Propagation of Normal Zones in Composite Superconductors

Lawrence Dresner

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OAK RIDGE NATIONAL LABORATORY

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PROPAGATION OF NORMAL ZONES IN COMPOSITE SUPERCONDUCTORS*

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Abstract

This paper describes calculations of propagation velocities of normal zones in composite superconductors. Full accounting is made for (1) current sharing, (2) the variation with temperature of the thermal conductivity of the copper matrix, and the specific heats of the matrix and the superconductor, and (3) the variation with temperature of the steady-state heat transfer at a copper-helium interface in the nucleate-boiling, transition, and film-boiling ranges. The theory, which contains no adjustable parameters, is compared with experiments on bare (uninsulated) conductors. Agreement is not good. It is concluded that the effects of transient heat transfer may need to be included in the theory to improve agreement with experiment.

*Research sponsored by the Energy Research and Development Administration under contract with Union Carbide Corporation.

PROPAGATION OF NORMAL ZONES IN COMPOSITE SUPERCONDUCTORS*

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Introduction

The earliest attempts to calculate the propagation velocity of normal zones were those of Cherry and Gittleman¹ and Broom and Rhoderick² in 1960. These authors considered propagation of a normal zone in an unclad superconductor in contact with boiling helium. Because superconductors of technological interest today are composites consisting of copper- or aluminum-clad superconducting strands, these early calculations are not of direct use. In 1967, Keilin, Klimenko, Kremlev, and Samoilov³ presented a calculation of the propagation velocity in a composite superconductor. The theory described in their paper takes into account current sharing between the substrate and the superconductor; but the results they presented were only for the case of no current sharing, i.e., the case of an instantaneous jump of all the current from the superconductor to the substrate at one point. In 1973, Altov, Kremlev, Sytchev, and Zenkevitch⁴ refined the results of Keilin et al. by including the effects of current sharing. Whereas all of the previous studies had produced results in the form of closed formulas, the work of Altov et al. had to be done numerically.

All of the above studies were carried out under the assumption of constant material properties (that is to say, constant specific heat and

* Research sponsored by the Energy Research and Development Administration under contract with Union Carbide Corporation.

thermal conductivity) and a constant heat transfer coefficient. Attempts were made in the past to include the temperature variation of the material properties. For example, Whetstone and Roos⁵ in their calculations of propagation velocities in Nb-Zr alloys included the temperature variation of the specific heat, the thermal conductivity, and the electrical conductivity. However, their experimental samples were potted or isolated by cellophane tape from the helium bath; and in their calculations they ignored transverse heat transfer to the helium altogether. Furthermore, their calculations referred to unclad wires and consequently did not need to take current sharing into account. Thus, while instructive, the work of Whetstone and Roos is not of direct use here.

Maddock, James, and Norris⁶ calculated the current at which the propagation velocity in composite superconductors changes sign, i.e., the current at which normal zones stop recovering and start propagating. They took into account the details of the heat transfer process in pool-boiling helium, as well as current sharing and the temperature variation of the thermal conductivity. The only drawback of their paper is that it is limited to the calculation of only one point on the curve of propagation velocity vs series current. Their method is extremely interesting, however, and will be used as the jumping-off point of the present work.

Recently, Dresner, Miller, and Donaldson⁷ presented measurements and calculations of normal zone propagation in bare, insulated, and potted composite superconductors. Their calculations took into account fully current sharing and the temperature variation of the material properties, but assumed the heat transfer coefficient to be constant. When they tried to fit curves of velocity vs current, they found

that "the experimental points for the insulated wire ... can be fitted fairly well by a single unique choice of overall heat transfer coefficient" but that "no single heat transfer coefficient can ever reproduce the experimental points for the bare wire." It seems clear from this that the next step in the comparison of theory and experiment, at least for bare conductors, is to incorporate into the theory a heat transfer coefficient more realistic than a simple constant.

This paper describes calculations of propagation velocities in composite superconductors in which this is done. Full accounting is made for (1) current sharing, (2) the variation with temperature of the thermal conductivity of the matrix (copper) and the specific heat of the matrix and the superconductor (NbTi); and (3) the variation with temperature of the heat transfer at a copper-helium surface in the nucleate-boiling, transition, and film-boiling ranges.

Theory - Part 1: Basic Equations

We begin by writing a heat balance for a length, Δx , of a bare composite superconductor in a helium bath (see Fig. 1). The local temperature $T(x,t)$ is assumed to vary with time, t , and with axial position, x , but to be uniform in the transverse direction. The heat balance is as follows:

$$A\Delta x S \frac{\partial T}{\partial t} = Q_J A \Delta x - Q_P \Delta x - A \Delta x \frac{d}{dx} \left(-k \frac{dT}{dx} \right) \quad (1)$$

where

- A = the cross-sectional area of the conductor (m^2)
- S = the specific heat of the conductor per unit volume
 $(J m^{-3} K^{-1}) [= f S_{Cu} + (1-f) S_{NbTi}]$, where f is the volume fraction of copper]

Q_J = the Joule heat produced per unit volume of conductor
($W m^{-3}$)

Q = the heat flux per unit surface area from the conductor
into the helium ($W m^{-2}$). Q is a function of the temperature
difference between the conductor and the helium bath.

P = the cooled perimeter of the conductor (m)

k = the thermal conductivity of the conductor ($W m^{-1} K^{-1}$)
($= f k_{cu}$).

We are looking for a traveling wave solution of Eq. 1 of the form
 $T = T(x + vt)$, which represents a wave traveling from right to left (see
Fig. 2). Far to the right of the wave front, T is asymptotic to the
constant temperature T_{as} ; far to the left of the wave front T is asymptotic
to the temperature T_b of the helium. For such a traveling wave $\partial T / \partial t = v \partial T / \partial x$,
and Eq. 1 becomes

$$\frac{d}{dx} \left(k \frac{dT}{dx} \right) - vS \frac{dT}{dx} + \left(Q_J - \frac{QP}{A} \right) = 0 \quad (2)$$

At this point we note that the last term on the right, $Q_J - QP/A$,
depends explicitly only on temperature. We can, therefore, reduce the
order of this differential equation by one by using the substitution of
Maddock, James, and Norris:⁶

$$s = k \frac{dT}{dx} \quad (3)$$

Equation 2 then becomes

$$s \frac{ds}{dT} - vSs + k \left(Q_J - \frac{QP}{A} \right) = 0 \quad (4)$$

The voltage drop in the distance Δx is $I_{cu} \rho \Delta x / fA$, where I_{cu} is the part of the series current I flowing in the conductor. In the superconducting region, $I_{cu} = 0$; in the normal region, $I_{cu} = I$; and in the current sharing region, I_{cu} can be calculated from the assumed linear relation between critical current and temperature:⁶

$$\frac{I_{cr}(T)}{I_{cr}(T_b)} = \frac{T_{cr} - T}{T_{cr} - T_b} \quad (5)$$

Since the voltage drop in the superconductor is the same as that in the copper,

$$A \Delta x Q_J = I \left(\frac{I_{cu} \rho \Delta x}{fA} \right) \quad (6a)$$

or

$$Q_J = \frac{I(I_{cu})\rho}{fA^2} \quad (6b)$$

Then Eq. 4 becomes

$$s \frac{ds}{dT} - vSs + \frac{k_b P}{T_b A} T \left[\frac{I(I_{cu})\rho}{fAP} - Q \right] = 0 \quad (7)$$

where we have taken k to be equal to $k_b T / T_b$, and where k_b is the value of k at $T = T_b$.⁶ More will be said about this assumption in a later section.

Theory - Part II: Boundary Conditions

Far to the left of the propagating front, the temperature of the conductor approaches the constant value T_b . There $s = 0$. The last term on the right must then also vanish. It does: for when $T = T_b$, $I_{cu} = 0$ and $Q = 0$.

Far to the right of the propagating front, the temperature approaches the constant value T_{as} . Again $s = 0$, and the first two terms in Eq. 7 vanish. Hence T_{as} is one of the solutions of the equation

$$\frac{I(I_{cu})\rho}{fAP} = Q \quad (8)$$

Equation 8 always has an even number of solutions, i.e., 0, 2, 4 (see Fig. 3). When more than one solution exists, we must determine which one to use. This we do by first noting that the solutions of Eq. 8 are the equilibrium points of Eq. 1 and that not all of them are stable. Only stable solutions of Eq. 8 can be used.

We linearize Eq. 1 in order to see which solutions of Eq. 8 are stable:

$$s \frac{\partial(\delta T)}{\partial t} = \frac{P}{A} \left[\frac{d}{dT} \left(\frac{I(I_{cu})\rho}{fAP} - Q \right) \right] \delta T + k \frac{d^2}{dx^2} (\delta T) \quad (9)$$

where δT represents a small departure from T_{as} . All other quantities are evaluated at $T = T_{as}$. If we impose a perturbation δT that is proportional to $\cos(kx)$ or $\sin(kx)$, we find

$$s \frac{\partial \delta T}{\partial t} = \left\{ \frac{P}{A} \left[\frac{d}{dT} \left(\frac{I(I_{cu})\rho}{fAP} - Q \right) \right]_{T=T_{as}} - k^2 \right\} \delta T \quad (10)$$

The perturbation will die away if

$$\frac{d}{dT} \left(\frac{I(I_{cu})\rho}{fAP} - Q \right)_{T=T_{as}} < \frac{A}{P} k^2 \quad (11)$$

Otherwise it will grow. The only solutions of Eq. 8 that are stable against all perturbations are those for which the left side of Eq. 11 is negative, i.e., those solutions at which the curve of heat production $I(I_{cu})\rho/fAP$ crosses from above the curve of heat loss Q to below it with increasing temperature. Thus in Fig. 3 only the points marked with arrows are stable solutions of Eq. 8.

Since $dQ/dT < 0$ in the transition boiling range, while the slope of the heat production curve is always ≥ 0 , no stable equilibrium solution of Eq. 8 can occur in the transition boiling range. However, as Maddock, James, and Norris⁶ point out, all temperatures between T_b and T_{as} occur in the temperature profile at the wave front; for dT/dx must not be infinite at any point in order to prevent the axial heat flux from becoming infinite.

The experiments analyzed in this paper correspond to either Fig. 3(a) or Fig. 3(d).

Theory - Part III: Numerical Method

We must solve the differential equation (7) under the boundary conditions (1) $s = 0$ at $T = T_b$ and (2) $s = 0$ at $T = T_{as}$. These boundary conditions can only be satisfied simultaneously for the correct value of v . We find the correct value of v by trial and error. We assume a value of v and integrate from T_{as} to T_b . If v is too small, s will be positive over the (open) interval $T_b < T < T_{as}$. If v is too large, s will change sign somewhere in this interval. Only for the correct v will s be zero at $T = T_b$ as well as at $T = T_{as}$.

In order to start a numerical integration of Eq. 7, we need a value of s away from the equilibrium point $T = T_{as}$, $s = 0$. We get one by

solving Eq. 7 in a small enough neighborhood of T around T_{as} that we can use the lowest-order non-vanishing approximation to the coefficients:

$$s \frac{ds}{d\tau} - v S_{as} s - \frac{k_b P}{T_b A} T_{as} \left(\frac{dQ}{dT} \right)_{as} \tau = 0 \quad (12)$$

where $\tau = T - T_{as}$ and the subscript "as" indicates that the quantity should be evaluated at T_{as} . [The derivative of $I(I_{cu})\rho/fAP$ drops out, since I_{cu} is constant and equal to I in the neighborhood of $T = T_{as}$]. This equation has the form

$$s \frac{ds}{d\tau} - as - b\tau = 0 \quad (12a)$$

where a and b are constant positive coefficients given by comparison with Eq. 12. If we introduce a new variable y , equal to $s/\sqrt{b}$, we find

$$\frac{dy}{d\tau} = m + \frac{\tau}{y}, \quad m = \frac{a}{\sqrt{b}} \quad (12b)$$

Owing to the nonlinearity of Eq. 12b, the best way to study its solutions is to plot its direction field, part of which is shown in Fig. 4 for $m = 1$. The derivative $dy/d\tau = \infty$ when $y = 0$; $dy/d\tau = 0$ on the line L_0 : $y = -\tau/m$; and $dy/d\tau = m$ when $\tau = 0$. The derivative $dy/d\tau = y/\tau$ on two perpendicular straight lines L_+ and L_- with respective slopes $\frac{1}{2}(m \pm \sqrt{m^2 + 4})$. These two lines are integral curves of the differential equation (12b). All other integral curves lie entirely in one of the four quadrants defined by L_+ and L_- , and asymptotically approach each of them. L_+ and L_- are the only integral curves which pass through the point $(0,0)$. Since we are looking for the one for which $s > 0$ when $\tau < 0$, we choose L_- . Hence we choose some point on this line as our starting value, e.g.,

$\tau = -|dT|$, $s = \frac{\tau}{2} \sqrt{b} (m - \sqrt{m^2 + 4})$, where $dT = -|dT|$ is the temperature increment used in the stepwise integration of Eq. 8b.

There is a difficulty with this straightforward procedure. As m grows larger, the slopes of the lines L_0 and L_- grow smaller and the two lines approach each other very closely. For example, in Fig. 4 (for which the scales on the y - and τ -axes are the same) L_0 and L_- are inclined at angles of 45° and 32° to the τ -axis, respectively. For $m = 4$, these two angles are 14.0° and 13.3° , respectively, and for $m = 10$, they are 5.65° and 5.71° , respectively. When L_0 and L_- are very close together and close to the negative τ -axis, starting the numerical integration of Eq. 12b at $\tau = -|dT|$ may lead to the development of an instability. The instability is caused by the rapid change of slope $dy/d\tau$ which occurs near the origin in the region included between L_0 and the τ -axis. Taking a starting point further along the line L_- avoids the instability; and because the integral curves eventually approach the line L_- as an asymptote, once away from the neighborhood of the origin the integration is very stable. A similar difficulty occurs in the integration of Eq. 7 and a similar remedy has been used, viz., the integration is started at $T = T_{as} - n|dT|$, $s = \frac{-n|dT|}{2} \sqrt{b} (m - \sqrt{m^2 + 4})$, where n is a sufficiently large integer to assure stability.

Shown in Fig. 5 are typical curves of s vs T obtained by Runge-Kutta integration of Eq. 7, starting as described above. When the assumed propagation velocity is too small, the curves remain above the axis for all $T < T_{as}$. For the correct value of v , the curve of s vs T would just fall to zero at $T = T_b$. If v is too large, the curve intersects the T -axis at some point $T > T_b$. Once the curve

has crossed the T -axis there is no reason to continue the integration, since the solution again becomes unstable.

To see this we again expand Eq. 7, this time around $T = T_b$. Since $I_{cu} = 0$ in small neighborhoods of T_b , the behavior of the last term in Eq. 7 is governed by the behavior of Q . In general, Q decreases with decreasing $\tau = T - T_b$ more rapidly than would a linear function, let us say it decreases as $Q_0 \tau^c$, where $c > 1$. Then, near $T = T_b$,

$$s \frac{ds}{d\tau} - v S_b s - \frac{k_b P}{A} Q_0 \tau^c = 0 \quad (13a)$$

which can be reduced to the form

$$\frac{dy}{d\tau} = m + \frac{\tau^c}{y}, \quad c > 1 \quad (13b)$$

We are interested in Eq. 13b only for $\tau \geq 0$.

The derivative $dy/d\tau = 0$ for $y = -\tau^c/m$ (curve L_0 in Fig. 6); $dy/d\tau = \infty$ for $y = 0$; and $dy/d\tau = m$ for $\tau = 0$. The derivative $dy/d\tau$ can only change sign on the τ -axis and on the curve L_0 ; since it is positive on the y -axis, it must be positive everywhere above the τ -axis and in the region between the negative y -axis and the curve L_0 . Elsewhere $dy/d\tau < 0$. There are three families of integral curves as shown in Fig. 6; they are separated by two separatrices S_1 and S_2 , which are the only integral curves to pass through the origin.

We are interested in finding the integral curve of Eq. 7 which becomes S_2 near $T = T_b$. If we choose a value of v too small, the curve of s vs τ lies above S_2 and thus remains positive right down to $\tau = 0$. On the other hand, if v is too large, the curve of s vs τ lies below S_2 . For a sufficiently small value of τ the curve of s vs τ crosses the τ -axis with infinite slope. Owing to the occurrence of very large slopes,

any finite-difference integration scheme becomes inaccurate near the crossing. However, any change in sign of s from positive to negative will be a signal that v is too large.

Material Properties

We take the temperature dependence of the material properties to be the following:

(1) Electrical Resistivity. In the experiments analyzed here, the maximum temperature of the conductor T_{as} was $\lesssim 20$ K. We have therefore taken the resistivity of the copper matrix to be independent of temperature over the temperature range of interest in normal-zone propagation.

(2) Thermal Conductivity. The thermal conductivity of copper is taken to vary directly with temperature, in accordance with the Wiedemann-Franz Law, which is assumed to apply in the background magnetic field.⁸

(3) Specific Heat. The specific heat of copper is taken as $0.628 \times 10^{-2} T + 0.859 \times 10^{-3} T^3$ ($J\ kg^{-1}\ K^{-1}$).⁹ The specific heat of NbTi has been taken from measurements of Zbasnik¹⁰ done at zero field for Nb-48.8 wt % Ti. If we assume that the thermodynamic critical field of the superconductor varies linearly with temperature (which it does for temperatures close to the zero-field critical temperature), it is then possible, using thermodynamic arguments, to calculate the specific heat at non-zero magnetic fields. Figure 7 shows the results of such calculations at 4 and 8 T. The superconducting-normal transition is shown as if it were sharp, although experimentally it is spread over several tenths of a degree.

(4) Heat Transfer at Helium-Copper Surface. Steady-state heat transfer in pool boiling is a complicated matter; and it is by no means clear that steady-state heat transfer coefficients rather than transient

heat transfer coefficients are the correct choice. In the experiments analyzed here, the transition zones at the normal front are only a few centimeters long. With propagation velocities of a few meters per second, the zones pass a fixed point in a few tens of milliseconds at most. Experiments on transient heat transfer¹¹⁻¹² have shown that for a given temperature difference, the heat fluxes which may be sustained for times as short as this are much higher than those which occur in steady state.

We do not have enough information to prescribe a curve of transient heat flux vs temperature difference, but we do have enough information to prescribe a curve of steady-state heat flux vs temperature. By use of the experimental steady-state curve, we obtain a model for comparison with experiment in which all the required data are known and for which all the calculations can be done accurately. Comparison of this model with experiment will tell us at once whether transient heat transfer is important or not.

There is wide variation even in steady-state heat transfer data, due to variations in the orientation and condition of the surface. In this paper we use three reported curves of heat flux vs temperature, viz., those of Brentari and Smith,¹³ of Butler et al.,¹⁴ and of Lyon¹⁵ (see Fig. 8).

Comparison with Experiment

Shown in Fig. 9 are plots of measured propagation velocities vs current, based on data reported in Ref. 7 for a 5.1×10^{-4} -m-diameter, 348-strand composite conductor with a Cu/SC volume ratio of 1.9:1. The resistivity of the copper was measured as 2.6×10^{-10} ohm-m at zero field; at a field of 6.5 T, at which the velocities were measured, the

resistivity is increased by the magneto-resistive effect to 6.06×10^{-10} ohm-m.¹⁶ The critical current has been taken from the measurements of Hampshire et al.¹⁷ and is 53 A.

Shown also are five curves labeled with numbers. These curves are taken from Ref. 7 and are calculations performed with constant heat transfer coefficients equal (in W cm^{-2}) to the labeling numbers. In addition there are three curves, labeled (a), (b), and (c), that have been calculated by the method of this paper using the boiling heat transfer curves of Brentari and Smith, Butler et al., and Lyon.

Neither family of curves agrees well with the experimental points. This seems to be an unequivocal indication that steady-state heat transfer coefficients are inadequate to describe the growth of normal zones (except possibly for very small velocities of propagation).

We can make a crude test of this conclusion by arbitrarily modifying the heat transfer curves in some way intended to simulate transient heat transfer. There is included in Fig. 9 a dotted curve obtained by modifying the heat transfer coefficient of Brentari and Smith by the factor $\exp [\ln 2 (T_{as} - T)/(T_{as} - T_b)]$. This causes the heat transfer coefficient to be twice as great as the steady-state value at the head of the transition zone and to approach the steady-state value far behind the transition zone. Agreement is much improved; and it seems clear that a velocity-dependent correction that increases with velocity could improve agreement even more. Further exertions in this direction seem unwarranted because they involve so much arbitrariness.

Finally, one more thing needs to be taken into account, namely, the latent heat of the normal-superconducting transition. When a superconductor goes from the superconducting state to the normal state

in the presence of an external magnetic field, it absorbs heat. Normal zones grow by axial conduction of heat from the normal towards the superconducting material. The absorption of latent heat by the superconductor decreases this axial transport. Inclusion of the latent heat in the theory should decrease the propagation velocity.

The amendment necessary to the theory already given is simple. At the temperature $T_{cr}(B)$ there is a step change in s given by

$$s_{sc} = s_n - v\ell(1-f) \quad (14)$$

where the subscripts indicate the superconducting and normal sides of the moving boundary at $T = T_{cr}(B)$, and ℓ is the latent heat per unit volume of superconductor. The latter can be calculated under the same assumptions as stated earlier for the calculation of the specific heat, with the following result:

$$\ell = \Delta S[T_{cr}(B)] [T_{cr}(B=0) - T_{cr}(B)] \quad (15)$$

where S is now the specific heat per unit volume of superconductor, ΔS is the jump in S at the transition temperature $T_{cr}(B)$, and $T_{cr}(B=0)$ is the transition temperature at zero external magnetic field.

When the effects of the latent heat modify curve A in Fig. 9, the velocity is decreased everywhere by a factor of 1.2, i.e., the slope of line A is decreased by a factor of 1.2. This is a small reduction compared with the reduction by a factor of 2.9 necessary to get agreement with theory. The effects of the latent heat thus appear to be small. Nevertheless, their inclusion is no burden on the theory and is recommended. It seems clear that the major cause of the discrepancy between theory and experiment is the effect of transient heat transfer.

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Figure Captions

- Fig. 1. Sketch describing the heat balance for a length Δx of a bare conductor.
- Fig. 2. Sketch of the traveling temperature wave in the conductor.
- Fig. 3. Sketch showing the steady states of a conductor in a pool of boiling helium. The ordinate has the dimensions of heat per unit volume; the abscissa is the temperature elevation above the helium bath temperature. The heavy line is the heat loss to the helium; the light line is the Joule heat production. The arrows mark the stable states.
- Fig. 4. Sketch of the direction field and integral curves of Eq. 12b.
- Fig. 5. Part of a family of integral curves obtained by numerical integration of Eq. 7, showing the variation with increasing propagation velocity.
- Fig. 6. Sketch of the direction field and integral curves of Eq. 13b.
- Fig. 7. Calculated specific heat of Nb-48.8 wt % Ti as a function of temperature and magnetic field.
- Fig. 8. Boiling correlations of Brentari and Smith, Lyon, and Butler et al.
- Fig. 9. Comparison of theory and experiment.

FIGURE 1

FIGURE 2

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FIGURE 3

FIGURE 4

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FIGURE 5

FIGURE 6

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FIGURE 7

FIGURE 8

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FIGURE 9

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