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ANOMALOUS SPATIAL DEPLETION OF
THE LOWER HYBRID CONE THROUGH
PARAMETRIC DECAY

BY

LIU CHEN AND R. L. BERGER

PLASMA PHYSICS
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Through Parametric Decay

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ABSTRACT

Analytic solution for the envelope structures of two nonlinearly coupled lower hybrid waves propagating along their respective cone trajectories are obtained. The coupling occurs through induced scattering by particles. The results indicate anomalous spatial pump depletion may be important for present and future large scale tokamaks. Implications to plasma heating are also discussed.

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1. INTRODUCTION

Recently, there have been active theoretical and experimental efforts in studying the r.f. plasma heating scheme using waves in the lower hybrid frequency range. It is now widely believed that nonlinear effects, such as pump self-induced filamentation and parametric decay, play important roles in this heating scheme [1,2]. One of the major concerns, which motivates the present work, is then how these nonlinear processes affect the transport of r.f. energy into the interior of a plasma. The nonlinear process considered here is the decay instability due to induced scattering by particles; i. e., nonlinear electron and/or ion-Landau damping [3].

In Sec. 2, we describe the theoretical model and approach used in this work. We then derive the set of nonlinearly coupled equations in terms of the action variables. The solutions to these equations are presented in Sec. 3. Section 4 contains a summary of the theoretical results as well as a discussion on the implications of these results to the plasma heating process.

2. THE NONLINEARLY COUPLED EQUATIONS

The plasma is assumed to be spatially homogeneous and uniformly magnetized with $\underline{B} = B \underline{e}_z$. In the steady state, we assume that the electrostatic potential consists of two parts, $\phi_0(\underline{x}, t)$ and $\phi_1(\underline{x}, t)$, oscillating at frequencies ω_0 and ω_1 , respectively, i. e.,

$$\phi_j(\underline{x}, t) = \tilde{\phi}_j(\underline{x}) \exp(-i\omega_j t) + c. c. ; \quad j = 0, 1 \quad (1)$$

Here, ω_0 and ω_1 are of the order of lower hybrid frequency, $\omega_{lh} = \omega_{pi} (1 + \omega_{pe}^2 / \omega_{ce}^2)^{-1/2}$ and $\omega_{pe} \sim \omega_{ce}$ and the other notation used here is standard. ϕ_0 and ϕ_1 are coupled to each other through a low-frequency mode at frequency $\omega_s = \omega_0 - \omega_1 > 0$. Thus, ϕ_0 corresponds to the pump wave excited by an external structure, and ϕ_1 is the daughter wave excited by the pump wave ϕ_0 through the decay process. The dominant coupling between ϕ_0 and ϕ_1 comes from the electron density perturbation of the ω_s mode. Let

$$n_{es}(\underline{x}, t) = \tilde{n}_{es}(\underline{x}) \exp(-i\omega_s t) + c. c. \quad (2)$$

It is then straightforward to derive the following set of coupled equations for $\tilde{\phi}_0$ and $\tilde{\phi}_1$;

$$(\nabla_{\perp} \cdot \epsilon_{\perp 0} \nabla_{\perp} - \nabla_z \epsilon_{z0} \nabla_z) \tilde{\phi}_0 = -\frac{4\pi e}{\omega_0} \nabla \cdot (\tilde{n}_{es} \underline{u}_1), \quad (3)$$

$$(\nabla_{\perp} \cdot \epsilon_{\perp 1} \nabla_{\perp} - \nabla_z \epsilon_{z1} \nabla_z) \tilde{\phi}_1 = -\frac{4\pi e}{\omega_1} \nabla \cdot (\tilde{n}_{es}^* \underline{u}_0); \quad (4)$$

where with $j = 0, 1$

$$\epsilon_{\perp j} = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega_j^2}, \quad (5)$$

$$\epsilon_{zj} = \frac{\omega_{pe}^2}{\omega_j^2} - 1 \simeq \frac{\omega_{pe}^2}{\omega_j^2}, \quad (6)$$

and $\underline{u}_j$ is mainly due to the electron $\underline{E} \times \underline{B}$ drift,

$$\underline{u}_j \simeq (-c/B) (\underline{\nabla}_\perp \tilde{\phi}_j \times \underline{e}_z) . \quad (7)$$

Also, we note that $\underline{\nabla}_\perp = (\partial/\partial x) \underline{e}_x + (\partial/\partial y) \underline{e}_y$ and $\nabla_z = \partial/\partial z$. For toroidal devices, x , y , and z correspond to the (minor) radial, poloidal and toroidal directions, respectively. Furthermore, in deriving Eqs. (3) and (4) we have ignored the thermal effects; this is justified as long as

$$|v_i^2 \nabla^2|, |v_e^2 \nabla_z^2| \ll |\omega_j^2|; |v_e^2 \nabla_\perp^2| \ll \omega_{ce}^2 \quad \text{and} \quad \omega_j \text{ is not close to } \omega_{lh}.$$

For regions near the plasma surface, these conditions are easily satisfied in typical experiments.

As to the low-frequency ω_s mode, the dominant coupling is due to the parallel ponderomotive force, $-m_e (\underline{u} \cdot \underline{\nabla}) \underline{u}_z$, produced by ϕ_0 and ϕ_1 which acts on the electrons along $\underline{B}$. Denoting the corresponding potential be $\tilde{\psi}_p$; i. e.,

$$\nabla_z \tilde{\psi}_p = -(m_e/e) [(\underline{u}_0 \cdot \underline{\nabla}) \underline{u}_{z1}^* + (\underline{u}_1^* \cdot \underline{\nabla}) \underline{u}_{z0}] , \quad (8)$$

we obtain, using Eq. (7) and noting $\underline{u}_{zj} = (ie/m_e \omega_j) \nabla_z \tilde{\phi}_j$,

$$\tilde{\psi}_p(\underline{x}) = -(ic/B\omega_0) [(\underline{\nabla}_\perp \tilde{\phi}_1^* \times \underline{\nabla}_\perp \tilde{\phi}_0) \cdot \underline{e}_z] . \quad (9)$$

Because $\omega_s \ll \omega_{ce}$, electron dynamics is mainly along $\underline{B}$. We then can use $\tilde{\psi}_p$ along with the self-consistent potential $\tilde{\phi}_s$ in the electron one-dimensional (in $\underline{B}$ direction) Vlasov equation to obtain the perturbed electron distribution function $\tilde{f}_{es}$:

$$(-i\omega_s + \mathbf{v} \cdot \nabla_z) \tilde{f}_{es}(\mathbf{x}, \mathbf{v}_z) = -\frac{e}{m_e} \frac{df_{0e}}{dv_z} \nabla_z (\tilde{\phi}_s + \tilde{\psi}_p). \quad (10)$$

As to the effectively unmagnetized ions ($\omega_s^L \gg \omega_{ci}^2$), they only respond linearly to $\tilde{\phi}_s$. Hence, we have

$$(-i\omega_s + \mathbf{v} \cdot \nabla) \tilde{f}_{is}(\mathbf{x}, \mathbf{v}) = \frac{e}{m_i} \frac{\partial f_{i0}}{\partial v} \cdot \nabla \tilde{\phi}_s. \quad (11)$$

Poisson's equation

$$-\nabla^2 \tilde{\phi}_s = 4\pi e (\tilde{n}_{is} - \tilde{n}_{es}), \quad (12)$$

then relates $\tilde{\phi}_s$ and, hence $\tilde{n}_{es} = n_0 \int dv_z \tilde{f}_{es}$ to ψ_p . Equations (3), (4), (9), (10), (11), and (12) thus, provide a complete description of the nonlinearly coupled system.

To proceed further analytically, we make the following WKB approximation:

$$\tilde{\phi}_{j,\underline{m}}(\mathbf{x}) = \Phi_j(\epsilon \mathbf{x}, \epsilon z) \exp(i\mathbf{k}_{\underline{m}j} \cdot \mathbf{x}); \quad j = 0, 1, \quad (13)$$

and

$$(\tilde{n}_{es}, \tilde{\psi}_p, \tilde{\phi}_s)(\mathbf{x}) = (N_{cs}, \psi_p, \Phi_s)(\epsilon \mathbf{x}, \epsilon z) \exp(i\mathbf{k}_{\underline{m}s} \cdot \mathbf{x}). \quad (14)$$

Here, $\epsilon \ll 1$, $\underline{k}_0 = k_{x0} \underline{e}_x + k_{z0} \underline{e}_z$, $\underline{k}_1 = k_{x1} \underline{e}_x + k_{y1} \underline{e}_y + k_{z1} \underline{e}_z$,

$\underline{k}_s = \underline{k}_0 - \underline{k}_1$, and $(\omega_j, \underline{k}_j)$ satisfies the lower hybrid dispersion relation

$$\omega_j = \omega_{lh} (1 + k_{zj}^2 m_i / k_{\perp j}^2 m_e)^{1/2}; \quad j = 0, 1. \quad (15)$$

Note, in Eqs. (13) and (14), because we are mainly interested in the spatial structures in the x - z plane, we have assumed the envelopes to be independent of y . Equations (3) and (4) then become

$$[k_x \epsilon_{\perp} (\partial/\partial x) - k_z \epsilon_z (\partial/\partial z)]_0 \Phi_0 = \lambda N_{es} \Phi_1, \quad (16)$$

$$[k_x \epsilon_{\perp} (\partial/\partial x) - k_z \epsilon_z (\partial/\partial z)]_1 \Phi_1 = -\lambda N_{es}^* \Phi_0, \quad (17)$$

where

$$\lambda = \frac{2\pi e c}{B \omega_0} [(k_1 \times k_0) \cdot e_z]. \quad (18)$$

For the induced scattering processes considered here, $\omega_s \sim k_s v_i$ or $k_{zs} v_e$ and the low-frequency mode is nonresonant. Thus, we can replace ∇ by ik_s in Eqs. (10), (11), and (12). We then obtain

$$4\pi e N_{es} = \beta \psi_p, \quad (19)$$

where

$$\beta(\omega_s, k_s) = k_s^2 \chi_e (1 + \chi_i) / \epsilon_s, \quad (20)$$

$$\epsilon_s = 1 + \chi_e + \chi_i, \quad (21)$$

$$\chi_e = - \frac{\omega_{pe}^2}{k_s^2} \int dv_z \frac{df_{0e}/dv_z}{v_z - \omega_s/k_{zs}}, \quad (22)$$

$$\chi_i = - \frac{\omega_{pi}^2}{k_s^2} \int dv \frac{df_{0i}/dv}{v - \omega_s/k_s}, \quad (23)$$

and, from Eq. (9),

$$\psi_p = -\frac{ic}{B\omega_0} [(\underline{k}_1 \times \underline{k}_0) \cdot \underline{e}_z] \Phi_0 \Phi_1^* . \quad (24)$$

Substituting Eq. (19) into Eqs. (16) and (17), and expressing in terms of the action variables $I_j = k_j^2 |\Phi_j|^2 / \omega_j$ with $j = 0, 1$, we obtain

$$\underline{\nabla} \cdot (\underline{v}_{g0} I_0) = -\alpha I_0 I_1 , \quad (25)$$

$$\underline{\nabla} \cdot (\underline{v}_{g1} I_1) = \alpha I_0 I_1 , \quad (26)$$

where

$$\alpha = \frac{1}{2} \left(\frac{Ck_{y1}}{k_{1B}} \right)^2 \left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \right)^{-1} \quad \text{Im } \beta > 0 , \quad (27)$$

and

$$\underline{v}_{gj} = \left(\frac{\partial \omega}{\partial k_x} \underline{e}_x + \frac{\partial \omega}{\partial k_z} \underline{e}_z \right)_j ; \quad j = 0, 1 , \quad (28)$$

is the group velocity. Note Eqs. (25) and (26) exhibit the conservation of action flux; i. e., $\sum_{j=0,1} \underline{\nabla} \cdot (\underline{v}_{gj} I_j) = 0$. Solutions to this set of non-linearly coupled equations are discussed in the next section.

3. SOLUTIONS OF THE COUPLED EQUATIONS

Equations (25) and (26) can be written as

$$\frac{\partial G_0}{\partial x} + C_0 \frac{\partial G_0}{\partial z} = -G_0 G_1, \quad (29)$$

$$\frac{\partial G_1}{\partial x} + C_1 \frac{\partial G_1}{\partial z} = G_0 G_1. \quad (30)$$

Here, $G_0 = \alpha I_0 / V_{gx1}$, $G_1 = \alpha I_1 / V_{gx0}$, and $C_j = (V_{gz} / V_{gx})_j$ with $j = 0, 1$. Equations (29) and (30) indicate that if nonlinear coupling is absent, $G_0(I_0)$ and $G_1(I_1)$ just propagate along these respective cone trajectories; i. e., G_0 and G_1 are only functions of $z - C_0 x$ and $z - C_1 x$, respectively. Nonlinear couplings, thus, make G_0 and G_1 functions of both $z - C_0 x$ and $z - C_1 x$. There exists a class of exact solutions to Eqs. (29) and (30), and is given by [4]

$$G_0(\tau, \xi) = \frac{T_\tau(\tau)}{Z(\xi) - T(\tau)}, \quad (31)$$

$$G_1(\tau, \xi) = \frac{Z_\xi(\xi)}{Z(\xi) - T(\tau)}, \quad (32)$$

where

$$\tau = (z - C_0 x) / V, \quad (33)$$

$$\xi = (z - C_1 x) / V, \quad (34)$$

and

$$V = C_0 - C_1. \quad (35)$$

Subscripts τ and ξ denote derivatives. Formally, T and Z are two arbitrary differentiable functions which, however, are uniquely defined by the boundary conditions at $x=0$. Denoting $G_0(x=0, z) = \bar{G}_0(z)$ and $G_1(x=0, z) = \bar{G}_1(z)$, we then have

$$T(\tau) = -\frac{1}{2} - \frac{1}{V} \int_0^{-V\tau} ds \bar{G}_0(s) \exp \left[\int_0^s dr G_t(r)/V \right], \quad (36)$$

$$Z(\xi) = \frac{1}{2} + \frac{1}{V} \int_0^{V\xi} ds \bar{G}_1(s) \exp \left[\int_0^s dr G_t(r)/V \right] \quad (37)$$

with $G_t(z) = \bar{G}_0(z) + \bar{G}_1(z)$.

To illuminate the physical meanings of the solutions, let us take the following simple boundary conditions

$$\bar{G}_0(z) = \begin{cases} A_0, & |z| \leq a \\ 0, & |z| > a \end{cases} \quad (38)$$

and similarly

$$\bar{G}_1(z) = \begin{cases} A_1, & |z| \leq a \\ 0, & |z| > a \end{cases} \quad (39)$$

Let $A_t = A_0 + A_1$, the solution then is:
for $|z - C_0 x|, |z - C_1 x| \leq a$,

$$G_0(\tau, \xi) = \frac{A_t A_0}{A_0 + A_1 \exp(A_t x)}, \quad (40)$$

$$G_1(\tau, \xi) = \frac{A_t A_1 \exp(A_t x)}{A_0 + A_1 \exp(A_t x)} ; \quad (41)$$

for $|z - C_0 x| \leq a$, $|z - C_1 x| > a$,

$$G_0(\tau, \xi) = \frac{A_t A_0}{A_0 + A_1 \exp[A_t(\tau + a/V)]} , \quad (42)$$

$$G_1(\tau, \xi) = 0 ; \quad (43)$$

and for $|z - C_0 x| > a$, $|z - C_1 x| \leq a$,

$$G_0(\tau, \xi) = 0 , \quad (44)$$

$$G_1(\tau, \xi) = \frac{A_t A_1 \exp[A_t(\xi + a/V)]}{A_0 + A_1 \exp[A_t(\xi + a/V)]} . \quad (45)$$

The above solutions, thus, suggests a typical scale length of pump depletion (or growth of the decay wave) in x given by

$$x_d = 1/A_t = [\alpha (I_{00}/V_{gx1} + I_{10}/V_{gx0})]^{-1} . \quad (46)$$

Here, I_{j0} is the value of I_j at $x=0$ for $j=0,1$. Note, from the definition of α in Eq. (27), x_d has a simple physical interpretation; i.e., $x_d =$ (effective group velocity in x)/(effective parametric growth rate). The anomalous pump depletion, however, does not occur until the pump wave penetrates a typical distance x_p into the plasma which is given by

$$x_p = x_d \ln(I_{00} V_{gx0} / I_{10} V_{gx1}) . \quad (47)$$

To estimate x_d and x_p , we note that I_{10} corresponds to fluctuations excited by steady-state sources. For a sufficiently intense pump, the dominant source is the enhanced induced emission due to interactions between the pump wave and the ion-acoustic thermal fluctuations. Inserting the emission term S_{kl} into Eq. (26), and examining the solutions near the boundary I_{10} is then estimated to be

$$I_{10} \simeq S_{kl} \omega_0 / \alpha |E_0|^2, \quad (48)$$

where $|E_0| = |k_0 \Phi_0|$ and S_{kl} is given by [5]

$$S_k = \left(\frac{\pi}{8}\right)^{1/2} T_e \left|\frac{CE_0}{BC_s}\right|^2 \frac{\omega_{lh}^2}{(2\pi)^3} \frac{\sin^2 \psi}{k_z v_e} \frac{k_z^2 m_i}{k^2 m_e} \exp(-m_e k^2 / 2m_i k_z^2) \\ \cdot I_0(k^2 \rho_e^2) \exp(-k^2 \rho_e^2),$$

where $\cos \psi = (\underline{k} \cdot \underline{E}_0) / k |E_0|$, $C_s^2 = T_e / m_i$, and $\rho_e^2 = T_e / m_e \omega_{ce}^2$.

In deriving this equation, we have assumed $T_e > 3T_i$ and hence, the ion-acoustic fluctuations are mainly dissipated by electron-Landau damping. From the frequency dispersion for lower hybrid waves and the frequency matching conditions for parametric decay instabilities, we have

$$k_{z1} \simeq (m_e / m_i)^{1/2} k_1 (\omega_1^2 / \omega_{lh}^2 - 1)^{1/2}$$

and

$$S_{kl} \simeq \frac{(\pi/2)^{1/2} \pi}{4(2\pi)^3 \omega_1} \frac{T_e}{\lambda_{De}^3} \left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \right)^{-1} \frac{\omega_{ce}^2}{\omega_{pe}^2} \left| \frac{CE_0}{BC_0} \right|^2 \left(\frac{\omega_1^2}{\omega_{lh}^2} - 1 \right)^{1/2} \cdot \exp \left[-\frac{1}{2} \left(\omega_1^2 / \omega_{lh}^2 - 1 \right)^{-1} \right] . \quad (49)$$

From Eqs. (48) and (49), we have $|I_{10} V_{gx1}| \ll |I_{00} V_{gx0}|$ and x_d can be further approximated to be

$$x_d \simeq \omega_0 |V_{gx1}| / \alpha |E_0|^2 \simeq (2\pi)^{1/2} \left(\frac{\omega_1^2}{\omega_{lh}^2} - 1 \right)^{1/2} \left(\frac{CE_0}{BC_s} \right)^{-2} (k_1 \lambda_{De})^{-1} \lambda_{De} . \quad (50)$$

Here, we have assumed $\omega_s \sim k_s C_s$ for maximum coupling. Note that the value of $k_1 \lambda_{De}$ is limited by the consideration of Landau damping which leads to a typical value $k_1 \lambda_{De} \simeq 0.25$ [6]. Meanwhile, x_p becomes approximately

$$x_p = x_d \ln \frac{|V_{gx0}| |E_0|^2}{\omega_0 S_{kl} x_d} = x_d \ln \left[2(4\pi)^3 n_0 \lambda_{De}^3 \left| \frac{CE_0}{B_0 C_s} \right|^2 \frac{m_e}{m_i} k_1 \lambda_{De} \frac{\omega_{pi}}{\omega_0} \left(\frac{m_e C^2}{T_e} \right)^{1/2} \right] . \quad (51)$$

It is interesting to obtain some quantitative estimates for x_d and x_p in the region near the plasma surface. Using typical tokamak parameters near the surface, $n_0 = 10^{12}$, $T_e = 100$ eV, $\omega_0 \sim 3\omega_{pi}$, and, as mentioned previously, $k_1 \lambda_{De} \simeq 0.25$, we find

$$x_d \simeq 0.2 \left| CE_0/BC_s \right|^{-2} \text{ cm} , \quad (52)$$

and

$$x_p \approx 0.2 \left| \frac{CE_0}{BC_s} \right|^{-2} \ln \left(5 \times 10^6 \left| \frac{CE_0}{BC_s} \right|^2 \right) \text{ cm} . \quad (53)$$

If, $|CE_0/BC_s|^2 \sim 1$, $x_p \sim 3$ cm. Currently, lower hybrid heating experiments are running with powers such that $|CE_0/BC_s|^2 \sim 0(1)$; hence, x_d is in the millimeter (or centimeter) range and x_p is in the centimeter (or ten centimeter) range.

The above estimates show that for typical tokamak parameters and lower hybrid wave heating experiments with $|CE_0/BC_s|^2 \sim 0(1)$, the pump penetration length in x (the minor radius) is in the centimeter (or ten centimeter) range. As pointed out to us by F. W. Perkins, the most pessimistic observation is that the scaling law [which, from Eqs. (50) and (51)] does not improve with the size of the device. It, therefore, indicates that, while in present generation of tokamaks with minor radii in the tens of centimeter range (e.g., ATC and PLT), a nonlinear decay process may or may not anomalously deplete the pump wave energy near the plasma surface; they will, however, play crucial roles in future large-scale (e.g. reactor-size) tokamaks.

SUMMARY AND DISCUSSION

In this paper, we consider the effects of parametric decay on the transport of lower hybrid pump wave energy into the interior of a plasma. Using the WKB approximation, a set of equations describing two nonlinearly coupled lower hybrid waves propagating along their respective cone trajectories are derived in terms of the action variables. The two waves correspond to, respectively, the pump and the decay waves. The nonlinear coupling considered here is due to nonlinear electron and/or ion-Landau damping. Exact solutions are then constructed satisfying the boundary conditions. Using simple but not unrealistic boundary conditions, a scale length for pump penetration is obtained which for typical tokamak parameters and pump power $|CE_0/BC_s|^2 \sim 0(1)$, is of the order of centimeters. The results, thus, indicate that for present and future large-scale tokamaks, the lower hybrid pump wave may anomalously deplete its energy close to the plasma surface.

We now briefly discuss the implications of the above results to the plasma heating process. We note that the depleted pump wave energy goes into the decay wave, which is parametrically amplified. However, due to the fact that the decay wave has a lower frequency and generally a finite k_y component, its cone trajectory in the $x-z$ plane is closer to the plasma surface than that of the pump wave; i.e., $C_1 > C_0$ using the notation of Eqs. (29) and (30). This observation suggests that the decay wave will have further problems in penetrating into the plasma and will deposit most of

its energy near the surface which implies, therefore, anomalous heating at the plasma surface.

Finally, we remark that the analysis presented here can be extended to include the effects due to plasma inhomogeneities. However, since the scale lengths due to nonlinear effects are shown to be of the order of centimeters and are generally comparable or shorter than those associated with inhomogeneities, the essential features of the results obtained here, therefore, are expected to remain valid.

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REFERENCES

- [1] MORALES, G., and LEE, Y. C., Phys. Rev. Letters 35 (1975) 930.
- [2] BERGER, R. L., CHEN, L., CHENG, C., KAW, P. K., and PERKINS, F. W., Proceedings of Annual Meeting on Theoretical Aspects of Controlled Thermonuclear Research, Madison, Wisconsin USA (1975).
- [3] PORKOLAB, M. Phys. Fluids 17, (1974) 1432.
- [4] CHU, F. Y. F., Phys. Letters 51A (1975) 129. Also, CHU, F. Y. F. and KARNEY, C. F. F., to be published.
- [5] LARSSON, J., STEFLO, L., and TEGEBACK, R., J. Plasma Physics 16 (1976) 37; BERGER, R. L., CHEN, L., and PERKINS, F. W., "Lower Hybrid Parametric Instabilities - Saturation by Cascading and Pump Depletion" (submitted to Phys. Fluids).