Scaling of End Loss Times in Linear Theta Pinches

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Abstract

The influence of energy loss on the particle loss from the ends of a linear theta pinch is examined. The basic formalism for the equations which have been used to scale linear theta pinches is first developed, then examined relative to inherent constraints. Characteristic times for pinch, compression, wave propagation, and loss for various experiments are noted to be not repetitively ordered, thus making data extrapolation difficult. Taking ion temperature as a decaying variable, the e-folding particle loss time (t_e) is evaluated in a self-consistent fashion, first for exponential T_i decay (t_{thl}) and also for linear T_i decay. Expressions for particle loss times are evaluated for $t_{thl} \lesseqgtr t_e$. Expressions for scaling loss time are presented. Millisecond particle loss times for moderate length (~150 meters) linear pinches are indicated because of low temperatures at the ends due to energy loss.

I. Introduction

In the evaluation of experiments with linear theta pinches¹⁻⁴, the determination and scaling of the time for plasma to be lost from the ends has received considerable attention. In order to correlate with interferometric experimental data indicative of the total number of electrons within the plasma column, $N(t)$, it has generally been assumed^{2,4} that $N(t) = N_0 \exp(-t/\tau)$, where N_0 is the number of electrons within the column at $t = 0$ and τ is referred to as the plasma containment time. The relationship of τ to pinch parameters has been discussed by Little et al.¹ and also by Morse⁵, and in general a scaling with functional dependence, $\tau \sim \ell/T_i^{1/2}$ has been presumed¹⁻⁵, where ℓ is the length of the theta coil and T_i is the ion temperature. The purpose of this note is to clarify the limitations and constraints inherent in the use of this loss scaling, and to identify other scaling relationships that may be more appropriate in a general sense.

II. Earlier Loss Scaling Efforts

First, in a model attributed¹ to Grad, the rate at which the total number of particles in the column changes, the rate at which the particles are lost out of the ends of the compression coil, was given as

$$\frac{dN}{dt} = -2A_E \left[\frac{1}{4} n \langle v \rangle \right]$$

where A_E is the effective area of the open end, n is particle number density, and $\langle v \rangle$ is the average velocity of the ions. With $N \equiv A_p \cdot \ell \cdot n$ then $\tau = 2 \frac{A_p}{A_E} \frac{\ell}{\langle v \rangle}$. Directly, if one takes $\langle v \rangle = [8kT_i/\pi m]^{1/2}$, where m is the particle mass, then

$$\tau = 2 \frac{A_p}{A_E} \ell \left[\frac{\pi m}{8kT_i} \right]^{1/2} \sim \frac{\ell}{T_i^{1/2}} .$$

Second, the mass flow rate from the end of a linear theta pinch when the plasma is collision dominated was expressed for steady flow by Taylor and Wesson⁶ as

$$J = \rho_o A_o \left(\frac{\gamma p_o}{\rho_o} \right)^{1/2} (1 - \beta) F(R) ,$$

where the o subscript refers to plasma conditions upstream of the throat, and for unsteady flow by Wesson⁷ as

$$J = \rho_o A_o \left(\frac{\gamma p_o}{\rho_o} \right)^{1/2} \left(\frac{2}{\gamma} \right)^{1/2} (1 - \beta)^{1/2}$$

where the o subscript refers to plasma conditions at the coil end ($x = 0$). Both of these results show a $T_i^{-1/2}$ dependence which can be manipulated into an $\ell/T_i^{1/2}$ dependence for τ . Third, the work presented by Morse⁵ directly addressed the question of scaling linear theta pinch data, and his results emphasized competition between thermal (energy) loss and particle (mass) loss in limiting the containment of fusion plasma. The loss time was categorized as, "the time required for a thermal ion to travel the distance ℓ_2 from the middle to the end of the system or for a rarefaction moving with the acoustic speed to travel from the end to the middle, depending on whether the ion mean free path is longer or shorter than ℓ_2 . In either case the streaming loss time is approximately

$$t_{s.l.} = \frac{7.2 \ell_2 \times 10^{-7} \text{ "}}{(T_i/A_i)^{1/2}} \sim \frac{\ell}{T_i^{1/2}}$$

Finally, the work of Freidberg and Weitzner⁸ simply utilizes the $\ell/T_i^{1/2}$ factor to scale the loss time, $\dot{N} = N/\tau$ with

$$\tau = \frac{\ell}{2} \left(\frac{m}{2 \langle T \rangle} \right)^{1/2} \eta$$

where η is a parameter determined as part of the solution.

Relative to the above results, several facts will be noted. In all of the above analyses, collisional or collisionless, it is implicitly presumed that the temperature at the center at time $t = 0$, a constant, is the appropriate scaling value. While Morse noted the competing effects of energy transfer and mass loss in limiting the linear pinch, the effect of energy loss or mass loss scaling through the decay in $T_i(t)$ was not explored. Attention has not been given to the fact that since T_i will naturally decrease from the center to the end of the coil, the lower T_i at the end of the coil can substantially inhibit mass loss. If mass loss can be controlled by the cooling at the ends, it would be interesting to consider what supplementary heating of the plasma in the center would be necessary to maintain fusion plasma.

III. Time Hierarchy of Linear Pinch Experiments

In terms of the ability to observe phenomena in linear pinch devices, it appears worthwhile to consider the different times that characterize the events in each device. In particular, there is a need to reinforce conclusions drawn from any one experiment by reviewing data from similar experiments before general conclusions can be drawn regarding end loss related phenomena. Some times of

interest are now listed:

- t_p - time for dynamic sheath implosion to reach discharge axis
- $t_{B_{max}}$ - time for magnetic field to reach maximum value, also the time for termination of adiabatic compression.
- t_{alfven} - time for an Alfvén wave to move from end to the middle of coil
- τ_T - time for an acoustic wave to move from end to the middle of coil
- τ_{exp} - time, experimentally determined, for the number of particles in the plasma column to decay to a value $N(t = 0)/e$

The above defined terms are now listed for the sequence of LASL linear pinch devices:

Scylla IV-1	93kG at 3.5 μ sec, no crowbar $n = 2.1 \times 10^{16} \text{cm}^{-3}$, $T_i = 3.2 \text{keV}$
	$t_p < t_{alfven} < \tau_T < \tau_{exp} < t_{B_{max}}$.5 .7 2.4 3.5 μ sec
Scylla IV-3	60kG at 3.2 μ sec, crowbar $n = 3.2 \times 10^{16} \text{cm}^{-3}$, $T_i = 1.4 \text{keV}$
	$t_p < t_{alfven} < \tau_T = t_{B_{max}} < \tau_{exp}$ 2.8 3.2 3.2 10.1 μ sec
5-m Linear Scyllac	60kG at 4.3 μ sec, crowbar $n = 1.5 - 2.0 \times 10^{16} \text{cm}^{-3}$, $T_i = 2.7 \text{keV}$
	$t_p < t_{alfven} < \tau_T < t_{B_{max}} < \tau_{exp}$ 3.4 3.8 4.3 11.5 μ sec
Scylla I-C	35kG at 2.0 μ sec, crowbar $n = 10^{17} \text{cm}^{-3}$, $T_i = 50 \text{eV}$
	$t_p < t_{B_{max}} < t_{alfven} < \tau_T < \tau_{exp}$ 2.0 2.8 5.6 14.4 μ sec

From the above figures, it can be seen that Scylla IV-1 was truly unique and has considerable dissimilarity with the other experiments. For Scylla IV-3 and linear Scyllac, t_{alfven} and τ_T were both less than $t_{B_{\text{max}}}$, and so it probably would be difficult to identify any wave transported, and initiated phenomena. An example of this type of event appears to occur in the Scylla I-C experiment with a definable temperature drop at the time for an acoustic wave to travel from the end. Further, with $t_{B_{\text{max}}}$ being a significant percentage of the loss time on Scylla IV-3 and linear Scyllac (32% and 37%, respectively), the relationship of the increasing field strength to the resultant time for end loss is not clear. The low temperature in Scylla I-C places that plasma uniquely in the collisional range.

IV. Evaluation of Loss Times - End Loss Scaling

In the work to be presented below, the particle loss rate is characterized as

$$\frac{dN}{dt} = -2Anv$$

with

$$N(t) = An\ell$$

where, at one axial point, A is the radius of the plasma column, n is the number density, and v is the plasma flow velocity; ℓ is the length of the plasma column. As noted above, the flow velocity can be anticipated to be on the order of the acoustic velocity, $c_s = (2\gamma kT_i/m)^{1/2}$, or the average thermal velocity of the ions, $\bar{c} = (8kT_i/\pi m)^{1/2}$. As these velocities differ by only a small constant factor, the acoustic velocity will be used in the analysis. Generally, the loss will be expressed

as a function of properties in the center of the coil. Accordingly, several factors that would enter into any exact calculation of particle loss for a given case will not be explicitly included, rather, it will be assumed that they can be accounted for by a correction factor on plasma loss rate; this correction factor would be based on the ratio of the variable involved. Some examples of this are: the difference of plasma area, density, and temperature between mid-coil and end-coil positions; the difference between c_s and $\bar{c}$.

1. Constant temperature plasma

The total number of particles in the plasma column is written as

$$N = n A_p \ell$$

where A_p represents the effective cross section area of the column, usually taken as the area at the center. The rate of change of the total number of particles can be written

$$\frac{dN}{dt} = -2(A_E n \langle v \rangle)$$

where A_E is the area at the end, n is taken to be the number density of plasma in the center and is some multiple of density at the end, $\langle v \rangle$ is taken to be acoustic velocity at the center and is some multiple of acoustic velocity at the end. Then

$$\begin{aligned} \frac{dN}{dt} &= -2A_E \langle v \rangle \cdot \frac{N}{A_p \ell} \\ \frac{dN}{N} &= - \frac{2A_E \langle v \rangle}{A_p \ell} dt \end{aligned}$$

which, when v is constant in time, i.e. $T = \text{constant}$,

$$\ln N = - \frac{2A_E \langle v \rangle}{A_p \ell} t + \ln \text{constant}$$

and with $N = N_0$ at $t = 0$

$$N = N_0 \exp \left\{ -\frac{2A_E \langle v \rangle}{A_p \cdot \ell} t \right\} .$$

Accordingly, we have a first order system with time constant

$$\tau = \frac{A_p \cdot \ell}{2A_E \langle v \rangle} ,$$

so

$$N = N_0 \exp \left(-t/\tau \right) .$$

Directly,

$$\frac{dN}{dt} = - \frac{N_0}{\tau} \exp \left(-t/\tau \right)$$

and

$$\left. \frac{dN}{dt} \right|_{t=0} = - \frac{N_0}{\tau} .$$

Accordingly, it can be seen that the time for the number of particles to be reduced to N_0/e in the system with the behavior as defined is

$$\tau = \frac{A_p \cdot \ell}{2A_E \langle v \rangle} = \frac{A_p \cdot \ell}{2A_E} \left(\frac{m}{2\gamma k T_i} \right)^{1/2} \sim \frac{\ell}{T_i^{1/2}}$$

and for $A_E = A_p$, the loss time, τ , becomes equal to the acoustic transit time

$$\tau_T \equiv \frac{\ell}{2} \left(\frac{m}{2\gamma k T_i} \right)^{1/2} .$$

Again, as this type of formulation for $N(t)$ has been assumed in identifying and scaling τ_{exp} , then there has been no account taken of the variation of plasma temperature with time. This could have considerable influence on the scaling of loss time as well as the understanding of loss processes. That influence will now be considered.

2. Variable temperature plasma - thermal loss time

The variation of temperature and the characteristic time related to thermal energy loss ($t_{\text{th}\ell}$) have been discussed by Morse⁵. It was

noted that a different thermal loss time scaling exists for short machines and long machines. For long machines, T_e approaches T_i and the loss time can be taken as the electron conduction time,

$$t_{\text{con}} = n(.2\ell)(.5\ell)\ell n\lambda \times 5.4 \times 10^{-21} T_i^{-5/2}.$$

However, for short machines where the electron energy end loss occurs quickly, and the ion-electron equilibration time controls the events, the thermal loss time can be taken as,

$$t_{\text{eq}} = \left(\frac{3.16 A_i T_e^{3/2}}{n\ell n\lambda} \right) 10^8$$

or

$$t_{\text{eq}} = \frac{3.16 A_i \times 10^8}{n\ell n\lambda} \left\{ .86 n^2 \frac{(.2\ell)(.5\ell) T_i (\ell n\lambda)^2}{A_i} \times 10^{-29} \right\}^{3/10}.$$

Following Morse, we can write

$$\frac{1}{T_i} \frac{dT_i}{dt} = \frac{1}{t_{\text{th}\ell}}$$

where $t_{\text{th}\ell}$ is the thermal loss time, or

$$\frac{dT_i}{T_i} = \frac{dt}{t_{\text{th}\ell}}.$$

We integrate with the assumption that $t_{\text{th}\ell}$ is a constant, then

$$T_i = T_{i0} \exp(-t/t_{\text{th}\ell})$$

where T_{i0} represents ion temperature at $t = 0$, a constant, and $t_{\text{th}\ell}$ represents the time for temperature to drop by the factor $1/e$.

With the above variation in temperature, it is possible to substitute into the loss relationship and evaluate the time variation of number of particles in the column, including temperature change. Specifically with

$$\begin{aligned}\frac{dN}{N} &= - \frac{2A_E}{A_p \cdot \ell} \left(\frac{2\gamma k}{m} \right)^{1/2} (T_i)^{1/2} dt \\ &= - \frac{2A_E}{A_p \cdot \ell} \left(\frac{2\gamma k}{m} \right)^{1/2} [T_{io} \exp(-t/t_{th\ell})]^{1/2} dt\end{aligned}$$

then

$$\frac{dN}{N} = - \frac{2A_E}{A_p \cdot \ell} \left(\frac{2\gamma k T_{io}}{m} \right)^{1/2} \exp(-t/2t_{th\ell}) dt .$$

This can be integrated and solved for N with the boundary condition,

$N = N_0$ at $t = 0$, to get

$$N = N_0 \exp \left\{ - \frac{2A_E}{A_p \cdot \ell} \left(\frac{2\gamma k T_{io}}{m} \right)^{1/2} (2t_{th\ell}) [1 - \exp(-t/2t_{th\ell})] \right\}$$

or

$$N = N_0 \exp \left\{ - \frac{2t_{th\ell}}{\tau} [1 - \exp(-t/2t_{th\ell})] \right\} .$$

The time when $N = N_0/e$ is defined as t_e ; there,

$$1 = \frac{2t_{th\ell}}{\tau} [1 - \exp(-t_e/2t_{th\ell})]$$

which can be solved to give

$$t_e = 2t_{th\ell} \ln \left(\frac{1}{1 - \tau/2t_{th\ell}} \right) .$$

Clearly this analytical form is distinctly different from the standard

scaling relationship, $t_e = \tau$, however, that result can be recovered

by taking an expansion for the limiting condition where $\tau < t_{th\ell}$. First, writing

$$t_e = -2t_{th\ell} \ln(1 - \tau/2t_{th\ell})$$

and expanding $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3$ ($-1 < x < 1$), get

$$t_e = \tau(1 + \tau/4t_{th\ell})$$

or for $\tau \ll t_{th\ell}$

$$t_e \approx -2t_{th\ell} (-\tau/2t_{th\ell}) = \tau ,$$

in standard fashion. However, when magnitudes of τ and $t_{th\ell}$ are comparable, then the time for particle loss does not scale simply as $\ell/T_i^{1/2}$; loss time increases due to temperature drop with the functional form given above.

In order to explore the significance of this result to experiments that have been carried out, values of τ_T , $t_{th\ell}$, τ_{exp} , and t_e calculated with $\tau = \tau_T$ are tabulated below. Note that $t_{th\ell} \equiv t_{eq}$ except for Scylla I-C which must have $t_{th\ell} \equiv t_{con}$ because $T_e \approx T_i$:

	$\tau_T(\mu\text{sec})$	$t_{th\ell}$	t_e	τ_{exp}
Scylla IV-I	0.7	9.4	0.71	2.4
Scylla IV-3	3.16	11.98	3.38	10.1
5-m Linear Scyllac	3.79	25.5	3.94	11.5
Scylla I-C	5.60	305.	5.66	14.4

Several facts are evident on considering the above figures. First, $t_{th\ell} > \tau_T$ and so the assumption that $t_e \sim \tau_T$ is reasonably correct for these experiments. However, τ_T , t_e are substantially lower than τ_{exp} , indicating that while the functional form of the relationships is appropriate, the constant correction factor to loss rate because of differences between mid-coil and end coil, referred to above, should be evaluated. If one accepts the new formalism as correct and defines a new constant temperature loss time derived from experiment as τ_L , then it can be determined from

$$t_e \equiv \tau_{exp} \equiv \tau_L (1 + \tau_L/4t_{th\ell})$$

Choosing 5m Linear Scyllac as base with $\tau_{exp} = 11.5$, then $\tau_L = 10.3$

from $t_{th\lambda} = 25.5$, and the factor by which end loss time is altered due to plasma changes from mid-coil conditions can be written as

$$K \equiv \frac{\tau_T}{\tau_L} = \frac{3.79}{10.3} = 0.37$$

Then we can write a scaling relationship for particle loss time including energy loss effects as

$$t_e = 2t_{th\lambda} \ln\left(\frac{1}{1 - \tau_T/2Kt_{th\lambda}}\right)$$

or, for scaling loss time as a function of $t_{th\lambda}$, τ_T and an experimental factor K, we write:

$$t_e = \tau_{Ls} \left(1 + \frac{\tau_{Ls}}{4t_{th\lambda}}\right) \text{ for } \tau_{Ls} < t_{th\lambda} \text{ with } \tau_{Ls} \equiv \frac{\tau_T}{K}$$

With this scaling relationship, taking $K = 0.37$, and expressing time in μsec :

	τ_{Ls}	t_e	τ_{exp}	t_e/τ_{Ls}
Scylla IV-1	1.89	1.98	2.4	1.05
Scylla IV-3	8.54	10.56	10.1	1.24
5m Linear Scyllac	10.30	11.50	11.5	1.10
Scylla I-C	15.14	15.32	14.4	1.06

In the above table, again: τ_{exp} is the experimentally derived time for N to drop to N_0/e ; τ_{Ls} is the scaled loss time (based on 5m Scyllac, $K = 0.37$) equivalent for constant T plasma, and is followed by a T change factor in t_e ; the expression for t_e can be considered a more correctly scaled loss time because temperature change has been included. The ratio t_e/τ_{Ls} provides an indication of the percentage difference in scaled loss time between that predicted with and without temperature effects; the percentage difference is between 5 and 24 percent.

$$\tau_L = \frac{\ell}{2} \left(\frac{m}{2kT}\right)^{1/2} \frac{1}{\gamma^{1/2} K} \text{ compares with other loss expressions.}^8$$

On the basis of the above calculations, when $t_{thl} \leq 2\tau_L$ the simple $\ell/T_i^{1/2}$ scaling relationship should not be expected to hold. When the relationships for these variables are considered, some straightforward calculations produce the following results. With

$$t_{thl} < 2\tau_L$$

$$\frac{3.16A_i \times 10^8}{n\ell n\lambda} \{ .86n^2 \frac{(.2\ell)(.5\ell)T_i(\ell n\lambda)^2}{A_i} \times 10^{-29} \}^{3/10} < \frac{\ell}{K} \left(\frac{m}{2\gamma k T_i} \right)^{1/2}$$

and taking dueterium, $A_i = 2$,

$$\frac{KT_i^{0.8}}{\ell^{2/5} n^{2/5}} < 4.44 \times 10^{-6}$$

and with $n = 10^{16} \text{ cm}^{-3}$, $T_i = 1 \text{ keV}$ produces the result that

$$\ell > 1.99 \text{ m}$$

Clearly, for even reasonable length linear pinches ($\ell \sim 10 \text{ m}$) thermal effects on particle loss would be significant. Accordingly, by specifying an n, T state of the plasma of interest, it is possible to examine the difference between loss time scaled on $\ell/T_i^{1/2}$ and loss time scaled with thermal loss included, with both a function of length. Taking $T_i = 2.7 \text{ keV}$ and $n = 1.7 \times 10^{16} \text{ cm}^{-3}$, standard scaling based on 5-m Linear Scyllac would be

$$\tau_s(\ell) = 11.5 \frac{\ell}{5} = 2.3\ell(\text{m}), \tau(\mu\text{sec})$$

Scaling including thermal effects would be written

$$t_e = 2t_{thl} \ln \left(\frac{1}{1 - \tau_s/2t_{thl}} \right)$$

where t_{thl} is evaluated as t_{eq} , which is appropriate for short pinch behavior with length under about 500 m^5 . The results of these calculations give:

$\ell(m)$	$\tau_s(\mu\text{sec})$	$t_{thl}(\mu\text{sec})$	$t_e(\mu\text{sec})$	$T(t_e)$
5	11.5	24.15	13.52	1.54keV
10	23.	36.6	27.08	1.28keV
100	230.	145.7	454.6	119eV
150	345.	185.5	986.	13.3eV

In the above figures, it can be seen that loss time including thermal loss, t_e , can be substantially different from τ_s with $\ell/T_i^{1/2}$ scaling; the reason for this is obviously the cooling of the plasma affecting the (acoustic) velocity of loss ($t_{thl} \ll \tau_s$ implies $t_e \rightarrow \infty$, no loss). It is interesting to note however, that long (100m) experimental machines can be expected to obtain very long confinement times if the natural energy loss processes proceed as anticipated. The plasma becomes cooler, of course, but there would be the ability to study the stability of the confinement. It is also interesting to consider the possibility of experiments with preferential cooling of the plasma at the ends to see if the mass loss is reduced, as well as supplementary heating of the plasma at mid-coil to see how the energy is distributed. The successful combination of both, of course, would solve the loss and heating problems in the linear theta pinch.

3. Variable temperature plasma - linear T decay

In some cases such as Scylla I-C⁴, it has been observed that temperature decays linearly rather than exponentially. Also in a scheme with some supplementary heating, it might be better to categorize T variation as linear rather than exponential. As an addition to the above formulations for loss with exponential temperature decay, the time for mass loss with linear temperature decay will now be presented.

Again, with

$$\frac{dN}{N} = -\frac{2A_E}{A_p \cdot \ell} \left(\frac{2\gamma k}{m}\right)^{1/2} (T_i)^{1/2} dt$$

and taking $T_i = T_{io} - \left(\frac{dT_i}{dt}\right)t \equiv T_{io} - K_t t$ where K_t is taken constant, get

$$\frac{dN}{N} = -\frac{2A_E}{A_p \cdot \ell} \left(\frac{2\gamma k}{m}\right)^{1/2} (T_{io} - K_t t)^{1/2} dt$$

which can be integrated to give, with $N = N_0$ at $t = 0$,

$$N = N_0 \exp\left[-\frac{2A_E}{A_p \cdot \ell} \left(\frac{2\gamma k}{m}\right)^{1/2} \left(\frac{2}{3K_t}\right) \{T_{io}^{3/2} - (T_{io} - K_t t)^{3/2}\}\right]$$

To get the time, $t \equiv t_e$, when $N = N_0/e$, set

$$1 = \frac{2}{\ell} \left(\frac{2\gamma k T_{io}}{m}\right)^{1/2} \left(\frac{2}{3K_t}\right) \left\{T_{io} - \frac{(T_{io} - K_t t_e)^{3/2}}{T_{io}^{1/2}}\right\}$$

and using $\tau_T = \frac{\ell}{2} \left(\frac{m}{2\gamma k T_{io}}\right)^{1/2}$ get

$$t_e = \frac{T_{io}}{K_t} \left\{1 - \left[1 - \frac{3}{2} \frac{K_t \tau_T}{T_{io}}\right]^{2/3}\right\}$$

Again, this would provide a distinctly different scaling than $\tau \sim \ell/T_i^{1/2}$

As before, this expression can be expanded and simplified for limiting conditions. Specifically considering

$$\left[1 - \frac{3}{2} \frac{K_t \tau_T}{T_{io}}\right]^{2/3} \text{ with } \tau_T/T_{io}/K_t \ll 1$$

and using

$$(1-x)^n = 1 - nx + \frac{n(n-1)x^2}{2!} + \dots \quad x^2 < 1$$

$$\approx 1 - nx$$

$$\left[1 - \frac{3}{2} \frac{K_t \tau_T}{T_{io}}\right]^{2/3} \approx 1 - \frac{K_t \tau_T}{T_{io}}$$

then

$$t_e \equiv \frac{T_{io}}{K_t} \{1 - [1 - \frac{K_t \tau_T}{T_{io}}]\} \approx \tau_T$$

Since $\tau_T/T_{io}/K_t \ll 1$ implies that $K_t \ll 1$ or $T \approx$ constant, this would be consistent with standard $\ell/T_i^{1/2}$ scaling.

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