OPTIMIZATION OF PLASMA PROFILES FOR IGNITED LOW-BETA TOROIDAL PLASMAS UTILIZING "ADVANCED FUELS"

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PLASMA PHYSICS LABORATORY

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ABSTRACT

The radial density and temperature profiles of ignited plasmas utilizing non-DT fuels can be optimized to maximize fusion power density and minimize required $\bar{n}_E$ under the constraint of a maximum average plasma pressure. Strong axial temperature peaking or strong density peaking is advantageous, according to whether the fusion reactivity increases faster or more slowly than quadratically with temperature, respectively. For tokamak plasmas with maximum beta limited to 10% or less by MHD instability, optimal profile tailoring allows ignition of catalyzed D-D or D-He fuels in devices of reasonable size ($R_o < 8$ m), and with first-wall power loadings exceeding 1 MW/m$^2$. 
I. INTRODUCTION

The plasma conditions for equilibrium ignition are markedly more severe for "advanced fusion fuels" than for deuterium-tritium.\textsuperscript{1,2} Recent studies using a global (i.e., zero-dimensional) analysis\textsuperscript{3} indicate that utilization of advanced fuels in tokamak plasmas results in economically interesting reactors only when the plasma beta ($\beta = \text{plasma pressure/magnetic field pressure}$) is of the order of several tens of per cent, a value which is forbidden by MHD stability considerations.\textsuperscript{4}

It is known that the \(nT_E\) requirement for ignition of a D-T plasma can be reduced markedly from that of a uniform plasma, if the density \((n)\) and temperature \((T)\) profiles have strong axial peaking.\textsuperscript{5} This result follows from the fact that the fusion reaction rate is proportional to \(n^2\langle v\rangle\), with \(\langle v\rangle\) increasing faster than \(T\), so that \(\int n^2\langle v\rangle dV\) is increased substantially even for the same average value of \(nT\). A similar result can be shown for $^1$CT-type plasmas with axially peaked temperature and beam-deposition profiles.\textsuperscript{6} Now the beta limitation in tokamak plasmas, which is determined by MHD instability, refers only to the average plasma pressure, whereas the beta in a local region such as the plasma center can be many times larger.\textsuperscript{4} Thus by profile tailoring, one can set up a small high-beta plasma region which produces most of the fusion power; this productive region is surrounded by a plasma of much lower beta, so that the volume-averaged plasma pressure remains within the MHD limit.

In this study, we examine the effects of profile shaping on the requirements for obtaining ignited plasmas with non-DT fusion fuels. We find that in general it is important to decouple the radial dependences of \(n\) and \(T\). By strong temperature peaking and relatively weak density peaking — as in experimental tokamak operation — it is possible to achieve ignition in D-D and D-$^3$He plasmas of reasonable size, with realistic beta-values.

II. PROFILE OPTIMIZATION

1. Fusion Power Density. Consider the two simple plasma pressure profiles shown in Fig. 1. Each plasma has the same average pressure \(p_0\), and the same average beta, \(\bar{\beta} = \bar{p}/\bar{B}^2\), provided...
that
\[ n^2 p_1 + (1-n^2)p_2 = p_0 \] (1)

where \( n < 1 \). As indicated in Fig. 2, the fusion reactivities in the temperature range of interest can be fitted reasonably well by simple formulas of the type \(<ov> = T^5 \). (See also Table 1.) Using this relation and \( p = 2nT \), the ratio of the spatially-averaged fusion power densities for the two pressure profiles of Fig. 1 is

\[ R_f = n^2 \left( \frac{p_1}{p_0} \right)^2 \left( \frac{T_1}{T_0} \right)^{s-2} + (1-n^2) \left( \frac{p_2}{p_0} \right)^2 \left( \frac{T_2}{T_0} \right)^{s-2}. \] (2)

The maximum value of \( R_f \) occurs when \( p_1/p_0 = n^{-2} \) and \( p_2 = 0 \); the outer region is now a cold dense plasma which still must carry a large fraction of the plasma current. Then \( R_{f\text{max}} = n^{-2}(T_1/T_0)^{s-2} \). For example, if \( n = 1/2 \) and \( s = 2 \), then the average fusion power density is increased by a factor of 4, when the plasma pressure is quadrupled at \( r/a < 1/2 \), and made nearly zero at \( r/a > 1/2 \) — for the same average pressure as in the uniform case.

Now if \( s > 2 \), it is clear from Eq. (2) that the increase in pressure \([\Delta p = \Delta(nT)]\) is obtained most favorably by increasing \( T_1 \) rather than \( n_1 \). Thus if \( \Delta T_1 = \Delta p \), with \( n_1 = n_2 \), we have \( R_{f\text{max}} = n^{2-2s} \). Table 1 shows the maximum possible gains in average fusion power density, for several important reactions.

On the other hand, if \( s < 2 \), it is evidently more advantageous to increase \( n_1 \) rather than \( T_1 \), in order to produce the desired pressure increase. If \( T_1 = T_0 \), then \( R_{f\text{max}} \) has the same value as for the \( s = 2 \) case. Of the more practical fusion reactions, only D-D has \( s < 2 \), and here \( s \) is sufficiently close to 2 so that temperature peaking does not lead to a marked reduction from the maximum possible \( R_f \).

The advantage of temperature peaking with a uniform density profile is especially marked for reactions with \( s >> 1 \), even when

Figure 2. Solid lines are fusion reactivities for Maxwellian ion velocity distributions. Dashed lines are linear fits (on the log-log scale). (773473)
Table 1. Effect of Spatial Peaking on Power Densities

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Temperature Range (keV)</th>
<th>Temperature Dependence of $&lt;v&gt;$</th>
<th>Fusion Power Density</th>
<th>Maximum Increase In Average Power Density*</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-D</td>
<td>15-70</td>
<td>$T^{3/2}$</td>
<td>$\beta T^{-1/2}$</td>
<td>4</td>
</tr>
<tr>
<td>D-T</td>
<td>8-25</td>
<td>$T^2$</td>
<td>$\beta T^0$</td>
<td>4</td>
</tr>
<tr>
<td>D-3He</td>
<td>15-60</td>
<td>$T^3$</td>
<td>$\beta T^2$</td>
<td>16</td>
</tr>
<tr>
<td>p-11B</td>
<td>55-95</td>
<td>$T^5$</td>
<td>$\beta T^3$</td>
<td>256</td>
</tr>
</tbody>
</table>

*Two-level pressure model. See Fig. 1.

$p_1/p_0 < n^{-2}$. For example, consider a p-11B plasma ($s = 5$) with $n = 1/2$ and $p_1/p_0 = 2$. The increase in pressure at $r/a < 1/2$ is best obtained by doubling $T$; to keep $\bar{p}$ the same, $T$ is reduced by $1/3$ at $r/a > 1/2$, with $n(r) = \text{constant}$. Then $R_f = 8.0$, even though the plasma at $r/a > 1/2$ contributes essentially no fusion power. However, the outer region does serve to enhance energy confinement, as well as to reduce the spatially-averaged pressure.

2. Confinement Parameter. Defining the energy confinement time $\tau_E$ to include all plasma transport and radiative losses, the overall plasma power balance at equilibrium ignition is

$$\frac{\int 3nTdv}{\tau_E} = \bar{p}_{f+} \times (\text{plasma volume})$$

where $\bar{p}_{f+}$ is the average fusion power density including only charge fusion-reaction products, and we have assumed $T_e = T_i$. Thus

$$\bar{n}\tau_E = \frac{3}{2} \frac{\bar{n}}{\bar{p}_{f+}} R_f$$

where $R_f$ is given by Eq. (2). (The last operation assumes that one operates a given plasma at the maximum possible $\bar{p}$.) If $\bar{n}$ does not change markedly when going to peak temperature profiles, as is the usual case, then the required $\bar{n}\tau_E$ is inversely proportional to $R_f$, whose maximum value is shown in Table 1.

III. NUMERICAL RESULTS

1. Profiles. The conclusions of the previous section are qualitatively valid for the smoothly varying pressure profiles that are encountered in practice. This section presents the results of numerical evaluations of fusion
power densities for D-D and D-^3\text{He} plasmas, as a function of realistic spatial profiles. Our assumptions are as follows:

(a) \(T_i(r) = T_e(r) = T(r) = T_c(1-r^2/a^2)^x\).

(b) All ion species in a given plasma have the same density profile as the electrons: \(n(r) = n_c(1-r^2/a^2)^y\).

(c) All tritium and helium-3 reaction products are burned up at the same rate as produced (catalyzed D-D operation\(^2\)). The tritium concentration is neglected. (It's actually \(< 0.5\%\).) The equilibrium \(^3\text{He} \) concentration is calculated for each temperature profile.

(d) The concentrations of \(^4\text{He} \) and H reaction products are neglected.

(e) Synchrotron radiation loss is neglected.

Figure 3 shows radial profiles of \(n, T\) and \(P_f\) for \(x = 3\) and \(y = 1\). Evidently, only a small fraction of the plasma volume contributes to the fusion power production — although the entire plasma contributes to energy confinement, if \(\bar{n}e = \bar{n}a^2\).

2. Ignition Criteria. The equilibrium ignition condition is

\[
\bar{n}e^{3/2}e = \frac{\bar{n}e^{3/2}eT}{P_f - bZ_{eff}n_e^{2/1/2}}
\]

Here the "bar" over a symbol represents the spatially averaged value; \(n = n_e + n_i\), where \(n_i\) is the total ion density; and the bremsstrahlung constant \(b = 3.65\times10^{-15}\), with \(T_e\) in keV and \(n_e\) in cm\(^{-3}\). It is most useful to plot \(\bar{n}e^{3/2}e\) versus the density-averaged temperature, \(\langle T\rangle\), where

\[
\langle T\rangle = \frac{\int_0^a n(r)T(r)2\pi r dr}{\int_0^a n(r)2\pi r dr} = \frac{\bar{P}}{\bar{n}}
\]

Thus plasmas of the same \(\bar{n}\) and \(\langle T\rangle\) have the same beta.

Figures 4 and 5 show the ignition criteria calculated
numerically from Eq. (5) with the fusion reactivities taken from Ref. 7. The required \( \bar{n}_E \) for D-D when \( x = 3 \) is reduced by a factor of 2.5 to 10 from the value for a uniform plasma. This reduction corresponds to a rapid increase in fusion power density, and is in the range predicted in Table 1. Furthermore, ignition can now be obtained with \( \langle T \rangle \) as low as 16 keV. (Analogous results were obtained previously\(^5\) for D-T).

Figure 5 shows that factor of 10 reductions in \( \bar{n}_E \) can be obtained for \( x = 3 \) in a 1:1 mixture of D-\(^3\)He (\( Z_{\text{eff}} = 1.67 \)). The gain in power density is not as dramatic as indicated in Table 1, because D-D reactions are still important, especially at smaller \( \langle T \rangle \). Whereas the catalyzed-D plasma contains an equilibrium concentration of 10 to 20% \(^3\)He, and 37% of the fusion power is produced in fast neutrons, for the 1:1 D-\(^3\)He mixture only 6 to 10% of the fusion power is produced in fast neutrons, depending on \( \langle T \rangle \).

3. Power Density. Figure 6 shows the spatially-averaged fusion power density (including neutron production) in catalyzed D-D plasmas with a fixed

\[ T = T_e (1-r^2/a^2)^x \]
\[ n = n_0 (1-r^2/a^2)^y \]

Figure 4. Ignition criteria for catalyzed D-D fuel, for various temperature and density profiles, with \( T_e(r) = T_i(r) \). The \(^3\)H and \(^3\)He are burned up at the same rate as produced.

Figure 5. Ignition criteria for D-\(^3\)He fuel, for various temperature and density profiles, with \( T_e(r) = T_i(r) \). Equal concentrations of D and \(^3\)He. (773473)
total pressure of 2.1 J/cm$^3$, corresponding to $\beta = 8\%$ at $B_T = 8.0$ T. (The pressure of decelerating charged fusion-reaction products increases the total pressure and $\beta$ by 20 to 25%.). Whilst $\bar{P}_f$ increases as the temperature profile narrows, the gain in $\bar{P}_f$ at a given $<T>$ is not as strong as the reduction in $\bar{n}_E$; as can be seen from Eq. (5), the different dependence is due to the bremsstrahlung radiation. Evidently, power densities exceeding 1 W/cm$^3$ are achievable when $x > 2$. The corresponding first-wall power loading would exceed 1 MW/m$^2$ in tokamaks with plasma half-widths of 2 m or greater.

4. Proton-Boron Reaction. We have found it impossible to ignite a $p-^{11}$B plasma with uniform density, but using temperature peaking as large as $x = 7$. (Synchrotron radiation loss was arbitrarily set equal to the bremsstrahlung loss.) For $T_i > 100$ keV, however, the temperature dependence of $<\alpha >$ is much less steep than $T^5$ (see Fig. 2), so that density peaking becomes increasingly preferred. Although ignition of $p-^{11}$B may always be unattainable, the advantages of spatial peaking would be retained in a beam-driven mode of operation.

IV. MINIMUM SIZE ADVANCED-FUEL TOKAMAK REACTORS

1. Restrictions. The results of Section III have been used to determine the minimum size of D-D or D-He reactors that produce a first-wall power loading $\phi_w > 1$ MW/m$^2$. The following restrictions were applied:

(a) The maximum $\beta$ determined by stability against MHD "ballooning" modes varies nearly inversely with plasma aspect ratio. At $R/a = 3$, $\beta \leq 10\%$.

(b) The degree of temperature peaking is limited to $x \leq 3$ by the maximum current-density peaking allowed for kink-mode stability, as well as for attaining the

![Figure 6. Spatially-averaged fusion power density for various temperature and density profiles, with a constant spatially-averaged plasma pressure: 2.1 J/cm$^3$, or $\beta = 8\%$ at $B_T = 8.0$ T. (Energetic ions not included in $\beta$.) The minimum ignition temperature is at the left end of each curve. (773490)]
highest possible beta.\(^4\)

(c) Synchrotron radiation loss\(^1\) may be intolerable at temperatures much exceeding 25 keV.

(d) High-\(N\) tokamak operation is achieved with relatively flat density profiles \((y \lesssim 1)\), as in recent PLT experiments with \(^4\)He-D mixtures.\(^11\)

(e) When impurity radiation loss is relatively unimportant, the empirical energy confinement scaling\(^12\) is \(\tau_E \approx 3 \times 10^{-19} q^{1/2} n_e^{2} a^2\). In the following examples, this scaling of \(\tau_E\) has been divided by 2 to allow for enhanced radiation loss. Then the required plasma size is \(\hat{n}_e a^2 = 6.7 \times 10^{18} q^{-1/2} n_e^{2} \tau_E\).

(f) The maximum practical magnetic field at the coil windings is 16 T, with Nb\(_3\)Sn conductor. Considering the relatively small neutron wall loading with advanced-fuel plasmas, a total thickness of 1.0 m between the first wall and the coil windings is adequate.

[Effects (b) and (c) can be ameliorated if it is possible to operate with \(T_i > T_e\) in the hot central region. But decoupling of \(T_i\) and \(T_e\) is not easily achieved in an ignited tokamak plasma, where one needs large \(\hat{n}\) to achieve large \(\hat{n} \tau_E\).]

2. **Examples.** Table 2 gives plasma parameters for ignited D-D and D-\(^3\)He reactors with fusion power productions 77% and 62% respectively of that of a reference high-field, high-density D-T reactor.\(^13\) The physical size of these reactors is quite practical, although the plasma current must be approximately four times that of the D-T plasma. The advanced-fuel plasmas have size and power outputs similar to those determined previously for plasmas of uniform profile\(^3\); but the present examples require a spatially-averaged beta only one-third as large, and \(\langle T \rangle\) only 60% as large, as the uniform-profile cases.

The first-wall power loadings are only about one-quarter that of the D-T reactor. This economic disadvantage must be weighed against the well-known advantages of the advanced fuels that result from the elimination of tritium breeding: a huge reduction in tritium inventory, the absence of lithium-fire hazard, and especially the flexibility in choosing blanket composition for minimization of long-term activation. The latter problem is relieved in any event because of the reduction in neutron fluence for a given production of fusion energy. Further reduction in reactor size — or an increase in fusion power output — can be anticipated if even larger magnetic fields eventually become available.
Table 2. Low-Beta Ignited Tokamak Reactors

<table>
<thead>
<tr>
<th></th>
<th>D-T</th>
<th>D-D</th>
<th>D-(^3)He</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major radius (m)</td>
<td>6.0</td>
<td>7.7</td>
<td>7.7</td>
</tr>
<tr>
<td>Plasma radius (m)</td>
<td>1.2</td>
<td>2.6</td>
<td>2.6</td>
</tr>
<tr>
<td>Vertical elongation of plasma</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>(B_t) at plasma (T)</td>
<td>7.1</td>
<td>7.8</td>
<td>8.0</td>
</tr>
<tr>
<td>(B_t) at coils (T)</td>
<td>13.1</td>
<td>15.4</td>
<td>15.8</td>
</tr>
<tr>
<td>Plasma current (MA)</td>
<td>6.8</td>
<td>26.0</td>
<td>27.0</td>
</tr>
<tr>
<td>(&lt;T&gt;) (keV)</td>
<td>8.0</td>
<td>25.0</td>
<td>25.0</td>
</tr>
<tr>
<td>Peak temperature (keV)</td>
<td>12.4</td>
<td>62</td>
<td>62</td>
</tr>
<tr>
<td>(\bar{n}_e) (cm(^{-3}))</td>
<td>3.4\times10^{14}</td>
<td>2.6\times10^{14}</td>
<td>2.7\times10^{14}</td>
</tr>
<tr>
<td>Peak density (cm(^{-3}))</td>
<td>5.7\times10^{14}</td>
<td>5.2\times10^{14}</td>
<td>5.4\times10^{14}</td>
</tr>
<tr>
<td>(\bar{n}_e)(\tau_E) (cm(^{-3})s)</td>
<td>4.0\times10^{14}</td>
<td>2.0\times10^{15}</td>
<td>2.2\times10^{15}</td>
</tr>
<tr>
<td>(\bar{\beta}) including energetic charged reaction products</td>
<td>0.046</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>(\bar{P}_f) (W/cm(^3))</td>
<td>9.2</td>
<td>1.18</td>
<td>0.96</td>
</tr>
<tr>
<td>Total fusion power (MW)</td>
<td>1995</td>
<td>1540</td>
<td>1230</td>
</tr>
<tr>
<td>First-wall power loading (MW/m(^2))</td>
<td>4.7</td>
<td>1.4</td>
<td>1.13</td>
</tr>
<tr>
<td>Neutron wall loading (MW/m(^2))</td>
<td>3.75</td>
<td>0.53</td>
<td>0.09</td>
</tr>
</tbody>
</table>

\(^{a}\) \(x = 3\)
\(^{b}\) \(y = 1\)
\(^{c}\) Energetic ions account for 20% of the total plasma pressure.
V. REFERENCES

   (1974).
8. Jassby, D. L., Nucl. Fusion 17 (1977) 328, Fig. 13.

ACKNOWLEDGMENT

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