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AN APPROACH TO THE ANALYSIS OF INTEGRATED PRODUCTION-
DISTRIBUTION SYSTEMS

by

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An Approach to the Analysis of Integrated Production-Distribution Systems

Many firms, in the general business setting, produce goods at spatially separated plants and meet spatially separated demands. These firms frequently have a volume such that intermediate distribution points are necessary. The location of these distribution points is a nontrivial problem. The firm needs to locate these points consistent with its production, demand, and transportation facilities. The fundamental problem is to establish a tradeoff between facility, transportation, and production costs.

Geoffrion and Graves have recently presented a model which handles a class of distribution problems. However, their formulation assumes that production of each commodity at each plant is fixed and invariant. This paper argues that production is a controllable variable and that the location of production should be considered simultaneously with the location of distribution points. The paper proceeds to first present and discuss the Geoffrion and Graves formulation. Specific algorithmic advantages of their formulation will be discussed along with a brief discussion of the short-comings of their model. From here the paper moves on to present a revised formulation which maintains many of the computational features of the Geoffrion and Graves approach. A computational, decomposition approach to this problem is presented followed by a formal algorithm statement. The final part of the paper presents an example problem.

Geoffrion and Graves

Geoffrion and Graves present a formulation of the warehouse location problem which is:

- $$\begin{aligned}
 (1) \quad & \text{Min.} \quad \sum_{ijKL} C_{ijKL} X_{ijKL} + \sum_K \left[F_K Z_K + V_K \sum_{iL} D_{iL} Y_{KL} \right] \\
 & X_{ijKL} \geq 0 \\
 & Y_L, Z_K \in (0,1) \\
 (2) \quad & \text{s.t.} \quad \sum_{KL} X_{ijKL} \leq S_{ij} \quad \text{for all } ij \\
 (3) \quad & \sum_j X_{ijKL} = D_{iL} Y_{KL} \quad \text{for all } iKL \\
 (4) \quad & \sum_K Y_{KL} = 1 \quad \text{for all } L \\
 (5) \quad & \underline{V}_K Z_K \leq \sum_{iL} D_{iL} Y_{KL} \leq \bar{V}_K Z_K \quad \text{for all } K
 \end{aligned}$$

where

- i denotes commodity
- j denotes production plant
- K denotes possible distribution points
- L denotes demand zones
- C_{ijKL} is the cost of shipping one unit of commodity i from plant j through distribution point K to demand zone L
- X_{ijKL} is the quantity of commodity i shipped from plant j via distribution point K to demand zone L
- f_K is the fixed cost of establishing (or closing) distribution point K
- Z_K is a zero one variable indicating whether distribution point K is opened or closed
- V_K is the variable cost of shipping one unit through distribution point K

- D_{iL} is the demand for commodity i at demand zone L
- Y_{KL} tells whether distribution point K serves demand zone L
- S_{ij} is the supply of commodity i available at plant j
- $\underline{V}_K$ is the lower bound on shipments through distribution point K
- $\bar{V}_K$ is the upper bound on shipments through distribution point K

In addition to the formal mathematical formulation (1) - (5) configuration constraints may be entered on the distribution points.

Geoffrion and Graves point out several advantages to their formulation which are:

1. Transportation variables are quadruply subscripted retaining the complete identity of the shipping pattern throughout the network. Thus certain transportation features (i.e., storage in transit) may be portrayed.
2. Constraint (4) requires that each demand zone be served by one and only one distribution point. If the restriction is unrealistic Geoffrion has developed a commodity bounding concept.^{1/}
3. Bounds are placed on distribution point thruput. Thereby economies of scale in distribution may be introduced through the creation of alternative distribution facilities at a site with different thruput restrictions.

The Geoffrion and Graves formulation is structured in such a way that the application of Benders Decomposition yields a series of problems which are of reduced computational complexity. Specifically the problem

breaks into an integer problem in Y and Z and a continuous problem in X. Further the continuous problem (due to constraint (4)) is in the form of independent transportation problems for each commodity. Finally these transportation problems contain several rows with zero demands ($D_{iL} Y_{KL}$ where $Y_{KL}=0$) which can be dropped and the information implicit in these rows can be recovered through an application of duality theory (Geoffrion and Graves pp. 820-830).

Firms Control of Production

The Geoffrion and Graves model assumes production at a plant (S_{ij}) is fixed and invariant. This restricts the firm to distribution planning in a distribution context only. However in a long run planning exercise spatial adjustments frequently occur in production consistent with demand and distribution. Thus when considering distribution, the firm may wish to consider changes in the location of production.

The underlying factors for spatial production adjustments lies in spatial resource availability, product resource requirements, production costs, demands and transportation costs. The firm does have direct input into all of these factors and thus distribution planning can be done with simultaneous consideration of production. The following model is an integrated production-distribution model developed based upon the Geoffrion and Graves formulation.

$$(6) \quad \text{Min} \quad \sum_{jm} e_{jm} W_{jm} + \sum_{ijKL} C_{ijKL} X_{ijKL} + \sum_K \left[F_K Z_K + V_K \sum_{iL} D_{iL} Y_{KL} \right]$$

$$(7) \quad \text{s.t.} \quad -\sum_m f_{jm} W_{jm} + S_{ij} \leq 0 \quad \text{for all } im$$

$$(8) \quad \sum_m a_{jmn} W_{jm} \leq b_{jn} \quad \text{for all } jn$$

$$(9) \quad \sum_{KL} x_{ijKL} s_{ij} \leq 0 \quad \text{for all } ij$$

$$(10) \quad \sum_j x_{ijKL} = D_{iL} Y_{KL} \quad \text{for all } iKL$$

$$(11) \quad \sum_K Y_{KL} = 1 \quad \text{for all } L$$

$$(12) \quad \frac{V_K}{K} Z_K \leq \sum_{iL} D_{iL} Y_{KL} \leq \bar{V}_K Z_K \quad \text{for all } K$$

$$S, W, X \geq 0 \quad Y, Z \in \{0, 1\}$$

where in addition to earlier definitions

m stands for production processes available

n stands for limits on production

e_{jm} is the cost of operating one unit of production process m at production plant j

W_{jm} is the number of units of production process m operated at plant j

f_{ijm} is the yield of commodity i from one unit of production process m operated at plant j (Note this allows joint products.)

a_{ijmn} is the use of production limit n by one unit of production process m at plant j

b_{jn} is the total available of production limit n at plant j

S_{ij} is an endogenous variable telling how much of commodity i is produced at plant j

This model minimizes production plus distribution cost (6)

subject to constraints which related commodity output to supply (7),

constraints which impose limits on production's use of resources (8), and the constraints from the Geoffrion and Graves model (9) - (12).

Decomposition

The above model, like the Geoffrion and Graves model, is a candidate for Benders Decomposition. Further, under decomposition the problem does have desirable computational features. The following discussion does assume knowledge of Benders Decomposition. Background references are Benders, Hu, McCarl and Harling, Geoffrion and Zionts.

The problem may be decomposed into two problems as follows:

Master Problem I

$$(13) \quad \text{Min} \quad \sum_{jm} C_{jm} W_{jm} + \sum_K \left[F_K Z_K + V_K \sum_{iL} D_{iL} Y_{KL} \right] + Z_0$$

$$(14) \quad -\sum_m f_{ijm} W_{jm} + S_{ij} \leq 0 \quad \text{for all } ij$$

$$(15) \quad \sum_m a_{ijmn} W_{jm} \leq b_{jn} \quad \text{for all } jn$$

$$(16) \quad \sum_K Y_{KL} = 1 \quad \text{for all } L$$

$$(17) \quad \underline{V}_K Z_K \leq \sum_{iL} D_{iL} Y_{KL} \leq \bar{V}_K Z_K \quad \text{for all } K$$

$$(18) \quad Z_0 \geq \sum_{ij} U_{ij}^h S_{ij} + \sum_{iKL} H_{iKL}^h D_{iL} Y_{KL} \quad \text{for all } h$$

$$Y, Z \in (0, 1)$$

$$W \geq 0$$

$$Z_0 \text{ arb}$$

Sub Problem I

$$(19) \quad \text{Min} \quad \sum_{ijKL} C_{ijKL} X_{ijKL}$$

$$(20) \quad \sum_{KL} X_{ijKL} \leq S_{ij} \quad \text{for all } ij$$

$$(21) \quad \sum_j X_{ijKL} = D_{iL} Y_{KL} \quad \text{for all } iKL$$

$$X_{ijKL} \geq 0$$

where the variables are defined as before and

Z_0 is the predicted value of (19) derived through linear programming theory

π_{iKL}^h is the dual variable associated with 20 in solution h

u_{ij}^h is the dual variable associated with 19 in solution h

The master problem minimizes (1) production plus distribution point plus predicted transportation costs (Z_0); subject to constraints on output (14), resource use (15), distribution centers (16) (17); and a Benders type constraint which controls the predicted value of the transportation cost (18). The subproblem is of the transportation type and decomposes into classical single commodity problems when the fact is considered that for each L there is a unique K . For discussion of this problem see Geoffrion and Graves.

Further Decomposition

Master problem I is a mixed integer programming problem which contains distribution and production characteristics. These problems

are largely disjoint with the exception of the Benders constraint (18). This problem is a candidate for further decomposition. Specifically we get two new problems.

Master Problem II

$$(22) \quad \text{Min} \quad \sum_K \left[F_K Z_K + V_K \sum_{iL} D_{iL} Y_{KL} \right] + Z_{oo}$$

$$(23) \quad \text{s.t.} \quad \sum_K Y_{KL} = 1 \quad \text{for all } L$$

$$(24) \quad \underline{V}_K Z_K \leq \sum_{iL} D_{iL} Y_{KL} \leq \bar{V}_K Z_K \quad \text{for all } K$$

$$(25) \quad Z_{oo} \geq \sum_h \psi_h^p \left(- \sum_{iKL} \Pi_{iKL}^h D_{iL} Y_{KL} \right) + \sum_{jn} s_{jn}^p \quad \text{for all } p$$

$$Y_{KL}, Z \in \text{integer}$$

$$Z_{oo} \text{ arb}$$

Subproblem II

$$(26) \quad \text{Min} \quad \sum_{jm} C_{jm} W_{jm} + Z_o$$

$$(27) \quad - \sum_m f_{ijm} W_{jm} + s_{ij} \leq 0 \quad \text{for all } i, j$$

$$(28) \quad - \sum_m a_{ijm} W_{jm} \leq b_{jn} \quad \text{for all } jn$$

$$(29) \quad \sum_{ij}^h U_{ij} s_{ij} - Z_o \leq \sum_{iKL} \Pi_{iKL}^h D_{iL} Y_{KL} \quad \text{for all } h$$

where the new variables are

Z_{oo} is the predicted rate of (26) obtained through duality theory

ψ_h^p is the dual variable associated with constraint (29) in master problem I decomposition iteration p

s_{ij} is the dual variable associated with constraint (28) in iteration p.

The master problem II minimizes distribution facility costs plus the predicted objective function for the rest of the problem subject to the distribution facility constraints (23), (24); and a constraint which reflects the prediction of subproblem II objective function. Subproblem II minimizes the cost of production plus the predicted cost of distribution, subject to constraints on output (27), resource usage (28) and a constraint which reflects the prediction of the distribution cost objective (29) from Subproblem I.

Formal Algorithm

The above decomposition leads to the following algorithm which we have implemented in a preliminary version.

- Step 0 Establish 0 feasible distribution point $(\bar{Z}, \bar{Y})$ and supply $(\bar{W}, \bar{S})$ --solutions choose a convergence parameter ϵ , set the solution upper bound (UB) to ∞ , lower bound (LB) to $-\infty$.
- Step 1 Solve Subproblem I as a set of independent transportation problems. Obtain $-\Pi$ and U if the original objective function value (6) evaluated at the optimal point to this problem $\bar{X}$ and $\bar{Z}, \bar{Y}, \bar{W}, \bar{S}$ is less than or equal to the current lower bound plus ϵ , stop. Otherwise compare this objective value with incumbent upper bound and reset if it is lower.

- Step 2 Set master problem bounds $SUB = \infty$, $SLB = -\infty$ and set $h = h+1$, $p = 0$.
- Step 3 Set up subproblem II given Π , U , $\bar{Y}$ and $\bar{Z}$. Solve. If the objective function value from (26) plus $\sum_K \left[f_K Z_K + v_K \sum_{iL} D_{iL} Y_{KL} \right]$ is greater than current global upper bound, stop. Otherwise $p = p+1$. If (26) plus integer part of the objective function is less than SUB reset SUB .
- Step 4 Set up master problem II. Given Π^L , U^L , ψ^P , s^P . Solve. If the objective function value is greater than $UB - \epsilon$, stop. If the objective function value (22) is less than $SLB + \epsilon$ then return to (1) with $\bar{Y}$, $\bar{Z}$, $\bar{W}$, $\bar{B}$. Otherwise if (22) is greater than SLB reset SLB . Go to (3).

The algorithm has worked well in trials. Convergence is guaranteed through the Benders convergence proofs and convergence does appear to be rapid.

Example

Conclusions

This paper has demonstrated a modeling and solution strategy to remove the stringent assumption that supplies are fixed at a plant. The formulation is currently undergoing large scale implementation.

With respect to the solution methodology several questions yet remain to be studied. First, is the two level Benders Decomposition worth the trouble? Second, as Geoffrion and Graves infer would suboptimization of the Master problem I (and thus MP_{II}, SP_{II}) be desirable? These questions are currently under study and we hope to know more about their answers in the future.

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Footnotes

- 1/ Geoffrion's commodity bundling concept involves the separation of demand zones into demand zones for each commodity bundle. Thus when a group of commodities requires special distribution facilities the group can be formed into a bundle and the bundles can go through special distribution facilities.