Clinch River Breeder Reactor
d,D	diameter
g	acceleration due to gravity
i	angle of incidence measured from the normal to the interface
k	correction factor, $\bar{u}_A / \bar{u}_{DIA}$
K	thermal conductivity (BTU/hr ft °F)
ln	natural logarithm
l,L	length, feet
LEFM	Leading Edge Flow Measurement
LMFBR	Liquid Metal Fast Breeder Reactor
n	a constant
p	pressure
PF	pipe factor, $\bar{u}_A / U$
PLBR	Prototypic Large Breeder Reactor
Q	average volumetric flow rate, ft ³ /sec
r	radius, feet
r _o ,R	inside diameter of a pipe, feet
Re	Reynolds' number ($\bar{u}D/\nu$)
t	time, seconds
T	local temperature
ΔT	difference between two local temperatures
u(r)	local velocity as a function of pipe radius, feet/sec
$\bar{u}_{DIA}$	velocity as averaged by an ultrasonic flowmeter path
$\bar{u}_A$	mean velocity averaged over the pipe area (Q/A)
U	maximum local velocity, usually at pipe center
u*	$\sqrt{\tau_o/p}$; for smooth pipe, $u^* = \bar{u}_A \sqrt{\lambda/8}$; friction velocity
y	r _o -r, distance measured from pipe wall, feet
z	distance along pipe axis
ν	kinematic viscosity of fluid, ft ² /sec
ρ	density, lb/ft ³
τ _o	shear stress between pipe wall and fluid
η	dimensionless distance from wall ($y u^* / \nu$)
λ	resistance coefficient of pipe flow near the wall

ABSTRACT

Assuming that ultrasonic flowmeters of suitable intrinsic accuracy are feasible, this report explores factors extrinsic to the flowmeter which affect the accuracy such as asymmetric flow profile, regions of high turbulence and thermal stratification. By integrating isovelocity flow profile maps, the predicted performance of various flowmeter configurations may be compared to experimental data. For the two pipe arrangements analyzed, the single diametral path flowmeter results were within 5 percent of true flow rate. Theoretical correction factors could reduce the error for the straight pipe but increased the error for asymmetrical flow. On the same pipe arrangements a four path ultrasonic flowmeter spaced for Gaussian integration gave less than 1 percent error. For more general conclusions a range of flow profiles produced by typical LMFBR piping arrangements must be analyzed.

EXTRINSIC FACTORS AFFECTING ACCURACY OF ULTRASONIC FLOWMETERS FOR LMFBRs

1.0 INTRODUCTION

1.1 Statement of Need

LMFBRs need flowmeters for safety purposes to establish that sodium is flowing at an adequate rate and that the flow is not suddenly changed; for operational purposes such as balancing the flows in several loops of loop type LMFBRs; for thermodynamic measurements and for regulation of flow through the reactor. Accuracy requirements are at the 5% level for safety purposes and 1% or better for thermodynamic use. The redundancy factor for safety channels is four.

1.2 Purpose of Report

The purpose here is to show that ultrasonic flowmeters can meet the needs of future LMFBRs under certain precautions, and to show what these precautions are, to introduce the subject, discuss the historical background and operating principles of ultrasonic flowmeters, review LMFBR operating conditions and flowmeter requirements, assess errors from the extrinsic factors, to investigate whether the redundancy of four offers the possibility of error reduction and summarize the present state of the art in flowmeter accuracy. A second purpose is to point out the need to plan now for future flow calibration facilities.

1.3 Feasibility

The feasibility of ultrasonic flowmeters is established. Ultrasonic flowmeters are commercially available,¹ their applications have been reviewed,^{2,3,4} and are well established at moderate temperatures. Their reliable application to future LMFBRs requires development of suitable high temperature transducers and system flow calibration capabilities. The design of high temperature transducers for ultrasonic flowmeters by Karplus is the subject of ERDA patent application file #44342 and will not be described here.⁵ The need for accurate flow calibration will be considered along with the problems of translating experience from water to sodium flow calibration.

1.4 The Weigh Tank Method of Flow Calibration

The average linear flow $\bar{u}$ through a piping system of cross-sectional area A is most accurately measured by collecting the volumetric flow Q in a large tank for a carefully measured period of time (t) and carefully measuring the weight of the total quantity ($Qt\rho$) of the fluid in the tank. The division of the total weight by the total time (t), pipe area (A), and density (ρ) gives the average linear flow $\bar{u}$ through the piping system in terms of fundamental units of mass, length and time, $Qt\rho/tA\rho = \bar{u}_A$. Note that $\bar{u}_A$ is not the linear flow $\bar{u}_{DIA}$ directly measured by an ultrasonic flowmeter. The relation between $\bar{u}_A$ and $\bar{u}_{DIA}$ will be discussed. It is convenient to have a frequent or continuous measure and readout of $\bar{u}_A$ even if less accurate than the weigh tank, whether it be from a differential pressure device, a turbine flowmeter, an ultrasonic flowmeter, or one of the many other types of flowmeters. The fundamental weigh tank method is not applicable to many situations which require continuous operation, high pressure operation or other considerations as with sodium in LMFBRs. To meet these needs a variety of flowmeters have been developed, each with advantages, limitations and errors.

1.5 Extrinsic Factors Affecting Accuracy of Flow Measurement

With each type of flowmeter the precaution is usually given that it be placed in a long straight pipe many diameters downstream from any flow disturbance where "fully developed turbulent flow exists." This is not practical in LMFBRs since the piping system must be fitted into a containment vessel and contains elbows, pumps, heat exchangers, valves and hydraulic equipment which perturb the local flow near the point of flow measurement. It is expected that in LMFBRs the longest straight sections of pipe will be less than eight diameters between bends. In the absence of leaks, continuity demands that the average flow be the same throughout the piping system. The calibration factors obtained by means of the fundamental weigh tank method have shown that the flow as measured by an ultrasonic flowmeter is dependent on the location and orientation of the flowmeter's ultrasonic path in the piping system.^{6,7,8} Certain multiple parallel path configurations (namely, those for Gaussian integration) are found to be intrinsically more accurate than others; hence, strongly

preferred where accuracy justifies added cost. These sources of variability or error will be separated from the flowmeter itself and referred to as extrinsic factors affecting the accuracy. These problems are not unique to ultrasonic flowmeters, but shared by all types to varying degrees.

1.6 LMFBR Safety, Redundancy and Accuracy Requirements

In the discussion following, accuracy is intended to refer to measurements against an absolute standard such as afforded for flow measurement by comparing flowmeter readings over a period of time against the quantity collected in a weigh tank. Error in accuracy is the difference between the reading of the instrument being tested and the reading of the absolute standard. Precision is intended to refer to the difference in measurements made with the same instrument at different times, under similar conditions, or differences between measurements with similar instruments at the same time under similar conditions. An example of the latter would be the balancing of the flows in several "similar" loops of a PLBR using "similar" ultrasonic flowmeters. Each flowmeter might be characterized by an absolute error of several percent but these are likely to be "similar"; hence, the balancing may be performed with "precision" such that the difference between the flows is much less than the absolute error of the individual flowmeters.

The accuracy needs in future LMFBRs fall into two levels. The first is the 2-5% level which is considered satisfactory for Plant Protection System service and for balancing the flows in the several loops of a loop type LMFBR. The second accuracy level is equal to, or less than, 1% which will permit satisfactory calorimetric or thermodynamic measurements. The above-mentioned extrinsic factors will be shown to cause error in the range 0-5%. Thus, even though small, they may not be neglected in terms of LMFBR flowmeter accuracy requirements.

1.7 Comparison of LMFBR Flowmeter Features

One consideration for LMFBR flowmeters is that they not penetrate the pipe (see Fig. 1 and Table I). This rules out the traditional differential pressure devices based on orifices and restricts the application of the eddy

FIG. 1 ULTRASONIC FLOWMETER CONFIGURATION

current probe-type and magnetometer probe-type flowmeters. A similar consideration is that there should be no obstruction to the flow in the pipe. This again eliminates the differential pressure orifice type, the eddy current probe type, and the magnetometer probe type flowmeters. The remaining candidate flowmeters then are the permanent magnet type and the permanent magnet with transit-time correlation. The concept of pulsed neutron activation as a means of determining absolute flow is considered to fall in the calibration category with the weigh tank and will not be considered here as a permanently installed long term operational flowmeter. Whereas the present generation of sodium test loops and the EBR-II reactor deal with four inch to twelve inch diameter pipes, the projected sizes for CRBR and the Prototypic Large Breeder Reactor (PLBR) may range up to 35 inch and 47 inch inside diameters with flows up to 80,000 or 120,000 gallons per minute, depending on whether 3 or 4 loops are used. (See Table II.) For these the ultrasonic flowmeter appears to have definite advantages over the permanent magnet type: the increase of cost with pipe diameter is less; the precision of the ultrasonic flowmeter improves with larger pipe diameter whereas it becomes increasingly difficult to maintain the necessary magnetic field and accuracy for the permanent magnet type; the ultrasonic flowmeter is applicable to ferritic pipe which the permanent magnet type is not; the ultrasonic flowmeter offers long term stability and a measure of absolute calibration (limits on this absolute accuracy will be discussed below).

2.0 HISTORY OF ULTRASONIC FLOWMETERS

2.1 General

An interesting review of ultrasonic flowmeter development prior to 1957 is due to del Grosso.⁹ The earliest reference he found on the use of sound to measure fluid flow on a pipe dated back to Rutten's German patent issued in 1928.

Another excellent historical review has been provided by McShane.³ He cites early patents and points out that while awkward in their described form, indicating the inadequate state of the transducer and electronic art, they did bring out many of the basic approaches and practical problems. He makes the interesting observation that the Russians have done extensive theoretical and practical work on ultrasonic flowmeters and published the first book on this subject in 1967. Because of its generality and interest, this historical review with references is attached as Appendix A.

FEATURES TYPE OF FLOWMETER	NON-PENETRATING	ABSOLUTE CALIBRATION	LONG TERM STABILITY	HIGH TEMPERATURE CAPABILITY	NO FLOW OBSTRUCTION	PRECISION IMPROVES WITH PIPE DIAMETER	COST GROWS SLOWLY WITH PIPE DIAMETER	APPLICABLE TO FERRITIC PIPE	RETROFIT CAPABILITY	SIMPLE READ OUT
PERMANENT MAGNET	X	X		X	X			X	X	
PERMANENT MAGNET WITH TRANSIT TIME CORRELATION	X	X	X	X	X			X		
DIFFERENTIAL PRESSURE (ORIFICE)			X	X		X	X		X	
EDDY CURRENT PROBE				X		X	X			
ULTRASONIC	X	X	X	X	X	X	X	X		
MAGNETOMETER PROBE			X	X		X	X			

Comparison of LMFBR Flowmeter Features

TABLE I

2.2 Applications

In addition to the paper by McShane,³ a more recent review has been presented by L. C. Lynnworth.¹ He reviews applications of ultrasonic flowmeters and presents a list of manufacturers as of 1975. A more extensive survey of most of these manufacturers with details on major specifications and prices for commercial ultrasonic flowmeters is furnished by Liptak and Kaminski.² Their listing of the vendors of ultrasonic level and flow instruments is included as Appendix B. Selected manufacturers have been added to update the list.

An international conference held at Harwell, sponsored by the Atomic Energy Research Establishment at Harwell, and the National Engineering Laboratory at East Kilbride, September 21-23, 1971, covered many flow measurement topics including ultrasonic flowmeters.⁶ Of particular interest in this proceedings were a Japanese and an American paper on industrial experience with accurate flow measurement. The article by Suzuki, Okobori and Jamamoto entitled, "Ultrasonic Method of Flow Measurement in Large Conduits and Open Channels," describes experience with more than 400 installations of ultrasonic flowmeters in Japan.¹⁰ Modified sing around systems, consisting of one pair of probes and one electronic unit, operate with high stability in all climatic conditions and with low sensitivity to temperature changes. One of the features of the ultrasonic flowmeter is that it is capable of measuring flow rate by using probes mounted on the outside of the conduit wall. Thus, there is no pressure loss, it can be installed on existing pipes, and installation costs are low compared with conventional meters. The authors describe the principal and practical applications of the ultrasonic flowmeters for conduits and open channels such as waterworks and power plants. The authors find the accuracy against other flow calibration methods to be better than $\pm 5\%$ and frequently 1% . One interesting result from their early work was that the ultrasonic method was specified in the Japanese electrical code as a standard means for measuring turbine efficiency and flow measurement in major Japanese power stations. The total number of measurements carried out exceeded 1,000 at the time the paper was presented. The work reported in this paper utilized single and multiple diameter paths.

"Ultrasonics as a Standard for Volumetric Flow Measurement," by S. G. Fisher and P. G. Spink, describes the operation and testing of a multiple parallel

path ultrasonic flowmeter with accuracies better than 1% and with the potential to realize accuracies of 0.1%.⁶ This paper has both theoretical and experimental work of a high order to back up these statements on the accuracy obtained. Their method is based on an elegant numerical integration procedure devised by Gauss¹¹ and has been patented.⁸

Fisher and Spink work for the Oceanic Division, Westinghouse, which has offered the high accuracy ultrasonic flowmeter since about 1968. For accuracies better than 1%, this method (referred to as Gaussian integration) is definitely preferred over any other ultrasonic flowmeter configurations. (Fig. 2)

2.3 EBR-II Feasibility Test

Of interest from the standpoint of history and feasibility of applying ultrasonic flowmeters to sodium systems in LMFBRs, is a series of internal memoranda concerning the proposal and the successful results of test EX48-70, "Ultrasonic Flowmeter-Performance Evaluation," performed at EBR-II in 1970 with Idaho Nuclear Corporation (INC) and reported in INC Report No. ARA-12-70. H. W. Buschman, EBR-II, was the test coordinator. He reports the device was installed, checked out, and later removed without affecting normal plant operation. The sodium pipe was thermally hot and the plant operated during these periods. No difficulties were encountered. These memoranda, dating from 1970, are attached as Appendix C. Use of waveguides to withstand temperature difference permitted use of room temperature piezoelectric transducers. The oscilloscope measurement method provided data which were not precise, but quite sufficient to demonstrate agreement with theory.

2.4 History of Extrinsic Factor Studies

It is appropriate to conclude the historical survey with a reference which returns us to the extrinsic factors affecting accuracy. In 1962 Birger reviewed the mathematical and theoretical equations for determining the flow profile in a pipe and the relationship between this flow profile, the average flow as measured by an ultrasonic flowmeter and the true average flow.¹² He shows the speed of turbulent flow measured by an ultrasonic flowmeter in the range of Reynolds' numbers 5×10^3 to 5×10^6 exceeds the mean speed of the flow (or true average flow) by 4-8%. He then makes the following interesting observation:

APPLICATION OF THEORY

$$Q = skD \sum_{i=1}^4 \frac{w_i \ell_i^2 \Delta T_i \tan \theta_i}{2T_{1i} (T_{1i} + \Delta T_i)}$$

s = Scaling for Residual Error in Gaussian Integration

k = Scaling for Operating Temperature and Pressure as They Affect the Measured As-Built Pipe Dimensions D , ℓ_i and θ_i .

(See Appendix D)

FIG. 2 MULTIPLE PARALLEL PATH ULTRASONIC FLOWMETER FOR GAUSSIAN INTEGRATION

"It is obvious that all these conclusions will hold only if the axial symmetry of the flow is maintained, for which purpose it is necessary and sufficient for the lengths of the straight pipe segments to meet the regulations for installing throttling devices for differential-manometer flowmeters. In the case of simple elbows and bends, this length must not be less than 5 diameters. In the case of greater disturbances of the flow (treble junctions, tees, with part of the additional flow coming at right angles to the main, etc.), the length of the straight segment must accordingly be increased.

The solution of specific problems in calibrating ultrasonic flowmeters meets with considerable difficulties of a practical nature, due to the fact that these flowmeters, as a rule, are intended for a use under conditions for which no other methods are suitable apart from measuring the volume.

The volumetric method is perfectly suitable for calibrating flowmeters, but in practice, in the majority of cases, it cannot be applied since it entails installation of additional heavy equipment, considerable disruption in the production process, etc. It is, therefore, advisable to calibrate ultrasonic flowmeters on a test rack by means of water, and then to apply corrections due to the difference in the acoustical and hydrodynamic characteristics of water and the tested medium."

The implications of this recommendation will be considered further under the need for flow calibration capabilities.

3.0 PLBR OPERATING CONDITIONS

The tentative PLBR design specifications needed for assessment of ultrasonic flowmeter accuracy were supplied by W-ARD.¹³ These are shown in Table II along with other pertinent physical data on sodium and 304 stainless steel.¹⁴

3.1 PLBR Flow Measurement Site Pipe Configuration

Plant arrangements being considered provide a maximum of 8 pipe diameters straight length of pipe for installation of the flowmeter assembly. Upstream

TABLE II

TENTATIVE PLBR FLOW SPECIFICATIONS[†]

	<u>COLD</u>	<u>HOT</u>
Loop temperature	(356°C)675°F	(510°C)950°F
Loop pressure (psia)	195	195
Loop flow (lb/sec)	10,750	10,750
Loop flow (ft ³ /sec)	198.7	207.5
Loop flow (gpm)	89,182	93,132
Loop flow (ft/sec)	29.38	30.5
Density, ρ_{Na} (lb/ft ³)**	54.1	51.2
Viscosity, ν_{Na} (ft ² /hr)**	.0135	.0113
Reynolds' number, Re	2.3×10^7	2.9×10^7
Refractive index ($n = C_{304SS}/C_{Na}$)**	1.217	1.209
Sodium conductivity (BTU/ft hr °F)**	43.5	38.6
Pipe* inside diameter (35 inches @75°F)	35.214	35.312
Pipe* inside area (ft ²)	6.763	6.801

*Pipe, 304 stainless steel, nominal 36 inch outside diameter x 1/2 inch wall,
thermal expansion coefficient $10.2 \times 10^{-6} \text{ } ^\circ\text{F}^{-1}$,
thermal conductivity 12.4 (BTU/ft hr °F) @932°F.

**Sodium - NaK Engineering Handbook Volume 1; O. J. Foust, Ed., Gordon and Breach, N. Y., 1972.

[†]For each of four loops.

of the 8 pipe diameters is a 90° elbow. Downstream is a 45° elbow. This is illustrated in Fig. 3 indicating the expected straight section will have a length, L , between 6 and 8 diameters. Also, the center of the flowmeter spool-piece will be located approximately $3/4 L$ downstream between elbows, a value based on common usage and the experimental observation that flow disturbances are felt much further downstream than upstream.

The preferred location (that is, the exact fraction of the length between elbows) for an ultrasonic flowmeter spoolpiece remains to be determined in a water or sodium mockup of the piping as shown in Fig. 3.

3.2 PLBR Flow Conditions

The PLBR flow conditions are unusual, not only for sodium, but also for water. The linear velocity is high at approximately 30 feet/sec compared to 10-20 feet/sec in EBR-II and sodium test loops. The inside diameter is large at 35 inch, while the density and viscosity are comparable to that in smaller systems.

Fluid flow conditions vary widely for changes in velocity $\bar{u}_A$, pipe diameter, (D), density, (ρ), and viscosity, (μ), or kinematic viscosity ($\nu = \mu/\rho$). Comparison of different fluid flows is greatly enhanced by use of Reynolds' number, the dimensionless ratio of the variables $\bar{u}_A D/\nu$. Laminar flow exists only in flows having a Reynolds' number less than a few thousand. For higher Reynolds' numbers, the flow is turbulent. One interpretation of this is that as the Reynolds' number increases to high values, the flow profile becomes increasingly uniform across the pipe, decreasing only very near the pipe wall.

The PLBR tentative Reynolds' numbers for 5% and 100% of full flow are 1.5×10^6 and 3×10^7 , respectively. This implies highly turbulent flow with small changes in flow profile from full flow down to five per cent of full flow. It also places the full flow an order of magnitude beyond the capabilities of most existing U. S. water flow calibration facilities, particularly those having weigh tanks.

3.3 PLBR Reactor Shutdown System Redundancy and Implications for Ultrasonic Flowmeters

A significant condition resulting from utilization of flowmeters by the PLBR Reactor Shutdown System (RSS) is the requirement of four independent (redundant) channels per loop. This raises the questions:

$$L : 6D_{ia} < L < 8D_{ia} \text{ Max}$$

$$F_L \approx 3/4 L$$

FIG. 3 PLBR FLOW MEASUREMENT SITE PIPE CONFIGURATION

1. How should four independent flowmeter assemblies be located along the straight section of pipe? Should the four be mounted on one spoolpiece with the ultrasonic beams approximately 45° apart?
2. Is there a preferred configuration of four flowmeter assemblies such that some combination of their signals yields superior accuracy? (See 4.6.2.)
3. Can the multiple parallel path (Gaussian integration with $n = 4$) flowmeter as embodied in the Westinghouse design be used to satisfy this requirement? (See 4.6.2.)

4.0 FLOW PROFILE EFFECTS

4.1 Differential, Integral and Average Flow Concepts

It is important to start with the concept of a mathematical average in order to consider correction terms which bring ultrasonically measured flow values into agreement with true average flows. The mathematical concept of an average is of the ratio of 2 integrals, the numerator being the product of two functions and the denominator one of the functions of the numerator product. For example, if one makes a trip at differing speeds, a graph of the velocity versus time would be characterized by a number of areas having the dimensions of velocity-seconds, which could be summed to give the total area in velocity-seconds. If this is divided by the total elapsed time in seconds, one then has the average velocity for the trip.

Similarly, if one conducts a flow calibration experiment, in which water flows through a piping system into a weigh tank, the flow may fluctuate slightly during the run - or it may vary from one part of the pipe to another, but the division of the total weight of water (Q_{tp}) collected in a tank by the total time of collection and density (t_p) will give a good true average flow (Q), in cubic feet per second. This average flow divided by pipe area is the true average linear velocity of the fluid, $\bar{u}_A$, in feet/sec.

A graph of this flow would give a perfectly flat profile across the diameter of the pipe resembling frictionless flow. (See Fig.12.) Studies of boundary-layer theory have shown that friction between the fluid and the wall reduces the velocity of the fluid near the wall.¹⁵ At very low flows,

laminar flow distribution occurs with the fluid velocity greatest in the center of the pipe with a parabolic shape from the center toward the wall. Given this theoretical curve, it is possible to calculate the relationship between the maximum velocity in the center and the average velocity, which times the area of the pipe, would give the total flow. However, we are only interested here in much higher flows with turbulent flow profiles.

4.2 Turbulent Flow Profile Laws

Both experiment and theory of the flow of fluids in pipes show that as the velocity increases, turbulent vortices are formed which carry energy and momentum radially, tending to slow the higher velocity in the center relative to that near the wall. The net effect of this turbulence is to produce a flattened flow profile which at very high velocities becomes much more nearly rectangular or constant across the pipe. Flow profiles in the turbulent regimes are characterized by a number of theoretical expressions having varying degrees of agreement with experimental results. These are:

(1) the inverse power law of Nikuradse,¹⁵ $u/U = (y/R)^{1/n}$;

(2) the logarithmic law,¹⁶ $u = U + 2.5 u^* \ln(y/R)$; and

(3) a variation of the logarithmic law due to Reichardt,¹⁷

$$u/u^* = 2.5 \ln(1 + 0.4 y^*) + 7.8 (1 - e^{-y^*/11} - (y^*/11) e^{-0.33y^*}).$$

$$\text{where } y^* = y u^* / \nu$$

$$u^* = \sqrt{\tau_o / \rho}, \text{ or for smooth pipe}$$

$$u^* = \bar{u}_A \sqrt{\lambda/8},$$

$$\lambda = \text{resistance coefficient and}$$

$$y = R - r$$

All these laws and the experimental data confirm that the correction required for a single diametral path ultrasonic flowmeter becomes less as the flow or Reynolds' number increases.

The relationship between these velocity profiles and the true average velocity $\bar{u}_A$ have been studied by a number of other authors: the inverse power

law by Schlichting,¹⁵ and the logarithmic law by Schlichting¹⁵ and Birger.¹² Before going into these relationships, however, it is necessary to apply the flow profile concept to the velocity averaging by ultrasonic flowmeters. The simplest of these, the inverse power law, will be used for estimating theoretical errors and corrections since asymmetry of the flow profiles in most practical cases renders the theoretical corrections doubtful or even harmful.

4.3 Velocity Averaging by Ultrasonic Flowmeters

Consider an acoustic path aligned with the x axis with transducers located at $x = 0$ and $x = d$. If fluid velocity in the x direction is given by an arbitrary function $v(x)$, and the velocity of sound in the fluid is c , the total velocity of acoustic waves in the $+x$ direction is

$$\frac{dx}{dt} = c + v(x) \quad (A1)$$

The assumption of an arbitrary function for $v(x)$ is for purposes of illustration only, since the flow is required to be constant along the pipe axis at any radius, especially in the case of ultrasonic flowmeters. The flow may vary along the pipe if there is a bend or the area changes, but this is not a good location for an ultrasonic flowmeter.

From this we can write the integral equation

$$\int_0^{t_A} dt = \int_0^d \frac{dx}{c + v(x)}, \quad (A2)$$

which shows that the "downstream" travel time t_A is the summation of time increments corresponding to each increment of length.

$$t_A = \int_0^d \frac{c - v(x)}{c^2 - v(x)^2} dx. \quad (A3)$$

Similarly the "upstream" travel time

$$t_B = \int_0^d \frac{c + v(x)}{c^2 - v(x)^2} dx. \quad (A4)$$

The difference,

$$\Delta t = t_B - t_A = 2 \int_0^d \frac{v(x)}{c^2 - v(x)^2} dx. \quad (A5)$$

If $v(x)^2 \ll c^2$, replacing $v(x)$ in the denominator by a constant (with respect to x) v_{eff} , the effective velocity should introduce little error, (Approximately 2×10^{-4}). Thus Eq. (A5) can be written

$$\Delta t = \frac{2d}{c^2 - v_{eff}^2} \left[\frac{1}{d} \int_0^d v(x) dx \right] \quad (A6)$$

$$= \frac{2d v_{eff}}{c^2 - v_{eff}^2} \quad (A7)$$

The flowmeter responds to the effective velocity along the acoustic path. The detailed derivation of these effects for acoustic paths at an angle to the pipe axis is given by McShane³ and by Fisher and Spink⁶. If the velocity profile is not a constant, then a correction factor is required.

4.4 Correction of Ultrasonic Flow $\bar{u}_{DIA}$ to Area Average Flow $\bar{u}_A$

The component of velocity $v(x)$ along an acoustic path at an angle θ with the pipe axis can be written in terms of the flow velocity $u(r)$ at a distance r from the axis of the pipe of radius R :

$$\begin{aligned} v(x) &= u(r) \cos \theta \\ x &= (R - r) / \cos \theta \end{aligned}$$

We now define the effective flow velocity $\bar{u}_{Dia}$ as measured with an ultrasonic flowmeter from equations A6 and A7 as

$$\bar{u}_{DIA} = \frac{1}{R} \int_0^R u(r) dr \quad (A9)*$$

To compare this effective velocity as interpreted by the flowmeter with an area average, $\bar{u}_A$, as measured with weigh tank we use the Blasius profile,

$$u(r) / U = (1 - r/R)^{\frac{1}{n}} \equiv (y/R)^{\frac{1}{n}} \quad (A10)$$

where U is the maximum velocity at $y = R$ and n a constant characterizing a

* This is not the usual definition of an average velocity as the instantaneous velocity averaged over time.

particular flow.¹⁵ Substituting for $u(r)$ and integrating,

$$\bar{u}_{DIA} = \frac{U}{R} R^{-1/n} \left(\frac{n}{n+1} \right) \left[y^{\frac{n+1}{n}} \right]_0^R$$

$$\bar{u}_{DIA} = U \left(\frac{n}{n+1} \right) \quad (A11)$$

Now the area average flow¹⁵ requires evaluation of the integral over the pipe area with the local flow $u(r)$ weighted by the annular area $2\pi r dr$. (Fig. 12)

$$\bar{u}_A = \frac{\int_0^R u(r) \cdot 2\pi r dr}{\int_0^R 2\pi r dr},$$

which after substitution and integration gives

$$\bar{u}_A = U \frac{2n^2}{(n+1)(2n+1)}, \quad (A12)$$

where U is the maximum velocity.

Hence, to convert the ultrasonically measured $\bar{u}_{DIA}$ to the desired $\bar{u}_A$, the correction factor k is

$$\begin{aligned} k_n \bar{u}_{DIA} &= \bar{u}_A, \text{ or} \\ k_n &= \frac{\bar{u}_{Area}}{\bar{u}_{DIA}} = \frac{U \frac{2n^2}{(n+1)(2n+1)}}{U \left(\frac{n}{n+1} \right)} \\ k_n &= \frac{2n}{2n+1} \end{aligned} \quad (A13)$$

In the sections following it was desired to have the average flow along many diametral or chordal paths as measured by an ultrasonic flowmeter, but without actually performing the experiment. This was simulated by integrating

along the desired path using actual experimental isovelocity contour maps of the flow velocity profile under two conditions: one that approximated "fully developed turbulent flow" in a long straight pipe, and the other being asymmetric flow between close bends (Figs. 5, 6, 7, 8, 9, 10). Four isovelocity contour maps were available for the two pipe configurations and two flow velocities.⁷

The path integration of the isovelocity contours to obtain the ultrasonic flowmeter velocity $\bar{u}_{DIA}$ was done in three parts, two small wall integrals and a large central integral. The flow changes slowly from the pipe center ($r = 0$) to about $r = .9r_o$ as shown in Fig. 12. In this range, it is well integrated by Simpson's Rule. However, near the wall the flow drops rapidly to zero at the wall. Including these wall regions ($r = .9r_o$ to $r = r_o$) as end terms in Simpson's Rule leads to errors of several percent. A procedure found satisfactory was to fit the wall segment data to the inverse power law and to integrate the wall segments analytically. The values obtained in this way were compared with theoretical values for "fully developed turbulent flow" and with each other. This provided a basis for studying errors which might result from choosing various paths or combinations of paths.

It has been shown¹⁵ that the inverse power formula fits experimental data fairly well over the Reynolds' number range from 4×10^3 to 3×10^6 , for which the corresponding values of n range from 8 to 12. In the work by Fisher and Spink,⁶ the values of Reynolds' numbers were extended to higher ranges and the values of n to approximately 16. For values of $n = 10$ and 20, the corresponding correction terms would be approximately 5 and 2.5% respectively. Since the PLBR case is expected to confine its attention to Reynolds' numbers above 2×10^6 , the expected correction factors will be less than 5%.

Two cautionary reminders should be kept in memory regarding this correction factor: 1) If referred to a full scale flow value, the error reaches a maximum near midscale. By slightly overcompensating the full scale value, the error may be divided into positive and negative components, reducing the net effect.³ 2) All of the above assumptions are based on fully developed turbulent flow, which implies radial symmetry of the flow profile. This is not usually attained in practical systems; hence, it is necessary to examine

experimental evidence for the effect of asymmetry of flow profile upon accuracy as noted in 2.4. [12]

4.5 Experimental Flow Calibration Studies

The piping configuration shown in Fig. 3, in which the flow measurement location is in a short straight section between two bends, represents one of the major causes of flow asymmetry of interest in PLBRs. In writing about the flow of real fluids in ducts, Olson¹⁸ gives a physical basis for the asymmetry downstream of an elbow, shown in Fig. 4. The pressure drop, or head loss, owing to energy dissipation in an elbow, is largely due to a secondary flow superimposed on the main flow. As a result of centrifugal effects, the fluid in the faster core is directed outwards, and in order to satisfy continuity (the outward flowing fluid must be replaced), the slower fluid near the boundaries is directed inwards as shown in Fig. 4.

Note that these vortex flows are orthogonal to the main flow. A test instrument inserted at various points in the flow would be exposed to the vector sum of the velocity components. It is customary in doing flow profile maps to insert along a diameter (or two diameters at right angles) streamlined support members from which pitot tubes project upstream into the flow. This is referred to as a pitot-tube rake, or as a diameter traverse.¹⁹ Other flow-sensing techniques such as hot wires, hot films or laser Doppler may be used to map the flow profile. An example of the flow profile downstream from a 40° elbow following a simulated heat exchanger is shown in Figs. 7 and 8.⁷

The asymmetry in the experimental pattern is characterized by an overall maximum on the downstream, outside of the bend, with small double maxima representing the two vortices as shown by Olson for the secondary flow. Also, there is a pronounced reduction in flow on the side near the inside of the bend.

The Reynolds' numbers of approximately 1×10^6 and 3×10^6 indicate that the flow is indeed highly turbulent. However, it is far from the symmetry implied by fully developed turbulent flow. It is instructive to consider what flow profile would be obtained in a moderately long straight section of pipe, rather than near the 40° bend. This experimental data is also available from the same report.

FIG. 4 SECONDARY FLOW IN AN ELBOW

TABLE III TRAVERSE DATA IN 19.25" ID STRAIGHT PIPE
EVALUATION OF WESTINGHOUSE LEFM

TRAV. POSITION		TIME OF DAY	Q BY W.T.	Ave DEFL H.	VH	RATIO TO Ave VH	TIME OF DAY	Q BY W.T.	Ave DEFL H.	VH	RATIO TO Ave VH
No.	% D	HRS.	CFS.	FT. OF WATER		%	HRS.	CFS.	IN. OF MERCURY		%
1	50.0	1510	16.045	1.2425	1.1147		1055	40.375	7.040	2.6533	
2				0.532	0.7294	74.1			4.0605	2.0151	82.8
3				0.766	0.8625	88.3			5.508	2.3469	96.4
4		1525	16.025	0.943	0.971	98.6	1118	40.421	6.389	2.5276	103.8
5				1.237	1.1122				7.047	2.6546	
6				1.167	1.0803	109.7			6.658	2.5803	106.0
7				1.2285	1.1084	112.5			6.959	2.6380	108.3
8		1540	16.014	1.2035	1.097	111.4	1134	40.349	6.999	2.6456	108.7
9				1.243	1.1149				7.008	2.6473	
10				1.117	1.0529	107.3			6.822	2.6119	107.3
11				1.0705	1.0346	105.1			4.910	2.1000	
12		1550	16.015	0.8745	0.9351	95.0	1201	40.493	4.056	2.014	82.7
13				0.6455	0.8084	81.6			7.061	2.6573	
14				1.249	1.1176				7.088	2.0219	83.0
15				0.629	0.7931	80.5			5.423	2.3394	96.1
16				0.8665	0.9369	94.5			6.3919	2.5204	103.8
17		1605	16.015	1.0670	1.0329	104.9	1212	40.323	6.972	2.6405	
18				1.244	1.1153				6.969	2.6209	107.6
19				1.1635	1.0787	109.5			7.005	2.6467	108.7
20				1.2360	1.1118	112.9			7.001	2.6459	108.7
21		1612	16.013	1.2458	1.1160		1225	40.249	6.989	2.6437	
22				1.1515	1.0781	109.0			6.746	2.5974	106.7
23				1.092	1.0450	106.1			6.295	2.570	103.1
24				0.9375	0.9682	98.3			5.645	2.3759	97.6
25		1628	16.011	0.6755	0.8219	83.5	1240	40.269	4.378	2.024	85.9
26				1.2665	1.1254				7.094	2.6685	
27			16.020								
28											
29											
30											
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NOTES:

10 READINGS AT EACH PITOT POSITION

PIPE ID. = 19.25" AREA = 2.021 SQ. FT.

Ave VH = 0.9848 Ave Q VH = 1.1166

PIPE FACTOR = .8802

Ave Q = 16.020 CFS. Ave VEL = 7.9263 FT/SEC

Ave. WATER TEMP. = 75°F KIN. VIS. OF WATER 1.0×10^{-5}

REYNOLDS' NUMBER = 1.272×10^6

PITOT COEFFICIENT = 1.003

20 READINGS AT EACH PITOT POSITION.

Ave VH = 2.4348 Ave Q VH = 2.6515 = 108.9%

PIPE FACTOR = 0.918

Ave Q = 40.354 CFS. Ave VEL = 19.967 FT/SEC.

REYNOLDS' No = 3.198×10^6

PITOT 'C' = 0.999

Note: A diametral line bounded by 1 at 0° and 10 at 0° indicate there are 10 p.p.c. positions exclusive of the centerline position. Positions Nos. 1 and 10 are nearest the pipe wall.

DISCHARGE 16 cfs

ISOVEL DIAGRAMS IN 19.25"

STRAIGHT PIPE

Looking Downstream

TEST OF WESTINGHOUSE LEFM

F Zone

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Fig. 5

Fig. 6

Note: A diametral line bounded by No. 1 at 0° and 10 at 0° indicates there are 10 pitot positions exclusive of the centerline position. Positions Nos. 1 and 10 are nearest the pipe wall.

ISOVEL DIAGRAMS IN 19.25"

STRAIGHT PIPE

Looking Downstream

TEST OF WESTINGHOUSE LEFM

F Zone

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TABLE IV TRAVERSE DATA IN 19.25" ID PIPE AT 0° REFERENCE ANGLE
EVALUATION OF WESTINGHOUSE LEFM

TRAV. POSITION		TIME OF DAY	Q BY WT	AVE DEFL H	$\sqrt{H}$	RATIO TO AVE $\sqrt{H}$	TIME OF DAY	Q BY WT	AVE DEFL H	$\sqrt{H}$	RATIO TO AVE $\sqrt{H}$		
No.	% D	HRS.	CFS	FT OF WATER		%	HRS	CFS	IN. OF MERCURY		%		
E	50.0	1040	15.491	0.927	0.963		1605	38.153	5.164	2.272			
1				0.694	0.833	87.47			3.77	1.942	82.64		
2				0.7385	0.859	90.20			4.030	2.0075	83.16		
3				0.778	0.882	92.62			4.247	2.0608	84.26		
E				0.944	0.972		1620	38.077	5.41	2.326			
4				0.814	0.902	94.72			4.390	2.095	89.16		
5				0.8765	0.935	98.29			4.748	2.179	92.73		
6				0.968	0.984	103.33			5.445	2.333	99.28		
E		1056	15.525	0.946	0.973		1637	38.069	5.274	2.297			
7				0.985	0.992	104.17			5.495	2.344	99.75		
8				0.976	0.988	103.75			5.415	2.327	99.03		
9				0.944	0.972	102.07			5.290	2.30	97.88		
10				0.827	0.909	95.45			4.810	2.193	93.33		
E		1110	15.433	0.925	0.962		1649	38.058	5.222	2.285			
11				0.909	0.953	100.08		38.090	5.475	2.340			
12				0.955	0.977	102.60			5.205	2.281	97.07		
13				0.9445	0.972	102.07			5.425	2.329	99.11		
E		1122	15.432	0.949	0.974		1705	38.076	5.455	2.315	98.52		
14				0.933	0.966	101.44			5.325	2.308			
15				0.925	0.962	101.02			5.485	2.342	99.67		
16				0.937	0.968	101.65		38.411	5.370	2.317	97.75	2.297	CORR
E		1135	15.345	0.928	0.963				5.405	2.325	98.05	2.304	
17				0.925	0.986	103.54			5.470	2.339		2.318	
18				0.959	0.979	102.81			5.57	2.36	99.54	2.339	
19				0.948	0.974	102.28			5.605	2.367	99.84	2.346	
20				0.7865	0.887	93.14			5.58	2.362	99.62	2.341	
E		1148	15.383	0.9165	0.957		1730	38.445	4.585	2.141	90.30	2.122	
									5.555	2.357		2.336	CORR

NOTES.

10 READINGS AT EACH PITOT POSITION
PIPE I.D. = 19.25" AREA = 2.021 SQ. FT.
AVE $\sqrt{H}$ = 0.944 AVE $\Sigma \sqrt{H}$ = 0.966
PIPE FACTOR = .977
AVE Q = 15.435 CFS AVE VEL = 7.6373 FT/SEC
AVE WATER TEMP = 74°F $\gamma = 1.01 \times 10^{-5}$
REYNOLDS' NUMBER = 1.21×10^6
PITOT COEFFICIENT = 1.009

10 READINGS AT EACH PITOT POSITION
AVE $\sqrt{H}$ = 2.243 AVE $\Sigma \sqrt{H}$ = 2.310
PIPE FACTOR = .971
AVE Q = 38.087 CFS AVE VEL = 18.8458 FT/SEC
AVE WATER TEMP = 78°F $\gamma = .96 \times 10^{-5}$
REYNOLDS' NUMBER = 3.15×10^6
PITOT COEFFICIENT = 1.023

TABLE V TRAVERSE DATA IN 19.25" ID PIPE AT 30° REFERENCE ANGLE
EVALUATION OF WESTINGHOUSE LEFM

TRAV. POSITION		TIME OF DAY	Q	AVE	RATIO	TIME OF DAY	Q	AVE	RATIO
No	% D	HRS	BY WT CFS	DEFL H FT OF WATER	$\sqrt{H}$ AVE $\sqrt{H}$ %	HRS	BY WT CFS	DEFL H IN OF MERCURY	$\sqrt{H}$ AVE $\sqrt{H}$ %
4	50.0	1403	15.594	0.9744	0.987	1040	42.787	6.734	2.595
1				0.5165	0.719			3.475	1.864
2				0.6450	0.803			4.390	2.095
3				0.831	0.912			5.625	2.372
4		1414	15.574	0.9661	0.983	1056	42.641	6.752	2.598
5				0.9070	0.952			6.170	2.484
6				0.904	0.951			6.180	2.486
7				0.9815	0.991			6.875	2.622
8		1428	15.550	0.9535	0.976	1112	42.638	6.791	2.601
9				1.017	1.008			6.973	2.641
10				1.30005	1.002			7.065	2.658
11				0.9645	0.982			7.050	2.655
12		1445	15.531	0.8845	0.940	1123	42.580	6.335	2.517
13				0.9730	0.986			6.741	2.596
14				0.9185	0.958			6.610	2.571
15				0.949	0.974			6.785	2.605
16		1457	15.525	0.952	0.976	1136	42.532	6.725	2.593
17				0.9680	0.984			6.700	2.588
18				0.968	0.984			6.620	2.573
19				0.953	0.976			6.840	2.615
20		1511	15.516	0.975	0.988	1148	42.574	6.875	2.622
21				0.9575	0.979			6.695	2.587
22				1.013	1.006			7.035	2.652
23				1.0175	1.007			7.055	2.656
24				1.0055	1.003			6.875	2.622
25		1526	15.507	0.8415	0.917	1203	42.557	5.935	2.436
26				0.9480	0.974			6.745	2.597

NOTES

ID READINGS AT EACH PITOT POSITION

PIPE I.D. = 19.25" AREA = 2.021 SQ FT

AVE $\sqrt{H}$ = 0.952 AVE $\epsilon \sqrt{H}$ = 0.980

AVE Q = 15.542 CFS AVE VEL = 7.6903 FT/SEC

AVE WATER TEMP = 81 OF $V = 0.925 \times 10^{-5}$

REYNOLDS' NUMBER = 1.33×10^6

PITOT COEFFICIENT = 1.007

PIPE FACTOR = 0.971

ID READINGS AT EACH PITOT POSITION

AVE $\sqrt{H}$ = 2.519 AVE $\epsilon \sqrt{H}$ = 2.595

AVE Q = 42.616 CFS AVE VEL = 21.0866 FT/SEC

AVE WATER TEMP = 84.5 OF $V = 0.88 \times 10^{-5}$

REYNOLDS NUMBER = 3.84×10^6

PITOT COEFFICIENT = 1.019

PIPE FACTOR = 0.971

FIG. 7

Note: A diametral line bounded by No. 1 at 0° and 10 at 0° indicates there are 10 pitot positions inclusive of the centerline position. Positions 1 and 10 are nearest the pipe wall.

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Fig. 8

Note: A diametral line bounded by No. 1 at 0° and 10 at 0° indicate there are 10 pitot positions exclusive of the centerline position. Positions 1 @ 30° Nos. 1 and 10 are nearest the pipe wall.

Figs. 5 and 6 show the flow profile as measured with the pitot tube rakes with a flow measurement section located 21 diameters downstream from the nearest obstruction and 6 diameters upstream from the nearest obstruction. Visual inspection reveals that there is asymmetry at the low flow rates, but with improvement at the higher rate. In the straight pipe at both flow rates, the approach to symmetry is much closer than near the 40° bend.

4.6 Comparison of Theoretical and Experimental Correction Factors

The flow profile theory is important for general understanding and for prediction of flow behavior in untested applications. However, the real world piping configurations (e.g., Fig. 3) for sodium flow in PLBRs do not permit realization of the conditions under which the theory can be applied with high accuracy.

This situation is well illustrated in the ARL experimental data (see Tables III, IV, V and Figs. 5-8) for flow in 19.25 inch inside diameter pipe at 8 and 20 feet per second, Reynolds' numbers of 1×10^6 and 3×10^6 approximately, under two different configurations described above and summarized here to compare the differences: a) straight pipe with flowmeter spoolpiece 21 diameters downstream and 6 diameters upstream from bends or obstructions, pipe factor = .882, (Fig. 9), and b) heat exchanger multijet discharge through a 40° elbow with the flowmeter spoolpiece bolted to the downstream flange of the elbow and exiting the spoolpiece into a 90° elbow. The pipe factor = .974 (Figs. 10, 11).

Application of theoretical correction to ultrasonic flowmeter data (diametral path) in Figs. 5, 6, 7, 8 permits comparison of the calibration factor k derived from experiment and theory and tabulated in Table VI.

Case a - 1) $\bar{u}_A = 8 \text{ ft/sec}$, $u_A / U = \text{pipe factor} = .8822$

Solving A12 for n , $n = 11.55$

Substituting in A13, $k_n = .966$

PIPING DIAGRAM

STRAIGHT PIPE TESTS

EVALUATION OF WESTINGHOUSE LEFM

FLOW MEASUREMENT SECTION

30

Fig. 9

ELEVATION VIEW

WESTINGHOUSE LEFM

EVALUATION TEST

ALDEN RESEARCH LABORATORIES

LJH

MARCH 1971

PIPING DIAGRAM
WESTINGHOUSE FLOW TEST MODEL

Fig. 10

WESTINGHOUSE LEFM
EVALUATION TEST
ALDEN RESEARCH LABORATORIES
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FLOW TEST MODEL
MODEL RADIO 1.636/1

Heat Exchanger Model
Diam. - 76.5"

20" L.R. Elbow

40° Elbow

90°

44"

30" Nipple

SIMULATED
TUBESHEET

DIVIDER PLATE
(SUPPORT PLATE)

1-3/4" Sq. Holes
on 3-1/2" C. L.

+ COUNTERCLOCKWISE
- CLOCKWISE

PARALLEL TO SUPPORT
PLATE WITH
PENETRATIONS
AS SHOWN

180°

90°

270°

FLOW MEASUREMENT SECTION
19.25" I.D. x 33-1/4" LONG

Fig. II

Integrating the diametral path from the isovel graph using Simpson's rule gives

$$\bar{u}_{DIA} / U = .9227 \quad \text{and}$$

$$k_{exp} = \frac{.8822}{.9227} = 0.956$$

which agrees within 1% of the calculated correction factor.

Case a - 2) $\bar{u}_A = 20$ ft/sec, Pipe Factor = .9183

solving for n from A12, $n = 17.19$, from A13 $k_n = .972$

Integrating the diametral path from the isovel graph gives

$$\bar{u}_{DIA} / U = .9462 \quad \text{and}$$

$$k_{exp} = \frac{.9183}{.9462} = .9705 ,$$

again within 1% of the theoretical value.

Because of the apparent agreement it is tempting to find a relationship between the experimental data and theory from which the correction factor may be estimated. This has been done.^{3,12,15} From the uncorrected flow measurement the Reynolds' number can be estimated with sufficient precision to obtain a good correction factor for "fully developed turbulent flow." Kivilis and Reshetnikov²⁸ give an equation for a correction factor versus Reynolds number,

$$m = 1.119 - 0.011 \log Re = \bar{u}_{DIA} / \bar{u}_A$$

The correction factors ($k_{Re} = 1/m$) derived from this equation compare within -1.7 percent of the experimental correction factors listed in Table VI.

Unfortunately, this theory is not accurate for unsymmetrical flow profiles or near elbows. Consider case b-1) relative to case a-1) where the average velocities $\bar{u}$ and Reynolds' numbers are nearly equal. Application of the correction factor from case a-1 to case b-1) would lead to

$$(\bar{u}_{DIA} / U) k_n = .9544 \times .958 = .9143$$

TABLE VI, Pipe Layout Effect on Ultrasonic Flowmeter Velocity,

 $\bar{u}_{DIA}$, and Calibration Factor, k

Case	Pipe Layout	Experimental Data			Derived		Calculated	
		Q ft ³ /sec	Re	$\bar{u}_A$ ft/Sec	$\bar{u}_{DIA}$ ft/Sec	k_{exp}	k_n	k_{Re}
a-1	Straight 21 dia. up 6 dia. down	16	1.3×10^6	8.0	8.37	.956	.958	.951
a-2	Straight 21 dia. up 6 dia. down	40	3.2×10^6	20	20.60	.971	.975	.955
b-1	40° elbow upstream 90° elbow downstream	16	1.2×10^6	8.0	7.84	1.021	.958	.951
b-2	40° elbow upstream 90° elbow downstream	40	3.1×10^6	20	19.64	1.018	.975	.955

or, a corrected Pipe Factor = .9143. This differs from the measured Pipe Factor = .974 by -6.5%.

At the higher flow rate, $(\bar{u}_{DIA} / U) k_n = .95355 \times .972 = .926$, or a corrected Pipe Factor = .926. This differs from the measured Pipe Factor = .971 by -4.8%.

This illustrates again the findings that: a) flow profile variation may cause errors in the 5% range even in turbulent flow, b) the error becomes less as the flow increases, c) theoretical corrections based on "fully developed turbulent flow" do not apply to assymetrical flow profiles, indeed such use may result in increased error. Therefore, d) testing with a mockup of the piping and other factors influencing the flow profile is essential if accuracies better than 5% are desired from single diameter path ultrasonic flowmeters. Similarly, mockup tests including effects of angular orientation are essential if accuracy better than 1% is desired from an ultrasonic flowmeter using Gaussian integration with $n = 4$ parallel paths. Full scale tests with Reynolds' numbers matching the expected application are advisable, if possible and not too costly.

4.6.1 Single Diametral Ultrasonic Path

The basis of comparison takes advantage of the experimental information in the isovelocity contours in Figs. 5, 6, 7, 8 to estimate the average velocity $\bar{u}_{DIA}$ along a diameter (or other chosen path) as it would be measured by an ultrasonic flowmeter. This diametral ultrasonic flowmeter average velocity requires a correction factor, k , to make it agree with the area average velocity $\bar{u}_A$. Note that in the convention of earlier flow profile work, U is defined as the maximum or centerline velocity, $U = \sqrt{\Delta p}$, read by the centerline pitot tube (i.e., where the theoretical velocity maximum occurs) even in the cases where the maximum velocity occurs off center. Thus, the Pipe Factor, or "pipe signature" of water flow is "defined" as $P.F. \equiv \bar{u}_A / U$. Operationally, the total mass, Q_{tp} , is collected in time, t , and weighed. Then $P.F. \equiv Q / AU$. In order to avoid misunderstanding, the comparison here will not use the centerline normalization, but will use the reported volumetric average flow Q , ft^3/sec . Thus, $k \bar{u}_{DIA} A = Q$, and $\bar{u}_A = Q/A$.

Now, the correction factor, k , may be obtained experimentally (ideally) by measuring $\bar{u}_{DIA}$ and Q simultaneously; that is, by performing a calibration. Alternately, a theoretical calculation of k from one of several theories can be made. In this case, a simulation of the calibration method was used by integrating the isovel diagram along chosen diameters to obtain $\bar{u}_{DIA}$.

Then

$$k = Q / \bar{u}_{DIA} A.$$

For different linear velocities, Reynolds' numbers or pipe configurations may be compared with a theoretical k value for "fully developed turbulent flows" at the same velocity or Reynolds' number (Table VII). As expected, the 21 diameter long straight pipe case agreed well between experiment and theory. However, the same correction factors when applied near a bend caused increased error. Note that in long straight pipes $\bar{u}_{DIA} > \bar{u}_A$. The error may change sign as well as magnitude as a bend is approached in highly turbulent flow.

A similar comparison can be made for multiple paths including averages or Gaussian integration.⁶

4.6.2 Multiple Ultrasonic Paths

Using the PLBR safety redundancy requirement, $n = 4$, for instruments such as flow meters, the question of optimum or preferred arrangement arises. Mechanical simplicity and having nearly identical instruments suggests two arrangements to compete with the Gaussian integration with $n = 4$. These other two are: 1) four diametral paths 45° apart in the same spoolpiece; 2) two orthogonal pairs, each pair spaced for Gaussian integration with $n = 2$.

The line integration of the isovelocity contours to obtain the ultrasonic flowmeter velocity $\bar{u}$ was done in three parts, two small end integrals and a large central integral. The flow changes slowly from the pipe center ($r = 0$) to about $r = .9r_o$ as shown in Fig. 12. In this range, it is well integrated by Simpson's Rule. However, near the wall the flow drops rapidly to zero at the wall. Including these wall regions as end terms in Simpson's Rule leads to errors of several

Table VII, COMPARISON OF ULTRASONIC FLOWMETER ACCURACY VERSUS
 CONFIGURATION, STRAIGHT PIPE, 21 DIA UPSTREAM,
 6 DIA DOWNSTREAM, $Re = 1 \times 10^6$, $\bar{u}_A = 8$ FT/SEC

CONFIGURATION Fig. 9	ERROR IN PERCENT		
	CONTOUR MAP INTEGRATION	CORRECTED, THEORY	
		k_n	k_{Re}
Single Diameter	+5.0	< 1	< 1
Four Diameters, 45° Apart	+4.6	< 1	< 1
GAUSSIAN INTEGRATION			
n=2, one pair	+3.5	< 1	}*
n=2, two pair othogonal	+3.5	< 1	
n=4	+0.2	< -.3	

*Correction factors derived as per method of Fisher and Spink, Ref. 6.

FIGURE 12. NORMALIZED POINT AND AREA WEIGHTED FLOW PROFILES

percent. A procedure found satisfactory was to fit the data to the inverse power law in the edge segments and to integrate the edge segments analytically. (See 4.4 and 4.6)

The importance of the edges for changes in flow profile can be seen in Fig. 12 in the area weighted flow profiles. Here the change is from Reynolds' number of 1.2×10^6 (8 ft/sec) to 3.2×10^6 (20 ft/sec). Most of the profile variation (crosshatched area) is near the wall. This helps explain the importance of sampling the flow near the edge to obtain accurate integration.

The results for the straight pipe layout are given in Table VII. There is little gain in averaging 4 diametral paths 45° apart or in using two orthogonal pairs for Gaussian integration. Gaussian integration with $n = 4$ is clearly superior, yielding errors of about a quarter of one percent. The theoretical error is about $+ 0.4\%$.

For the flowmeter location between a 40° and a 90° elbow, Table VIII, the errors were similar in magnitude but worse in that theoretical correction increased the error! Again, the Gaussian integration with $n = 4$ is superior, particularly if attention is paid to angular orientation.

It can be judged that accuracy of 5% is readily attainable with single diametral paths. With weigh tank calibration check using scale model mockup of pipe layout, accuracy of 1% is possible. Note that precision rather than high accuracy is required for balancing flow in several mechanically identical loops of an LMFBR since (very nearly) the same correction factor applies to each flowmeter. Thus single diametral path ultrasonic flowmeters are adequate for balancing flows in the several loops of a loop type LMFBR.

For accuracy better than 1% for thermodynamic measurement, the Gaussian integration with $n = 4$ is recommended in spite of added mechanical difficulties and cost. Again, calibration using a mockup of the proposed pipe layout is strongly recommended.

5.0 U. S. WATER CALIBRATION FACILITIES

A number of water flow calibration facilities were contacted in testing the feasibility of conducting a full scale (35") (or fractional size

Table VIII, COMPARISON OF ULTRASONIC FLOWMETER ACCURACY VERSUS CONFIGURATION
 BETWEEN 40° ELBOW UPSTREAM AND 90° ELBOW DOWNSTREAM,
 $Re = 1.2 \times 10^6$, $\bar{u}_A = 8$ FT/SEC. ERROR VARIES WITH ANGULAR ORIENTATION

CONFIGURATION Fig. 10	ERROR IN PERCENT		
	CONTOUR MAP INTEGRATION	CORRECTED, THEORY k_n k_{Re}	
Single Diameter	-0.4 to -3.7	-5.4 to -8.7	-6.1 to -9.4
Four diameters, 45° Apart	-2.0	-7.0	-7.7
GAUSSIAN INTEGRATION			
n=2, one pair	+4.0	$\left. \begin{matrix} <1 \\ <1 \\ <1 \end{matrix} \right\} *$	
n=2, two pair orthogonal	+3.0		
n=4	-0.4 to +1.2		

*Correction factor derived as per method of Fisher and Spink, Ref. 6.

scale) water mockup at the tentative PLBR design Reynolds' number of 3×10^7 using the tentative PLBR pipe configuration at the flowmeter locations to determine the flow profile, the preferred flowmeter location between bends and the preferred flowmeter orientation with respect to the pipe bends. Availability of absolute calibration by weigh tank was a major consideration to ensure that data from a sequence of tests were truly comparable. Capability of calibrating at Reynolds' number of 3×10^7 and with water temperatures around 190°F to simulate the kinematic viscosity of 400°F sodium was considered important.

Most of the better facilities were eliminated on the basis of limited flow in gallons per minute, limited or no weigh tank capacity or similar limitations. One had suffered earthquake damage to its weigh tank, severely limiting its capacity. The two candidates were located at the Pennsylvania State University, State College, Pa., and at the Alden Research Laboratories, Worcester Polytechnic Institute, Worcester, Mass.

The facility at Pennsylvania State University was described by Dr. Robert Henderson as a four foot diameter water tunnel operated at linear velocities up to 80 feet/sec like a wind tunnel. The hydrodynamic design has been published.²⁰ This provides flows with Reynolds' number up to 3×10^7 . Water temperature rises to 105°F during sustained high flow. Glass windows permit laser doppler velocity profiling. No weigh tank is available, nor is it possible to operate at 190°F . This facility is capable of simulating the PLBR flows both in gallons per minute and Reynolds' number, but without the absolute accuracy of a weigh tank. Careful, continuous measurement of the laser doppler flow profile during a calibration run may partially offset the absence of the weigh tank. This facility would definitely be useful in carrying out a full scale test of angular orientation effects.

The Alden Research Laboratories (ARL) facilities at the Worcester Polytechnic Institute was described by Dr. Lawrence C. Neale.^{21,6} The newest and largest ARL calibration facility includes a 100,000 pound weigh tank and a maximum flow capacity of about 20,000 gallons per minute or 45 cubic feet per second at 100 foot head from two 300 horsepower pumps. The highest Reynolds' number obtainable is about 3×10^6 at room temperature and 1×10^7 at 190°F . Operation at 190°F is possible with special arrangement.

The absolute accuracy of this facility is on the order of 0.1% as shown by comparisons of orifice plate assemblies between National Engineering Laboratory, Glasgow, Scotland, U. K., and ARL.²¹ ARL appears to be the best water flow calibration facility in the U. S. within its equipment limitations. It is recommended that a scaled mockup of PLBR pipe layout as in Fig. 3, or similar, be tested at ARL to determine the preferred location and orientation of the spoolpiece and ultrasonic paths. It is recommended that tests be conducted at ARL to determine the absolute accuracy with which flow measurements in water can be made using laser doppler velocity profile measurement. If satisfactory accuracy is demonstrated, this method could be applied to larger water facilities which lack weigh tanks.

There is need to extend U. S. water flow calibration capabilities to higher velocities and larger diameters (i.e., higher Reynolds' numbers) while maintaining and/or improving the absolute accuracy attainable. Capability of making absolute flow calibrations of high accuracy in sodium should be planned for the needs of a future LMFBR age, even if flowmeters other than ultrasonic are used.

6.0 THERMAL STRATIFICATION

The thermal buoyancy of sodium produced by spatial density variations with temperature (see Table II) is much more pronounced than for water. Thermal buoyancy in a liquid is analogous to the concept in gases that "hot air rises." Different system configurations and operating conditions might be expected to lead to stratification, and, indeed, some of these have been observed. Three cases will be discussed. None of these examples indicate that there will be any difficulty in applying ultrasonic flowmeters to LMFBRs as presently contemplated. However, they will point to potential sources of difficulty, related to location of an ultrasonic flowmeter, which should be kept in mind and given further study. Thermal stratification may impact other LMFBR components, such as pumps.

6.1 Heat Exchanger

An early heat exchanger design for sodium used a horizontal, corrugated (chevron) configuration. When tested, it was found that only a few percent

of the calculated heat exchange occurred. Investigation into this unexpected behavior revealed that the chevron configuration afforded stagnation points which established back-flow eddies such that only a few percent of the region along the midline of the heat exchanger actually had sodium flowing from the inlet to the outlet, thus accounting for the small heat exchange. The stagnation and back-flow was enhanced by the low viscosity and high thermal buoyancy of sodium relative to water, with which this difficulty had not been observed.

6.2 Mixing Tee

A more recent observation of thermal stratification has been made at ANL by Kasza and Uherka in the Components Technology Mixing Components Test Facility (MCTF) while studying turbulence in mixing tees when water of different temperatures was mixed in the tee.^{22,23} Four inch standard schedule 80 pipe and tee, having an inside diameter of 3.82 inches, were used. The cold water had a temperature of 100°F. The hot water flow had an initial temperature of 200°F. The two flows were 180° apart as they entered the mixing tee inlet legs. The approximately 25 feet long hot leg from flowmeter to mixing tee is "U" shaped with a 10 foot separation between two 90° elbows and an 8 foot separation between the mixing tee and the second 90° elbow. (See Fig. 13) The flows were approximately 24.9 gallons per minute for the cold and 2.9 gallons per minute for the hot water. The Reynolds' number for the cold water was 3×10^4 . The Reynolds' number for the hot water, because of the lower flow rate, was approximately 1/4 that of the Reynolds' number for the cold flow. There was insulation from the flowmeter to the second elbow. Observations shown graphically in Figs. 14 and 15 may be summarized as follows:

Starting on the hot leg side approximately 25 feet upstream from the mixing tee, the hot temperature was observed to be 200°F. In the vicinity of the first bend, thermocouples at the top and bottom of the outside pipe wall measured $\Delta T \approx 15\text{--}20^\circ\text{F}$. Proceeding around the first bend, ten feet to the second bend and then approximately four feet past the second bend, a pair of thermocouples on the outside of the pipe showed a $\Delta T = 40^\circ\text{F}$, while thermocouples on the inside diameter of the pipe showed a $\Delta T = 70^\circ\text{F}$. This is a major fraction of the total ($\Delta T = 100^\circ\text{F}$) available between the hot water and the cold water.

A study was made²² as part of the ANL Mixing Components Program which indicates that it is possible at these moderate flows, where a sharp temperature

FIG. 13 MIXING COMPONENTS TEST FACILITY

FIG. 14 RESULTS OF POTENTIAL FLOW SOLUTION FOR STREAM INTRUSION IN A SIMULATED PIPE TEE

FIG. 15 SCHEMATIC OF FLOW MODEL USED TO DETERMINE RECIRCULATION EDDY CRITERIA

interface exists, to develop a flow instability which could cause backstreaming and hence thermal stratification. This study, which is a theoretical analysis on modeling of mixing components, brings out the interesting finding that the thermal stratification observed is most likely due to buoyancy effects rather than to intrusion of the cold leg flow into the hot inlet flow pipe. The analysis shows that a dimensionless Froude number may be used to characterize the situation. For Froude numbers equal to or less than $1/\pi$, the results may be interpreted as corresponding to a recirculation eddy that extends back from the cold front to $x = \text{infinity}$; that is, the eddy motion produced by the thermal buoyancy associated with the hot-cold interface in the mixing tee influences the entire lengthwise flow field of the hot inlet channel, and strong thermal stratification was observed.

The Froude number can be thought of as the ratio of the inertia forces to the gravity forces and is further described in standard texts on fluid mechanics and heat transfer.^{24,25} Studies of thermal mixing and thermal turbulence characteristics in sodium, potassium, mercury and ethylene glycol in pipe flow have been described by Hochreiter and Sesonske²⁶ and by Rust and Sesonske.²⁷

Other dimensionless numbers which bear on heat transfer in fluid flow are the Grashof number and the Prandtl number. The Grashof number is a measure of the vigor of the induced thermal buoyancy flow. It is analogous to the Reynolds' number of forced flow; that is, the square root of the Grashof number corresponds to the Reynolds' number. The Grashof number arises inevitably in the force-momentum balance as the ratio of the relative magnitudes of viscous force and the convection of momentum terms. Low Grashof number flows are very weak and very thick flows; high Grashof number flows are very thin and vigorous, leading to laminar instability and transition. Such characteristics are also found in plumes and in internal flows. The Prandtl number occurs in the energy equation and indicates the steepness of temperature gradients in the flow field.²⁵

In trying to extend observations from a water system to a sodium system, at least four material variables should be considered: 1) viscosity, 2) buoyancy, 3) thermal conductivity, and 4) specific heat. In addition, two different locations (i.e., upstream versus downstream from the flow disturbance or thermal mixing) need to be considered.

1. The viscosity of sodium is less than that of hot water by a factor of 2-4 over the temperature range of interest in LMFBRs.²⁴ This can be expected to enhance the probability of thermal stratification.
2. Buoyancy. The change of density with temperature is much greater in sodium than in water (see Table II) and may be expected to enhance the likelihood of thermal stratification.
3. Thermal conductivity. The thermal conductivity of sodium is approximately two orders of magnitude greater than that of water.^{14,24} This is expected to have a very strong upstream effect on reducing the likelihood of thermal stratification compared to a water system. This is particularly true in highly turbulent flows, characterized by high Reynolds' numbers, and where the piping is not characterized by long straight runs. Since the piping in LMFBRs has frequent bends, the added turbulence coupled with the high thermal conductivity is expected to reduce the likelihood of thermal stratification. Downstream of thermal mixing, sodium will approach thermal equilibrium faster than water.
4. The specific heat of 538°C (1000°F) sodium is about 1/3 that of 38°C (100°F) water. The effect on stratification depends on being upstream or downstream of the point of thermal mixing.

6.3 Estimate of Thermal Stratification Required for Loss of Signal Across a Pipe

It is useful to consider the temperature differential representing thermal stratification of sodium in an LMFBR pipe, which would be required to cause loss of signal. Referring to the Fig. 16, stratification at the midline has been assumed for simplicity and symmetry. A dimension along the length of the pipe comparable to that of the transducer is assumed, the implication being that if the deflection is greater than one transducer length, there will be loss of signal. Using the velocities of sound in sodium as a function of temperature,¹⁴ one calculates an expression for the velocity ratio which is the acoustic refractive index, applies Snell's law to obtain the refractive angle at the thermal interface and projects the acoustic beam to a new intercept

Figure 16

Temperature Gradient Across the Pipe Required to Cause Loss of Signal

with the pipe wall. For a beam displacement $\Delta S = 0.4$ inch and pipe radius 17.5 inches, the required velocity ratio would be 1.011. If one uses the tabulated velocity for the first velocity at 500°C (932°F),¹⁴ the second velocity then differs about 1.1%, and the temperature would be 450°C (842°F) for $\Delta T = 50^\circ\text{C}$ (90°F).

Loss of signal resulting from thermal stratification can be avoided by transducer design. A modified transducer design using lower frequency crystals and consequent broader acoustic beam would require many times the axial length ΔS and temperature difference ΔT estimated above.

6.4 Change in Wall-Sodium Refraction Angle with Temperature

The reader should not conclude that a temperature change of several hundred degrees Centigrade will cause troublesome refraction of the acoustic path. While this is a problem intrinsic to the shear wave transducer and not the subject of this report, the velocity of sound, C_s , in the steel pipe wall and in the sodium, C_ℓ , change at similar rates with change in temperature. The result is that their ratio, which determines the acoustic refractive index $n = C_s/C_\ell$, changes very slowly over the range 205-538°C (400-1000°F). For example, over the projected PLBR operating range (Table II, 357-510°C [675-950°F]) the acoustic beam deflection is approximately 0.16 inches along the pipe wall.

Temperature compensation for effects of refraction, thermal expansion of the pipe, etc., is assumed to be intrinsic to the flowmeter.

7.0 CONCLUSIONS

Experimental isovelocity flow profile contour maps may be used to predict the probable performance of various ultrasonic flowmeter configurations for straight pipe with nearly "fully developed turbulent flow" and for asymmetric, highly turbulent flow near pipe bends. A variety of ultrasonic flowmeter diametral and chordal paths at several orientations were integrated to obtain the effective flow velocity over the path, which was compared to the experimentally measured average flow velocity over the area.⁷

For the cases studied the single diametral path flowmeter should measure on effective flow velocity that is within 5 percent of the true average flow. Application of correction factors based on theoretical profile or Reynolds number will reduce errors to about 1 percent for straight pipes but may in-

crease the error for asymmetric flow profiles typical of pipe bends.

An ultrasonic flowmeter using four parallel paths spaced for Gaussian integration was predicted to give errors of less than 0.5 percent for the long straight pipe. On the asymmetric flow the predicted errors were $-.4$ to $+1.2\%$ depending on orientation. By judicious selection of orientation the error should be less than 1 percent. These predictions confirmed the measured performances.⁷ Some other configurations of four paths predicted greater errors.

These results cover only two pipe arrangements. Other cases typical of probable LMFBR applications must be studied before general conclusions can be drawn. Full scale tests in sodium as well as water will be necessary to demonstrate that results from water may be transferred to sodium with confidence.

Loss of signal due to thermal stratification across the pipe will apparently be a minor problem. Based on a simplified model, 50°C stratification is necessary to shift the beam by 10 mm in a 900 mm diameter pipe.

This report has addressed only extrinsic factors and not problems of realizability of an ultrasonic flowmeter such as circuitry and transducers. Solutions of problems in this area may also effect the choice of layout, number of paths, etc.

REFERENCES

No.

1. L. C. Lynnworth, "Industrial Applications of Ultrasound - A Review, II Measurements, Tests, and Process Control Using Low-Intensity Ultrasound," IEEE. Trans Sonics and Ultrasonics SU-22, 2, (March, 1975) 71.
2. B. G. Liptak and R. K. Kaminski, "Ultrasonic Instruments for Level and Flow," Instrumentation Technology, (September, 1974) 49.
3. J. L. McShane, "Ultrasonic Flowmeters," Flow - Its Measurement and Control in Science and Industry, 1, ISA (1974) 897-960.
4. V. A. Nosov, "Ultrasonics in the Chemical Industry," (Soviet Progress in Applied Ultrasonics, Vol. 2). Consultants Bureau, New York, (1965) 119.
5. H. B. Karplus, "Ultrasonic Transducer," Patent Application, ERDA File No. 44342.
6. S. G. Fisher and P. G. Spink, "Ultrasonics as a Standard for Volumetric Flow Measurement," in: C. G. Clayton, Ed., Modern Developments in Flow Measurement, Peregrinus (1972), pp 139-159.
7. L. J. Hooper, "Evaluation of the Westinghouse Leading Edge Flowmeter System," Report ARL-M71-83, Alden Research Laboratories, Worcester Polytechnic Inst., Holden, Mass. (March, 1971).
8. J. T. Malone and D. K. Whirlow, "Fluid Flow Measurement System," U. S. Patent 3,564,912 (Feb. 23, 1971).
9. A. del Grosso, Naval Research Laboratory 4967 (Nov. 1957); A. D. 149409.
10. N. Suzuki, H. Nakabori and M. Yamamoto, "Ultrasonic Method of Flow Measurement in Large Conduits and Open Channels," in: C. G. Clayton, Ed., Modern Developments in Flow Measurement, Peregrinus (1972) 115.
11. L. Lapidus, "Digital Computation for Chemical Engineers," McGraw-Hill, 1962.
12. G. I. Birger, "Certain Problems in Calibrating Ultrasonic Flowmeters," Measurement Techniques, Vol. 1962, (Oct. 1962) 872-874.
13. Letter: "ARD/ANL Meeting," W. Adkins, W-ARD, March 3, 1976.

No.

14. O. J. Foust, Ed., "Sodium - NaK Engineering Handbook Volume 1," Gordon and Breach, New York, (1972).
15. H. Schlichting, "Boundary-Layer Theory," 4th Ed., McGraw-Hill, New York, (1958), 593.
16. J. Nikuradze, "Gesetzmässigkeit der Turbulenten Stromung in Glatten Rohren," Forschungsheft 356 (1932).
17. H. Reichardt, "Vollständige Darstellung der Turbulenten Geschwindigkeitsverteilung in Glatten Leitungen," Z. Angew. Math. Mech., Bd 31, NAT (Juli 1951) 208.
18. R. M. Olsen, "Engineering Fluid Mechanics," International Textbook Co., Scranton, Pa., 242-243, 1970.
19. W. F. Coxon, "Flow Measurement and Its Control," Heywood and Co., Ltd., London, 286-294, 1959.
20. D. Ross, J. M. Robertson, and R. B. Power, "Hydrodynamic Design of the 48-Inch Water Tunnel at the Pennsylvania State College," Soc. Naval Arch and Marine Eng., Trans, 56, 5-29, 1948.
21. E. A. Spencer and L. C. Neale, "The Reliability of Test Data from Flow Measurement Calibration Laboratories," in: R. B. Dowdell, Ed., Flow, Its Measurement and Control in Science and Industry, 1, part 3, 1255-1266, ISA, Pittsburgh, Pa., 1974.
22. ANL-RDP-33 pp 3.8-3.9.
23. ANL-RDP-44 pp 3.18-3.20.
24. J. D. Parker, J. H. Boggs, E. F. Blick, "Introduction to Fluid Mechanics and Heat Transfer," Addison-Wesley. Reading, Mass., (1969).
25. B. Gebhart, "Heat Transfer, 2nd Ed.," McGraw-Hill, New York (1971).
26. L. E. Hochreiter and A. Sesonske, "Thermal Turbulence Characteristics in Sodium-Potassium," Int. J. Heat Mass Transfer, 12, (1969) 114.
27. J. H. Rust and A. Sesonske, "Turbulent Temperature Fluctuations in Mercury and Ethylene Glycol in Pipe Flow," Int. J. Heat Mass Transfer 9 (1966) 215.
28. S. S. Kivilis and V.A. Reshetnikov, "Measurement Techniques, 1965" 1057 (Nov. 1965).

APPENDIX A

Section 3, Background

Section 4, Advantages

References

From

Reference 3, J. L. McShane, "Ultrasonic Flowmeters,"
Flow - Its Measurement and Control in Science and Industry,
1, ISA (1974) 897-960.

3. BACKGROUND

The history of acoustic methods for measuring fluid flow dates back at least to 1919 when Kunze⁷ described a wind velocity measuring system in which an observer adjusted the length of a listening tube going to one ear to compensate for a shift in seeming location of an audible sound source. The length of tube needed to make the source of sound appear to come from directly ahead was a measure of velocity. In 1928, Rutten⁸ described a travel time difference system requiring three probes (center transmitter and two receivers) in the fluid. Early U.S. patents were obtained in 1932 by Chilowsky,⁹ for a Doppler vessel speed measuring device utilizing reflections from fixed objects, and by Hartig and Wilcox,¹⁰ for a system whereby frequencies of upstream and downstream signals were adjusted to maintain a constant phase between receivers. Frequency difference was a measure of velocity. Even though these and other early systems were somewhat awkward in their described form, indicating the inadequate state of the transducer and electronic art, they did bring out many of the basic approaches and practical problems.

The Doppler technique using return signals from scatterers in the fluid was investigated in 1942 and 1943 using ship's sonar. This work¹¹ (originally classified) was part of a marine speedometer development program, which included work on a phase difference system. The need for higher frequencies and narrower beams than those that could be obtained with current transducer designs was recognized. Other early vessel speed devices used the phase difference approach.

It was not until the 1950's that ultrasonic flowmeters were described in published literature and practicable devices were built. In 1949,

however, the sing-around technique was described for sound velocity measurement (in air).¹² Papers on wind velocity measurement¹³ and water flow measurement in large ducts¹⁴ appeared in 1950. Kalmus¹⁵ is often credited with developing the first usable device in 1953. The design of transducers and the test setup for the Kalmus instrument was appropriate for blood flow measurement in small tubes. In 1955 a commercial instrument for measuring liquid flow rate in pipes was announced by J. Kritz of the Maxson Company.¹⁶ This meter had been developed for metering in-flight refueling operations for the Air Force. Since that time, dozens of successful ultrasonic flow-meter applications have been reported in the literature. Several medical-engineering teams^{1,17,18} have had good results, using variations of the Kalmus system and developing entirely different systems. The principle was applied to wind velocity measurement for atmospheric research with good results.

By 1955 development was in progress on flowmeters for industrial, medical, oceanographic, river flow, and reactor applications. In all of these areas, successful results were reported, and the investigators were unanimously enthusiastic about the future of ultrasonic flow measurement. During this same period, the sing-around velocimeter¹⁹ became a standard instrument for sound velocity measurement in the ocean. The techniques developed apply equally well to fluid velocity measurement. A recent commercial instrument, announced by Lockheed in 1966, was a combination of a two sing-around velocimeters.²⁰

A beam deflection flowmeter⁴ was offered by Gulton Industries in 1960. In England and in Russia, industrial flowmeters have been developed starting in 1959 and 1960. In non-industrial applications, such as blood

flow and wind velocity measurement, the ultrasonic method has become accepted for research work. For large pipes and channels, ultrasonic flow measurement has become increasingly accepted,²¹ as a result of the capability to determine flow rate accurately without calibration.

Since 1961, judging by the published literature, there has been greater emphasis in this country on ocean current measurement, large conduit application, and blood flow than on industrial applications. In Russia, however, the situation is quite different. Recent Russian publications have contained many excellent theoretical papers which have expanded the well known relationships for travel time difference flowmeters to include significant secondary effects. Their investigations cover the basic variations of the travel time difference method specifically applied to liquid flow in cylindrical pipes. Their insight implies considerable practical experience, and this is verified by many references to particular models. One publication²² presents basic information on metering section design for five specific instruments and electronic block diagrams for four of these.

The Russians have classified ultrasonic flowmeters extensively and have rated them for various types of service.²³ They have concentrated on the difficult applications, such as metering corrosive liquids and slurries. Consistent with their emphasis on difficult fluids, smooth-walled metering sections, with resulting refraction of the acoustic beam, are incorporated in many of the specific designs,²² and considerable analysis has been devoted to this method.^{24,25} They produced the first book on ultrasonic flowmeters²⁶ in 1967.

4. ADVANTAGES

An ideal flowmeter should provide an output proportional to volume flow rate, should be applicable to any fluid, and should not obstruct the flow. It should also be accurate and fast responding. Ultrasonic flowmeters can meet these requirements. As with any instrument, however, the overall performance depends on how well the principle is implemented. Precision in design and manufacture of the mechanical and acoustic portions of the system, and accuracy and stability in the electronics, are necessary to realize the inherent good performance.

Since ultrasonic meters will probably be considered for applications similar to the ones for which turbine and magnetic meters are now used, a comparison with these types is meaningful. Both of these other types of meters are linear and accurate, and magnetic meters are completely obstructionless. But, while these devices have some of the desirable features of ultrasonic meters, they also have serious limitations which ultrasonic meters do not have. Magnetic meters require an electrically conductive fluid and turbine meters present some flow obstruction and have moving parts. Rated accuracy for turbine meters applies only over a 10 to 1 range of flow rates, and velocities in excess of rated cause excessive pressure drop, accelerated bearing wear, and the possibility of the rotor breaking loose. Liquids must be clean and noncorrosive for most turbine meters.

The ultrasonic meter gains advantage over both these other meters as pipe size increases, because transducer mountings add little to size and weight over that for a section of straight pipe. For larger pipes, transducer size can be made relatively smaller and will cause less disturbance of the inside pipe contour. For very small pipes, transducers cannot be

scaled down proportionately because of handling and mounting difficulties. The electronic problem does not vary much in difficulty over the industrial pipe size range. If anything, the larger pipes present an easier problem because time differences are larger for a given velocity.

Ultrasonic flowmeters can measure the flow of any fluid through which sound can be transmitted. The only fluids for which there is any doubt about the sound transmitting ability are those which contain two or more phases. Liquids mixed with gases pose the greatest problem, because gas bubbles effectively block or scatter sound waves. Slurries and pulps are more suitable, because the acoustic impedance mismatch between the particles and the liquid is not excessive, and some sound is transmitted. High attenuation can be overcome by changing some of the system parameters.

The configuration of the metering section can be made truly ideal when acoustic signals are transmitted through the walls, and the inside pipe surface is not modified. To the liquid, the meter is a straight, smooth pipe. The acoustic problem is complicated in this case by refraction, and some tradeoff in accuracy is likely consequence of having this configuration, which can be matched by no other known type of meter. This form is especially suitable for slurries, since any irregularities of contour are subject to excessive wear.

The statement that volume flow rate is usually desired reflects the fact that no flowmeter in widespread use inherently measures mass flow, which often is the ultimately desired quantity. Most meters respond to velocity, averaged in some way over the pipe cross section, and are calibrated in terms of volume flow rate. One of the few flowmeters which actually measures mass flow rate consists of a driven set of tubes which impart

rotational inertia to the fluid, and a set of restrained tubes which measure the torque required to straighten the flow again.²⁷ The need for a mechanical drive connection, the flow obstruction, and the unwieldy nature should limit the use of this meter to applications where mass flow must be measured directly. Because most liquids can be considered incompressible, density is mainly a function of temperature, and compensation can be made on the basis of a temperature measurement. If density does change widely and rapidly, it can be measured separately by ultrasonic means. The ultrasonic density compensation technique described in Section 2 would not be highly accurate, however.

Ultrasonic flowmeters respond to the average fluid velocity along the acoustic path. When transducers are installed in pipe walls, the average velocity in a path crossing the pipe is measured. This average can be shown to vary only slightly relative to the average over the entire cross sectional area. The response of the ultrasonic meter to flow rate is therefore extremely linear, and the whole range can be accurately calibrated by checking one point. This is not to say that individual calibration is required for normal accuracy. An important advantage of the ultrasonic method is that measured time differences are related to velocity by quantities that are accurately known, principally the geometry of the metering section. A single calculated correction factor can convert measured velocity to volume flow rate. Calibration could eliminate small secondary errors.

In summary the advantages of the ultrasonic flowmeter are:

1. High accuracy.
2. Fast response.

3. Negligible flow obstruction.
4. Linearity over wide ranges.
5. Suitability for all fluids.
6. Independence of calibration for normal accuracy.
7. Absence of moving parts.
8. Size and weight advantage increasing with pipe size.
9. Velocity variation averaging across a duct.
10. Suitability for bi-directional or pulsating flows.

REFERENCES

1. D. L. Franklin, D. W. Baker, R. M. Ellis, and R. F. Rushmer, A Pulsed Ultrasonic Flowmeter, IRE Transactions on Medical Electronics, Vol. ME-6, pp. 204-206, December 1959.
2. H. P. Kalmus, Electronic Flowmeter System, The Review of Scientific Instruments, Vol. 25, pp. 201-206, March 1954.
3. H. Suzuki, H. Nakabori, and M. Yamamoto, Ultrasonic Method of Flow Measurement, Water Power, pp. 266-269, July 1968.
4. H. E. Dahlke and W. Welkowitz, A New Ultrasonic Flowmeter for Industry, ISA Journal, Vol. 7, pp. 60-63, October 1960.
5. P. L. Smith and R. W. Fitter, Remote Measurement of Wind Velocity by the Electromagnetic-acoustic Probe, Report No. MR1419 and 420, Midwest Research Institute, Kansas City, Mo., 1961.
6. G. G. Gibney, Device for Measuring Liquid Flow, U. S. Patent 2,936,619, May 17, 1960.
7. W. Kunze, German patent, 1919.
8. O. Rutten, German patent 520484, 1928.
9. C. Chilowsky, Method and Means for the Observation and Measurement of the Speed of a Vessel by Directed Beams of Ultra-audible Waves, U. S. Patent 1,864,638, June 28, 1932.
10. H. E. Hartig and H. B. Wilcox, Fluid Meter, U. S. Patent 1,881,543, October 11, 1932.
11. Completion Report on Acoustic Marine Speedometer, Sect. No. 6.1-sr287-2074, For Office of Scientific Research and Development by Harvard Underwater Sound Laboratory, August 15, 1945.

12. E. W. Barrett and V. E. Suomi, Preliminary Report on Temperature Measurements by Sonic Means. Journal of Meteorology, Vol. 6, pp. 273-276, 1949.
13. R. E. Corby, Acoustic Anemometer-Anemoscope, Electronics, Vol. 23, pp. 88-90, January 1950.
14. W. B. Hess, R. C. Swengel, and S. K. Waldorf, An Ultrasonic Method for Measuring Water Velocity, AIEE Miscellaneous Paper 50-214, August 1950.
15. Electronic Flowmeter, National Bureau of Standards Technical News Bulletin, Vol. 37, pp. 30-31, February 1953.
16. J. Kritz, Ultrasonic Flowmeter, Instruments and Automation, Vol. 28, pp. 1912-1913, November 1955.
17. M. G. Haugen, W. R. Farrall, J. F. Herrick, and E. J. Baldes, An Ultrasonic Flowmeter, Proceedings of the National Electronics Conference, Vol. 11, pp. 465-475, October 1955.
18. W. C. Zarnstorff, C. A. Castillo, and C. W. Crumpton, A Phase-Shift Ultrasonic Flowmeter. IRE Transactions on Bio-Medical Electronics, pp. 199-203, July 1962.
19. M. Greenspan and C. E. Tschiegg, Sing-Around Velocimeter for Liquids. The Review of Scientific Instruments, Vol. 28, pp. 897-901, November 1957.
20. A. E. Brown, Dual Path Ultrasonic Measurement of Fluid Flow, The Review of Scientific Instruments, Vol. 37, pp. 1181-1186, September 1966.

21. C. R. Hastings, The LE Flowmeter: An Application to Discharge Measurement, Presented at Annual Convention of the New England Water Works Assoc., Boston, Mass., September 21-24, 1969.
22. V. A. Nosov, Ultrasonics in the Chemical Industry. (Soviet Progress in Applied Ultrasonics, Vol. 2). Consultants Bureau, New York, pp. 119-128, 1965.
23. G. I. Birger, Elements of the General Theory of Ultrasonic Flowmeters, Measurement Techniques, Vol. 1961, pp. 309-317, April 1961.
24. N. I. Brazhnikov, Contactless Acoustical Measuring Converters for Ultrasonic Flowmeters, Measurement Techniques, Vol. 1965, pp. 925, 932, October 1965.
25. N. I. Brazhnikov, Theoretical Analysis of the Basic Parameters of Ultrasonic Flowmeters. Measurement Techniques, Vol. 1966, pp. 1053-1060, August 1966.
26. I. N. Kanevskii, Review of a book "Ultrasonic Flow Meters" by G. I. Birger and N. I. Brazhnikov, Soviet Physics - Acoustics, Vol. 12, pp. 441-442, April - June 1967.
27. J. D. Goodrich, Measuring Fluid Flow, Control Engineering, Vol. 9, pp. 111-115, September 1962.
28. J. Obraz, An Ultrasonic Pulse Flowmeter, Technical Digest, pp. 632-636, October 1968.
29. R. A. Lester, Double Frequency Transducer, U. S. Patent 3,109,112, October 29, 1963.
30. R. E. Fischbacher, Ultrasonic Flowmeter Design, Control, Vol. 5, pp. 93-97, January 1962.

31. J. R. Frederick, Ultrasonic Engineering, John Wiley and Sons, Inc., New York, 1965.
32. B. Banks, G. E. Oldfield, and H. Rawliding, Ultrasonic Flaw Detection in Metals, Illiffe Books, London, 1962.
33. D. K. Whirlow and L. G. Wright, LE (Leading Edge) Flowmeter: A Unique Device for Open Channel Discharge Measurement, Presented at Symposium on Hydrometry, Koblenz, Fed. Rep. of Germany, September 13-19, 1970.
34. J. Kritz, An Ultrasonic Flowmeter for Liquids, Paper No. 55-16-3, I.S.A. Proceedings, Vol. 10, Part II, pp. 55-16-3-1 to 55-16-3-6.
35. G. I. Birger, Certain Problems in Calibrating Ultrasonic Flowmeters, Measurement Techniques, Vol. 1962, pp. 872-874, October 1962.
36. S. S. Kivilis and V. A. Reshetnikov, Hydromechanical Error in Ultrasonic Flowmeters, Measurement Techniques, Vol. 1965, pp. 1057-1059, November 1965.
37. R. E. Fischbacher, An Ultrasonic Flowmeter for Process Liquids, Paper G-2, Proceedings of International Symposium on Flow Measurement in Closed Circuits, East Kilbride, pp. 687-700, 740-743, September 1960.
38. C. R. Hastings, LE Flowmeter - A New Device for Measuring Liquid Flow Rates, Westinghouse Engineer, Vol. 28, pp. 183-186, November 1968.
39. L. J. Hooper, Evaluation of the Westinghouse Leading Edge Flowmeter System, Report ARL-M71-83, Alden Research Laboratories, Worcester Polytechnic Inst., Holden, Mass., March 1971.
40. J. T. Malone and D. K. Whirlow, Fluid Flow Measurement System, U. S. Patent 3,564,912, Feb. 23, 1971.

APPENDIX B

A partial list of Manufacturers of Ultrasonic Flowmeters. "Reprinted from the September 1974 issue of Instrumentation Technology, Copyright 1976 by Instrument Society of America."

Table I: Vendors of Ultrasonic level and flow instruments

Badger Meter, Inc. Precision Products Div. 6116 East 15th St. Tulsa, OK 74115	(918) 836-8411 LT,FT	National Sonics Div. Envirotech Corp. 120 Milbar Blvd. Farmingdale, NY 11735	(516) 694-5110 LS
Bindicator Co. 1915 Dove St. Port Huron, MI 48060	(313) 987-2700 LS,LT	NUSonics, Inc. P.O. Box 248 9 Keystone Place Paramus, NJ 07652	(201) 265-2400 FT
Conrac Corp. P.O. Box 32 Caldwell, NJ 07006	(201) 575-8000, ext. 262 LS	Oceanography International Corp. P.O. Box 2980 College Station, TX 77840	(713) 846-7721 FT
Controlotron Corp. 111 Bell St. West Babylon, NY 11704	(516) 249-4400 LS,LT,FT	REFAC Technology Development Corp. 122 East 42nd St. New York, NY 10017	(212) 687-4741 FT
Datasonics Corp. 1437 Water St. Port Huron, MI 48060	(313) 982-8551 LS,LT,FT	Saratoga Systems, Inc. P.O. Box 266 Cupertino, CA 95014	(408) 257-7120 FT
Delavan Electronics, Inc. 14441 North 73rd St. Scottsdale, AZ 85254	(602) 948-6350 LS	Scarpa Laboratories, Inc. Brainy Boro Station Metuchen, NJ 08840	(201) 549-4260 FS,FT
Endress + Hauser Inc. 2350 Endress Place Greenwood, IN 46142	(317) 535-7138 LS,LT	C. W. Stevens, Inc. P.O. Box 619 Kennett Square, PA 19348	(215) 444-0616 LT
Inventron Industries, Inc. 4005 W. Jefferson Blvd. Los Angeles, CA 90016	(213) 731-2507 LS,LT	Western Marine Electronics (WESMAR) 905 Dexter Ave. North Seattle, WA 98109	(206) 285-2420 LS,LT,FS
Milltronics Ltd. 730 The Kingsway Peterborough, Ont. K9J 6W6 Canada	(705) 745-2431 LT	Notes: LS level switches LT level transmitters FS flow switches FT flow transmitters	

Westinghouse Oceanic Division (301) 765-5253
P. O. Box 1488, MS 9R30
Annapolis, MD 21404

APPENDIX C

A collection of early correspondence and memoranda relating to feasibility tests of applying the ultrasonic flowmeter to sodium flow in the EBR-II reactor.

ARGONNE NATIONAL LABORATORY
IDAHO FACILITIES

July 15, 1970

To: H. Hooker

From: H. W. Buschman *HWB*

Subject: Proof Test of Ultrasonic Flowmeter - Results of Test EX 48-70

Transmitted for your information are the results of EBR-II Experimental Procedure No. EX 48-70, "Ultrasonic Flowmeter - Performance Evaluation." The test was performed at EBR-II with Idaho Nuclear Corporation (INC) personnel having responsibility for providing the test equipment, conducting the test, and reporting the results. The attached INC report, ARA-12-70, presents their evaluation of the test. As observer and coordinator of the test, some comments by myself appear appropriate.

First, the test device was installed, checked out, and later removed without affecting normal plant operation. The sodium pipe was hot and the plant operating during these periods. No difficulties were encountered.

Second, the intent of the test was for the purpose of demonstrating the feasibility of using the ultrasonic technique for measuring flow rates in sodium piping systems without having to penetrate the pipe wall. No attempt was made to optimize the system from an accuracy viewpoint. However, before running the test, it was predicted that the accuracy would be within 10% of the actual maximum flow. It may be observed on the attached figure that this accuracy objective was met.

Third, the best accuracy that can be obtained using the ultrasonic technique depends on three things:

- (1) The resolution time of the instrumentation.
- (2) How well the geometry of the installation is known.
- (3) The error associated with not knowing the exact velocity profile in the pipe.

Obviously, the larger the pipe and the higher the flow rate, the more accurate the measurement may be made. At present, it appears impractical to use this technique on piping less than 4" in diameter. With piping 1-ft in diameter and larger, it would appear possible to measure flow rates more accurately by the ultrasonic technique than by any of the presently available ordinary techniques.

Fourth, the ultrasonic flowmeter is no longer a developmental item nor is it a laboratory plaything. At least one company (Westinghouse) is marketing a system based on the ultrasonic technique.

In conclusion, the results of the test which was conducted at EBR-II verify that flow measurement may be accurately made in sodium piping systems. Further, it would appear that this technique will soon find much wider application than it currently is receiving. It has the distinct advantages of:

- (1) No head loss
- ? (2) No calibration required
- (3) No penetration required in the pipe wall

The cost of an ultrasonic system varies greatly with its application and it may or may not be an advantage. With systems containing large pipes and correspondingly high volumetric flow rates where several measurements are to be made, it is possible to time share the electronic and read-out equipment, therefore, the cost will be attractive. On small piping, where only one measurement is to be made, the cost probably is prohibitively high.

HWB:md

cc: B. C. Cerutti
G. E. Deegan
J. F. Koenig
H. Lawroski
K. J. Moriarty
R. E. Rice
C. Scheibelhut
W. R. Wallin

TEST RESULTS (EX 43-10)

ULTRASONIC FLOWMETER

IDAHO NUCLEAR CORPORATION

P. O. BOX 1845

IDAHO FALLS, IDAHO 83401

208-522-6640

LD-208-526-0111

June 25, 1970

Ultrasonic Flowmeter for
Liquid Sodium
ARA-12-70

Mr. H. W. Buschman
Argonne National Laboratory
Idaho Site E T-6
P. O. Box 2528
Idaho Falls, Idaho 83401

Dear Mr. Buschman:

Please find attached the report "A Test at EBR II to Determine the Feasibility of the Ultrasonic Pulse Propagation Time Technique to Measure the Flow of Liquid Sodium", which completes the study asked for on Work Request 26-2081.

The test showed the ultrasonic method for measuring flow to be possible and advantageous. With a little development, instrumentation could be designed to furnish an analog or digital output directly proportional to flow. Using the accuracy equation in the report, for an 8 in. pipe with a velocity of 15 ft/sec, the accuracy would be approximately $\pm .6$ ft/sec. As outlined in the report, there are many advantages over present methods. The pipe walls do not have to be penetrated; therefore, no reactor downtime is required for installation. Also, calibration is not required.

Very truly yours,

A. E. Arave
Instrumentation & Control Branch

AEA:md

Attachment

cc: H. W. Buschman
C. M. Rice

A TEST AT EBR II TO DETERMINE
THE FEASIBILITY OF THE ULTRASONIC PULSE
PROPAGATION TIME TECHNIQUE TO
MEASURE THE FLOW OF LIQUID SODIUM

INTRODUCTION

Ultrasonics is a powerful tool for making measurements in environments where conventional detectors will not operate. Ultrasonic signals may be transmitted by metal rods and vessel walls into the media to be measured. Environmentally sensitive components associated with ultrasonic systems may be located remotely from hostile environments. Also, penetration of a scaled system is simplified by not needing electrical penetrations.

Flow can be measured by transmitting ultrasonic signals across the pipe at an angle. The signal is transmitted through the pipe wall into the flow media. Changes in the transmission time across the pipe due to flow velocity are detected. Temperature effects are readily cancelled by comparing two signals obtained by transmitting in opposite directions over the same transmission path.

Tests with ultrasonic transducers attached to the outside wall of the ATR primary piping have established that water flow can be measured by this technique. To determine if this technique was also possible with liquid sodium, the test described below was conducted at EBR II on a 12 inch liquid sodium line.

TEST DESCRIPTION

A sketch of the transducer mounting configuration on the pipe is shown in Figure 1. The transducer is a 1 in. diameter, 6 in. long carbon steel transmission rod with a 4 MHz compressional stress wave introduced into one end of the rod with a 20 mil thick, 0.5 in. diameter PZT-5 disc; at the other end of the rod a 45° cut allows the transmitted signal in the rod to enter the pipe at an angle to transmit a signal across the pipe at an angle. A 5/8 in. diameter rod is welded to the transmission rod perpendicular to the 45° cut to hold the transmission rod in place and to apply enough pressure to couple the ultrasonic signal into the pipe wall. The PZT-5 crystal is kept at operating temperature by placing a cover over the transmission rod and forcing cooling air over the rod. A small air flow keeps the crystals below 135°F.

The block diagram in Figure 2 shows the instrumentation used to measure the

difference in the propagation time in the liquid sodium due to flow and to measure the propagation velocity in the liquid sodium. The difference in the propagation time was obtained by switching the transducers manually back and forth between the transmit and receive mode. The signals were delayed and expanded on the oscilloscope screen to obtain good resolution. A polaroid film was exposed with the switch in both positions for different transmission directions across the pipe. The time difference was then obtained from two traces exposed on the film.

The transmission time in the liquid was obtained by measuring the transmission time of the echo in the rods, adding to it a calculated transmission time in the pipe wall and subtracting the total from the transmission time between crystals.

The above procedure was repeated for each of the four flow rates and recorded as shown in Table I. The approximate time required to establish a constant flow, after changing reactor power to change flow, was 45 minutes.

ANALYSIS OF DATA OBTAINED

It must be noted that the purpose of this test was to determine the ability to obtain signals that are sensitive to flow, not to determine the accuracy of the ultrasonic method for measuring flow. The accuracy of a measurement is determined by the electronic design and can be predicted by theoretical calculations. To keep the cost down for this feasibility study, an oscilloscope was used to measure the transmission time with expectations of only being able to make a flow measurement within 10% of maximum flow. In this light the test was a success in that flow was measured to within 10% of the loop electromagnetic flowmeter measurement.

THEORETICAL EXAMINATION OF ERRORS

Examination of the ultrasonic flow equation, $V = \frac{\Delta t c^2}{2d_2}$ (see Appendix I) shows that the accuracy with which the velocity can be measured can be expressed as

$$\frac{\Delta Q}{Q} = \frac{\Delta V}{V} = \sqrt{x_t^2 + K_d^2 + K_c^2 + K_v^2}$$

where

$$x_t = \frac{\Delta(\Delta t)}{\Delta t} = f(d, V) \quad \text{This can be expressed as } \pm .34/d_3 V \text{ ft}^2/\text{sec or as}$$

$$\pm \frac{120d_3}{Q} \text{ gpm/ft where } d_3 \text{ is the diameter of the pipe in ft, } V \text{ is the velocity being}$$

TABLE I
Measured and Calculated Data

Reactor Power	0	81.2%	63.6%	48.0%	30.8%
Venturi Flow Measurement (gpm)	0	4750	4250	3720	2730
EM Flow Measurement (gpm)	0	4740	4240	3680	2770
Ultrasonic Time At (nsec)	0	126	109	98	80
Measurement time in liquid (μsec)	135.58	140.70	139.10	138.50	138.30
(a) Calculated Flow from Time (gpm)	0	4483	3952	3591	2947
Measurements (ft/sec)	0	12.74	11.22	10.20	8.37
(b) Calculated Propagation Velocity from Time Measurements (ft/sec)	7690	7400	7170	7510	7530
(c) Propagation Velocity from Literature (ft/sec)	7792	7658	7683	7711	7738
Liquid Sodium Temperature (°F)	700	845	818	788	760

Footnote:

(a) Volumetric Flow Rate = $\frac{At \cdot c^2 \cdot K_1 K_2}{2d}$ (Sec Appendix I)

where At = the difference in propagation time

c = transmission path length/propagation time in liquid.
(path length = 12.48 in.)

d = the distance along the pipe (3.10 in.)

K₁ = velocity profile correction factor (.954)

K₂ = velocity to volumetric conversion factor for 12 in. pipe
(352 gal, sec/ft, min)

(b) See "c" in (a)

(c) ANL Report 7323

measured in ft/sec or when using volumetric flow rate Q is in gpm. These constants assume 150 MHz circuitry and the use of shear waves.

$$K_d = \frac{\Delta d_2}{d_2}$$

This is the ^{error in} accuracy of placing the transducer along the pipe.

This constant will be less than .01.

$$K_c = \frac{2\Delta c}{c}$$

The accuracy of the velocity of propagation measurement is a function of how well the distance across the pipe is known and how well the propagation time across the pipe and in the transmission rod can be measured. The constant, K_c , is less than .01.

K_v = The error due to not obtaining the proper average of the velocity profile.

The velocity profile is present in the pipe in cylindrical configuration and the ultrasonic signal averages the profile as if it were flow between two parallel plates. For turbulent flow, theoretical calculations show that a correction factor can be applied to the measurement that will reduce the error to less than .02 of the flow being measured. By placing more transducers on the pipe wall the velocity profile can be measured and the effect upon the flow measurement automatically compensated for to better than .01 of the flow being measured.

CONCLUSIONS

From the results of the test it is evident that ultrasonic technique employed is an excellent way of measuring flow. As the pipe size increases the accuracy increases for constant velocity but decreases for constant flow rate. The advantages are quite obvious:

1. No penetration of the pipe wall.
2. Temperature effects on the detection technique are compensated for.
3. No calibration is required.
4. Can be repaired without reactor downtime.
5. No head loss.
6. Linear output.

The design of the experimental test section was sufficient to show the capability of the technique. Improvements need to be made for future installations:

1. For optimum coupling and accurate definition of distance between transducers, the face of the transducer coupling to the pipe must be reduced to a smaller area.
2. Transmission must take place in both directions over the same path at the same time to prevent temperature effect time shifts while switching and to allow a time comparison without using memory techniques.
3. By using shear wave transducers the path length along the pipe between transducers can be doubled. This will double the accuracy of the Δt measurement.
4. By employing high temperature crystals the need for air cooling can be eliminated.

By using the improvements outlined above, an ultrasonic flowmeter can be designed for liquid sodium that will have an approximate percent accuracy of:

$\pm \sqrt{\left(\frac{.34}{d_3 V}\right)^2} + .006 \times 100$ or in volumetric units $\pm \sqrt{\left(\frac{120 d_3}{Q}\right)^2} + .006 \times 100$. Using the pipe size and maximum gpm of the test this would be approximately a 3.5% measurement.

Figure 1. EBR II Ultrasonic Flow Meter Configuration

Figure 2. Ultrasonic Flow Detection Test Instrumentation

APPENDIX I

Ultrasonic Flowmeter Theory

Referring to the sketch in Figure 2:

d_1 = transmission path length in liquid sodium

c = speed of sound in liquid sodium, which is a function of temperature

t^- = time of travel along path "d" when flow is decreasing the transmission time

t^+ = time of travel along path "d" when flow is increasing the transmission time

t_1 = time of transmission in transmission rod number one

t_2 = time of transmission in rod number two

V = flow velocity of the liquid sodium

θ = the angle between the transmission path and the direction of flow

The time of transmission in the liquid sodium is

$$t^+ = \frac{d_1}{c - V \cos \theta}, \quad t^- = \frac{d_1}{c + V \cos \theta} \quad (1)$$

The total transmission time in one direction, including transmission time in the transmission rods, is

$$t_t^+ = t_1 + t^+ + t_2 \quad (2)$$

The time of transmission in the opposite direction is

$$t_t^- = t_1 + t^- + t_2 \quad (3)$$

Subtracting equation 3 from 2

$$t_t^+ - t_t^- = t^+ - t^- = \Delta t \quad (4)$$

or

$$t = \frac{2d_1 V \cos \theta}{c^2} \quad \text{where } V \ll c \text{ and } d_1 \cos \theta \text{ is the distance, } (5)$$

d_2 , along the pipe between transducers.

This is the flowmeter equation where the difference in transmission time is directly proportional to the flow velocity of the liquid sodium. All temperature effects, except one second order effect, are cancelled out by using the same path for the bi-directional transmission. This second order effect as a

APPENDIX I

result of changes in the speed of sound in the liquid sodium can be compensated for by measuring the speed of sound. When the temperature of the liquid sodium changes from 300°F to 1200°F, as much as 20% change in the speed of sound squared takes place. To compensate for this temperature effect the speed of sound is measured by subtracting the rod transmission time (t_1, t_2) from the total transmission time, (t_t^+ or t_t^-), obtaining $c = \frac{d_1}{t^+}$ where $V \ll c$.

ATR Ultrasonic Flowmeter ProposalI. Summary

The theory of ultrasonic flow measurements is well established.^{1,2} The reduction of theory to practice is an art in overcoming transmission and noise problems for a particular application. The continuous transmission phase difference concept and the pulse transmission, time difference concept are compared in this report. The results of this comparison, and the evaluation of experience from previous successful ultrasonic sensor development programs^{2,3,4} lead to the conclusion that the pulse transmission, time difference concept is preferable for this application. The problems associated with the "pulse" method are discussed herein. It is concluded that ultrasonic flow sensors and readout electronics that meet the requirements of the ATR flow measurements can be developed.

II. Introduction

Ultrasonics is a powerful tool for making measurements in environments which conventional detectors cannot tolerate. Ultrasonic signals may be transmitted by metal rods and through pipe and vessel walls into the media to be measured. Environmentally sensitive components associated with ultrasonic systems may be located remotely from hostile environments. Also, penetration of a sealed system is simplified by not having electrical penetrations.

Flow can be measured by transmitting ultrasonic signals at an angle across the pipe. Changes in the transmission time due to flow velocity can be detected. Certain temperature effects are readily cancelled by comparing two signals obtained by alternately transmitting in opposite directions along a single transmission path.

Development of a flow sensor hinges upon development of an ultrasonic transducer that will give the desired resolution, accuracy and reliability required for ATR flow measurements. Ultrasonic signal transmission by undesirable alternate paths, such as around the pipe, result in "false" signals that can mask out the desired signal. To eliminate this problem, consideration must be given to designing a transducer mounting which is ultrasonically decoupled from the pipe walls.

The continuous frequency technique is compared to the pulse technique below. It is shown that the pulse method has many advantages, and is the technique proposed for development of an ultrasonic flow sensor.

III. Pulse Method

This method is characterized by transmitting a tone burst of ultrasonic energy across the pipe at an angle, allowing a velocity component to be in the direction of flow (Figure 1).

Only two transducers are required as the signals will be alternately switched electronically between transducers. In each case, the one transducer will be transmitting while the other acts as a receiver and vice versa on alternate cycles. The same path of transmission is used in both cases and the transmission time will be either increased or decreased by the flow of liquid depending on the direction of transmission.

Let

- d = transmission path length in liquid sodium
- c = speed of sound in liquid sodium, which is a function of temperature
- t^- = time of travel along path "d" when flow is decreasing the transmission time
- t^+ = time of travel along path "d" when flow is increasing the transmission time
- t_1 = time of transmission in transmission rod number one
- t_2 = time of transmission in rod number two
- V = flow velocity of the liquid sodium
- θ = the angle between the transmission path and the direction of flow.

The time of transmission in the liquid sodium is

$$t^+ = \frac{d}{c - V \cos \theta}, \quad t^- = \frac{d}{c + V \cos \theta} \quad (1)$$

The total transmission time in one direction, including transmission time in the transmission rods, is

$$t_t^+ = t_1 + t^+ + t_2 \quad (2)$$

The transmission in the opposite direction is

$$t_t^- = t_1 + t^- + t_2 \quad (3)$$

Subtracting equation 3 from 2

$$t_t^+ - t_t^- = t^+ - t^- = \Delta t \quad (4)$$

or

$$\Delta t = \frac{2dV \cos \theta}{c^2} \quad \text{where } V \ll c \quad (5)$$

This is the flowmeter equation where the difference in transmission time is directly proportional to the flow velocity of the media. All temperature effects, except one second order effect, are cancelled out by using the same transmission path. This second order effect, as a result of changes in the speed of sound in the liquid sodium, can be compensated for by measuring the change in the speed of sound. When the temperature of the water changes from 60°F to 170°F, a 5% error is introduced by the change in the speed of sound. It is measured by subtracting the rod transmission time (t_1, t_2) from the total transmission time, (t_t^+ or t_t^-), obtaining $t^+ = \frac{d}{c}$ where $V \ll c$.

The envelope shape of the tone bursts is exponential in rise and fall with a sinusoidal carrier. The carrier frequency is the resonant frequency of the transmitting transducer. Time sharing of the transmitting and receiving components aids in the extraction of undesirable noise, such as reflections and ultrasonic bursts received by other paths. The system receivers are gated on only during the time interval when a received signal is anticipated. The use of this technique becomes more useful when measuring flow in large pipes.

Another feature of the pulse technique is its inherent freedom from zero drift; changes in transmission rod path length are cancelled as previously described and errors due to differences in transmission path do not exist. Also, the pulse technique has a large dynamic range capability. The range is only limited by ultrasonic noise signals by other paths, which in many cases can be eliminated.

The pulse technique does have disadvantages; paramount is the limitation on resolution. Resolution is dictated by the rise time of the transmitted pulse envelope. This disadvantage can be overcome by development of a transducer with a fast rise time. This is done by designing a transducer with a high resonant frequency to obtain the desired resolution for a given path length at the same time taking into consideration that attenuation increases with frequency. Larger pipes make these design requirements much easier to arrive at because of the increase in transmission time.

IV. Continuous Frequency Method Theory

Transmission of ultrasonic energy across the pipe is along two paths. One transmitter and two receivers are used (see Figure 2).

Let d_1, d_2 = transmission path length in liquid sodium

c = speed of sound in liquid sodium which is a function of temperature

t_1^+ = time of travel along path " d_1 " when flow is increasing the transmission time

t_2^- = time of travel along path " d_2 " when flow is decreasing the transmission time

t_3 = time of transmission in the transmission rod

t_4 = time of transmission in the receiving rod #1

t_5 = time of transmission in the receiving rod #2

V = flow velocity of the liquid sodium

θ = the angle between the transmission path and the direction of flow.

The time of transmission in the liquid sodium is

$$t_1^+ = \frac{d_1}{c - V \cos \theta}, \quad t_2^- = \frac{d_2}{c + V \cos \theta}. \quad (6)$$

The total transmission time along path d_1 is

$$t_t^+ = t_3 + t_1^+ + t_4 \quad (7)$$

The total transmission time along path d_2 is

$$t_t^- = t_3 + t_2^- + t_5 \quad (8)$$

Subtracting equation 8 from 7

$$t_t^+ - t_t^- = t_1^+ - t_2^- + t_4 - t_5 = \Delta t \quad (9)$$

Substituting equation 6 into 9

$$\Delta t = \frac{(d_1 + d_2)V\cos\theta}{c^2} + \frac{d_1 - d_2}{c} + (t_4 - t_5) \quad (10)$$

where $V \ll C$.

This is the flowmeter equation for two separate transmission paths as required for the continuous frequency method (phase methods). Time may be converted to degrees if desired. The continuous frequency technique is limited to a phase comparison of 360° between the two signals. When the phase difference goes beyond 360° the zero reference is ambiguous. The first term of the equation is essentially the same as for the pulse method, equation 5. The two additional terms of the equation exist because separate paths are used. In most cases they are of significant magnitude to contribute to errors as temperature changes take place. An idea of how troublesome these errors can be is obtained by letting the velocity of flow become zero. In equation 5, Δt becomes zero. In equation 10

$$\Delta t = \frac{d_1 - d_2}{c} + (t_4 - t_5) \quad (11)$$

If $(d_1 - d_2)$ is 0.012 inch, " c " is 7×10^3 ft/sec and $(t_4 - t_5)$ is zero then $\Delta t = 135$ nanoseconds. If d_1 is 18 inches, then 135 nanoseconds is approximately the time difference generated by a flow of three feet per second. This time offset can be compensated for, but the temperature dependence still introduces an error. A 5% change in " c " will, by this one term alone, introduce an error of 1.25 ft/sec in measuring flow. The term $(t_4 - t_5)$ introduces another error of the same type.

A continuous frequency flowmeter has a drawback not apparent by examination of the theory, but which has always plagued developers of the device. Reflections in the water and signals via the pipe structure with different phase and amplitude add to the flow signal to introduce an error. If all these signals are of constant phase and amplitude, the effect can be compensated for, but the amplitude and phase relationship change with temperature and flow rate, resulting in large errors.

V. Method Comparisons

In comparing the pulse to the continuous frequency method, several significant advantages of the pulse method are apparent.

- A. The temperature effects are minimized by the use of a common transmission path.
- B. The speed of sound is easily measured and can be used to correct the second order effect of temperature variations.
- C. The spurious noise caused by unwanted reflections can be gated out on a time basis.
- D. The effects of changes in transmission rod effective path length are cancelled by the use of the common transmission path resulting in good zero stability.
- E. The pulse method has a larger dynamic range.

The most significant disadvantage of this method is the limited resolution which is improved in large pipes.

VI. Preliminary Tests

A preliminary study was made to evaluate the effect of the ATR water flow conditions on an ultrasonic signal. Signals were sent across the 18" outlet pipe under flow and no flow conditions. The ultrasonic signals were extremely stable, and when transmitting at an angle across the pipe the time of transmission changes observed agreed with theoretical calculations.

VII. Developments

First, a detailed study will be made of the specifications placed upon the system. Then the piezoelectric transducers will be designed to best meet these specifications. The transducers will be tested on a no-flow test section in the laboratory. When a prototype system has been developed, testing under flow conditions will be done at ATR on one of the coolant pipes. Necessary improvements will be made on this prototype and tried before construction of four self-contained flowmeters is started.

The flowmeter instrumentation will be made from reliable integrated circuits with logic circuitry to greatly reduce the possibility of a failure signal due to electrical noise. Interlocks and fail-safe logic will be designed into the system as specified in the work request. Analog and digital outputs will be provided for continuous signal monitoring. Also, a calibration circuit will be provided for the electronics.

REFERENCES

1. A. E. Brown, "Dual Path Ultrasonic Measurement of Fluid Flow", The Review of Scientific Instruments, Vol. 37, No. 9 (September 1966).
2. A. E. Arave, An Ultrasonic Anemometer for Measuring Steam Convection Currents, IDO-17293 (January 1969).
3. A. E. Arave, An Ultrasonic Channel Measuring System, IDO-17290 (January 1969).
4. A. E. Arave, "An Ultrasonic and Sonic System to Make Measurements for the Ex-Reactor Heat Transfer Experiment", Instrument Development Branch Annual Report - 1967, IDO-17269 (August 1968).

Figure 1. Pulsed Ultrasonic Flowmeter Schematic

Figure 2. Ultrasonic Continuous Frequency Flowmeter Schematic

NUMERICAL INTEGRATION OF FLOW PROFILES OVER THE PIPE AREA USING GAUSS'S FORMULA

The classical "on line" flow measurement method used a point by point pitot tube sampling of differential pressure or point velocity across the flow profile. This typical flow analysis extended the point velocity measurements to the full pipe area by assuming uniform velocity in small equal areas around the samples which are summed using polar coordinates. See Fig. D1.

Ultrasonic flowmeters measure the average velocity along a line (beam path). See Fig. D2. Hence, it is appropriate to use rectangular coordinates for summing the contributions across the pipe as shown in Fig. D2. Numerical integration is referred to as quadrature in older mathematical literature. Derivation of Gauss's quadrature formula may be found in Lapidus¹ and Scarborough.²

The most accurate of the quadrature formulas in ordinary use is known as Gauss's formula. In Simpson's and Weddle's formulas the ordinates are equally spaced, but it occurred to Gauss that some other spacing might give a better result. Hence he set for himself this problem:

If the definite integral $\int_a^b f(x)dx$ is to be computed from a given number of values of $f(x)$, just where should these values be taken in order to get a result of the greatest possible accuracy? In other words, how shall the interval (a, b) be subdivided so as to give the best possible result?

It turns out that the points of subdivision should not be equidistant, but they are symmetrically placed with respect to the mid-point of the interval of integration. Gauss found they were the zeros of the Legendre polynomials. The tabulated abscissas and the weight factors are found in Fig. D5 to $n = 24$.³ This elegant result simply requires that the experimenter shall:

- 1 - Select a number of samples n in the interval $-1 \leq x \leq +1$ appropriate to the accuracy desired.
- 2 - Note the abscissa locations, x_i , for n and the corresponding weighting factors, w_i , as tabulated.³ Data samples must be obtained at the x_i locations.

- 3 - Multiply each velocity data sample, u_i , by its weight, w_i , to obtain $w_i u_i$.
- 4 - Sum the weighted data samples and multiply by scaling and correction factors as shown in Figs. D3, D4.

An example for $n = 4$ is shown in Figs. D3, D4.

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1. L. Lapidus, "Digital Computation for Chemical Engineers," McGraw-Hill, 1962.
 2. J. B. Scarborough, "Numerical Mathematical Analysis," Johns Hopkins Press, Baltimore, MD, 1930, 131-139
 3. M. Abramowitz and I. A. Stegun, Eds., N.B.S. Handbook of Mathematical Functions, AMS 55, pp. 887, 916.

TYPICAL FLOW ANALYSIS

FIG. D1

- Point Velocity Measurements
- Equal Areas
- Polar Coordinates

ULTRASONIC FLOW ANALYSIS

FIG. D2

- Line Velocity Measurements
- Fewest Measurements for Maximum Accuracy
- Rectangular Coordinates

APPLICATION OF THEORY

$$Q = skD \sum_{i=1}^4 \frac{w_i \ell_i^2 \Delta T_i \tan \theta_i}{2T_{1i} (T_{1i} + \Delta T_i)}$$

s = Scaling for Residual Error in Gaussian Integration

k = Scaling for Operating Temperature and Pressure as They Affect the Measured As-Built Pipe Dimensions D , ℓ_i and θ_i .

FIG. D3 MULTIPLE PARALLEL PATH ULTRASONIC FLOWMETER FOR GAUSSIAN INTEGRATION

NUMERICAL AREA INTEGRATION

Using Gauss's Quadrature:

$$Q = [w_1 v_1 l_1 + w_2 v_2 l_2 + w_3 v_3 l_3 + w_4 v_4 l_4] D$$

$$w_1 = w_4 = 0.173925$$

$$w_2 = w_3 = 0.32807$$

FIG. D4

NUMERICAL ANALYSIS

Table 25.4 ABSCISSAS AND WEIGHT FACTORS FOR GAUSSIAN INTEGRATION

$$\int_{-1}^{+1} f(x) dx \approx \sum_{i=1}^n w_i f(x_i)$$

Abcissas = $\pm x_i$ (Zeros of Legendre Polynomials)			Weight Factors = w_i		
$\pm x_i$	w_i		$\pm x_i$	w_i	
$n=2$			$n=8$		
0.57735 02691 89626	1.00000 00000 00000		0.18343 46424 95650	0.36268 37833 78362	
$n=3$			0.52553 24099 16329	0.31370 66458 77887	
0.00000 00000 00000	0.88888 88888 88889		0.79666 64774 13627	0.22238 10344 53374	
0.77459 66692 41483	0.55555 55555 55556		0.96028 98564 97536	0.10122 85362 90376	
$n=4$			$n=9$		
0.33998 10435 84856	0.65214 51548 62546		0.00000 00000 00000	0.33023 93550 01260	
0.86113 63115 94053	0.34785 48451 37454		0.32425 34234 03809	0.31234 70770 40003	
$n=5$			0.61337 14327 00590	0.26061 06964 02935	
0.00000 00000 00000	0.56888 88888 88889		0.83603 11073 26636	0.18064 81606 94857	
0.53846 93101 05683	0.47862 86704 99366		0.96816 02395 07626	0.08127 43883 61574	
0.90617 98459 38664	0.23692 68850 56189		$n=10$		
$n=6$			0.14887 43389 81631	0.29552 42247 14753	
0.23861 91860 83197	0.46791 39345 72691		0.43339 53941 29247	0.26926 67193 09996	
0.66120 93864 66265	0.36076 15730 48139		0.67940 95682 99024	0.21908 63625 15982	
0.93246 95142 03152	0.17132 44923 79170		0.86506 33666 88985	0.14945 13491 50581	
$n=7$			0.97390 65285 17172	0.06667 13443 08688	
0.00000 00000 00000	0.41795 91836 73469		$n=12$		
0.40584 51513 77397	0.38183 00505 05119		0.12523 34085 11469	0.24914 70458 13403	
0.74153 11855 99394	0.27970 53914 89277		0.36783 14989 98180	0.23349 25365 38355	
0.94910 79123 42759	0.12948 49661 68870		0.58731 79542 86617	0.20316 74267 23066	
$n=16$			0.76990 26741 94305	0.16007 83285 43346	
0.09501 25098 37637 440185			0.90411 72563 70475	0.10693 93259 95318	
0.28160 35507 79258 913230			0.98156 06342 46719	0.04717 53363 86512	
0.45801 67776 57227 386342			$n=20$		
0.61787 62444 02643 748447			0.18945 06104 55068 496285		
0.75540 44083 55003 033895			0.18260 34150 44923 588867		
0.86563 12023 87831 743880			0.16915 65193 95002 538189		
0.94457 50230 73232 576078			0.14959 59888 16576 732081		
0.98940 09349 91649 932596			0.12462 89712 55533 872052		
$n=24$			0.09515 85116 82492 784810		
0.07652 65211 33497 333755			0.06225 35239 38647 892863		
0.22778 58511 41645 078080			0.02715 24594 11754 094852		
0.37370 60887 15419 560673			$n=24$		
0.51086 70019 50827 098004			0.15275 33871 30725 850698		
0.63605 36807 26515 025453			0.14917 29864 72603 746788		
0.74633 19064 60150 792614			0.14209 61093 18382 051329		
0.83911 69718 22218 823395			0.13168 86384 49176 626898		
0.91223 44282 51325 905868			0.11819 45319 61518 417312		
0.96397 19272 77913 791268			0.10193 01198 17240 435037		
0.99312 85991 85094 924786			0.08327 67415 76704 748725		
$n=24$			0.06267 20483 34109 063570		
0.06405 68928 62605 626085			0.04060 14298 00386 941331		
0.19111 88674 73616 309159			0.01761 40071 39152 118312		
0.31504 26796 96163 374387			$n=24$		
0.43379 35076 26045 138487			0.12793 81953 46752 156974		
0.54542 14713 88839 535658			0.12583 74563 46828 296121		
0.64809 36519 36975 569252			0.12167 04729 27803 391204		
0.74012 41915 78554 364244			0.11550 56680 53725 601353		
0.82000 19859 73902 921954			0.10744 42701 15965 634783		
0.88641 55270 04401 034213			0.09761 86521 04113 888270		
0.93827 45520 02732 758524			0.08619 01615 31953 275917		
0.97472 85559 71309 498198			0.07334 64814 11080 305734		
0.99518 72199 97021 360180			0.05929 85849 15436 780746		
			0.04427 74388 17419 806169		
			0.02853 13886 28933 663181		
			0.01234 12297 99987 199547		

Compiled from P. Davis and P. Rabinowitz, Abcissas and weights for Gaussian quadratures of high order, J. Research NBS 56, 35-37, 1956, RP2645; P. Davis and P. Rabinowitz, Additional abscissas and weights for Gaussian quadratures of high order. Values for $n=64, 80$, and 96 , J. Research NBS 60, 613-614, 1958, RP2875; and A. N. Lowan, N. Davids, and A. Levenson, Table of the zeros of the Legendre polynomials of order 1-16 and the weight coefficients for Gauss' mechanical quadrature formula, Bull. Amer. Math. Soc. 48, 739-743, 1942 (with permission).