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INTRODUCTION

Numerical methods are capable of solving very difficult problems in solid mechanics and gas dynamics. In the design of engineering structures, critical decisions are possible if the behavior of materials is correctly described in the calculation. Problems of current interest require accurate analysis of stress-strain fields that range from a very small elastic displacement to very large plastic deformation. Described here is a finite difference program that solves problems over this range and in two and three space-dimensions and time. A series of experiments and calculations serve to establish confidence in the plasticity formulation. Then the program can be used to design high pressure systems where plastic flow occurs. The purpose is to identify material properties, strength and elongation, that meet the operating requirements. An objective is to be able to perform destructive testing on a computer rather than on the engineering structure. Examples of typical interest are given.

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METHOD

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The important objective of a computer simulation program is to obtain the fundamental understanding of physical phenomena that will permit critical decisions to be made on the design and operation of physical systems. Continuum mechanics offers an approach for describing a large range of physical phenomena. There are three fundamental equations; the conservation of momentum, conservation of mass, and conservation of energy. Closure to the system is provided by constitutive models that describe the behavior of materials. The fundamental conservation laws are expressed by sets of partial differential equations. These equations can be organized so changes in variables associated with a mass point can be interpreted as resulting from a flux through a surface surrounding the mass point. Finite difference operators can be organized in the same way, i.e., in the spirit of divergence theorem. Space derivatives in three dimensions are defined as the summation of the normal components of the flux around an enclosed volume. Thus, for a vector F representing a flux, e.g., heat flow, velocity, stress, and the like, the following integral definitions of the partial derivatives are used:

$$\frac{\partial F}{\partial x} = \lim_{V \rightarrow 0} \int_{(S)} \frac{F \cdot (n \cdot i)}{V} dA,$$

$$\frac{\partial F}{\partial y} = \lim_{V \rightarrow 0} \int_{(S)} \frac{F \cdot (n \cdot j)}{V} dA,$$

$$\frac{\partial F}{\partial z} = \lim_{V \rightarrow 0} \int_{(S)} \frac{F \cdot (n \cdot k)}{V} dA.$$

ef

where dA = element of surface area, $\vec{n}$ = outward normal to surface,
 (s) = boundary of volume v , and $\vec{i}, \vec{j}, \vec{k}$ = unit vectors.

The physical object is characterized by a calculational grid that moves with the flow. The intersection of grid lines are called zone node points and the interior space of the grid is called the zone. The integration path for evaluating the partial derivatives is defined in the grid in two ways. Figure 1a shows the integration path for evaluating the components of the operator, $\nabla \cdot \Sigma'$, where Σ' represents the total stress tensor and is defined in the interior of a zone. The conservation of momentum expressed by $\frac{d\vec{w}}{dt} = \frac{1}{\rho} \nabla \cdot \Sigma'$ relates zone interior parameters to changes in the velocity $\vec{w}$ defined at zone nodes. Space derivatives of the node velocities, Fig. 1b, are used to evaluate the rate of strain tensor, which in turn changes the stress tensor through a constitutive model. Thus, space derivatives of zone interior parameters change zone node parameters and vice versa. The use of this double grid system permits the difference equations to be centered in space and time, giving second-order accuracy to the numerical method.

The finite difference programs HEMP ^[1] and HEMP3D ^[2] solve the three fundamental equations in two or three space-dimensions and in time for given initial and boundary conditions. With the appropriate constitutive model, the computer programs are capable of simulating phenomena ranging from elasticity to dynamic plasticity to compressible gas flow. The constitutive model that describes the material behavior presents the greatest uncertainty. If the material behavior is known from previous experiments, a simulation can be applied to other physical systems that involve this material.

COLLISION OF PLATES

Figure 2 shows a calculation simulating the collision of a copper plate with a steel plate. The elastic-plastic properties of the two materials were previously determined experimentally. This problem was also calculated with hydrodynamic theory by Godunov [3] and represents the geometry for explosive welding. The geometry is also useful for the compaction of materials by the sudden application of a high pressure [4]. A critical design parameter of interest is the collision angle γ , chosen so plates do not rebound after collision. Other important information that can be obtained is the time history of the pressure at the collision interface. Results from explosive welding research [5] show that for this geometry the velocity of the point of contact (closing velocity) should be subsonic compared to the sound speed of the materials. The simulation program can be used to study this requirement.

Figure 3 shows a calculation in two space-dimensions of the collision of two aluminum plates described by an elastic plastic constitutive law. In the one case, the collision angle is chosen so that the closing contact point moves faster than the aluminum sound speed. The two plates are seen to rebound. For the second calculation shown in Fig. 3, the angle is chosen such that the closing velocity is subsonic with respect to the aluminum sound speed. Now the two plates remain in contact after collision.

The explanation is found in Fig. 4, which shows, for the two calculations, the material velocities for positions behind the contact points. For the supersonic closing velocity, a high-pressure region (A) is created behind the closure point. The high pressure is not being attenuated by release waves from the plate boundaries. The release waves travel at approximately the speed of sound of the material and cannot keep up with the closing velocity. The

unattenuated high pressure turns the flow around and directs it outward (B) from the closing point. For the subsonic closing velocity, the release waves from the boundaries can reach the closing position, decrease the pressure, and turn the material flow in the direction of the closing velocity (C).

The next step for this application of the simulation program is to determine the experimental set-up that will provide the desired plate-closing velocity. High explosives that burn tangentially to a plate provide a means for accelerating the plate to high velocities. With the high-explosive equation of state previously determined from calculations and experiments ⁽⁶⁾, it is possible to accurately calculate the space-time history of metal systems accelerated by high explosives. Figure 5 shows a calculation for determining the critical design parameters for establishing a desired closing angle. Note that subsonic and supersonic closing velocities are not necessarily the critical considerations for explosive-welding applications. What is critical is the design of a collision system to avoid local high pressures that lead to catastrophic release waves.

PLASTICITY

For the optimum use of materials in an engineering structure, a description is required of the material behavior after the elastic limit has been exceeded. A first requirement in designing with plasticity is a high confidence in the plasticity formulation used in the simulation program. The programs here use the von Mises flow-stress hypothesis with an associated flow law. The flow stress can increase with the equivalent plastic strain (work hardening) and increase with the applied hydrostatic pressure. A general constitutive model for the flow stress provides for a decrease with increasing temperature until

the melting point of the material is approached. The flow stress is then not equal to zero, and the material assumes a hydrodynamic behavior. For problems where only the work-hardening character of the flow stress is required, a constitutive model can be developed by simulating a simple tension test (see Fig. 6 for results with aluminum). The flow-stress model can then be checked by simulation of other experimental geometries, for example, the tension test of a flat plate (Fig. 7). Taylor [7] devised a very simple means of obtaining flow stress information for small to very large plastic strains. In this method, the change in length of a right circular cylinder after impact with a rigid boundary is related to the flow stress by a simple theory [7]. Computer simulations of these experiments have established that the method gives a very sensitive measure of the flow stress of a material. Figure 8 shows some results taken from Ref. 8 for the same aluminum used above. The calculated final length and shape of the cylinder agree closely with the experimental results.

An extreme test of the simulation program and the constitutive model is provided by the penetration of an aluminum target by a sharp, high-strength, steel projectile. Figure 9 compares experimental results, from a shadowgraph framing camera, with a calculation. The deceleration of the projectile is sensitive to the flow stress of the target. From these examples, it is apparent that close agreement can be attained between calculation and experiment once the constitutive law has been established.

Similar procedures were used to determine the flow stress of SA-533B-1 steel. Figure 10 shows a comparison between calculation and experiment for a simple tension test using this steel [9]. The plastic strain at the geometric center of the specimens exceeds 100% before ductile fracture occurs. The constitutive model required a much more complicated form to fit the experimental data than the simple power law that adequately described the flow stress for

aluminum. Figure 10 compares the calculated curve for load vs engineering strain with the experimental data.

PRESSURE VESSELS

Once we are provided with knowledge of the material behavior in the plastic regime, it becomes possible to design with plasticity. Figure 11 shows the calculated results for three pressure vessels. The calculations model a cylinder with a plate at one end where a fillet weld joins the two sections. As the base plate thickness is decreased, the stress concentration at the fillet increases. In this study, the internal pressure caused yielding in all three vessels. In this application, an objective is to find the design that best utilizes material properties by balancing the flow stress and the elongation behavior so that the best economy of materials can be attained. If the ductile fracture properties of the material were known, then the burst pressure could be determined by the simulations. With this information, destructive testing could be done on the computer rather than on the physical object.

The great uncertainty in designing with plasticity is to know under what conditions ductile fracture occurs. For example, it is known that elongation to fracture differs according to the combination of stresses occurring during loading history of the material. The constitutive model for predicting ductile fracture would be complex and could only come from a calculational and experimental program. For determining failure conditions for pressure vessels with cracks, the familiar linear-fracture-mechanics approach can be used.

Figure 12 shows a pressure vessel pipe used for studying the effect of surface cracks. Figure 13 shows the grid used to simulate cracks on the outside and on the inside of the pressure vessel wall after a 1-kb pressure is suddenly applied to the inside surface. Figure 14 shows the calculated stress-intensity factor for the two cracks as a function of time. In the utilization of linear-fracture mechanics, the material is assumed to be linear elastic. For ductile materials, this approach leads to overly conservative design limits.

It is interesting to study the stress-strain distribution in this pressurized pipe without the presence of cracks and using the constitutive model for SA-533B-1 steel given in Figure 10. Figure 15a shows the stress distribution at steady-state conditions when the internal pressure is 1.25 kb. This pressure is not sufficient to cause the material to exceed the elastic limit given by the constitutive law. Figure 15b shows results for the same pipe when the pressure is increased to 1.75 kb. At this internal pressure, the elastic limit, $Y = 3.81$ kb, is exceeded. The plastic strain is $\bar{\epsilon}^P = 0.3\%$ on the inside surface and falls off to zero at the outside surface. The stress distribution is reversed from the case where the elastic limit was not exceeded. After slight plastic flow, tensile stresses are greater on the outside surface rather than the inside surface. Work of Bridgman [10] on internally pressurized pipes showed that fracture starts from the outside surface when the material is ductile. With the constitutive model for plasticity determined, it should be possible to find the stress-strain distribution at failure by simulating experiments similar to Bridgman's.

ISOSTATIC PRESS

In the design and operation of an isostatic press, accurate knowledge of the stress-strain distribution for a given application can lead to the best selection of material properties for the press design. Figure 16 shows a calculational grid for studying the stress-strain field in a press. A tungsten carbide piston compresses liquid hydrogen in a steel cylinder. The equation of state of the liquid hydrogen and the elastic plastic properties of the piston and cylinder are assumed to be accurately known. The calculation can then determine the stress load on the containment vessel as a function of the piston displacement. Figure 17 shows the stress distributions in the press when the hydrogen has been compressed to 12 kb. The flow stress of the steel has not reached the plastic limit, $Y = 14.7$ kb, used in this calculation. The radial stress in the steel cylinder is compressive for positions opposite the hydrogen cavity. For positions on the piston side of the gas-piston interface, the stress in the steel becomes tensile. This calculation can select the best combination of material properties for the press design.

SUMMARY

In engineering design, computer simulations can assist in making decisions when the material behavior is known. In the examples shown, the important behaviors of materials were the elastic limit and the flow stress that is associated with increasing plasticity. Simulations of tension tests offer a simple means for developing constitutive models for plasticity. With the plasticity model verified by experiment, it becomes possible to simulate stress-strain conditions prior to catastrophic failure by fracture. This information may then be used to do destructive testing by computer.

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FIGURE CAPTIONS

Fig. 1. Schematic of three-dimensional calculational grid: (a) volume element to calculate gradient from flux of 8 zone-centered parameters; (b) volume element to calculate gradient from flux of 8 node-centered parameters.

Fig. 2. Calculation of a copper plate colliding with a steel anvil. Elastic plastic constitutive models describe the two materials. The collision angle γ is 10° ; the velocity of the copper plate U_p is 0.05 cm/ μ s; time is measured from first contact.

Fig. 3. Calculation of the collision of aluminum plates. Time is measured from the first contact. The plates are 1-cm thick and 8-cm long; plate velocity $U_p = 0.07$ cm/ μ s; longitudinal sound speed = 0.635 cm/ μ s. (The grid is omitted for clarity.)

Fig. 4. Enlarged view of the calculation shown in Fig. 3. The direction of material velocities in the colliding plates for each grid node point are indicated. The times after contact are 6 and 14 μ s.

Fig. 5. Calculation of the throw angle θ when a layer of Composition B is detonated along a plate. From knowledge of the throw angle, the set-up angle α that is required to provide the desired collision angle γ can be determined.

Fig. 6. Tension test of an aluminum 6061T6 cylinder. Original diameter $D^0 = 15.85$ mm; flow stress $\bar{\sigma} = 2.84 [1 + 125 \bar{\epsilon}^P]^{0.1}$ (kb); $\bar{\epsilon}^P$ = equivalent plastic strain; bulk modulus $k = 728$ kb; shear modulus $\mu = 248$ kb; density $\rho^0 = 2.7$ g/cm³.

Fig. 7. Tension test of aluminum plate with the same constitutive model given in Fig. 6.

Fig. 8. Impact of aluminum cylinders on a rigid boundary.

Fig. 9. Penetration of an aluminum target by a steel cylinder. The projectile flow stress $Y = 20$ kb. The work hardening of the aluminum was slightly over 3 kb. Projectile impact velocity = 0.084 cm/ μ s; exit velocity = 0.068 cm/ μ s.

Fig. 10. Tension test of SA-533B-1 steel bar. Flow stress $Y = 3.81[1 + 463 \bar{\epsilon}^P]^{0.148}$ (kb) for $\bar{\epsilon}^P < 0.10$; bulk modulus $k = 1650$ kb; shear modulus $\mu = 820$ kb; density $\rho^0 = 7.85$ g/cm³.

Fig. 11. Calculations for cylindrical pressure vessels with different end-plate thicknesses. A 0.5-kb pressure was applied to the inside surfaces.

Fig. 12. Computational grid for studying results of axial cracks in a pipe wall. Inner radius is 135 mm; outer radius is 195 mm.

Fig. 13. Grid to simulate cracks in the wall of the pipe shown in Fig. 12. Zero-stress boundary conditions are applied to those elliptical regions representing cracks in the axial direction; and A, B, and C are planes of symmetry. For semi-major axis: outer crack = 42.9 mm; inside crack = 42.9 mm. For semi-minor axis: outside crack = 12.0 mm; inside crack = 13.3 mm.

Fig. 14. Stress-intensity factor after the sudden application of 1-kb pressure to the inner surface of the pipe shown in Fig. 13.

Fig. 15. Stress profiles in a cylinder wall. In (a) the pipe is pressurized below the elastic limit; $Y = 3.81$ kb; internal pressure = 1.25 kb. In (b), the pipe is pressurized above the elastic limit; the constitutive model is given in Figure 10; internal pressure = 1.75 kb. Notation: σ_r = radial stress, σ_z = axial stress, σ_θ = hoop stress, and P = hydrostatic pressure.

Fig. 16. Computational grid to simulate the static compression of liquid hydrogen.

Fig. 17. Stress profiles for a section of the steel wall of the press shown in Fig. 16 when the liquid hydrogen is compressed 2 1/2 times and the pressure is 12 kb: (a) contours of the hoop stress minus the radial stress with stresses measured positive in tension; (b) contours of hydrostatic pressure p measured positive in compression.

(a) OCTAHEDRON

i, j, k = LAGRANGE COORDINATE

(b) CUBE

SUBSONIC CLOSING VELOCITY ($0.29 \text{ cm}/\mu\text{sec}$)

HEMP CALCULATION OF A COPPER PLATE COLLIDING WITH A
STEEL ANVIL.

Wilkins - Fig. 2

SUPERSONIC CLOSING VELOCITY ($0.67 \text{ cm}/\mu\text{sec}$)

SUBSONIC CLOSING VELOCITY ($0.42 \text{ cm}/\mu\text{sec}$)

CALCULATIONS OF THE COLLISION OF ALUMINUM PLATES

SUPERSONIC CLOSING VELOCITY

SHOCK

SUBSONIC CLOSING VELOCITY

SHOCK

MATERIAL VELOCITIES IN COLLIDING PLATES

Wilkins - Fig. 4

Wilkins - Fig. 5

TENSION TEST OF AN ALUMINUM 6061-T6 BAR

TENSION TEST OF A THIN ALUMINUM 6061-T6 PLATE

IMPACT OF ALUMINUM CYLINDERS ON A RIGID BOUNDARY

EXPERIMENT

$t = 0$

$t = 40 \mu\text{sec}$

$t = 52 \mu\text{sec}$

CALCULATION

TENSION TEST OF A A533 STEEL BAR

CALCULATION OF PRESSURE VESSELS WITH DIFFERENT BASE PLATES.
INTERNAL PRESSURE 0.5 kb.

STEEL PRESSURE VESSEL CYLINDER
INSIDE RADIUS = 135 mm
OUTSIDE RADIUS = 195 mm

Wilkins - Fig. 12

**GRID TO STUDY EFFECT OF CRACKS
IN A STEEL PRESSURE VESSEL CYLINDER**

Wilkins - Fig. 14

STRESS PROFILES IN A CYLINDER WALL

(a) BELOW THE ELASTIC LIMIT

(b) ABOVE THE ELASTIC LIMIT

VPRESS3

CYCLE = 0. TIME = 0.

COMPUTER SIMULATION OF A STATIC PRESS

Wilkins - Fig. 16

CONTOURS OF STRESS (kb) WHEN THE H₂ IS
COMPRESSED 2.5, PRESSURE 12kb